Title: Majorana Ghosts: From topological superconductor to the origin of neutrino mass, three generations and their mass mixing

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Abstract: The existence of three generations of neutrinos and their mass mixing is a deep mystery of our universe. On the other hand, Majorana's elegant work on the real solution of Dirac equation predicted the existence of Majorana particles in our nature, unfortunately, these Majorana particles have never been observed. In this talk, I will begin with a simple 1D condensed matter model which realizes a T^2=-1 time reversal symmetry protected superconductors and then discuss the physical property of its boundary Majorana zero modes. It is shown that these Majorana zero modes realize a T^4=-1 time reversal doubelets and carry 1/4 spin. Such a simple observation motivates us to revisit the CPT symmetry of those ghost particles--neutrinos by assuming that they are Majorana zero modes. Interestingly, we find that a topological Majorana particle will realize a P^4=-1 parity symmetry as well. It even realizes a nontrivial C^4=-1 charge conjugation symmetry, which is a big surprise from a usual perspective that the charge conjugation symmetry for a Majorana particle is trivial. Indeed, such a C^4=-1 charge conjugation symmetry is a Z_2 gauge symmetry and its spontaneously breaking leads to the origin of neutrino mass. We further attribute the origin of three generations of neutrinos to three distinguishable types of topological Majorana zero modes protected by CPT symmetry. Such an assumption leads to an S3 symmetry in the generation space and uniquely determines the mass mixing matrix with no adjustable parameters! In the absence of CP violation, we derive \theta_12=32degree, \theta_23=45degree and \theta_13=0degree, which is intrinsically closed to the current experimental results. We further predict an exact mass ratio of the three mass eigenstate with m_1/m_3~m_2/m_3=3/\sqrt{5}.

Majorana Ghosts:

From topological superconductor to the origin of neutrino masses, three generations and their mass mixing

Zheng-Cheng Gu (Caltech/PI)

(Z.C. Gu, 2013, to appear)

Emergence & Entanglement II

PI

May 6 - 10, 2013

Outlines

- The definition of a Majorana ghost and its realization in condensed matter systems.

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- Unconventional CPT symmetry for Majorana ghosts.

- The origin of three generations of neutrinos.

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- Unconventional CPT symmetry for Majorana ghosts.

- The origin of three generations of neutrinos.

- The origin of neutrino masses and their mass mixing. Compute the mass mixing matrix with no adjustable parameters.

Neutrino

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- Extremely weak interactions with other particles.
- Almost vanishing rest mass.
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Ghosts hunting: neutrino oscillation

- There exist three generations of (light) neutrinos, identified through their weak doublets: electron, muon and tauon.

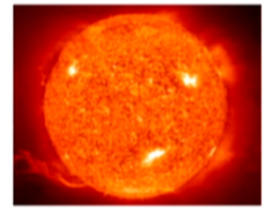
$$L_\ell = \begin{pmatrix} \nu_{L\ell} \\ \ell_L^- \end{pmatrix}, \quad \ell = e, \mu, \tau. \qquad \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

- Weak eigenstates do not coincide with mass eigenstates, oscillation happens during their propagation.

$$|\psi(0)\rangle = |\nu_e\rangle = \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle$$

$$|\psi(z)\rangle = \cos\theta|\nu_1\rangle e^{-i(E_1-|\vec{p}_1|)z} + \sin\theta|\nu_2\rangle e^{-i(E_2-|\vec{p}_2|)z}$$

