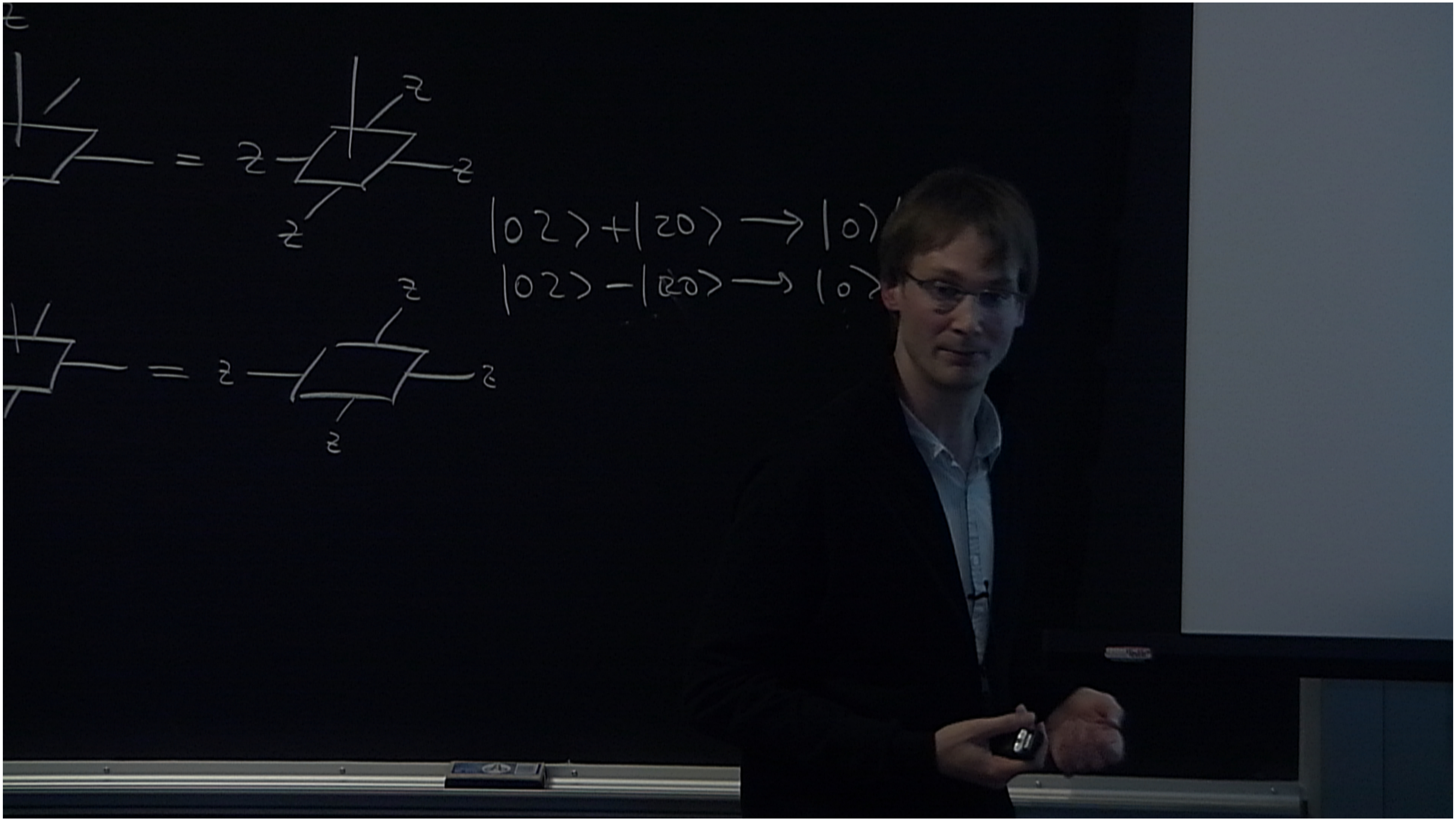
Title: Spin-orbital quantum liquid on the honeycomb lattice

Date: May 09, 2013 04:00 PM

URL: http://pirsa.org/13050045

Abstract: The symmetric Kugel-Khomskii can be seen as a minimal model describing the interactions between spin and orbital degrees of freedom in certain transition-metal oxides with orbital degeneracy, and it is equivalent to the SU(4) Heisenberg model of four-color fermionic atoms. We present simulation results for this model on various two-dimensional lattices obtained with infinite projected-entangled pair states (iPEPS), an efficient variational tensor-network ansatz for two dimensional wave functions in the thermodynamic limit. We find a rich variety of exotic phases: while on the square and checkerboard lattices the ground state exhibits dimer-N\'eel order and plaquette order, respectively, quantum fluctuations on the honeycomb lattice destroy any order, giving rise to a spin-orbital liquid. Our results are supported from flavor-wave theory and exact diagonalization. Furthermore, the properties of the spin-orbital liquid state on the honeycomb lattice are accurately accounted for by a projected variational wave-function based on the pi-flux state of fermions on the honeycomb lattice at 1/4-filling. In that state, correlations are algebraic because of the presence of a Dirac point at the Fermi level, suggesting that the ground state is an algebraic spin-orbital liquid. This model provides a possible starting point to understand the recently discovered spin-orbital liquid behavior of Ba_3CuSb_2O_9. The present results also suggest to choose optical lattices with honeycomb geometry in the search for quantum liquids in ultra-cold four-color fermionic atoms.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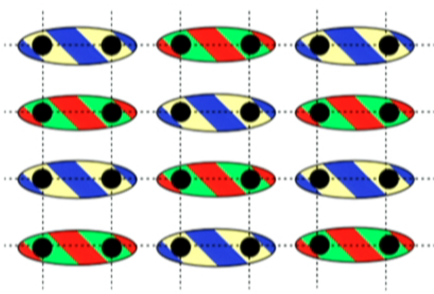
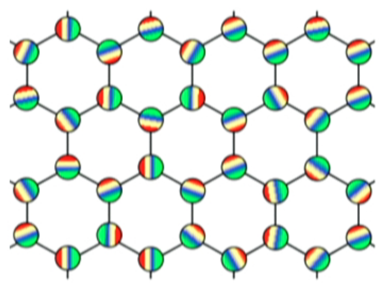
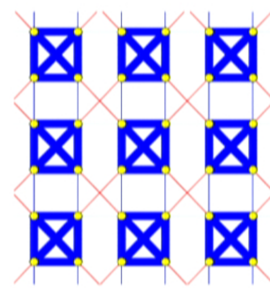
Outline

- Motivation: Spin-orbital systems & ultra-cold atoms
  - ✦ Equivalent: The *symmetric Kugel-Khomskii model* & the *SU(4) Heisenberg model*
- Methods
  - ✦ linear flavor-wave theory, exact diagonalization, variational Monte Carlo
  - ✦ **iPEPS** (infinite projected entangled-pair states): *tensor network method*
- Results for various lattices in 2D: Rich variety of ground states







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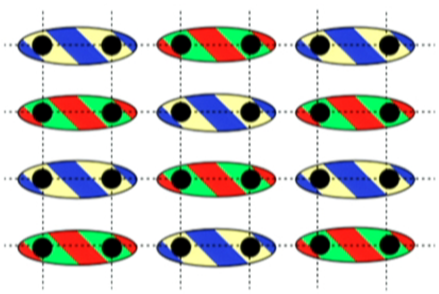
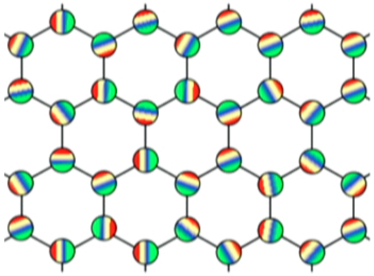
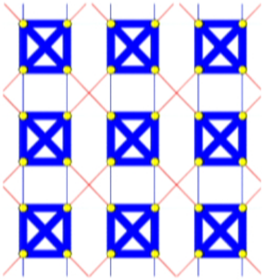




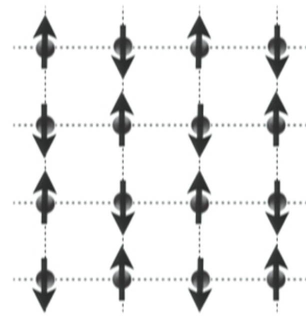
Effective models of Mott insulators

SU(2) Heisenberg model:

Effective model of the Hubbard model in the limit of U>>t and 1 particle per site (Mott insulating state)

$$H = \sum_{\langle i,j \rangle} S_i S_j$$

spin degrees of freedom $|\uparrow\rangle, |\downarrow\rangle$



Néel order

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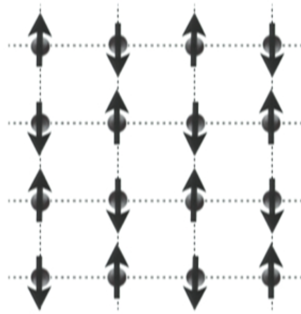

Néel order

- What happens in the case of an orbital degeneracy on each site?

two **orbital** states per lattice site $|a\rangle, |b\rangle$ (e.g. two degenerate e_g orbitals)

Symmetric Kugel-Khomskii model

Simple model for Mott insulating state in systems with two-fold orbital degeneracy

$$H = \sum_{\langle i,j \rangle} (2\mathbf{S}_i \cdot \mathbf{S}_j + \frac{1}{2})(2\mathbf{T}_i \cdot \mathbf{T}_j + \frac{1}{2})$$

spin degrees of freedom $|\uparrow\rangle, |\downarrow\rangle$

orbital degrees of freedom $|a\rangle, |b\rangle$

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spin degrees of freedom $|\uparrow\rangle, |\downarrow\rangle$

orbital degrees of freedom $|a\rangle, |b\rangle$

$\longrightarrow$

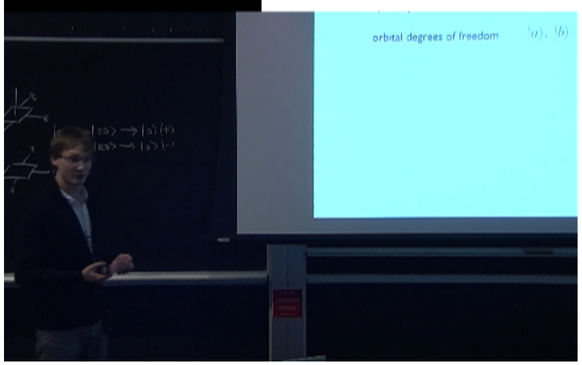
$|\uparrow\rangle|a\rangle$

$|\downarrow\rangle|a\rangle$

$|\uparrow\rangle|b\rangle$

$|\downarrow\rangle|b\rangle$

4 basis states per site



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SU(4) symmetry

spin degrees of freedom $|\uparrow\rangle, |\downarrow\rangle$

orbital degrees of freedom $|a\rangle, |b\rangle$

→ $|\uparrow\rangle|a\rangle$, $|\downarrow\rangle|a\rangle$, $|\uparrow\rangle|b\rangle$, $|\downarrow\rangle|b\rangle$ — 4 basis states per site

