Title: Characterizing quantum non-locality: a histories approach

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Abstract: Characterising quantum non-locality using simple physical principles has become a hot topic in quantum foundations of late. In the simpler case of local hidden variable models, the space of allowed correlations can be characterised by requiring that there exists a joint probability distribution over all possible experimental outcomes, from which the experimental probabilities arise as marginals. This follows from Bellâ€™s causality condition. But the existing characterisations of quantum correlations are far from being so straightforward.
Motivated by a histories outlook, we propose the following condition: there exists a positive semi-definite matrix in which the indices run over all possible experimental outcomes, from which the experimental probabilities arise as â€œmarginalsâ€• in a similar way. This is a much simpler condition than the usual statement of the existence of a quantum model for the probabilities, and suggests an underlying connection with Bellâ€™s derivation of his bound on local correlations. I will outline existing proofs that this condition places strong bounds on correlations consistent with QM, and ask whether it could completely characterise quantum non-locality

Bounding quantum nonlocality by generalising Bell locality

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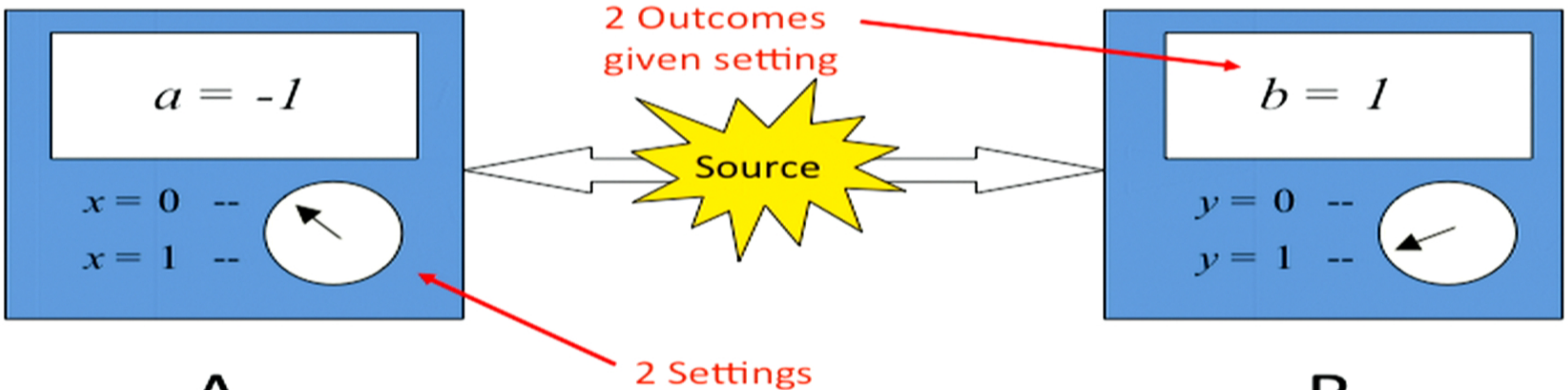
Bounding quantum correlations

- Some correlations allowed by QM violate Bell locality (= are weird)
- Not all correlations allowed by the "no-signalling condition" are allowed by QM
- Task: Look for simple/physically compelling principle(s) behind this limitation of QM
- Principles for QM have many uses: understanding, reformulating, testing, generalising...

