

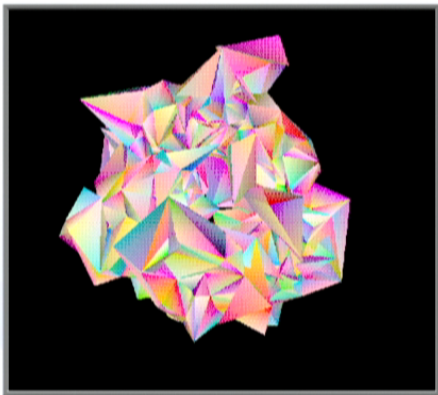
Title: Quantum Gravity from Causal Dynamical Triangulations

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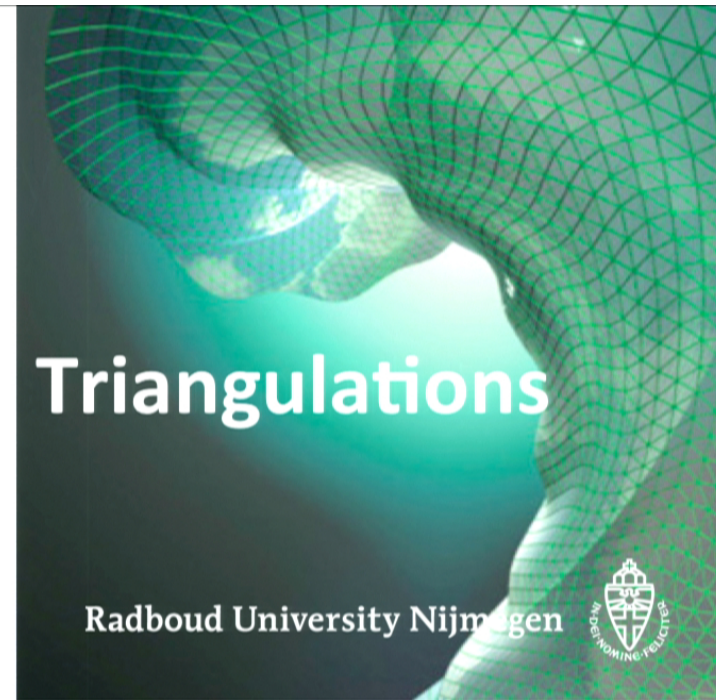
Abstract: By way of presenting some classic and many new results, my talk will indulge shamelessly in<br>advertising "Causal Dynamical Triangulations (CDT)" as a hands-on approach to nonperturbative quantum gravity that reaches where other approaches currently don't. After summarizing the rationale and basic ingredients of CDT quantum gravity and some of its key findings (like the emergence of a classical de Sitter space), I will focus on some very recent results: how we uncovered the presence of a second-order phase transition (so far unique in 4D quantum<br>gravity), news on the controversy between doing things the Lorentzian or the Euclidean way, and new results on the role of the preferred time slicing in CDT - all hopefully worth your while!

# Quantum Gravity from Causal Dynamical Triangulations



triangulated model of quantum space

Waterloo, 23 May 2013



**Renate Loll**

Institute for Mathematics, Astrophysics and  
Particle Physics (IMAPP),  
Radboud University Nijmegen (NL) &  
Perimeter Institute

## Challenges for quantum gravity:

- What are the quantum laws underlying General Relativity?
- Can we explain gravitational attraction from first principles?
- What are the quantum origins of space and time?
- What is the quantum microstructure of spacetime?
- Which observables capture its properties?

Apart from their choice of elementary *degrees of freedom* and a *dynamical principle*, different approaches to quantum gravity can be distinguished by how much background structure they use, e.g. whether metric/differentiable/manifold structure, topology, dimension etc. are fixed a priori or part of dynamics.

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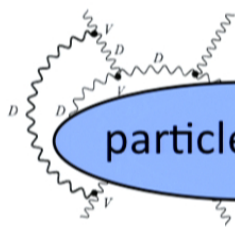
# Where are we in the search for Quantum Gravity?



R. Feynman

*“gravity is like any other field theory”*

“historical dichotomy” in quantum gravity



particle physics



A. Einstein

*“gravity is special; gravity = geometry”*

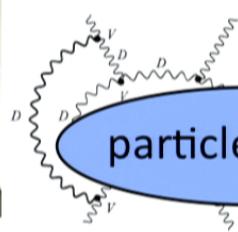
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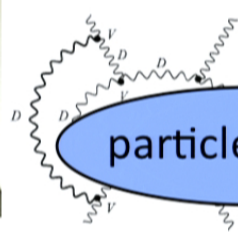
Division is **rarely** a good idea!

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In the last ~10 years, progress in quantum gravity has come from combining insights and methods from both sides of this divide, by

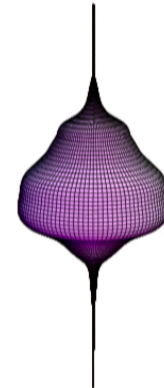
- (i) being minimalist in terms of ingredients and prior assumptions, with little background structure,
- (ii) using standard quantum field-theoretic methods and
- (iii) nonperturbative computat. tools for quantitative evaluation.

# Quantum Gravity from Causal Dynamical Triangulations (CDT)<sup>★</sup>

... is a *nonperturbative* implementation of the gravitational path integral, much in the spirit of lattice quantum field theory, but based on *dynamical* lattices, reflecting the dynamical nature of spacetime geometry.

CDT is currently the only candidate quantum theory of gravity which can generate *dynamically* a spacetime with semiclassical properties from pure quantum excitations, without using a background metric.

(C)DT has also given us crucial *new* insights into nonperturbative dynamics and pitfalls.