Examples of materials with unusual behavior:

$LiNiO_2$

Triangular S=1/2 system with orbital degeneracy

No orbital and magnetic ordering down to low T?

spin-orbital liquid?

still debated

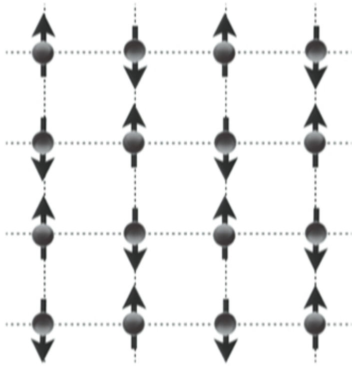
Kitaoka, *et al*, J. Phys. Soc. Jpn. **67** (1998)

Mostovoy, Khomskii, PRL **89** (2002)

SU(N) Heisenberg models

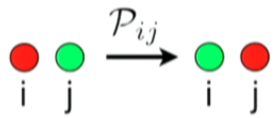
- N=2: $H = \sum_{\langle i,j \rangle} S_i S_j$

  local basis states: $|\uparrow\rangle, \; |\downarrow\rangle$

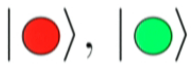


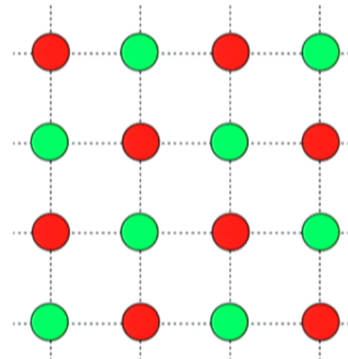
Néel order

SU(N) Heisenberg models



- N=2: $$H = \sum_{\langle i,j \rangle} P_{ij}$$

  local basis states: 



Néel order

- N=3 

- N=4 

Motivation II: Ultra-cold atoms in optical lattices

nature physics | ARTICLES
PUBLISHED ONLINE: 28 FEBRUARY 2010 | DOI: 10.1038/NPHYS1535

Two-orbital *SU*(*N*) magnetism with ultracold alkaline-earth atoms

A. V. Gorshkov[1]*, M. Hermele[2], V. Gurarie[2], C. Xu[1], P. S. Julienne[3], J. Ye[4], P. Zoller[5,6], E. Demler[1,7], M. D. Lukin[1,7] and A. M. Rey[4]

see also: Cazalilla, Ho, Ueda, New J. Phys. **11** (2009)

Nuclear spin

$$^{87}Sr: \quad I = 9/2 \quad \longrightarrow \quad N_{max} = 2I + 1 = 10$$

related proposals with Alkali fermions: Wu, et al, PRL '03; Honerkamp&Hofstetter, PRL '04

Two equivalent models

Symmetric Kugel-Khomskii model

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SU(4) symmetry

spin degrees of freedom $|\uparrow\rangle, |\downarrow\rangle$

orbital degrees of freedom $|a\rangle, |b\rangle$

(e.g. two degenerate e_g orbitals)

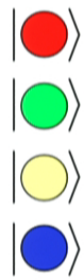
$\longrightarrow$ $|\uparrow\rangle|a\rangle$, $|\downarrow\rangle|a\rangle$, $|\uparrow\rangle|b\rangle$, $|\downarrow\rangle|b\rangle$ — 4 basis states per site

$\updownarrow$ equivalent

SU(4) Heisenberg model

$$H = \sum_{\langle i,j \rangle} P_{ij}$$

4 basis states per site



$\leftrightarrow$ $|\uparrow\rangle|a\rangle$

$\leftrightarrow$ $|\downarrow\rangle|a\rangle$

$\leftrightarrow$ $|\uparrow\rangle|b\rangle$

$\leftrightarrow$ $|\downarrow\rangle|b\rangle$

Methods

- Get first insights with *linear-flavor wave theory* (LFWT)

  product state + harmonic quantum fluctuations on top

  K. Penc, F. Mila

Methods

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- Exact diagonalization for clusters up to N=24 sites (A. Läuchli)

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Accurate simulation of large 2D systems?

- Quantum Monte Carlo: suffers from the **negative sign problem**

Methods

- Get first insights with *linear-flavor wave theory* (LFWT)

  product state + harmonic quantum fluctuations on top

  K. Penc, F. Mila

- Exact diagonalization for clusters up to N=24 sites (A. Läuchli)

Accurate simulation of large 2D systems?

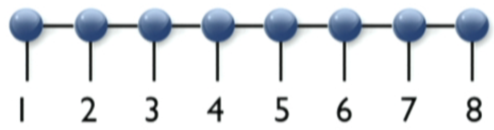
- Quantum Monte Carlo: suffers from the **negative sign problem**
- Variational Monte Carlo: relies on good trial wave function (M. Lajko, K. Penc)

Overview: tensor networks in 1D and 2D

1D

MPS

Matrix-product state



Underlying ansatz of the density-matrix renormalization group (**DMRG**) method

Overview: tensor networks in 1D and 2D

1D

MPS
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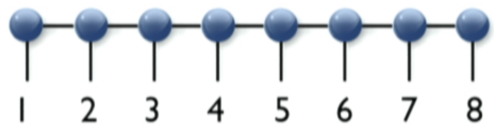
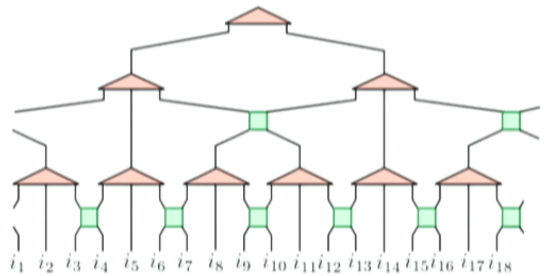


Underlying ansatz of the density-matrix renormalization group (**DMRG**) method

1D MERA
Multi-scale entanglement renormalization ansatz

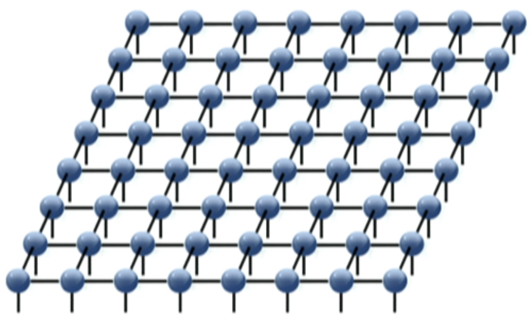


and more

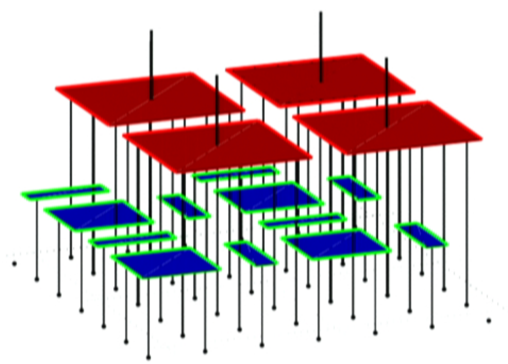
- 1D tree tensor network
- ...

2D

PEPS
projected entangled-pair state



2D MERA



and more

- Entangled-plaquette states
- 2D tree tensor network
- String-bond states
- ...

Tensor network ansatz for a wave function

State: $$|\Psi\rangle = \sum_{i_1 i_2 i_3 i_4 i_5 i_6} \Psi_{i_1 i_2 i_3 i_4 i_5 i_6} |i_1 \otimes i_2 \otimes i_3 \otimes i_4 \otimes i_5 \otimes i_6\rangle$$

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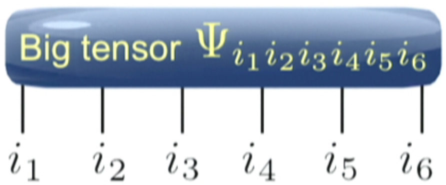
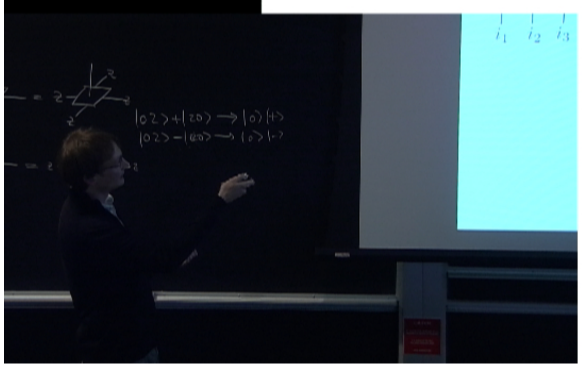
Tensor/multidimensional array





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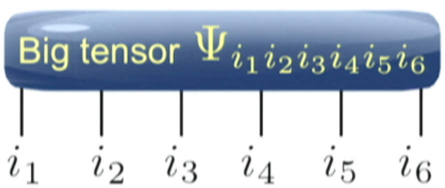
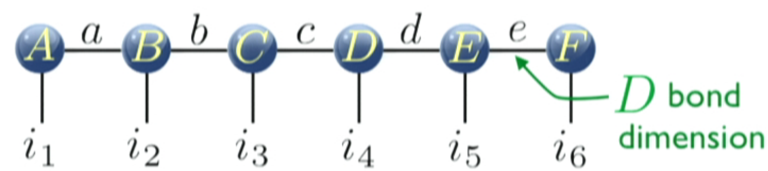
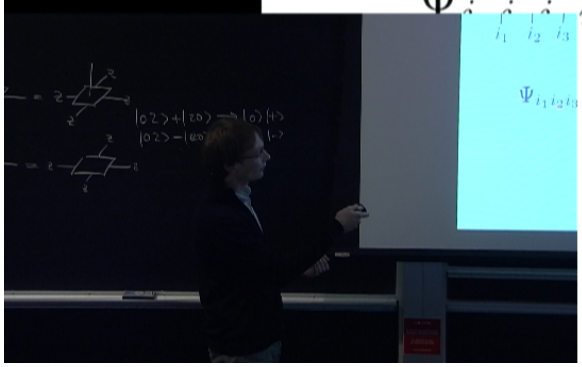


