Title: Conjugates, Correlation and the Jordan Structure of Finite-Dimensional Quantum Theory

Date: May 30, 2013 02:00 PM

URL: http://pirsa.org/13050070

Abstract: The last few years have seen a spate of operational or information-theoretic
derivations (or \reconstructions") of nite-dimensional quantum mechanics [1]. These rest of some strong assumptions. In particular, most assume the state of a composite system is entirely determined by the joint probabilities it assigns to the outcomes of measurements on the component systems. This condition, often called local tomography, is satis ed by neither real nor quaternionic quantum theory. If we are interested in the possibilities for theories more general than orthodox quantum mechanics, we might wish to relax this constraint.
In this talk I will discuss a weaker system of assumptions, involving correla-
tions between a probabilistic system and an isomorphic conjugate system, that leads to a representation of such systems in terms of euclidean Jordan algebras [2]. These have a well-known classi cation as direct sums of real, complex or quaternionic quantum systems, possibly the exceptional Jordan algebra, and spin factors. The last are a form of \bit", characterized by a family of two-valued observables, parametrized by antipodal vectors in a sphere of arbitrary dimension. Orthodox quantum mechanics can be singled out by imposing local tomography, plus the existence of a qubit as additional axioms [3]. However, there is a natural way to form non-signaling, but generally non-locally tomo-graphic, composites of systems based on special euclidean Jordan algebras (that is, excluding the exceptional one). This yields a probabilistic theory strictly, but not wildly, more general than orthodox nite-dimensional QM; one that elegantly uni es real, complex and quaternionic quantum theory, has a simple operational basis, and allows for a spectrum of bits more general than permitted in orthodox quantum theory. Parts of this talk reflect ongoing joint work with Howard Barnum, Matthew Graydon and Cozmin Ududec.
References:
[1] G. Chiribella, M. D'Ariano and P. Perinotti, Phys. Rev. A 84, (2011), 012311-
012350; B. Dakic and C. Brukner, in H. Halvorson (ed.), Deep Beauty, Cambridge,
2011; P. Goyal, Phys. Rev. A 78 (2008), 052120-052146; L. Hardy, arXiv:quant-
ph/0101012 (2001); L. Masanes and M. Mueller, New J. Phys. 13 (2011) ; J. Rau,
Annals of Physics 324 (2009) 2622{2637;
[2] A. Wilce, Conjugates, correlation and quantum mechanics, arXiv:1206.2897 (2012)
[3] H. Barnum and A. Wilce, Local tomography and the Jordan structure of quantum
theory, arXiv:1202.4513, (2012)

Conjugates, Correlation & s.d. the Jordan Structure of QM

(Parts joint w/ Howard Barnum & Matthew Graydon)

(correct)

HB+AW → arXiv: 1202.4513

AW → arXiv: 1206.2897

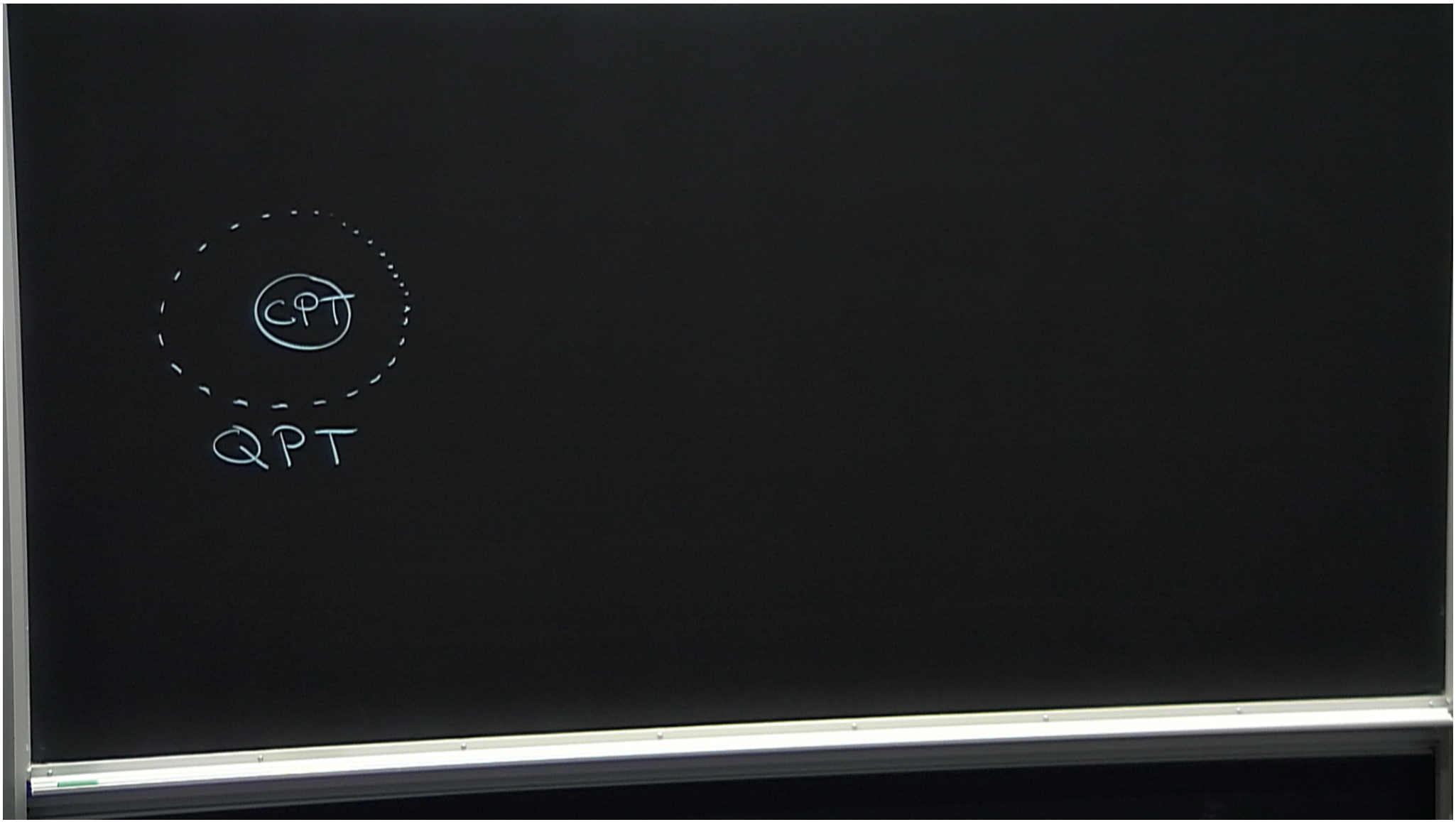
I. Intro (Pt of View)

