Title: Toward a Higher-Spin Dual of Interacting Field Theories

Date: Jun 11, 2013 10:00 AM

URL: http://pirsa.org/13060007

Abstract: We show explicitly how the exact renormalization group
equation of interacting vector models in the large N limit can be
mapped into certain higher-spin equations of motion. The equations of
motion are generalized to incorporate a multiparticle extension of the
higher-spin algebra, which reflects the "multitrace" nature of the
interactions in the dual field theory from the holographic point of view.

Toward a Higher-Spin Dual of Interacting Field Theories

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Based on arXiv:1303.6641 LPZ and Cheng Peng
Work in Progress, R. Leigh, D. Minic, LPZ, O. Parrikar, D. Vaman

Outline

- Motivation: The inspiring tale of the duality between Sine-Gordon and massive Thirring.
- Some attemps at proving Holography (AdS/CFT).
- Wilson-Polchinski: Exact Renormalization Group Equations.
- DMR: ERG of free VM $\mapsto$ HS (not Vasiliev's).
- Interactions (PZP): Multi-trace deformations $\mapsto$ Multiparticle HS algebra.
- Conclusions and open problems.

Duality $\mapsto$ Bosonization (tool)

- Precursor: Skyrme (Soliton modes of the sine-Gordon equation are fermions whose interaction is Thirring.)
- Understood beyond perturbation theory, including solitons: Mandelstam.
- Non-abelian bozonization: Witten and Polyakov-Wiegmann.
- The role of symmetries (2d algebras): $\partial_+ J_- = 0$ and $\partial_- J_+ = 0$.
- Remark: The Fermi field ψ has a complicated non-local expression in term of ϕ. Fermion bilinears take a simple form.

Some approaches to proving Holgraphic dualities

- Gopakumar: Constructing the string theory a la 't Hooft, "From free fields to AdS," 2003.
- Berkovits: From string theory to a field theory (zero radius limit): Berkovits-Vafa "Towards a Worldsheet Derivation of the Maldacena Conjecture," (2007) and Berkovits "Perturbative Super-Yang-Mills from the Topological $AdS_5 \times S^5$ Sigma Model" (2008)
- Jevicki et al. "Bi-local Model and Higher Spin Gravity" (2003-)
- Douglas-Mazzucato-Razamat: "Holographic Dual of Free Field Theory," (2010) [ERG $\mapsto$ HS]

AdS/CFT: What do we know?

- Main Statement of AdS/CFT

$$Z_{string}[\phi \to \phi_0] = \langle \exp\left(-\int \phi_0 \mathcal{O}\right)\rangle_{QFT}.$$

- Scale invariance:
  Field Theory $(\vec{x} \to \lambda \vec{x}) \Leftrightarrow$ AdS spacetime $(\vec{x} \to \lambda \vec{x}, r \to \lambda r)$

$$ds^2 = \frac{dr^2 + d\vec{x}^2}{r^2}.$$

- The UV/IR relation (Susskind): $r \Leftrightarrow 1/\Lambda$
  Holographic renormalization: Regularization, counterterms; n-point functions.

Higher Spin/Vector Model Duality

- Klebanov-Polyakov, Sezgin-Sundell 2002.
- Vasiliev's A-type minimal bosonic theory in AdS$_4$ (with $\Delta = 1$) is holographically dual to the free $O(N)$ vector model.
- Vasiliev's A-type minimal bosonic theory in AdS$_4$ (with $\Delta = 2$) is holographically dual to the critical $O(N)$ vector model (Wilson-Fisher).
- Vasiliev's B-type minimal bosonic theory in AdS$_4$ (with $\Delta = 2$) is holographically dual to the free $O(N)$ fermion model.
- Vasiliev's B-type minimal bosonic theory in AdS$_4$ (with $\Delta = 1$) is holographically dual to the critical $O(N)$ fermion model (Gross-Neveu).