Current experimental progress toward ghost hunting

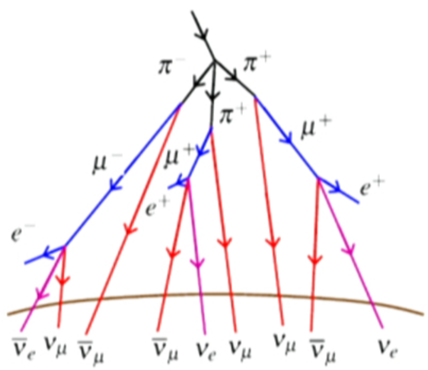


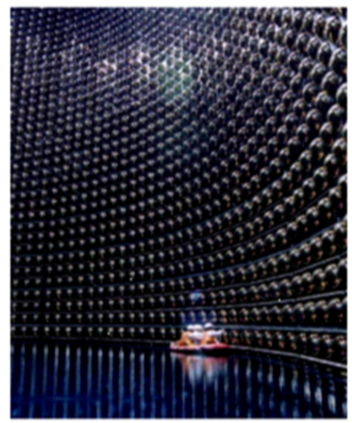
Three-generation mixing matrix:

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{"Atmospheric"}} \times \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix} \times \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{"Solar"}}$$

Dominates:

$$\approx \begin{pmatrix} 0.85 & 0.53 & 0 \\ -0.37 & 0.60 & 0.71 \\ 0.37 & -0.60 & 0.71 \end{pmatrix} \quad \theta_{23} = 45° \pm 8 \quad \theta_{13} \approx 0 \quad \theta_{12} = 32.3° \pm 1.6$$





(Japan)

(Canada)

(from Michaelmas Term 2009, Prof Mark Thomson)

An elegant proposal by Ettore Majorana

Real solution of Dirac equation(in standard representation)

$$\begin{aligned}\gamma_0 &= -i\rho_x \otimes \sigma_y; \quad \gamma_1 = I \otimes \sigma_z; \\ \gamma_2 &= \rho_y \otimes \sigma_y; \quad \gamma_3 = -I \otimes \sigma_x, \\ \mathcal{L}_0 &= \frac{1}{4}\overline{\psi}(x) i\gamma_\mu \partial_\mu \psi(x) + im\overline{\psi}(x)\psi(x),\end{aligned}$$

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- Those ghosts like neutrinos are promising candidates of Majorana particles.
- But why there are three generations and where do those mystery mixing angles come from?

The main goal of this talk:

- **Try to understand the origin of neutrino masses, three generations and compute their mass mixing matrix by investigating the topological nature of a Majorana particle.**
- We refer a topological Majorana particle carrying unusual CPT symmetry as a **Majorana ghost.**

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How to identify a ghost?

The typical answer from a physicist:

- There is no ghost in our world!!!

But if we insist on identifying a ghost through physical law:

- It seems not easy to identify a ghost, from most of the ghost movies ...... they just look like usual people.
- However, they can appear and disappear suddenly.
- It seems that a ghost will have unusual behavior under time reversal symmetry.

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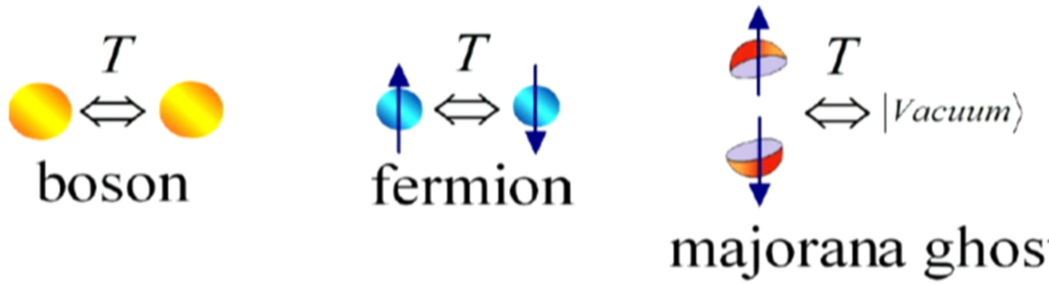
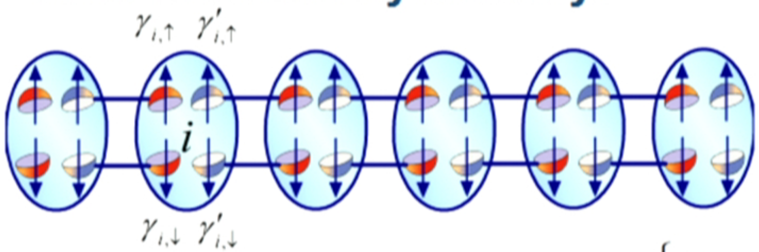
If a particle disappears under time reversal symmetry, we can call it a ghost.

