Title: Energy diffusion from relativistic spontaneous localization

Date: May 30, 2013 10:00 AM

URL: http://pirsa.org/13050068

Abstract: We discuss energy diffusion due to spontaneous localization (SL) for a relativistically-fast moving particle. Based on evidence from relativistic extensions of SL we argue that non-relativistic SL should remain valid in the particle rest frame. This implies that calculations can be performed by transforming non relativistic results from the particle rest frame to the frame of the observer. We demonstrate this by considering a relativistic stream of non-interacting particles of cosmological origin and showing how their energy distribution evolves as they traverse the Universe. We present a solution and discuss the potential for astrophysical observations.

Energy diffusion from relativistic spontaneous localization

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Quantum Landscape 2013

Outline

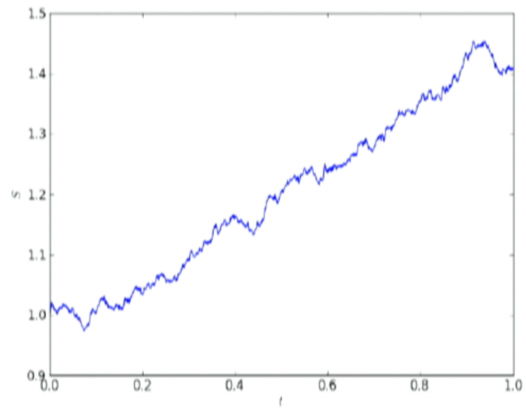
- Overview of continuous spontaneous localization (CSL).
- Taking the localized single particle limit.
- Steady state solutions.
- Features of relativistic models.
- Relativistic energy diffusion.

Stochastic processes

Stochastic differential equation

$$dS = \underbrace{\mu dt}_{\text{drift}} + \underbrace{\sigma dW_t}_{\text{stochastic}} \ .$$

W_t is Wiener process/Brownian motion process.



Continuous spontaneous localization (CSL) model

State diffusion:

$$d|\psi\rangle = \left[-i\hat{H}dt - \frac{\gamma}{2}\int dx(\hat{N}(x) - \langle\hat{N}(x)\rangle)^2 dt + \gamma^{1/2}\int dx(\hat{N}(x) - \langle\hat{N}(x)\rangle)dB_t(x)\right]|\psi\rangle,$$

where

$$\hat{N}(x) = \int dy \exp\left\{-\frac{\alpha(x-y)^2}{2}\right\} \hat{a}^\dagger(y)\hat{a}(y),$$

and

$$\mathbb{E}[dB_t(x)] = 0 \quad ; \quad dB_t(x)dB_{t'}(y) = \delta_{tt'}\delta(x-y).$$

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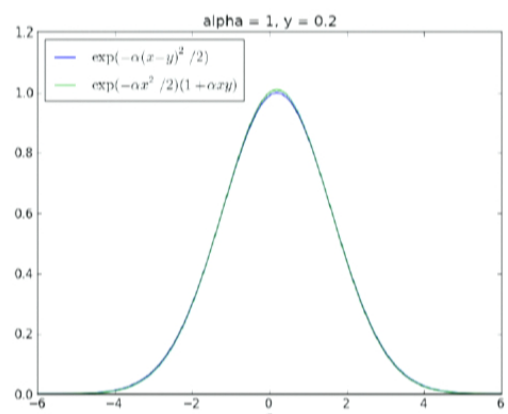
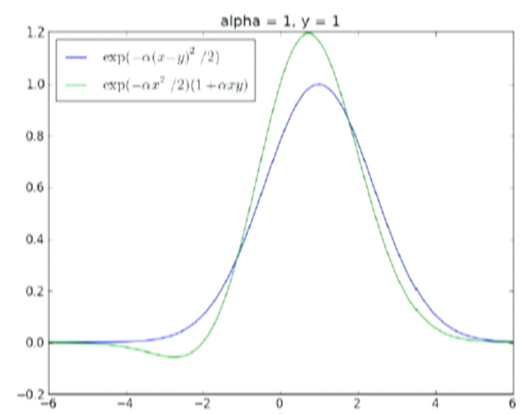
Sufficiently localized approximation

Assume following

$$\exp\left\{-\frac{\alpha(x-y)^2}{2}\right\} \simeq \exp\left\{-\frac{\alpha x^2}{2}\right\}(1+\alpha xy),$$

valid when $y \ll 1/\sqrt{\alpha}$.





