

Title: Path Integrals, Reality and Generalizations of Quantum Theory

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Abstract: I describe some tentative new ideas on modified versions of quantum theory motivated by the path integral formalism, and on other generalizations, and comment on possible experimental implications.

Path Integrals, Reality and Generalizations of Quantum Theory

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talk at Perimeter Institute 100029052013

Reference

"Path Integrals and Reality "

(A.K. arxiv: 1305.6565)

If time allows ⁸ may also mention ideas from

"Beyond Boundary Conditions: General Cosmological theories "

(A.K. in Proceedings of COSMO97 (1997), arxiv: 0905.0632)

"Beable-Guided Quantum Theories: Generalising Quantum Probability Laws"

(A.K., Phys Rev A 87 022105 (2013))

* it probably won't

Some motivations for working on dynamical collapse models and other generalisations of quantum theory:

1. A combination of theoretical curiosity and the desire to test quantum theory as well as possible (and, pace Weinberg, optimism that this can be done in many interesting ways).

"[I]t is very difficult to find any logically consistent generalization of quantum mechanics."
(Steven Weinberg, in Testing Quantum Mechanics, Annals of Physics (1989))

2. The incompatibility of quantum theory and general relativity and maybe also a hunch that neither is completely correct and (to some degree) quantum theory needs to be "gravitised" as well as vice versa.

3. Belief that the quantum reality problem -- or, if you prefer, our inability to explain the appearance of a quasiclassical world from within quantum theory -- implies that even non-relativistic quantum theory is necessarily either incomplete or not completely correct.

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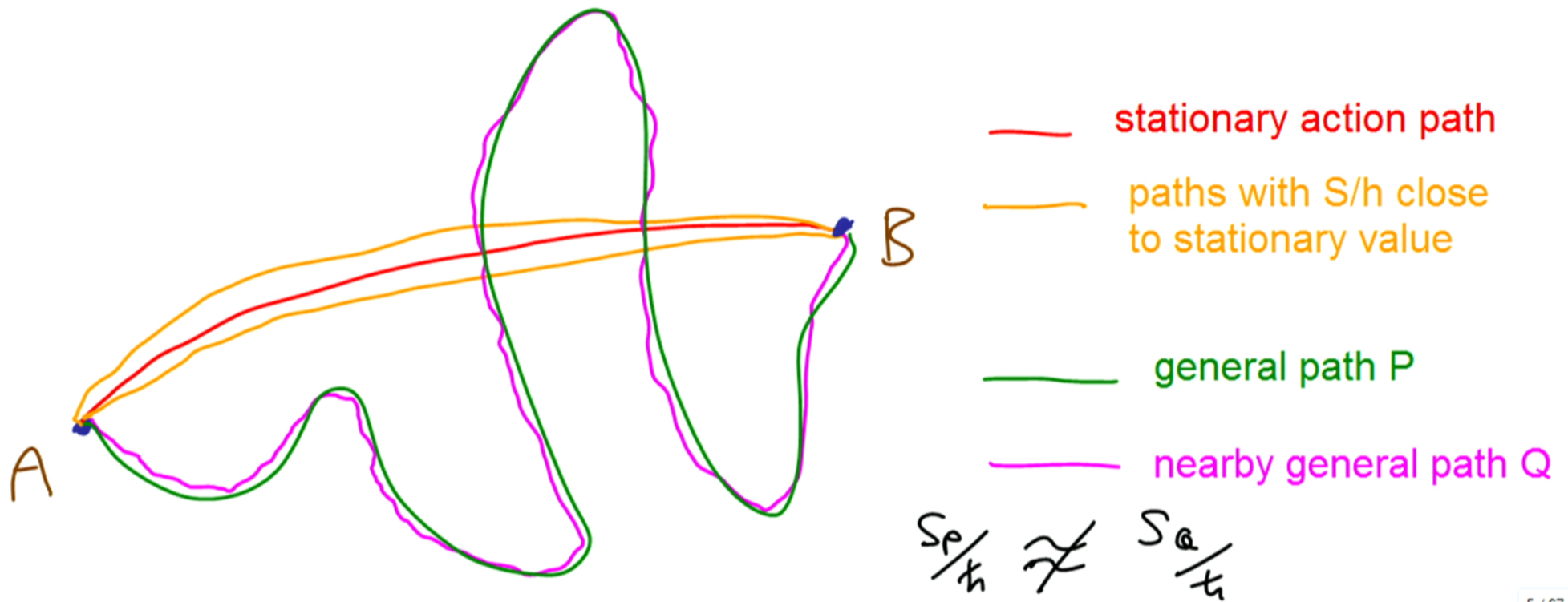
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“For those people who insist that the only thing that is important is that the theory agrees with experiment, I would like to imagine a discussion between a Mayan astronomer and his student. The Mayans were able to calculate with great precision predictions, for example, for eclipses and for the position of the moon in the sky, the position of Venus, etc. It was all done by arithmetic. They counted a certain number, and subtracted some numbers, and so on. There was no discussion of what the moon was. There was no discussion even of the idea that it went around. They just calculated the time when there would be an eclipse, or when the moon would rise at the full, and so on. Suppose that a young man went to the astronomer and said ‘I have an idea. Maybe those things are going around, and there are balls of something like rocks out there, and we could calculate how they move in a completely different way from just calculating what time they appear in the sky’, ‘Yes’, says the astronomer, ‘and how accurately can you predict eclipses?’ He says, ‘I haven’t developed the thing very far yet’, Then says the astronomer, ‘Well, we can calculate eclipses more accurately than you can with your model, so you must not pay any attention to your idea because obviously the mathematical scheme is better’. There is a very strong tendency, when someone comes up with an idea and says, ‘Let’s suppose that the world is this way’, for people to say to him, ‘What would you get for the answer to such and such a problem?’ And he says ‘I haven’t developed it far enough’. And they say, ‘Well, we have already developed it much further, and we can get the answers very accurately’. So it is a problem whether or not to worry about philosophies behind ideas.”

(Richard Feynman, *Seeking New Laws* [10])

Consider a path integral description of the centre-of-mass motion of a non-relativistic massive object that travels from A to B



