Title: Novel s-wave superconducting phase of doped topological insulators.

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Abstract: Many of the topological insulators, in their naturally
available form are not insulating in the bulk. It has been shown that some of these metallic compounds,
become superconductor at low enough temperature and the nature of their
superconducting phase is still widely debated. In this talk I show that even
the s-wave superconducting phase of doped topological insulators, at low
doping, is different from ordinary s-wave superconductors and goes through a
topological phase transition to an ordinary s-wave state by increasing the
doping. I show that the critical doping is determined using the
SU(2) Berry phase on the fermi surface of doped
topological insulator and can be modified by different tunable features of the
material. At the end I present the results of a recent experiment on the
Josephson junctions made of thin films of Bismuth selenide , which can be
explained using our theory of doping induced phase transition in topological
insulators.

Novel s-wave superconducting phase of doped topological insulators

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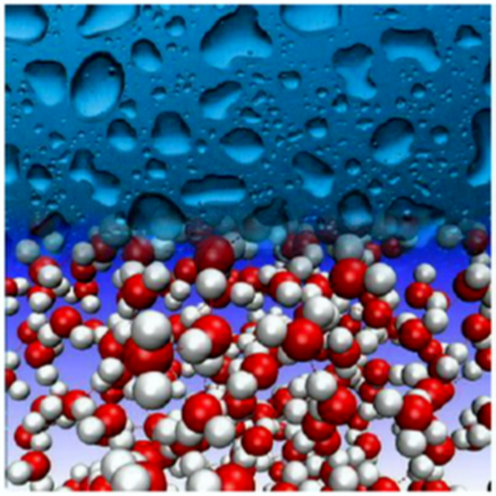
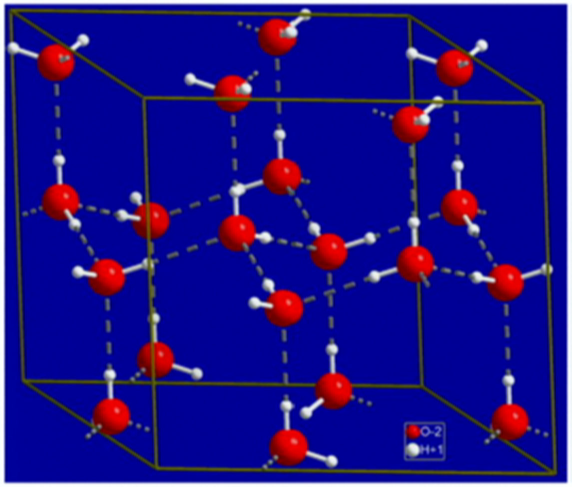
Phases of matter

- Classification based on long range symmetries!





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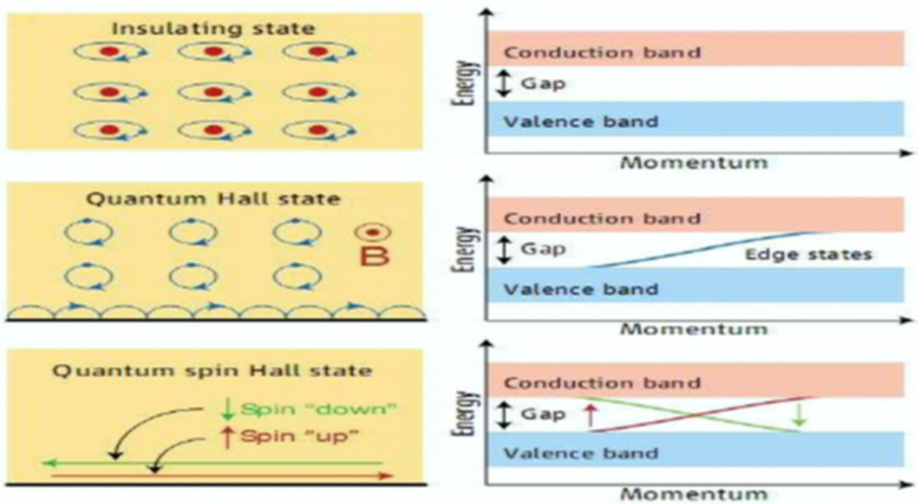
Topological Phases

- Quantum Hall state
- Quantum spin Hall state

Gapped in the bulk, conducting on the edge



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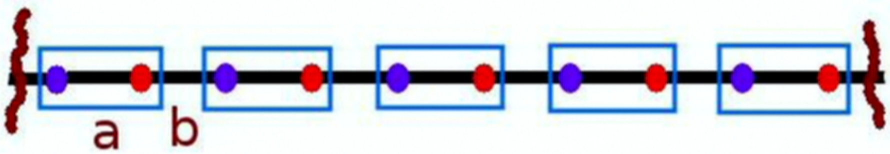
As usual: *Any simple model?*

- One dimensional solid with orbital for spinless fermions

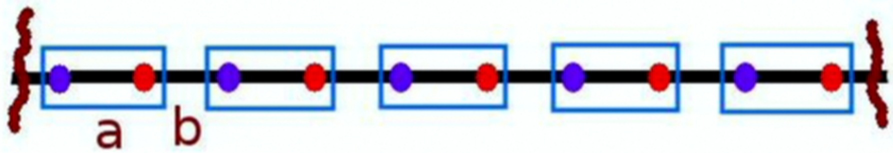


- Wave functions: $\begin{bmatrix} |\eta_i\rangle \\ |\nu_i\rangle \end{bmatrix}$

- Hamiltonian:

$$H = \sum_i \begin{bmatrix} 0 & a|\eta_i\rangle\langle\nu_i| + b|\eta_{i-1}\rangle\langle\nu_i| \\ a|\nu_i\rangle\langle\eta_i| + b|\nu_{i+1}\rangle\langle\eta_i| & 0 \end{bmatrix}$$

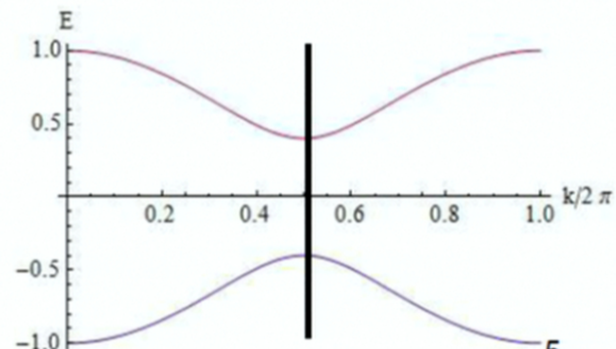
Periodic lattice: *Momentum space!*



$$\begin{bmatrix} |\eta_k\rangle \\ |\nu_k\rangle \end{bmatrix} = \sum_i e^{-ik} \begin{bmatrix} |\eta_i\rangle \\ |\nu_i\rangle \end{bmatrix} \qquad H = \sum_k \begin{bmatrix} 0 & (a + e^{-ik}\ b)\ |\eta_k\rangle\langle\nu_k| \\ (a + e^{ik}\ b)\ |\nu_k\rangle\langle\eta_k| & 0 \end{bmatrix}$$

$$H_k = [a + b\ Cos(k)]\ \sigma_x + b\ Sin(k)\sigma_y$$

$$E(k) = \pm\sqrt{a^2 + b^2 + 2ab\ Cos(k)}$$



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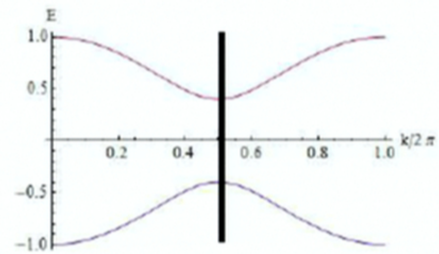
Band inversion

$$H_k = [a + b\ Cos(k)]\ \sigma_x + b\ Sin(k)\sigma_y$$

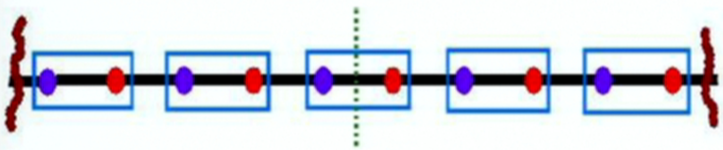
Time Reversal

$$\Theta k = -k$$

$$H_0 = [a+b]\,\sigma_x,\ H_\pi = [a-b]\,\sigma_x$$



Inversion



$$P\ k = -k \qquad P \equiv \sigma_x$$

$$p = \pm 1 \qquad E_0 = \pm(a+b),\ E_\pi = \pm(a-b)$$

At $k=0$ lower band $p=-1$

At $k=\pi$ lower band parity depends on (a-b)

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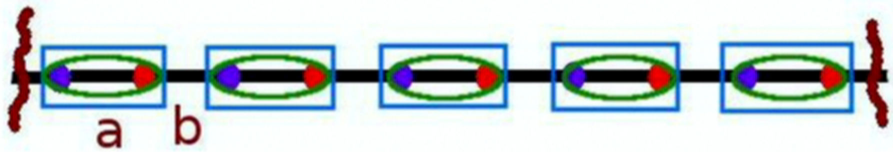
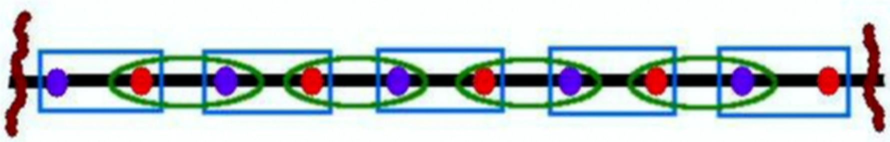
What is happening?

