Title: Neutrino as Majorana zero modes

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Abstract: The existence of three generations of neutrinos and their mass mixing is a deep mystery of our universe. Majorana's elegant work on the real solution of Dirac equation predicted the existence of Majorana particles in our nature, unfortunately, these Majorana particles have never been observed. In this talk, I will begin with a simple 1D condensed matter model which realizes a T^2=-1time reversal symmetry protected superconductors and then discuss the physical property of its boundary Majorana zero modes. It is shown that these Majorana zero modes realize a T^4=-1 time reversal doubelets and carry 1/4 spin. Such a simple observation motivates us to revisit the CPT symmetry of those ghost particles--neutrinos by assuming that they are topological Majorana particles made by four
Majorana zero modes. Interestingly, we find that Majorana zero modes will realize a P^4=-1 parity symmetry as well. It can even realize a nontrivial C^4=-1 charge conjugation symmetry, which is a big surprise from a usual perspective that the charge conjugation symmetry for a Majorana particle is trivial. Indeed, such a C^4=-1 charge conjugation symmetry can be promoted to a Z_2 gauge symmetry and its spontaneously breaking leads to the origin of neutrino mass. We further attribute
the origin of three generations of neutrinos to three distinguishable ways of defining two complex fermions from four Majorana zero modes.
The above assumptions lead to a D2 symmetry in the generation space and uniquely determine the mass mixing matrix with no adjustable parameters! In the absence of CP violation, we derive
\theta_12=32degree, \theta_23=45degree and \theta_13=0degree, which is intrinsically closed to
the current experimental results. We further predict an exact mass ratio of the three mass eigenstate with m_1/m_3=m_2/m_3=3/\sqrt{5}.

Neutrino as Majorana zero modes

Zheng-Cheng Gu (Caltech/PI)
(Z.C. Gu, 2013, to appear)

Emergence & Entanglement II
PI
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Outlines

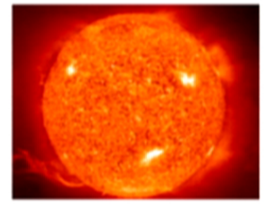
- The definition of Majorana zero modes and its realization in condensed matter systems.

Neutrino

Neutrino as a ghost particle:

- Extremely weak interactions with other particles.
- Almost vanishing rest mass.
- Billions of neutrinos surrounding us!

Current experimental progress toward ghost hunting

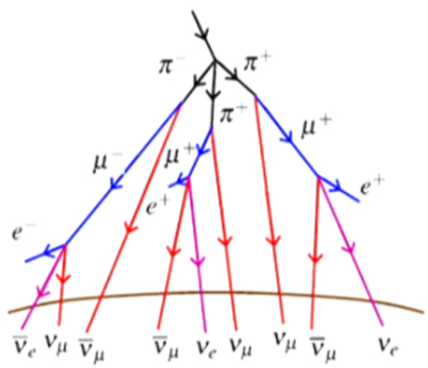


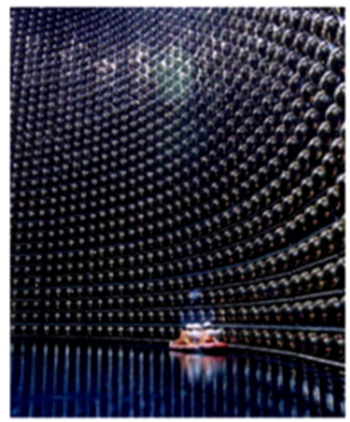
Three-generation mixing matrix:

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}} \times \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \times \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}$$

Dominates: "Atmospheric" "Solar"

$$\approx \begin{pmatrix} 0.85 & 0.53 & 0 \\ -0.37 & 0.60 & 0.71 \\ 0.37 & -0.60 & 0.71 \end{pmatrix} \quad \theta_{23} = 45° \pm 8 \quad \theta_{13} \approx 0 \quad \theta_{12} = 32.3° \pm 1.6$$





(Japan)

(Canada)

(from Michaelmas Term 2009, Prof Mark Thomson)

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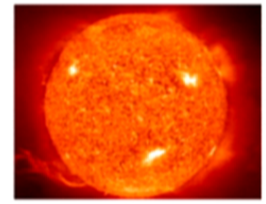

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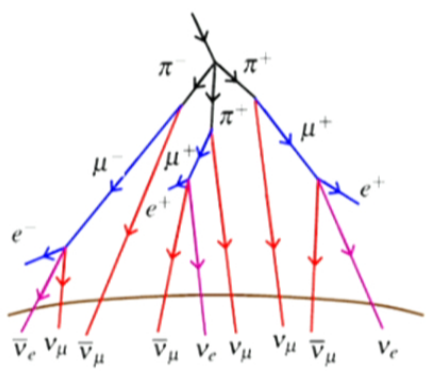

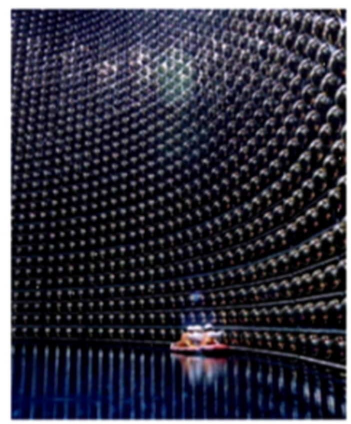

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Stirring neutrino and see-saw mechanism

Stirring neutrino and see-saw mechanism

Standard model predicts exactly zero mass for neutrino!