Majorana zero modes can have such a strange behavior!



Majorana ghosts in condensed matter systems.

A 1+1D topological superconductor protected by T^2=-1 time reversal symmetry.



$$H = \sum_{i=1}^{N} \sum_{\sigma} i\sigma \gamma'_{i,\sigma} \gamma_{i+1,\sigma},$$

$$\{\gamma_{i,\sigma}, \gamma'_{i',\sigma'}\} = 0; \quad \{\gamma_{i,\sigma}, \gamma_{i',\sigma'}\} = 2\delta_{ii'}\delta_{\sigma\sigma'}$$

T^4=-1 time reversal symmetry

The classification of 1+1D symmetry protected topological (SPT) orders provides us with a very good understanding

Hilbert space for interacting systems: (Phys. Rev. B 83, 035107 (2011))

$$c_\uparrow^\dagger|0\rangle, c_\downarrow^\dagger|0\rangle, |0\rangle, c_\uparrow^\dagger c_\downarrow^\dagger|0\rangle$$

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- $T^2=-1$for fermion parity odd sector and $T^2=1$ for fermion parity even sector. The total symmetry group is extended over fermion parity symmetry group $\{I,P^f\}$, which is indeed $T^4=1$.

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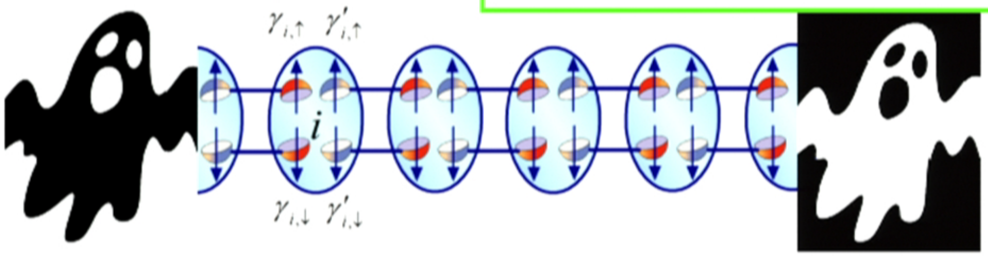
- According to the classification of 1+1D SPT phases, the edge majorana modes should carry projective representation with T^4=-1

Representation theory and 1/4 spin

Majorana ghosts arise on the edge:

$c_L = \frac{1}{2}(\gamma_\uparrow + i\gamma_\downarrow);\quad c_R = \frac{1}{2}(\gamma'_\uparrow - i\gamma'_\downarrow)$

$$T c_L T^{-1} = -i c_L^\dagger; \quad T c_R T^{-1} = i c_R^\dagger$$



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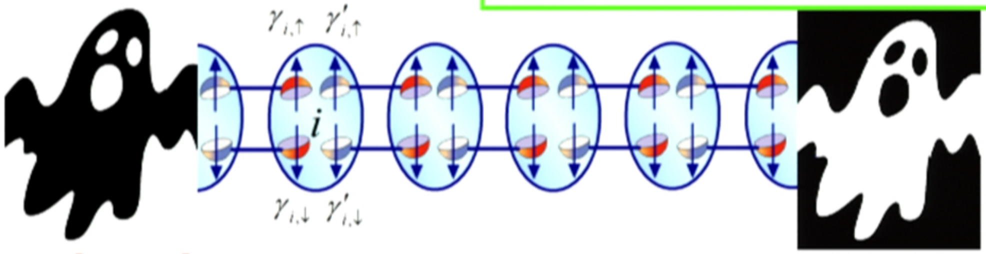