(PRL 93 (2004) 131301, PRD 72 (2005) 064014, PLB 607 (2005) 205)

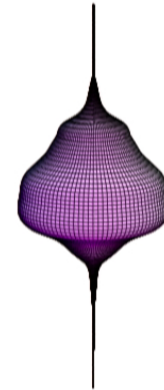
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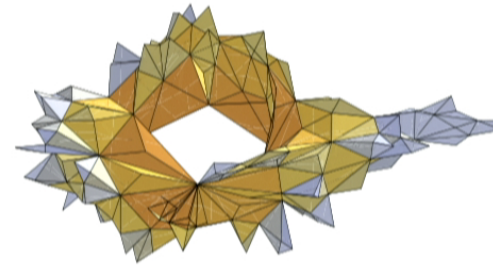
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# Key points of the CDT approach:

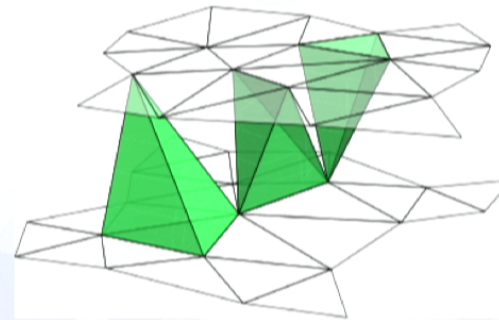
- Few ingredients/priors:
  - quantum superposition principle
  - locality and causal structure (*not* Euclidean quantum gravity)
  - notion of (proper) time
  - Wick rotation
  - standard tools of quantum field theory
- Few free parameters ( $\Lambda$ ,  $G_N$ ,  $\Delta$ )
- Robustness of construction; universality
- At intermediate stage, approximate curved spacetimes by triangulations
- Crucial: nonperturb. computational tools to extract quantitative results

I will briefly describe the set-up, then look at some of the results produced:

- “emergence” of spacetime
- scale-dependent dimensionality
- nontrivial phase structure
- several **new** results



triangulated torus



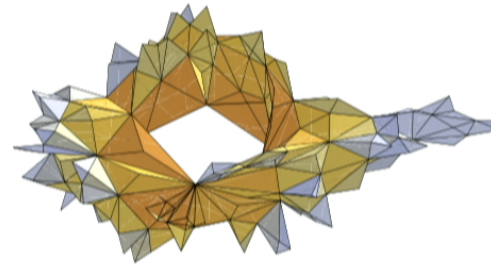
piece of causal triangulation

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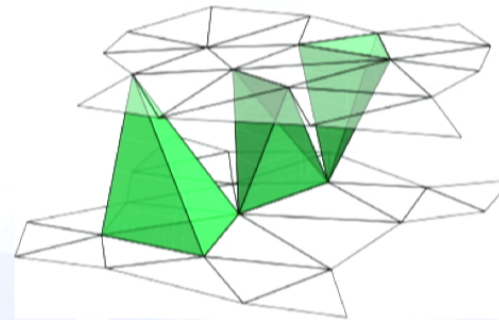
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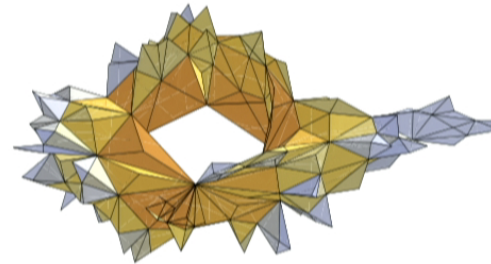
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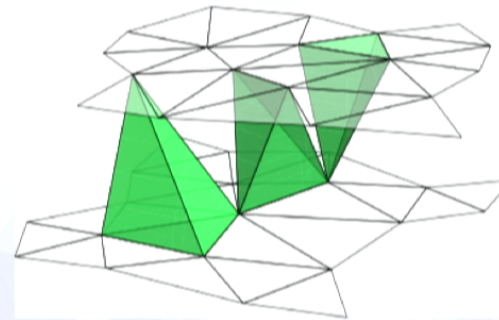
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piece of causal triangulation

# Our dynamical principle: Feynman path integral, “Sum over Histories”

Newton's constant

cosmological constant

$$Z(G_N, \Lambda) = \int_{\text{spacetimes } g \in \mathcal{G}} \mathcal{D}g \, e^{iS_{G_N, \Lambda}^{\text{EH}}[g]}$$

Each “path” is a four-dimensional, curved spacetime geometry  $g$ , which can be thought of as a three-dimensional, spatial geometry developing in time. The weight associated with each  $g$  is given by the Einstein-Hilbert action  $S^{\text{EH}}[g]$ ,

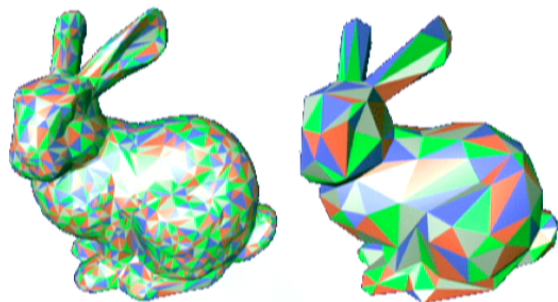
$$S^{\text{EH}} = \frac{1}{G_N} \int d^4x \sqrt{-\det g} (R[g, \partial g, \partial^2 g] - 2\Lambda)$$

How can we make  $Z(G_N, \Lambda)$  into a meaningful, well-defined quantity?