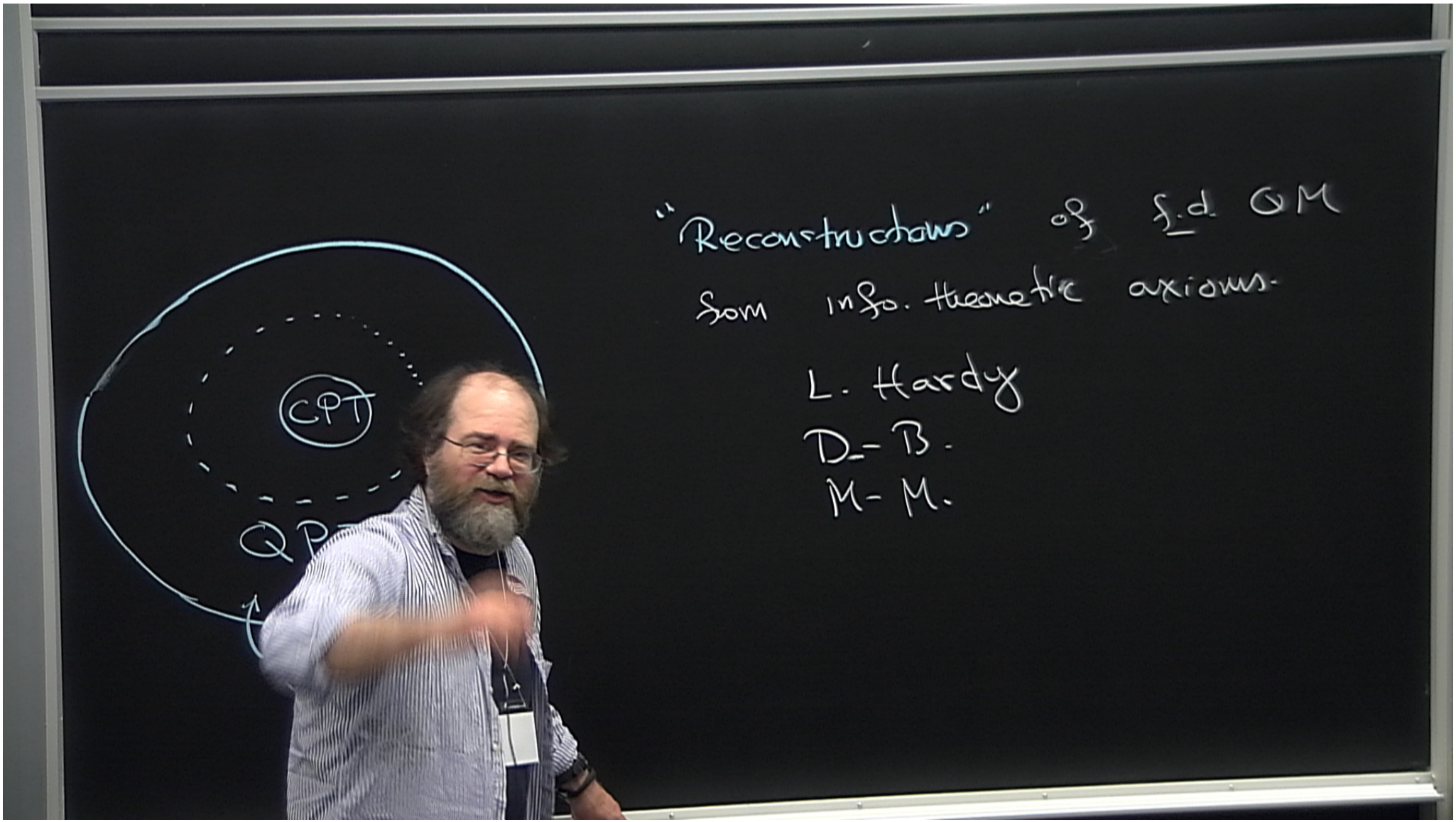
II. Probability Theory

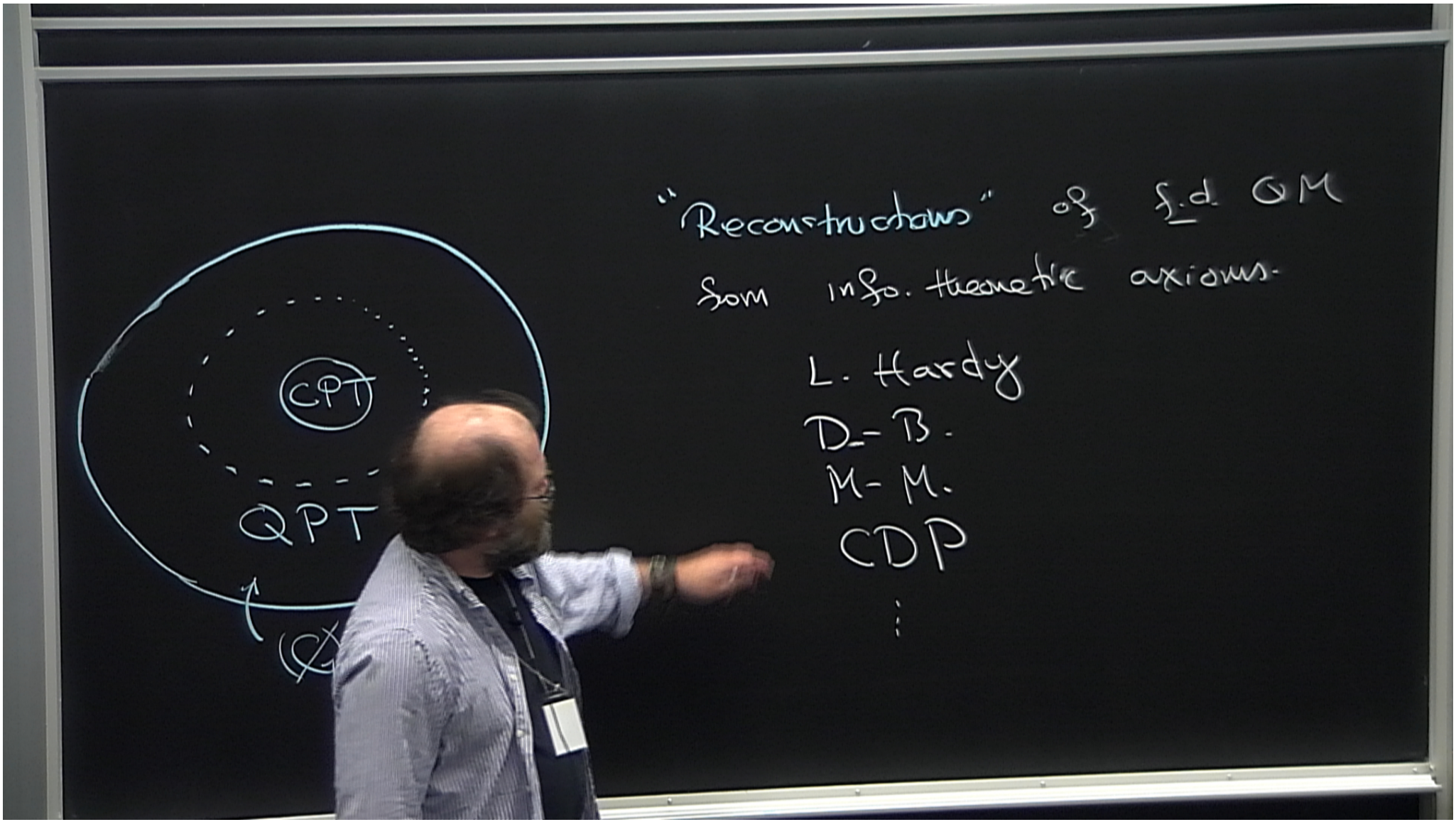
III. Axiomatics for Jordan Models

IV. Composites of " "


CPT


CPT
QPT


"Reconstructions" of f.d. QM
from info. theoretic axioms.
L. Hardy
D.-B.
M-M.
CPT
QP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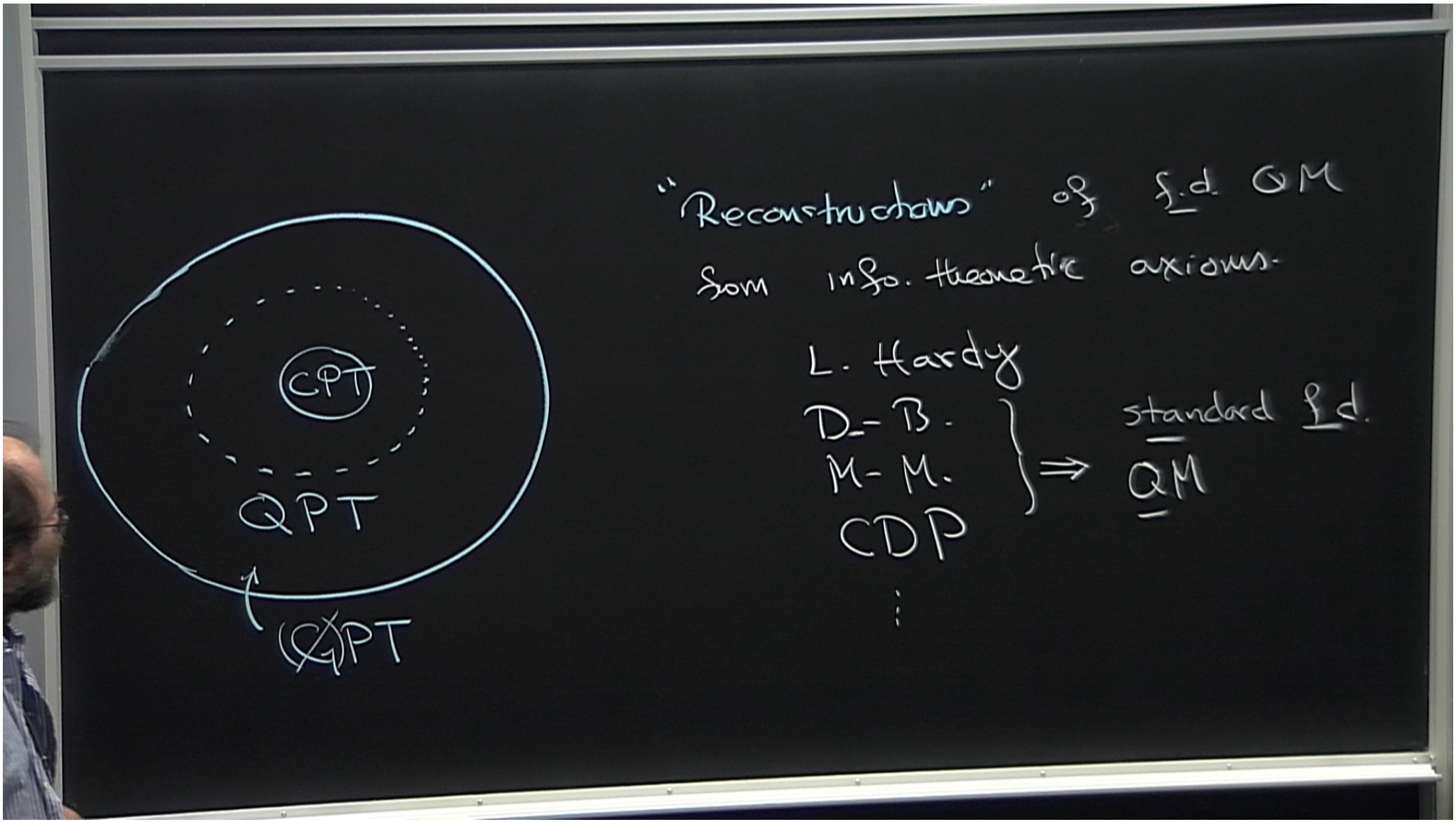
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<u>But</u>:

$\mathbb{R}$-QM, $\mathbb{H}$-QM are plausible theories.

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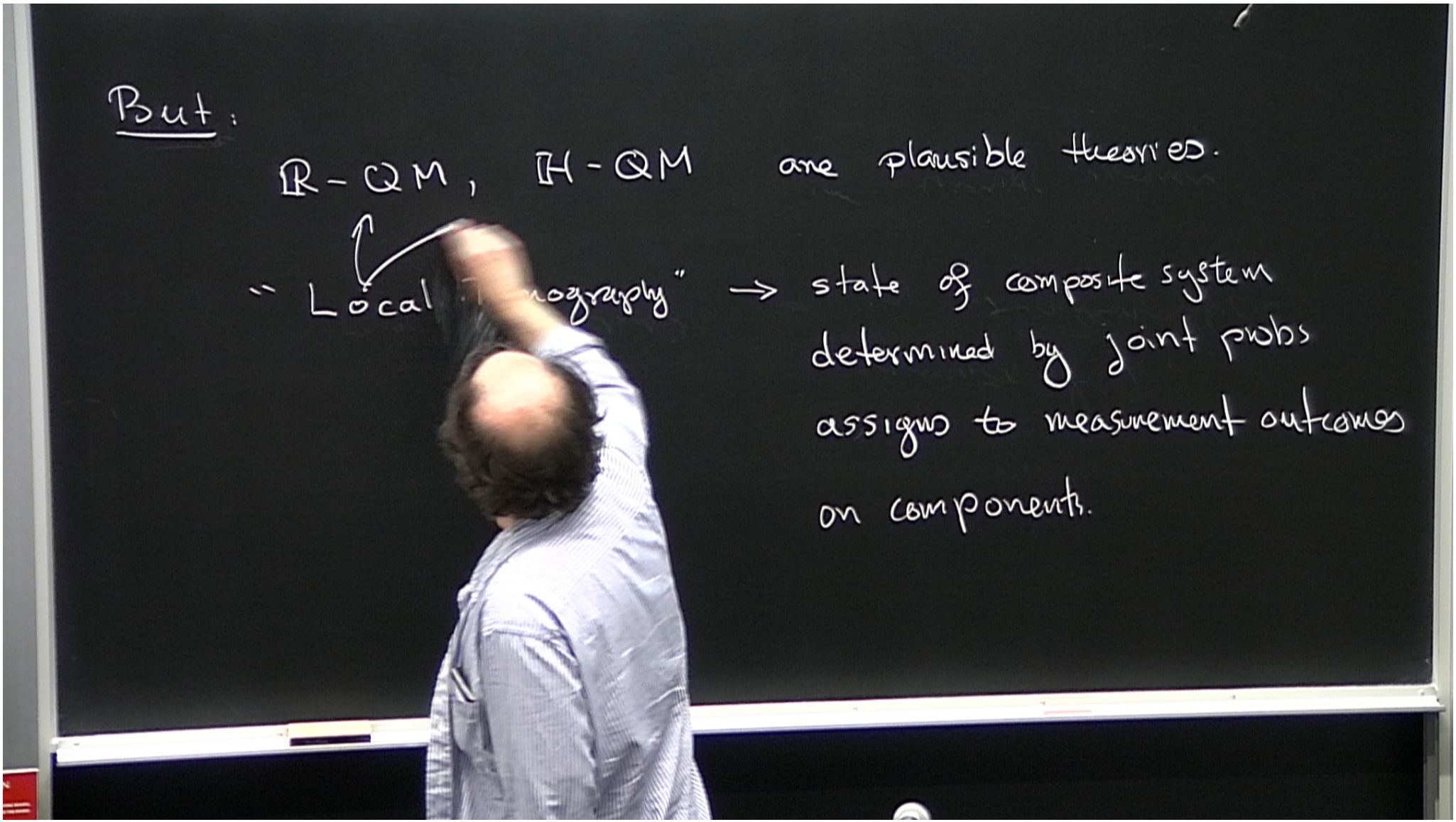
"Local Tomography" $\rightarrow$ state of composite system determined by joint probs assigns to measurement outcomes on components.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"Local Tomography" $\rightarrow$ state of composite system determined by joint probs assigns to measurement outcomes on components.

$\mathbb{R}/\mathbb{H}$ QM.

$\mathbb{R}$-Hilbert space $\mathcal{H}_{\mathbb{R}} \longleftrightarrow (\mathcal{H}_{\mathbb{C}}, J)$, $J^2 = -\mathbb{1}$

anti-unitary

$\mathbb{R}/\mathbb{H}$ QM.

$\mathbb{R}$ - Hilbert space $\mathcal{H}_{\mathbb{R}} \longleftrightarrow (\mathcal{H}_{\mathbb{C}}, J)$, $J^2 = +\mathbb{1}$

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$\mathbb{H}$ - Hilbert space $\mathcal{H}_{\mathbb{H}}$

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$\{ i, J, K = iJ \}$

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$(\mathcal{H}_1, J_1)$ $(\mathcal{H}_2, J_2)$

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$$(\mathcal{H}_1, J_1) \otimes (\mathcal{H}_2, J_2)$$

