Title: Entanglement at strongly-interacting quantum critical points in 2+1D

Date: May 10, 2013 04:00 PM

URL: http://pirsa.org/13050051

Abstract: In two or more spatial dimensions, leading-order contributions to the scaling of entanglement entropy typically follow the "area" or boundary law. Although this leading-order scaling is non-universal, at a quantum critical point (QCP), the sub-leading behavior does contain universal physics. Different universal functions can be access through entangling regions of different geometries. For example, for polygonal shaped regions, quantum field theories have demonstrated that the subleading scaling is logarithmic, with a universal coefficient dependent on the number of vertices in the polygon. Although such universal quantities are routinely studied in non-interacting field theories, it often requires numerical simulation to access them in interacting theories. In this talk, I discuss quantum Monte Carlo (QMC) and numerical Linked-Cluster Expansion (NLCE) calculations of the Renyi entropies at the transverse-field Ising model QCP on the 2D square lattice. We calculate the universal coefficient of the vertex-induced logarithmic scaling term, and compare to non-interacting field theory calculations by Casini and Huerta. Also, we examine the shape dependence of the Renyi entropy for finite-size toroidal lattices with smooth boundaries. Such geometries provide a sensitive probe of the gapless wave function in the thermodynamic limit, and give new universal quantities that could be examined by future field-theoretical studies in 2+1D.

Entanglement at strongly-interacting quantum critical points in 2+1D

Roger Melko

Entanglement scaling in two-dimensional gapless systems, **Hyejin Ju, Ann Kallin**, Paul Fendley, Matthew B Hastings, RGM, Phys. Rev. B 85, 165121 (2012)

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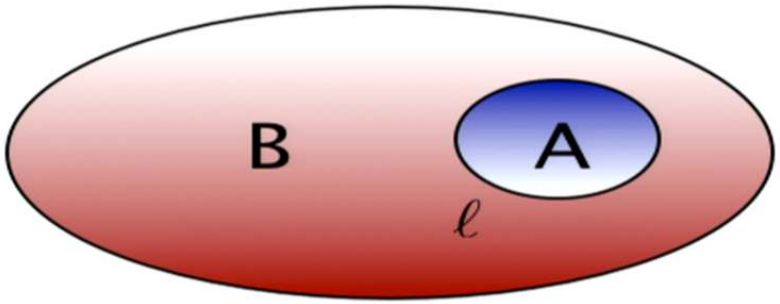
Renyi entropies

$$S_n(\rho_A) = \frac{1}{1-n} \ln \left[\mathrm{Tr}(\rho_A^n) \right]$$



- We have been very successfully in characterizing (via theory/ numerics) exotic gapped systems – e.g. Topological entanglement entropy:

$$S_n = A\ell + \gamma$$

- Similarly, will it be possible to classify gapless systems (spin liquids,quantum critical points) by subleading scaling terms?

- For example, subleading terms are predicted to be a resource in distinguishing conventional from fractional critical points

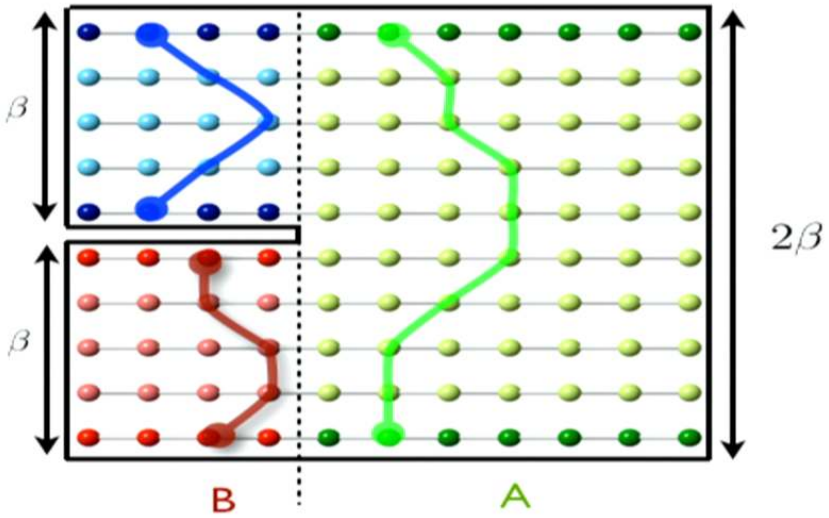
$$\gamma_{XY*} = \gamma_{XY} + \gamma_{Z_2}$$

Swingle, Senthil, Phys. Rev. B 86, 155131 (2012)

- In gapless systems in D>1, we are still learning how γ depends on subregion shape and topology

- Ultimately, want to compare field theory and numerics. However, some entangled shapes amenable to continuum theory don't work with lattice simulations, and vice versa

- Here, we will examine the shape dependence of the subdeading γ using numerics (mostly QMC) at the simplest interacting QCP in 2D



$$H = -J \sum_{\langle i,j \rangle} \sigma_i^z \sigma_j^z - h \sum_i \sigma_i^x$$

$(h/J)_c = 3.044$

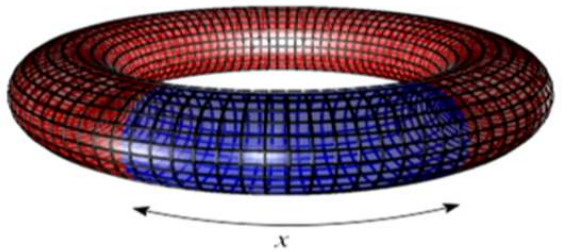
n-sheeted Riemann surface:

sometimes integer
$n > 2$ is all we have

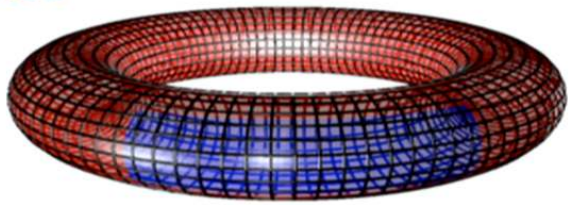
Outline

- Review of entanglement entropy scaling in 1D

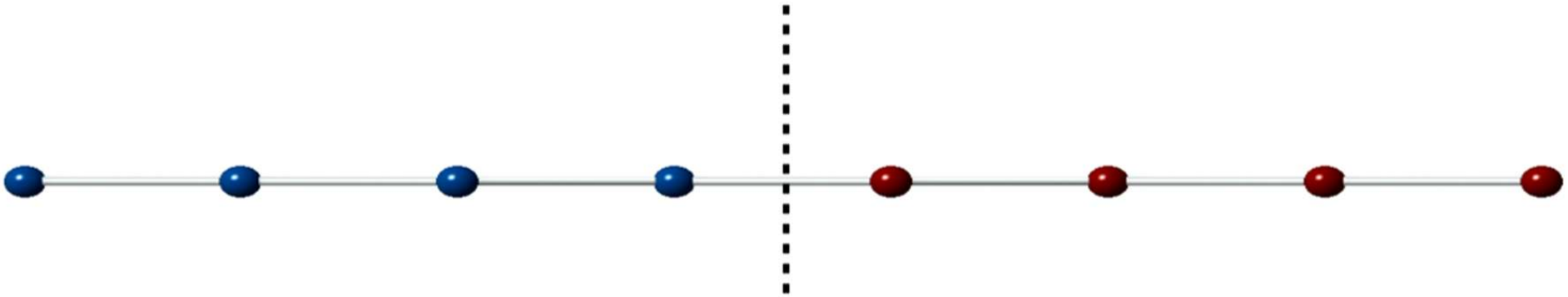
- 2D: area law and beyond
  - Two-cylinder entropies



  - Universal contributions due to vertices



• Critical systems in 1D... Scale invariance: assume $\mathcal{O}(1)$ unit of entanglement entropy at **each** length scale



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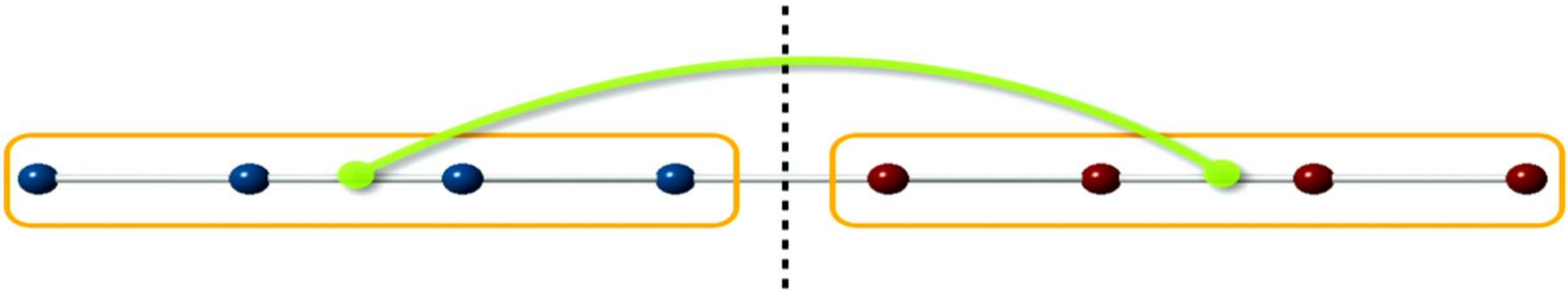

$$\mathcal{O}(1) \times (1+1+1+\cdots) = \log(L)$$

C. Holzhey, F. Larsen, and F. Wilczek, Nucl. Phys. B424, 443 (1994)
G. Vidal, J. I. Latorre, E. Rico, and A. Kitaev, Phys. Rev. Lett. 90, 227902 (2003)
Calabrese and Cardy, J. Stat. Mech: Theory Exp. P06002 (2004)

- Proper way: conformal mapping from a cylinder to a plane:

$$S_n = \frac{c}{3\eta}\left(1+\frac{1}{n}\right)\log\left[\frac{\eta L}{\pi a}\sin\frac{\pi x}{L}\right] + \cdots$$

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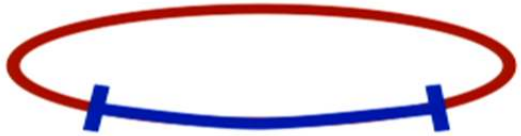
- Finite-size scaling form allows a detailed comparison with numerics