A direct majorana mass term for light neutrino is not allowed in Standard Model(SM) since it breaks electric-weak symmetry.

See-saw mechanism and massive stirring neutrino.

$$M_{total} = \begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix} \quad m_D \ll M \quad \text{GUT scale}$$

$$m_1 \sim m_D^2/M; \quad m_2 \sim M$$

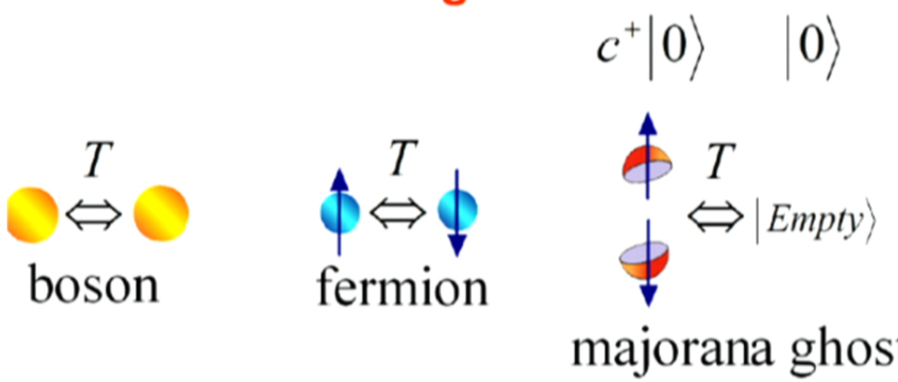
- But if the stirring neutrino does not carry any gauge charge, why it interacts with the light neutrino?
- Why it is so massive and where does the mass come from?

What's the time reversal symmetry of a Majorana particle

- Boson has $T^2=1$ time reversal symmetry.
- Fermion=half boson, therefore it has $T^2=-1$ time reversal symmetry.
- But can we say Majorana fermion=half fermion and it should carry $T^4=-1$ symmetry?

No. It is just different representation of complex fermion and of course has $T^2=-1$ time reversal symmetry.

Yes. Majorana ghosts formed by a pair of Majorana zero modes do have such a strange behavior!



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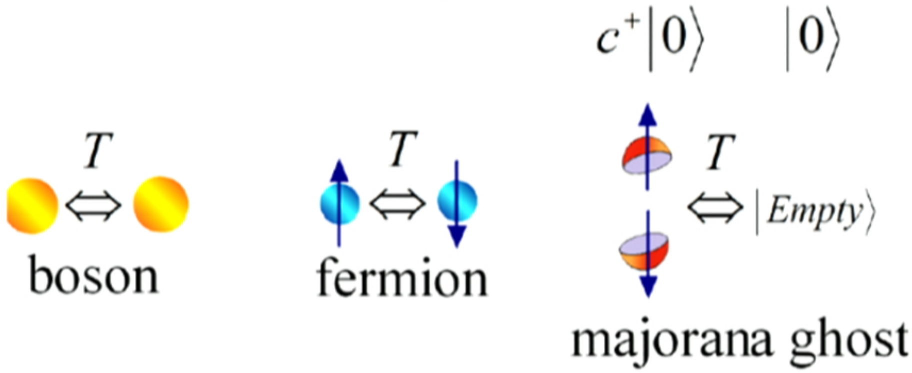


Symmetry fractionalization and symmetry protected topological order

Spin one Haldane chain realizes 1D topological order(even with strong interaction)

$$H = \sum_i (\mathbf{S}_i \cdot \mathbf{S}_{i+1} + U(S_i^z)^2)$$

- stable up to U~1

But Haldane phase requires symmetry protection!

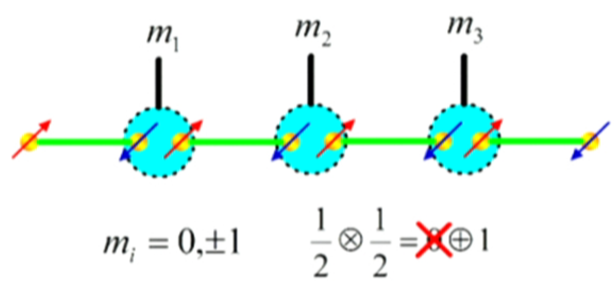
- Haldane phase can be protected by many kinds of symmetries: e.g. time reversal

Z C Gu, *et al*, 2009, F Pollmann, *et al*, 2010

AKLT model realizes Haldane phase

$$H = \sum_i P_2(\mathbf{S}_i + \mathbf{S}_{i+1})$$
$$= \sum_i [\tfrac{1}{2}\mathbf{S}_i \cdot \mathbf{S}_{i+1} + \tfrac{1}{6}(\mathbf{S}_i \cdot \mathbf{S}_{i+1})^2 + \tfrac{1}{3}].$$