Single particle approximation

Assume $|\psi\rangle = \int dx \psi(x) \hat{a}^\dagger(x)|0\rangle$; $|x\rangle = \hat{a}^\dagger(x)|0\rangle$.
Then define the position operator

$$\hat{x} = \int dy \, y \hat{a}^\dagger(y)\hat{a}(y).$$

Single particle state satisfies the quantum state diffusion (QSD)

$$d|\psi\rangle = \left[-i\hat{H}dt - D(\hat{x} - \langle\hat{x}\rangle)^2 dt + \sqrt{2D}(\hat{x} - \langle\hat{x}\rangle)dW_t\right]|\psi\rangle.$$

$W_t = \frac{\sqrt{2}\alpha^{3/4}}{\pi^{1/4}} \int dx \int^t dB_{t'}(x) e^{-\alpha x^2/2} x$ is a Wiener process:

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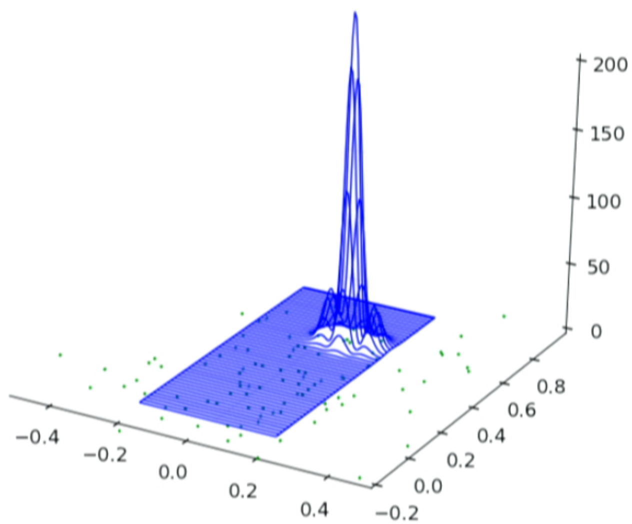
Double slit experiment - with collapses



Some examples

In 3D the diffusion coefficient is given by

$$D = \frac{\gamma \alpha^{5/2}}{32\pi^{3/2}}.$$

GRW parameters: $\gamma_{proton} = 10^{-30} cm^3 s^{-1}$; $\alpha^{-1/2} = 10^{-5} cm$
$\implies D \sim 0.001 m^{-2} s^{-1}$.
Assume $D = \frac{m^2}{m_{proton}^2} \times 0.001 m^{-2} s^{-1}$.

Particle	σ_∞	t_{loc}
neutrino ($0.1 eV/c^2$)	$1500 km$	$400 yrs$
electron	$14 m$	$60 days$
proton	$5 cm$	$35 hrs$
Fe nucleus	$2 mm$	$5 hrs$
10,000 a.u. cluster	$50 \mu m$	$20 mins$

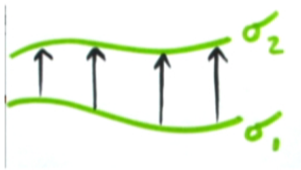
Adding (special) relativity - things to consider

- Need to work with relativistic quantum fields.
- Require a covariant expression of state evolution (Lorentz invariance of predictions for experiments).
- No preferred frame or foliation.
- No faster than light signalling.