$\eta = 1$

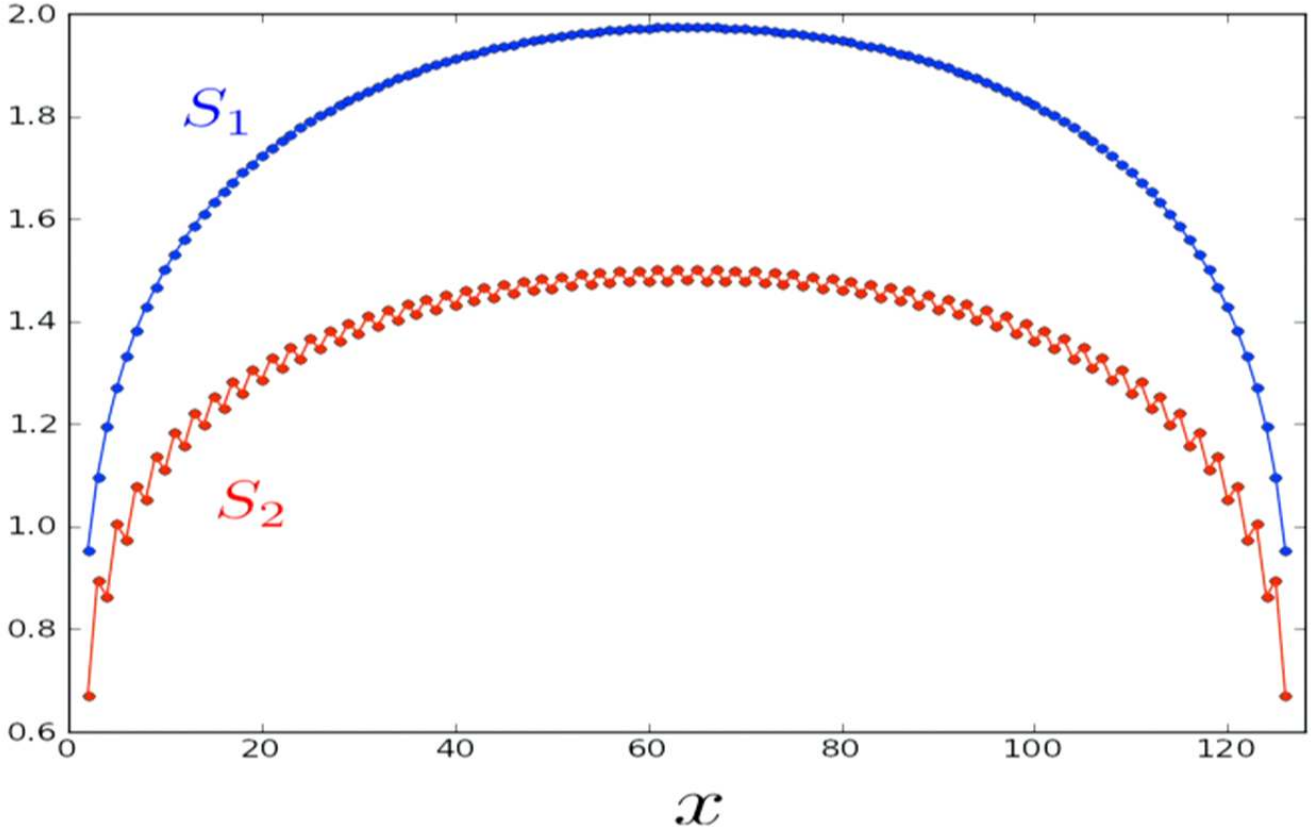


$\eta = 2$

- There is a lot of synergy between theory and DMRG in 1+1...

$$H = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

$x = 4$



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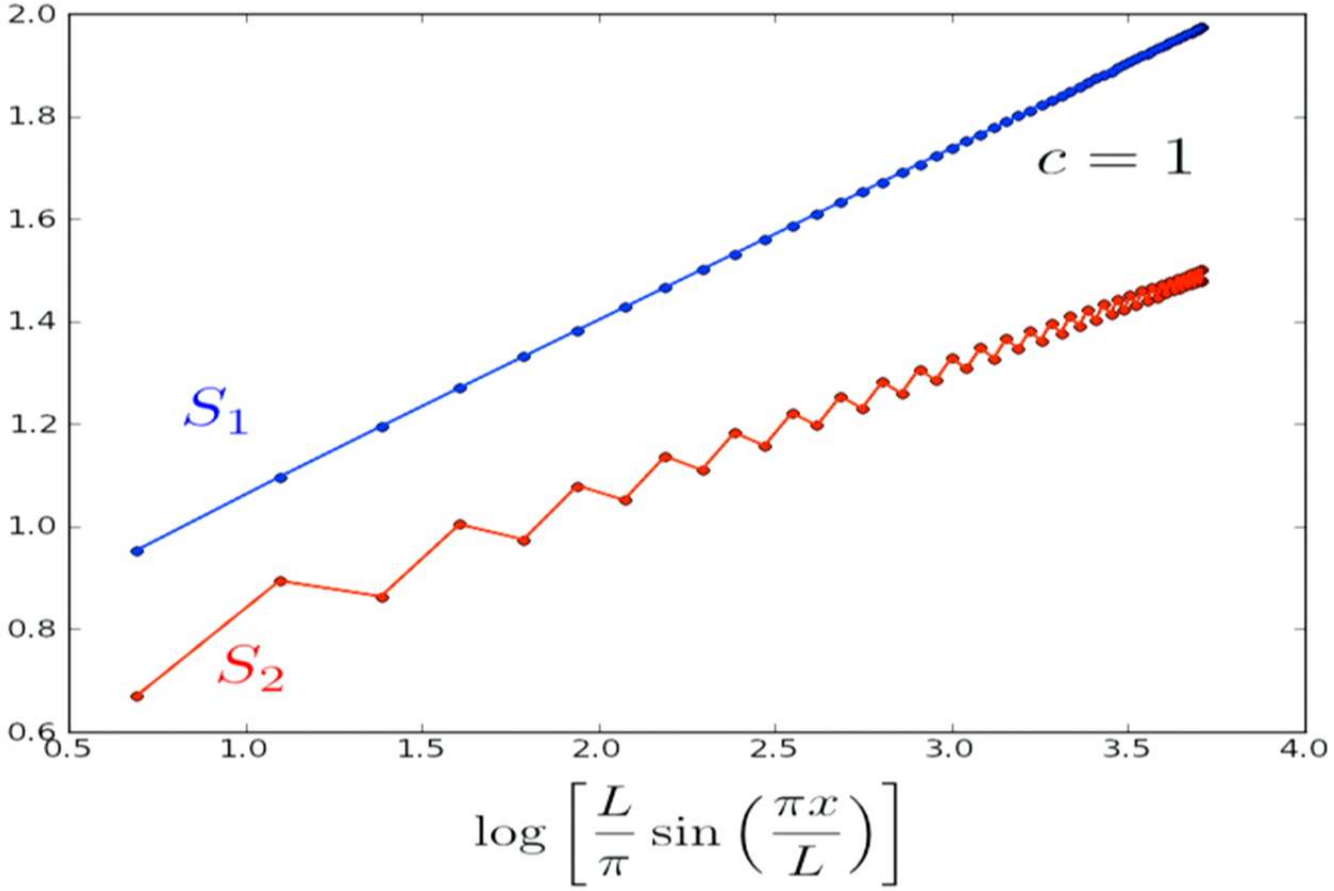

- In a critical system in 2D, the leading-order scaling is area-law:

$$S_n = A\ell + \cdots$$

this is confirmed numerically on a variety of boson models

- General expectation: geometry-dependent subleading term

S. Ryu and T. Takayanagi
Y. Zhang, T. Grover, Vishwanath
Casini, Huerta, Myers

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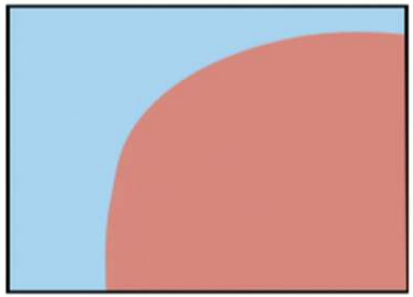
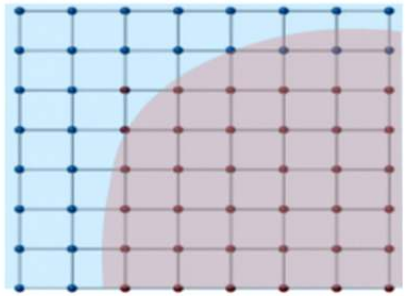
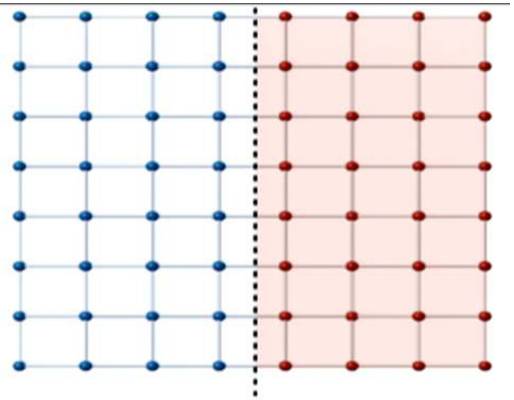
smooth, curved surfaces have defects on the lattice scale





- Want to avoid studying pixelization effects at this stage...

