Title: TBA

Date: May 10, 2013 09:00 AM

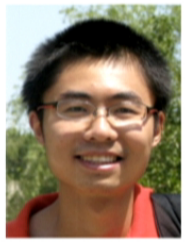
URL: http://pirsa.org/13050046

Abstract:

Topological Phases with of Bosons with Short Range Quantum Entanglement

Ashvin Vishwanath
UC Berkeley

Acknowledgements



YuanMing Lu
(Berkeley)

arXiv:1205.3156 (PRB 2012)
`Integer' topological phases in 2D: A Chern-Simons approach
Yuan-Ming Lu, AV

T. Senthil
(MIT)

arXiv:1209.3058 (PRX 2013)
Physics of 3D bosonic topological insulators:
AV, T. Senthil

Xie Chen
(Berkeley)

arxiv:
SPT phases from decorated domain walls:
Xie Chen, Y.M. Lu, AV.

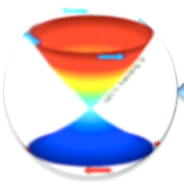
REVIEW: arXiv:1301.0330

Beyond Topological Band Insulators. Ari Turner, AV

Other collaborators:

Fiona Burnell, Lukas Fidkowski, Tarun Grover.

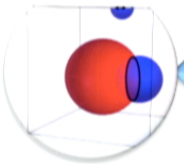
OUTLINE



Introduction

2. Field theory of Bosonic Topological Phases in 2+1 Dimensions.

3. Field Theory of Bosonic SPT in 3+1 Dimensions



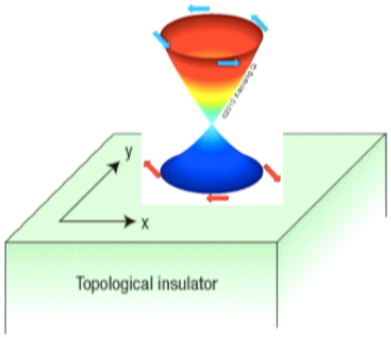
4. Intuitive Description from Decorated domain walls.

Introduction - Free Fermion Topological Insulators

3D Topological Insulators

(Fu, Kane, Mele; Moore, Balents; Fu, Kane; Roy; Hsieh, Hasan, Cava et al.; Xia et al.; Chen et al.; Qi, Hughes, Zhang)

- Bulk gapped. Gapless surface states, not realizable in a 2D system
- Protected by Time reversal and charge conservation symmetry.



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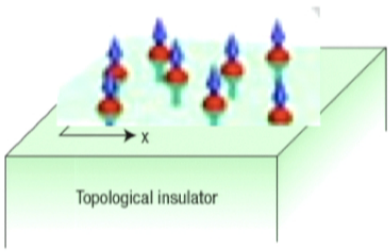

Break time reversal *only* at the surface.
Unusual insulating state. "σ_{xy}=1/2"

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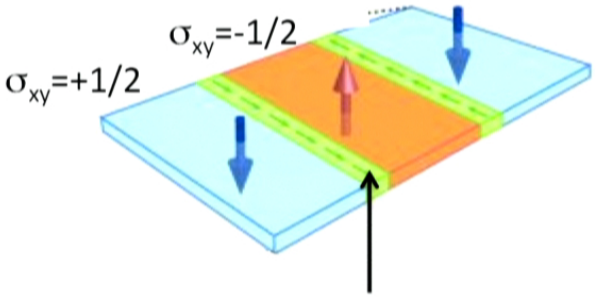
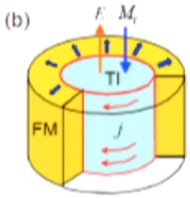
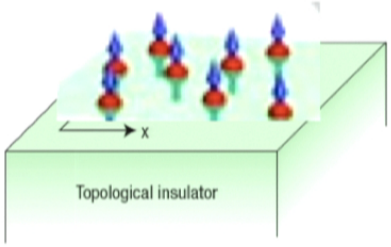


One chiral mode on surface domain walls

OR Quantized Magneto-electric polarizability.