Tensor network: matrix product state (**MPS**)



$$\Psi_{i_1 i_2 i_3 i_4 i_5 i_6} \approx \sum_{abcde} A^{a}_{i_1} B^{ab}_{i_2} C^{bc}_{i_3} D^{cd}_{i_4} E^{de}_{i_5} F^{e}_{i_6} = \tilde{\Psi}_{i_1 i_2 i_3 i_4 i_5 i_6}$$



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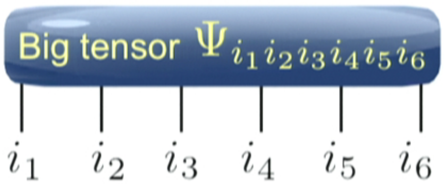
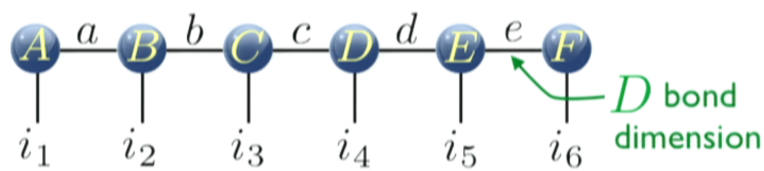
Tensor/multidimensional array



Tensor network: matrix product state (**MPS**)



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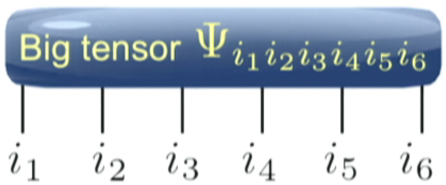

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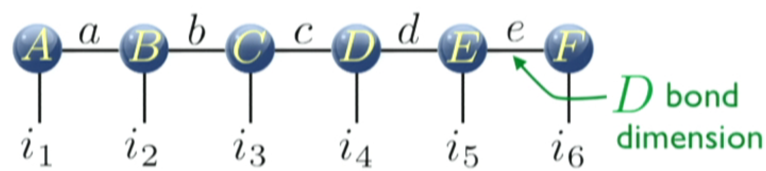

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exp(N) many numbers **VS** poly(D,N) numbers

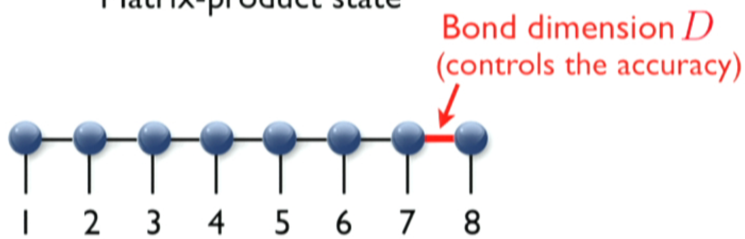
Efficient representation!

MPS and PEPS

1D

MPS

Matrix-product state



Physical indices (lattices sites)

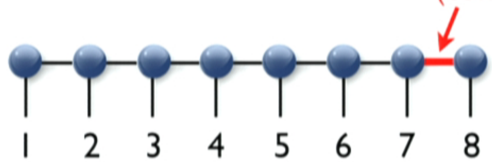
MPS and PEPS

1D

MPS

Matrix-product state

Bond dimension D
(controls the accuracy)



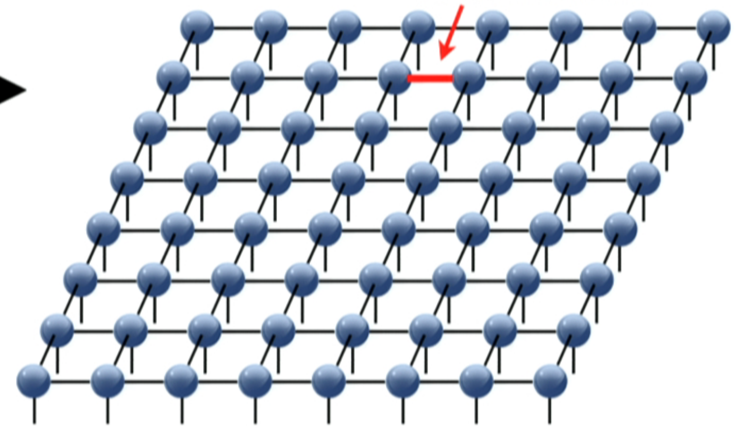
Physical indices (lattices sites)

2D

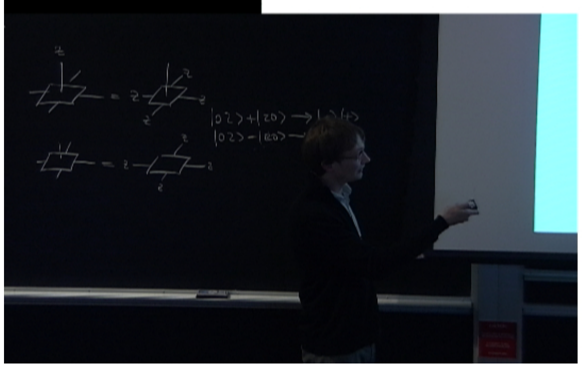
PEPS

projected entangled-pair state

Bond dimension D



Verstraete and Cirac, cond-mat/0407066



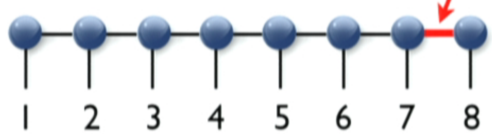
Infinite PEPS

1D

MPS

Matrix-product state

Bond dimension D
(controls the accuracy)

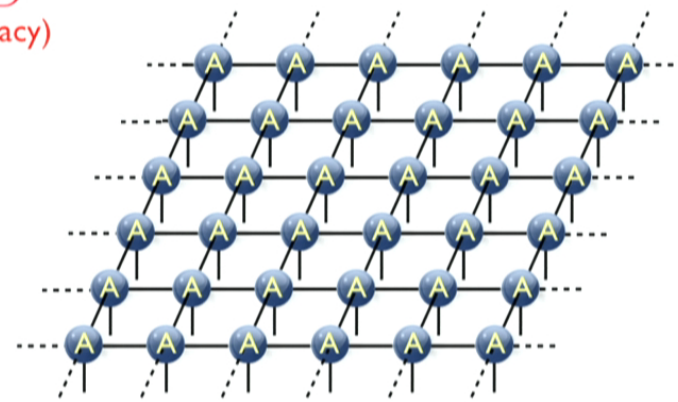


Physical indices (lattices sites)

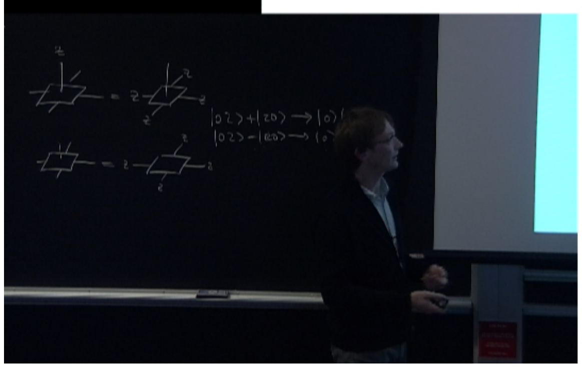
2D

iPEPS

infinite projected entangled-pair state



Jordan, Orus, Vidal, Verstraete, Cirac, PRL (2008)



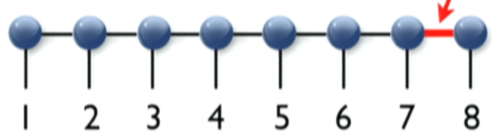
iPEPS with arbitrary unit cells

1D

MPS

Matrix-product state

Bond dimension D
(controls the accuracy)

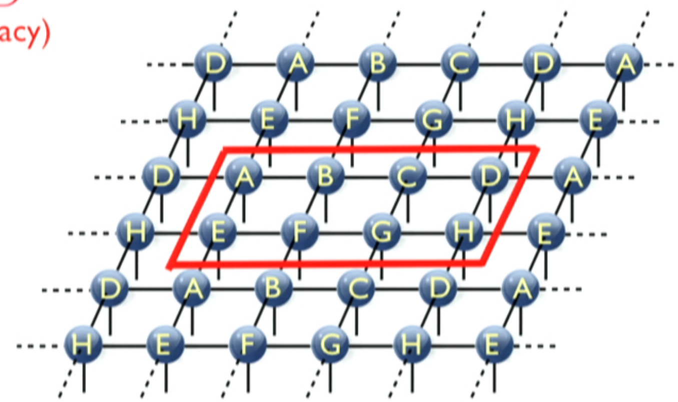


Physical indices (lattices sites)

2D

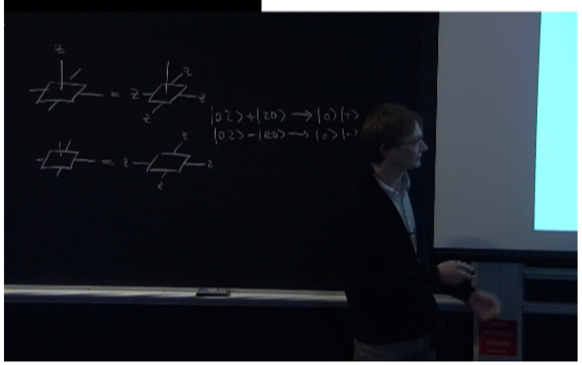
iPEPS

with arbitrary unit cell of tensors



here: 4x2 unit cell

Corboz, White, Vidal, Troyer, PRB 84 (2011)



iPEPS with arbitrary unit cells

1D

MPS

Matrix-product state

Bond dimension D
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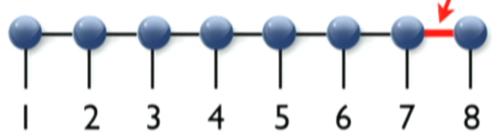

