Title: Gravitational waves from compact binaries with comparable masses using black hole perturbation theory

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Abstract: The direct detection of gravitational waves promises to open up a new spectrum that is otherwise mostly closed to electromagnetically based astronomical observations. Detecting gravitational waves from binary black holes and neutron stars, as well as estimating their parameters, requires a sufficiently accurate prediction for the expected waveform signal. Unfortunately, the state of the art for the pillars of gravitational wave theory -- numerical relativity, the post-Newtonian approximation, and linear black hole perturbation theory -- have yet to cover accurately the entire parameter space even when taken together. In this talk, I present a new direction for systematically describing compact binaries that is valid over a potentially very large portion of the parameter space.

The approach uses high-order black hole perturbation theory in the mass ratio together with ideas and techniques borrowed from effective field theory for incorporating the physics of extended masses like spin and tidal effects. I discuss recent advances, future prospects, and potential impacts in this direction.

Gravitational waves from compact binaries with comparable masses using black hole perturbation theory

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Strong Gravity seminar
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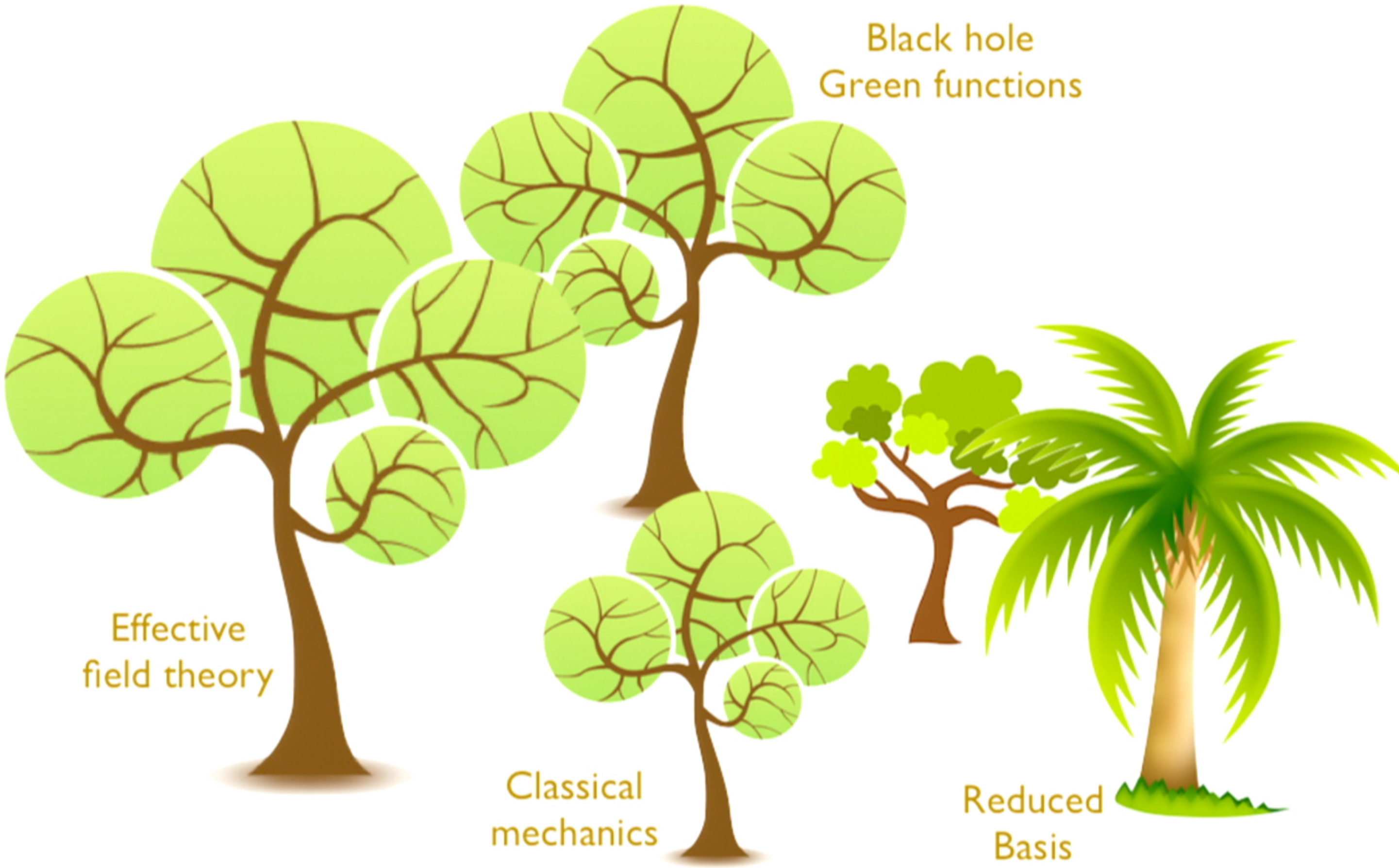
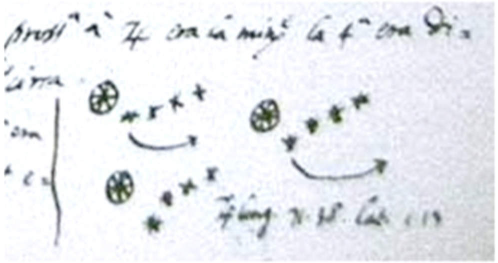
Research forest