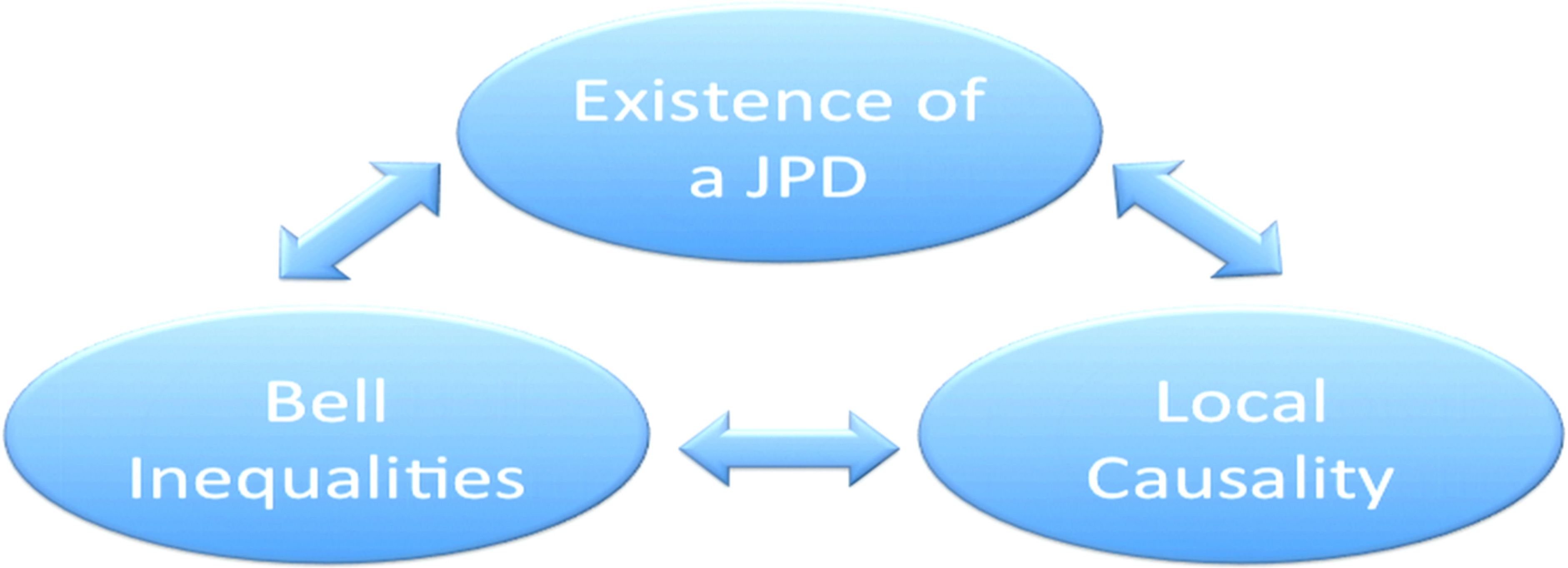
What correlations?



$$P_{xy}(a,b)$$

A **behaviour** for a Bell scenario specifies the probabilities for all global outcomes given the global setting.

Classical correlations



The 3 C's: classical case

Now consider a sample space of all the outcomes of all possible local experiments:

$$\Omega_J = \{a_0\, a_1\, b_0\, b_1\} = \{-1,1\} \times \{-1,1\} \times \{-1,1\} \times \{-1,1\}$$

Joint Probability Distribution (JPD) :

$$P_J(a_0\, a_1\, b_0\, b_1),$$

$$P_{xy}(a_x b_y) = P_J(a_x b_y) = \sum_{a_{\not x} b_{\not y}} P_J(a_0\, a_1\, b_0\, b_1)$$

Non-Contextuality

The 3 C's

Consider the Correlators, assuming the existence of a JPD:

$$E_{xy} = \sum_{ab} ab\, P_{xy}(ab) = \sum_{a_0\, a_1\, b_0\, b_1} a_x b_y\, P_J(a_0\, a_1\, b_0\, b_1)$$

Now,

$$|E_{00} + E_{01} + E_{10} - E_{11}| =$$
$$|\sum_{a_0\, a_1\, b_0\, b_1} P_J(a_0\, a_1\, b_0\, b_1)\,[(a_0 + a_1)b_0 + (a_0 - a_1)b_1]| \leq 2$$