• One may induce a universal subleading term from a smooth boundary on the infinite plane:



finite correlation length

Metlitski, Fuertes, Sachdev, PRB 80, 115122 (2009)

$$S_n = A\ell + r_n \frac{L^{D-1}}{\xi^{D-1}},$$

ϵ , large $-N$: r_n changes sign (!) near $n \approx 1$ at the W–F fixed point

$$r_n \propto \frac{(n-1)}{\epsilon^2}$$

violates some entanglement monotonicity theorem?

H. Casini and M. Huerta, Phys. Rev. D 85, 125016 (2012)

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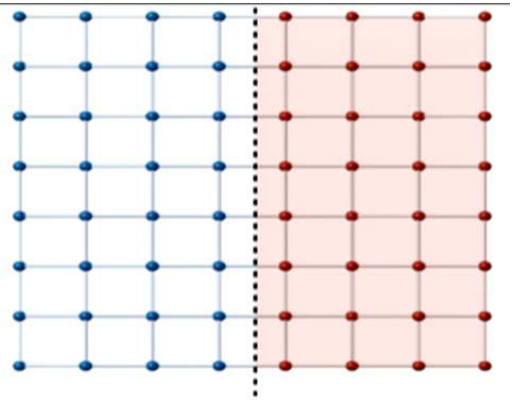

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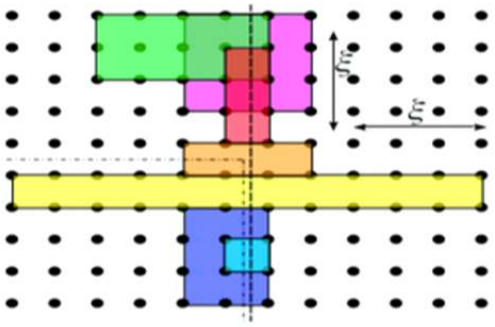

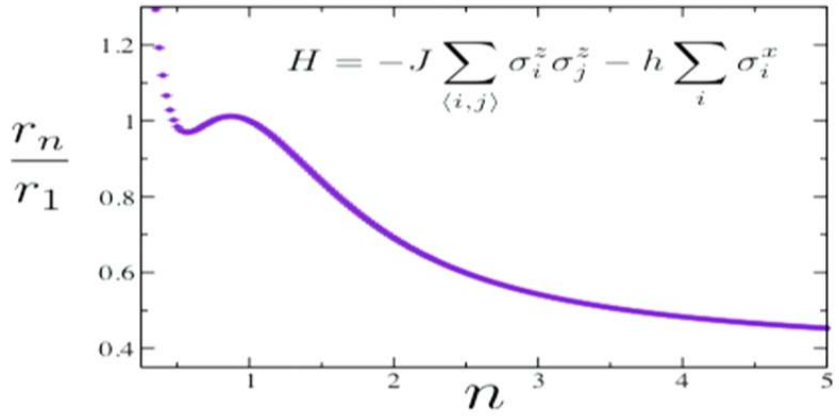

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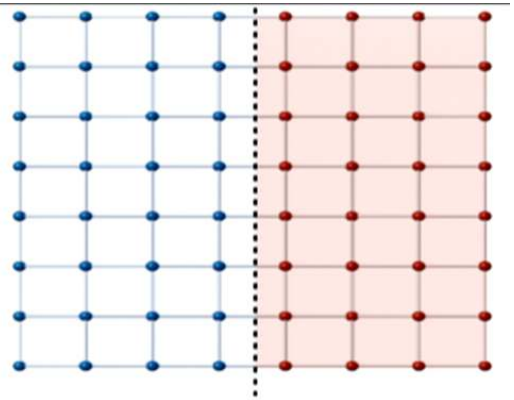

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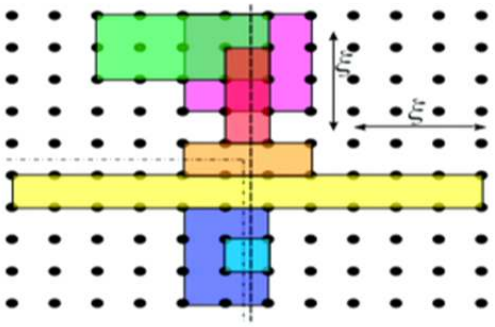
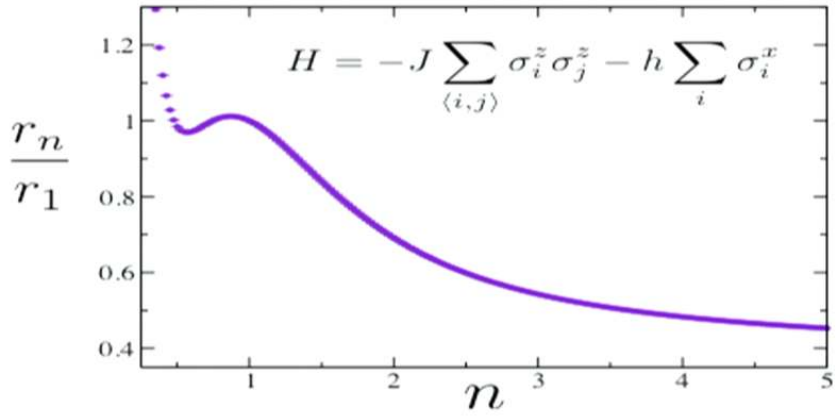
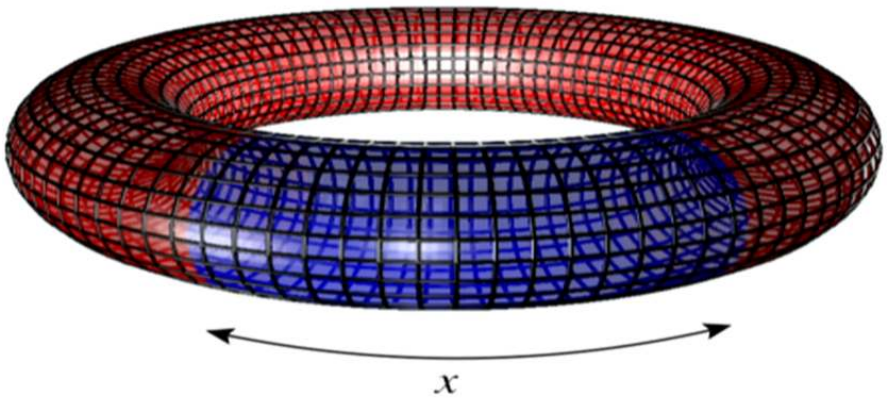