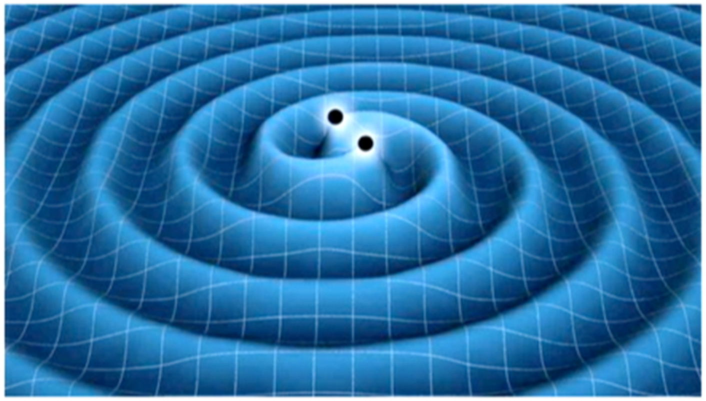
Astronomy highlights



from Galileo's notebook

Gravitational waves

- Gravitational waves appear in Einstein's general relativity as ripples of space-time that propagate at the speed of light

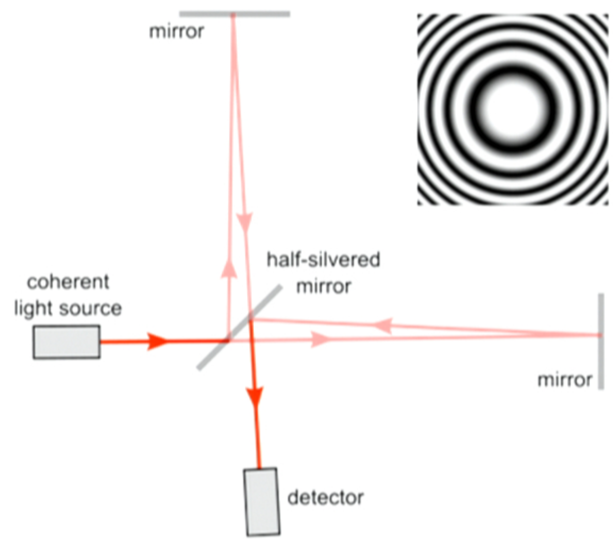


- Are generated by dynamical processes in strong gravitational fields
- Propagate through the universe essentially undistorted
- We could "see" very deep into processes near black holes and neutron stars

How can we detect gravitational waves?