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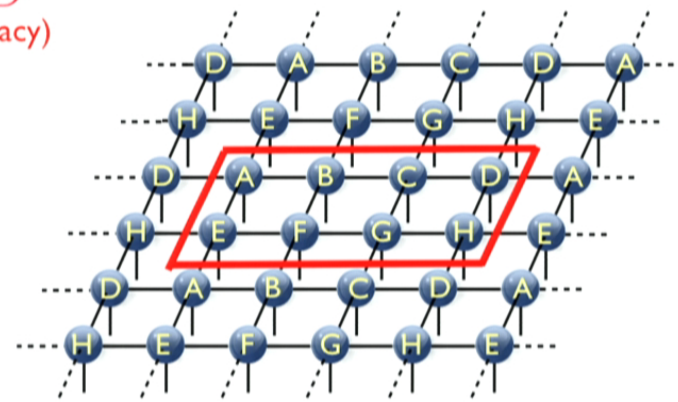
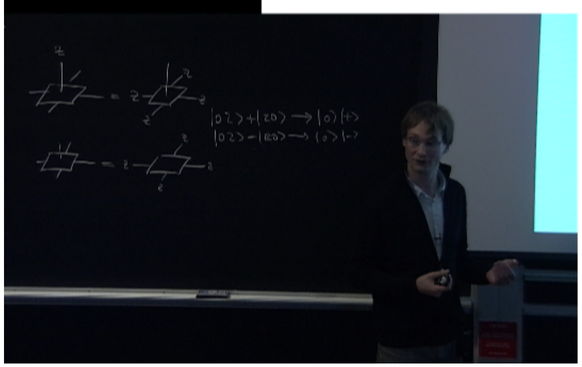


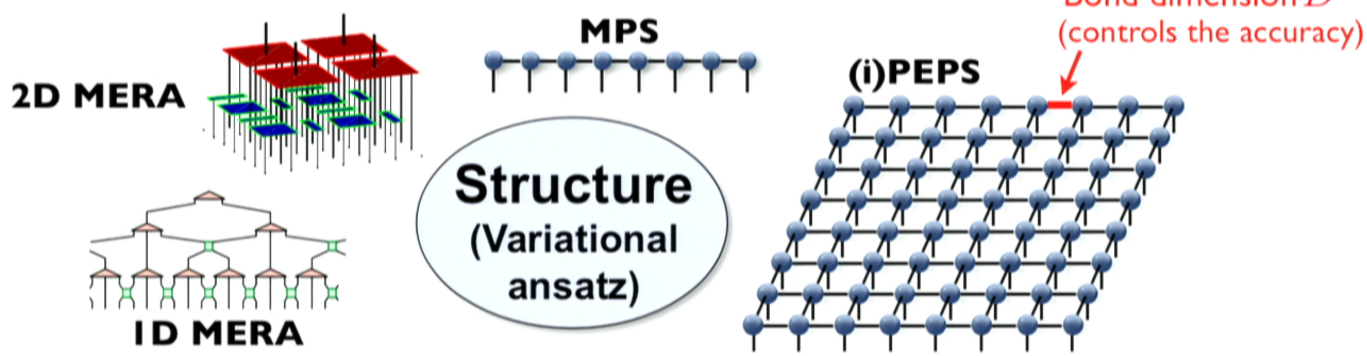
here: 4x2 unit cell

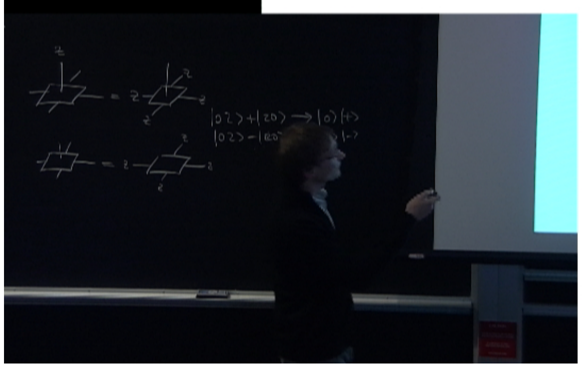
Corboz, White, Vidal, Troyer, PRB 84 (2011)

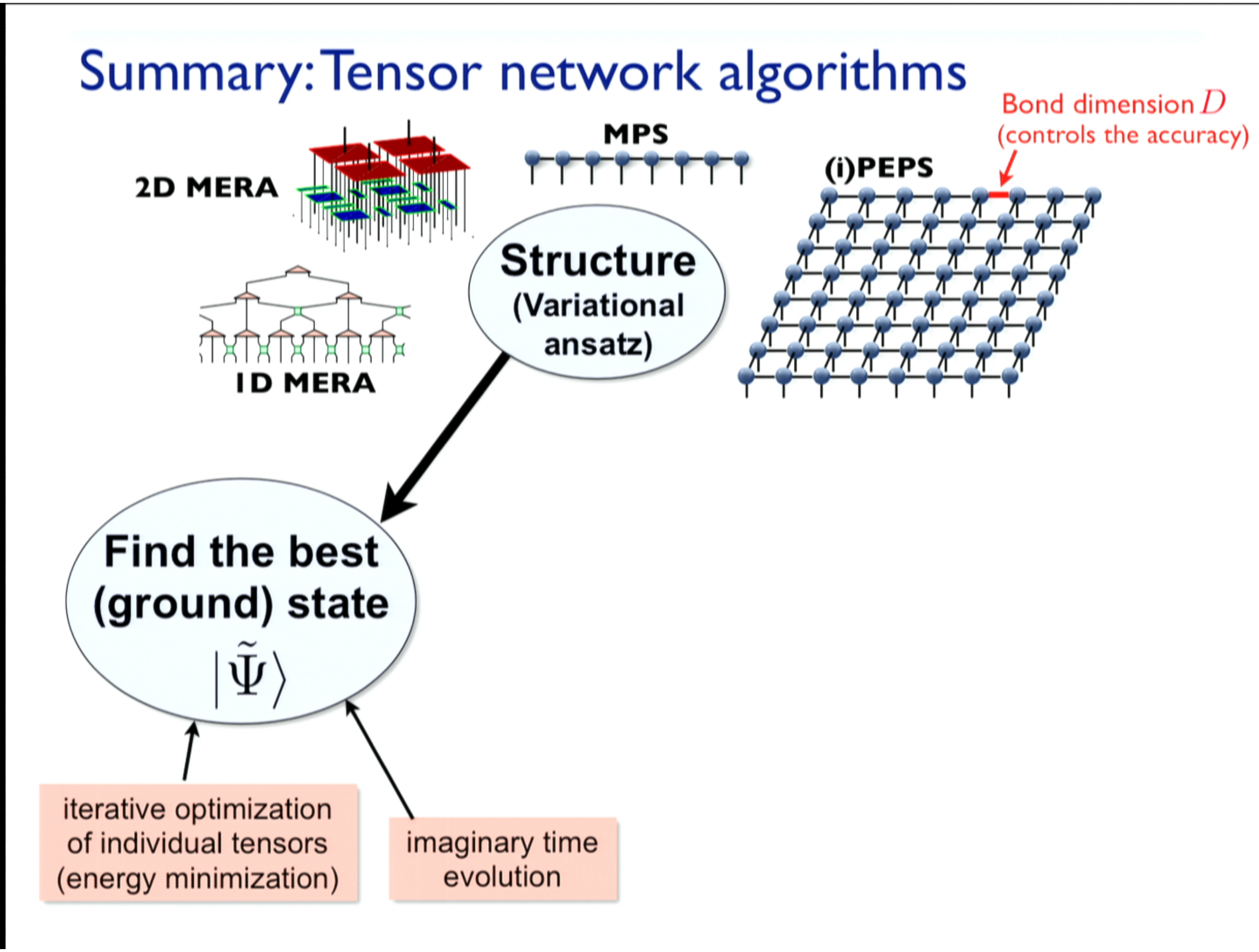
★ Run simulations with different unit cell sizes and compare variational energies



Summary: Tensor network algorithms






Summary: Tensor network algorithms
2D MERA
MPS
(i)PEPS
Bond dimension D
(controls the accuracy)
1D MERA
Structure
(Variational
ansatz)
Find the best
(ground) state
$|\tilde{\Psi}\rangle$
iterative optimization
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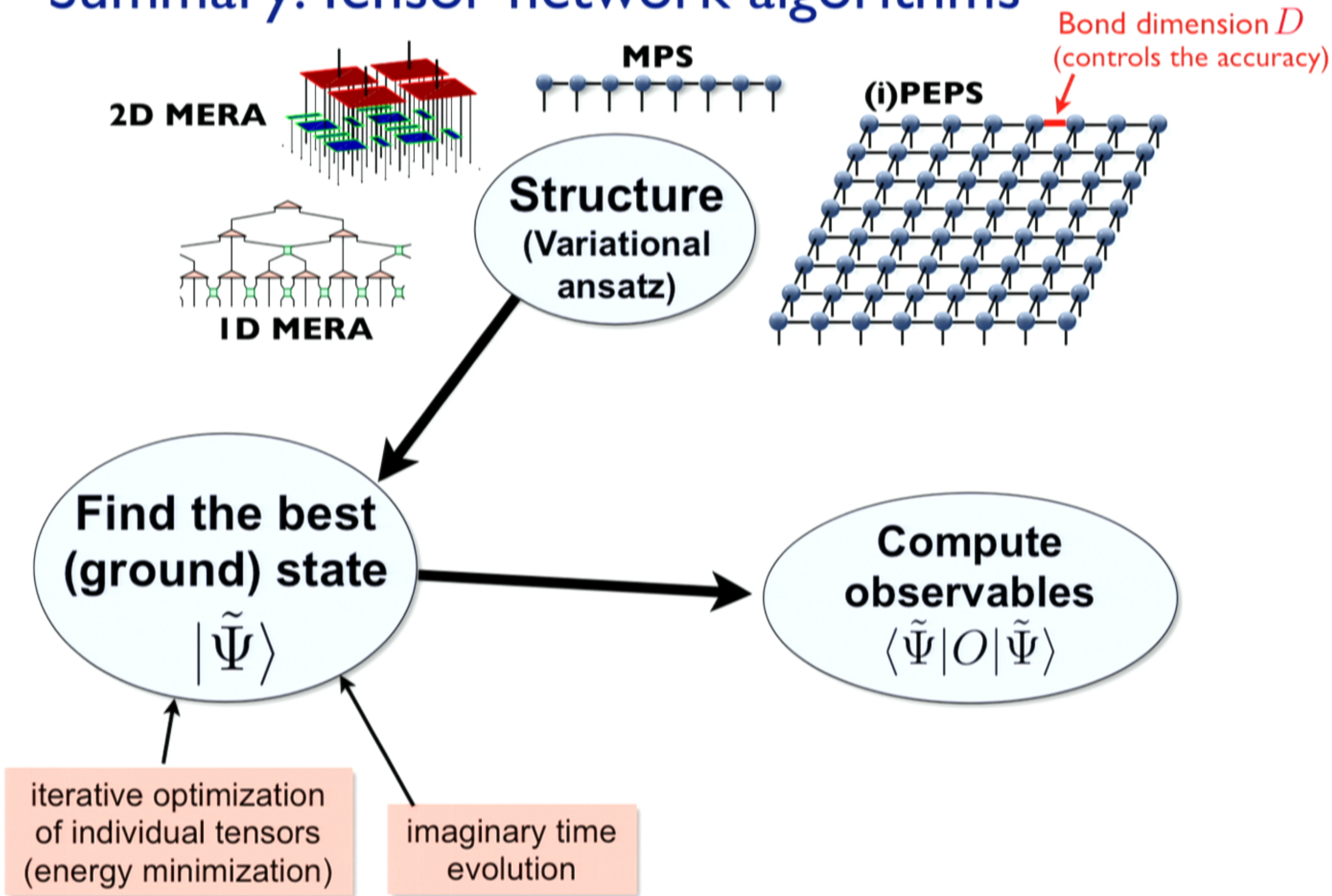