$$= (\mathcal{H}_1 \otimes \mathcal{H}_2, J_1 \otimes J_2)$$

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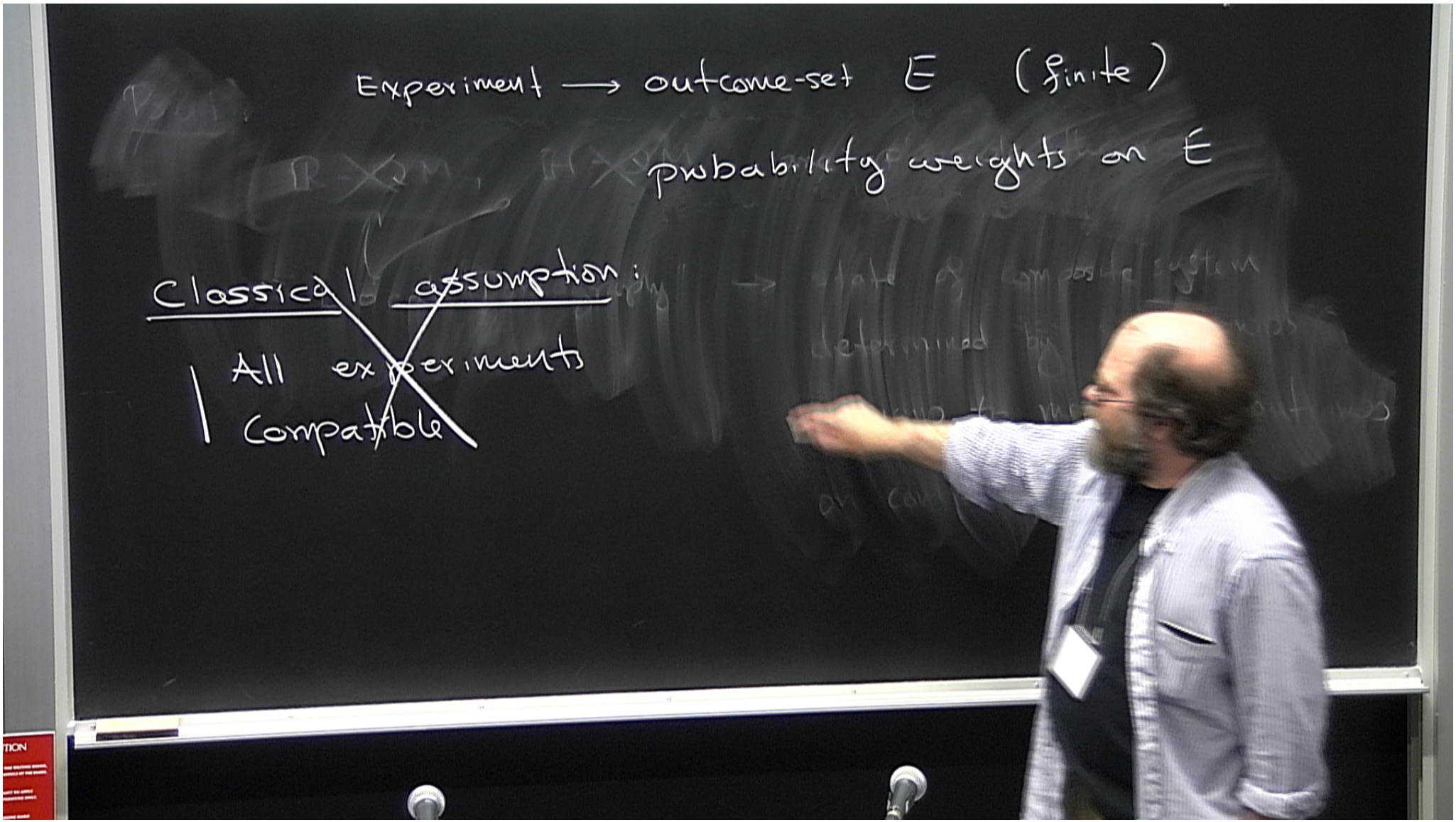
Experiment $\longrightarrow$ outcome-set E (finite)

probability weights on E

<u>Classical</u> <u>assumption</u>:

All experiments
Compatible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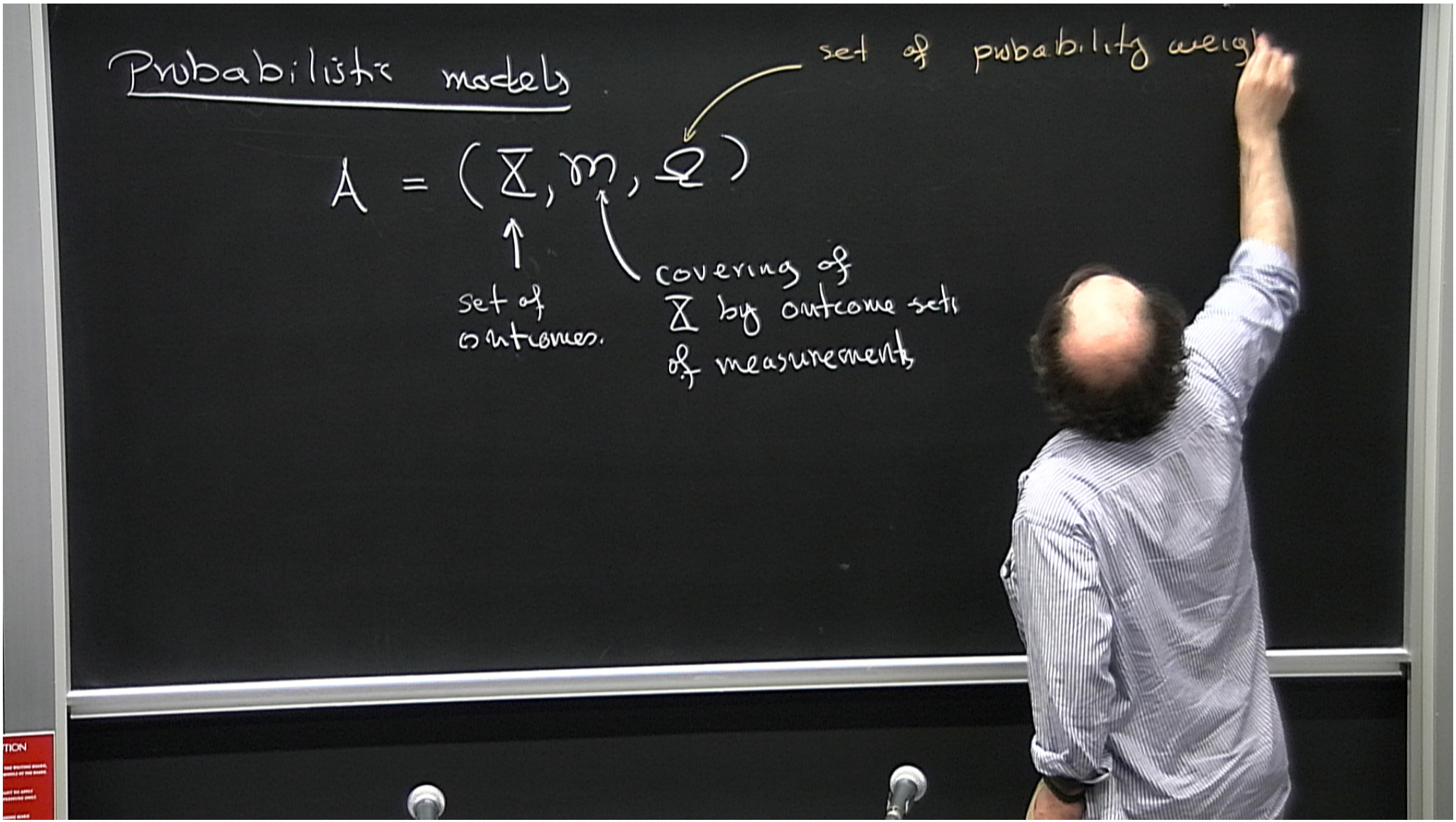
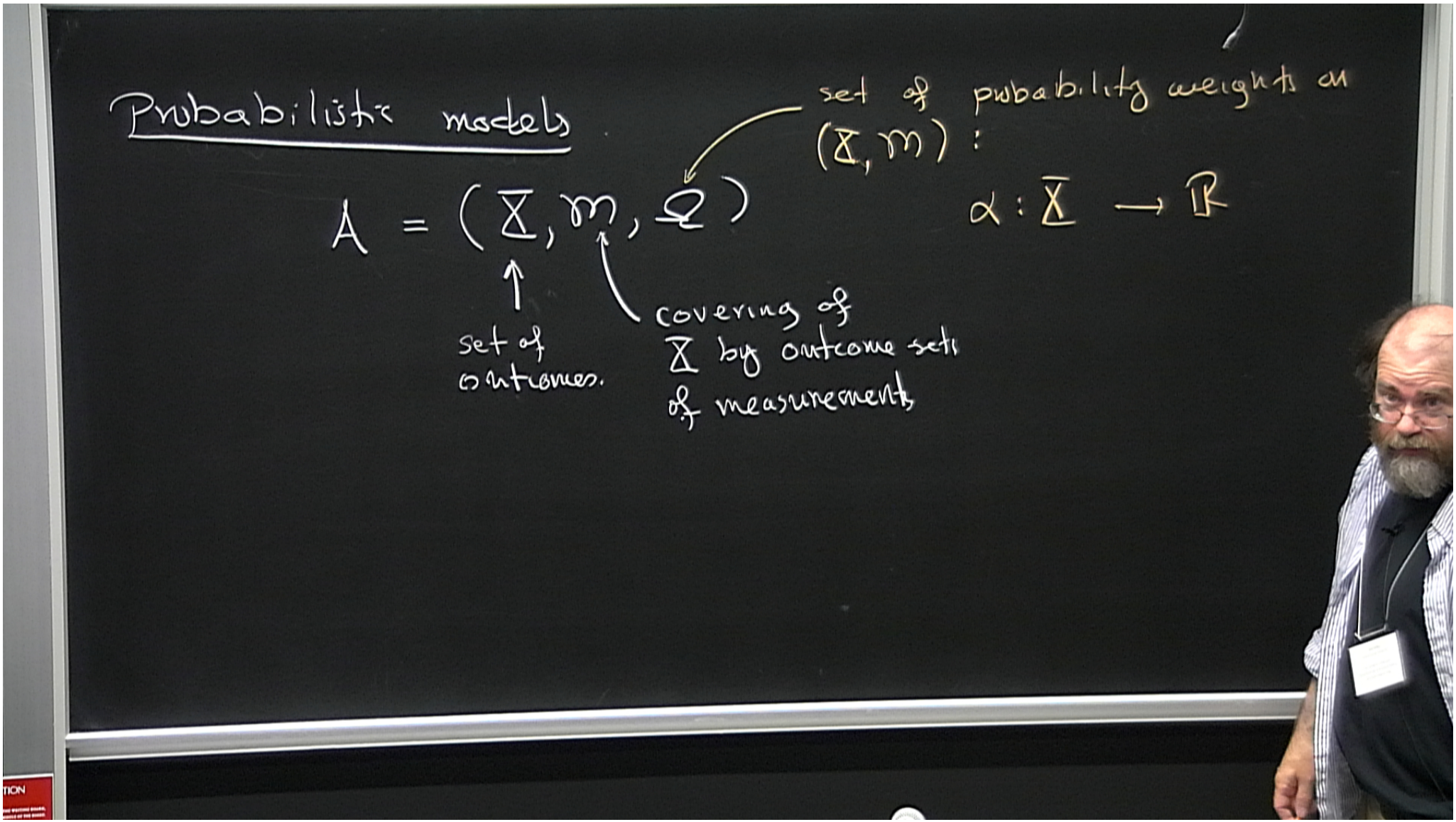
Classical assumption:
All experiments
Compatible

Probabilistic models

$$A = (\underline{X}, \mathcal{M}, \underline{Q})$$

set of outcomes.


Probabilistic models
set of probability weig
A = (X, M, Ω)
set of outcomes.
covering of X by outcome sets of measurements


Probabilistic models
set of probability weights on (X, m) :
A = (X, m, Q)
α : X → R
set of outcomes.
covering of X by outcome sets of measurements

Probabilistic models

$$A = (\mathcal{X}, \mathcal{M}, \Omega)$$

set of probability weights on $(\mathcal{X}, \mathcal{M})$:

$\alpha : \mathcal{X} \to \mathbb{R}$

$\alpha(x) \geq 0 \quad \forall x \in \mathcal{X}$

$\sum_{x \in E} \alpha(x) = 1$

$\forall E \in \mathcal{M}$

set of outcomes.