Representation theory:

$$T|0\rangle = UK|0\rangle = U|0\rangle = |1\rangle \equiv c^\dagger_{L(R)}|0\rangle \qquad U = \begin{pmatrix} 0 & 1 \\ \pm i & 0 \end{pmatrix}$$

$$\begin{aligned} T|1\rangle &= UKc^\dagger_{L(R)}|0\rangle = Uc^\dagger_{L(R)}|0\rangle \\ &= Tc^\dagger_{L(R)}T^{-1}T|0\rangle = \pm i c_{L(R)}c^\dagger_{L(R)}|0\rangle = \pm i|0\rangle \end{aligned}$$

1/4 spin:

$$S_L = \frac{|S|}{2}\sum_{\sigma\sigma'}\gamma_\sigma\sigma^y_{\sigma\sigma'}\gamma_{\sigma'} = \frac{|S|}{2}\sum_{\sigma\sigma'} i\gamma_\sigma\epsilon_{\sigma\sigma'}\gamma_{\sigma'} = \frac{i|S|}{2}(\gamma_\uparrow\gamma_\downarrow - \gamma_\downarrow\gamma_\uparrow) = i|S|\gamma_\uparrow\gamma_\downarrow = -|S|P^f_L$$

$$S_y = \frac{i}{2}(c^\dagger_\uparrow c_\downarrow - c^\dagger_\downarrow c_\uparrow) = \frac{i}{2}\left[(\gamma_\uparrow - i\gamma'_\uparrow)(\gamma_\downarrow - i\gamma'_\downarrow) - (\gamma_\downarrow + i\gamma'_\downarrow)(\gamma_\uparrow + i\gamma'_\uparrow)\right] = -|S|(P^f_L + P^f_R)$$

Representation theory in the zero energy subspace

What really happens?

- The fermion parity even (vacuum) states carry T²=-1 representation of time reversal symmetry

$$U = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$T|00\rangle = |11\rangle;$$

$$T|11\rangle = Tc_L^\dagger T^{-1} T c_R^\dagger T^{-1} T|00\rangle = c_L c_R |11\rangle = c_L c_R c_L^\dagger c_R^\dagger |00\rangle = -|00\rangle$$

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- The fermion parity odd (one particle) states carry $T^2=1$ representation of time reversal symmetry

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$$T|10\rangle = i|01\rangle; \quad T|01\rangle = i|10\rangle$$

Representation theory in the zero energy subspace

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$$U = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$$

The ghosts just "hide" in the Kramers doublet vacuum. Total "ghost" number is conserved!(entanglement & emergence)

Majorana ghosts in high dimensions

Where can we find Majorana ghosts in high dimensions?

- The majorana zero modes inside the vortex core of a 2D $T^2=-1$ symmetry protected topological superconductor carry $T^4=-1$ time reversal symmetry.

- Exactly solvable fix point Hamiltonian and its corresponding topological term can be constructed for such a 2D topological superconductor.(Z C Gu, 2013)

- Majorana zero modes in the Hedgehogs of a 3D $T^2=-1$ topological superconductor carry $T^4=-1$ symmetry.

(C L Kane's group Phys. Rev. Lett. 104,(2010), Z C Gu etal, in progross)

P⁴=-1 parity symmetry for Majorana ghosts

It is not a surprise that we can define a P⁴=-1 parity symmetry for a pair of Majorana spinons as well.