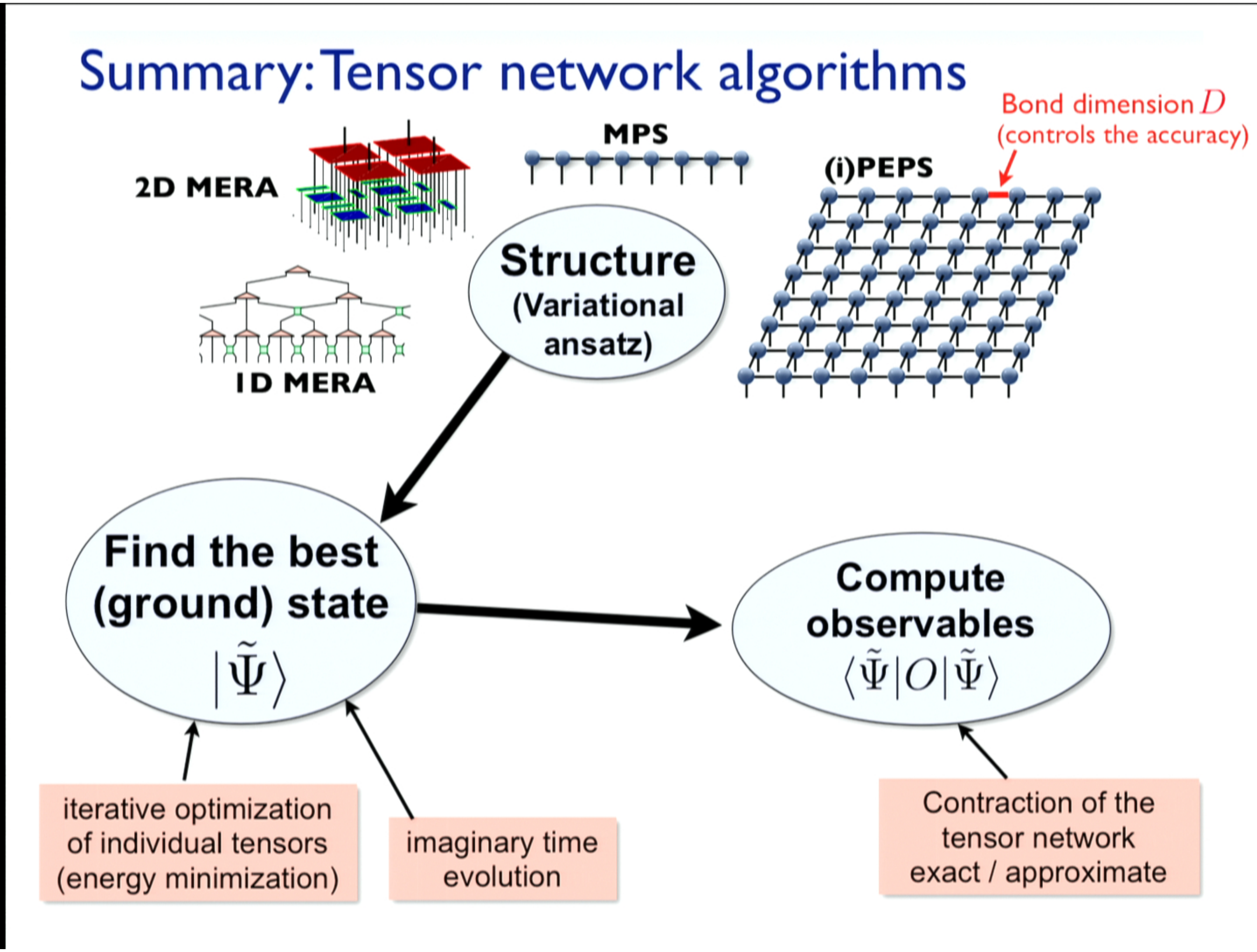
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Structure
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Find the best (ground) state
$|\tilde{\Psi}\rangle$
Compute observables
$\langle\tilde{\Psi}|O|\tilde{\Psi}\rangle$
iterative optimization of individual tensors (energy minimization)
imaginary time evolution
Contraction of the tensor network exact / approximate

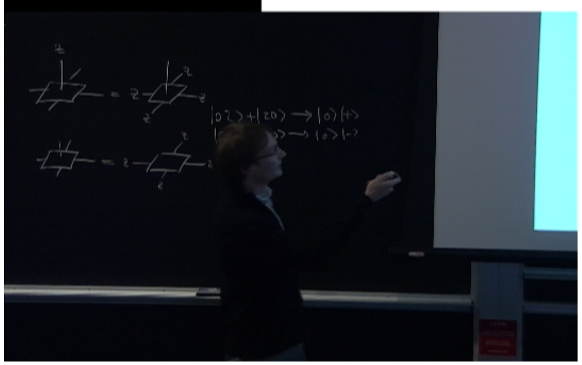
iPEPS: Benchmark results

- iPEPS can *outperform* the best-known variational methods for large 2D systems

  t-J model: Corboz, et al., PRB **84** (2011); Corboz, Troyer, Rice, *in preparation*

- SU(2) Heisenberg model: QMC (extrap.): -0.669437(5)J A. Sandvik, PRB **56** (1997)

  iPEPS (D=10): -0.66939J rel. error < 10^{-4}



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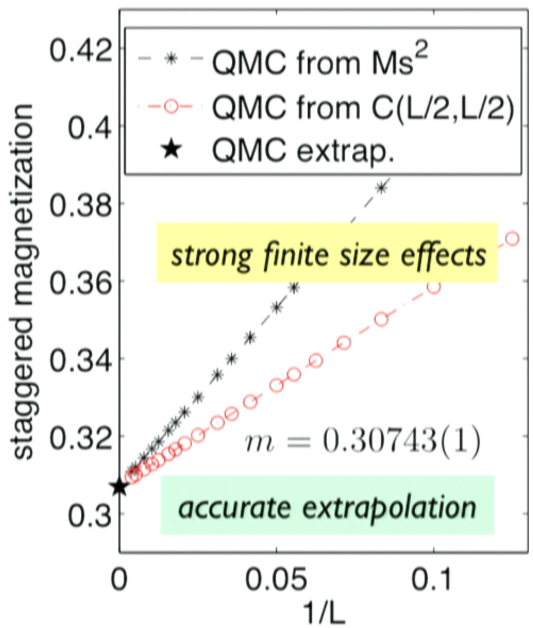

Sandvik & Evertz, PRB **82** (2010)

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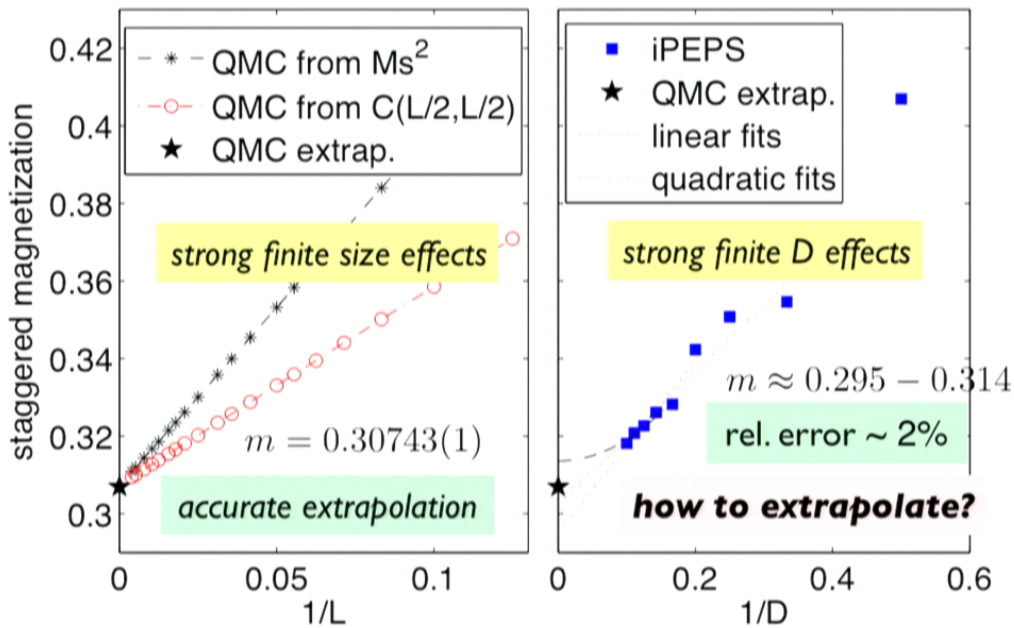
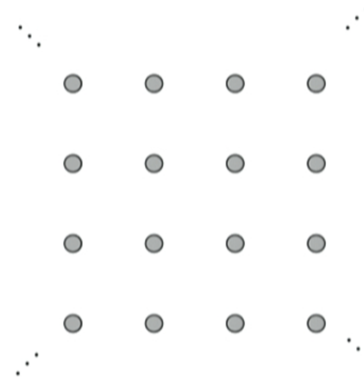


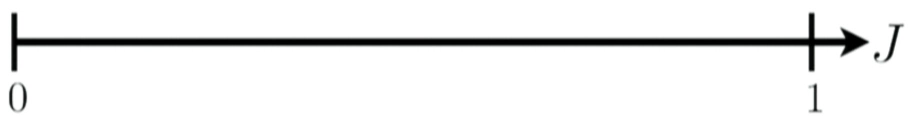
Sandvik & Evertz, PRB **82** (2010)