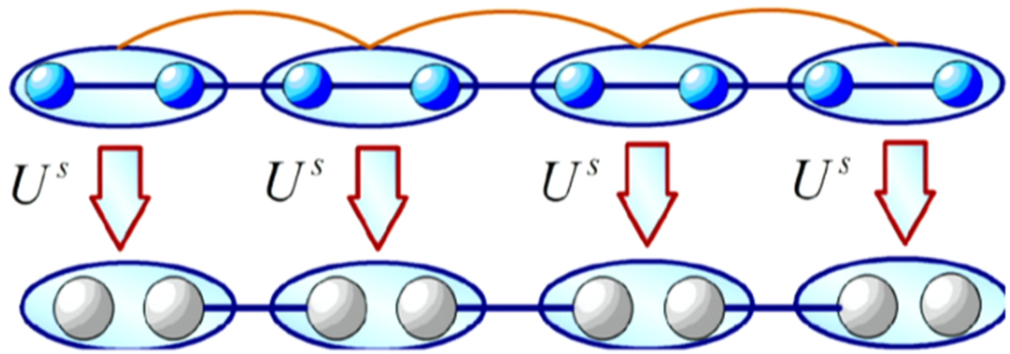
FDM Haldane, 1983, Ian Affleck,*et al,* 1987



RG Fixed point wavefunction of 1D SPT phases

correlation length $\xi = 2$

SPT phase is a gapped quantum phase with unique ground state



AKLT state decouples to spin-(1/2,1/2) dimer model

Projective representation on each dimer end

$$u_L(g_1)u_L(g_2) = \omega(g_1,g_2)u_L(g_1,g_2); \quad u_R(g_1)u_R(g_2) = \omega(g_1,g_2)^{-1}u_R(g_1,g_2)$$

$$u_{L(R)}(g) \sim \beta_{L(R)}(g)u_{L(R)}(g); \quad \beta_{L(R)}(g) \in U(1)$$

T^4=-1 time reversal symmetry

The classification of symmetry protected topological (SPT) orders leads to the concept of T^4=-1 time reversal symmetry.

- Hilbert space for interacting systems: (Phys. Rev. B 83, 035107 (2011))

$$c_\uparrow^\dagger|0\rangle, c_\downarrow^\dagger|0\rangle, |0\rangle, c_\uparrow^\dagger c_\downarrow^\dagger|0\rangle$$

- T^2=-1for fermion parity odd sector and T^2=1 for fermion parity even sector. The total symmetry group is extended over fermion parity symmetry group {I,P^f}, which is indeed T^4=1.

$\{I, T, T^2, T^3\}$ (Phys. Rev. B 84, 235128 (2011))

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- According to the classification of 1+1D SPT phases, the edge majorana modes should carry projective representation with T^4=-1

Explicit time reversal operator for a Majorana doublet:

$$T = UK \quad \boxed{U = \frac{1}{\sqrt{2}}(1 + \gamma_\uparrow\gamma_\downarrow) = e^{\frac{\pi}{4}\gamma_\uparrow\gamma_\downarrow}}$$

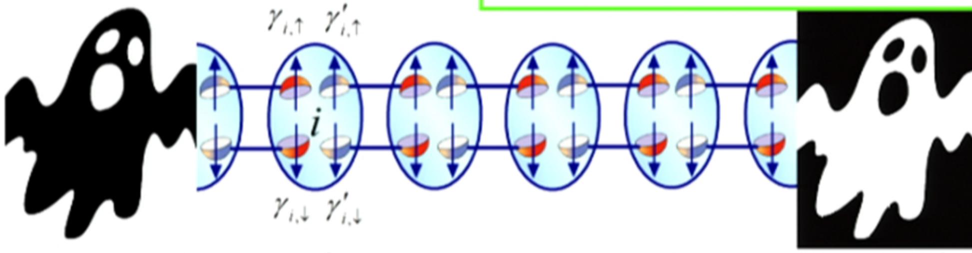
- a phase gate with T^4=-1

Representation theory and 1/4 spin

Majorana ghosts arise on the edge:

$$c_L = \frac{1}{2}(\gamma_\uparrow + i\gamma_\downarrow); \quad c_R = \frac{1}{2}(\gamma'_\uparrow - i\gamma'_\downarrow)$$

$$Tc_LT^{-1} = -ic_L^\dagger; \quad Tc_RT^{-1} = ic_R^\dagger$$



Representation theory(must be two dimensional):

$$T|0\rangle = UK|0\rangle = U|0\rangle = |1\rangle \equiv c^\dagger_{L(R)}|0\rangle \qquad U = \begin{pmatrix} 0 & 1 \\ \pm i & 0 \end{pmatrix}$$

$$\begin{aligned} T|1\rangle &= UKc^\dagger_{L(R)}|0\rangle = Uc^\dagger_{L(R)}|0\rangle \\ &= Tc^\dagger_{L(R)}T^{-1}T|0\rangle = \pm ic_{L(R)}c^\dagger_{L(R)}|0\rangle = \pm i|0\rangle \end{aligned}$$