PATH INTEGRALS AND THE PRINCIPLE OF STATIONARY ACTION

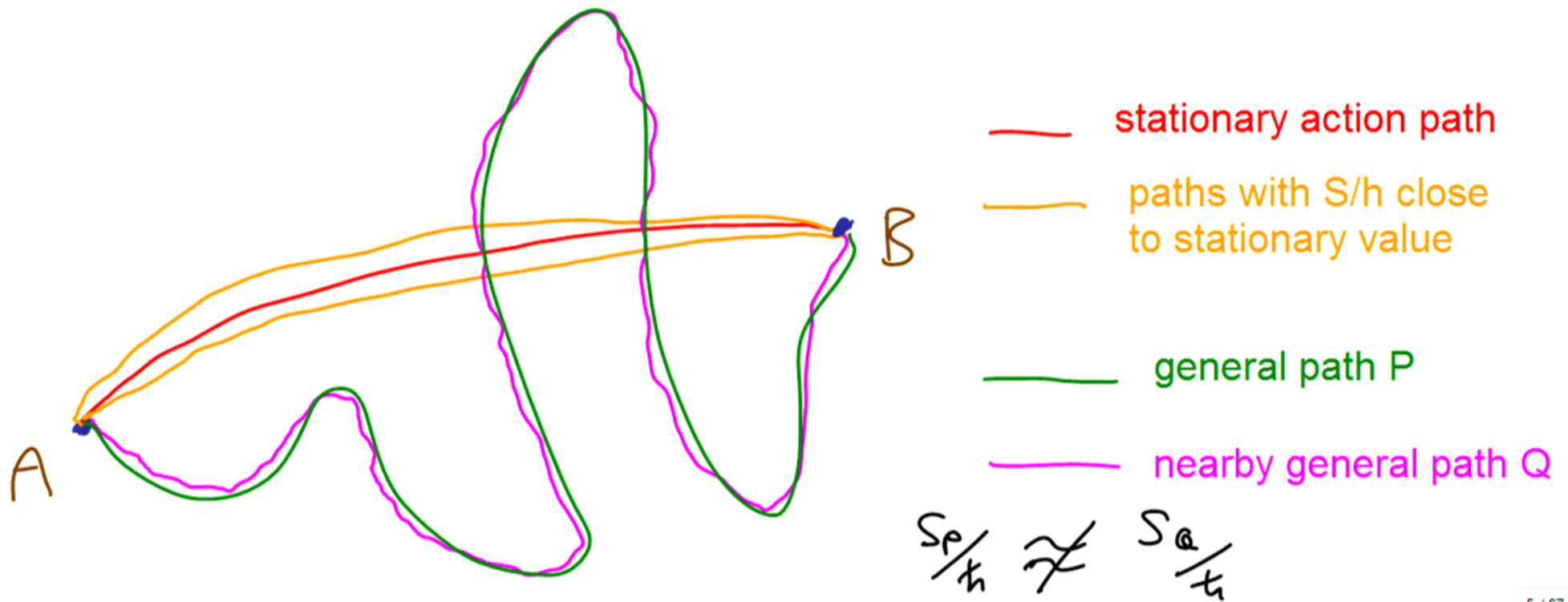
“Before we go on making the mathematics more complete, we shall compare this quantum law with the classical rule. At first sight, from Eq. (2.15), all paths contribute equally, although their phases vary, so it is not clear how, in the classical limit, some particular path becomes most important. The classical approximation, however, corresponds to the case that the dimensions, masses, times, etc., are so large that S is enormous in relation to \hbar . Then the phase of the contribution S/\hbar is some very, very large angle. The real (or imaginary) part of ϕ is the cosine (or sine) of this angle. This is as likely to be plus as minus. Now if we move the path as shown in Fig 2-1 by a small amount δx , *small on the classical scale*, the change in S is likewise small on the classical scale, but not when measured in the tiny units of \hbar . These small changes in path will, generally, make enormous changes in phase, and our cosine or sine will oscillate exceedingly rapidly between plus and minus values. The total contribution will then add to zero; for if one path makes a positive contribution, another infinitesimally close (on a classical scale) makes an equal negative contribution, so that no net contribution arises.

Therefore, no path really needs to be considered if the neighbouring path has a different action; for the paths in the neighbourhood cancel out the contribution. But for the special path $\bar{x}(t)$, for which S is an extremum, a small change in path produces, in the first order at least, no change in S . All the contributions from the paths in this region are nearly in phase, at phase S_{cl}/\hbar , and do not cancel out. Therefore, only for paths in the vicinity of $\bar{x}(t)$ can we get important contributions, and in the classical limit we need only consider this particular trajectory as being of importance. **In this way the classical laws of motion arise from the quantum laws.**

We may note that trajectories which differ from $\bar{x}(t)$ contribute as long as the action is still within about \hbar of S_{cl} . The classical trajectory is indefinite to this slight extent, and this rule serves as a measure of the limitations of the precision of the classically defined trajectory.”

(Feynman and Hibbs, [10]; italics original, bold face added)

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Examining the Feynman-Hibbs argument in a toy model

To see the problem with the Feynman-Hibbs argument more clearly, it is very helpful to separate conceptual questions from the problems of rigorously defining any path integral. To this end we define a toy discrete path integral model (which we will call $M1$) for the centre of mass motion of a single massive object in position space, involving some large finite number of paths from A to B . Because the number of paths is finite, we can rigorously define the path integral and related quantities. We define the model to have a set of paths that have mathematical properties analogous to those of quasiclassical trajectories – those in the neighbourhood of the stationary action path – in the standard quantum path integral. This allows us to focus on the conceptual question of what conclusions about quasiclassical physics do or do not follow from the path integral.

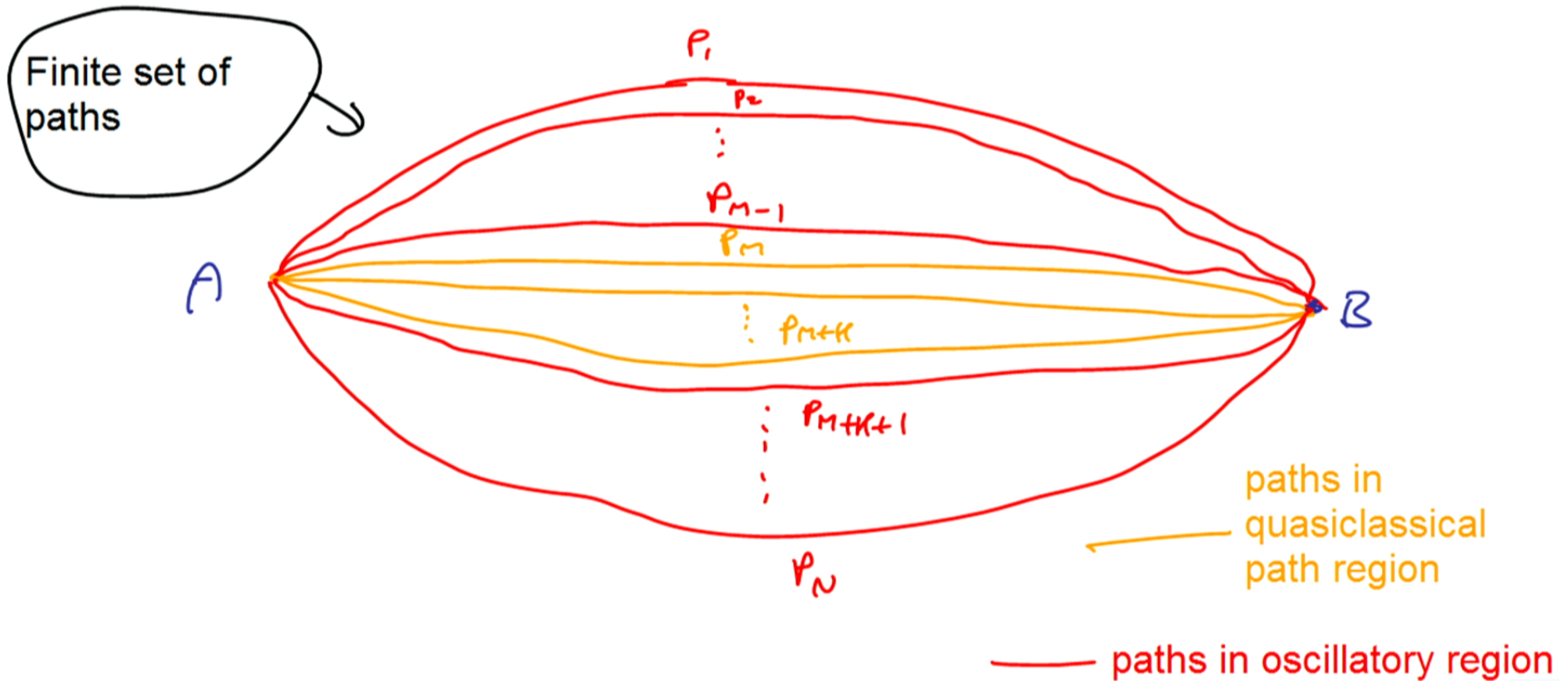
For simplicity, we define the model $M1$ so that each path has phase ± 1 , and we take the paths to have some natural ordering P_1, \dots, P_N , in which paths P_i and P_{i+1} are supposed to be physically adjacent. This is not generally entirely realistic, even in the simple versions of path integrals defined in discrete models of $1+1$ dimensional space-time. However, it simplifies the model while still allowing it to illustrate a key point that can be replicated in more geometrically realistic models.