Relativistic collapse framework

Covariant expression of unitary evolution

As we advance through spacetime from σ_1 to σ_2,



$$|\Psi(\sigma_1)\rangle \to |\Psi(\sigma_2)\rangle = Te^{-i\int_{\sigma_1}^{\sigma_2} d^4x\, H_{\text{int}}(x)}|\Psi(\sigma_1)\rangle.$$

$[H_{\text{int}}(x), H_{\text{int}}(y)] = 0$ for spacelike separated x, y

Including hits

Consider a sprinkling of Poisson distributed points in spacetime.
If the surface σ passes through one of these points (at x say), a hit event occurs:

$$|\Psi(\sigma)\rangle \to |\Psi(\sigma_+)\rangle = L(Z_x)|\Psi(\sigma)\rangle.$$

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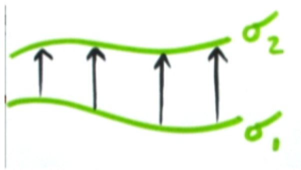

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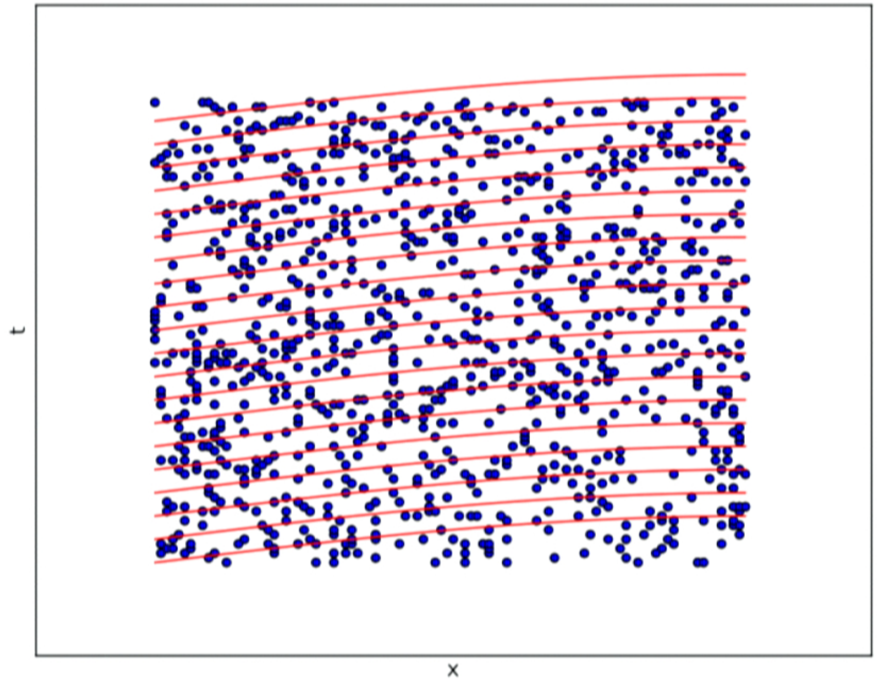
Picture to have in mind



Relativistic hits

Localization operator

$$L(Z_x) = \frac{1}{(2\pi s^2)^{1/4}} e^{-\frac{(N(x)-Z_x)^2}{4s^2}}.$$

Must have

- $N(x)$ is a Lorentz scalar.
- $[N(x), N(y)] = 0$ for spacelike separated x, y.
- $[N(x), H_{\text{int}}(y)] = 0$ for spacelike separated x, y.

Proposal

Try $N(x) = \phi^2(x)$ scalar field operator.
Problem occurs as we cross a hit location (going from σ to σ_+):
Expected energy increase is infinite.