QMC geometries – 2D torus, two smooth boundaries

$$S_n = A\ell + \gamma(x/L_x, L_x/L_y)$$

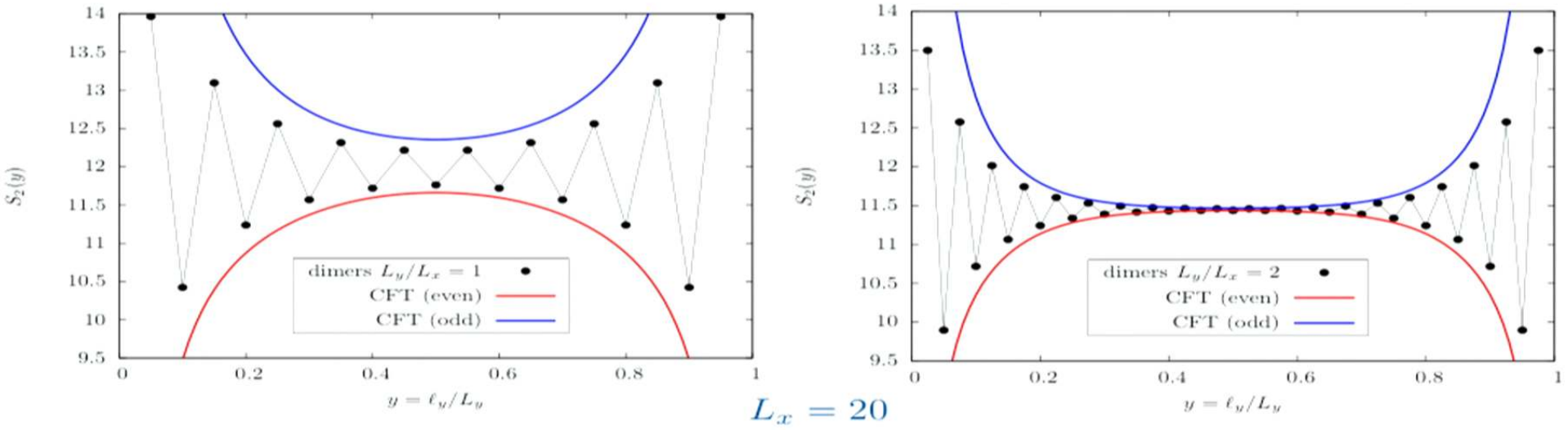
Stéphan, Ju, Fendley, RGM, New J. Phys. 15, 015004 (2013)

- Jean-Marie Stéphan: "RVB-shape function", derived in a quantum Lifshitz free scalar field theory

$$J_n(y) = \frac{n}{1-n} \log \left[\frac{\eta(\tau)^2}{\theta_3(2\tau)\theta_3(\tau/2)} \frac{\theta_3(2y\tau)\theta_3(2(1-y)\tau)}{\eta(2y\tau)\eta(2(1-y)\tau)} \right]$$

$$y = x/L$$

$$\tau = iL_x/L_y$$



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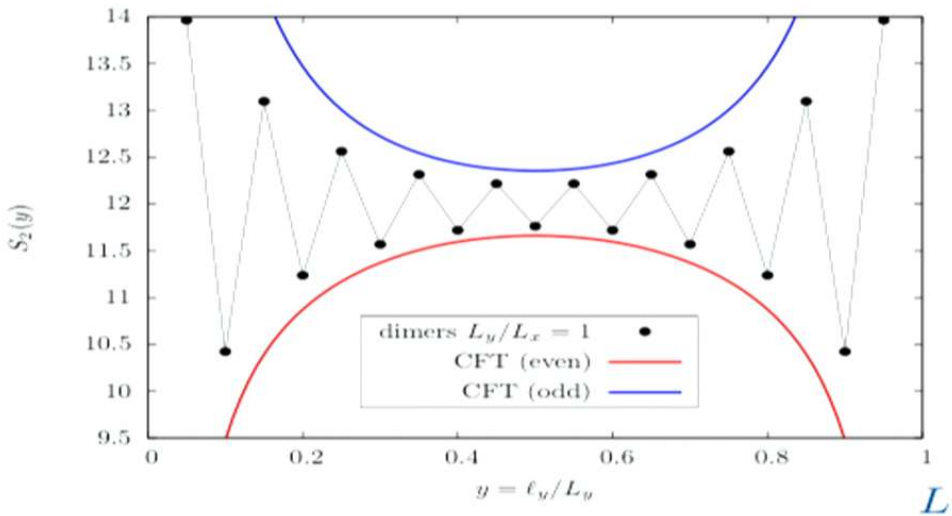