- We only consider the parity action on internal space here, we will include its spacial action in quantum field theory later.

$$P_{\uparrow\uparrow'} = \frac{1}{\sqrt{2}}(1 + \gamma_\uparrow \gamma'_\uparrow); \quad P_{\downarrow\downarrow'} = \frac{1}{\sqrt{2}}(1 - \gamma_\downarrow \gamma'_\downarrow),$$

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$$P\gamma'_\uparrow P^{-1} = \gamma_\uparrow; \quad P\gamma'_\downarrow P^{-1} = -\gamma_\downarrow,$$

- Parity symmetry acts on the spin basis and chiral basis in an expected way.

$$c_\uparrow = \frac{1}{2}(\gamma_\uparrow + i\gamma'_\uparrow); \quad c_\downarrow = \frac{1}{2}(\gamma_\downarrow - i\gamma'_\downarrow) \quad Pc_\uparrow P^{-1} = ic_\uparrow; \quad Pc_\downarrow P^{-1} = ic_\downarrow$$

C^4=-1 charge conjugation symmetry for Majorana ghosts

Since a Majorana particle does not carry charge, it has been believed the charge conjugation symmetry must be trivial.

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- To our surprise It turns out that Majorana ghosts formed by Majorana zero modes will have a nontrivial charge conjugtion symmetry. $C_{\uparrow\downarrow'} = \frac{1}{\sqrt{2}}(1 + \gamma_\uparrow \gamma'_\downarrow); \quad C_{\downarrow\uparrow'} = \frac{1}{\sqrt{2}}(1 + \gamma_\downarrow \gamma'_\uparrow)$

$$\begin{aligned} C\gamma_\uparrow C^{-1} &= -\gamma'_\downarrow; \quad C\gamma_\downarrow C^{-1} = -\gamma'_\uparrow \\ C\gamma'_\uparrow C^{-1} &= \gamma_\downarrow; \quad C\gamma'_\downarrow C^{-1} = \gamma_\uparrow, \end{aligned}$$

- Charge conjugation symmetry acts on the spin basis(particle-hole) and chiral basis(particle-antiparticle) in an expected way.

$$\begin{aligned} Cc_\uparrow C^{-1} &= ic^\dagger_\downarrow; \quad Cc_\downarrow C^{-1} = -ic^\dagger_\uparrow \\ Cc_L C^{-1} &= -ic_R; \quad Cc_R C^{-1} = -ic_L \end{aligned}$$

CPT super algebra for Majorana ghosts

The action of CPT symmetries on four Majorana zero modes forms a super algebra

$$
\begin{aligned}
C^2 &= P^f; \quad P^2 = P^f; \quad T^2 = P^f; \quad (P^f)^2 = 1 \\
TP^f &= P^f T; \quad PP^f = P^f P; \quad CP^f = P^f C \\
TP &= P^f PT; \quad TC = P^f CT; \quad PC = P^f CP,
\end{aligned}
$$

Relativistic field theory

CPT symmetry for Majorana ghosts(in chiral representation)

$$\gamma_0 = -i\rho_z \otimes \sigma_y; \quad \gamma_1 = I \otimes \sigma_z; \quad \gamma_2 = -\rho_y \otimes \sigma_y; \quad \gamma_3 = -I \otimes \sigma_x,$$

$$\gamma_5 = \gamma_0\gamma_1\gamma_2\gamma_3 = i\rho_x \otimes \sigma_y$$

$$\xi(x) = \begin{pmatrix} \gamma_\uparrow(x) \\ \gamma_\downarrow(x) \end{pmatrix}; \quad \eta(x) = \begin{pmatrix} -\gamma'_\uparrow(x) \\ \gamma'_\downarrow(x) \end{pmatrix}$$