Distinguish between ordered / disordered phase?

$$H = J \sum_{\langle i,j \rangle_A} S_i S_j + \sum_{\langle i,j \rangle_B} S_i S_j$$



Phase diagram



Wenzel, Janke, PRB 79 (2009)

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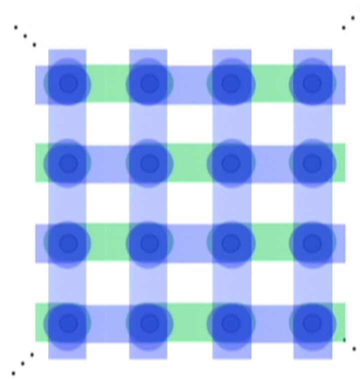

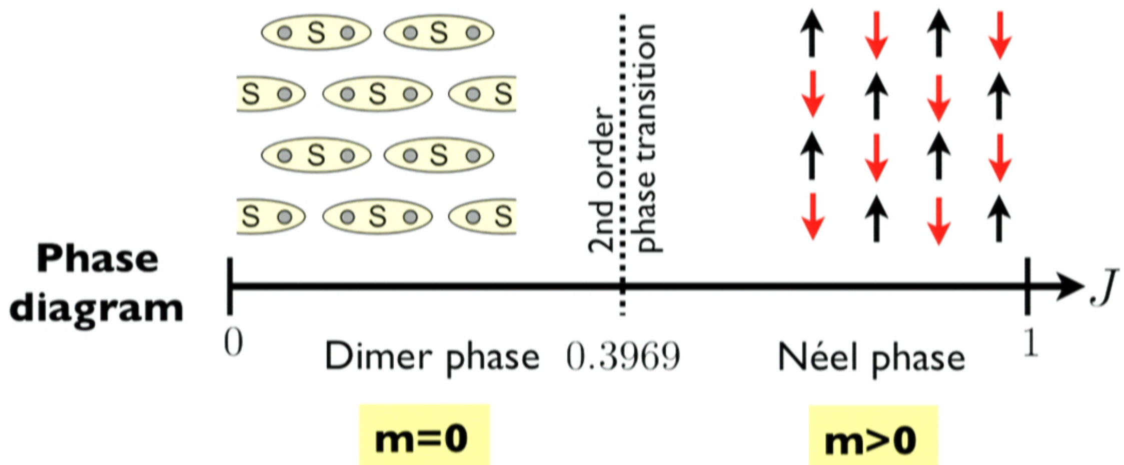

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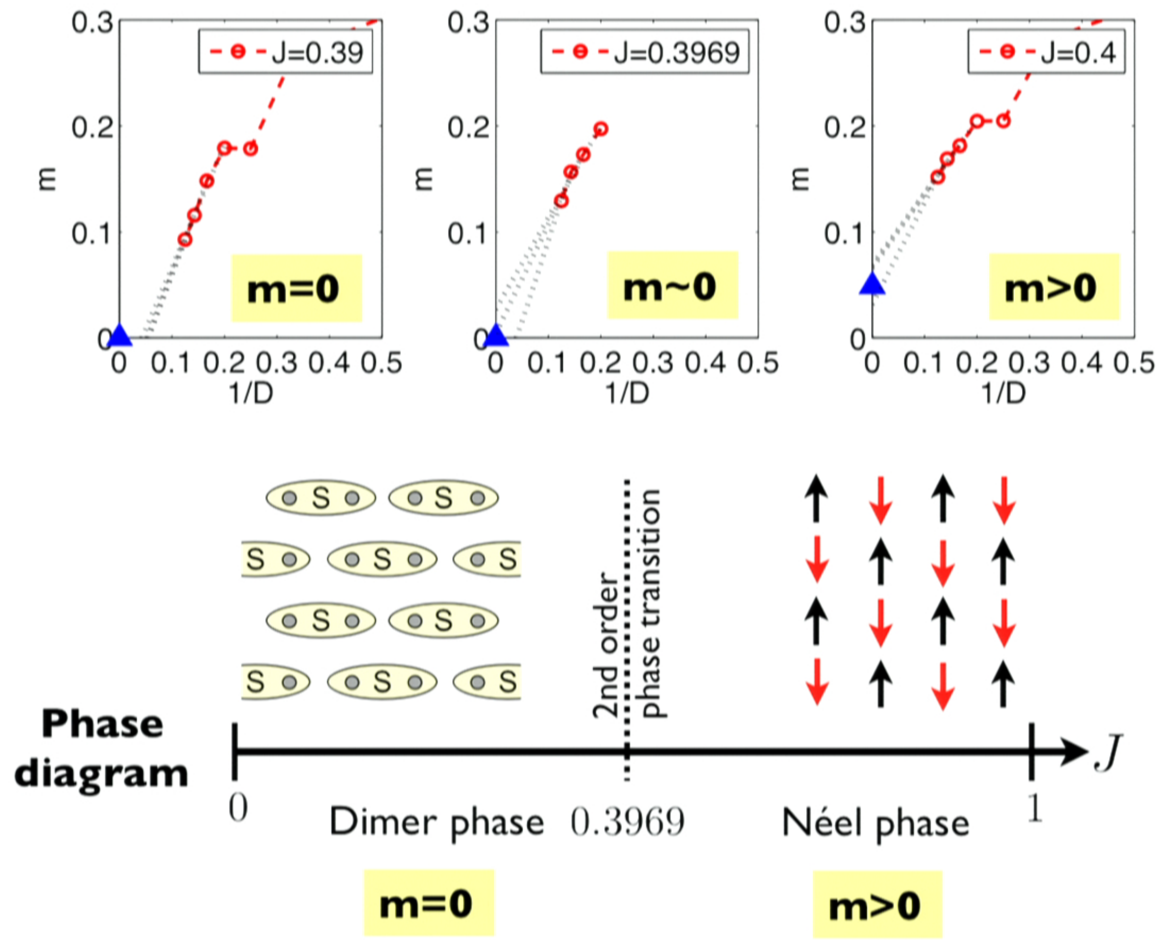
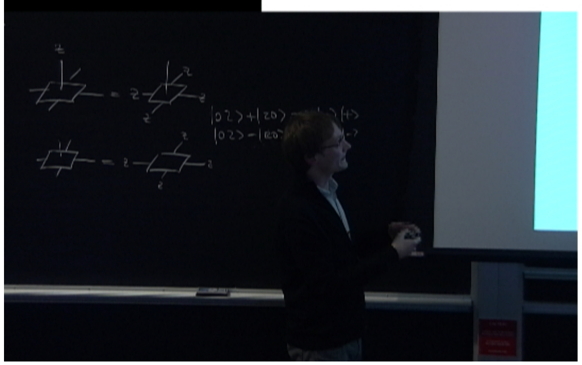


Classical solution

$$H = \sum_{\langle i,j \rangle} P_{ij} \xrightarrow{\text{product state}}$$

states on neighboring sites orthogonal (different colors)



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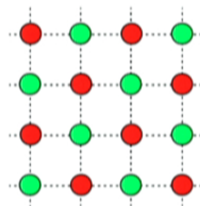
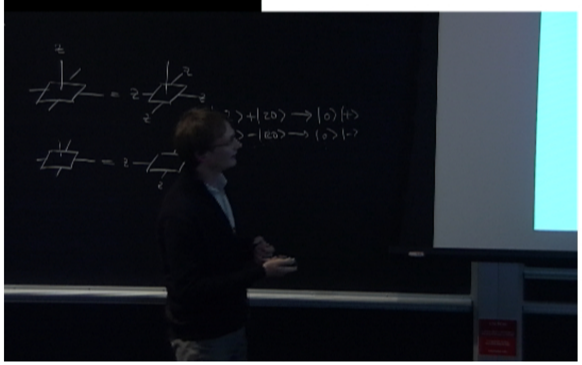


Different colors (spins) on the two sublattices



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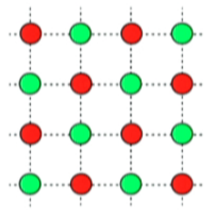

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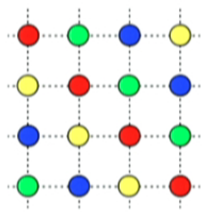

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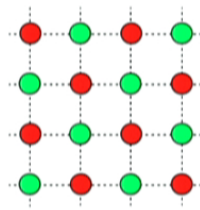

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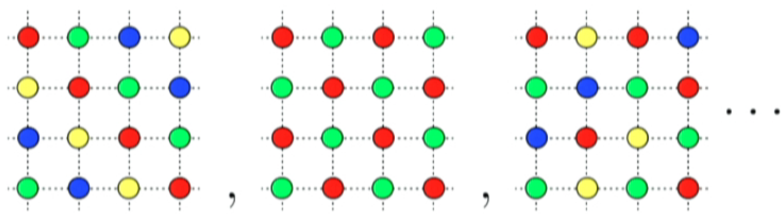

Same energy on the mean-field level

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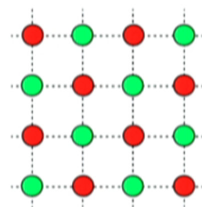

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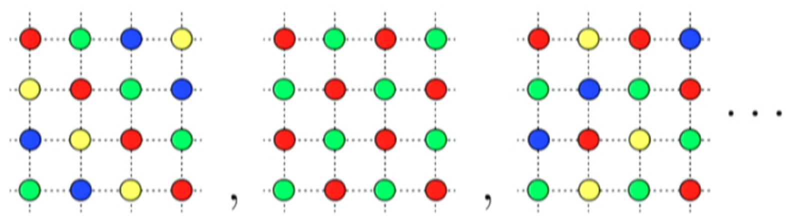


Same energy on the mean-field level

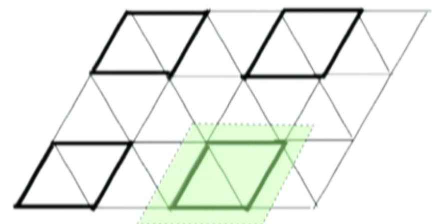
Quantum fluctuations lift the extensive degeneracy!