1/4 spin:

$$S_L = \frac{|S|}{2}\sum_{\sigma\sigma'}\gamma_\sigma\sigma^y_{\sigma\sigma'}\gamma_{\sigma'} = \frac{|S|}{2}\sum_{\sigma\sigma'}i\gamma_\sigma\epsilon_{\sigma\sigma'}\gamma_{\sigma'} = \frac{i|S|}{2}(\gamma_\uparrow\gamma_\downarrow - \gamma_\downarrow\gamma_\uparrow) = i|S|\gamma_\uparrow\gamma_\downarrow = -|S|P^f_L$$

$$S_y = \frac{i}{2}(c^\dagger_\uparrow c_\downarrow - c^\dagger_\downarrow c_\uparrow) = \frac{i}{2}\left[(\gamma_\uparrow - i\gamma'_\uparrow)(\gamma_\downarrow - i\gamma'_\downarrow) - (\gamma_\downarrow + i\gamma'_\downarrow)(\gamma_\uparrow + i\gamma'_\uparrow)\right] = -|S|(P^f_L + P^f_R)$$

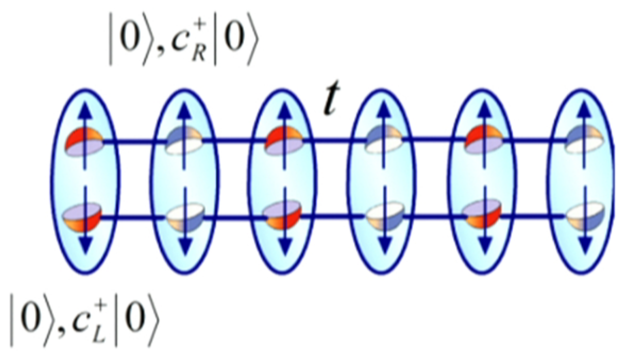
Representation theory in the zero energy subspace

What really happens?

Majorana zero modes in high dimensions

Deconfinement of 1/4 spin at critical point - a baby neutrino model in 1D

Similar to the Haldane chain, where 1/2 spin deconfines at critical point. 1/4 spin also deconfines at critical point.



$$H = it \sum_{\langle i\in A j\in B\rangle} (c^\dagger_{L,i} c_{R,j} - c^\dagger_{R,j} c_{L,i})$$

$$E_k = \pm 2t|\cos\frac{k}{2}|$$

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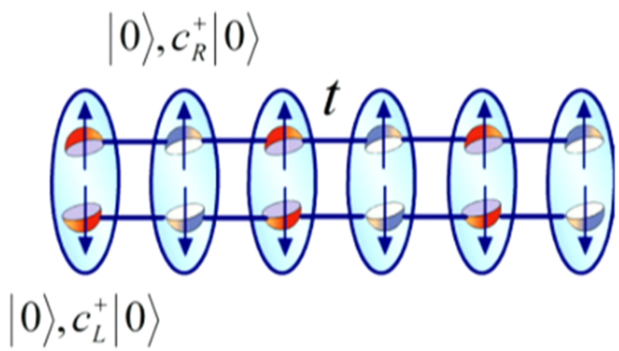

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$$E_k = \pm 2t |\cos \tfrac{k}{2}|$$

P^4=-1 parity symmetry for Majorana zero modes

It is not a surprise that we can define a P^4=-1 parity symmetry for a pair of Majorana spinons as well.

- We only consider the parity action on internal space here, we will include its spacial action in quantum field theory later.

$$P_{\uparrow\uparrow'} = \frac{1}{\sqrt{2}}(1+\gamma_\uparrow\gamma'_\uparrow) = e^{\frac{\pi}{4}\gamma_\uparrow\gamma'_\uparrow}; \quad P_{\downarrow\downarrow'} = \frac{1}{\sqrt{2}}(1-\gamma_\downarrow\gamma'_\downarrow) = e^{-\frac{\pi}{4}\gamma_\downarrow\gamma'_\downarrow},$$

$$\begin{aligned} P\gamma_\uparrow P^{-1} &= -\gamma'_\uparrow; \quad P\gamma_\downarrow P^{-1} = \gamma'_\downarrow \\ P\gamma'_\uparrow P^{-1} &= \gamma_\uparrow; \quad P\gamma'_\downarrow P^{-1} = -\gamma_\downarrow, \end{aligned}$$

- Parity symmetry acts on the spin basis and chiral basis in an expected way.

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- Parity symmetry acts on the spin basis and chiral basis in an expected way.

$$c_\uparrow = \frac{1}{2}(\gamma_\uparrow + i\gamma'_\uparrow); \quad c_\downarrow = \frac{1}{2}(\gamma_\downarrow - i\gamma'_\downarrow) \quad Pc_\uparrow P^{-1} = ic_\uparrow; \quad Pc_\downarrow P^{-1} = ic_\downarrow$$

$$c_L = \frac{1}{2}(\gamma_\uparrow + i\gamma_\downarrow); \quad c_R = \frac{1}{2}(\gamma'_\uparrow - i\gamma'_\downarrow) \quad Pc_L P^{-1} = -c_R; \quad Pc_R P^{-1} = c_L$$

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Nontrivial "charge conjugation" symmetry for Majorana zero modes

Since a Majorana particle does not carry charge, it has been believed the charge conjugation symmetry must be trivial.