We suppose also that we can identify paths P_M, \dots, P_{M+K} that correspond approximately to the quasiclassical trajectories for the particle, where M is odd, $N - M - K$ is even, $1 < M < M + K < N$ and $K \ll M, N$. We call these the *quasiclassical paths* in the model. Finally, we suppose that the path amplitudes $A(P_i)$ obey

$$\begin{aligned} A(P_i) &= (-1)^{i-1} \text{ for } 1 \leq i \leq M-1, \\ A(P_i) &= 1 \text{ for } M \leq i \leq M+K, \\ A(P_i) &= (-1)^{i-M-K} \text{ for } M+K < i \leq N. \end{aligned} \tag{1}$$

In other words, the amplitudes alternate in pairs before and after the quasiclassical paths, which all have amplitude

Toy model of paths for massive object



oscillatory
regionquasiclassical
regionoscillatory
region

paths

 p_1, p_2, \dots, p_{M-1}
 $p_M, p_{M+1}, \dots, p_{M+K}$
 p_{M+K+1}, \dots, p_N

amplitudes

 $|, -1, |, \dots, -1$
 $|, |, \dots, |$
 $-1, |, \dots, |$

alternating sign

constant (+1)

alternating sign

Since the path amplitudes either side of the quasiclassical paths cancel in pairs, we have the arithmetical identity

$$\sum_{i=1}^N A(P_i) = \sum_{i=M}^{M+K} A(P_i) = K+1$$

So we can indeed calculate the total sum – our discrete version of the path integral – by summing the amplitudes of the quasiclassical paths and ignoring the rest. But notice that this property *per se* does not single out the quasiclassical paths in our model as special. For example, we could also write

$$\sum_{i=1}^N A(P_i) = \sum_{i=1}^{K+1} A(P_{2i-1}) = K+1$$

More generally,

$$\sum_{i=1}^N A(P_i) = \sum_{i \in I} A(P_i) = K+1$$

for any size $(K+1)$ subset I of the set

$$\{1, 3, \dots, M, M+1, \dots, M+K, M+K+2, \dots, N\} = \{i : A(P_i) = +1\},$$

that is, the set of paths with amplitude $+1$.

It is true that the quasiclassical paths are all adjacent in our ordering, while the other paths are not. But nothing in the definition of the path integral or any standard presentation of its physical implications gives a special *ontological* status to subsets of adjacent paths. To derive classical laws of motion, we need to be able to make a statement about the actual trajectories of macroscopic objects. In particular, here, we need to be able to derive that the object follows one of the quasiclassical paths from A to B . This does not follow from the rules of standard path integral quantum theory, as set out in Feynman and Hibbs or elsewhere.

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The path probability postulate

If we can't extract a sensible explanation of quasiclassical physics (or indeed do anything more than calculate transition probabilities) from the path integral in its present form, then perhaps we need to change the definition of the path integral, or add additional postulates, or both. The difficulty in making physical sense of the path integral seems to be connected with the fact that it hints at an interpretation in which paths have probabilities, while at the same time suggesting conflicting intuitions about these physical path probabilities. We thus propose to explore the implications of explicitly assigning probabilities to paths via a new postulate. The idea here is that we define a quantity $\text{Prob}(P)$ that has the standard properties of a probability:

New postulate
for path
probability

$$\text{Prob}(P) \geq 0 \quad \int_{\text{paths } P} dP \text{Prob}(P) = 1.$$

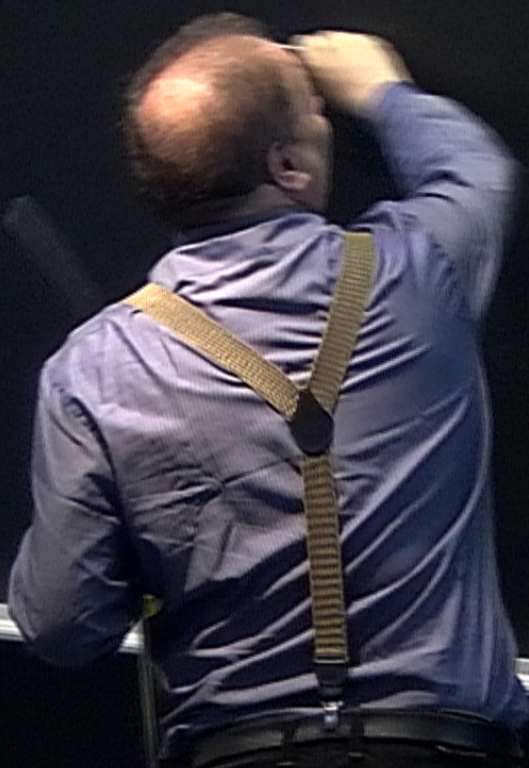
This quantity represents the probability that the given path P was actually followed. Physical reality – in an experiment, or, in principle, in the evolution of the cosmos from initial to final state – is given by the chosen path.

Specifically, we will consider a postulate of the form:

$$\text{Prob}(P) = C \left| \int dQ \exp(-iS(Q)) \exp(-d(P, Q)) \right|^2 \left(\int dQ \exp(-d(P, Q)) \right)^{-1}. \quad (2)$$

needs
distance
measure
 $d(P, Q)$

Here and below we take $\hbar = 1$. For the moment we take the integrals in this expression to be over all paths Q that have the same endpoints (A and B) as P . (Note that we would need to allow larger classes of paths to obtain an effective description of experiments with an extended initial wave function or to discuss the general possibilities allowed in cosmology.) We take $d(P, Q)$ to be some distance measure defined between paths P and Q . This measure d is supposed in some natural sense (to be elaborated) to say how distinct the paths are.