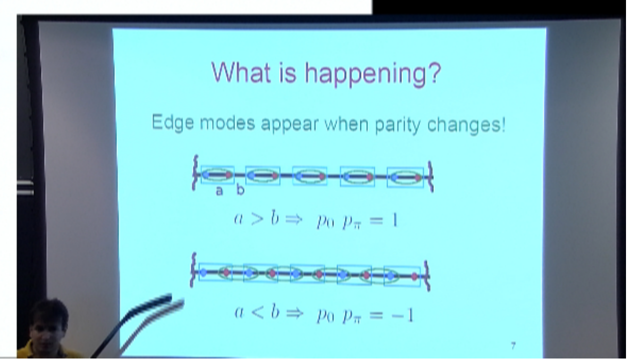
Edge modes appear when parity changes!



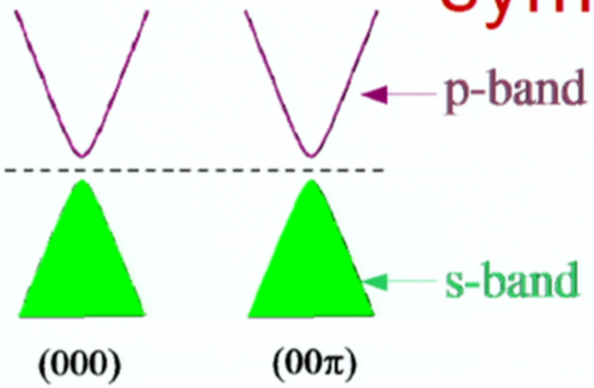
$$a > b \Rightarrow \; p_0 \, p_\pi = 1$$



$$a < b \Rightarrow \; p_0 \, p_\pi = -1$$

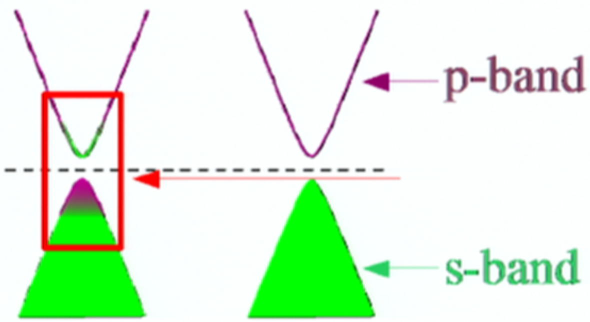


Bulk band properties inversion symmetric TI



Trivial insulator

$$P(000)P(00\pi) = (+1)(+1) = +1$$



Topological insulator

$$P(000)P(00\pi) = (+1)(-1) = -1$$

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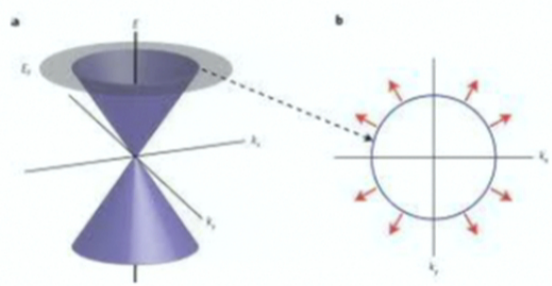
Topological insulators

- Strong spin-orbit coupling:

  Heavy elements

  Small band-gap

- Preserved time reversal symmetry.
- Gapped in the bulk, gapless chiral edge modes.



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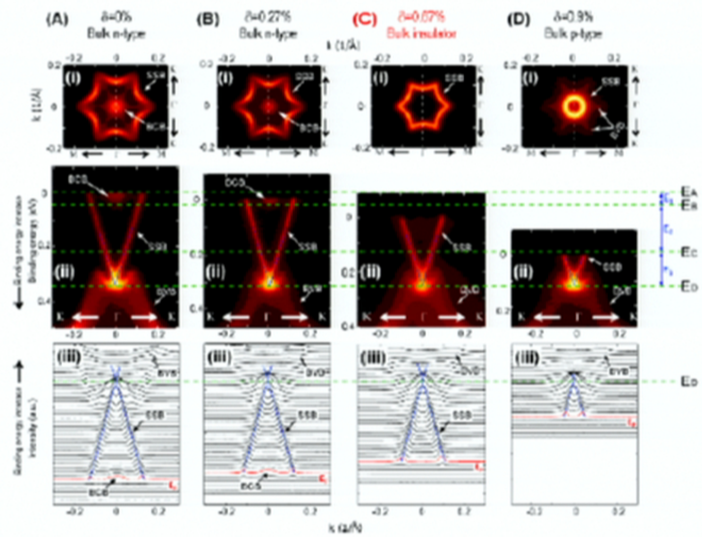
Experimental signature

- Symmetry properties of the bulk identifies the topological insulators. Surface states are experimental signature.



*Y. Chen, et. Al., Science, **325,** 178*

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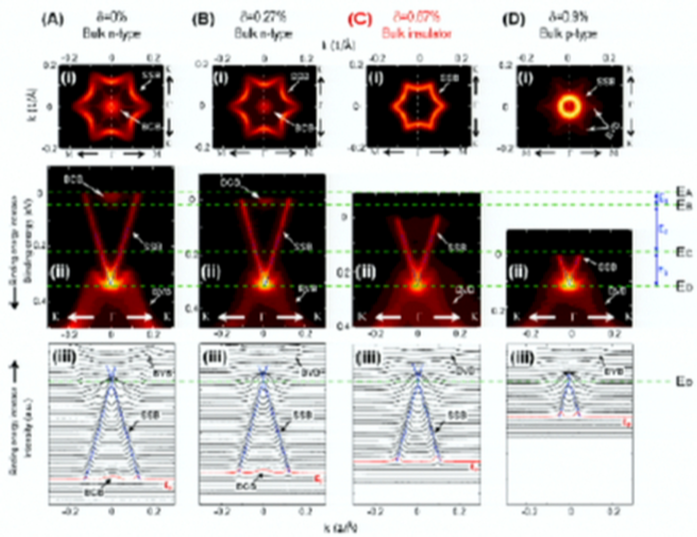

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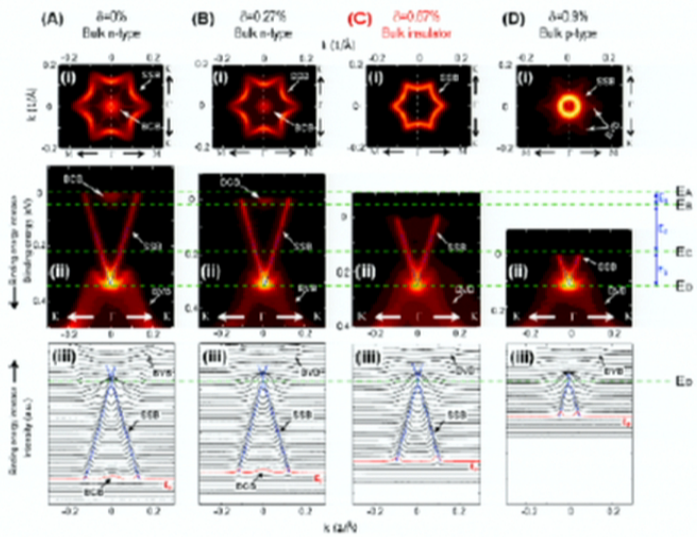
Is there signature of topological band structure left even in the conducting phase?



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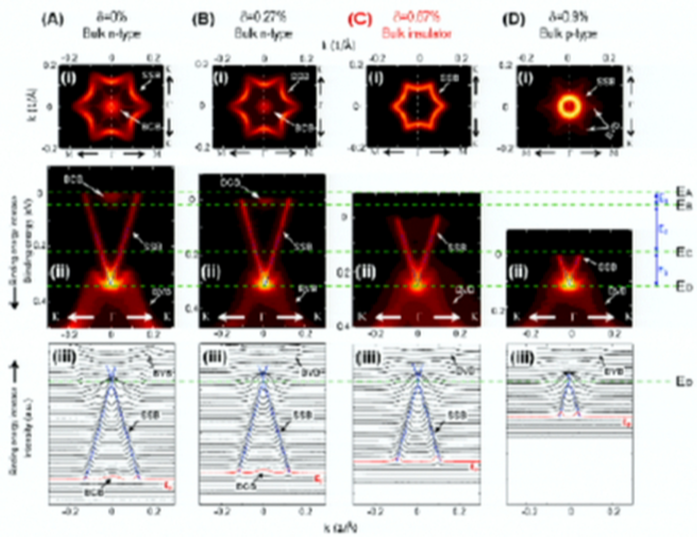
SC phase, Majorana vortex modes
Even appear in some non TI!