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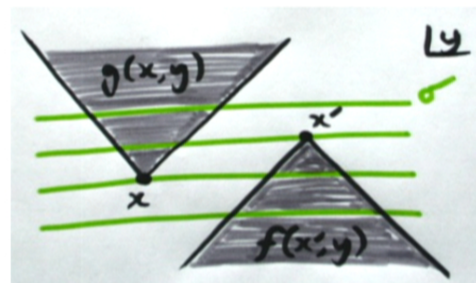
Operator properties

$$[N(x), N(x')] = 0, \text{ and } [A(x), A(x')] = 0 \;\; \forall x, x',$$

$$[A(x), N(x')] = \int d^4y \, f(x', y) g(x, y) \left[a(y) - a^\dagger(y) \right].$$

If f and g only non zero in these domains:



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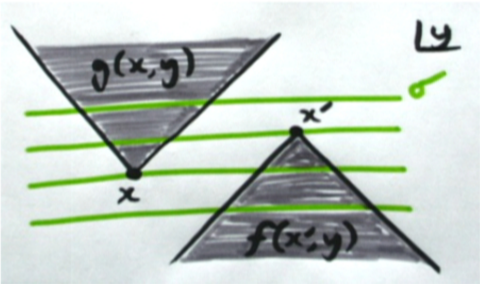

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Relativistic reduction model

Unitary background

Interaction picture with interaction Hamiltonian

$$H_{\text{int}}(x) = J(x)A(x).$$

$J(x)$ is some scalar current operator for a conventional quantum field, e.g., $J(x) = \phi^2(x)$ (sort of matter density distribution).

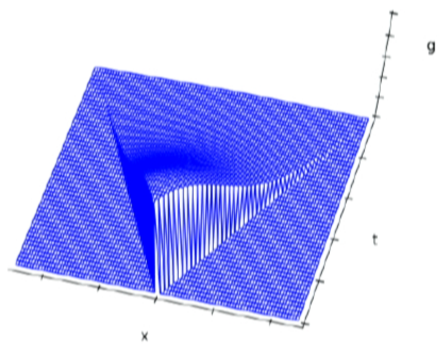
Random hits

Poisson distributed in spacetime with localization operator

$$L(Z_x) = \frac{1}{(2\pi s^2)^{1/4}} e^{-\frac{(N(x)-Z_x)^2}{4s^2}}.$$

Acting only on the mediating field.

Eg 1: the smearing functions



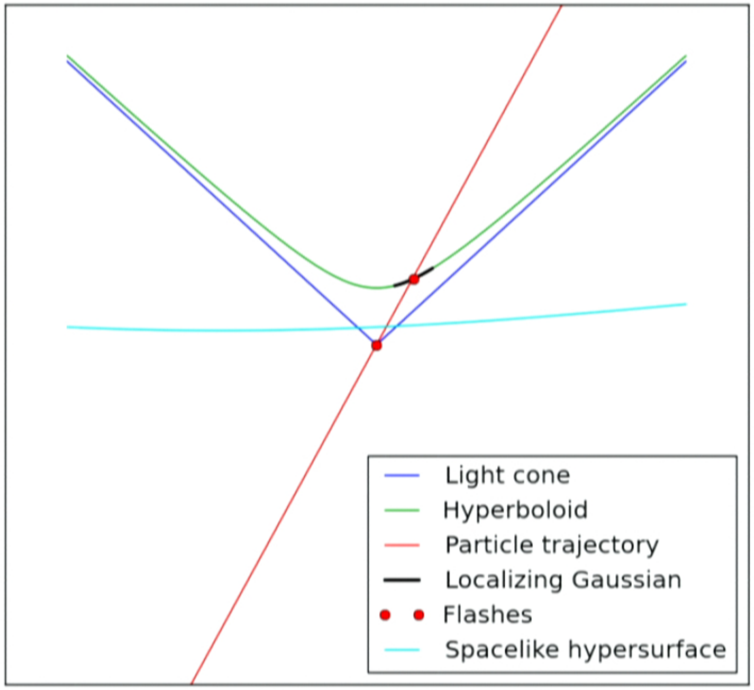
Smeared interaction

$$J(z) \int d^4x \; g(z,x) \left[a(x) + a^\dagger(x) \right]$$

Suggested form for g in future light cone:

$$g(z,x) \sim \exp\left\{ -K \langle T^{\mu\nu}(z) \rangle (x_\mu - z_\mu)(x_\nu - z_\nu) \right\}$$