A man with short brown hair, wearing a blue long-sleeved shirt and tan suspenders, is seen from the back, writing on a blackboard. He is holding a piece of chalk in his right hand, which is raised towards the equation. The blackboard is large and dark, with the equation written in white chalk. To the right of the blackboard, a portion of a projection screen is visible, showing text in pink and blue.
$$\text{Prob}(P) = c \left| \int dQ e^{-iS(Q)} \exp(-d(P, Q)) \right|^2$$

needs
distance
measure
 $d(P, Q)$

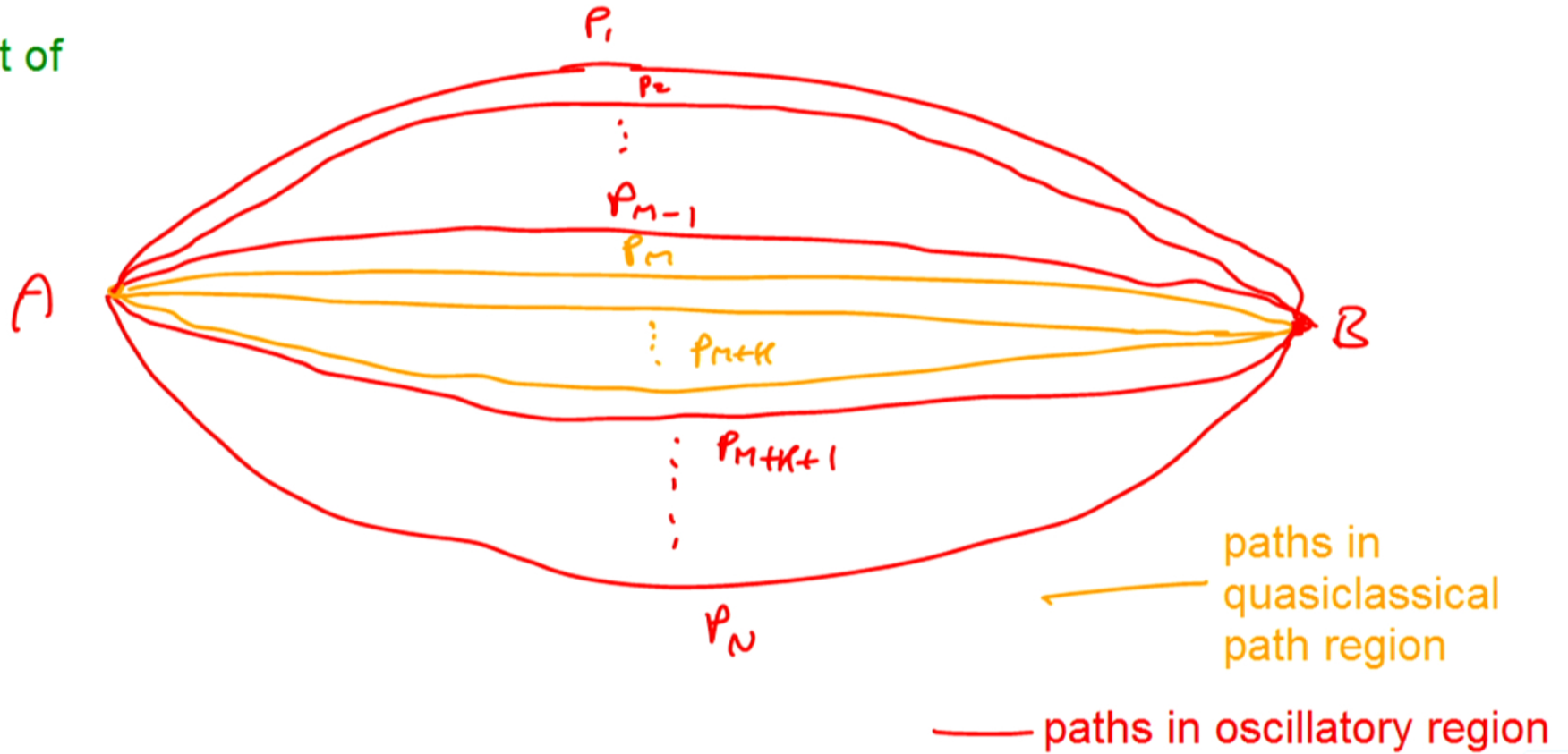
We'll try:

$$\text{Prob}(P) = \frac{1}{\left(\int dQ e^{-\beta S(Q)} \exp(-d(P, Q)) \right)^2}$$