- By measuring slight displacements of separated masses as a GW stretches and contracts the intervening space-time

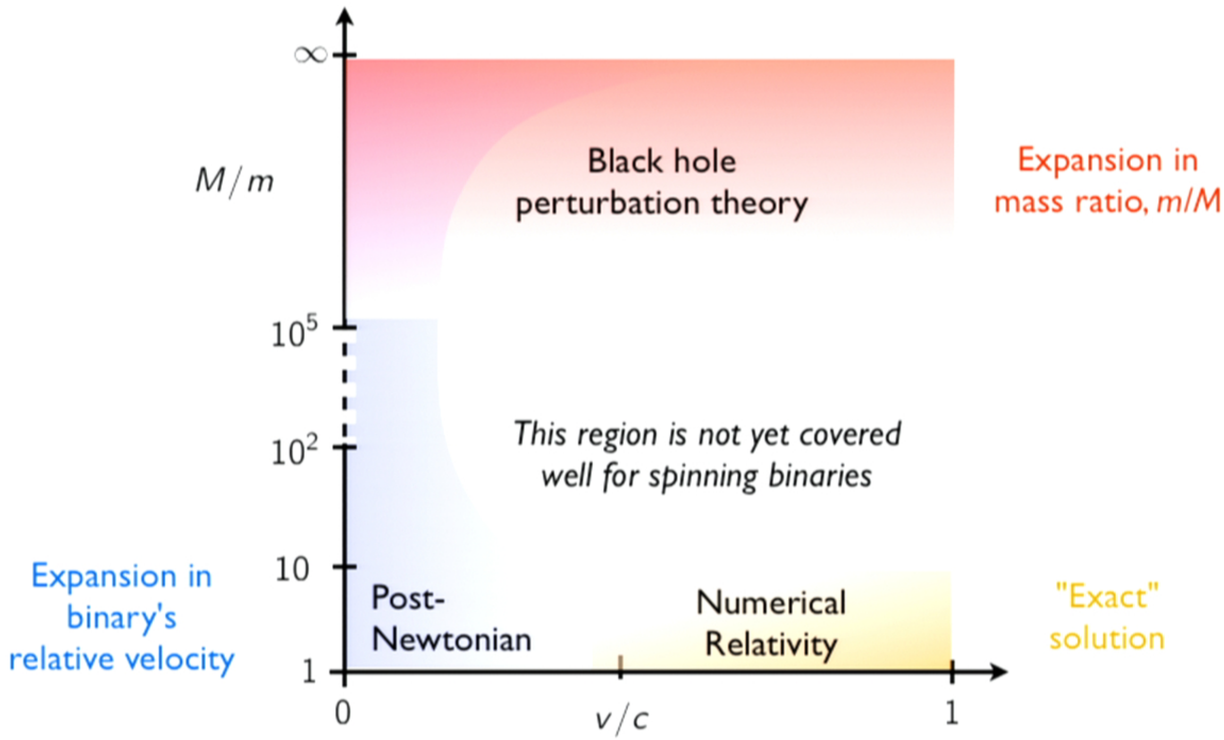
"Plus" (+) polarization "Cross" (x) polarization



- Interferometers are naturally suited for this purpose

Snapshot of gravitational wave theory

(e.g., for quasi-circular inspirals)



Issues to consider with larger mass ratios

- ☐ Higher order perturbative corrections are needed for accuracy
  - How many orders are needed? Is series asymptotic?...
- ☐ The finite size of smaller object becomes increasingly important
  - To what extent does the finite size matter?

Motion of an extended mass in EFT

- For an otherwise spherical extended mass:

General coordinate invariance

Rotation invariance, SO(3)

Reparametrization invariance

- EFT action for extended mass & general relativity is

$$S[z^\mu, h_{\alpha\beta}] = S_{\mathrm{EH}}[h_{\alpha\beta}] - m\int d\tau \sqrt{1 - h_{\alpha\beta}(z)u^\alpha u^\beta} + C_E \int d\tau\, \mathcal{E}_{\alpha\beta}(z)\mathcal{E}^{\alpha\beta}(z) + \cdots$$

Many such terms but each enter at a definite order in R/M

Finite-size effects

$$S[z^\mu, h_{\alpha\beta}] = S_{\mathrm{EH}}[h_{\alpha\beta}] - m \int d\tau \sqrt{1 - h_{\alpha\beta}(z) u^\alpha u^\beta} + C_E \int d\tau \, \mathcal{E}_{\alpha\beta}(z) \mathcal{E}^{\alpha\beta}(z) + \cdots$$