covering of $\mathcal{X}$ by outcome sets of measurements

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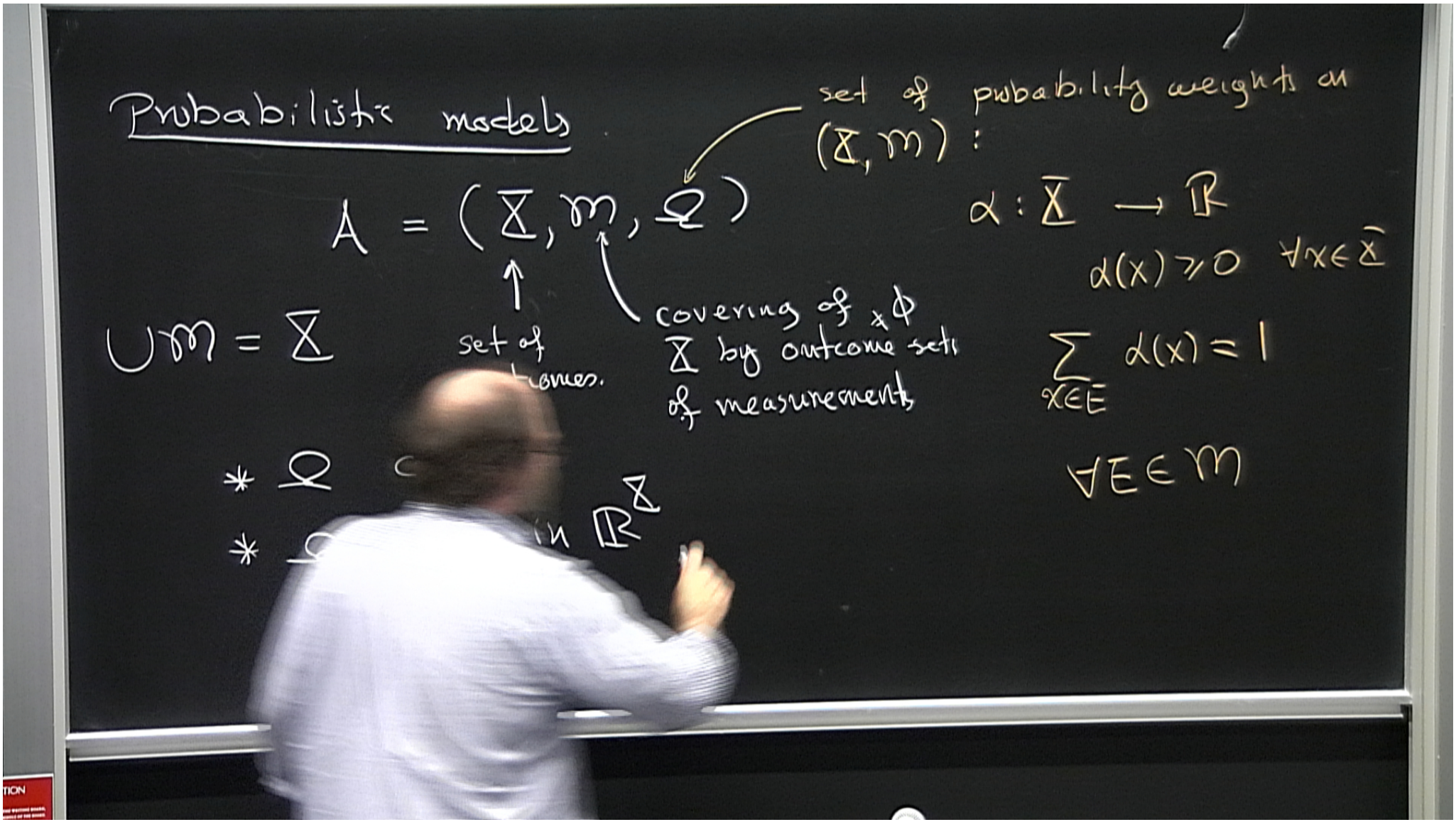
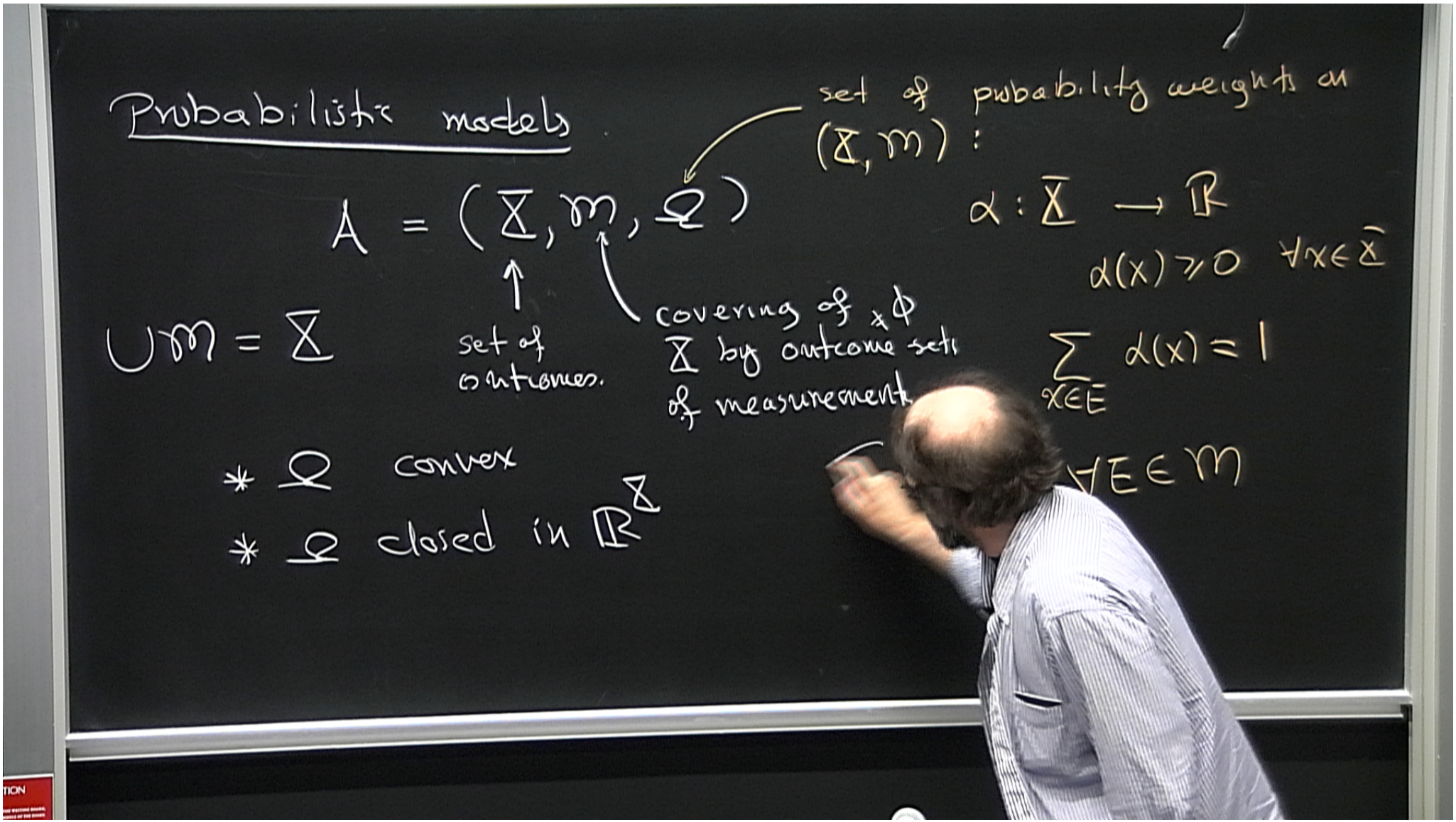
* $\mathcal{Q}$ convex
* $\mathcal{Q}$ closed in $\mathbb{R}^{\mathcal{X}}$


Probabilistic models
set of probability weights on (X, M) :
A = (X, M, Ω)
α : X → ℝ
α(x) ≥ 0 ∀x ∈ X
∪M = X
set of outcomes.
covering of X by outcome sets of measurements
Σ_{x∈E} α(x) = 1
∀E ∈ M
* Ω
in ℝ^X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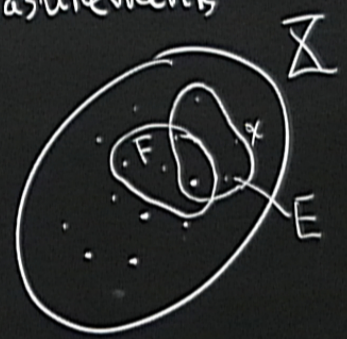
$\forall E \in \mathcal{M}$

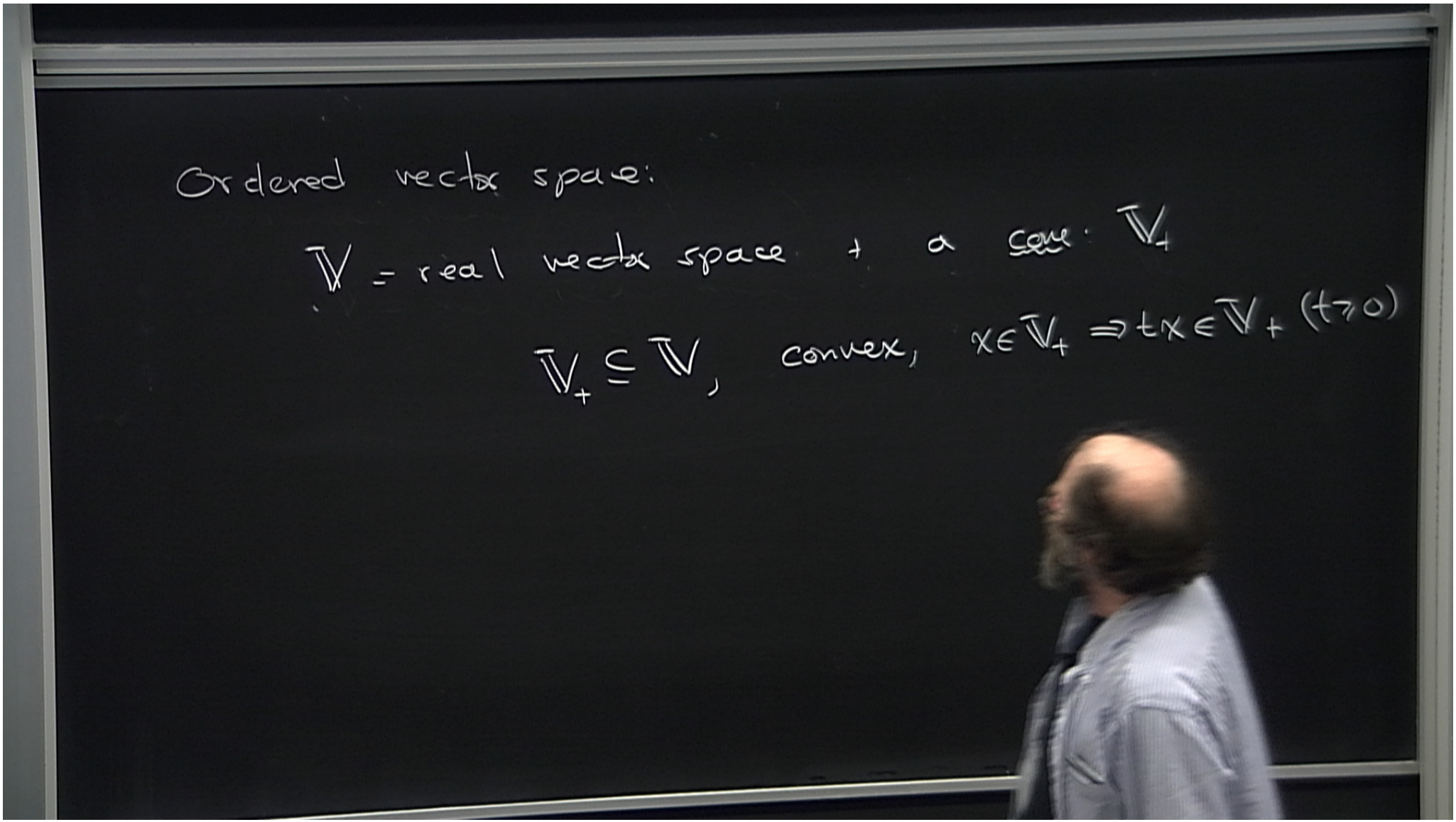
$\mathcal{X}$: set of outcomes.

$\mathcal{M}$: covering of $\mathcal{X}$ by outcome sets of measurements

$\bigcup \mathcal{M} = \mathcal{X}$

* $\mathcal{Q}$ convex
* $\mathcal{Q}$ closed in $\mathbb{R}^{\mathcal{X}}$




Ordered vector space:
$\mathbb{V}$ = real vector space + a cone: $\mathbb{V}_+$
$\mathbb{V}_+ \subseteq \mathbb{V}$, convex, $x \in \mathbb{V}_+ \Rightarrow tx \in \mathbb{V}_+$ $(t \geq 0)$

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$\mathbb{V}_+ \cap -\mathbb{V}_+ = \{0\}$, $\mathbb{V} = \mathbb{V}_+ - \mathbb{V}_+$