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Majorana Modes

- Fermionic mode which is its own anti-particle

$$C^{-1}\gamma C = \gamma^{\dagger} = \gamma$$

Electrons pairs: No cost to add a pair

Adding and removing the electrons can be equvalant???

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Vortex in *p+ip* superconductor (Read, Green '00)
Semiconductor-ferromagnet-superconductor sandwich (Lutchyn *et. al.* '10)
Superconductor-topological insulator interface (Fu, Kane '08)

Majorana modes in TI/SC interface

- Surface Hamiltonian: $H_0 = \psi^\dagger(-iv\vec{\sigma}\cdot\nabla - \mu)\psi$
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$$\Psi = ((\psi_\uparrow, \psi_\downarrow), (\psi_\downarrow^\dagger, -\psi_\uparrow^\dagger))^T$$

Jackiw and Rossi (1981)

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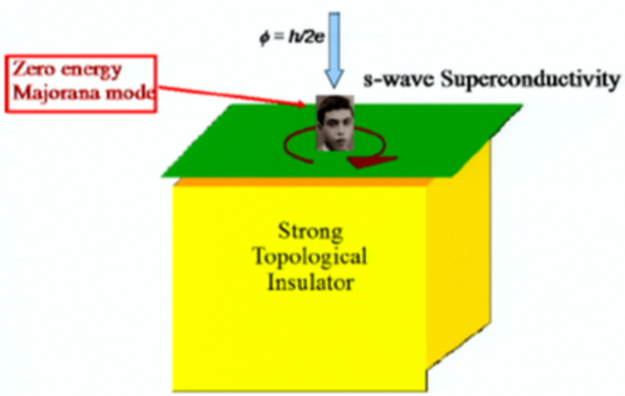

Ghaemi and Wilczek 2007

Fu and Kane 2008

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Superconducting TIs

- TIs could be doped and be pushed to SC phase.
  (e.g. Cu doped Bismuth Selenide, *Hor et. Al. 2010*)

Is there Majorana mode on the edge in the vortex?

Large chemical potential: Material does not know about band inversion, lost information about topological features so NO Majorana mode!

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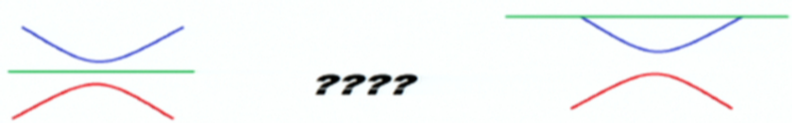
Chemical potential in bulk gap, only pairing on surface, like proximity induced case so Majorana mode.

What happens in between?



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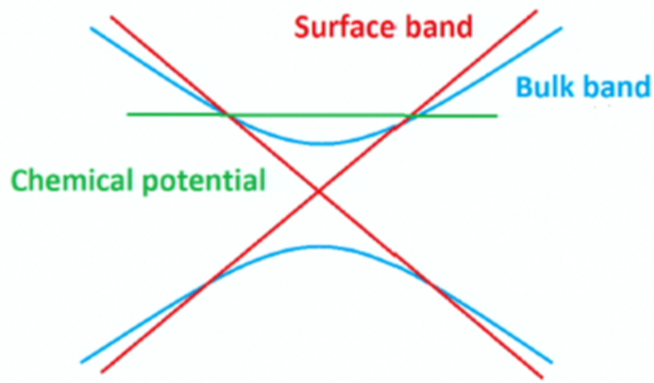
First guess!

When surface band cross the bulk band?

If so it would depends on the surface properties.

A single zero modes needs another partner to decay!



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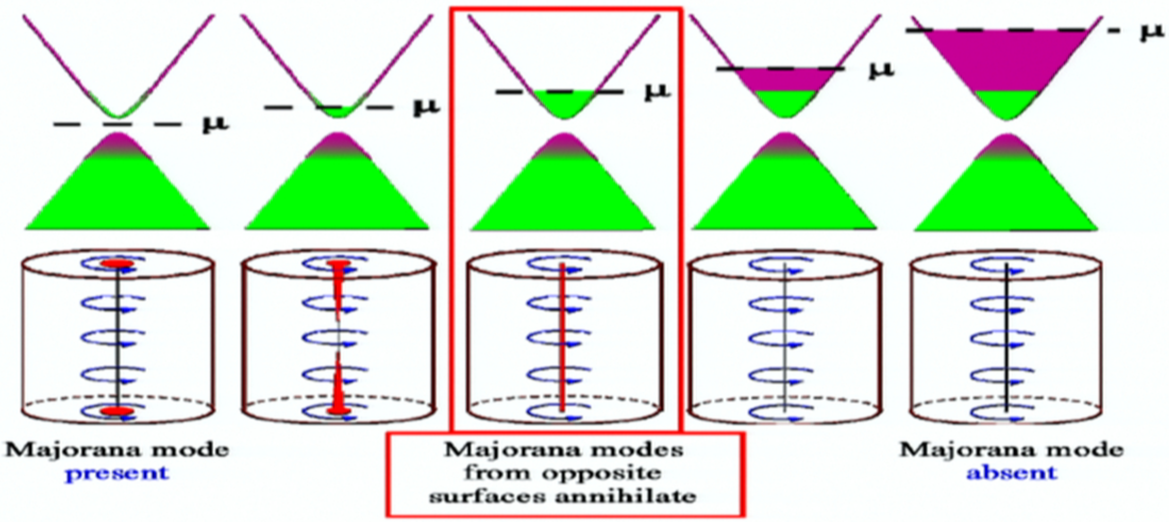
Physical picture!



When the transition happens the vortex line becomes gapless.
Lets look for the zero energy mode!

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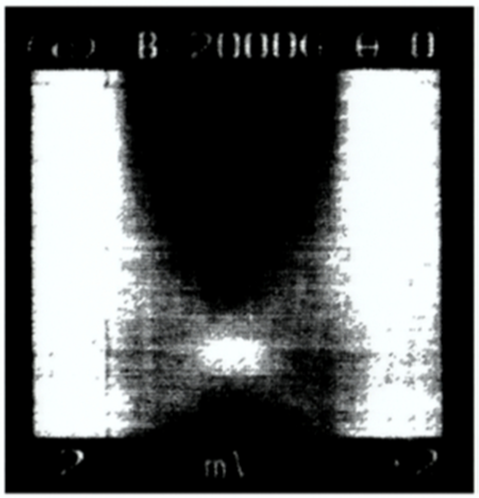
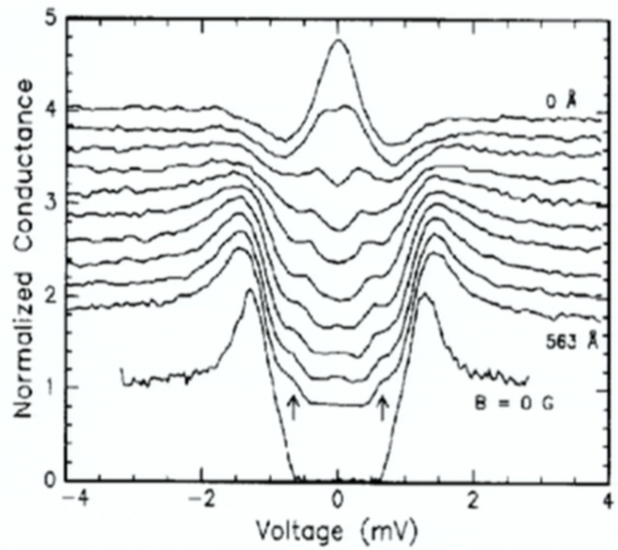
In gap states in SC vortices

- Simple *s-wave SC* vortex (Caroli et. Al. 1964) $E_n \sim (n+\frac{1}{2})\frac{\Delta^2}{\mu}$
- Experimental observation (Hess et. Al. 1990)





Lattice toy model

- Cubic lattice with *s* and *p* orbitals total spin 1/2

$$S_+ = s\uparrow, \quad P_+ = \frac{1}{\sqrt{3}} p_0 \uparrow - \sqrt{\frac{2}{3}} p_1 \downarrow,$$

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(Hosur et. Al. 2010) 28

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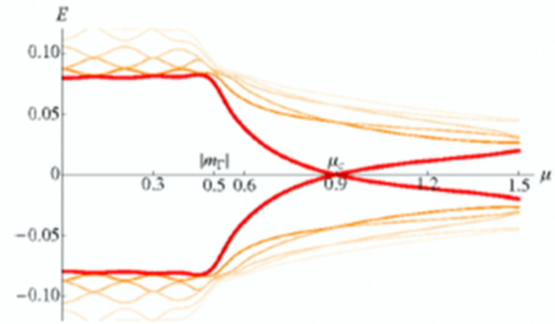
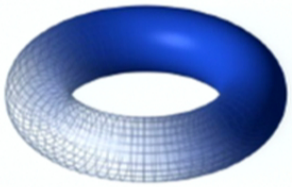
Low energy effective Hamiltonian