ERG: Polchinski

- Non-perturbative RG equations: $S = S_K + S_{int}$.
- $K(\Lambda)$ cutoff function: vanishes for $p^2 > \Lambda^2$ and $K \to 1$ for $p^2 < \Lambda^2$
- $d_\Lambda Z = 0 \Rightarrow$

$$d_\Lambda S_{\rm int} = -\int d^{(D)}p \frac{d_\Lambda K(p^2/\Lambda^2, \vec{e})}{K(p^2/\Lambda^2, \vec{e})} \left(1 + \frac{K(p^2/\Lambda^2, \vec{e})}{p^2}\left(\frac{\partial S_{\rm int}}{\partial \bar{\phi}(p)}\frac{\partial S_{\rm int}}{\partial \phi(p)} + \frac{\partial^2 S_{\rm int}}{\partial \bar{\phi}(p)\partial \phi(p)}\right)\right),$$

$d_\Lambda = \Lambda \partial / \partial \Lambda$

- 1: Field independent choice of normalization.

- What is the "meaning" of ERG? (Diagrammatically speaking)
- Non-pertubative approach!

$$\frac{\partial \mathcal{S}_{int}}{\partial \phi}\frac{\partial \mathcal{S}_{int}}{\partial \phi} \quad :$$

$: 3 \to 4$

$$\frac{\partial^2 \mathcal{S}_{int}}{(\partial \phi)^2} \quad :$$

$: 6 \to 4$

- A given interaction (4-point) gets contributions from lower $(\partial S_{int}/\partial \phi)^2$ (3-point) and higher $\partial^2 S_{int}/\partial^2 \phi$ (6-point) interactions.

Covariantizing ERG

- Allow the cutoff function to depend on a reference point $\vec{e}$.
- A GR motivated move toward ERG covariantization but it makes sense on its own

$$d_{e^a} S_{\rm int} = -\int d^{(D)}x \frac{d_{e^a} K(\Box/\Lambda^2, \vec{e})}{K(\Box/\Lambda^2, \vec{e})} \left(1 + \frac{K(\Box/\Lambda^2, \vec{e})}{\Box} \left(\frac{\partial S_{\rm int}}{\partial \bar{\phi}(x)} \frac{\partial S_{\rm int}}{\partial \phi(x)} + \frac{\partial^2 S_{\rm int}}{\partial \bar{\phi}(x) \partial \phi(x)}\right)\right),$$

$\Box \equiv \partial_b \partial^b$

- Just a delta function for momentum conservation in the vertex in the standard textbook case.

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DMR: ERG for Free VM

- N free complex scalar fields $\phi^A(x)$ in D dimensions (Momentum space)

$$S = \int d^D p\, d^D q\, (P(p,q) - B(p,q))\, \bar{\phi}^A(q)\phi^A(p)\,,$$

$$P(p,q) \;=\; p^2 K^{-1}(p^2/\Lambda^2)\delta^{(D)}(p-q),$$

- Quadratic action for "interacting" piece as:

$$S_{int} = \int d^D p d^D q B(p,q)\bar{\phi}^A(q)\phi^A(p).$$

- Bi-local interaction, collective notation $J_s \sim \phi^A \partial_{\mu_1} \dots \partial_{\mu_s} \phi^A$, a set of singlet conserved currents in the free field theory.

$$\Lambda \frac{\partial B(p,q)}{\partial \Lambda} = -\int \frac{d^D s}{s^2} \Lambda \frac{\partial K(s^2/\Lambda^2, \vec{e})}{\partial \Lambda} B(s,q) B(p,s).$$

$$\begin{aligned} \alpha_r &= \frac{d_\Lambda K(p^2/\Lambda^2, \vec{e})}{p^2} \delta^{(D)}(p-q)\,, \\ B(p/\Lambda, q/\Lambda) &= \Lambda^{2-D-|s|-|t|}\, B_{\underline{st}}\, p^{\underline{s}} q^{\underline{t}}\ , \\ \alpha_r^{\underline{st}} &= \Lambda^{2-D-|s|-|t|} \int d^D p \int d^D q\ \alpha_r(p,q) p^{\underline{s}} q^{\underline{t}}\,, \end{aligned}$$

$$\frac{d}{d\Lambda} B_{\underline{st}} = -B_{\underline{si}}\, \alpha_r^{\underline{ij}}\, B_{\underline{jt}} + \Lambda^{-1}(|s| + |t| + D - 2)\, B_{\underline{st}}\,.$$

- First term dynamic; second term kinematic (scaling).