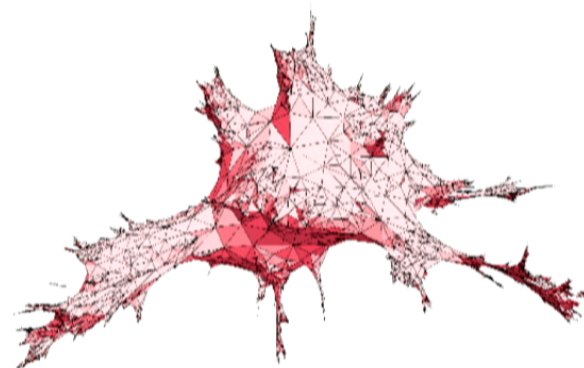
How can we get a handle on the *space of all spacetimes*?

# Key input in dynamical triangulations: “General Relativity without Coordinates” (Regge)

triangulation = regularization



approximate *classical* curved surfaces through triangulation



A typical path integral history  
(2d quantum gravity)

*Quantum Theory:* approximate the space of *all* curved geometries by a space of triangulations - the space we need to integrate over<sup>(\*)</sup>!

<sup>(\*)</sup> *by Monte Carlo simulations* (for CDT models in  $d=2, 3$  have also *exact* stat. mech. solutions methods, see e.g. [D. Benedetti, F. Zamponi, R.L., PRD 76 \(2007\) 104022](#); in  $d=2$ , the problem is exactly soluble - nontrivial propagator, Hamiltonian, ..., work by [J. Ambjørn, R.L., P. di Francesco, E. Guitter, C. Kristjansen, B. Durhuus, ...](#))

# Regularizing the path integral via CDT

$$Z(G_N, \Lambda) := \lim_{\substack{a \rightarrow 0 \\ N \rightarrow \infty}} \sum_{\substack{\text{inequiv.} \\ \text{triangul.s} \\ T \in \mathcal{G}_{a,N}}} \frac{1}{C(T)} e^{iS_{G_N, \Lambda}^{\text{Regge}}[T]}$$

$\swarrow$   
 $|\text{Aut}(T)|$

Each triangulated manifold  $T$  represents a different curved spacetime, consisting of  $N$  four-simplices, which can be “Wick-rotated” to a Riemannian space.

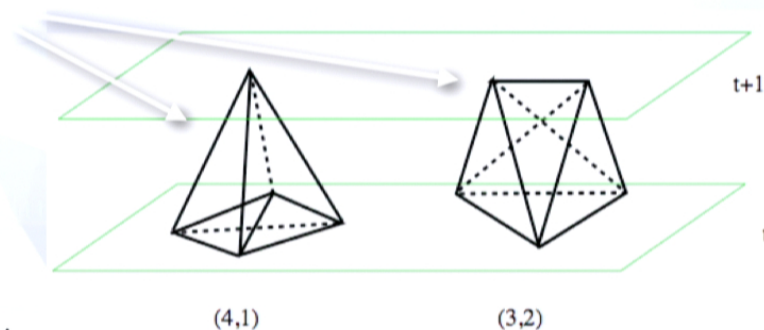
‘democratic’, regularized sum over piecewise flat spacetimes;

continuum limit required to obtain universal results independent of the regularization

Elementary four-simplex, building block for a causal dynamical triangulation

edge length  $a$  = a diffeomorphism-invariant UV regulator

N.B.: the causal structure of CDT is essential!  
This does not work in Euclidean signature - get only branched polymers (~mid-90s).



CDT's proper-time slicing -  
*time and space are not equivalent*

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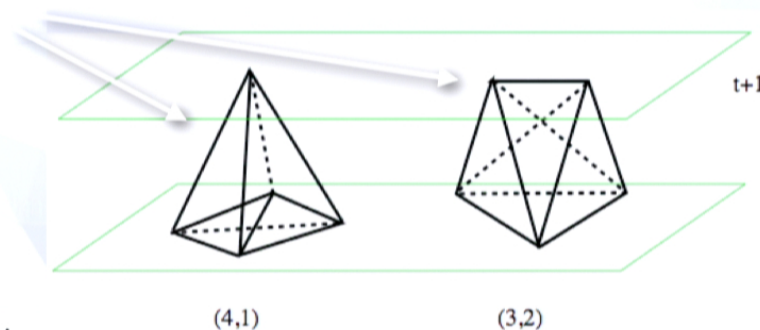
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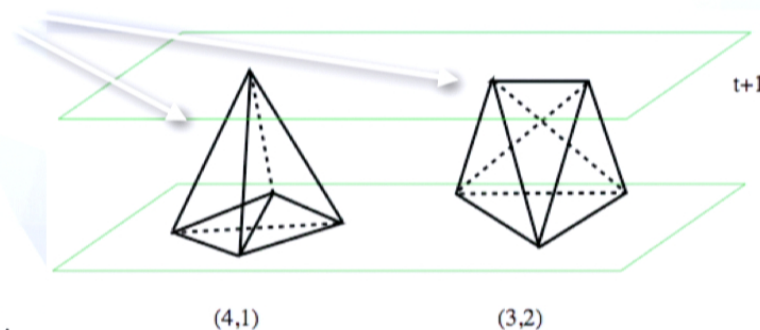
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**This is our toolbox - now for some results,  
starting with CDT “classics”.**

# Dynamical emergence of spacetime as we know it

For suitable bare coupling constants, CDT quantum gravity produces a “quantum spacetime”, that is, a ground state, whose macroscopic scaling properties are ***four-dimensional*** and whose macroscopic shape is that of a well known cosmology, ***de Sitter space***.

Evidence: When, from all the gravitational degrees of freedom present, we monitor only the average spatial three-volume  $\langle V_3(t) \rangle$  of the universe as a function of (discrete, proper) time  $t$ , we find a characteristic “volume profile”.