$$\begin{aligned} C\psi(x)C^{-1} &= C\begin{pmatrix} \xi(x) \\ \eta(x) \end{pmatrix} C^{-1} = \begin{pmatrix} -\epsilon\eta(x) \\ -\epsilon\xi(x) \end{pmatrix} = -\gamma_5\psi(x); \\ P\psi(x)P^{-1} &= P\begin{pmatrix} \xi(x) \\ \eta(x) \end{pmatrix} P^{-1} = \begin{pmatrix} \eta(\widetilde{x}) \\ -\xi(\widetilde{x}) \end{pmatrix} = \gamma_0\gamma_5\psi(\widetilde{x}); \\ T\psi(x)T^{-1} &= T\begin{pmatrix} \xi(x) \\ \eta(x) \end{pmatrix} T^{-1} = \begin{pmatrix} -\epsilon\xi(-\widetilde{x}) \\ \epsilon\eta(-\widetilde{x}) \end{pmatrix} = \gamma_0\psi(-\widetilde{x}), \end{aligned}$$

$$\psi(x) = \begin{pmatrix} \xi(x) \\ \eta(x) \end{pmatrix}$$

$$\widetilde{x} = (t, -\mathbf{x})$$

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The origin of three generations

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Out of four Majorana zero modes, there are *only* three different ways to form complex fermions, corresponding to the *only* three different projective representations of super CPT algebra with $T^4=-1$, $(TP)^4=1$, $(TC)^4=-1$

$$d_L = \frac{1}{2}(\gamma_\uparrow - i\gamma'_\downarrow); \quad d_R = \frac{1}{2}(\gamma'_\uparrow - i\gamma_\downarrow),$$

$$\begin{aligned} Cd_LC^{-1} &= -id_L; \quad Cd_RC^{-1} = id_R \\ Pd_LP^{-1} &= -d_R; \quad Pd_RP^{-1} = d_L \\ Td_LT^{-1} &= id_R^\dagger; \quad Td_RT^{-1} = id_L^\dagger, \end{aligned}$$

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carry half Z_2 charge!

$$f_L = \frac{1}{2}(\gamma_\uparrow + i\gamma'_\uparrow) = c_\uparrow; \quad f_R = \frac{1}{2}(\gamma_\downarrow + i\gamma'_\downarrow) = c_\downarrow^\dagger$$

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A majorana ghost has three different faces

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From Majorana fermion to chiral fermion

$$\mathcal{L}_0 = \frac{1}{4}\overline{\psi}(x)i\gamma_\mu\partial_\mu\psi(x)$$

$$\xi(x) = \begin{pmatrix} \gamma_\uparrow(x) \\ \gamma_\downarrow(x) \end{pmatrix}; \quad \eta(x) = \begin{pmatrix} -\gamma'_\uparrow(x) \\ \gamma'_\downarrow(x) \end{pmatrix}$$

$$i\partial_0\xi(x) - i\sigma_1\partial_1\xi(x) - i\partial_2\eta(x) - i\sigma_3\partial_3\xi(x) = 0;$$
$$i\partial_0\eta(x) + i\sigma_1\partial_1\eta(x) - i\partial_2\xi(x) + i\sigma_3\partial_3\eta(x) = 0,$$

$$\Psi_d(x) \equiv \begin{pmatrix} d_L(x) \\ id_R^\dagger(x) \end{pmatrix} = \frac{1}{2}[\xi(x) + \sigma_2\eta(x)]$$

$$i\partial_0\Psi_d(x) - i\sigma_1\partial_1\Psi_d(x) - i\sigma_2\partial_2\Psi_d(x) - i\sigma_3\partial_3\Psi_d(x) = i\partial_\mu\overline{\sigma}_\mu\Psi_d(x) = 0$$

$$\widetilde{\xi}(x) = \begin{pmatrix} \gamma_\uparrow(x) \\ \gamma'_\downarrow(x) \end{pmatrix}; \quad \widetilde{\eta}(x) = \begin{pmatrix} \gamma'_\uparrow(x) \\ \gamma_\downarrow(x) \end{pmatrix}$$