$x \leq y \Leftrightarrow y - x \in \mathbb{V}_+$

$(\mathbb{V}_+ = \{x \mid x \geq 0\})$

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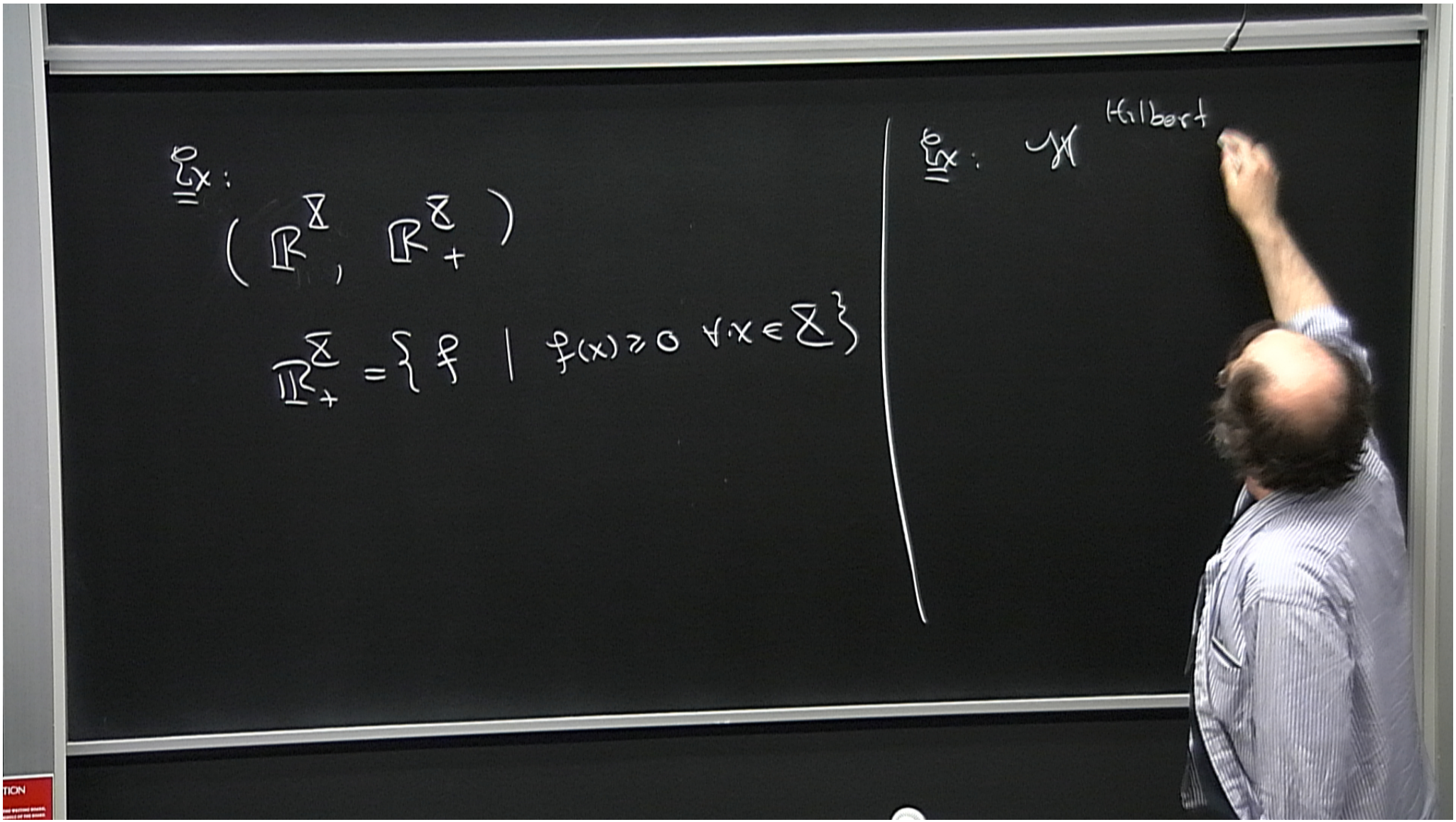
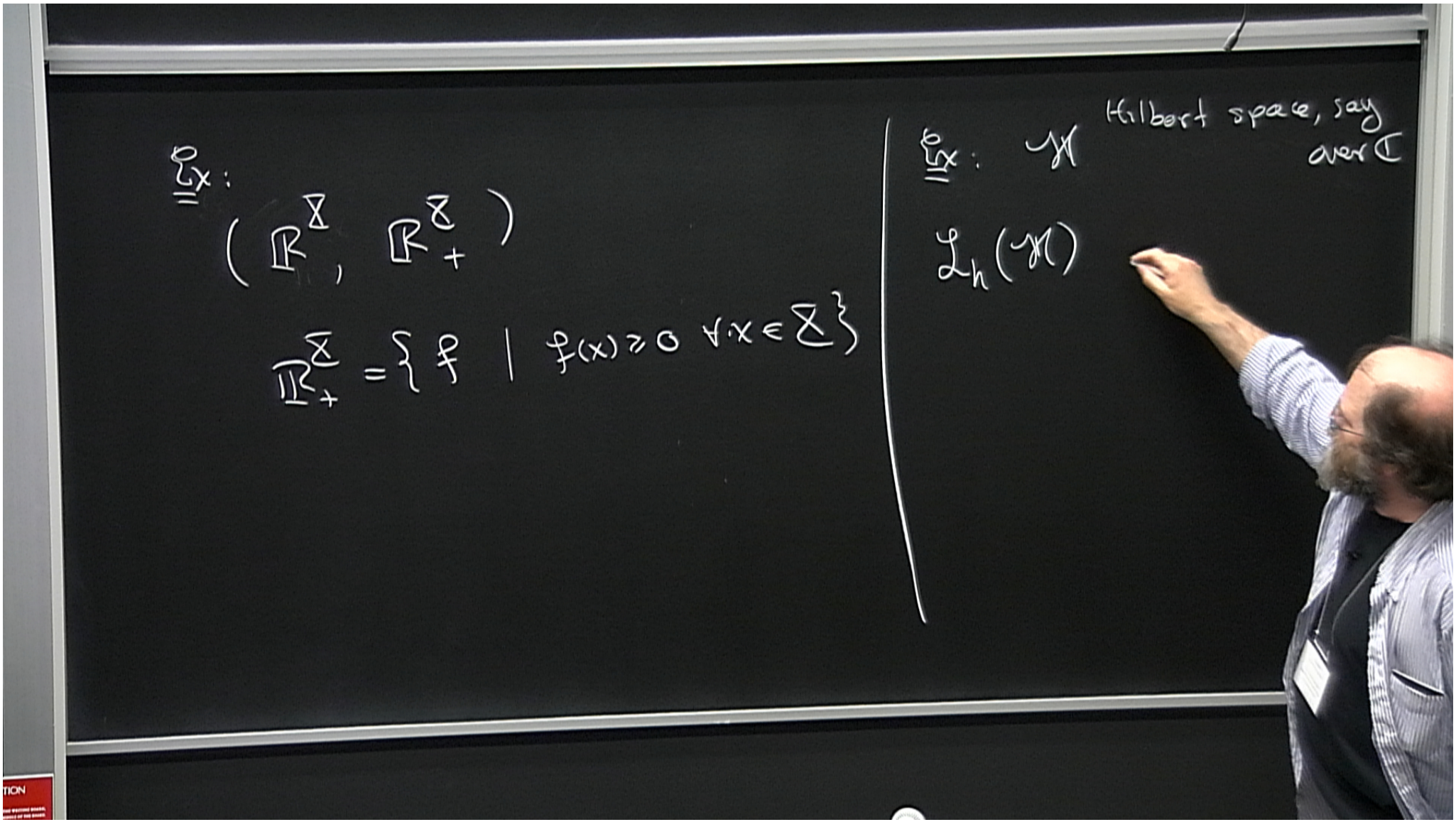

Ex:
$(\mathbb{R}^{\mathcal{X}}, \mathbb{R}^{\mathcal{X}}_{+})$
$\mathbb{R}^{\mathcal{X}}_{+} = \{ f \mid f(x) \geq 0 \ \forall x \in \mathcal{X} \}$
Ex: $\mathcal{H}$
Hilbert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Ex: $\mathcal{H}$ Hilbert space, say over $\mathbb{C}$
$\mathcal{L}_h(\mathcal{H})$

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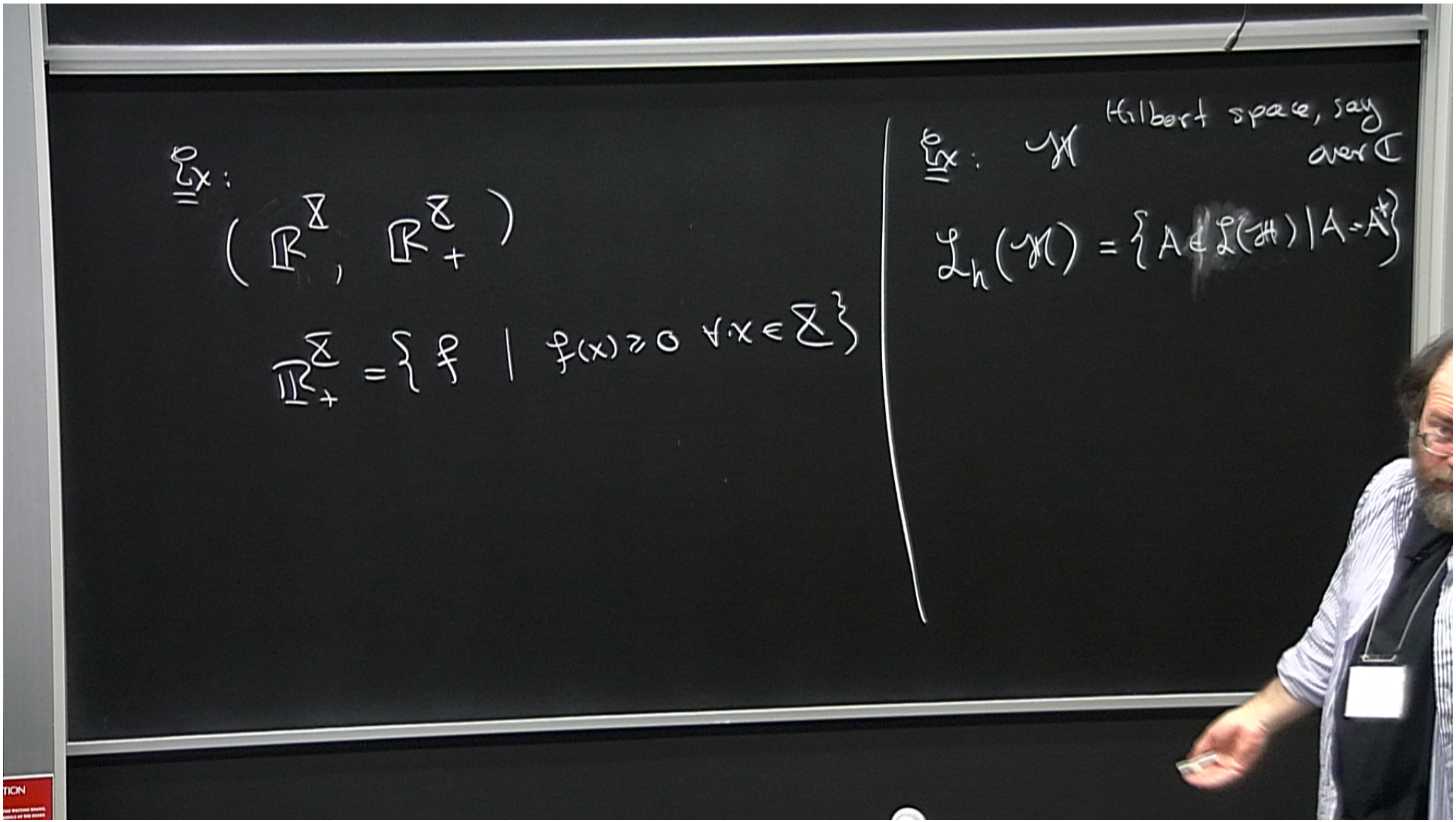
$$\mathcal{L}_h(\mathcal{H}) = \{$$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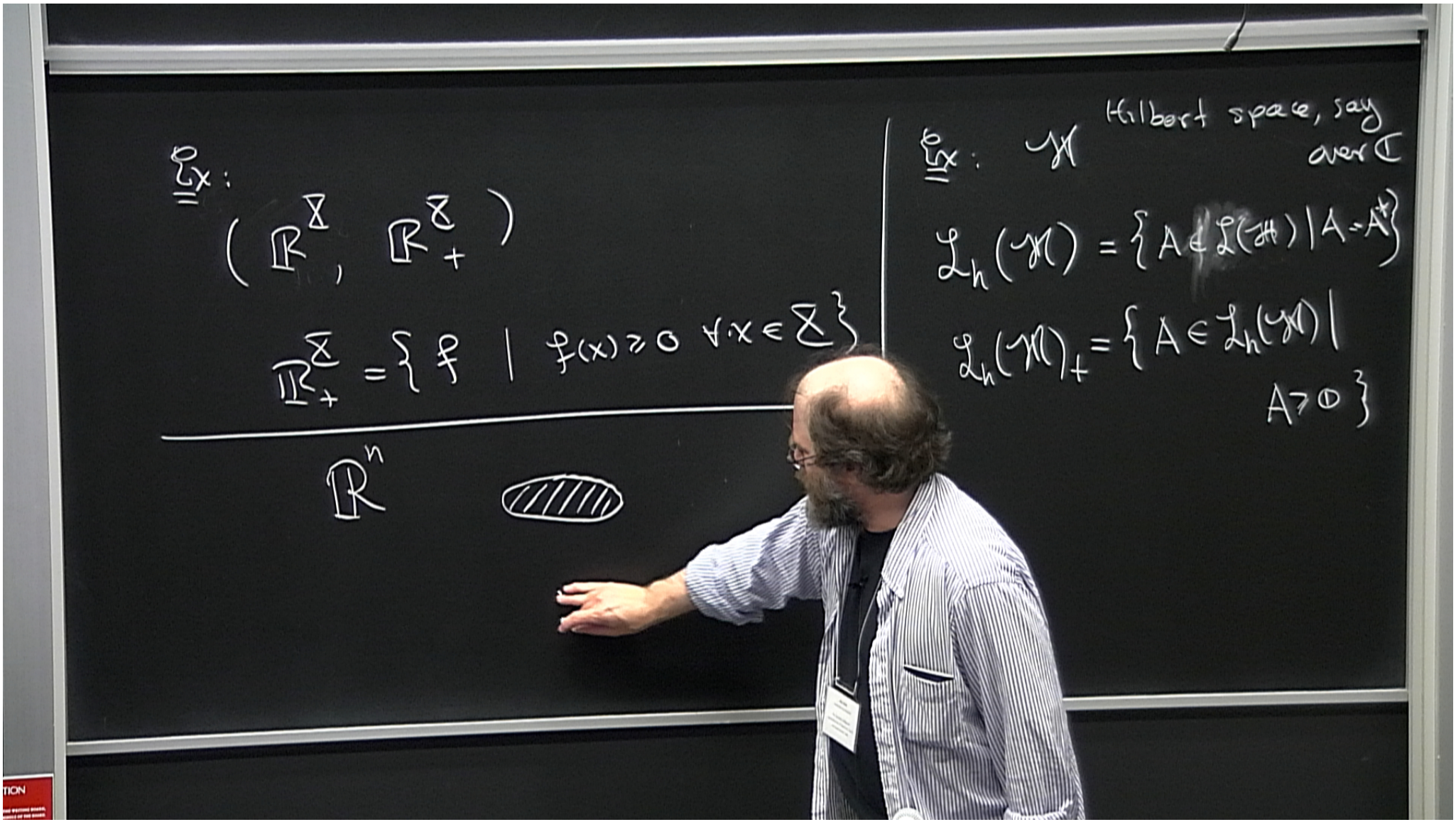
Ex:
$(\mathbb{R}^X, \mathbb{R}^X_+)$
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$\mathbb{R}^n$
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$$\mathbb{A} = (\mathcal{X}, \mathcal{M}, \mathcal{S})$$