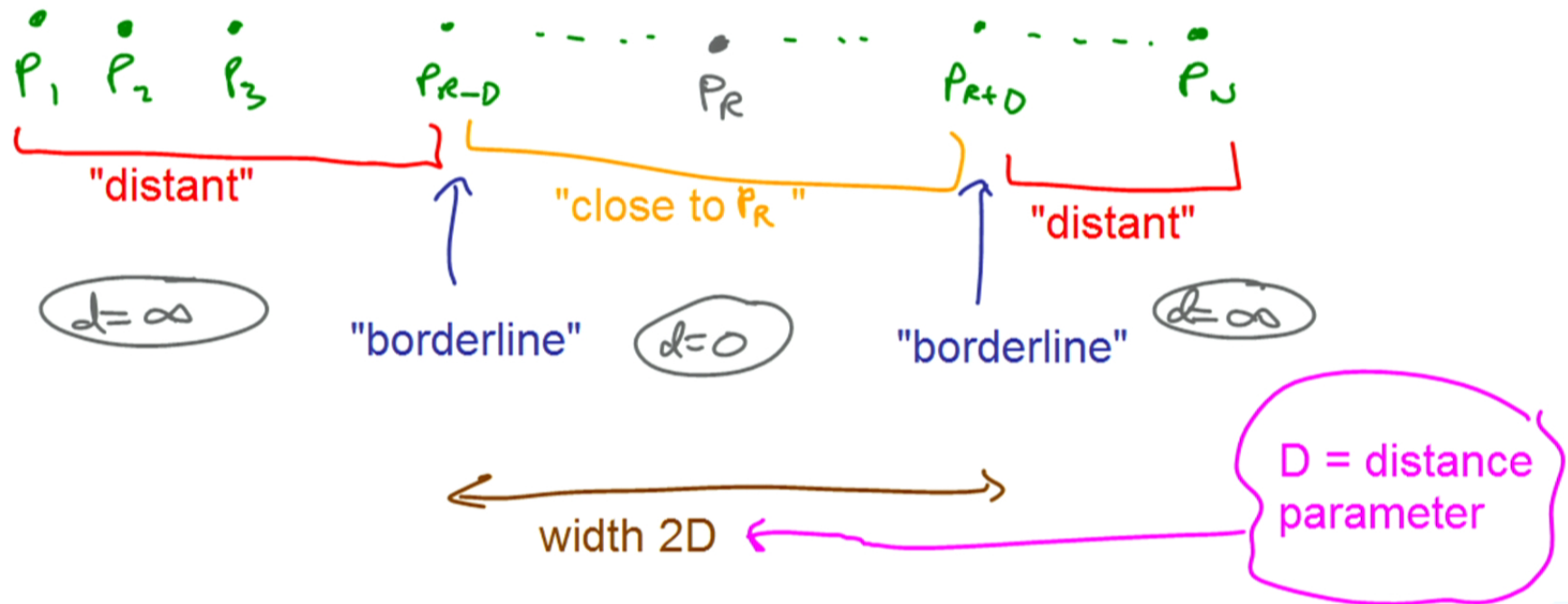
$$\int dQ \exp(-d(P, Q))$$

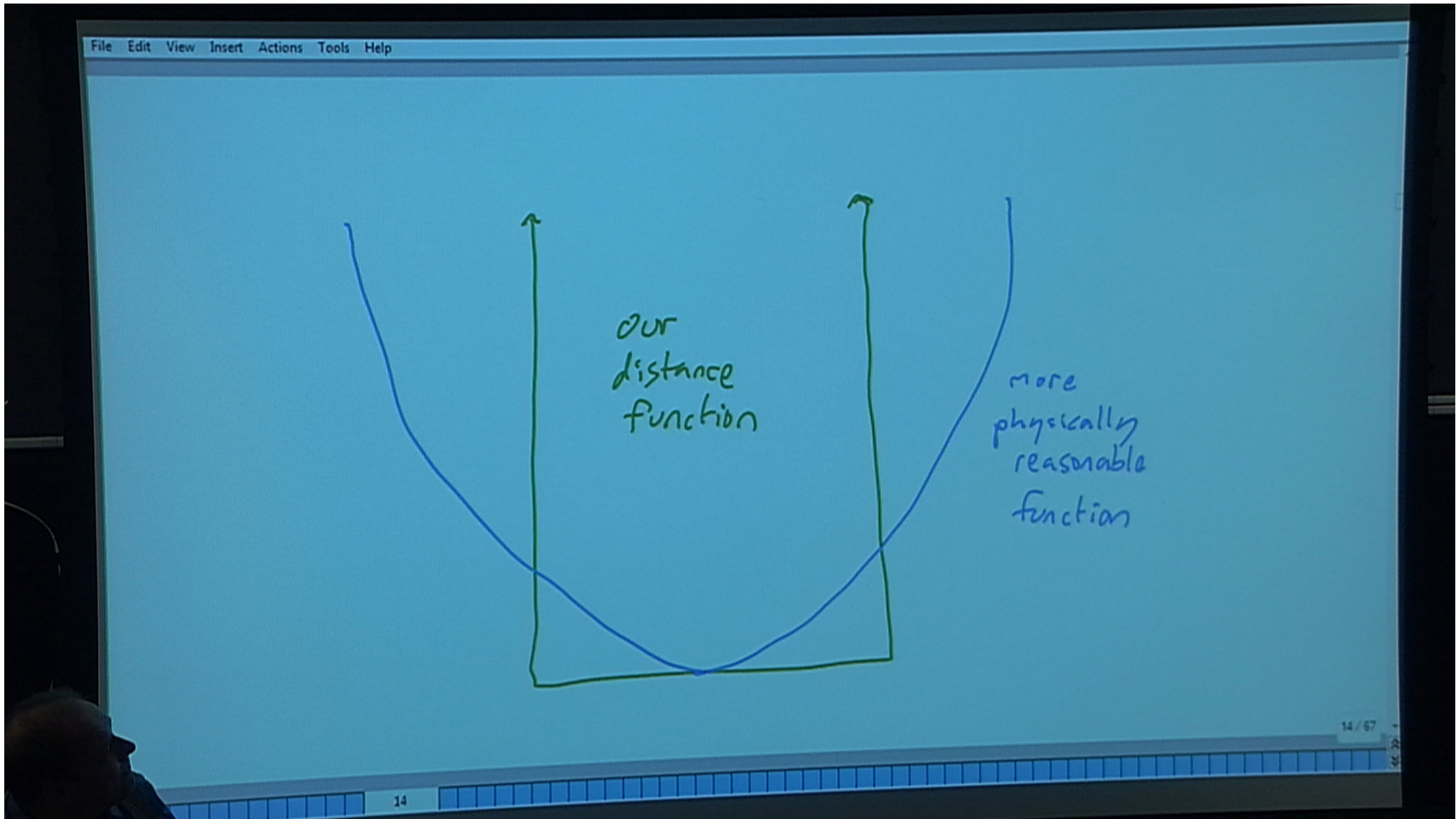
needs
distance
measure
 $d(P, Q)$

Toy model of paths for massive object

Finite set of
paths

A simple distance function for a toy model with linearly ordered paths





Toy model path amplitudes

$$\begin{aligned} A(P_i) &= (-1)^{i-1} && \text{for } 1 \leq i \leq M-1, \\ A(P_i) &= 1 && \text{for } M \leq i \leq M+K, \\ A(P_i) &= (-1)^{i-M-K} && \text{for } M+K < i \leq N, \end{aligned}$$

as before (6)

and thus the path amplitudes listed in order from A_1 to A_N are

$$1, -1, \dots, 1, -1, 1, 1, \dots, 1, -1, 1, \dots, -1, 1.$$

Toy model
distance
function

We now take the distance function to be

$$\begin{aligned} d(P_i, P_j) &= 0 && \text{if } |i-j| < D, \\ d(P_i, P_j) &= \infty && \text{if } |i-j| > D, \\ d(P_i, P_j) &= \log(1/2) && \text{if } |i-j| = D. \end{aligned} \quad (7)$$

This infinite step function is meant as a simplifying approximation to something more natural, such as

$$d(P_i, P_j) = \exp(|i-j|/D).$$

Here we require $D \ll N$, $M > 2D + 1$, and $N - M - K > 2D + 1$.


Since we are now working within real path quantum theory, we can apply (2) to obtain an explicit expression for the probability of the particle following any given path:


$$\text{Prob}(P_i) = C(2D)^{-1} \left| \sum_{|j-i| < D} A(P_j) + \sum_{|j-i|=D} \frac{1}{2} A(P_j) \right|^2, \quad (8)$$