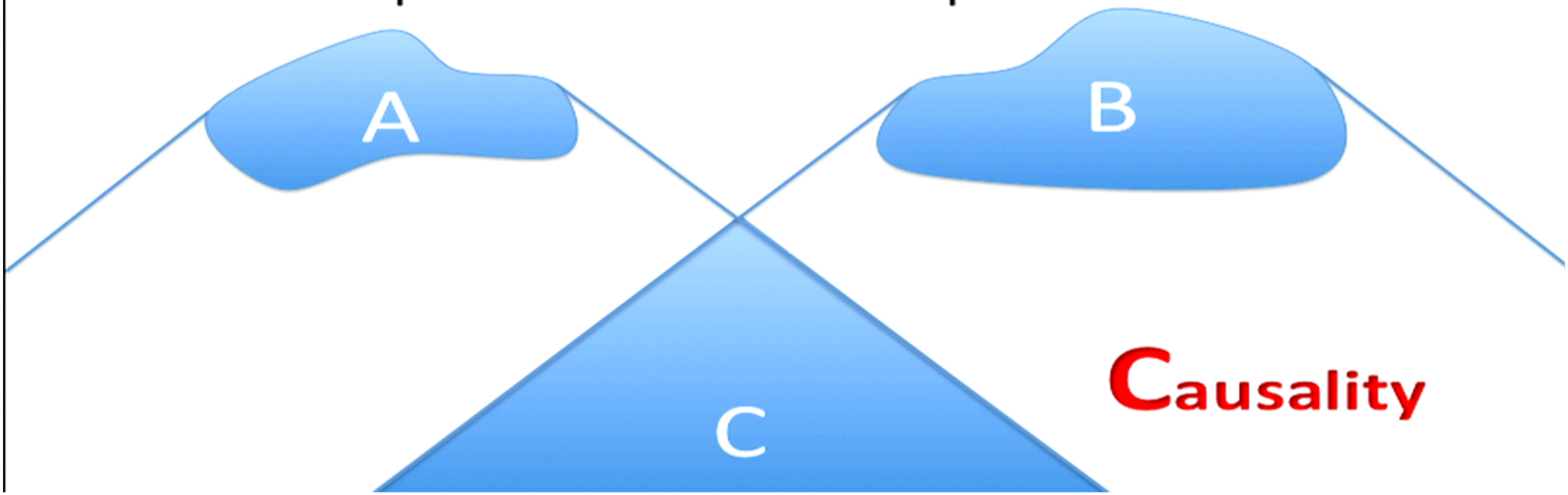
Correlations

The 3 C's

Motivation for assuming a JPD?

$$P(A|C)P(B|C) = P(A \cap B|C)$$

for all full specifications of the past



Causality

The 3 C's

Theorem: all these are equivalent:

Quantum Mechanics

Ordinary Quantum Models

An "ordinary quantum model" (OQM) for a behaviour gives a state and (spacelike commuting) Von Neumann measurements on some Hilbert space that reproduce the probabilities:

$$P_{xy}(ab) = tr(P_a P_b \, \rho)$$

This can't do everything:

$$E_{xy} = \sum_{ab} ab \, P_{xy}(ab) \qquad E_x^A = \sum_{ab} a \, P_{xy}(ab)$$

The PR box:

$$E_{xy} = (-1)^{xy}, \quad E_x^A = E_y^B = 0$$

$$E_{00} + E_{01} + E_{10} - E_{11} = 4$$

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Quantum behaviours

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What does QM imply about Bell experiments?

$$|E_{00} + E_{01} + E_{10} - E_{11}| \leq 2\sqrt{2}$$

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$$|\arcsin(E_{00}) + \arcsin(E_{01}) + \arcsin(E_{10}) - \arcsin(E_{11})| \leq \pi$$

Q-1: $|\arcsin(\tilde{E}_{00}) + \arcsin(\tilde{E}_{01}) + \arcsin(\tilde{E}_{10}) - \arcsin(\tilde{E}_{11})| \leq \pi$

$$\tilde{E}_{xy} = \frac{E_{xy} - E_x^A E_y^B}{\sqrt{(1 - E_x^{A\,2})(1 - E_y^{B\,2})}}$$

The NPA hierarchy

An "ordinary quantum model" (OQM) for a behaviour gives a state and (spacelike commuting) Von Neumann measurements that reproduce the probabilities.

Consider $$\Gamma_{ij} = \mathrm{Tr}(P_i \rho P_j)$$

where *i,j* range over the set of all projectors from the OQM.

$$\Gamma_{a_x b_y} = P_{xy}(ab)$$
$$\Gamma_{a_x \bar{a}_x} = P_x(a)\delta_{a\bar{a}}$$
$$\Gamma \geq 0$$

Quantum mechanics $\Longrightarrow$ the existence of a matrix with these properties $\Leftrightarrow$ Q-1.

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The NPA hierarchy

An "ordinary quantum model" (OQM) for a behaviour gives a state and (spacelike commuting) Von Neumann measurements that reproduce the probabilities.

Now consider $$\Gamma^n_{ij} = \mathrm{Tr}(S^n_i \, \rho \, S^{n\dagger}_j)$$

where *i,j* range over the set of all **length *n* sequences** of projectors from the OQM. These expressions also form a matrix with special properties.