This is brought about by a ***nonperturbative*** mechanism, with “energy” (the bare action) and “entropy” (the measure, i.e. number of microscopic spacetime configurations) contributing in equal measure.

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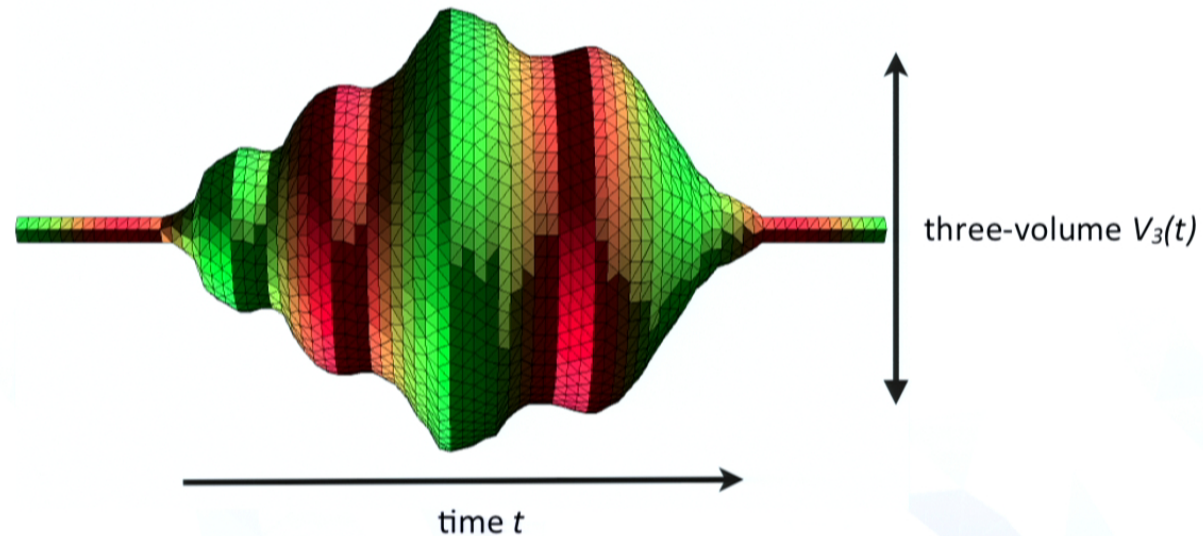
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# Dynamically generated 4D quantum universe, from a path integral over causal spacetimes

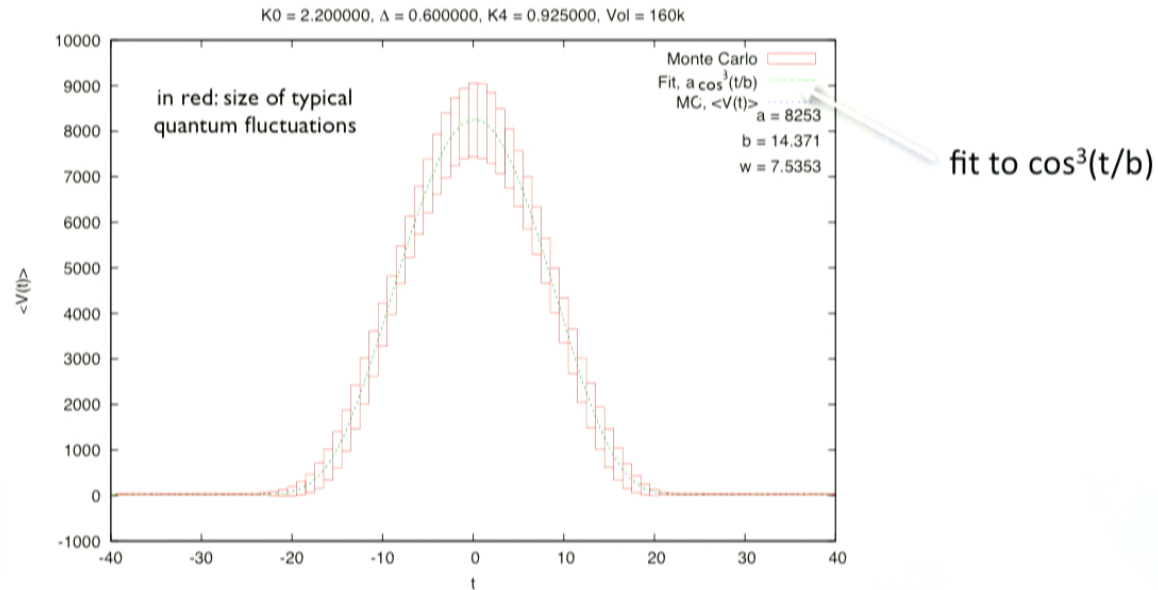


This is a Monte Carlo “snapshot” of spacetime shape (a single volume profile  $V_3(t)$ ) - we still need to average to obtain its *expectation value*  $\langle V_3(t) \rangle$ .★

N.B.: we are *not* doing quantum cosmology, i.e. do not impose symmetries by hand

★ careful, triangles in figure do *not* represent the actual local geometry

# The quantitative evidence for de Sitter space



The volume profile  $\langle V_3(t) \rangle$ , as function of Euclidean proper time  $t=i\tau$ , perfectly matches that of a Euclidean *de Sitter space*, with scale factor  $a(t)^2$  given by

$$ds^2 = dt^2 + a(t)^2 d\Omega_{(3)}^2 = dt^2 + c^2 \cos^2\left(\frac{t}{c}\right) d\Omega_{(3)}^2 \leftarrow \text{volume el. } S^3$$

(J. Ambjørn, A. Görlich, J. Jurkiewicz, RL, PRL 100 (2008) 091304, PRD 78 (2008) 063544, NPB 849 (2011) 144 (with J. Gizbert-Studnicki, T. Trzesniewski))

## Are there more local ways of characterizing quantum geometry<sup>(\*)</sup>?

Yes, its dimension, which in quantum gravity can behave in unexpected ways.