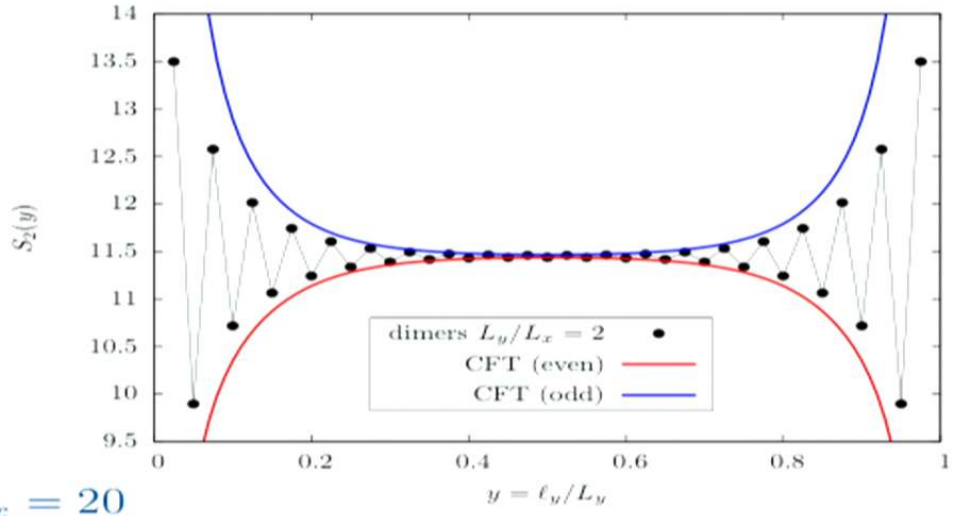

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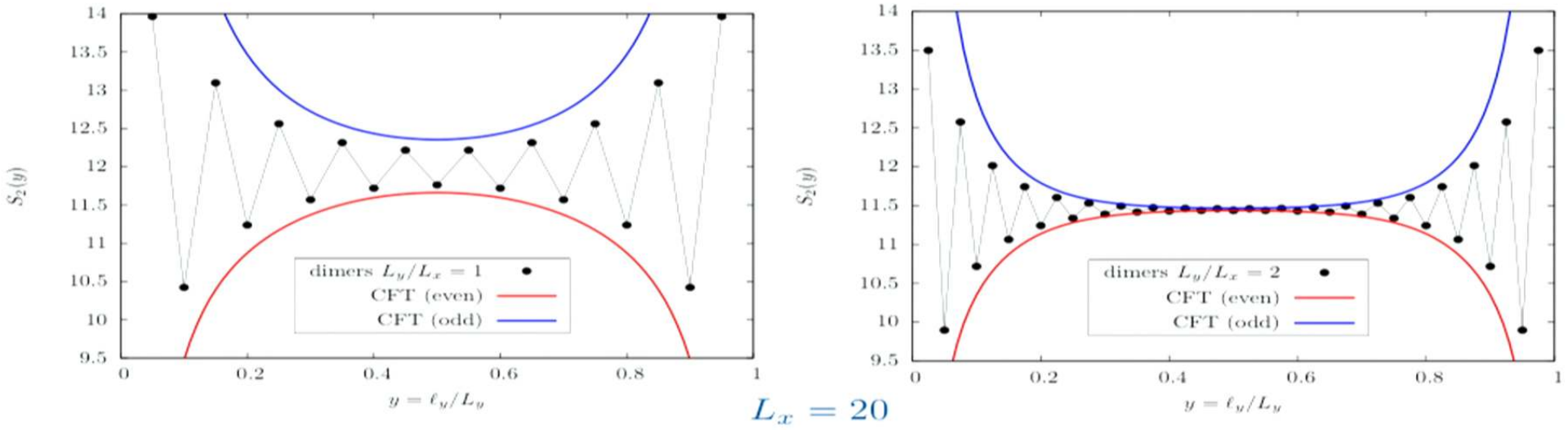

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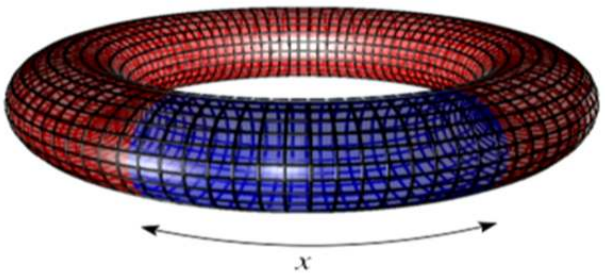
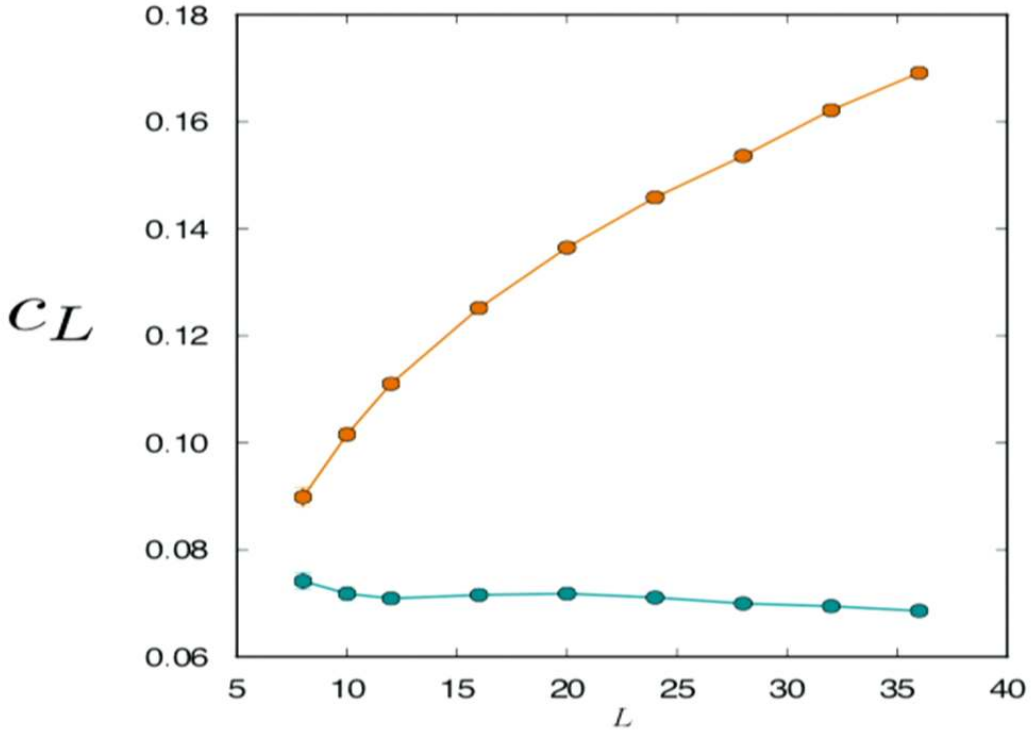