The existence of such a matrix ⇔ Q-n. Satisfaction of all Q-n ⇔ the existence of an OQM.

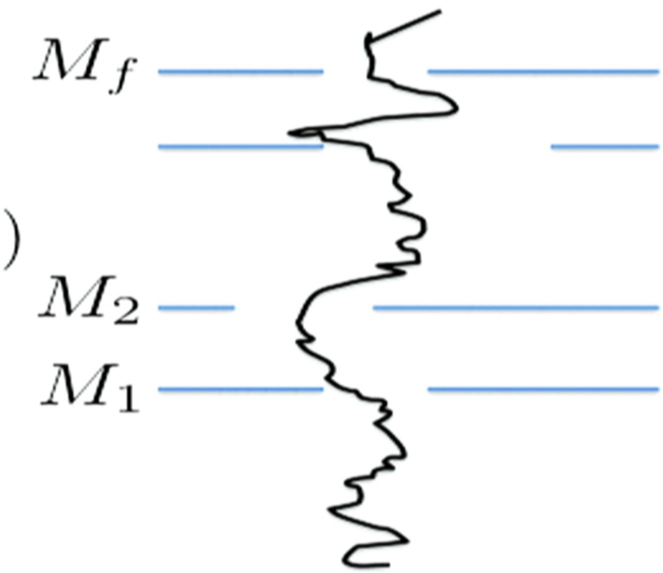
Bounding quantum correlations

- Task: Look for simple/physically compelling principle(s) behind this limitation of QM
- Principles for QM have many uses: understanding, reformulating, testing, generalising...
- Use a generalisation of the 3 C's ???

Probabilities of sequences of Events

Probability for a sequence of measurement outcomes :

$$\mu(M_1, \ldots, M_f) = \mathrm{Tr}(P_f \ldots P_1 \, \rho \, P_1 \ldots P_f)$$



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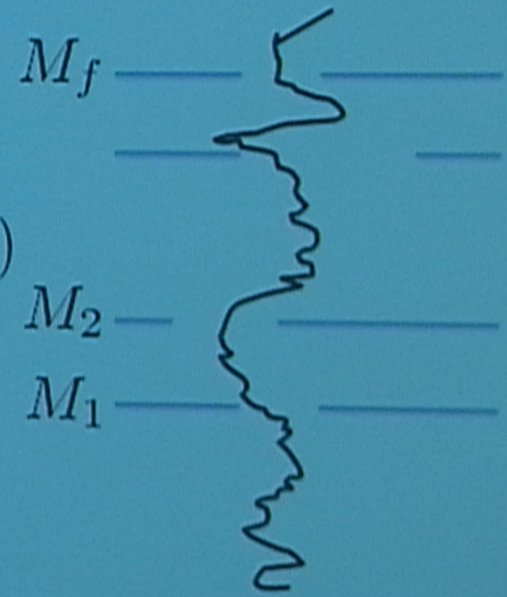

$$\mu(A) = \int_{\gamma \in A} \int_{\bar{\gamma} \in A} d\nu(\gamma) d\nu(\bar{\gamma}) \; \rho(\gamma(0), \bar{\gamma}(0)) \; e^{-iS(\gamma)} e^{iS(\bar{\gamma})} \; \delta(\gamma(f), \bar{\gamma}(f))$$

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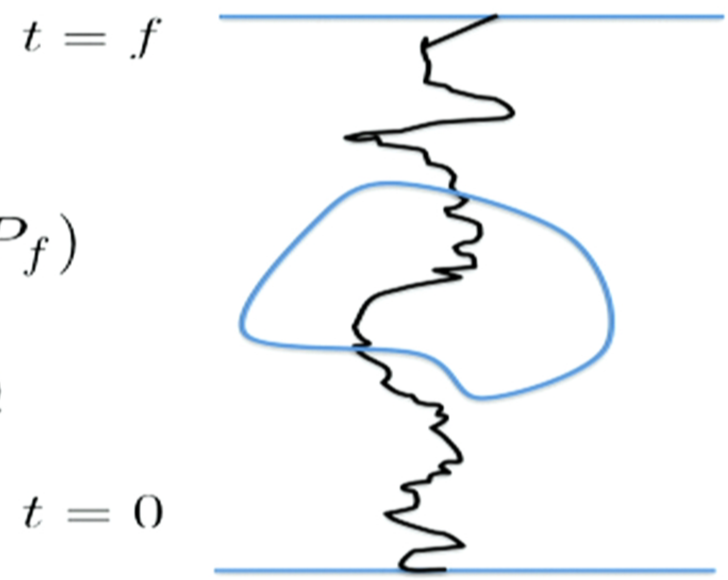

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What is essential?