Previous results for the symmetric Kugel-Khomskii model

Triangular lattice:

- Spin-orbital liquid of resonant plaquettes?

  variational + mean-field study: Li, et al., PRL **81** (1998)



fluctuating SU(4) singlets

Square lattice

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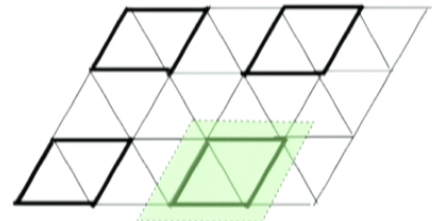

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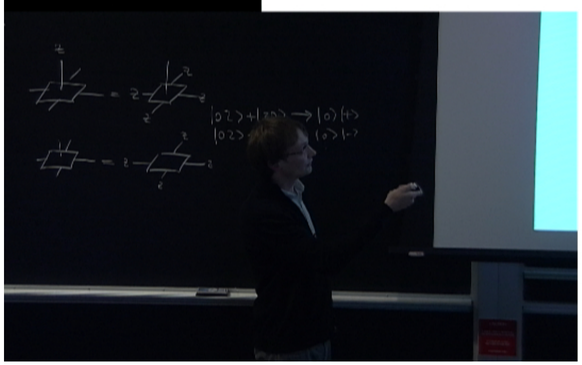

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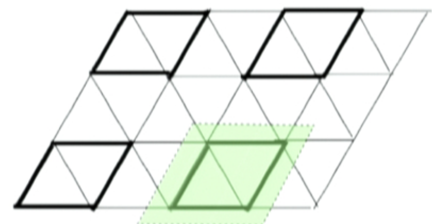

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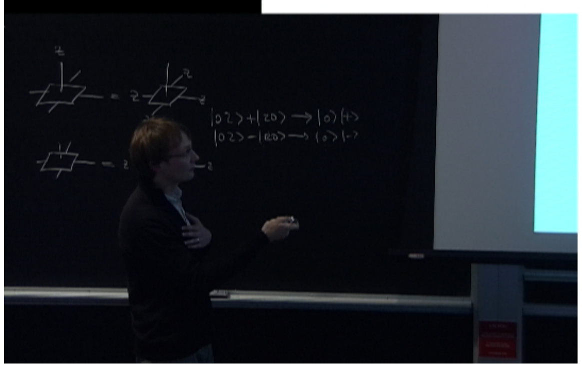

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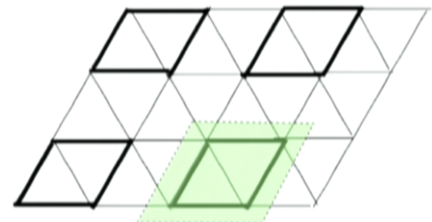

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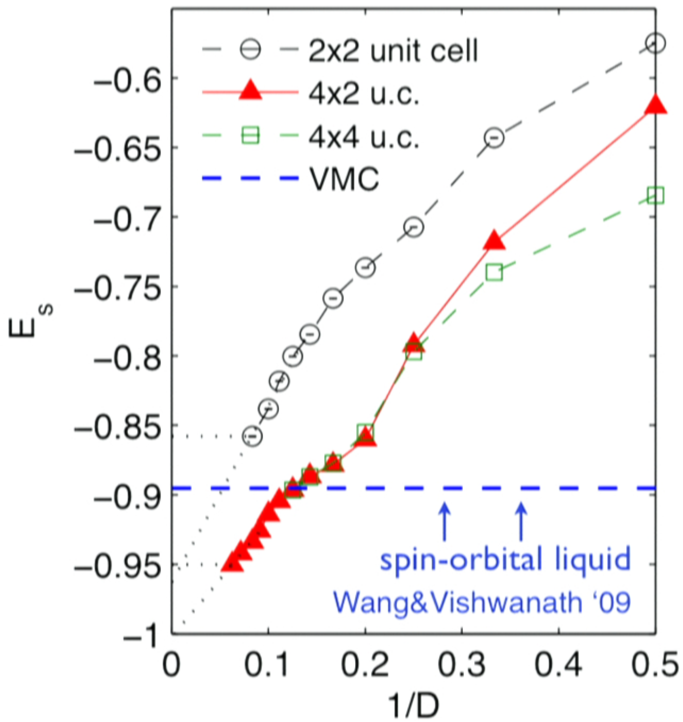
- Z_2 Spin-orbital liquid Wang, Vishwanath, PRB **80** (2009)

  VMC study based on projected wave functions using Majorana fermion representation

  Variational wave function with lowest energy: π-*flux* state

Square lattice: iPEPS results

Corboz, Läuchli, Penc, Troyer, Mila, PRL **107**, 215301 (2011)



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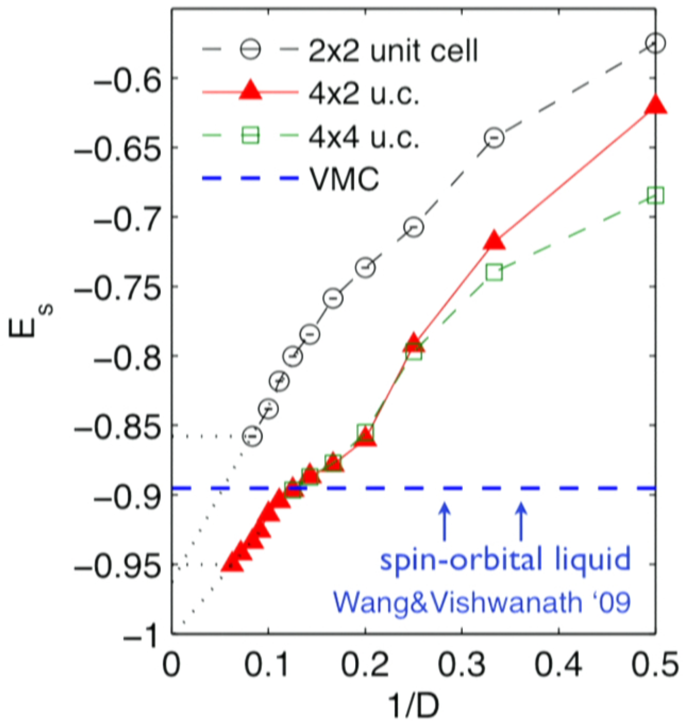

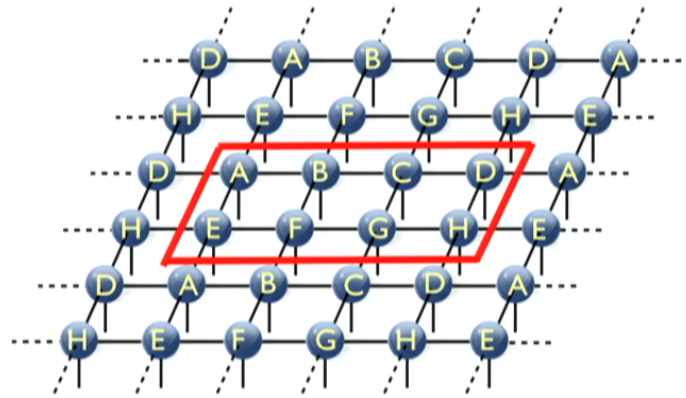

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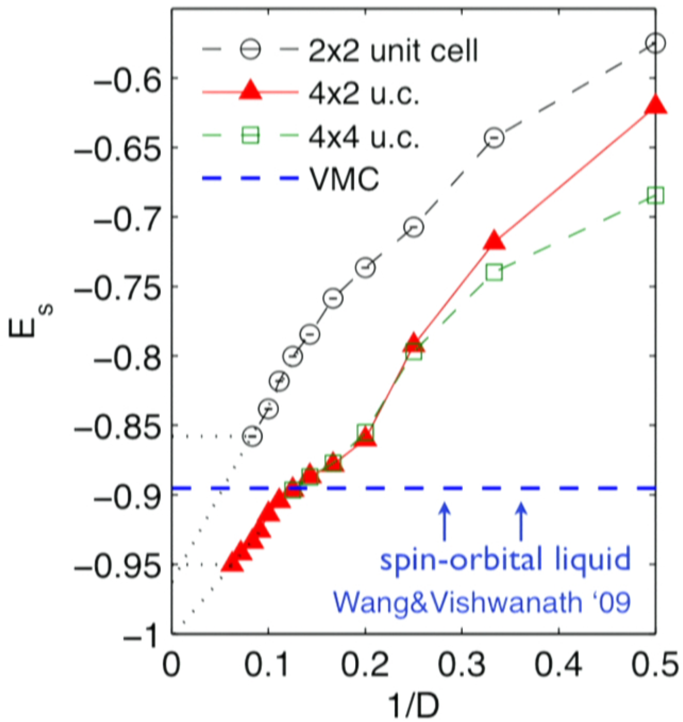

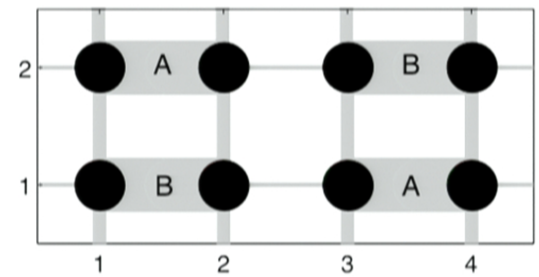

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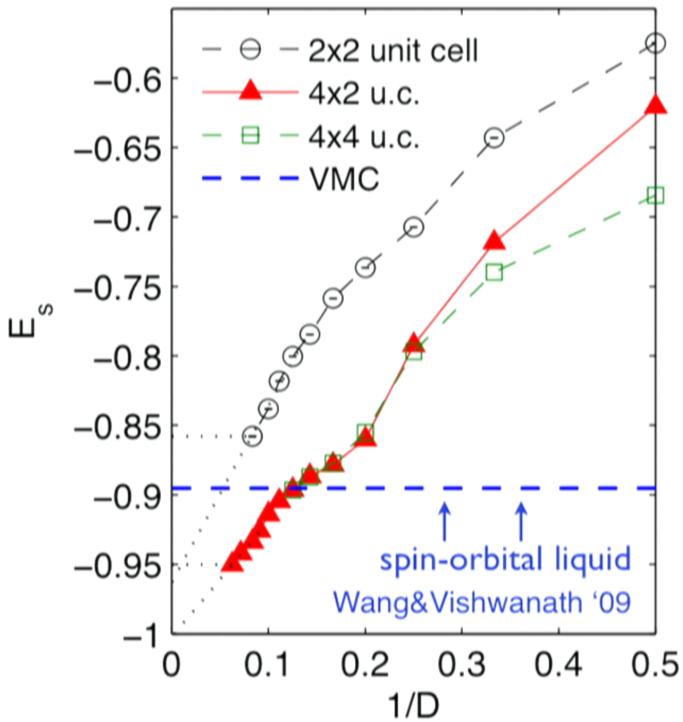