- To our surprise It turns out that Majorana ghosts formed by Majorana zero modes can have a nontrivial charge conjugation symmetry. $\overline{C}_{\uparrow\downarrow'} = \frac{1}{\sqrt{2}}(1+\gamma_\uparrow\gamma'_\downarrow) = e^{\frac{\pi}{4}\gamma_\uparrow\gamma'_\downarrow}; \quad \overline{C}_{\downarrow\uparrow'} = \frac{1}{\sqrt{2}}(1+\gamma_\downarrow\gamma'_\uparrow) = e^{\frac{\pi}{4}\gamma_\downarrow\gamma'_\uparrow}$

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CPT super algebra for Majorana ghosts

The action of CPT symmetries on four Majorana zero modes forms a super algebra

$$\overline{C}^2 = P^f; \quad P^2 = P^f; \quad T^2 = P^f; \quad (P^f)^2 = 1$$
$$TP^f = P^f T; \quad PP^f = P^f P; \quad \overline{C}P^f = P^f \overline{C}$$
$$TP = P^f PT; \quad T\overline{C} = P^f \overline{C}T; \quad P\overline{C} = P^f \overline{C}P,$$

Our new definition of CPT symmetries commutes with spin rotation

$$T\mathbf{S}T^{-1} = -\mathbf{S}; \quad P\mathbf{S}P^{-1} = \mathbf{S}; \quad \overline{C}\mathbf{S}\overline{C}^{-1} = \mathbf{S}$$

$$S^\alpha = \frac{1}{2}\sum_{\sigma,\sigma'} c_\sigma^\dagger \tau^\alpha_{\sigma\sigma'} c_{\sigma'}; \alpha = x, y, z$$

The origin of three generations

Out of four Majorana zero modes, there are *only* three different ways to form complex fermions, corresponding to the *only* three different projective representations of super CPT algebra with $T^4=-1$, $(TP)^4=-1$, $(T\underline{C})^4=-1$

$$d_L = \frac{1}{2}(\gamma_\uparrow - i\gamma'_\downarrow); \quad d_R = \frac{1}{2}(\gamma'_\uparrow - i\gamma_\downarrow),$$

$$\begin{aligned} \overline{C} d_L \overline{C}^{-1} &= -id_L; \quad \overline{C} d_R \overline{C}^{-1} = id_R \\ P d_L P^{-1} &= -d_R; \quad P d_R P^{-1} = d_L \\ T d_L T^{-1} &= id_R^\dagger; \quad T d_R T^{-1} = id_L^\dagger, \end{aligned}$$

carry half Z_2 charge!

$$f_L = \frac{1}{2}(\gamma_\uparrow + i\gamma'_\uparrow) = c_\uparrow; \quad f_R = \frac{1}{2}(\gamma_\downarrow + i\gamma'_\downarrow) = c_\downarrow^\dagger$$

$$\begin{aligned} \overline{C} f_L \overline{C}^{-1} &= if_R; \quad \overline{C} f_R \overline{C}^{-1} = if_L \\ P f_L P^{-1} &= if_L; \quad P f_R P^{-1} = -if_R \\ T f_L T^{-1} &= -f_R^\dagger; \quad T f_R T^{-1} = f_L^\dagger, \end{aligned}$$

A majorana ghost has three different faces

$$\begin{aligned} (TP) d_L (TP)^{-1} &= -id_L^\dagger; \quad (TP) d_R (TP)^{-1} = id_R^\dagger \\ (T\overline{C}) f_L (T\overline{C})^{-1} &= -if_L^\dagger; \quad (T\overline{C}) f_R (T\overline{C})^{-1} = if_R^\dagger, \end{aligned}$$

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$$\overline{C}\gamma'_\uparrow\overline{C}^{-1} = \gamma_\downarrow; \quad \overline{C}\gamma'_\downarrow\overline{C}^{-1} = \gamma_\uparrow,$$

- Charge conjugation symmetry acts on the spin basis(particle-hole) and chiral basis(particle-antiparticle) in an expected way.

Relativistic field theory

CPT symmetry for Majorana ghosts(in chiral representation)

$$\gamma_0 = -i\rho_z \otimes \sigma_y; \quad \gamma_1 = I \otimes \sigma_z; \quad \gamma_2 = -\rho_y \otimes \sigma_y; \quad \gamma_3 = -I \otimes \sigma_x,$$

$$\gamma_5 = \gamma_0\gamma_1\gamma_2\gamma_3 = i\rho_x \otimes \sigma_y$$

$$\xi(x) = \begin{pmatrix} \gamma_\uparrow(x) \\ \gamma_\downarrow(x) \end{pmatrix}; \quad \eta(x) = \begin{pmatrix} -\gamma'_\uparrow(x) \\ \gamma'_\downarrow(x) \end{pmatrix}$$

$$\begin{aligned} \overline{C}\psi(x)\overline{C}^{-1} &= \overline{C}\begin{pmatrix} \xi(x) \\ \eta(x) \end{pmatrix}\overline{C}^{-1} = \begin{pmatrix} -\epsilon\eta(x) \\ -\epsilon\xi(x) \end{pmatrix} = -\gamma_5\psi(x); \\ P\psi(x)P^{-1} &= P\begin{pmatrix} \xi(x) \\ \eta(x) \end{pmatrix}P^{-1} = \begin{pmatrix} \eta(\widetilde{x}) \\ -\xi(\widetilde{x}) \end{pmatrix} = \gamma_0\gamma_5\psi(\widetilde{x}); \\ T\psi(x)T^{-1} &= T\begin{pmatrix} \xi(x) \\ \eta(x) \end{pmatrix}T^{-1} = \begin{pmatrix} -\epsilon\xi(-\widetilde{x}) \\ \epsilon\eta(-\widetilde{x}) \end{pmatrix} = \gamma_0\psi(-\widetilde{x}), \end{aligned}$$

$$\psi(x) = \begin{pmatrix} \xi(x) \\ \eta(x) \end{pmatrix}$$

$$\widetilde{x} = (t, -\mathbf{x})$$

mass term breaks charge conjugation symmetry!