Consider the probability for a an outcome of a sequence of measurements, e.g. double slit:

A = "Particle went through slit 1 and ended up at point x".
B = "Particle went through slit 2 and ended up at point x".
A or B = "Particle ended up at point x."

$$\mu(A) + \mu(B) \neq \mu(A \cup B)$$

However,

$$\mu(A) + \mu(B) + \mu(C) - \mu(A \cup B) - \mu(A \cup C) - \mu(B \cup C) + \mu(A \cup B \cup C) = 0$$

$$\mu(\Omega) = 1$$
$$\mu(A) \geq 0 \quad \forall A \subset \Omega$$

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Pairwise Interference

In quantum theory, defining

$$D(A,B) = \mathrm{Tr}(P_f^{t=T} \ldots P_A^{t=t^*} \ldots P_1^{t=0}\, \rho\, P_1^{t=0} \ldots P_B^{t=t^*} \ldots P_f^{t=T})$$

$$\begin{aligned} \mu(A) &= D(A,A) \\ \mu(B) &= D(B,B) \\ \mu(A \cup B) &= D(A,A) + D(B,B) + D(A,B) + D(B,A) \end{aligned}$$

The Decoherence Functional

Equivalent formulation: $\mu(A) = D(A;\ A)$

Where: $D(A;B) = D^*(B;A)$

$$D(\Omega;\Omega) = 1$$

$$D(A;A) \geq 0 \quad \forall\, A \subset \Omega$$

$$D(A \sqcup B;C) = D(A;C) + D(B;C)$$

But this is not closed under composition. We need:

$$D(\gamma, \bar{\gamma}) = D \geq 0$$

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Back to Bell

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Joint Probability Distribution (JPD) :

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Joint Quantum measure (JQM) :

$$P_{xy}(a_x b_y) \qquad\qquad = \mu_J(a_x b_y)$$

Non-Contextuality

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Joint Quantum measure (JQM) :

$$P_{xy}(a_x b_y)\delta(a_x, \bar{a}_x)\delta(b_y, \bar{b}_y) = \mu_J(a_x b_y)\delta(a_x, \bar{a}_x)\delta(b_y, \bar{b}_y)$$

$$= \sum_{a_{\not x}, \bar{a}_{\not x}, b_{\not y}, \bar{b}_{\not y}} D_J(a_0\, a_1\, b_0\, b_1; \bar{a_0}\, \bar{a_1}\, \bar{b_0}\, \bar{b_1}),$$

Non-Contextuality

Existence of a JQM => Q1

$$P_{xy}(a_x b_y)\delta(a_x, \bar{a}_x)\delta(b_y, \bar{b}_y)$$
$$= \sum_{a_{\not x}, \bar{a}_{\not x}, b_{\not y}, \bar{b}_{\not y}} D_J(a_0\, a_1\, b_0\, b_1; \bar{a_0}\, \bar{a_1}\, \bar{b_0}\, \bar{b_1}),$$

$$\Gamma_{a_x \bar{a}_{\bar{x}}} := \sum_{\text{everything but } a_x \bar{a}_{\bar{x}}} D_J(a_0\, a_1\, b_0\, b_1; \bar{a_0}\, \bar{a_1}\, \bar{b_0}\, \bar{b_1}),$$

$$\Gamma_{b_y \bar{b}_{\bar{y}}} := \sum_{\text{everything but } b_y \bar{b}_{\bar{y}}} D_J(a_0\, a_1\, b_0\, b_1; \bar{a_0}\, \bar{a_1}\, \bar{b_0}\, \bar{b_1}),$$