Eg 2: Tumulka relativistic model



Energy process

Near the particle rest frame $\bar{\mathcal{O}}$ the energy is given by

$$\bar{E} = \sum_i \frac{\langle \bar{p}_i \rangle^2}{2m}.$$

We find for the energy process in the particle rest frame

$$d\bar{E} = \frac{3D}{m} d\tau + \frac{\sqrt{2D}}{m} \sum_i \langle \bar{p}_i \rangle dW_{i,\tau} = \frac{3D}{m} d\tau.$$

Convert to observer's time coordinate t

Lorentz transformation of coordinates

$$dt = \gamma \left(d\tau - \frac{v d\langle \bar{x}_1 \rangle}{c^2} \right) \simeq \gamma \left(d\tau - \frac{d\langle \bar{x}_1 \rangle}{c} \right).$$

Approximate by $dt = \gamma d\tau$, so that $dW_t = \gamma^{1/2} dW_{1,\tau}$
($\implies (dW_t)^2 = dt$).
Then

$$dE = \frac{3D}{m} dt + \sqrt{\frac{2D}{m} E}\, dW_t,$$

where we have used $E = \gamma mc^2$.

Relativistic energy process

Over cosmological timescales should also add a friction term to describe effects of expansion.
Resulting energy process is

$$dE = \frac{3D}{m}dt - \underbrace{\frac{\dot{a}}{a}Edt}_{\text{friction term}} + \sqrt{\frac{2D}{m}E}\,dW_t,$$

where a is the scale factor of the Universe.
This is a Cox-Ingersoll-Ross process.

Forward equation

Describes the behaviour of $p_t(E|E_0)$ - probability distribution function for E at time t conditional on a value E_0 at time 0:

$$\frac{d}{dt}p_t(E|E_0) = \left\{\frac{D}{m}E^2\frac{\partial^2}{\partial E^2} + \left(\frac{\dot{a}}{a} - \frac{D}{m}\right)\frac{\partial}{\partial E} + \frac{\dot{a}}{a}\right\} p_t(E|E_0).$$

Initial condition is $p_0(E|E_0) = \delta(E - E_0)$.

Forward equation solution

Assume Hubble constant is constant then

$$p_t(E|E_0) = \frac{\alpha}{\beta}\frac{E}{E_0}e^{-\alpha(E+\beta E_0)}I_2\left(2\alpha\sqrt{\beta E E_0}\right),$$

where

$$\beta = \exp\left\{-\frac{\dot{a}}{a}t\right\} \simeq 1 - \frac{\dot{a}}{a}t,$$
$$\alpha = \frac{\dot{a}}{a}\frac{m}{D}\frac{1}{1-\beta} \simeq \frac{m}{Dt},$$

and I_2 is a modified Bessel function of the first kind of order 2.

Put in real numbers

Write diffusion coefficient as

$$D = D_0 \frac{m^2}{m_{proton}^2}.$$

Diffraction experiments with C_{60} suggest $D_0 < 10^8 m^{-2} s^{-1}$.
(GRW suggestion corresponds to $D_0 = 10^{-3} m^{-2} s^{-1}$.)
Using the C_{60} limit and $t = 10^{17} s$ (age of Universe) we find

$$\mathbb{E}_t[E|E_0] \simeq E_0 + (10^{-5} mc^2) \simeq E_0,$$
$$\mathrm{Var}_t[E|E_0] \simeq (10^{-5} mc^2) E_0.$$

Summary and conclusions

- Worked in localized, single particle limit of CSL.
- There is a steady state solution for wavepacket which diffuses through phase space.
- This leads to diffusion of kinetic energy.
- Assumed non relativistic equations hold in particle rest frame and transformed to the cosmological frame.
- Find narrow spreading in comparison to initial energy of particle (assumed relativistic).
- Would need a source of particles with very precise energy to observe.

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