Compare with a Dirac particle

$$\begin{aligned} C\psi(x)C^{-1} &= C\begin{pmatrix} \xi(x) \\ \eta(x) \end{pmatrix}C^{-1} = \begin{pmatrix} \epsilon\eta^*(x) \\ -\epsilon\xi^*(x) \end{pmatrix}; \\ P\psi(x)P^{-1} &= P\begin{pmatrix} \xi(x) \\ \eta(x) \end{pmatrix}P^{-1} = \begin{pmatrix} \eta(\widetilde{x}) \\ \xi(\widetilde{x}) \end{pmatrix}; \\ T\psi(x)T^{-1} &= T\begin{pmatrix} \xi(x) \\ \eta(x) \end{pmatrix}T^{-1} = \begin{pmatrix} \epsilon\xi^*(-\widetilde{x}) \\ \epsilon\eta^*(-\widetilde{x}) \end{pmatrix}, \end{aligned}$$

Relativistic field theory

CPT symmetry for Majorana ghosts(in chiral representation)

$$\gamma_0 = -i\rho_z \otimes \sigma_y; \quad \gamma_1 = I \otimes \sigma_z; \quad \gamma_2 = -\rho_y \otimes \sigma_y; \quad \gamma_3 = -I \otimes \sigma_x,$$

$$\gamma_5 = \gamma_0\gamma_1\gamma_2\gamma_3 = i\rho_x \otimes \sigma_y$$

$$\xi(x) = \begin{pmatrix} \gamma_\uparrow(x) \\ \gamma_\downarrow(x) \end{pmatrix}; \quad \eta(x) = \begin{pmatrix} -\gamma'_\uparrow(x) \\ \gamma'_\downarrow(x) \end{pmatrix}$$

$$\begin{aligned}
\overline{C}\psi(x)\overline{C}^{-1} &= \overline{C}\begin{pmatrix} \xi(x) \\ \eta(x) \end{pmatrix}\overline{C}^{-1} = \begin{pmatrix} -\epsilon\eta(x) \\ -\epsilon\xi(x) \end{pmatrix} = -\gamma_5\psi(x); \\
P\psi(x)P^{-1} &= P\begin{pmatrix} \xi(x) \\ \eta(x) \end{pmatrix}P^{-1} = \begin{pmatrix} \eta(\widetilde{x}) \\ -\xi(\widetilde{x}) \end{pmatrix} = \gamma_0\gamma_5\psi(\widetilde{x}); \\
T\psi(x)T^{-1} &= T\begin{pmatrix} \xi(x) \\ \eta(x) \end{pmatrix}T^{-1} = \begin{pmatrix} -\epsilon\xi(-\widetilde{x}) \\ \epsilon\eta(-\widetilde{x}) \end{pmatrix} = \gamma_0\psi(-\widetilde{x}),
\end{aligned}$$