There are several notions of dimension, which in the Planckian regime need not coincide.

“Dimension” in nonperturbative quantum gravity is no longer fixed a priori, but reflects a particular quantum dynamics. It is *not* pre-determined by the dimensionality of the triangular building blocks used.

As we have already noted, to dynamically generate a *four*-dimensional extended geometry is highly nontrivial.

(\*) part of quantum gravity’s quest for observables, especially those that allow us to quantify genuine *quantum* properties of spacetime

# Getting a handle on Planckian physics (via “dimensions”)



(or, another nonperturbative surprise!)

A diffusion process is sensitive to the dimension of the medium where the “spreading” takes place. We have implemented such a process on the quantum superposition of spacetimes. By measuring a suitable “observable”<sup>★</sup>, we have extracted the spectral dimension  $D_s$  of the quantum spacetime.

Quite remarkably, we find that it depends on the length scale probed:  $D_s$  changes smoothly from 4 on large scales to  $\sim 2$  on short scales.

★ average return probability:

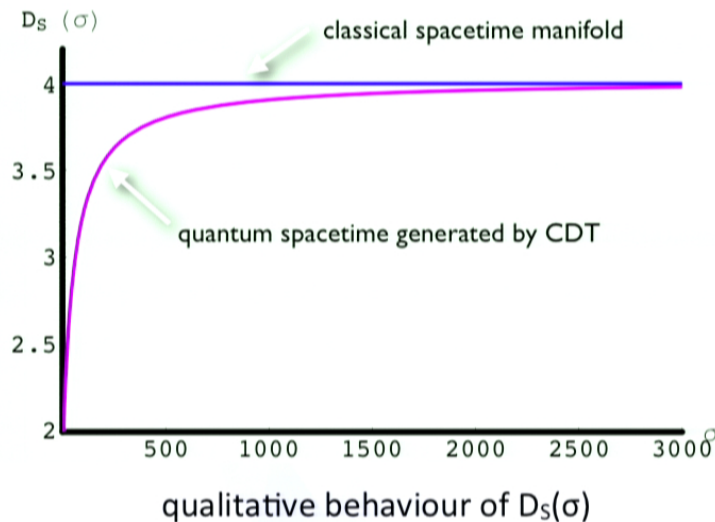
$$\mathcal{R}_V(\sigma) := \frac{1}{V(M)} \int_M d^d x P(x, x; \sigma) \propto \frac{1}{\sigma^{D_s/2}}$$

diffusion time

sol.n to heat equation



## $D_S(\sigma)$ as probe of geometry on linear scales $\sim \sigma^{1/2}$



→ on short scales, our “ground state of geometry” is definitely *not* a classical manifold.

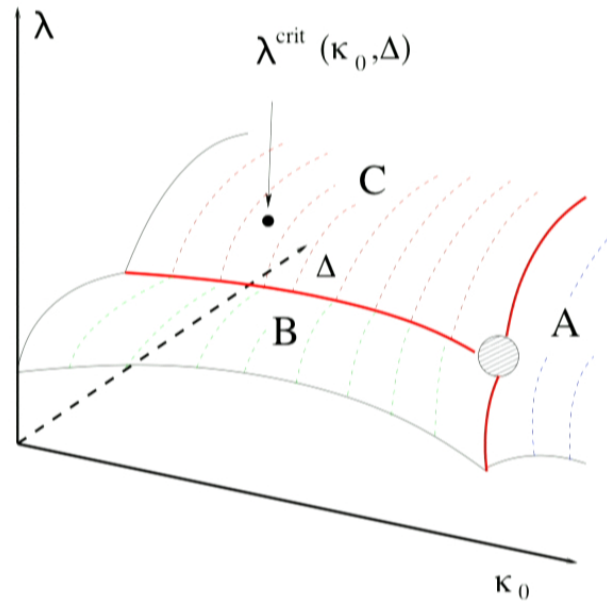
Instead, we find evidence for the presence of a random fractal structure.

(J. Ambjørn, A. Görlich, J. Jurkiewicz, RL, PLB 690 (2010) 420)

Intriguingly, the short-scale “*dynamical dimensional reduction*” of CDT has also been found in two rather different (also quantum field-theoretic) approaches:

- nonperturbative renormalization group flow analysis  
(M. Reuter, O. Lauscher, JHEP 0510:050, 2005, M. Reuter, F. Saueressig, JHEP 1112 (2011) 012)
- nonrelativistic “Lifshitz quantum gravity” (P. Hořava, PRL 102 (2009) 161301)

# Phase diagram of Causal Dynamical Triangulations



The CDT gravitational action is *simple*:

$$S_{\text{eu}}^{\text{Regge}} = -\kappa_0 N_2 + N_4(c\kappa_0 + \lambda) + \Delta(2N_4^{(4,1)} + N_4^{(3,2)})$$