Schematic story

$$d_\Lambda B(p,q) = -B(p,s) \bullet_s B(s,q).$$

$$\alpha_r(p,q) \Longrightarrow \alpha_r^{\underline{st}},$$
$$B(p/\Lambda, q/\Lambda) \Longrightarrow B_{\underline{st}}$$

- Where is geometry hiding in the RG equations?
- First equation looks like a **flat connection** where the product $\bullet_s$ is convolution.
- $\bullet_s$ is already pointing to a **star product** (integral formulation).

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Higher Spin Theories

- First order gravity $(e^a_\mu dx^\mu, \omega^{ab}_\mu dx^\mu)$:

$$de^a + \omega^a{}_b \wedge e^b = 0, \quad R^{ab} = d\omega^{ab} + \omega^{ac} \wedge \omega_c{}^b.$$

- Unifying theme: $W = e^a_\mu dx^\mu \, P_a + \omega^{ab}_\mu dx^\mu \, M_{ab}$:

$$dW + W \wedge W = \begin{cases} \left(de^a + \omega^a{}_b \wedge e^b\right) & P_a \\ + \left(d\omega^{ab} + \omega^{ac} \wedge \omega_c{}^b\right) & M_{ab} \end{cases}$$

- HS theories: Promote (P_a, M_{ab}) to a HS algebra, introduce the $*$-product and impose flatness conditions:

$$W_\mu = e^a_\mu P_a + \omega^{ab}_\mu M_{ab} \mapsto W = \sum_n W_{i_1 \ldots i_n} y^{i_1} \ldots y^{i_n},$$

$$dW + W * W = 0.$$

- MacDowell-Mansouri and Stelle-West: Bundle, C
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- MacDowell-Mansouri and Stelle-West: Bundle, Connection, Section, Young Tableaux, etc.

The DMR HS System

- Master fields: $(B, W, \tilde{W})$
- Equations of Motion:

$$dB + W * B - B * \tilde{W} = 0,$$
$$dW + W \wedge * W = 0, \qquad d\tilde{W} + \tilde{W} \wedge * \tilde{W} = 0,$$

- Gauge transformations:

$$\delta W = d\epsilon + [W, \epsilon]_*, \quad \delta\tilde{W} = d\tilde{\epsilon} - [\tilde{W}, \tilde{\epsilon}]_*,$$
$$\delta B = B * \tilde{\epsilon} - \epsilon * B.$$

AdS as a solution (with lowest generators)

$$W_\mu^{(0)} = \frac{1}{r}\, P_\mu \,,$$

where

$$\begin{aligned} P_r &= \bar{z}_r z^\bullet - \bar{z}_\bullet z^r , \\ P_a &= \bar{z}_a \,(z^\bullet - z^r) - (\bar{z}_\bullet - \bar{z}_r)\, z^a \,. \end{aligned}$$

The connection satisfies the flatness condition evaluated using the star product

$$dW^{(0)} + W^{(0)} * W^{(0)} = 0 \,.$$

Interactions in Vector Models

- Turning on an irrelevant interaction; only singlet sector (A_j)

$$S = \int d^D p d^D q \big(P(p,q) - B^{(1)}(p,q)\big)\bar{\phi}(q)^A \phi(p)^A$$
$$-\frac{1}{N^{n-1}} \int \left(\prod_{j=1}^{n} d^D p_j d^D q_j \bar{\phi}^{A_j}(q_j)\phi^{A_j}(p_j)\right) B^{(n)}(p_1,\ldots,p_n,q_1,\ldots,q_n)\,.$$