(Hosur et. Al. 2010) 29

Numerical approach

Low energy states





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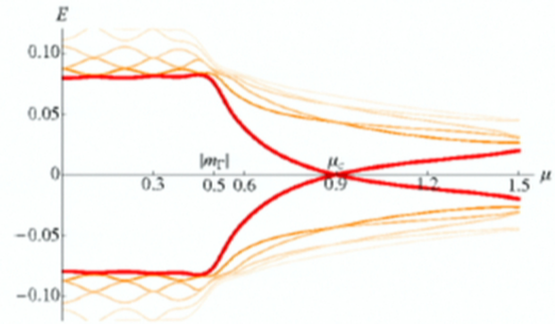

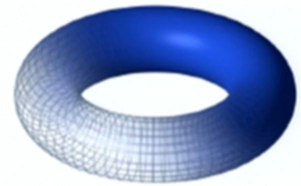

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Numerical approach

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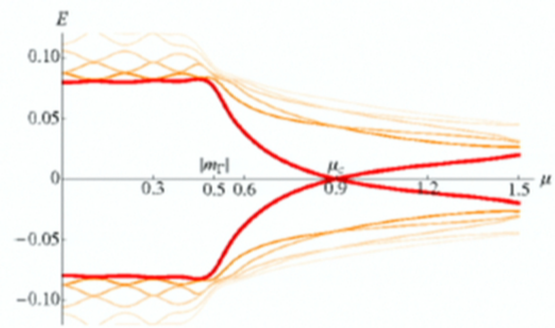
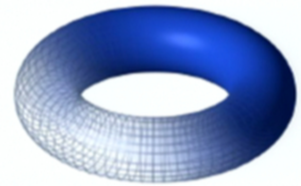
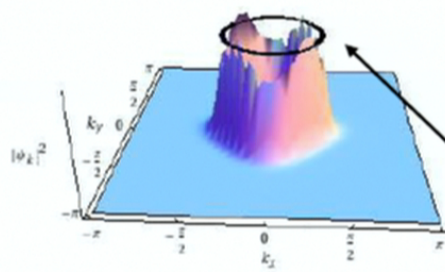




Fourier transform of zero energy state



$$H_k = v_F(\tau_x\sigma_y \sin k_x - \tau_x\sigma_x \sin k_y + \tau_y \sin k_z)$$
$$+ [M + m(\cos k_x + \cos k_y + \cos k_z)]\tau_z$$
$$+ n(\cos k_x + \cos k_y + \cos k_z).$$

$$M + m(Cos(k_x) + Cos(k_y) + Cos(k_z)) = 0$$

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Analytical approach

$$\mathcal{H}_{\boldsymbol{k}} = v_D \tau_x \boldsymbol{\sigma} \cdot \boldsymbol{k} + (m - \epsilon k^2)\tau_z - \mu$$

- Add pairing and the vortex, solve for zero energy solution:

$$J(kr) \quad m - \epsilon k^2 = 0 \qquad \mu = \sqrt{m/\epsilon}$$

- This condition is symmetry dependent. What if we don't have circular symmetry?

- Important observation: When $\mu = \sqrt{m/\epsilon}$, Simple Dirac Hamiltonian, Berry phase around the Fermi surface is π

Is this a more general condition?

Zero energy states

$$\tilde{\mathcal{H}}_{\boldsymbol{k}}^{\mathrm{BdG}} = \begin{bmatrix} E_{\boldsymbol{k}} - \mu & i\frac{\Delta_0}{\xi}(D_{k_x} - iD_{k_y}) \\ i\frac{\Delta_0}{\xi}(D_{k_x} + iD_{k_y}) & -E_{\boldsymbol{k}} + \mu \end{bmatrix}$$

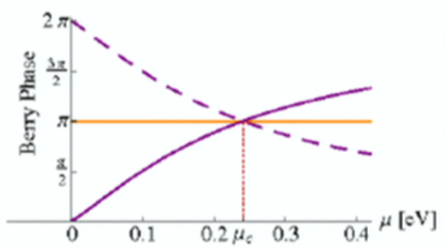
$$D_{k_\alpha} = \partial_{k_\alpha} - i\mathbf{A}_\alpha(\boldsymbol{k}) \qquad [\mathbf{A}]_\alpha^{\mu\nu}(\boldsymbol{k}) = i\langle\varphi_{\boldsymbol{k}}^{\mu}|\partial_{k_\alpha}|\varphi_{\boldsymbol{k}}^{\nu}\rangle$$

Effect of gauge potential phase of $U = e^{i\oint \mathbf{A}.d\mathbf{l}}$

Nonabelian Berry phase!

Semiclassical Bohr-Sommerfield quantization:

$$E_n = \frac{\Delta_0}{l_F\xi}(2\pi n + \pi \pm \phi_B)$$



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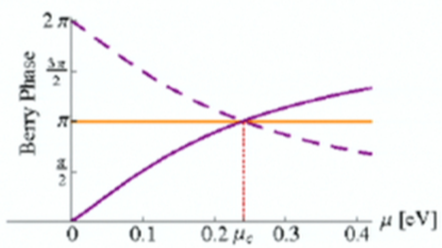

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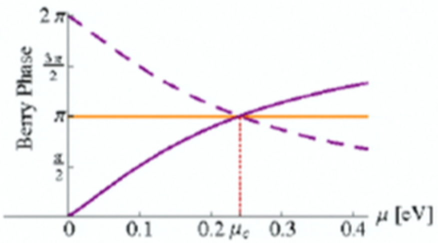
Confirmed using self-consistent BdG solutions numerically as well



Zero energy states

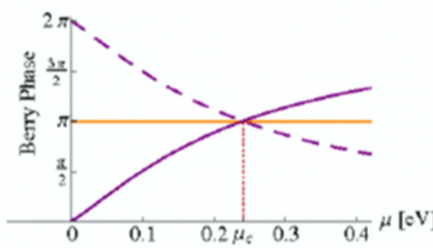
$$\tilde{\mathcal{H}}_{\boldsymbol{k}}^{\mathrm{BdG}} = \begin{bmatrix} E_{\boldsymbol{k}} - \mu & i\frac{\Delta_0}{\xi}(D_{k_x} - iD_{k_y}) \\ i\frac{\Delta_0}{\xi}(D_{k_x} + iD_{k_y}) & -E_{\boldsymbol{k}} + \mu \end{bmatrix}$$

$$D_{k_\alpha} = \partial_{k_\alpha} - i\mathbf{A}_\alpha(\boldsymbol{k}) \qquad [\mathbf{A}]_\alpha^{\mu\nu}(\boldsymbol{k}) = i\langle\varphi_{\boldsymbol{k}}^{\mu}|\partial_{k_\alpha}|\varphi_{\boldsymbol{k}}^{\nu}\rangle$$

Effect of gauge potential phase of $U = e^{i\oint \mathbf{A}.d\mathbf{l}}$

Nonabelian Berry phase!

Confirmed using self-consistent BdG solutions numerically as well



Candidate Materials

- Among doped topological insulators:
  - *p*-doped $TlBiTe_2$
  - *p*-doped Bi_2Te_3 under pressure
  - *n*-doped Bi_2Te_3
  - **not *n*-doped Bi_2Se_3**
- Among ordinary insulators
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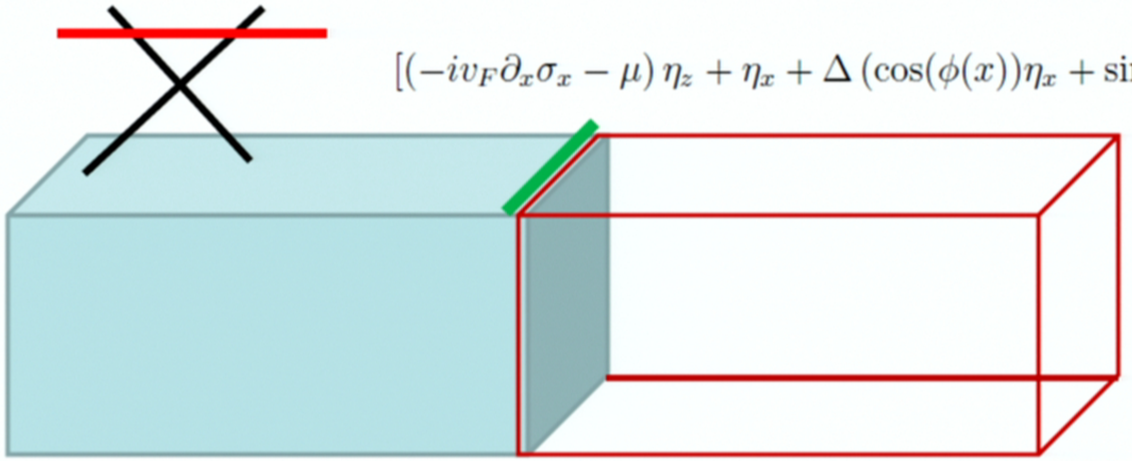
Josephson Junction