$\lambda \sim$  cosmological constant

$\kappa_0 \sim 1/G_N$  inverse Newton's constant

$\Delta \sim$  relative time/space scaling

$c \sim$  numerical constant,  $>0$

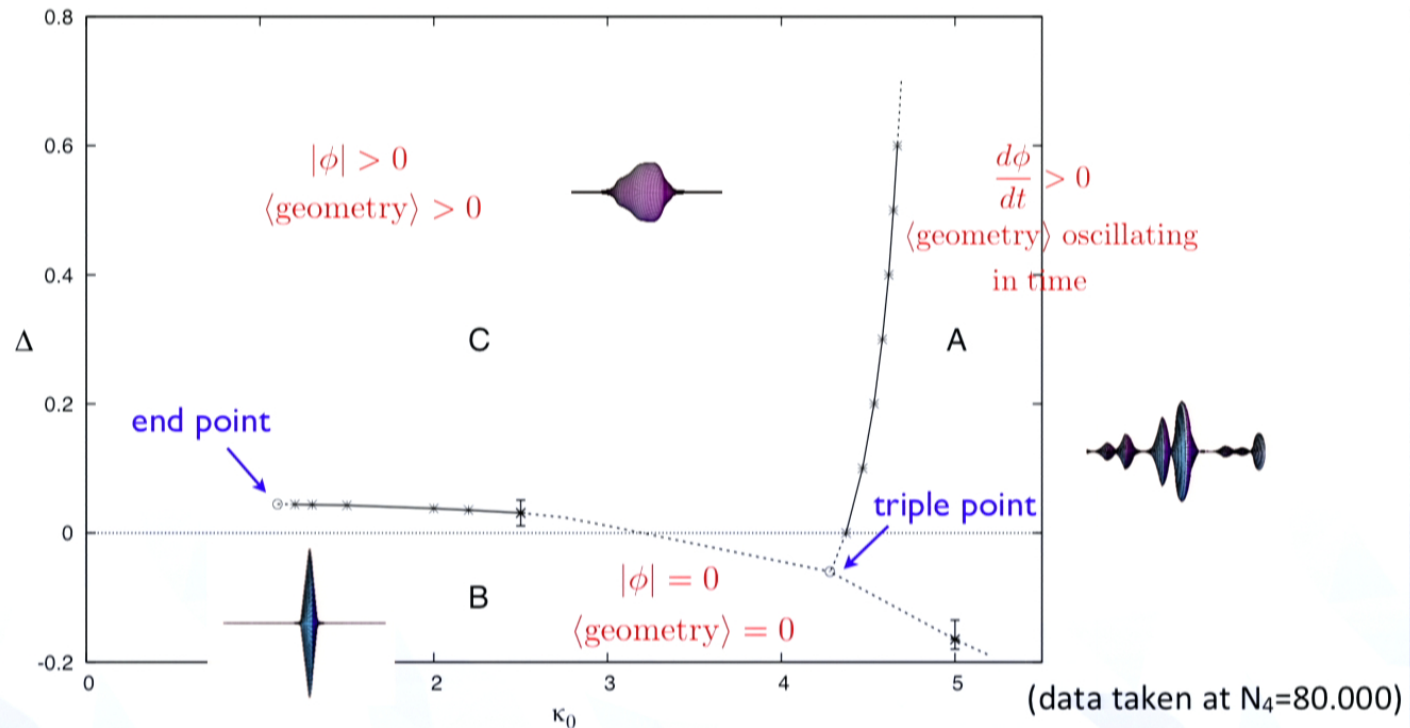
$N_i \sim$  # of triangular building blocks of dimension  $i$

The partition function is defined for  $\lambda > \lambda^{\text{crit}}(\kappa_0, \Delta)$ ;  
 approaching the critical surface from above = taking infinite-volume limit.  
 red lines  $\sim$  phase transitions

(J. Ambjørn, J. Jurkiewicz, RL, PRD 72 (2005) 064014;

J. Ambjørn, A. Görlich, S. Jordan, J. Jurkiewicz, RL, PLB 690 (2010) 413)

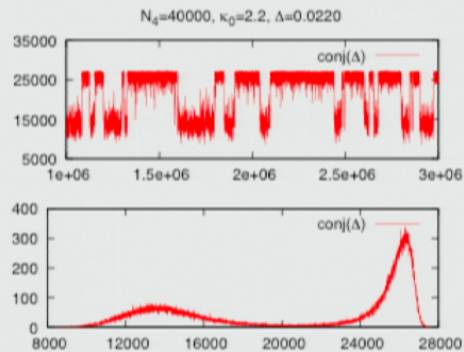
# The phase diagram of CDT in the $\kappa_0$ - $\Delta$ plane



The *average* geometry in phases A and B is degenerate and does *not* have a classical, four-dimensional limit. The interesting physics happens in phase C.

**!New!** the B-C transition appears to be of “second order” - unprecedented!

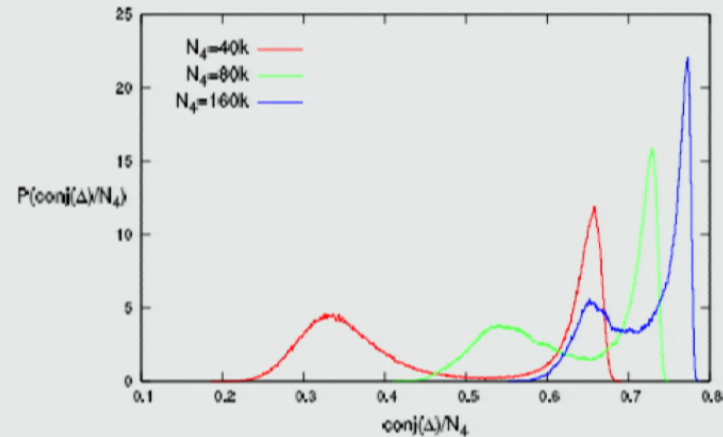
## The evidence:



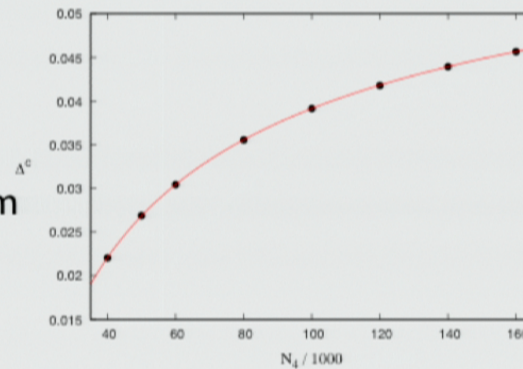
Monte Carlo time evolution of  $\text{conj}(\Delta)$  at the B-C transition with associated histogram

extracting the shift exponent  $\nu$  from measuring the location of the maximum of the susceptibility of  $\text{conj}(\Delta)$ :

$$\Delta^c(N_4) = \Delta^c(\infty) - C N_4^{-1/\tilde{\nu}}$$



peaks move closer with increasing volume

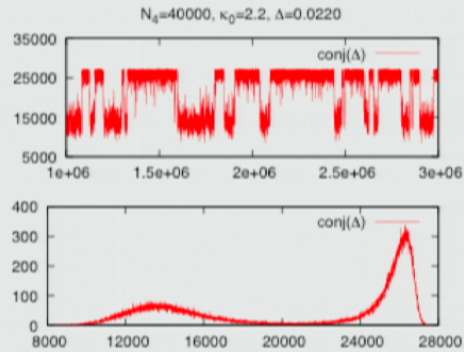


$$\nu=2.51(3)$$

(should be =1 for a first-order trans.)

(J. Ambjørn, S. Jordan, J. Jurkiewicz, R.L., PRL 107 (2011) 211303; PRD 85 (2012) 124044)

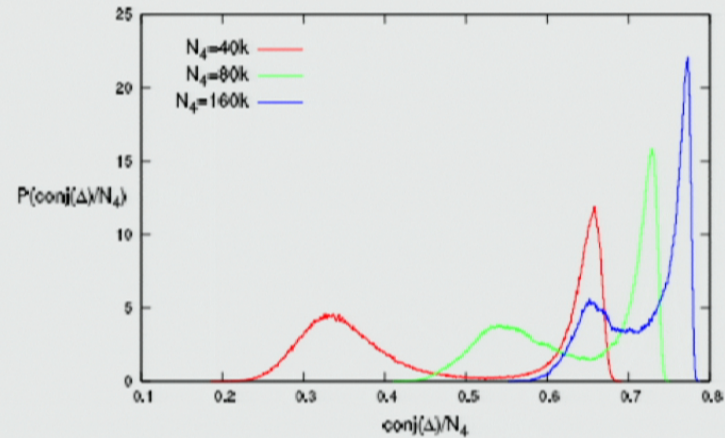
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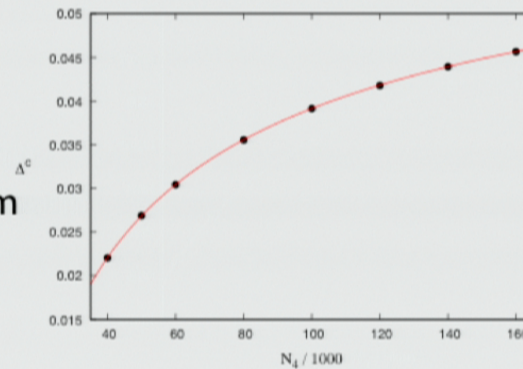
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## New (and old): Lorentzian versus Euclidean DT

- standard Euclidean DT has only a first-order transition, from a crumpled phase to a branched polymer phase
- resurrecting the “crinkled phase” of Euclidean DT quantum gravity, based on the (old) idea of changing the measure ( $o_i$  is the order of the  $i^{\text{th}}$  triangle):

$$\sum_{\text{triangul.s } T} \frac{1}{C(T)} \longrightarrow \sum_{\text{triangul.s } T} \left[ \prod_{i=1}^{N_2} o_i^\beta \right] \frac{1}{C(T)}$$

- there is evidence of a scale-dependent spectral dimension in the crinkled phase; perhaps this is just the nice “phase C” of CDT in disguise, and the only important ingredient is a third coupling constant, here related to higher-curvature terms (J. Laiho, D. Coumbe, PRL 107 (2011) 161301)
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## New (and old): Lorentzian versus Euclidean DT

- standard Euclidean DT has only a first-order transition, from a crumpled phase to a branched polymer phase
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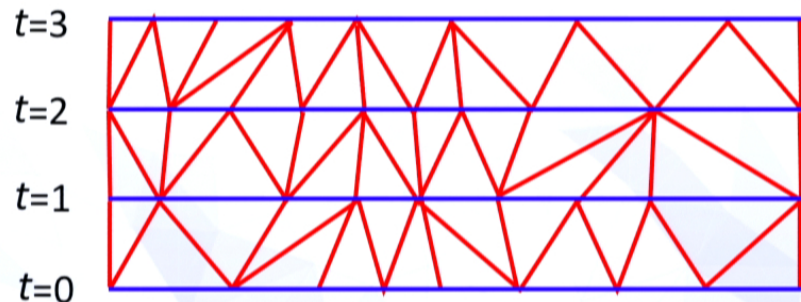
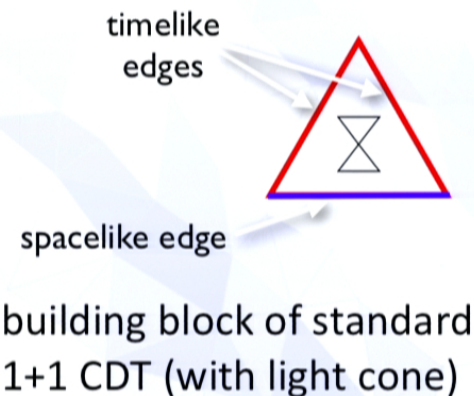
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## NEW: CDT quantum gravity without distinguished foliation