$$P = \theta \frac{e^2}{2\pi h} B \qquad \theta = \pi$$



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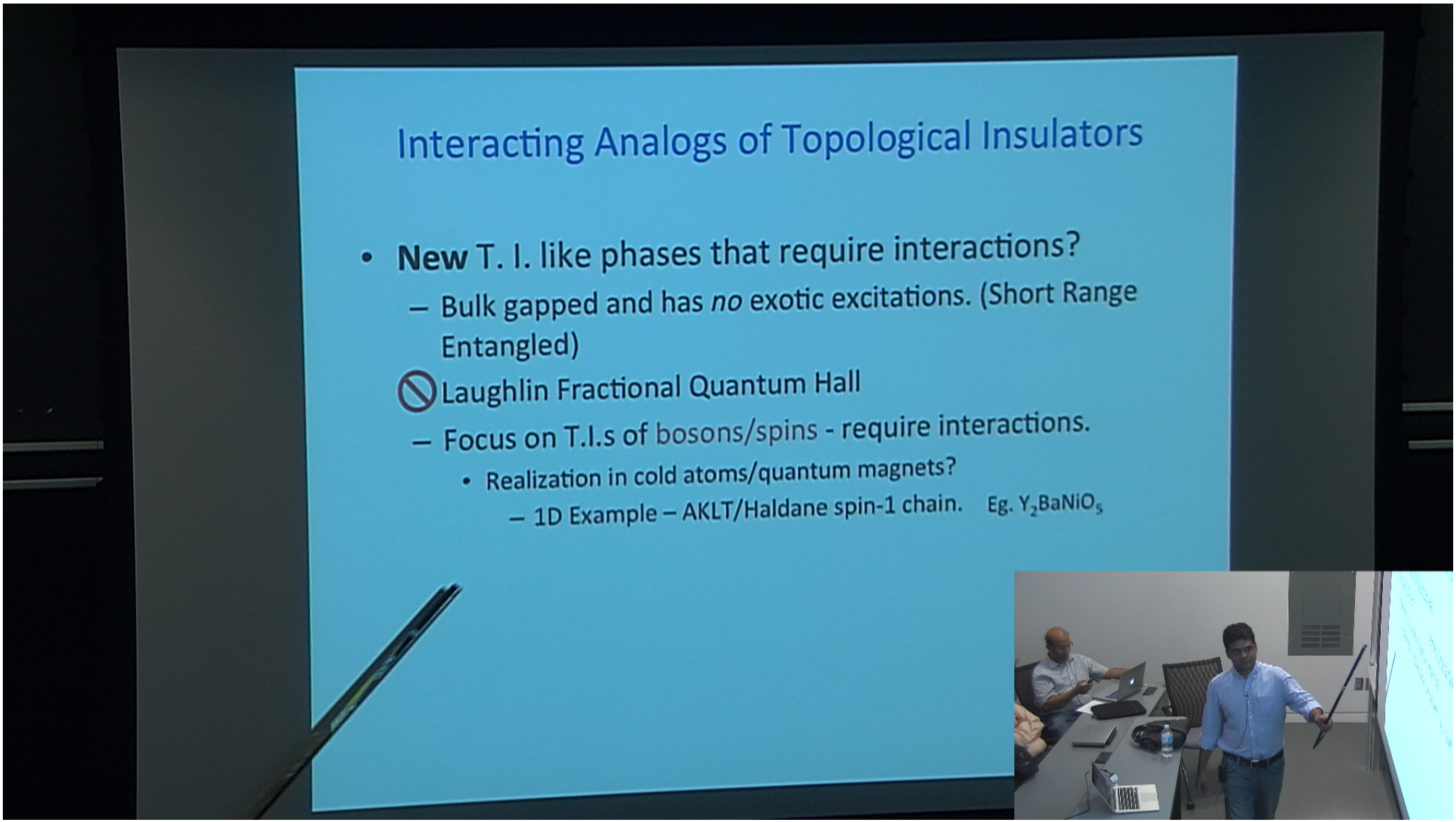
Interacting Analogs of Topological Insulators

- **New** T. I. like phases that require interactions?
  - Bulk gapped and has *no* exotic excitations. (Short Range Entangled)
  - ⊘ Laughlin Fractional Quantum Hall
  - Focus on T.I.s of bosons/spins - require interactions.
    - Realization in cold atoms/quantum magnets?
      - 1D Example – AKLT/Haldane spin-1 chain. Eg. Y_2BaNiO_5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`Ising model' of interacting topological phases – new insights.
Eg. `Impossible' Topological order on the surface of 3D Topological phases.