$$S_{\log} = a\ell + c_L \log(\sin(\pi y)) + d$$

$$S_{\mathrm{RVB}} = a\ell + c_L J(y) + d$$

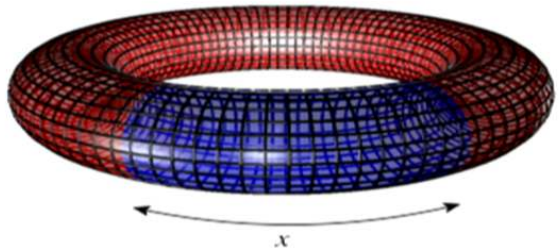


clear L-dependence remaining

consistent with L-independence

$c \approx 0.07$

Two-cylinder Renyi entropies



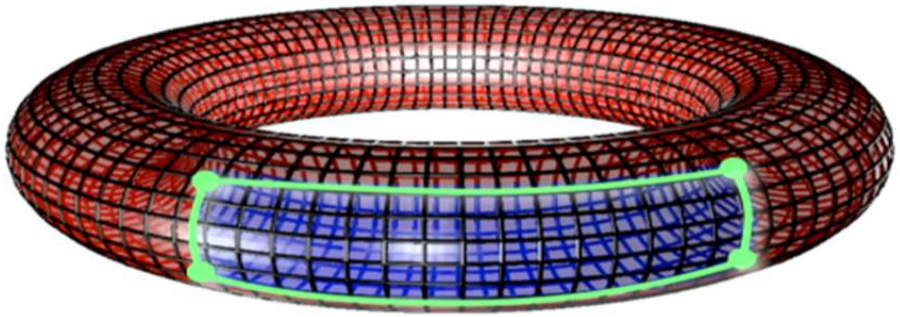
- Numerical results at quantum critical point of TFIM are consistent with NO additive logarithm
  - also true in RVBs, pi-flux fermions
  - in clear contrast to case with broken continuous symmetry:

$$\frac{N_G(D-1)}{2}\log(\ell)$$

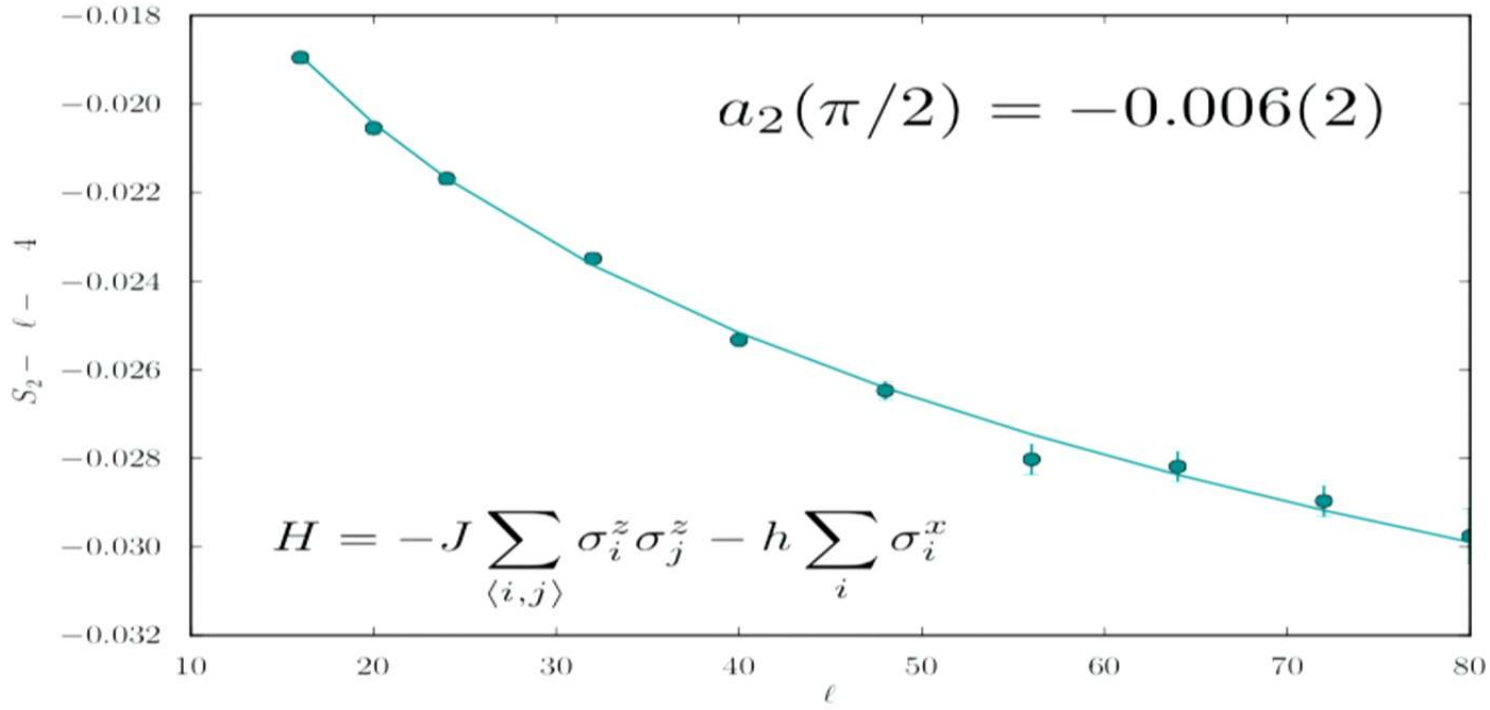
Kallin, Hastings, RGM, Singh, PRB 84, 165134 (2011)
Metlitski, Grover arXiv:1112.5166 (2011)