- standard path integral formulation needs a time  $t$ ; propagator  $G(g_{in}, g_{out}; t)$  satisfies

$$G(g_{in}, g_{out}; t) = \sum_g G(g_{in}, g; t_1) G(g, g_{out}; t_2), \quad t = t_1 + t_2$$

- *proper time* is a natural geometric choice; in standard CDT, slices of constant proper time  $t=0, 1, 2, 3, \dots$  coincide with simplicial submanifolds, consisting of purely spatial (d-1)-simplices



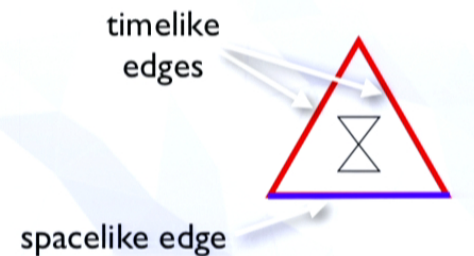
building causal spacetimes from proper-time strips in standard CDT quantum gravity

# NEW: CDT quantum gravity without *distinguished foliation*

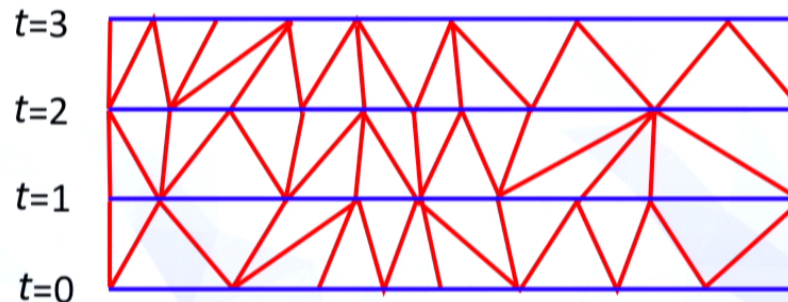
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building block of standard  
1+1 CDT (with light cone)



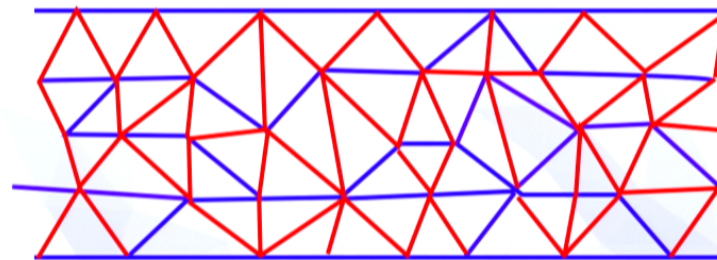
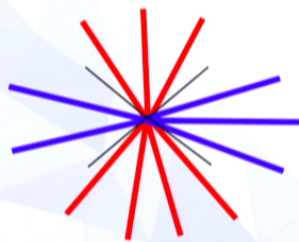
building causal spacetimes from proper-time  
strips in standard CDT quantum gravity

# Relax the strict slicing while retaining causality

- introduce additional elementary building blocks, e.g. in 1+1 dimensions



- impose local causality conditions at vertices:



building causal spacetimes in generalized CDT from these two building blocks

## Recovering standard CDT

- analogously, can introduce additional elementary building blocks in 2+1
- simulating the generalized 2+1 CDT model requires new Monte Carlo moves
- after fine-tuning  $\lambda$ , the model has a two-dimensional phase space, parametrized by  $(\kappa_0, \Delta)$
- there is a region in phase space where geometry is “almost foliated”
- in this region, volume profiles w.r.t. an averaged geodesic time are compatible with three-dimensional de Sitter space
- have recovered standard foliated CDT quantum gravity dynamically!

The strict time foliation of CDT appears to be a dispensable (albeit convenient) part of its background structure.

→ for more details, attend Samo Jordan's talk next week!

(S. Jordan, R.L., 1305.4582 (hep-th), and to appear)

# Causal Dynamical Triangulations - Outlook

CDT is a path integral formulation of gravity, incorporating the dynamical and causal nature of geometry. It depends on little background structure, few assumptions and few free parameters. Its toolbox provides an “experimental lab” - a nonperturbative calculational handle on (near-)Planckian physics.

- We can make quantitative statements about *quantum* geometry (properties of the ground state of quantum gravity).
- We obtained a derivation from first quantum principles of the shape of the universe, illustrating the emergence of classicality from quantum dynamics and the crucial role of “entropy” (number of quantum states).
- The dynamics of spacetime on Planckian scales is counterintuitive and nonclassical, as illustrated by the dynamical behaviour of “dimension”.

The hunt for more observables is on, with great scope for quantitative results and comparison with other approaches like the “asymptotic safety scenario”.

(e.g. G. Calcagni, A. Eichhorn, F. Saueressig, 1304.7247 (hep-th), J. Cooperman, J. Miller, 1305.2932 (gr-qc), T.G.Budd, R.L., 1305.4702 (hep-th), ...)

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## Where to learn more

For a comprehensive review of Causal Dynamical Triangulations, check out “Nonperturbative Quantum Gravity”, our new Physics Report 519 (2012) 127-212 [arXiv: 1203.3591]!

Links to other review material (from technical to popular) and lecture notes can be found at <http://www.hef.ru.nl/~rloll>.