(AV-Senthil, Wang-Senthil, Burnell-Chen-Fidkowski-AV, Metlitski-Kane-Fisher. See talks by Xie Chen and Senthil)

Short Range Entangled Topological Phases of Bosons in D=2,3

- A. Kitaev: E_8 state in d=2. No symmetry.

- Chen, Gu, Liu, Wen (2011): Symmetry protected topological phases.
  - Symmetry group G -> set of topological phases related to group cohomology.

Symmetry → Dimension↓	Time Rev. $\mathcal{T}$	Charge Consv. U(1)	Charge Consv. +Time Rev. U(1) |x T
d=1	Z_2	Z_1	Z_2
d=2	$\mathbf{Z_1}$	**Z**	$\mathbf{Z_2}$
d=3	$\mathbf{Z_2}$	$\mathbf{Z_1}$	$\mathbf{Z_2 x Z_2}$

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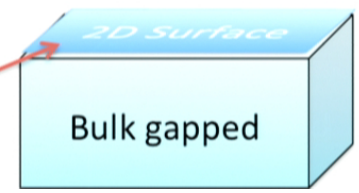
Physical properties? Nature of surface states?



Field Theory Approach to 2D Symmetry Protected Topological Phases

2D Topological Phases: Chern-Simons Approach

- K-matrix description of topological phases.
  (eg. Hierarchy quantum Hall states, bilayer states etc.).

Theoretically
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Physical properties
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$$(\rho, j_x, j_y) = \frac{\nabla \times a}{2\pi} \quad \text{where} \quad (\partial_t, \partial_x, \partial_y) = \nabla$$

$$\mathcal{L} = \sum_{I,J} \frac{K_{IJ}}{4\pi} a^I \cdot \nabla \times a^J$$

- Multiple components I,J=1,2.
- Mutual statistics

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2D Topological Phases: Chern-Simons Approach

- K-matrix directly related to edge states:

$$\mathcal{L} = \sum_{I,J} \frac{K_{IJ}}{4\pi} a^I \cdot \nabla \times a^J \quad \Rightarrow \quad \mathcal{L}_{\text{edge}} = \frac{1}{4\pi} K_{IJ} \dot{\phi}_I \partial_x \phi_J$$

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K=1 not allowed.

‘Bosonic’ Topological Superconductor in d=2.

1. Bosonic ‘topological Sc.’
(like p+ip)

- chiral state, no symmetry.

MIN Dimension: 8x8

- 8 chiral edge modes.

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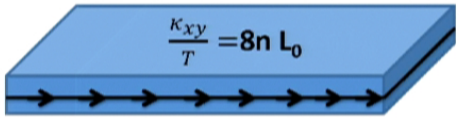
   - K matrix (E_8 Cartan matrix)
   - Kitaev E_8 state.

$$K^{E_8} = \begin{pmatrix} 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 2 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$



1. No Symmetry.
   Z classes. Chiral Edge States.
   Quantized Thermal Hall Conductivity

Bosonic Integer Quantum Hall Phase

Simplest non-chiral state:

$$K = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\mathcal{L} = \frac{1}{2\pi} a_1 \cdot \nabla \times a_2$$

2. Bosonic 'Integer Hall' state: U(1) symmetry. Charge vector: q= (1 n)

- Quantized σ_{xy}=2n (q^2/h).
- EVEN Integers only! (Lu-AV 2012, Senthil-Levin)

Input: DVI - 1920x1080p@60Hz
Output: SDI - 1920x1080i@60Hz

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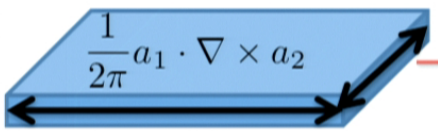
- Edge Luttinger Liquid with 'impossible' symmetry transformations.

Edge States and Symmetry

- Edge - 1D Luttinger Liquid

  But Symmetries act in a way that is *impossible* in 1D. (Lu-AV, Chen-Lu-Wen.)



$$\mathcal{L}_{\text{edge}} = \dot{\phi}_1 \frac{\partial_x \phi_2}{2\pi}$$

ϕ_1 Boson phase,

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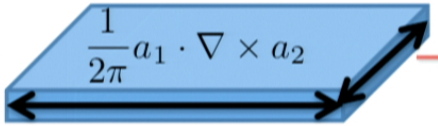