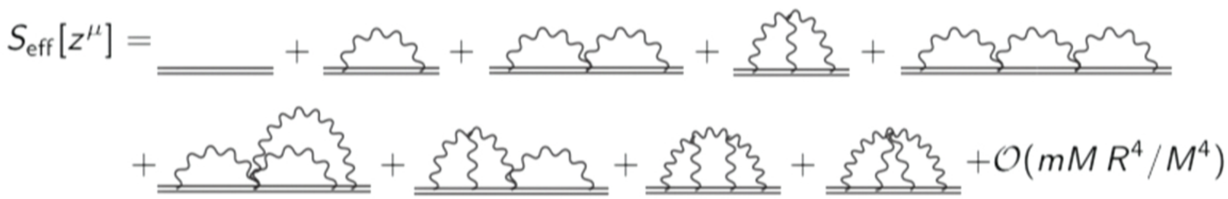
- Effacement Principle *CRG & Hu, (2009)*

Geodesic deviation due to the finite size of the non-spinning extended object begins at fourth order in m/M

$$C_E \sim m^5$$

Deriving self-force expressions

- Self-force can be derived by integrating out the gravitational perturbation *h* at the level of the action using Feynman diagrams



Gravitational self-force in the ultra-relativistic regime

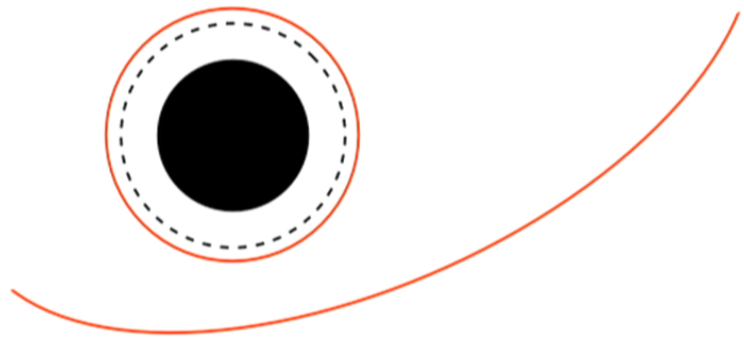
CRG & Porto (2013)

- Motivations:

  - What is the nature of self-force at ultra-relativistic speeds?
  - Does the perturbation theory simplify at all?
  - Can help to calibrate semi-analytical models of binary black hole mergers (e.g., Effective One Body models *Buonanno & Damour (2000)*)

- Contexts:

  - Circular orbit near Schwarzschild light ring
  - Fast "fly-by's"



Power counting

- Ultra-relativistic speeds parametrized by boost factor

$$\gamma = \frac{1}{\sqrt{-g_{\alpha\beta} v^\alpha v^\beta}} \qquad v^\mu = \frac{dz^\mu}{dt} = (1, \vec{v}) \sim 1$$

- Power counting shows that

$$h_{\alpha\beta} \sim \gamma q = \frac{\gamma m}{M} \equiv \epsilon$$

$$S[z^\mu, h_{\alpha\beta}] = S_{\text{EH}}[h_{\alpha\beta}] - m \int d\tau \sqrt{1 - \underbrace{\gamma^2 h_{\alpha\beta}(z) v^\alpha v^\beta}_{\sim \gamma^2 \epsilon = \gamma^3 q \ll 1}} + \text{finite size corrections}$$

- As the boost factor increases
  we require the gravitational
  perturbation to decrease in amplitude