where C is the normalisation factor ensuring that

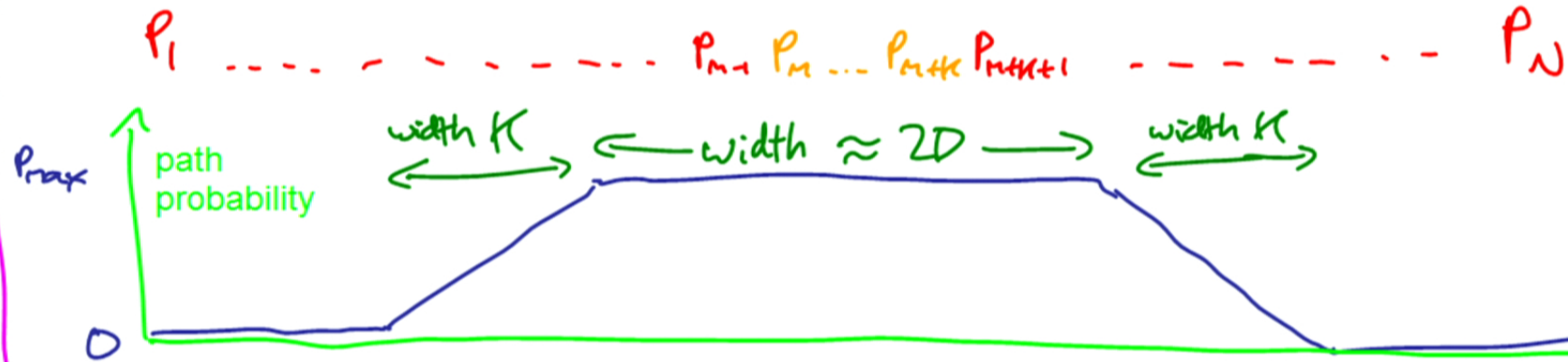
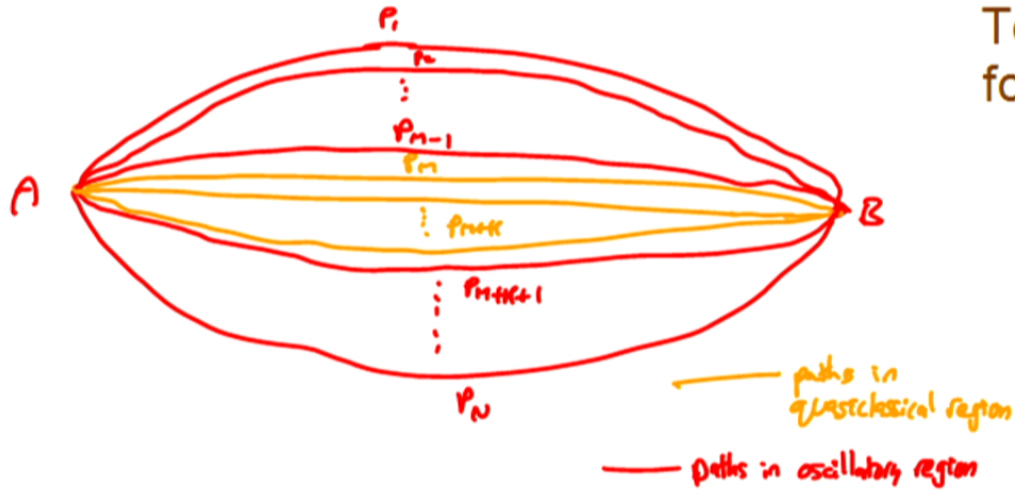
$$\sum_{i=1}^N \text{Prob}(P_i) = 1.$$

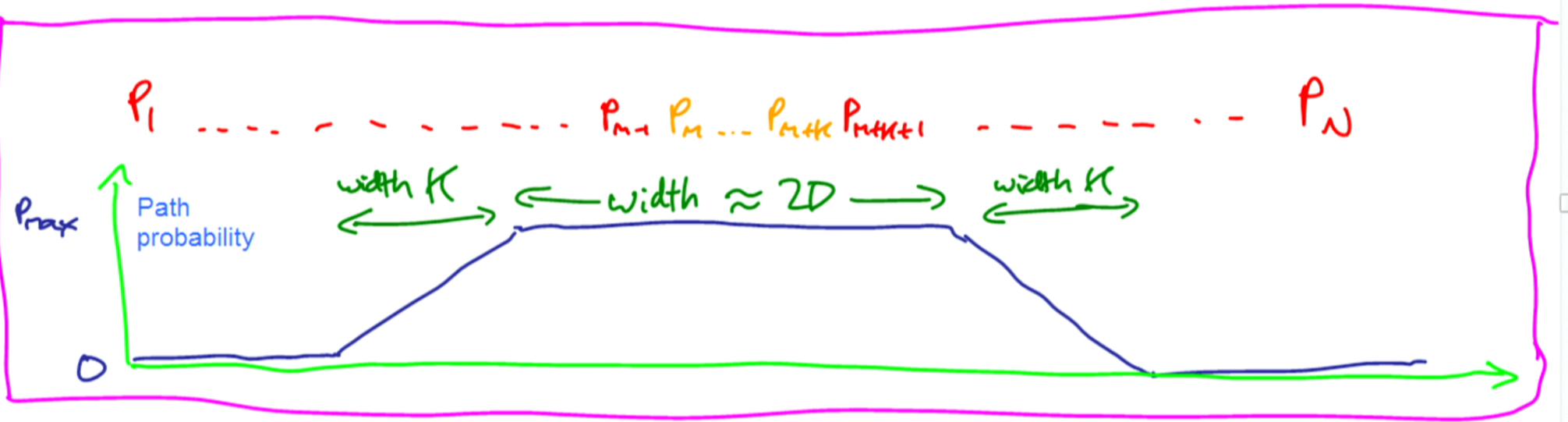
Given distance
function and
amplitudes,
our postulate
defines path
probabilities.

paths	$P_1, P_2 \dots P_{M-1} P_M P_{M+1} \dots P_{M+K} P_{M+K+1} \dots P_N$
amplitudes	$1 \ -1 \ 1 \ \dots \ -1 \ 1 \ 1 \ \dots \ 1 \ -1 \ 1 \ \dots \ 1$ 
$\exp(-d(P_i, P_R))$ ($R < M-D$)	$0 \dots 0 \ \frac{1}{2} \ 1 \ \dots \ 1 \ \frac{1}{2} \ 0 \dots 0 \ - \ - \ - \ - \ - \ - \ - \ - \ - \ - \ 0$
$A(P_i) \exp(-d(P_i, P_R))$	$0 \dots 0 \ -\frac{1}{2} \ 1 \ -1 \ \dots \ -1 \ 1 \ -\frac{1}{2} \ 0 \ - \ - \ - \ - \ - \ - \ - \ - \ - \ - \ 0$
$\text{Prob}(P_R) = \frac{C}{2D} \left \sum_{i=1}^N A(P_i) \exp(-d(P_i, P_R)) \right ^2 = 0$	

paths	$P_1, P_2 \dots P_{M-1} P_M P_{M+1} \dots P_{M+K} P_{M+K+1} \dots P_N$
amplitudes	$1 \ -1 \ 1 \ \dots \ -1 \ 1 \ 1 \ \dots \ 1 \ -1 \ 1 \ \dots \ 1$ 
$\exp(-d(P_i, P_R))$ ($M < R < M+K$)	$0 \dots 0 \quad \frac{1}{2} \ 1 \ 1 \ 1 \ \dots \ 1 \ \frac{1}{2} \ 0 \ \dots \ 0$
$A(P_i) \exp(-d(P_i, P_R))$	$0 \dots 0 \quad \dots \ -\frac{1}{2}(-1 \ 1 \ \dots \ 1 \ \frac{1}{2} \ 0 \dots 0 \ \dots \ 0$
(Here we take $2D > K$)	$Prob(P_R) = \frac{C}{2D} \left(\sum_{i=1}^N A(P_i) \exp(-d(P_i, P_R)) \right)^2 = \frac{C}{2D} (R+D-M)^2$