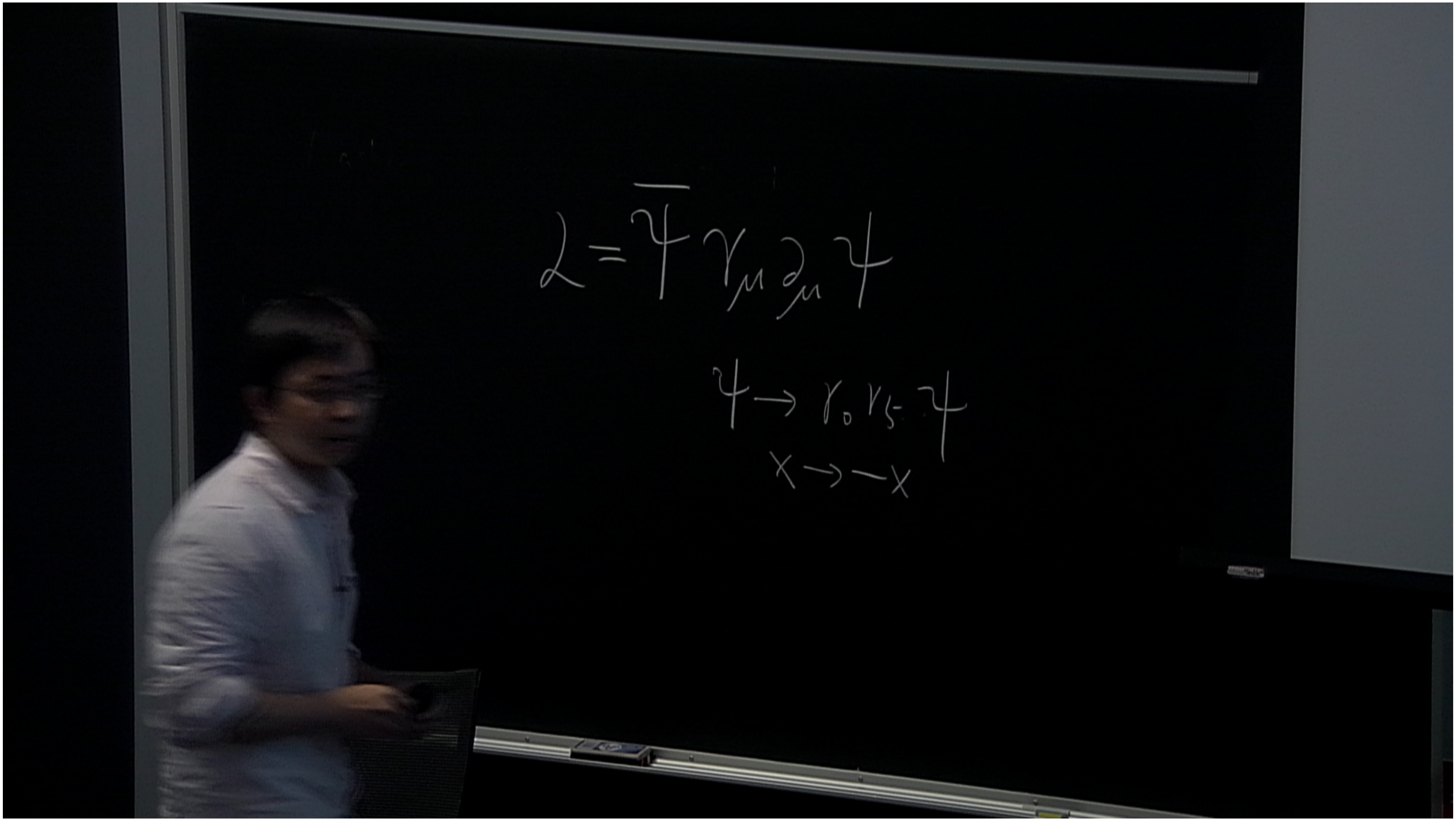
$$\psi(x) = \begin{pmatrix} \xi(x) \\ \eta(x) \end{pmatrix}$$

$$\widetilde{x} = (t, -\mathbf{x})$$

mass term breaks charge conjugation symmetry!

Compare with a Dirac particle

$$\begin{aligned}
C\psi(x)C^{-1} &= C\begin{pmatrix} \xi(x) \\ \eta(x) \end{pmatrix}C^{-1} = \begin{pmatrix} \epsilon\eta^*(x) \\ -\epsilon\xi^*(x) \end{pmatrix}; \\
P\psi(x)P^{-1} &= P\begin{pmatrix} \xi(x) \\ \eta(x) \end{pmatrix}P^{-1} = \begin{pmatrix} \eta(\widetilde{x}) \\ \xi(\widetilde{x}) \end{pmatrix}; \\
T\psi(x)T^{-1} &= T\begin{pmatrix} \xi(x) \\ \eta(x) \end{pmatrix}T^{-1} = \begin{pmatrix} \epsilon\xi^*(-\widetilde{x}) \\ \epsilon\eta^*(-\widetilde{x}) \end{pmatrix},
\end{aligned}$$


$\mathcal{L} = \bar{\psi} \gamma_\mu \partial_\mu \psi$
$\psi \to \gamma_0 \gamma_5 \psi$
$x \to -x$

Three inequivalent Majorana spinons with Lorentz and (super) CPT symmetry

$$\psi_c(x) = \begin{pmatrix} \xi(x) \\ \eta(x) \end{pmatrix} \qquad \xi(x) = \begin{pmatrix} \gamma_\uparrow(x) \\ \gamma_\downarrow(x) \end{pmatrix}; \quad \eta(x) = \begin{pmatrix} -\gamma'_\uparrow(x) \\ \gamma'_\downarrow(x) \end{pmatrix}$$

$$\overline{C}\psi_c(x)\overline{C}^{-1} == -\gamma_5\psi_c(x); \quad P\psi_c(x) = \gamma_0\gamma_5\psi_c(\widetilde{x}); \quad T\psi_c(x)T^{-1} = \gamma_0\psi_c(-\widetilde{x}),$$

$$\psi_f(x) = \begin{pmatrix} \widetilde{\xi}(x) \\ \widetilde{\eta}(x) \end{pmatrix} \qquad \widetilde{\xi}(x) = \begin{pmatrix} \gamma_\uparrow(x) \\ \gamma'_\uparrow(x) \end{pmatrix}; \quad \widetilde{\eta}(x) = \begin{pmatrix} \gamma_\downarrow(x) \\ \gamma'_\downarrow(x) \end{pmatrix}$$

$$\overline{C}\psi_f(x)\overline{C}^{-1} = -\gamma_5\psi_f(x); \quad P\psi_f(x)P^{-1} = \gamma_0\psi_f(\widetilde{x}); \quad T\psi_f(x)T^{-1} = -\gamma_0\gamma_5\psi_f(-\widetilde{x})$$