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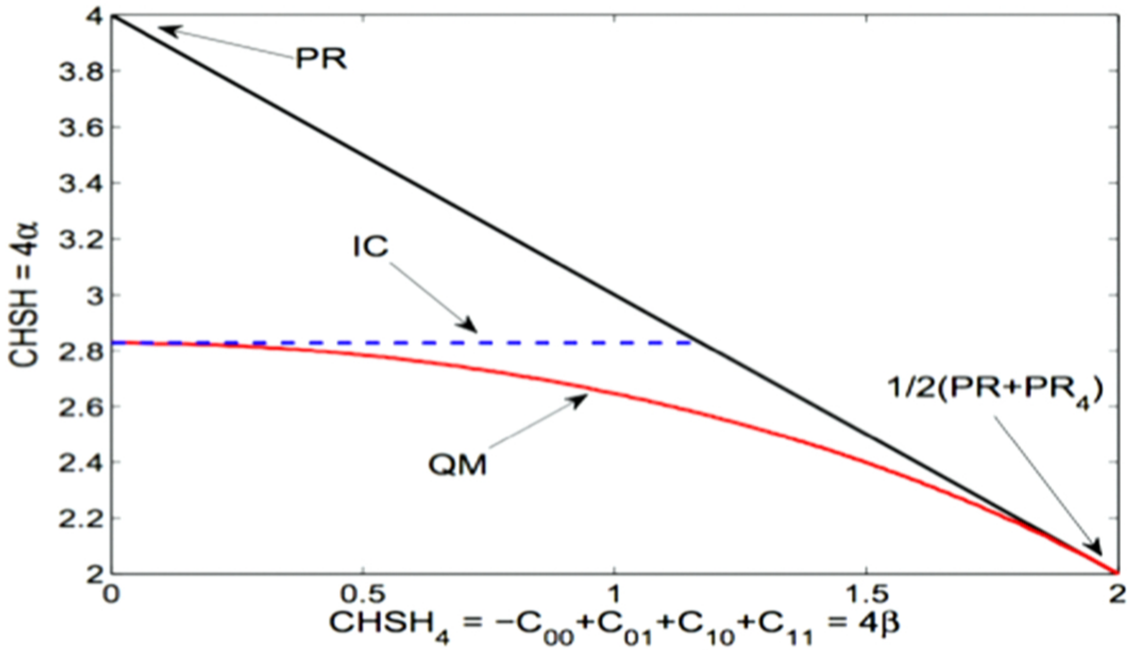
Q1 vs IC



Recovering part of the quantum boundary from information causality

Jonathan Allcock, Nicolas Brunner, Marcin Pawlowski, Valerio Scarani

Journal reference: Phys. Rev. A 80, 040103(R) (2009)
DOI: 10.1103/PhysRevA.80.040103
Cite as: arXiv:0906.3464 [quant-ph]

Q4 => Existence of a JQM

Given an Ordinary Quantum Model, can define joint QM:

$$D_J(a_0\, a_1\, b_0\, b_1; \bar{a_0}\, \bar{a_1}\, \bar{b_0}\, \bar{b_1})$$
$$= Tr(\underbrace{P_{a_0} P_{a_1} P_{b_0} P_{b_1}}_{\text{4 operators}}\, \rho\, P_{\bar{b}_1} P_{\bar{b}_0} P_{\bar{a}_1} P_{\bar{a}_0})$$

I.e. requiring the existence of a JQM is requiring that there exists a matrix with **some** of the properties of the matrix defined by all sequences of 4 projectors from the OQM.

Q4 requires that there exists a matrix with **all** of those properties.

A JQM does not imply an OQM?

$$E_{xy} = \sum_{ab} ab\, P_{xy}(ab) \qquad E_x^A = \sum_{ab} a\, P_{xy}(ab)$$

Computation evidence: semidefinite programing algorithm spits out a JQM such that

$$E_x^A = 0.2$$

$$|E_{00} + E_{01} + E_{10} - E_{11}| \approx 2\sqrt{2}$$

QM does not allow this.

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Correlations

The 3 C's

Motivation for assuming a JPD?

$$P(A|C)P(B|C) = P(A \cap B|C)$$

for all full specifications of the past.

Let's try: $\mu(A \cap C)\mu(B \cap C) = \mu(A \cap B \cap C)\mu(C)$

JQM implies this, **but is not implied by it**.


CAUTION

Loose ends

- More results on correlations and JQM?
- What are the relations between various principles for restricting correlations?
- Can the gap between JQMs and ordinary QM be tested?
- Other interesting physical/informational consequences of the generalisation?
- Is there a “limited causality” condition equivalent to the existence of a JQM?

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