$$\mathbb{A} = (\mathcal{X}, \mathcal{M}, \mathcal{Q})$$

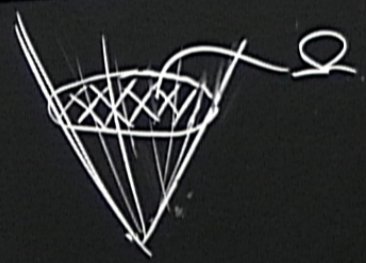
$$\mathbb{V}(\mathbb{A}) = \text{span of } \mathcal{Q} \text{ in } \mathbb{R}^{\mathcal{X}}$$

Ex:

$$(\mathbb{R}^{X}, \mathbb{R}^{X}_{+})$$

$$\mathbb{R}^{X}_{+} = \{ f \mid f(x) \geq 0 \ \forall x \in X \}$$

$\mathbb{R}^{X}$



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$$\mathbb{A} = (\mathcal{X}, \mathcal{M}, \mathcal{S})$$

* $\mathbb{V}(\mathbb{A})$ = span of $\mathcal{S}$ in $\mathbb{R}^{\mathcal{X}}$

$$\mathbb{V}(\mathbb{A})_+ = \{ t\alpha \mid \alpha \in \mathcal{S},\ t \geq 0 \}$$

* $\forall x \in \mathcal{X},\ \hat{x} \in \mathbb{V}(\mathbb{A})^*$

$$\mathbb{A} = (\mathcal{X}, \mathcal{M}, \mathcal{Q})$$

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$*$ $$\mathbb{V}(\mathbb{A})_+ = \{ t\alpha \mid \alpha \in \mathcal{Q},\ t \geq 0 \}$$

$$\forall x \in \mathcal{X},\ \hat{x} \in \mathbb{V}(\mathbb{A})^* \qquad \hat{x}(\alpha) = \alpha(x)$$

$*$ $$\mathbb{E}(\mathbb{A}) = \text{span}_n \text{ of } \{ \hat{x} \mid x \in \mathcal{X} \} \text{ in } \mathbb{V}(\mathbb{A})^*$$

$$\mathbb{E}(\mathbb{A})_+ = \left\{ \sum_{i=1}^{n} t_i \hat{x}_i \mid t_i \geq 0 \right\}.$$

$$\mathbb{A} = (\mathcal{X}, \mathcal{M}, \mathcal{S})$$

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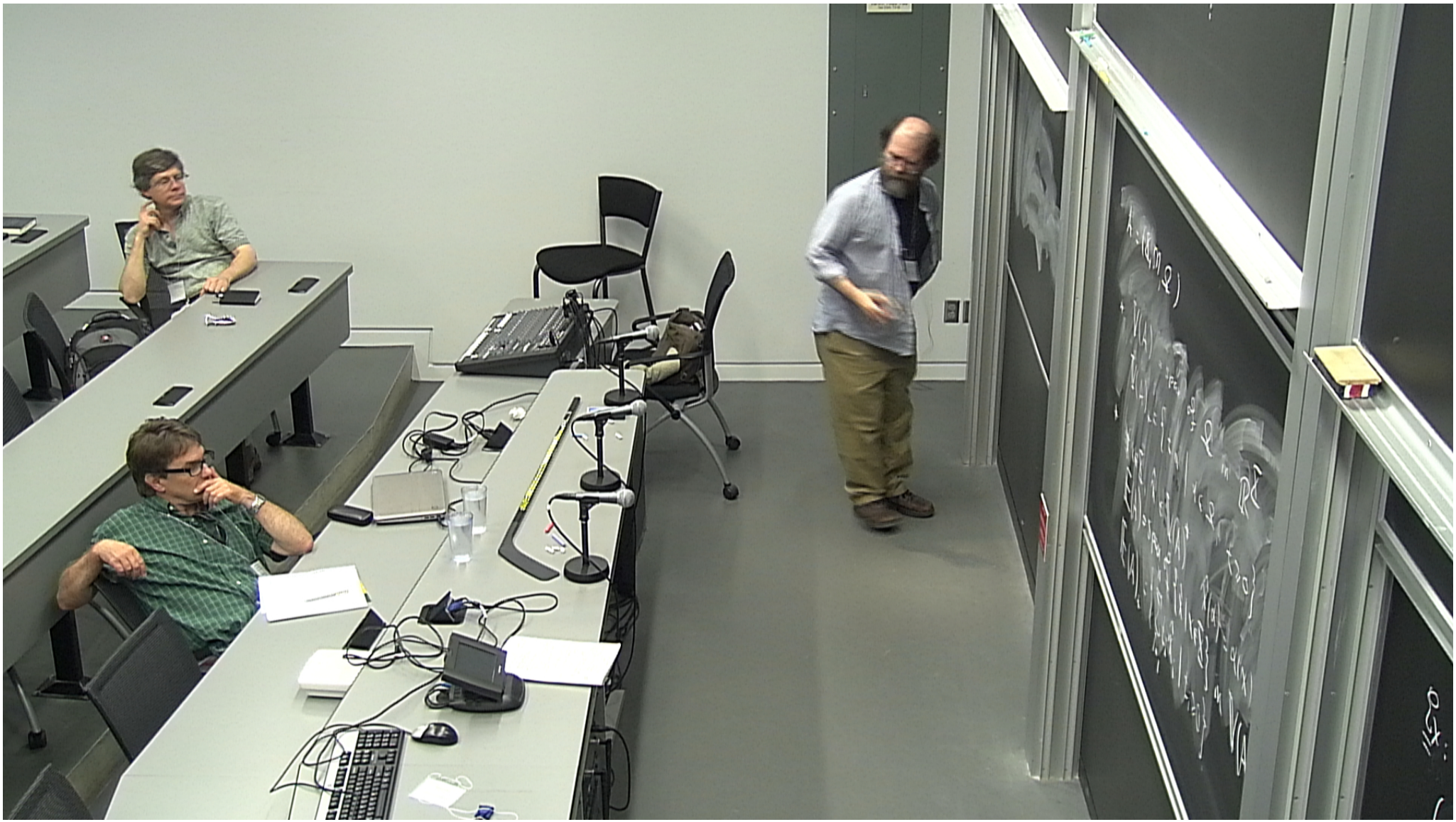

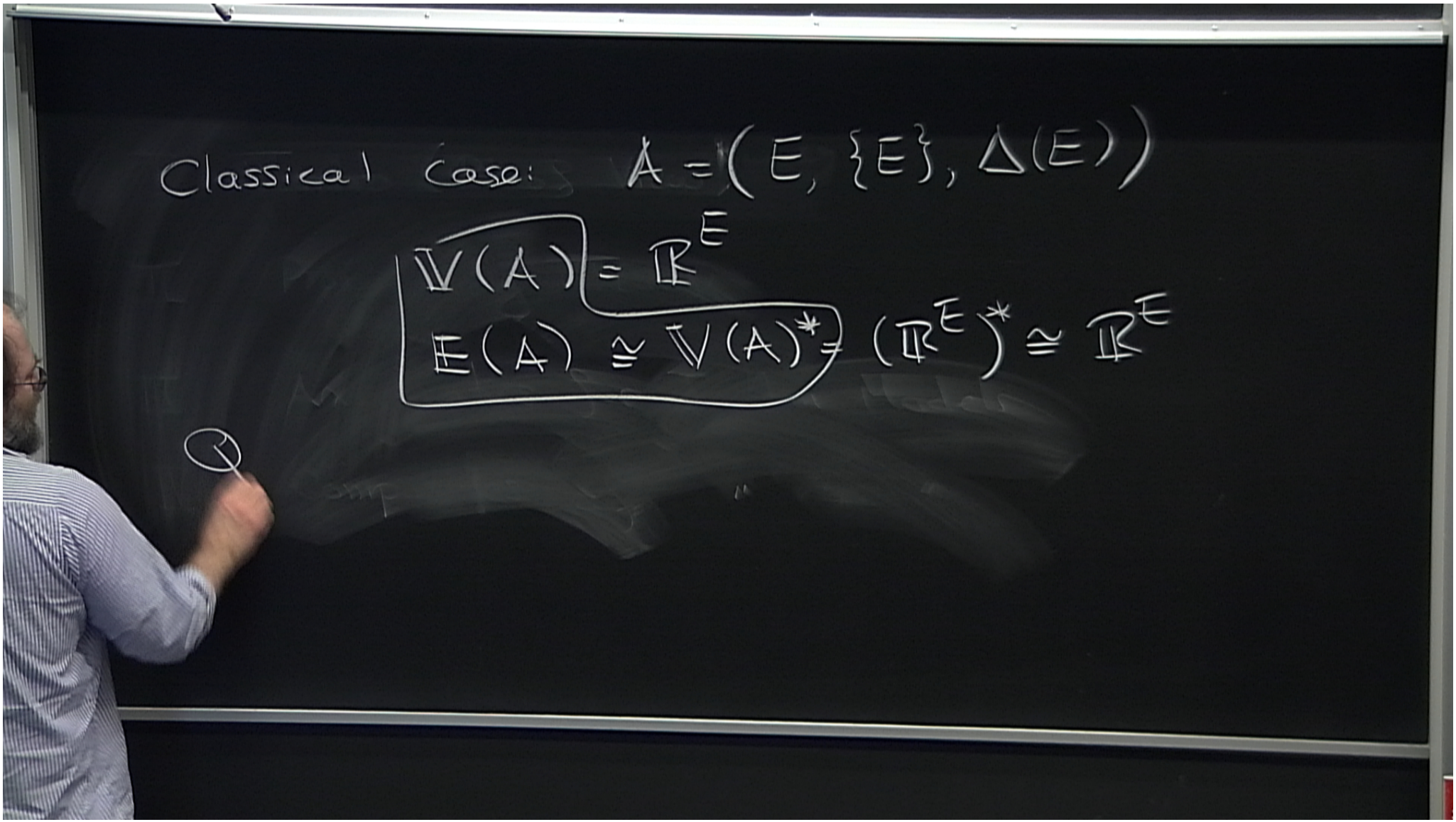
Classical case: $A = (E, \{E\}, \Delta(E))$
$\mathbb{V}(A) = \mathbb{R}^E$
$\mathbb{E}(A) \cong \mathbb{V}(A)^* = (\mathbb{R}^E)^* \cong \mathbb{R}^E$