Local quantum fields are defined in the eigen basis of C

$$\Psi_c(x) \equiv \begin{pmatrix} c_L(x) + c_R(x) \\ ic_L^\dagger(x) - ic_R^\dagger(x) \end{pmatrix} = \tfrac{1}{2}[\widetilde{\psi}_+(x) + \sigma_2\widetilde{\psi}_-(x)] \quad \widetilde{\psi}_\pm(x) = \frac{1}{\sqrt{2}}[\widetilde{\xi}(x) \pm \widetilde{\eta}(x)]$$

$$\hat{\xi}(x) = \begin{pmatrix} \gamma_\uparrow(x) \\ \gamma'_\downarrow(x) \end{pmatrix}; \quad \hat{\eta}(x) = \begin{pmatrix} -\gamma_\downarrow(x) \\ \gamma'_\uparrow(x) \end{pmatrix}$$

Three kinds of spinon basis can not be connected by rotation

$$\Psi_f(x) = \tfrac{1}{2}[\hat{\psi}_+(x) + \sigma_2\hat{\psi}_-(x)] = \begin{pmatrix} f_L(x) - f_R(x) \\ if_L^\dagger(x) + if_R^\dagger(x) \end{pmatrix} \quad \hat{\psi}_\pm(x) = \frac{1}{\sqrt{2}}[\hat{\xi}(x) \pm \hat{\eta}(x)]$$

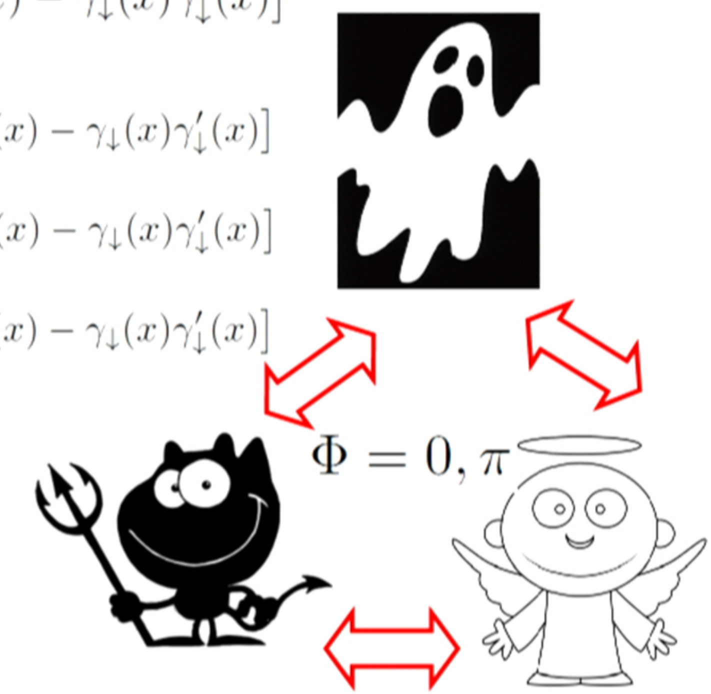
Universal gauge coupling

$$\begin{aligned}
\frac{ig}{4}\phi(x)\overline{\psi}_f(x)\psi_f(x) &= \frac{ig\phi(x)}{2}\left[\gamma_\uparrow(x)\gamma'_\uparrow(x) - \gamma_\downarrow(x)\gamma'_\downarrow(x)\right] \\
\frac{ig}{4}\phi(x)\overline{\psi'}_d(x)\psi'_d(x) &= \frac{ig\phi(x)}{2}\left[\gamma_\uparrow(x)\gamma'_\uparrow(x) - \gamma_\downarrow(x)\gamma'_\downarrow(x)\right] \\
\frac{ig}{4}\phi(x)\overline{\psi'}_c(x)\psi'_c(x) &= \frac{ig\phi(x)}{2}\left[\gamma_\uparrow(x)\gamma'_\uparrow(x) - \gamma_\downarrow(x)\gamma'_\downarrow(x)\right]
\end{aligned}$$