- Bulk insulating: Andreev modes on the boundary

$$[(-iv_F\partial_x\sigma_x - \mu)\,\eta_z + \eta_x + \Delta\,(\cos(\phi(x))\eta_x + \sin(\phi(x))\eta_y)]$$



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Josephson Junction

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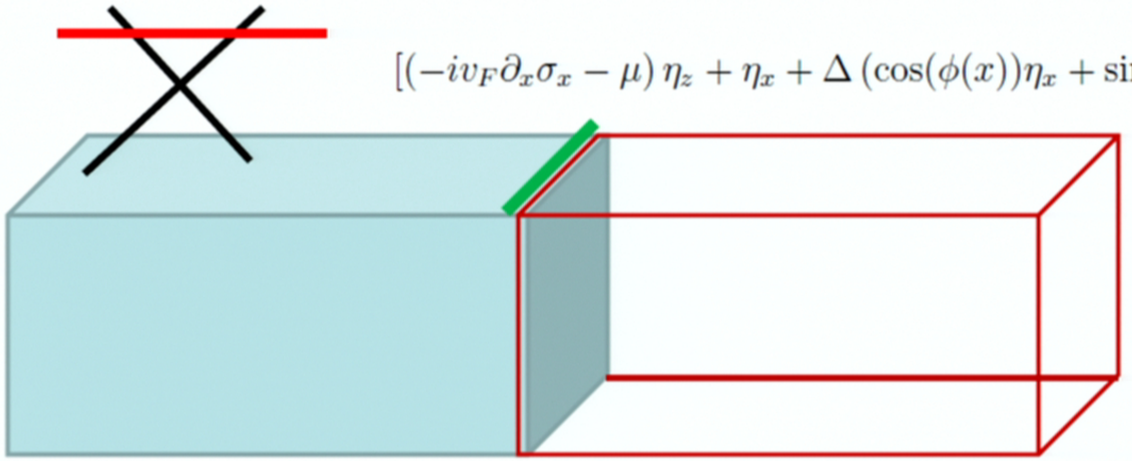

σ_x commutes with the Hamiltonian

$$[(\mp iv_F\partial_x - \mu)\,\eta_z + \Delta\,(\cos(\phi(x))\eta_x + \sin(\phi(x))\eta_y)]\,|v\rangle = E|v\rangle$$

41

Josephson Junction

- Bulk insulating: Andreev modes on the boundary

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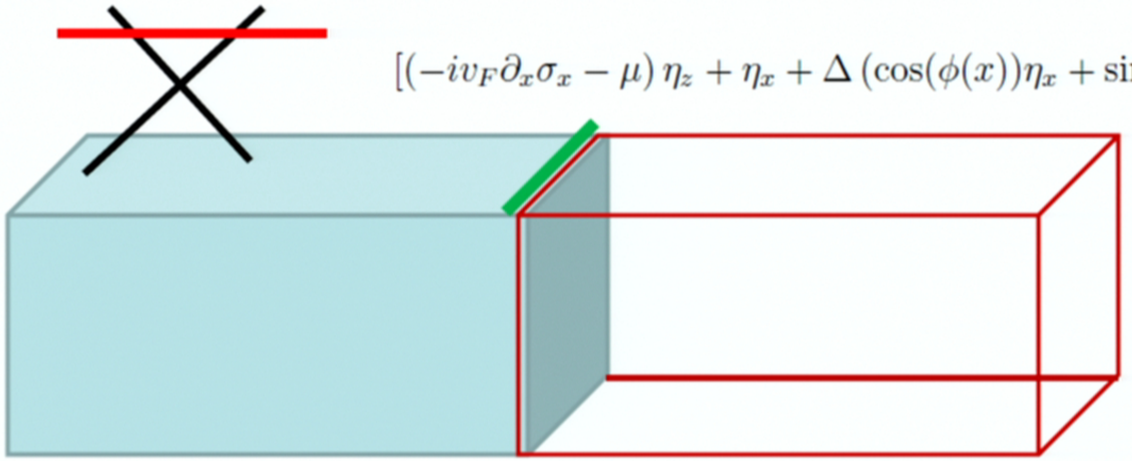

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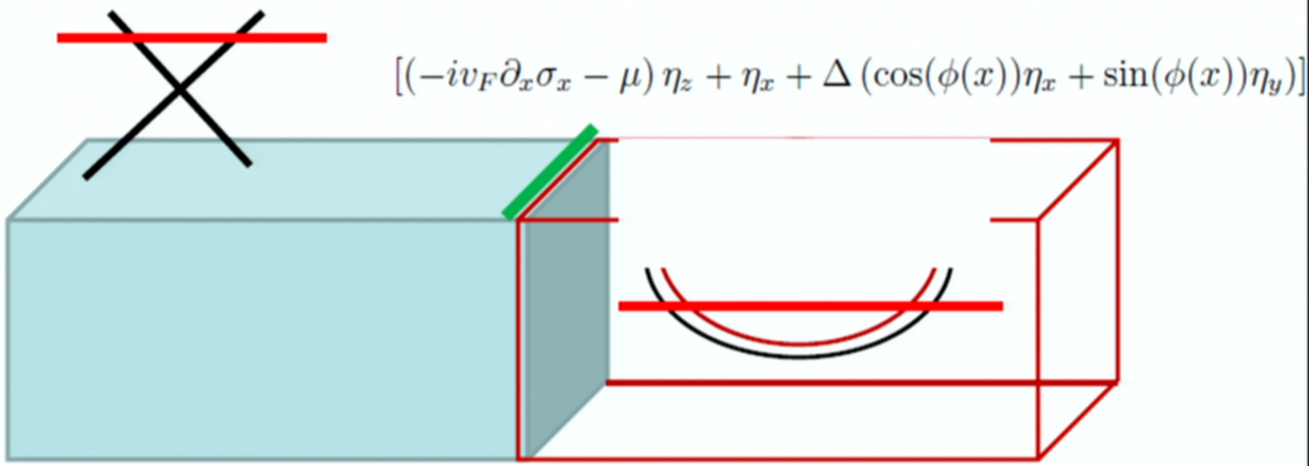

$$H_b=[v_Fk_x\sigma_x\tau_x+m\,(|\,\mathbf{k}|)\,\tau_z-\mu]\,\eta_z+\Delta_R(x)\eta_x+\Delta_I(x)\eta_y$$

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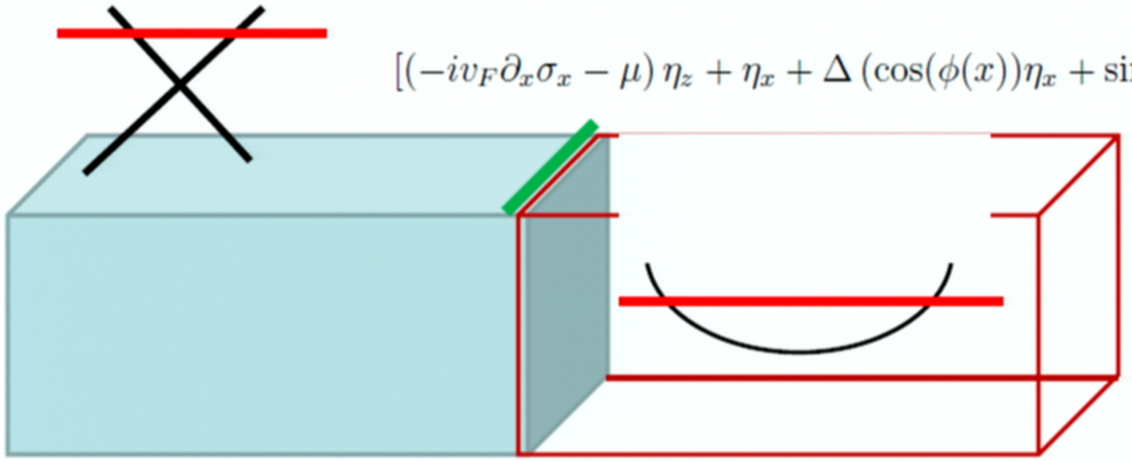

$$H_b = [v_F k_x \sigma_x \tau_x + m\,(|\,\mathbf{k}|)\,\tau_z - \mu]\,\eta_z + \Delta_R(x)\eta_x + \Delta_I(x)\eta_y$$