$$\psi_d(x) = \begin{pmatrix} \hat{\xi}(x) \\ \hat{\eta}(x) \end{pmatrix} \qquad \hat{\xi}(x) = \begin{pmatrix} \gamma_\uparrow(x) \\ \gamma'_\downarrow(x) \end{pmatrix}; \quad \hat{\eta}(x) = \begin{pmatrix} \gamma_\downarrow(x) \\ \gamma'_\uparrow(x) \end{pmatrix}$$

$$\overline{C}'\psi_d(x)(\overline{C}')^{-1} = -\bar{\gamma}_5\psi_d(x); \quad P'\psi_d(x)(P')^{-1} = \bar{\gamma}_0\psi_d(\widetilde{x}); \quad T\psi_d(x)(T')^{-1} = -\bar{\gamma}_0\bar{\gamma}_5\psi_d(-\widetilde{x})$$

Universal gauge coupling

$$\frac{ig}{4}\phi(x)\overline{\psi}_f(x)\psi_f(x) = \frac{ig\phi(x)}{2}\left[\gamma_\uparrow(x)\gamma'_\uparrow(x) - \gamma_\downarrow(x)\gamma'_\downarrow(x)\right]$$

$$\frac{ig}{4}\phi(x)\overline{\psi'}_d(x)\psi'_d(x) = \frac{ig\phi(x)}{2}\left[\gamma_\uparrow(x)\gamma'_\uparrow(x) - \gamma_\downarrow(x)\gamma'_\downarrow(x)\right]$$

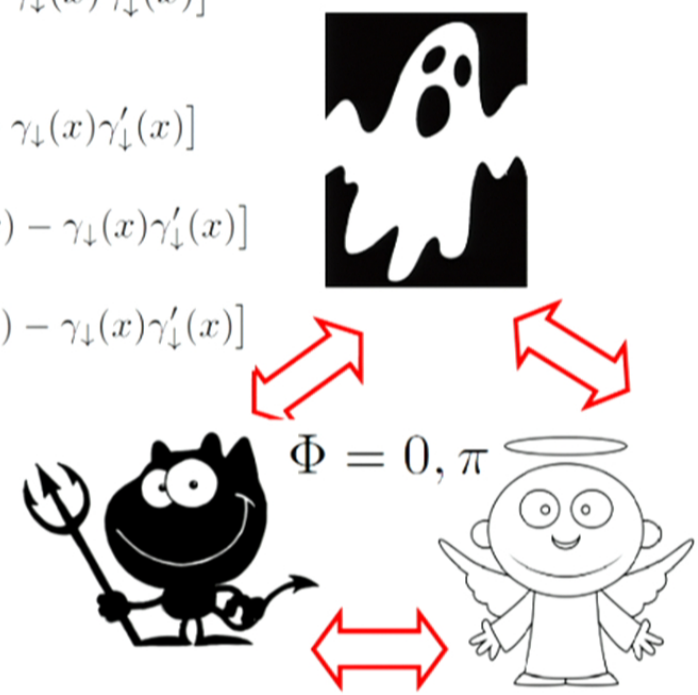
$$\frac{ig}{4}\phi(x)\overline{\psi'}_c(x)\psi'_c(x) = \frac{ig\phi(x)}{2}\left[\gamma_\uparrow(x)\gamma'_\uparrow(x) - \gamma_\downarrow(x)\gamma'_\downarrow(x)\right]$$

$$g = \pm g'$$

$$\frac{ig_{cd}}{4}\phi(x)\overline{\psi'}_d(x)\psi'_c(x) = \frac{ig_{cd}\phi(x)}{4}\left[\gamma_\uparrow(x)\gamma'_\uparrow(x) - \gamma_\downarrow(x)\gamma'_\downarrow(x)\right]$$

$$\frac{ig_{cf}}{4}\phi(x)\overline{\psi}_f(x)\psi'_c(x) = \frac{i\sqrt{2}g_{cf}\phi(x)}{4}\left[\gamma_\uparrow(x)\gamma'_\uparrow(x) - \gamma_\downarrow(x)\gamma'_\downarrow(x)\right]$$

$$\frac{ig_{df}}{4}\phi(x)\overline{\psi}_f(x)\psi'_d(x) = \frac{i\sqrt{2}g_{df}\phi(x)}{4}\left[\gamma_\uparrow(x)\gamma'_\uparrow(x) - \gamma_\downarrow(x)\gamma'_\downarrow(x)\right]$$



$$g_{cd} = \sqrt{2}g_{cf} = \sqrt{2}g_{df} = 2g'$$

Beyond Standard model and prediction of neutrino masses and CP phase

If we assume uniform Dirac mass

$$m_D = diag(m_D, m_D, m_D)$$

$$m_3 \simeq 0.054ev$$

$$\boxed{m_3/m_1 = m_3/m_2 = \sqrt{5}/3} \quad m_1 = m_2 \simeq 0.075ev$$

Conclusions and future works

- Neutrino as topological Majorana modes.
- CPT symmetry for Majorana modes.
- The origin of three generations of neutrinos.
- The origin of neutrino masses and their mass mixing.
- Compute CP angle from first principle.
- Quark CKM matrix(in progress).
- Lattice models in 3D and possible realization in lab.
- Shed new light on quantum gravity.(super cohomology /super fiberbundle)