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Usual Tomonaga-Luttinger Liquid	Edge Tomonaga-Luttinger Liquid
$(\varphi) \quad \phi_1 \to \phi_1 + \epsilon$	
$(2\theta) \quad \phi_2 \to \phi_2$	
Δ L=Cos ϕ_2 (can be gapped)	*No gapping term*

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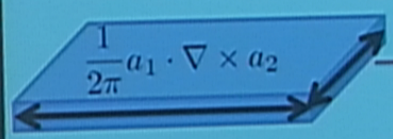

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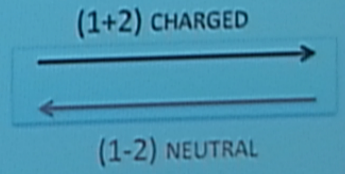

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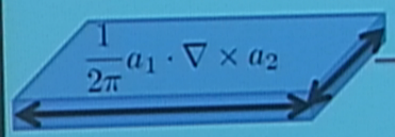

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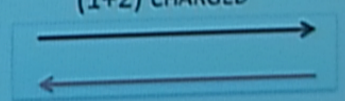
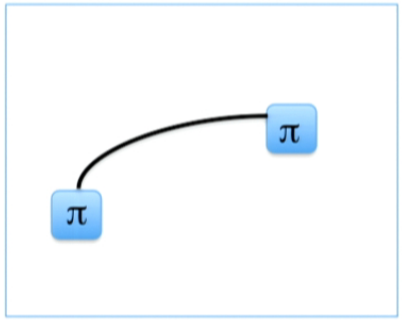


Topological Phase with Z_2 symmetry.

- Edge theory can be used to extract all universal properties.
  - Eg. Z_2 symmetry (Levin-Gu). $\Psi(\alpha) = (-1)^{N_{\text{domain-wall}}}$

Pi-fluxes. End points of Z2 domain walls.



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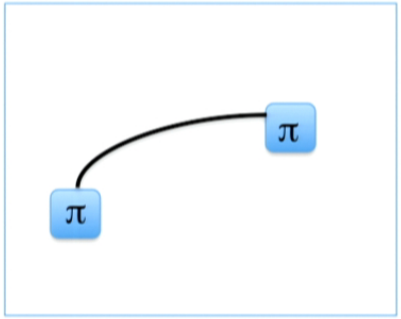
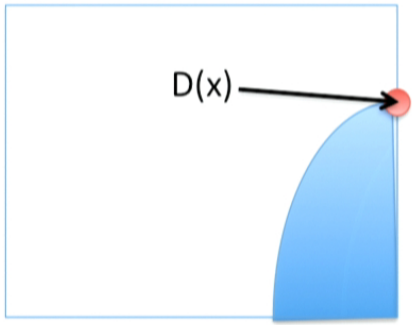


Pi fluxes are semions.

$$\phi_{1,2} \to \phi_{1,2} + \pi$$

Or – end domain wall at edge



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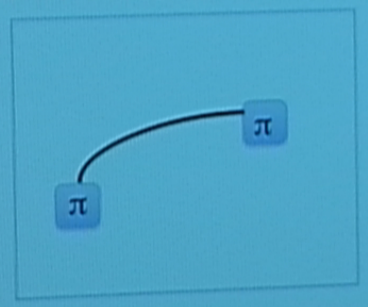

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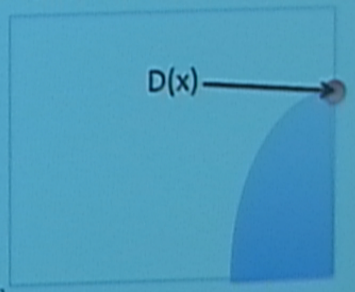


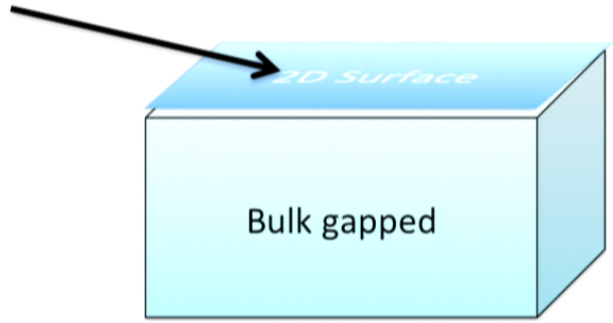
$$D(x) = e^{i\pi \frac{\int^x dx\,[\partial_x \phi_1 + \partial_x \phi_2]}{2\pi}}$$

$$= e^{\frac{i}{2}(\phi_1(x) + \phi_2(x))}$$

$$D(x)D(x') = i\,\mathrm{Sign}(x - x')D(x')D(x)$$

3D BOSONIC `TOPOLOGICAL INSULATORS'

- Surface: 2D Bosons but symmetries act in a special way.