- Fun to speculate regarding the universality of $J(y)$ and its coefficient

- Need systematic QMC calculations at other fixed points



$$S_n = A\ell + 4a_n(\theta)\log(\ell)$$



Results for 90-degree corner coefficient from the literature

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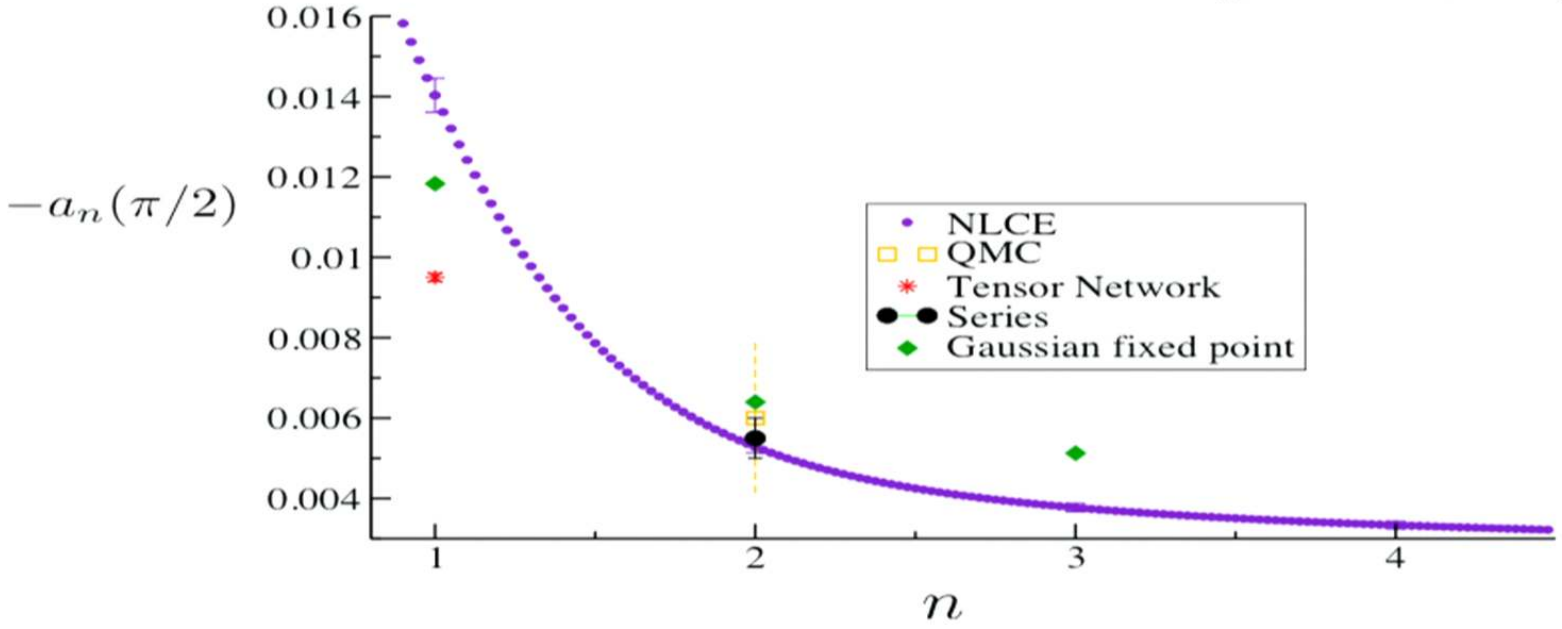

- Tagliacozzo, Evenbly, Vidal Phys. Rev. B 80, 235127 (2009)
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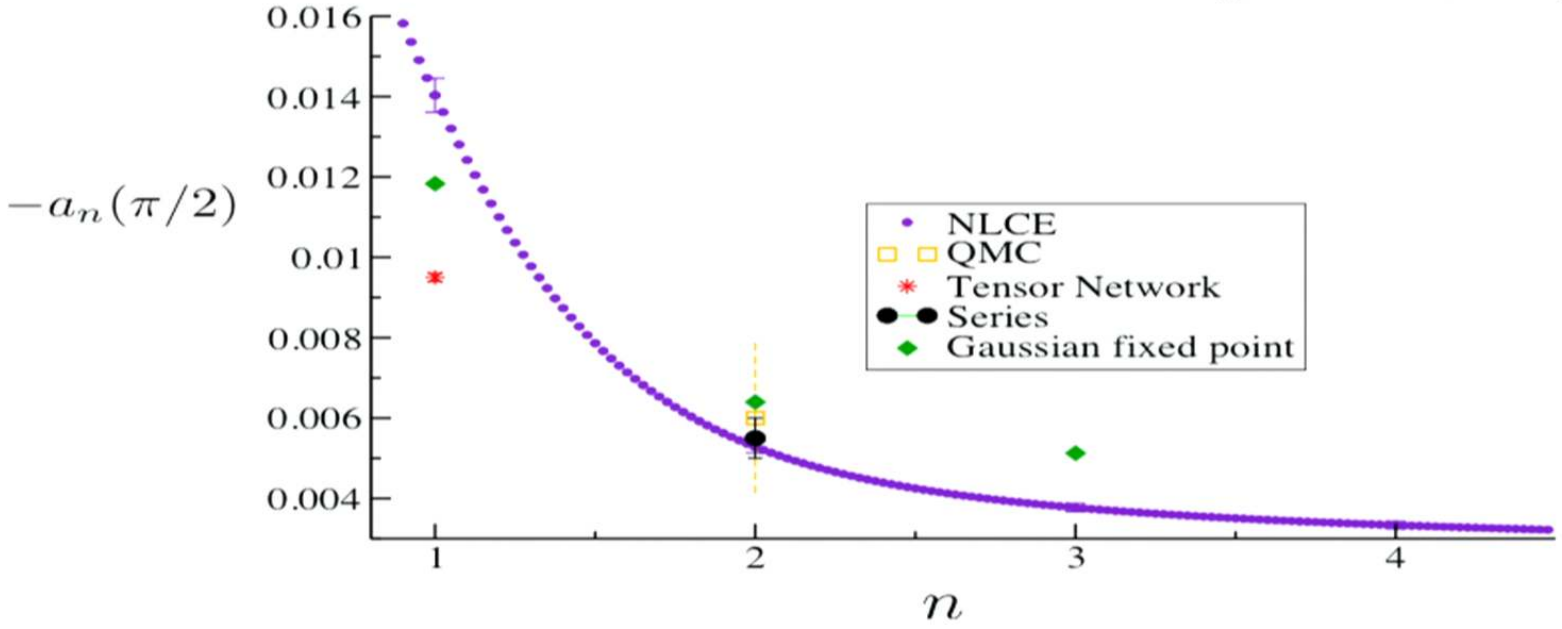

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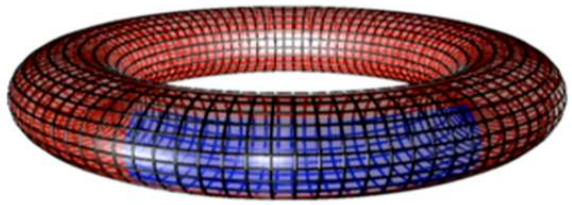
universal corner coefficient



- We're starting to get a handle on these numerical values for the Ising QCP

- Can we use this number to distinguish Gaussian from Ising? Which numerical method is working best?

- Wish List:

  - Coefficient of the corner log in ϵ and/or $1/N$ expansion at W-F fixed point

  - More tensor network/series expansion calculations

  - Next: QMC/series and field theory calculations for the O(N) transition

Discussion

- We have the technology to take a more serious look at entanglement in gapless systems.

- Numerics/field theory/holography has potential to give c,F-theorems working relevance for condensed-matter systems

- See Tarun Grover- best talk that never happened

11/07, 1:45 p.m.	Tarun Grover KITP	Quantum Entanglement and Stability of Gapless Spin-liquids[Podcast][Aud][Cam]

arXiv:1211.1392

- Renyi entropies will be the quantities eventually measured in experiments

Cardy, PRL 106, 150404 (2011)
Abanin, Demler, PRL 109, 020504 (2012)
Pichler, Bonnes, Daley, Läuchli, Zoller arXiv:1302.1187