- ERG equation for $B^{(n)}$

$$d_\Lambda B^{(n)}(p_1,\ldots,p_n,q_1,\ldots,q_n) = -\int d^D r \frac{d_\Lambda K(r^2/\Lambda^2,\vec{e})}{r^2} \times$$
$$\sum_{k=1}^{n}\left(B^{(n)}(\underline{p},\underline{q})\big|_{q_k=r} B^{(1)}(r,q_k) + B^{(n)}(\underline{p},\underline{q})\big|_{p_k=r} B^{(1)}(p_k,r)\right).$$

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Multiparticle HS Algebra

- $$M(A) = \sum_{n=0}^{\infty} \oplus \mathrm{Sym}\, \underbrace{A \otimes A \otimes \ldots \otimes A}_{n}. \qquad (1)$$
- The multiplication rules among them are defined
  $\Xi_i = (1, y_i^\mu, z_i^\mu, y_i^\mu y_i^\nu, \ldots)$
  $$\Xi_i^\alpha * \Xi_i^\beta = f_\gamma^{\alpha\beta} \Xi_i^\gamma \,, \qquad \Xi_i^\alpha * \Xi_j^\beta = \Xi_j^\beta * \Xi_i^\alpha \,,$$
  where i, j labels different sets of auxiliary variables and α, β labels different basis in one set of the basis.
- The multiparticle algebra can be realized as a subalgebra of the enveloping algebra of HS whose generators are polynomials of Ξ_i that are symmetric under the S_n permutation group of different species.
- Multiparticle algebra for any HS algebra, beyond Vasiliev's HS algebra.

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The Calculation

$$
\begin{aligned}
&(d_r\, B + W_r * B - B * \widetilde{W}_r)\, r^2 \\
&= \sum_{\underline{s},\underline{t}} \left[d_\Lambda B_{\underline{st}} + B_{\underline{sx}}\, \alpha^{\underline{xy}}\, B_{\underline{yt}} - \Lambda^{-1}(|s| + |t| + D - 2)\, B_{\underline{st}} \right] \sum_j C(\Xi_j, s, t) \\
&+ \frac{1}{N^{n-1}} \sum_{\sigma \in S_N} \sum_{\underline{s_i},\underline{t_i}} \Big[d_\Lambda B^{(n)}_{\underline{s_1},\ldots,\underline{t_n}} + \Lambda^{-1}(n(2-D) - \sum_i (|\underline{s_i}| + |\underline{t_i}|)) B^{(n)}_{\underline{s_1},\ldots,\underline{t_n}} \\
&- \sum_{k=1}^{n} ((B^{(n)}_{\underline{s_1},\ldots,\underline{t_n}}\big|_{\underline{t_k}=\underline{a}})\, \alpha^{\underline{a},\underline{b}} B^{(1)}_{\underline{b},t_k} + B^{(1)}_{\underline{s_k},\underline{a}}\, \alpha^{\underline{a},\underline{b}} (B^{(n)}_{\underline{s_1},\ldots,\underline{t_n}}\big|_{\underline{s_k}=\underline{b}})) \Big] \prod_{j=1}^{n} C(\Xi_{\sigma(j)}, s_j, t_j) \\
&+ \frac{1}{N^{2n-2}} \sum_{\sigma,\sigma' \in S_N} \left(\sum_{k=1}^{n} \sum_{\underline{s_i},\underline{t_i},\underline{u_i},\underline{v_i}} B^{(n)}_{\underline{s_1},\ldots\underline{s_n},\underline{t_1},\ldots\underline{t_n}}\big|_{\underline{t_k}=\underline{a}}\, \alpha^{\underline{a},\underline{b}} B^{(n)}_{\underline{u_1},\ldots\underline{u_n},\underline{v_1},\ldots\underline{v_n}}\big|_{\underline{u_k}=\underline{b}} \right) \\
&\qquad \times \prod_{j=1}^{n} C(\Xi_{\sigma(j)}, s_j, t_j) C(\Xi_{\sigma'(j)}, u_j, v_j)\,,
\end{aligned}
$$

Comments on the calculation

- Structure of the HS equation:

$$HS(B^{(1)}, B^{(n)}) = [ERG(B^{(1)})] + [ERG(B^{(n)})] + \mathcal{O}(1/N). \qquad (2)$$

- The square brackets vanish by virtue of the ERG equations.
- The last term vanishes in the large$-N$ limit for $n \geq 3$, irrelevant deformations of the vector model.
- HS side: The connection is only flat in the large$-N$ limit.