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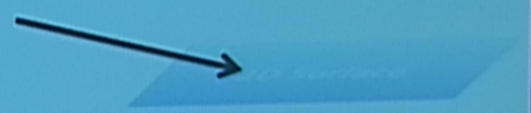

- Example: Symmetry: U(1) (S_z) + T
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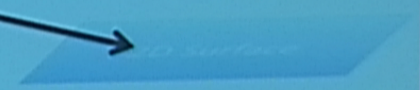

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$$\langle S^+ \rangle \stackrel{\rightarrow}{=} e^{i\phi_1}$$

$$\text{vortex} = (\psi_\uparrow, \psi_\downarrow)$$

- Describe two SPT phases:
  - PHASE 1: break U(1) on surface. Vortices are Kramers pairs. $T^2 = -1$
  - PHASE 2: break T on surface. Insulator with $\sigma_{xy}=1$

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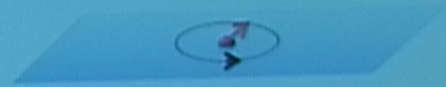

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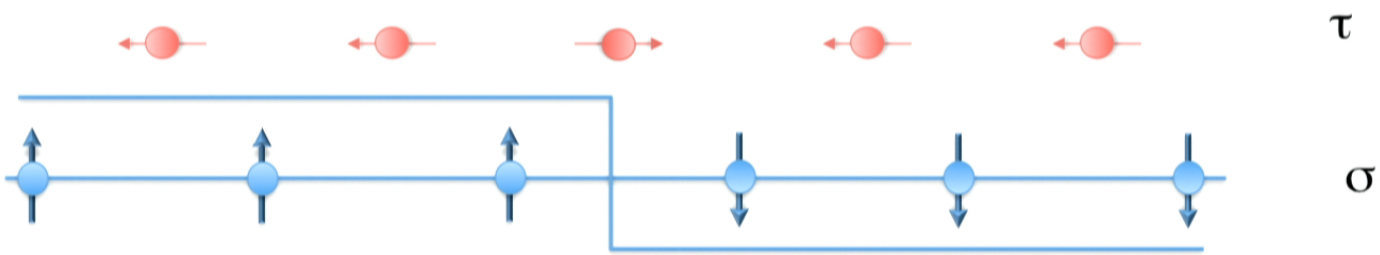
Two U(1)s needed to obtain fermionic vortices – `framing'.

Xu and Senthil

Physical Picture – Condensate of Decorated Defects

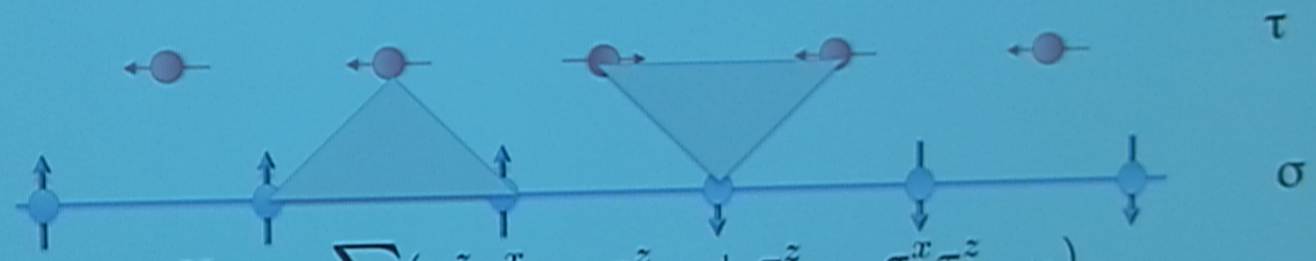


- 1D topological phase with G =Z_2xZ_2 symmetry.
  - Two Ising models. Rotn. by 180 degrees about X,Y,Z. Analog of Haldane phase.