σ_x commutes with the Hamiltonian

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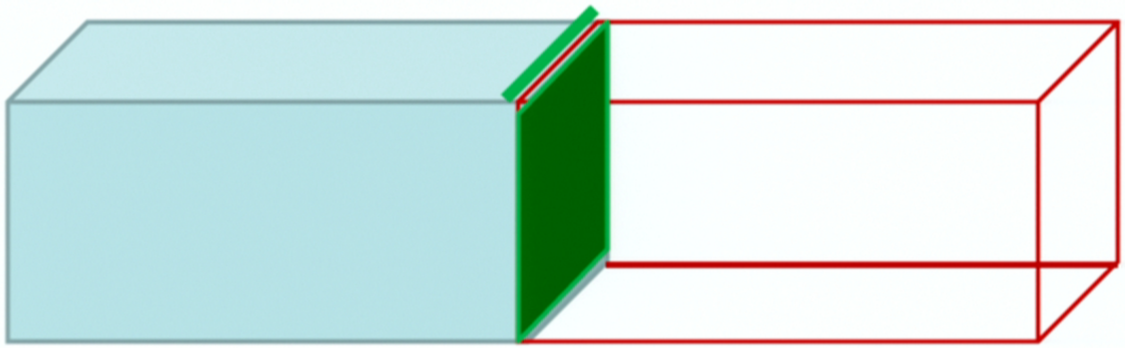
Josephson Junction

- Bulk conducting: Andreev modes on the boundary AND in the bulk



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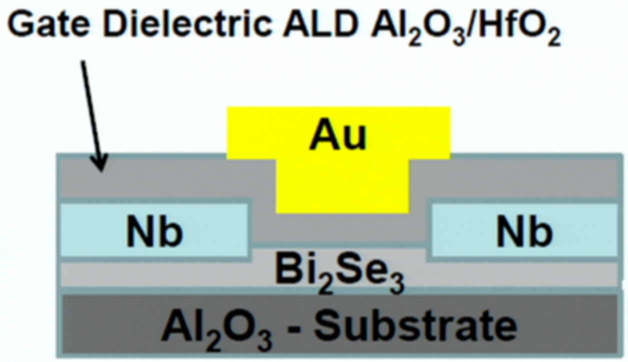
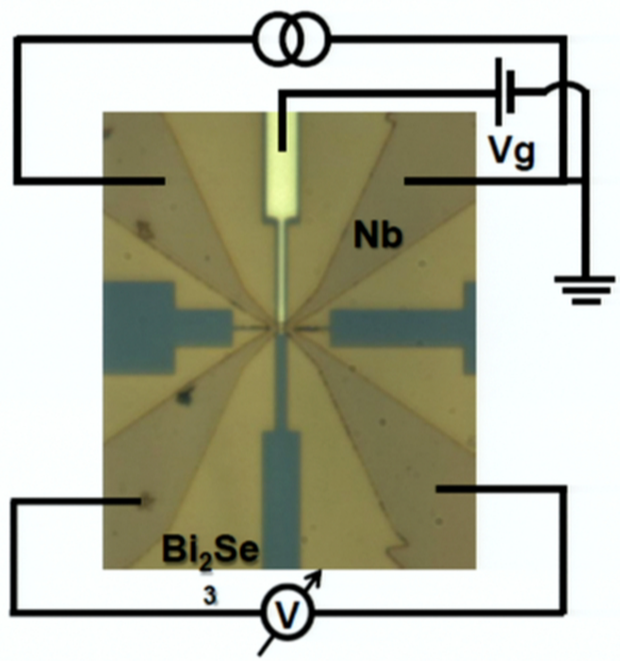
Nb/Bi_2Se_3/Nb Josephson Junctions

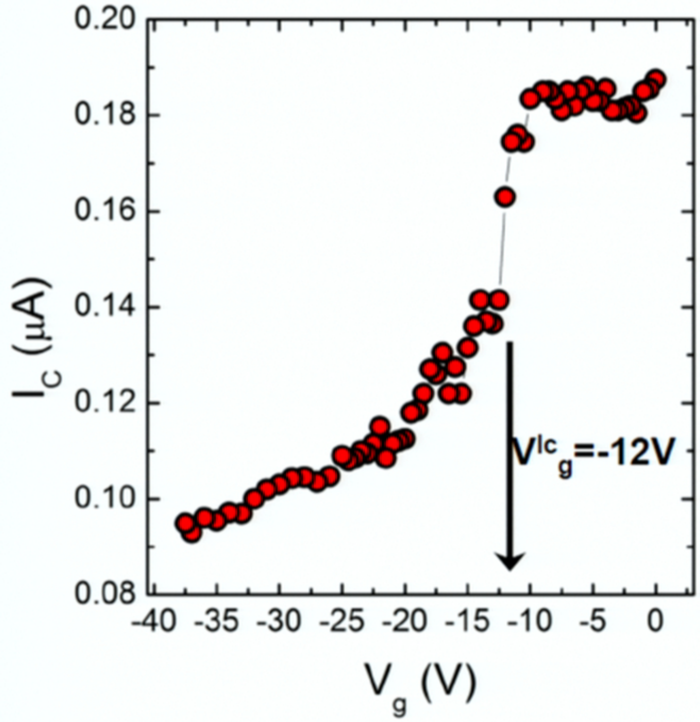


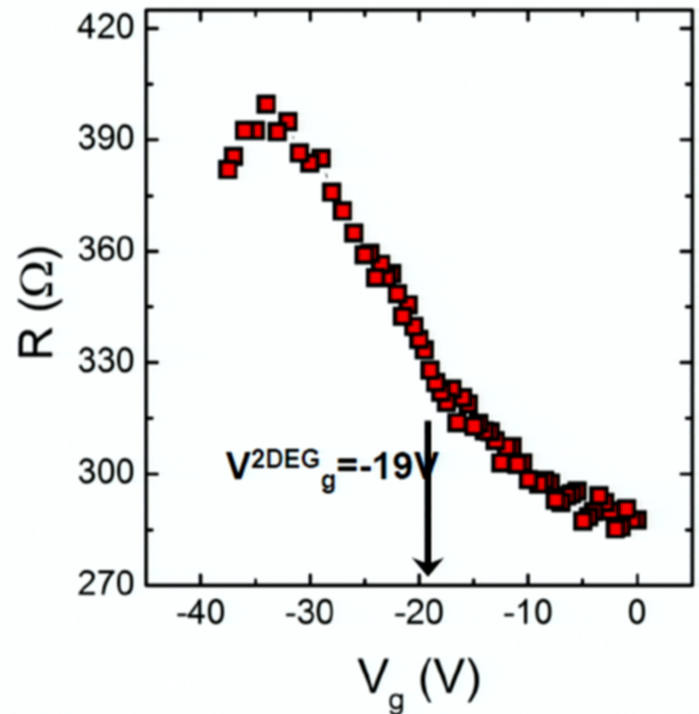


- E-beam lithography
- Ion milling
- Evaporation and sputtering

<u>Typical Dimensions</u>:
Superconductor - Nb
Length ~ 100-150nm
W = 0.3-1 μm
Top gate dielectric - (ALD Al_2O_3/HfO_2) ~ 35-40nm