Toy model path probabilities for our choice of distance function





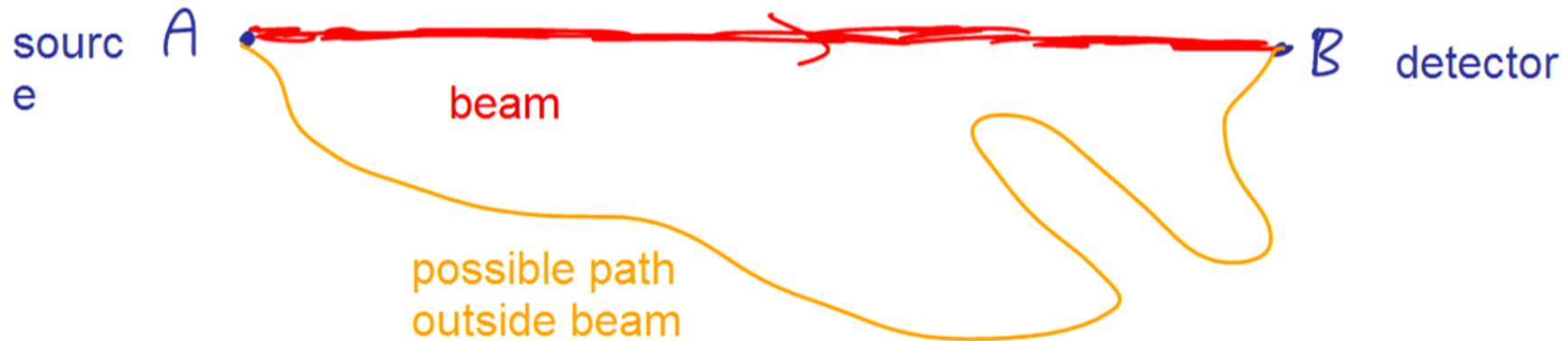
In this toy model, our path probability postulate succeeds where the Feynman-Hibbs argument fails:

It gives a realist ontology.

It explains why a approximately quasiclassical path is followed.

N.B. it requires a new parameter D (more generally a new distance function $d(P, Q)$).

The same toy model and discussion applies to a single microscopic particle beam (we require $2D \gg K$ for a sensible model here)



If we take the toy model seriously as an ontological guide here, it suggests the possible realised paths need only be D -close to the beam, with $D \gg K \sim \text{beam width}$

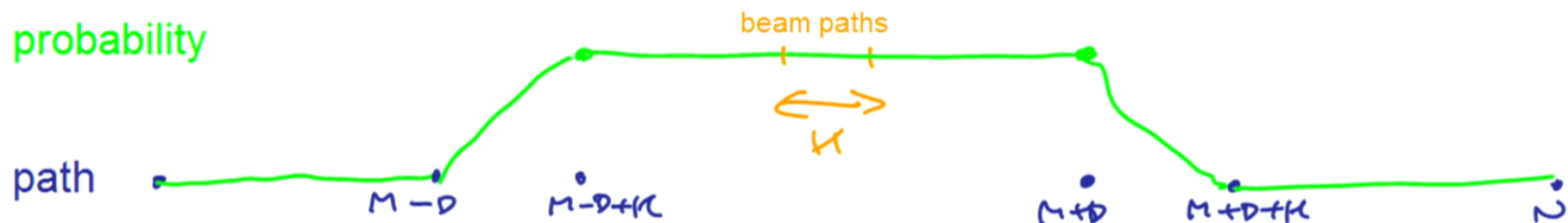
Modelling a single particle beam

If we think of $M1$ as modelling a single microscopic particle beam between source A and detector B , the most immediately interesting case for us is $2D > K$ (in particular $2D \gg K$, though the calculations depend only on whether $2D$ or K is larger). This parameter choice captures the intuition that any model that alters the predictions of quantum theory by introducing some intrinsic decoherence should ensure that paths lying within a single beam of a microscopic quantum particle are very far from decohering. (We see no decoherence for microscopic particles even in interference experiments involving multiple separated beams – a scenario we will model later.)

For $2D > K$, and for i further than D from the ends of the list of paths, i.e. in the range $D < i < N - D$, this gives

$$\begin{aligned} \text{Prob}(P_i) &= 0 && \text{if } D+1 < i < M-D \text{ or } N-(D+1) > i > M+K+D, \\ \text{Prob}(P_i) &= C(2D)^{-1}|K|^2 && \text{if } M+K-D < i < M+D, \\ \text{Prob}(P_i) &= C(2D)^{-1}|(i+D-M)|^2 && \text{if } M < i+D < M+K, \\ \text{Prob}(P_i) &= C(2D)^{-1}|(M+K-i+D)|^2 && \text{if } M < i-D < M+K. \end{aligned} \quad (9)$$

Calculations in toy model for $2D > K$



Recap

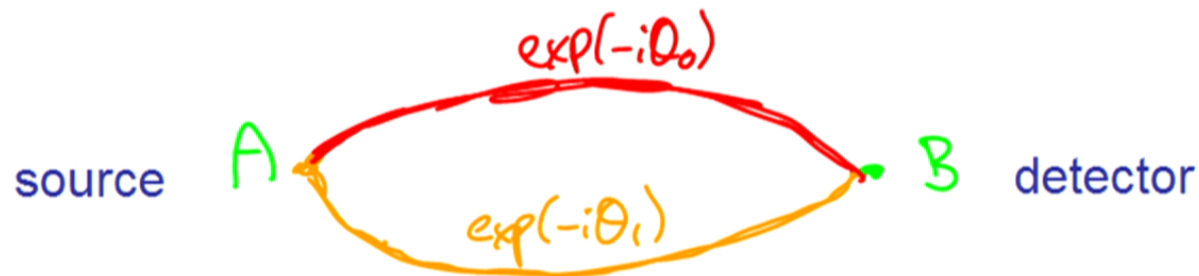
This toy model thus gives a realist ontology that tells us that, if the particle goes from A to B , then it follows a definite path. It respects physical intuition, in that the realised path will be close to the region of constant phase in path space. How close depends on the parameter D that characterizes our chosen distance function in this model.

Although our realist ontology is not part of standard quantum theory, its dependence on K in this model is consistent with standard intuitions. Our parameter K here models (in a loose intuitive sense, since we are not considering measures on the set of realistic paths here) the size of the set of paths around the stationary path for which (S/\hbar) is approximately constant in standard path integral quantum theory. The Feynman-Hibbs argument discussed earlier also aims to select a set of roughly equally relevant paths of similar phase around the stationary point of the action.