- In the ultra-relativistic regime the perturbation series simplifies

$$L \sim \gamma m M \sim -m \int d\tau \qquad N = \gamma^2$$

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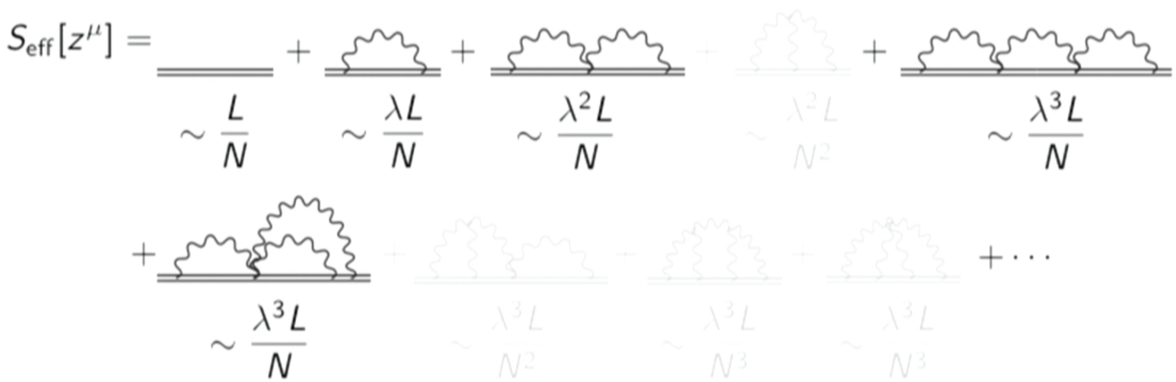


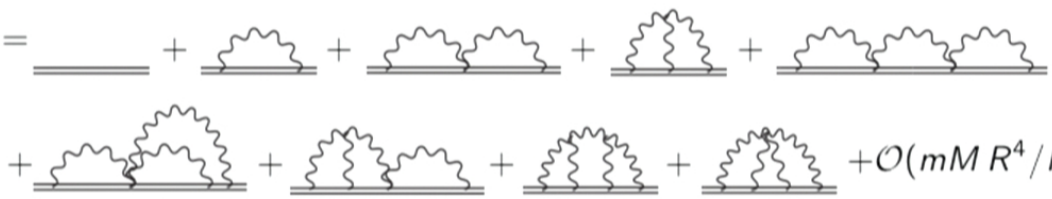
- Dominant contributions in ultra-relativistic regime are LO in $1/N$

Only diagrams without interactions in the bulk contribute
in the ultra-relativistic regime

BH perturbation theory and comparable masses

- What needs to be calculated in the general 2-body problem?

$S_{\text{eff}}[z^\mu] =$



$+\mathcal{O}(mM\,R^4/M^4)$

BH perturbation theory and comparable masses

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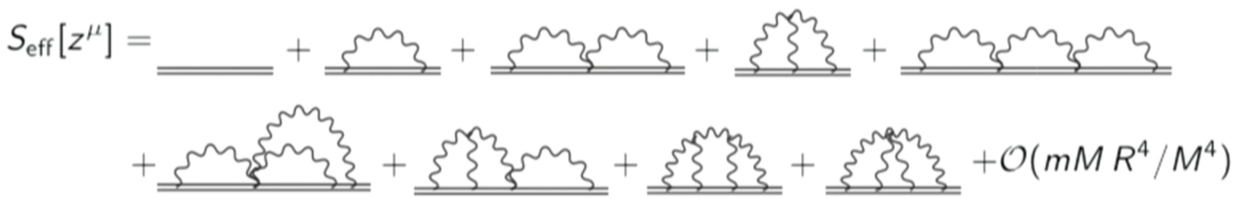


- Need to include spin of the small mass *Yee & Bander (1993); Porto (2006)*

- Possibly need to include effects from the finite size of the small mass
  CRG & Hu (2009)

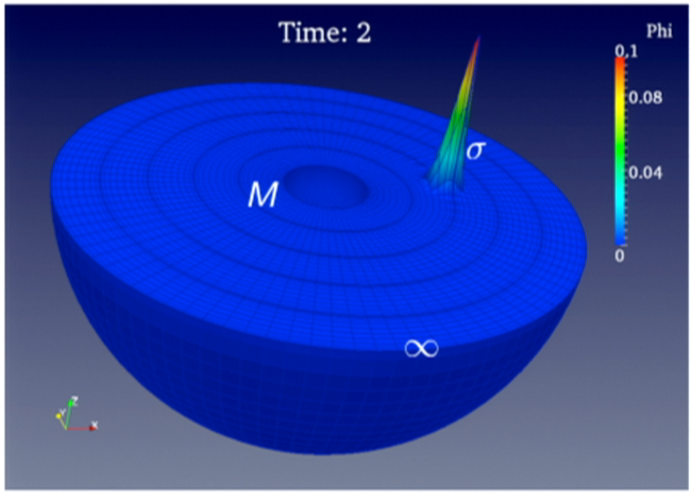
Method

Zenginoglu & CRG (2013)