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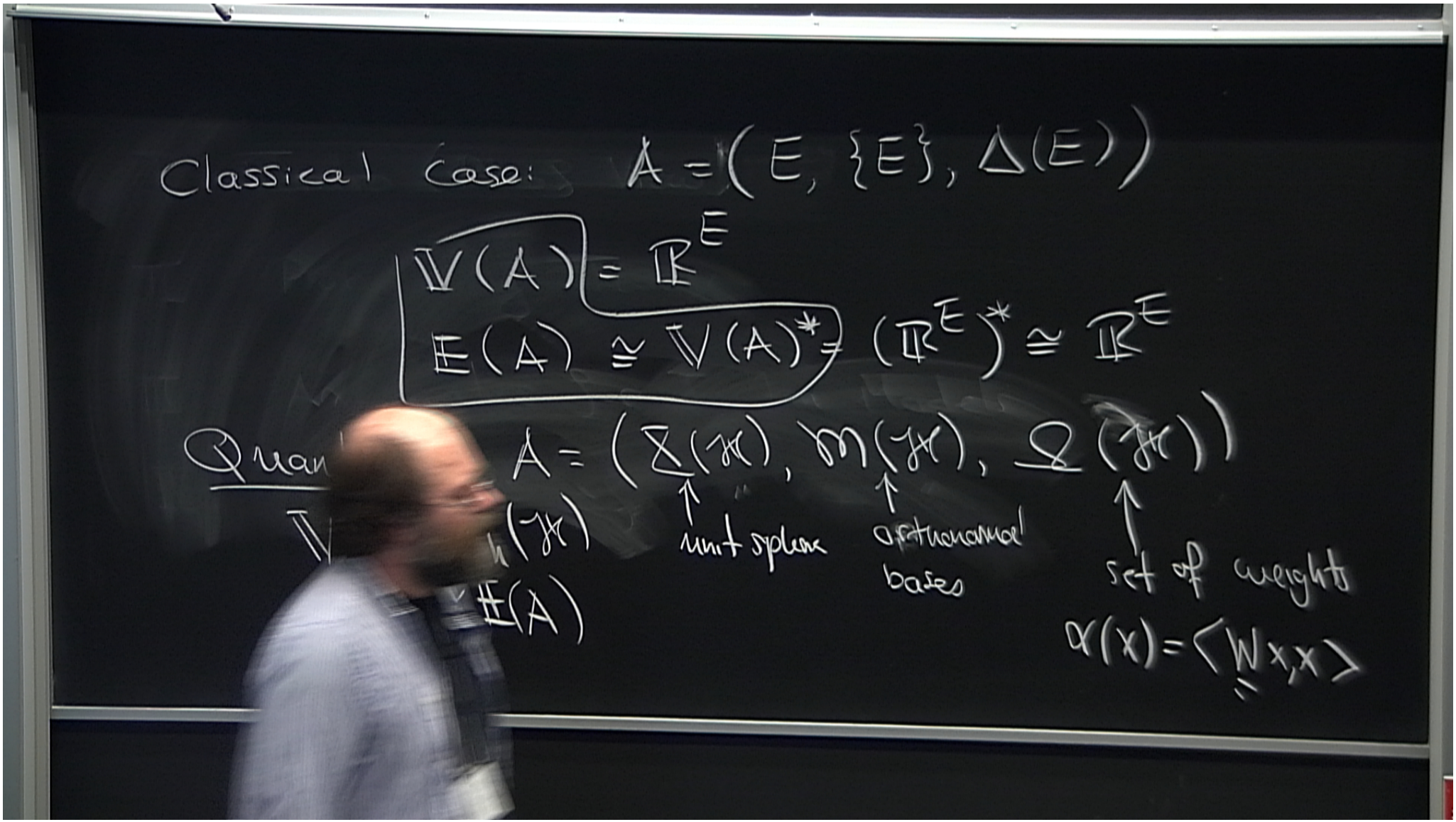
<u>Quantum</u>: $A = (\mathcal{S}(\mathcal{H}), \mathcal{M}(\mathcal{H}), \mathcal{Q}(\mathcal{H}))$

unit sphere

orthonormal bases

set of weights

$$\alpha(x) = \langle \underline{W} x, x \rangle$$


Classical case: $A = (E, \{E\}, \Delta(E))$
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unit sphere
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<u>Quantum</u>: $A = (\mathfrak{S}(\mathcal{H}), \mathfrak{M}(\mathcal{H}), \mathfrak{Q}(\mathcal{H}))$

unit sphere

orthonormal bases

set of weights

$$\mathbb{V}(A) \cong \mathcal{L}_h(\mathcal{H}) \cong \mathbb{E}(A)$$

$$\alpha(x) = \langle \underline{W} x, x \rangle$$

$\mathbb{E}$ ordered vector space is <u>self</u> <u>dual</u>

$\Leftrightarrow \exists \langle , \rangle$ on $\mathbb{E}$:

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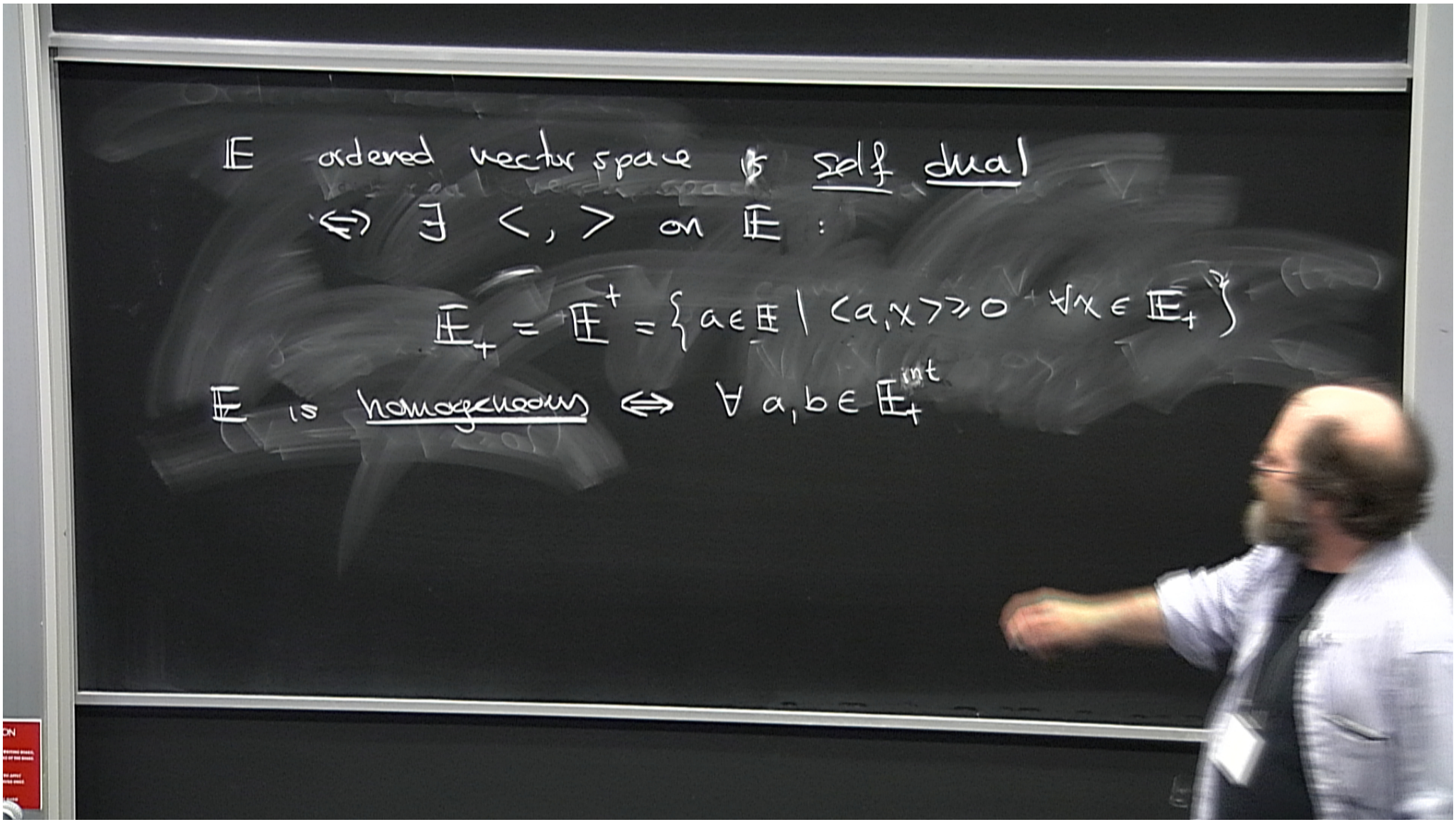
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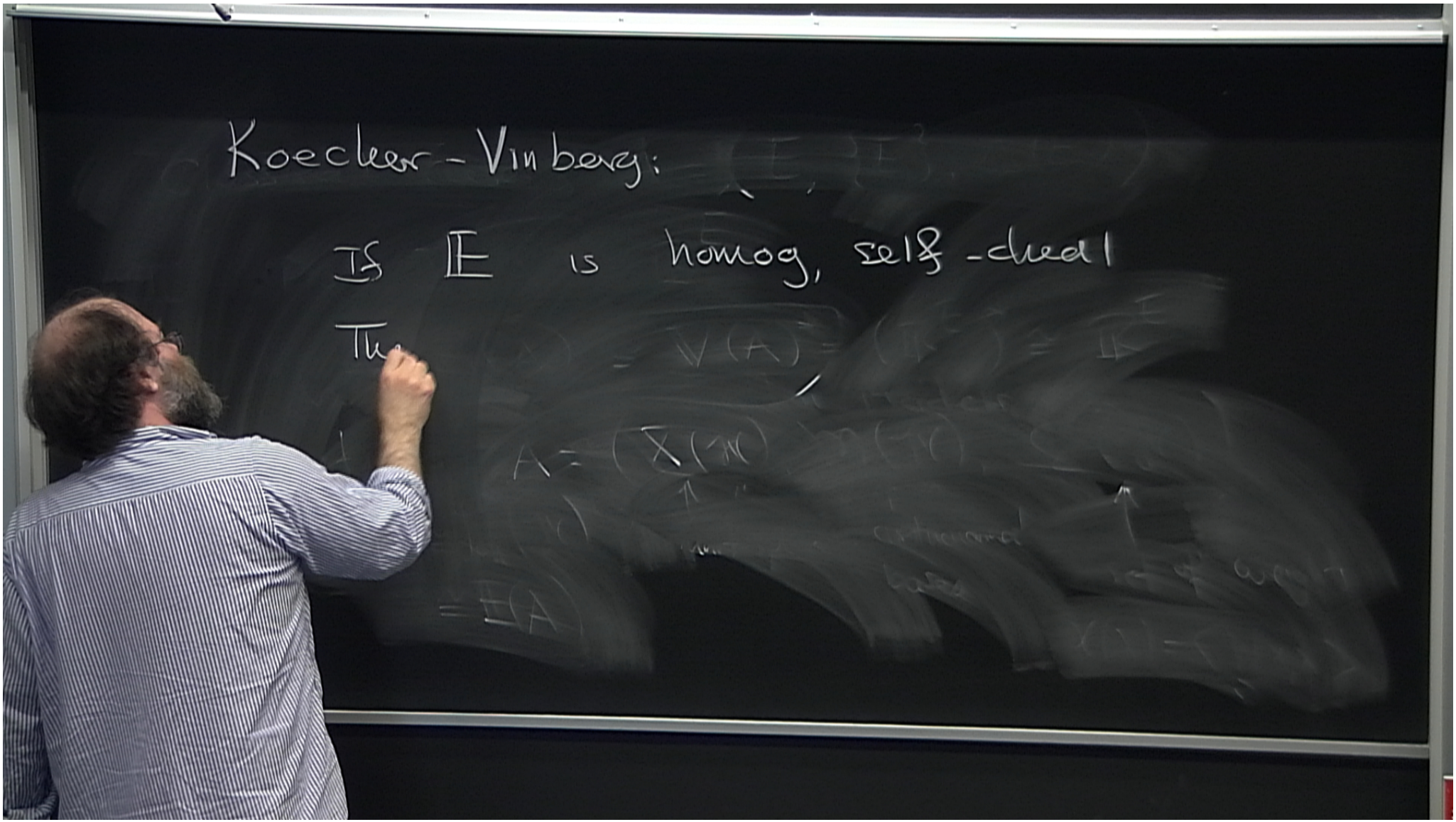
$\mathbb{E}$ is <u>homogeneous</u> $\Leftrightarrow \forall a, b \in \mathbb{E}_+^{int}$ $\exists$ an isomorphism φ