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- Full list (Ansatz) of Master Fields: ERG data $\mapsto$ Master Fields

$$B = \sum_i B^{(1)}(\Xi_i) + \frac{1}{N^{n-1}} \sum_{j_1,..,j_n} B^{(n)}(\Xi_{j_1}, \ldots, \Xi_{j_n})$$

$$W_r^{(0)} = \frac{1}{r} \sum_{j=1}^{n} \left((\bar{z_j})_r (z_j)^\bullet - (\bar{z_j})_\bullet (z_j)^r \right)$$

$$W_a^{(0)} = \frac{1}{r} \sum_{j=1}^{n} \left((\bar{z_j})_a \left((z_j)^\bullet - (z_j)^r \right) - \left((\bar{z_j})_\bullet - (\bar{z_j})_r \right) (z_j)^a \right)$$

$$\widetilde{W} = W^{(0)} , \qquad W = W^{(0)} + \delta W , \quad \delta W = B * \sum_k \alpha_\mu(\Xi_k) ,$$

The problem with a relevant interaction

- ERG with a $B^{(2)}$ coupling:

$$\begin{aligned} d_\Lambda B^{(1)} &= B^{(1)} \cdot \alpha \cdot B^{(1)} + \alpha \cdot B^{(2)} \\ d_\Lambda B^{(2)} &= B^{(1)} \cdot \alpha \cdot B^{(2)} + B^{(2)} \cdot \alpha \cdot B^{(1)} + \alpha \cdot B^{(3)}. \end{aligned} \tag{3}$$

- Going to HS with $B^{(2)}$

$$B = \sum B^{(1)} + \frac{1}{N^{n-1}} \sum_{j_1,\dots,j_n} B^{(n)}(\Xi_{j_1}\dots\Xi_{j_n})$$

- Your flat connection (from $B^{(1)}$) is no longer flat as in DMR.
- $B^{(2)}$ enters B above with $1/N$ but it contributes with a trace to $d_\Lambda B^{(1)}$, thus $\mathcal{O}(1)$.
- $B^{(n)}$ for $n \geq 3$: Does not affect the $B^{(1)}$ equation and the trace still leads to $1/N^{n-1}$, general case.

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Ansatz for the Master Fields

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$$\widetilde{W} = W^{(0)}, \qquad W = W^{(0)} + \delta W, \quad \delta W = B * \sum_k \alpha_\mu(\Xi_k),$$

Conclusions

- Explicit example of: Multi-trace deformations $\Leftrightarrow$ multi-particle states in the bulk.
- Large-N limit: Extra terms in the ERG and the curvature in the HS vanish only in large N. Introduction of interactions in the VM breaks the HS symmetry [MZ].
- Example of the "RG=GR" equation with GR=HS.
- Constructive duality: start with the ERG and move towards a HS equation of motion.
- A step toward a "covariantization" of the ERG equations which is needed to make full contact with any covariant gravity theory.

Some Open Questions

- Is DMR $\Leftrightarrow$ Vasiliev? (This is equivalent to a proof of the VM/HS duality)
- DMR (ERG) $\Leftrightarrow$ Jevicki?
- 3-point functions in DMR and the free $O(N)$ model.
- Full Covariant version of RG (Sung-Sik Lee).
- Fermionic version of HS/VM duality.
- What about 2d: CFT_2/HS on AdS_3?
- 80's: 2d NLSM $\beta = 0 \Leftrightarrow$ Sugra EoM. Not an argument for holography but worth revisiting.
- Applications to $\mathcal{N} = 4$ SYM toward IIB.