$$g = \pm g'$$

$$\begin{aligned}
\frac{ig_{cd}}{2}\phi(x)\overline{\psi'}_d(x)\psi'_c(x) &= \frac{ig_{cd}\phi(x)}{2}\left[\gamma_\uparrow(x)\gamma'_\uparrow(x) - \gamma_\downarrow(x)\gamma'_\downarrow(x)\right] \\
\frac{i\sqrt{2}g_{cf}}{4}\phi(x)\overline{\psi}_f(x)\psi'_c(x) &= \frac{ig_{cf}\phi(x)}{2}\left[\gamma_\uparrow(x)\gamma'_\uparrow(x) - \gamma_\downarrow(x)\gamma'_\downarrow(x)\right] \\
\frac{i\sqrt{2}g_{df}}{4}\phi(x)\overline{\psi}_f(x)\psi'_d(x) &= \frac{ig_{df}\phi(x)}{2}\left[\gamma_\uparrow(x)\gamma'_\uparrow(x) - \gamma_\downarrow(x)\gamma'_\downarrow(x)\right]
\end{aligned}$$

$$g_{cd} = g_{cf} = g_{df} = g'$$



Neutrino mass mixing matrix

$$M = \begin{pmatrix} g & \sqrt{2}g' & \sqrt{2}g' \\ \sqrt{2}g' & g & 2g' \\ \sqrt{2}g' & 2g' & g \end{pmatrix} = U^\dagger \begin{pmatrix} (1-\sqrt{5})g' + g & 0 & 0 \\ 0 & (1+\sqrt{5})g' + g & 0 \\ 0 & 0 & -2g' + g \end{pmatrix} U$$

$$U^\dagger = \begin{pmatrix} \sqrt{\frac{5+\sqrt{5}}{10}} & -\sqrt{\frac{5-\sqrt{5}}{20}} & -\sqrt{\frac{5-\sqrt{5}}{20}} \\ \sqrt{\frac{5-\sqrt{5}}{10}} & \sqrt{\frac{5+\sqrt{5}}{20}} & \sqrt{\frac{5+\sqrt{5}}{20}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \simeq \begin{pmatrix} 0.85 & -0.37 & -0.37 \\ 0.53 & 0.6 & 0.6 \\ 0 & -0.71 & 0.71 \end{pmatrix}$$

$$g = -g'$$

$$\theta_{23} = -45^\circ; \quad \theta_{13} = 0; \quad \theta_{12} = 31.7^\circ (\tan^2\theta_{12} = \frac{\sqrt{5}-1}{2}) \quad m_1 = m_2 = \sqrt{5}g$$

$$m_3 = 3g$$

Beyond Standard model and prediction of neutrino masses

Standard model predicts exact zero mass for neutrino!

A direct majorana mass term for light neutrino is not allowed in Standard Model(SM) since it breaks electric-weak interactions.

See-saw mechanism and massive stirring neutrino.

$$M_{total} = \begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix}$$

$m_D \ll M$ GUT scale

$m_1 \sim m_D^2/M; \quad m_2 \sim M$

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$m_D \ll M$ **GUT scale**

$m_1 \sim m_D^2/M; \quad m_2 \sim M$

$$m_D = diag(m_D, m_D, m_D)$$

$m_3 \simeq 0.054ev$

$$\boxed{m_3/m_1 = m_3/m_2 = \sqrt{5}/3}$$

$m_1 = m_2 \simeq 0.075ev$

Without massive neutrino

$$\boxed{m_3/m_1 = m_3/m_2 = 3/\sqrt{5}}$$

If a direct majorana mass appears, a big challenge for SM.

Conclusions and future works

- We propose a physical definition of a ghost.
- CPT symmetry for Majorana ghosts.
- The origin of three generations of neutrinos.
- The origin of neutrino masses and their mass mixing.
- CP violation physics and CP angle.
- Quark CKM matrix.
- Lattice models in 3D and possible realization.
- Shed new light on quantum gravity.(super cohomology /super fiberbundle)