However, a new feature of our models is that the ontology and hence the physical predictions also depend on the distance function d – in this case via the parameter D . The physically relevant paths in our ontology – those with nonzero probability – are not only those in the stationary phase region, but also those d -close to that region. A more fundamental new feature of our models, of course, is that they *have* a realist ontology: as already noted, the standard Feynman-Hibbs intuitions have no logical justification in standard path integral quantum theory.

Modelling particle beam interferometry

Consider two beam case (works equally well for multiple beams)



amplitudes	$ \dots $	$e^{-i\theta_0} \dots e^{i\theta_0}$	$ \dots $	$e^{-i\theta_1} \dots e^{i\theta_1}$	$ \dots $
paths	$p_1 \dots p_{M_0-1}$	$p_{M_0} \dots p_{M_0+N_0}$	$p_{M_0+N_0+1} \dots p_{M_1}$	$p_{M_1+N_1} \dots p_{M_1+N_1+N_1}$	$p_{M_1+N_1+N_1+1} \dots p_N$
"location"	"outside"	"paths in beam 0"	"between"	"paths in beam 1"	"outside"

Toy model predictions for $2D < M1 - M0 - K0$ (i.e. where beams are d-separated)

$$\text{Prob}(P_i) \approx C \left| (K_j + 1) \exp(-i\theta_j) \right|^2 \frac{1}{2D} = C (K_j + 1)^2 \frac{1}{2D}$$

for typical model paths P_i that are D-dose to beam j ($=0$ or 1)

$$\text{Prob}(P_i) \approx 0$$

for other paths.

i.e. the model's real path ontology gives decoherence for d-separated beams
- the real path "chooses" to be associated with one beam
or the other.

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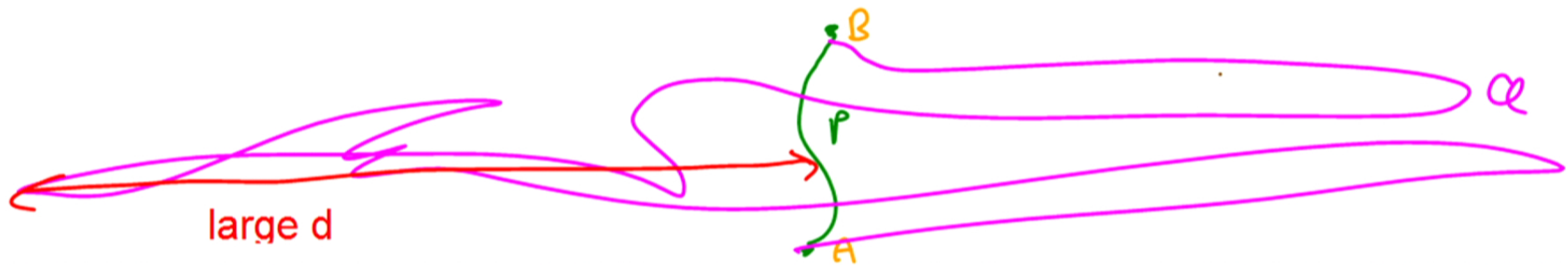
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Comment on realistic models and rigour

Obviously, finding an ontology that gives sensible and suggestive results in discrete toy models is no guarantee that the real path probability postulate (with any choice of path distance function) can produce well-defined and sensible and analogous results in realistic models.

Still, it's worth noting that natural choices of distance function can suppress contributions from (some? many ?? most??? almost all????) "pathological" paths:



Comment on realistic models and rigour (cont.)

The formalism also lends itself to other ideas for suppressing pathological paths (see 1305.3565 for some comments).

I don't know if any of them produces rigorously defined path probabilities in realistic continuous models.

(As Piet Hein reminds us, T.T.T.)

Put up in a place
where it's easy to see
the cryptic admonishment
T.T.T.

When you feel how depressingly
slowly you climb,
it's well to remember that
Things Take Time.

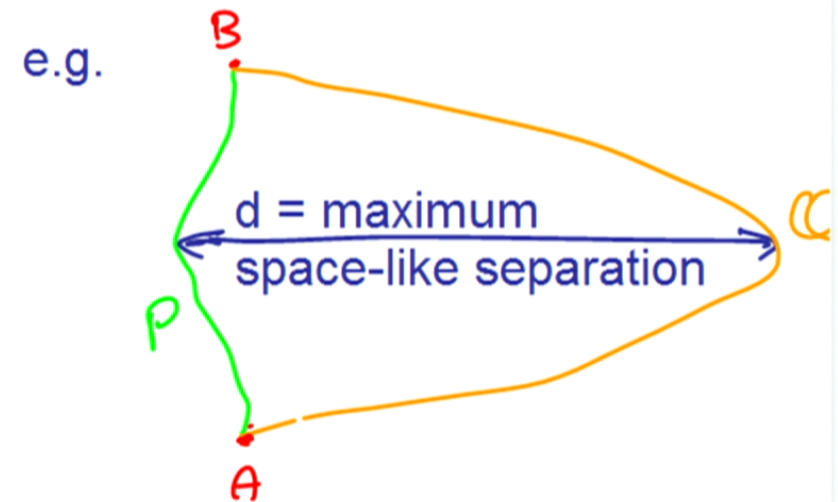
(Piet Hein, from *Collected Grooks*)



Comments on distance function 1

It's easy to find seemingly natural Lorentz or generally covariant options for distance functions in Minkowski or other Lorentzian space-times.

Whether the idea of real path quantum theory works technically is unknown, but at least there is **no purely conceptual obstacle to relativistic versions.**



Comments on distance function 2

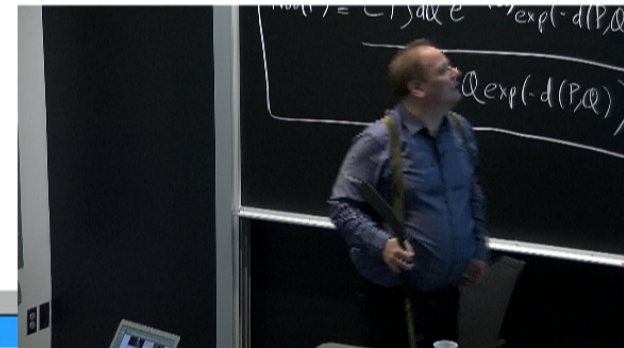
With the same crucial caveat about lack of technical understanding of the possibility or otherwise of rigorous definitions of path probabilities

... one can also imagine natural seeming choices of distance function for multi-particle configuration space paths, field configuration paths, perhaps (?) even paths in geometry space relevant to quantum gravity

Comments on distance function 3

We do not have any strong constraints on the choice of distance function. If one takes real path quantum theory seriously, even if only as a metaphor, and if no stronger constraints on d are found, it suggests testing quantum interference in any unexplored range defined by possibly natural parameters:

(Just to mention a few examples:
maximum beam separation in space,
average beam separation in space,
object mass,
beam separation time,)



Ontological comment

(Once again with the caveat that we don't presently have a theory outside toy models)

If the idea worked in realistic models, there would be no issue with double ontologies and no problem of tails.

The wave function has no fundamental role.

We simply have one real path drawn from a probability distribution.