$\varphi : \mathbb{E} \cong \mathbb{E}$

$\varphi(a) = b$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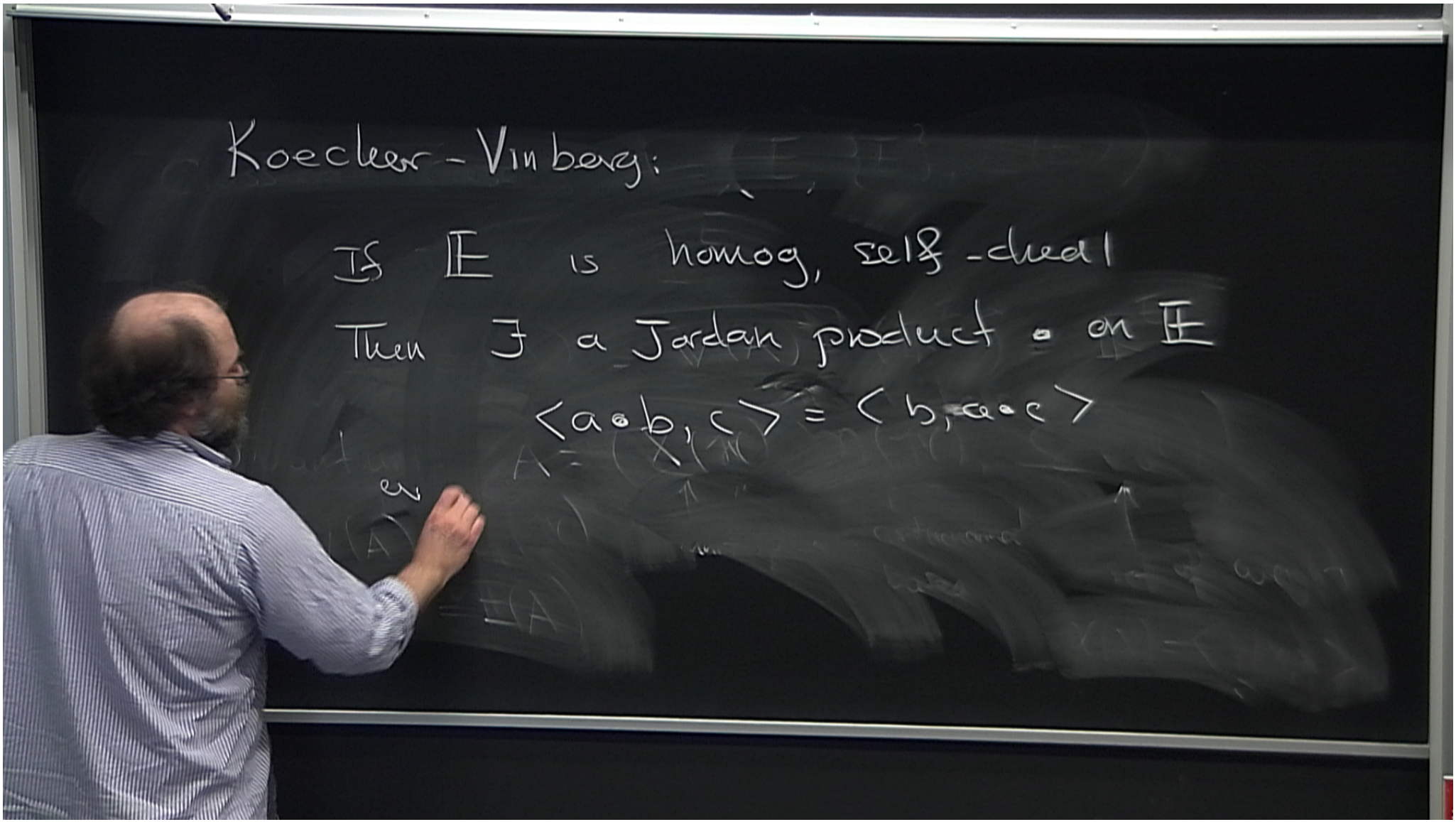
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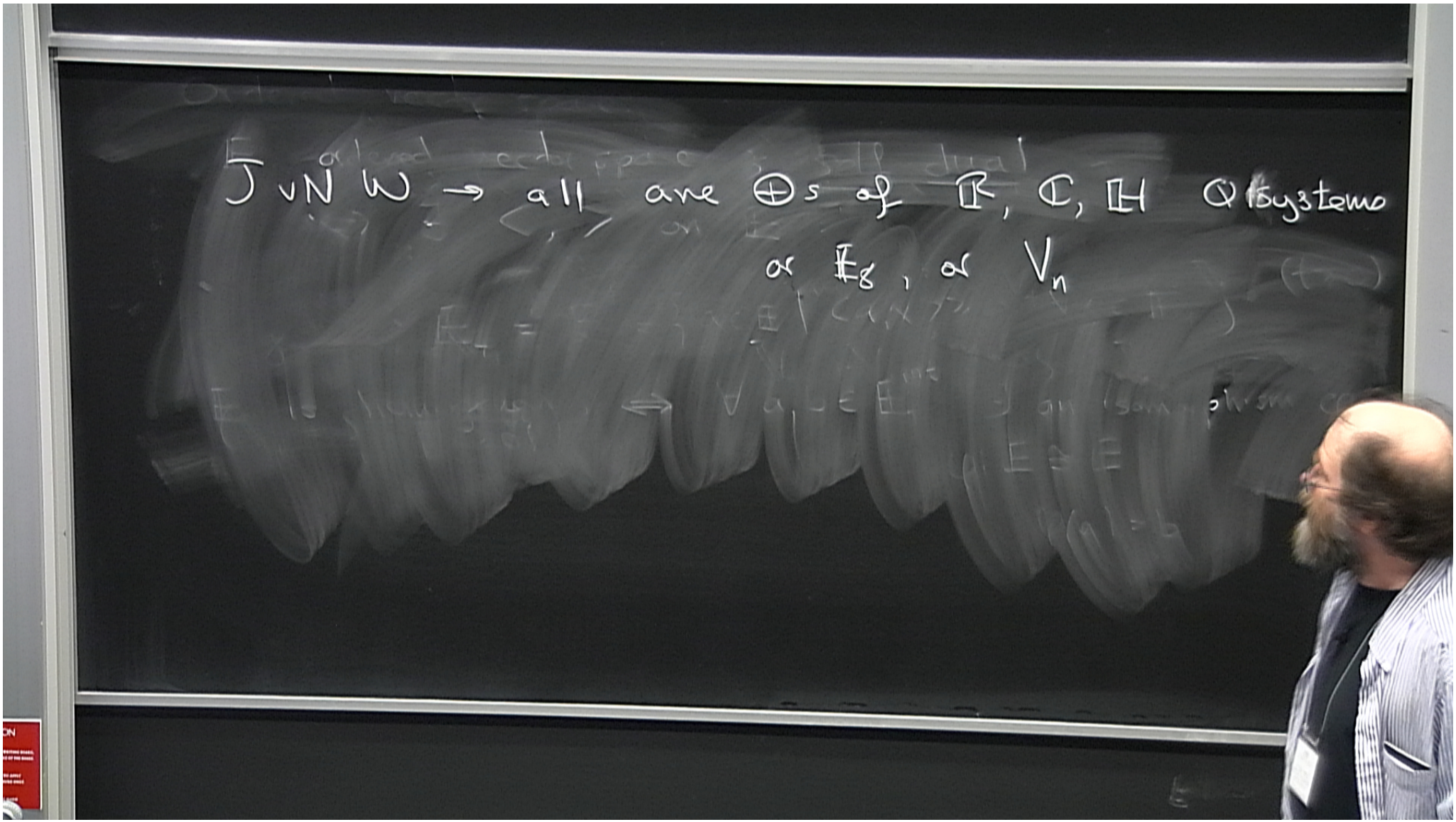
w.r.t which $\mathbb{E}_+ = \{ \underbrace{a \circ a}_{a^2} \mid a \in \mathbb{E} \}$


J vN W → all are ⊕s of ℝ, ℂ, ℍ Q systems
or E_8, or V_n

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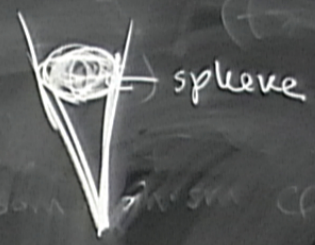

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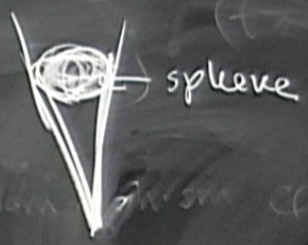
A few ~~simple~~ axioms about

conjugates & correlations ⇒ HSP



$A \otimes$ for euclidean Jordan alg.

(Hanche-Olsen)

A $\otimes$ for special euclidean Jordan alg.

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$$\mathbb{E} \leq C^*(\mathbb{E})$$

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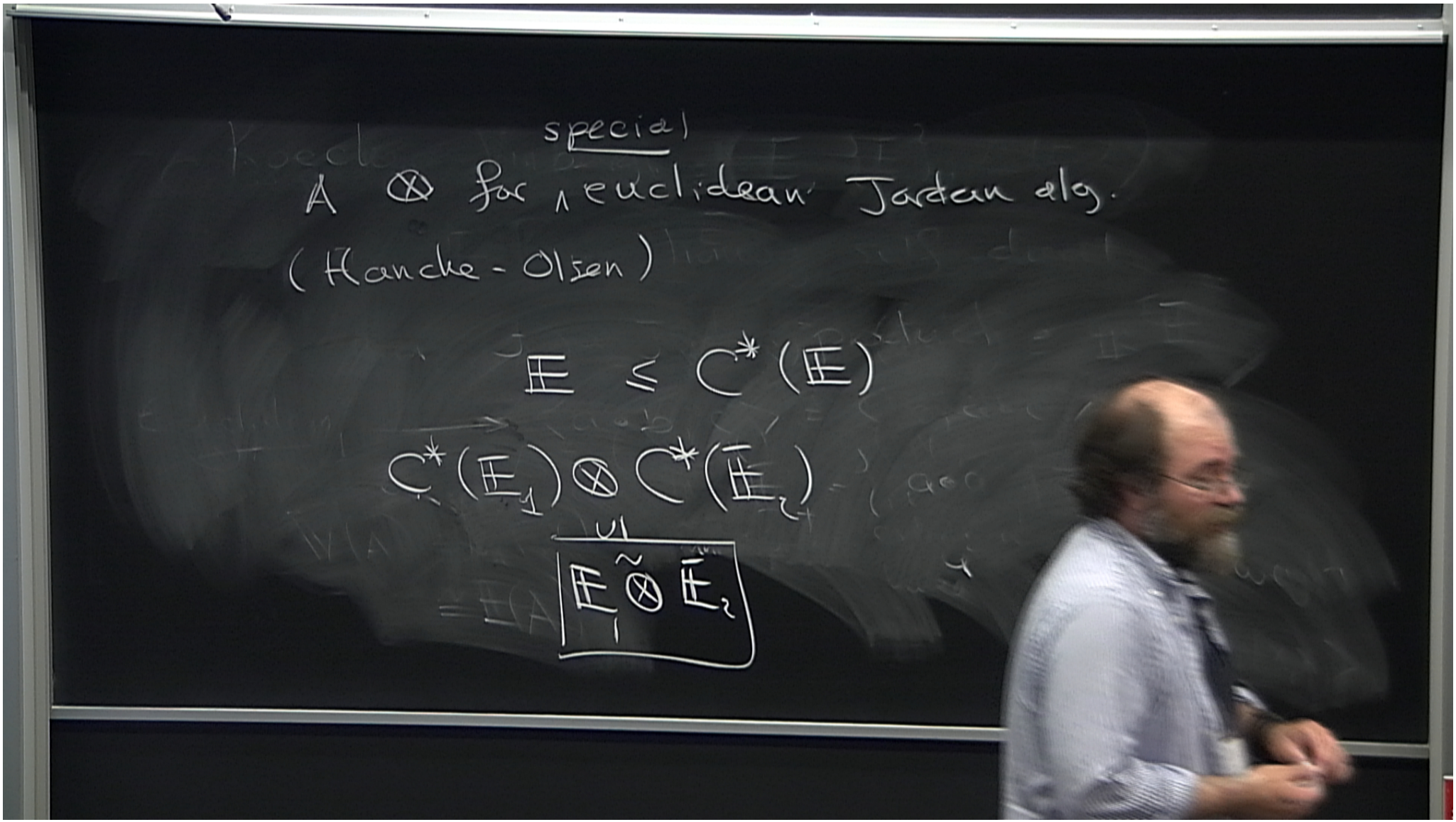
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