Gate Voltage dependence



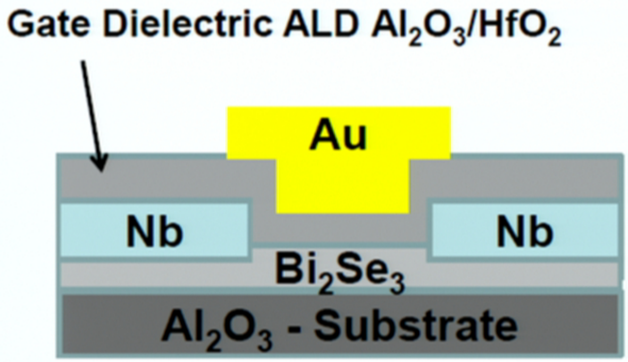


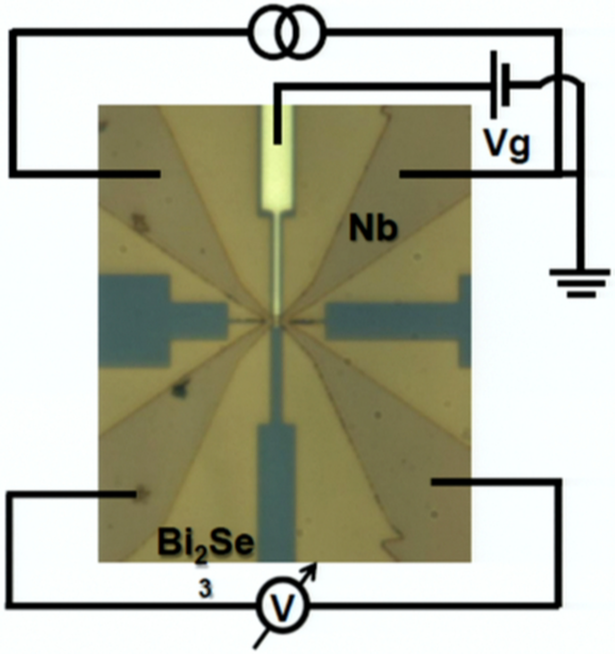
$|V^{Ic}_g| < |V^{2DEG}_g|$

"Jump" in Ic

depopulation of 2DEG

Nb/Bi_2Se_3/Nb Josephson Junctions





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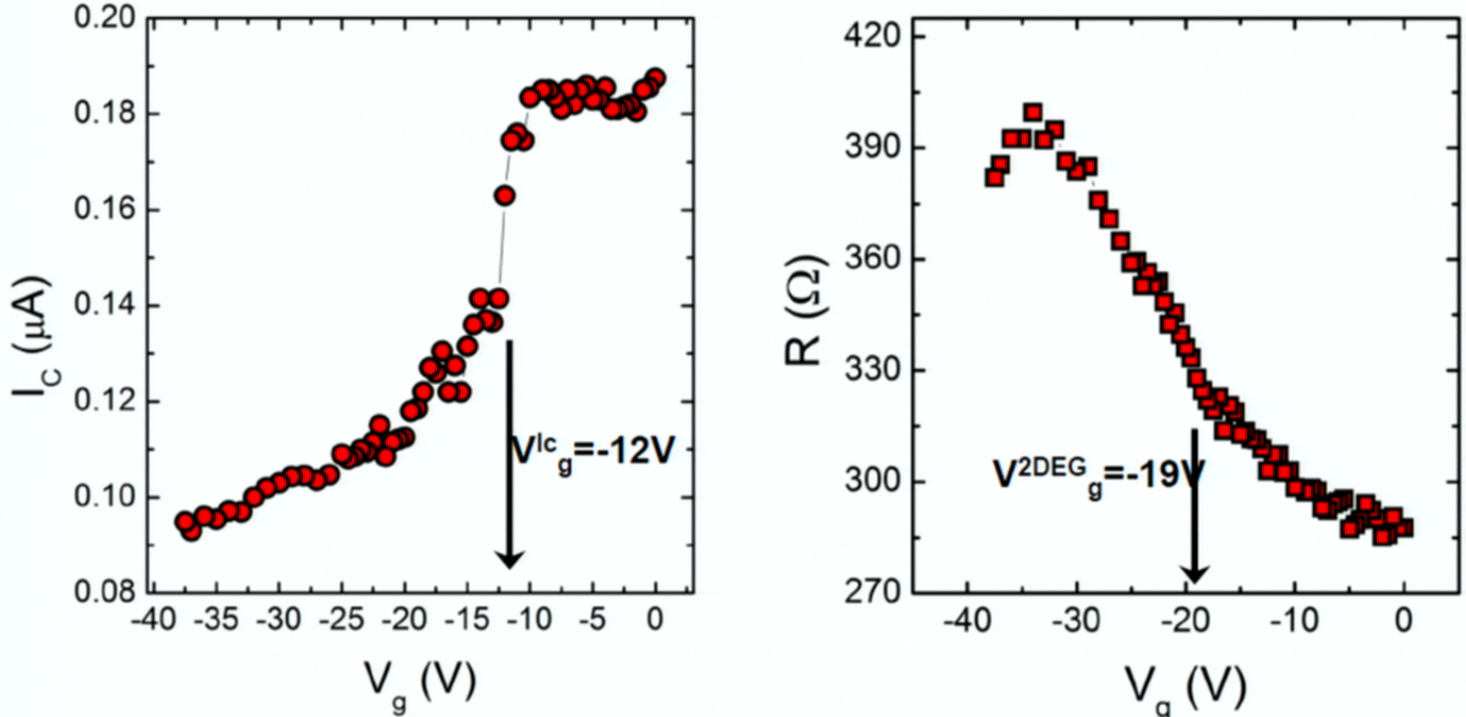

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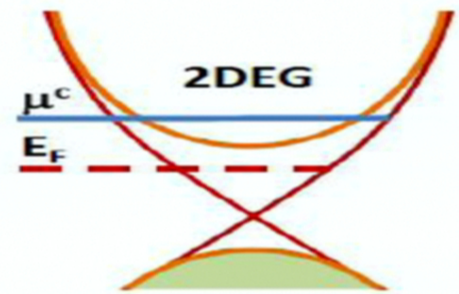
Model



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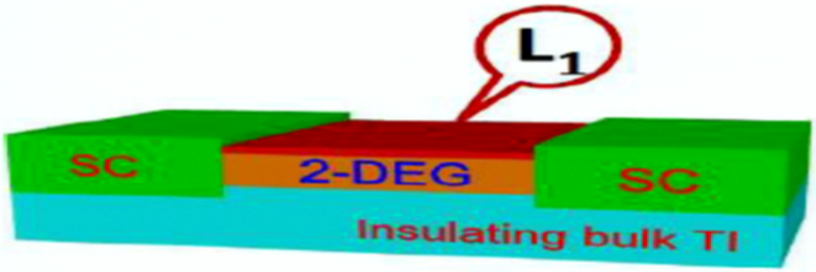
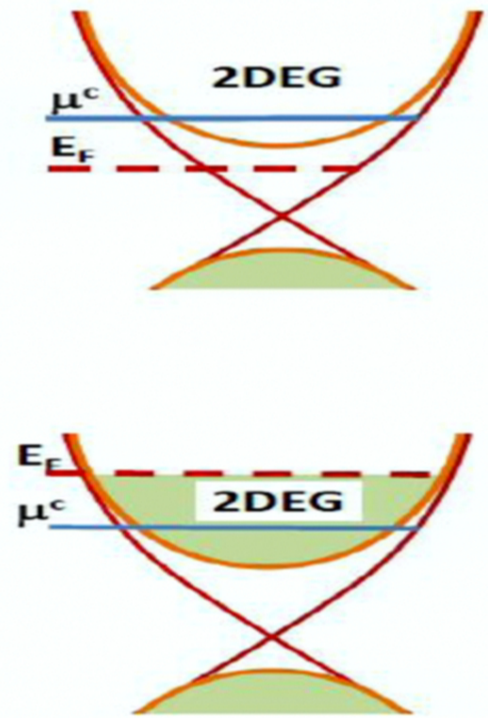
Model





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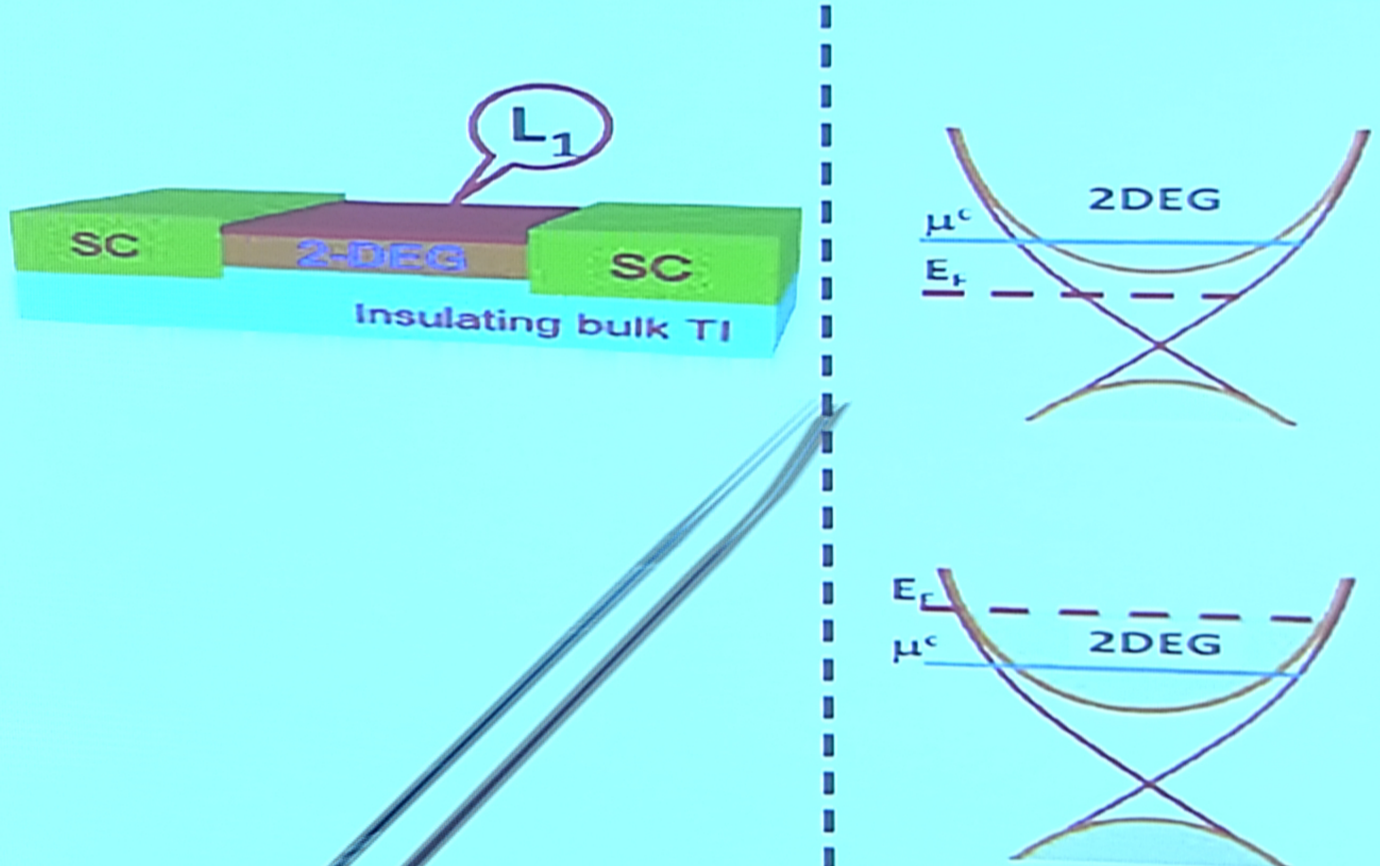
Model