- Model

$$\Box_x G_{\rm ret}(x, x') \approx \frac{4\pi}{\sqrt{-g}} \frac{1}{(2\pi\sigma^2)^2} \exp\left[-\frac{(x-x')^2}{2\sigma^2} \right]$$

- Problem has 3 scales
  - Width of Gaussian $\sigma \sim 0$
  - Mass of black hole $M \sim 1$
  - Ideal observer $\text{obs} \sim \infty$



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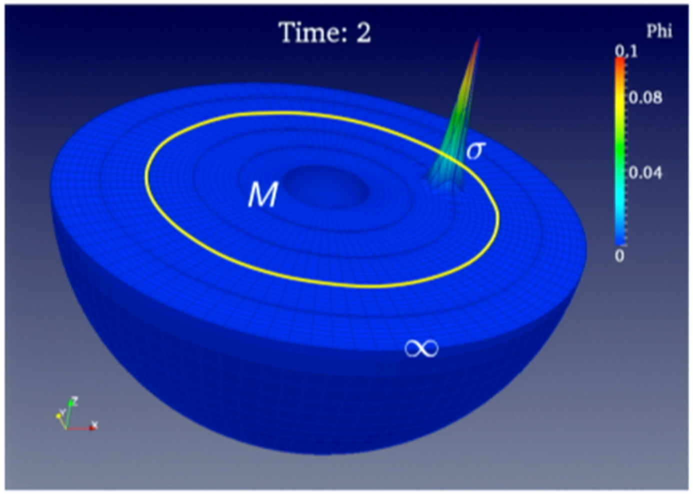

- Used a layered hyperboloidal compactification

Zenginoglu (2008); Zenginoglu (2011)

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Worldline convolution integrals

- Worldline convolutions to compute

$$\phi_{\text{reg}}(\tau) \propto \lim_{\epsilon \to 0^+} \int_{-\infty}^{\tau-\epsilon} d\tau' \, G_{\text{ret}}(z(\tau), z(\tau'))$$

$$F_\mu^{(1)}(\tau) \propto \lim_{\epsilon \to 0^+} \int_{-\infty}^{\tau-\epsilon} d\tau' \, \nabla_\mu G_{\text{ret}}(z(\tau), z(\tau'))$$

Take-home points

- Gravitational waves comprise an important spectrum for probing strong gravity dynamics

  - Compact binary inspirals and mergers most anticipated sources

My goal:

- Calculate and utilize higher order self-force corrections in black hole perturbation theory to:
  - Find its practical domain of validity (up to what mass ratio?)
  - Complement existing approaches (numerical relativity & post-Newtonian)
  - Provide sufficiently accurate waveforms of spinning binaries (for LIGO)
  - Provide "tests" of general relativity using space-based detectors (for eLISA)
  - Understand the basic physics of black hole binaries

My approach:

- Derive higher order self-force expressions using effective field theory techniques

Conclusion & Outlook

- The Effective Field Theory (EFT) approach offers a systematic way to calculate formal self-force expressions
  - Regularization and renormalization are naturally incorporated
  - Finite size effects of the smaller mass are systematically accounted for

- EFT has already been "field-tested" with high order self-force calculations
  - To 3rd order in a nonlinear scalar field model *CRG (2010a), (2010b)*
  - To 4th order in the ultra-relativistic limit for gravity *CRG & Porto (2013)*
  - Need to complete 2nd order expressions with spin effects for gravity

- The resulting formal self-force expressions can be numerically evaluated using global approximations for the retarded Green function
  - Proof of principle already demonstrated at 1st order *Casals, CRG + (in prep)*
  - Work ongoing to improve numerical accuracy and precision
  - Need to compute Green function for Kerr and for gravity