Condense domain walls of Z_2 with Z_2 charge.

$$H = -\sum_i \left(\sigma_i^z \tau_{i+1/2}^x \sigma_{i+1}^z + \tau_{i-1/2}^z \sigma_i^x \tau_{i+1/2}^z\right)$$

Topological Phase from Condensing Decorated Defects



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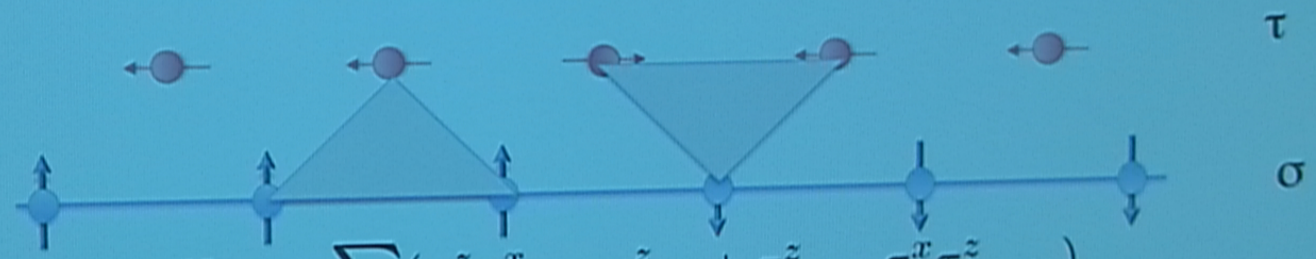

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Condensate of domain wall&spin flip =string order parameter: $\langle \tau_{1/2}^z \left[\prod_{i=1}^N \mu_i^x\right] \tau_{N+1/2}^z \rangle$

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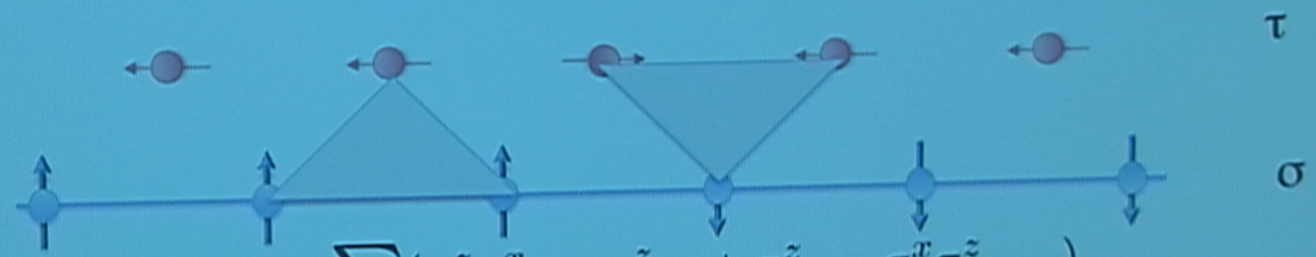

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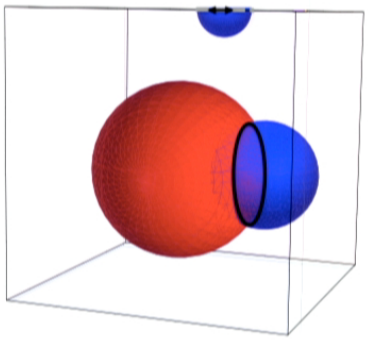

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3. Classification – phases labeled by excluded symmetry: $Z_2 x Z_2$

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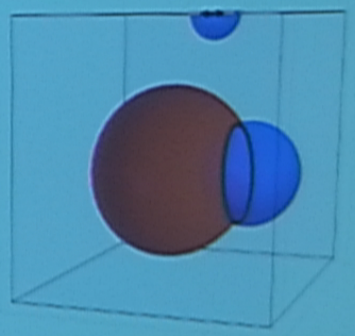

- Symmetries: $Z_2 x Z_2$
- Domain walls of eg. Z_2 in an SPT phase protected by Z_2 symmetry.

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$$\Psi = (-1)^{N_{\mathrm{dw-intersect}}}$$

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Conclusions and Future Directions

Symmetry Dimension	Time Rev. $\mathcal{T}$	Charge Consv. U(1)	Charge Consv. +Time Rev. U(1) |x T
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d=2	Z_1	Z	Z_2
d=3	Z_2	Z_1	$Z_2 x Z_2$

X Z_2

- Physical description of bosonic topological insulators.
- New topological phase *beyond* group cohomology. Surface related to Kitaev E8 state.

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- Simple physical realizations in magnets/Bose-Hubbard models?