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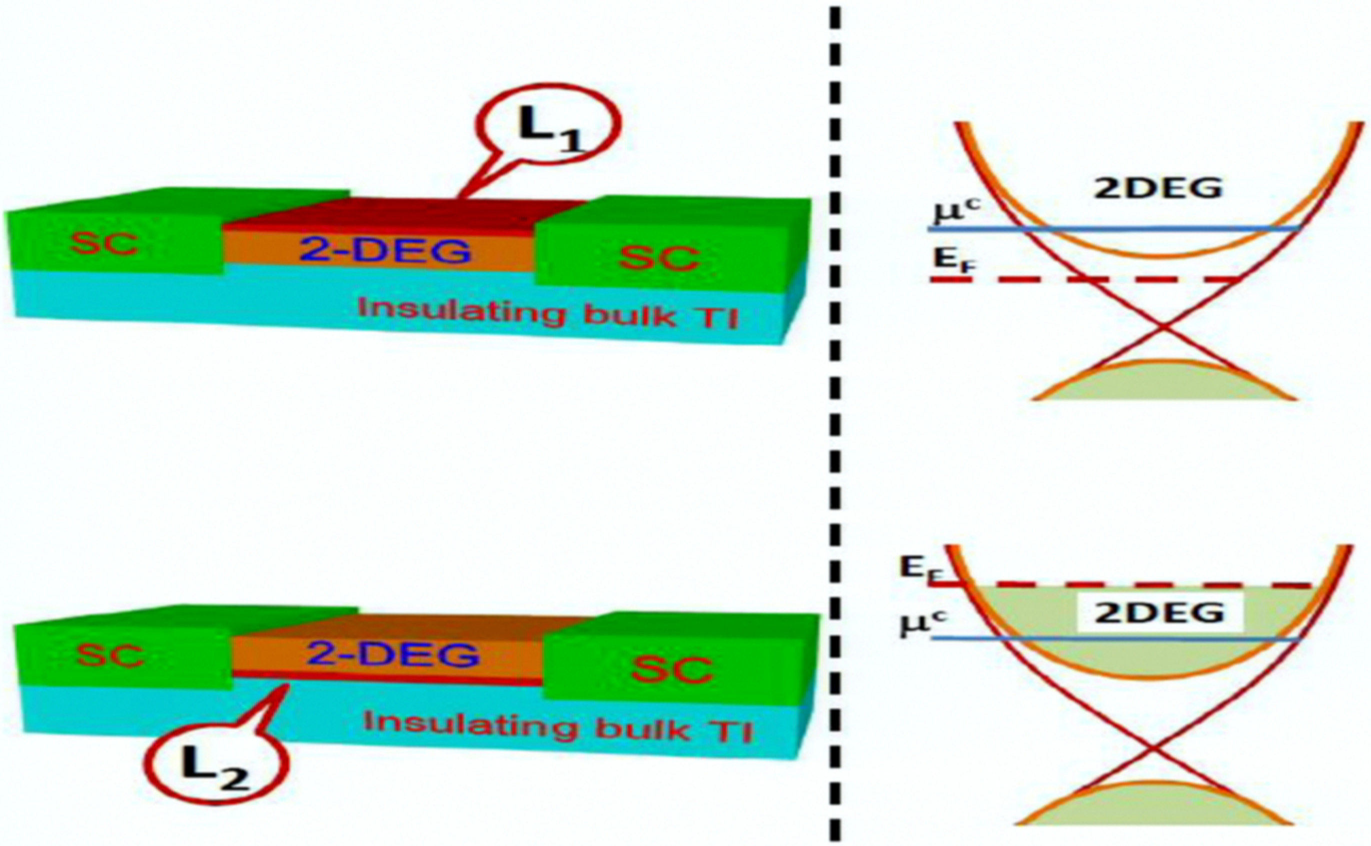
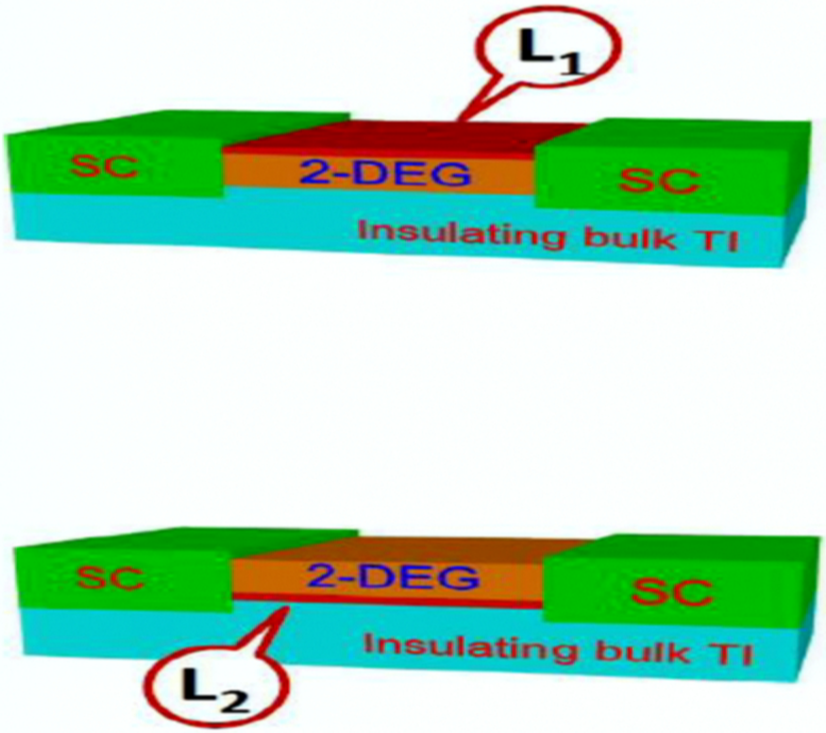
Model

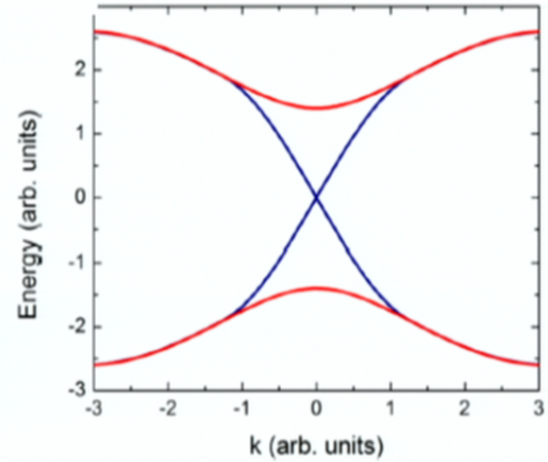


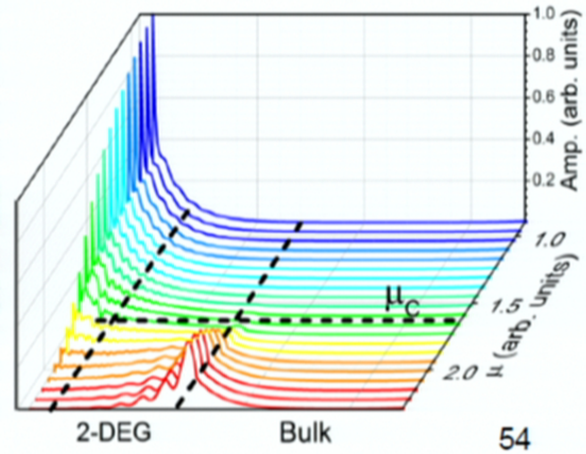
L1
SC
2-DEG
SC
Insulating bulk TI
2DEG
μc
EF
EF
2DEG
μc
SC
2-DEG
SC
Insulating bulk TI
L2

53

Model


L1
SC
2-DEG
SC
Insulating bulk TI
SC
2-DEG
SC
Insulating bulk TI
L2


Energy (arb. units)
k (arb. units)


Amp. (arb. units)
μ (arb. units)
μ_c
2-DEG
Bulk

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Conclusion

- Doped topological insulator retains signatures of topological parent compound
- Critical doping where the transition into normal metal happens, determined by properties of the doped TI fermi surface
- Simple s-wave paring in TIs have non-trivial features.

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References

- V. Orlyanchik, M. P. Stehno, C. D. Nugreho and P. Ghaemi, D. J. Van Harlingen, ***To appear soon***
- Phys. Rev. B **87**, 035401 (2013)
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