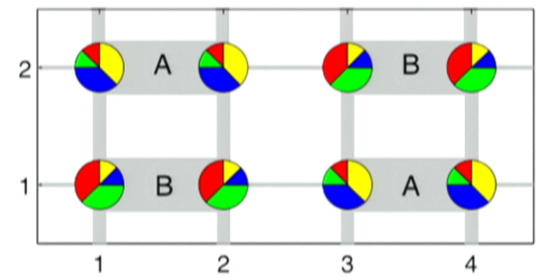

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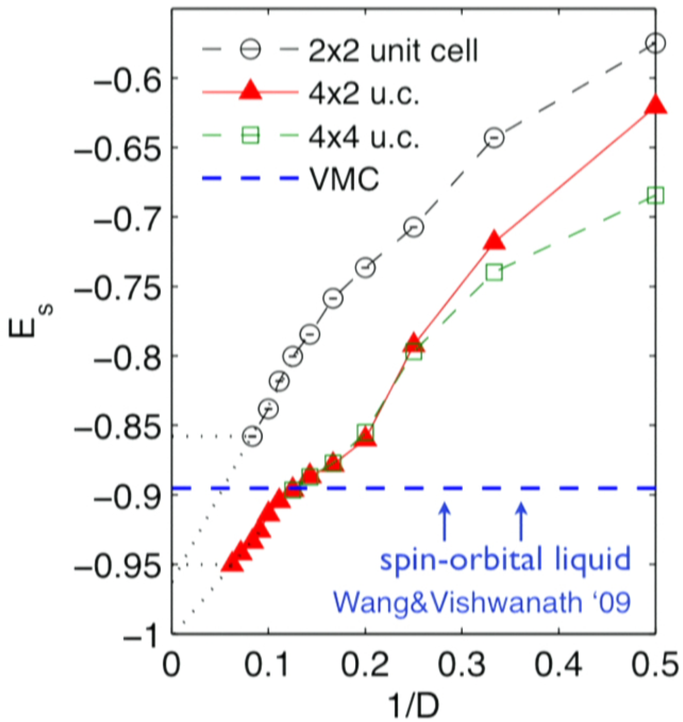
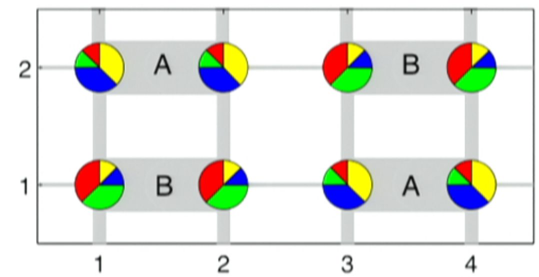
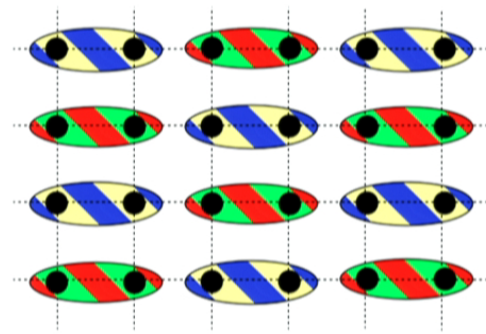




"Dimer-Néel" order



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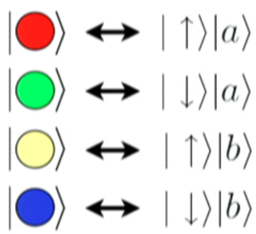
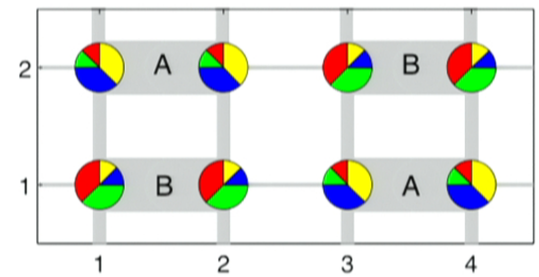
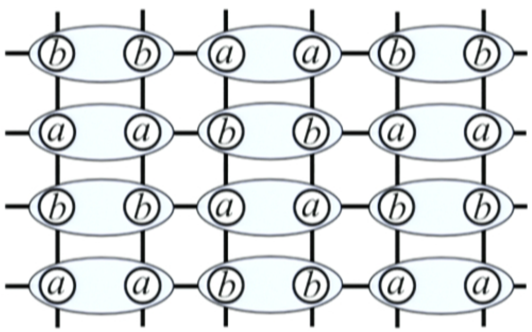




in spin-orbital language:

Spin singlets + orbital order



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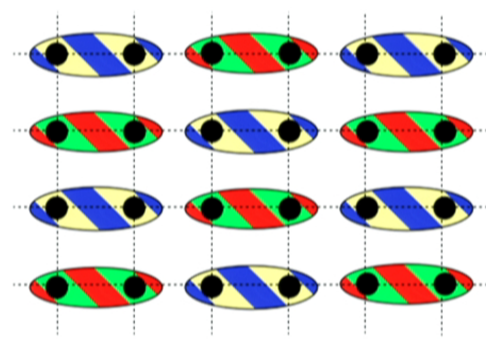

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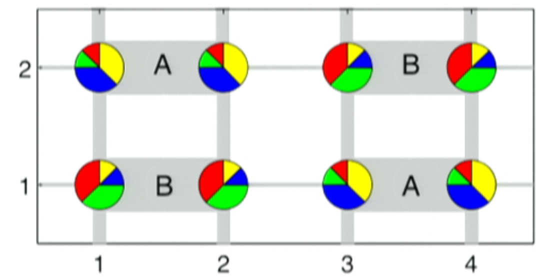
Corboz, Läuchli, Penc, Troyer, Mila, PRL **107**, 215301 (2011)

Why is this a good state?



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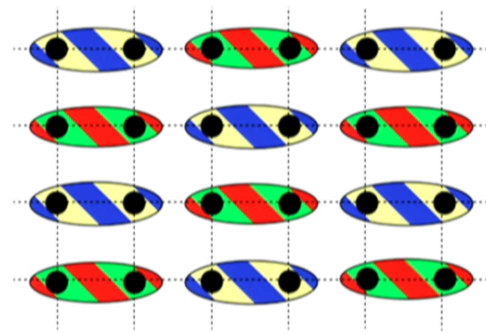

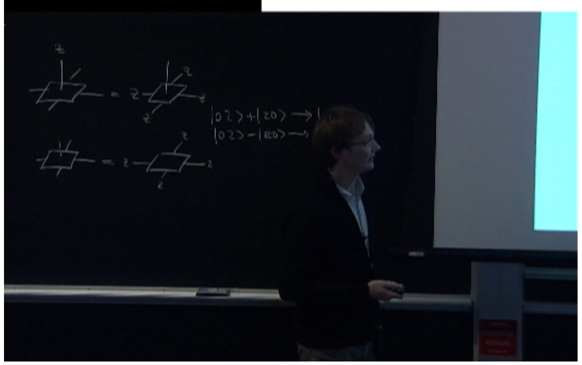

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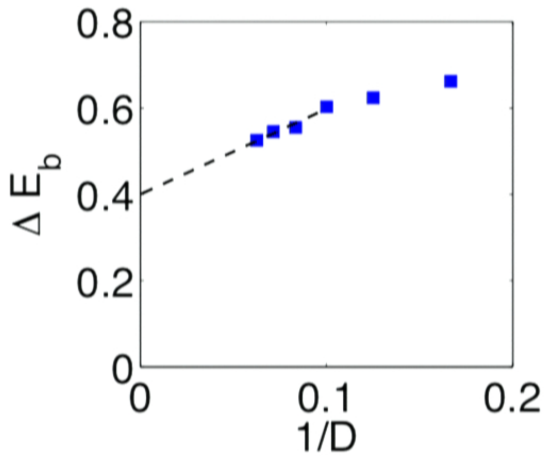

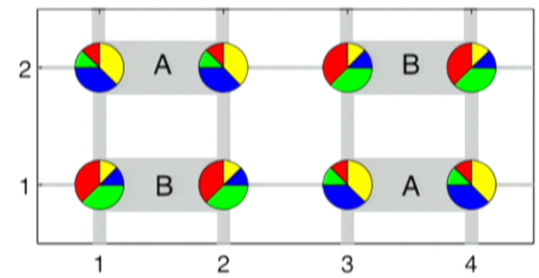

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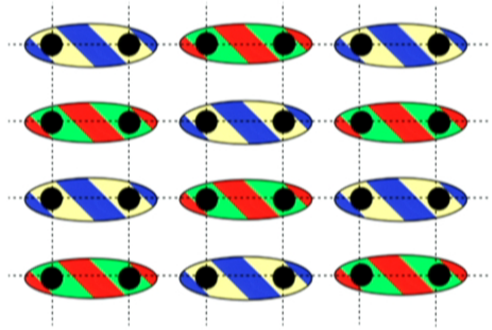

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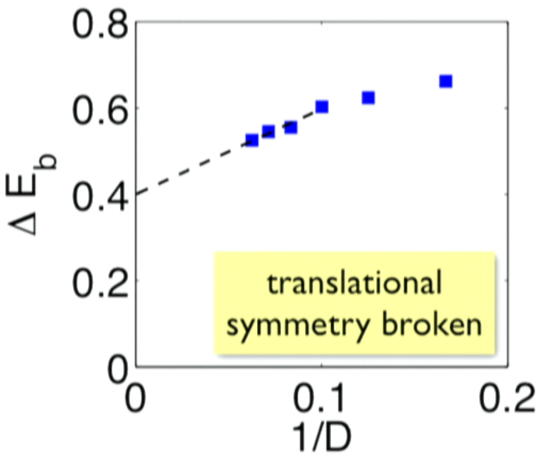

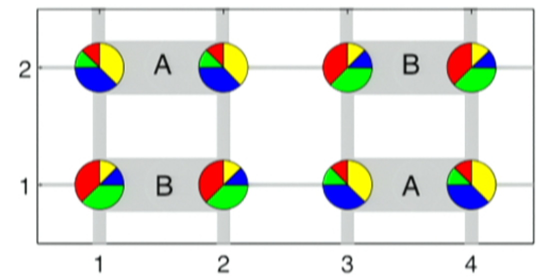

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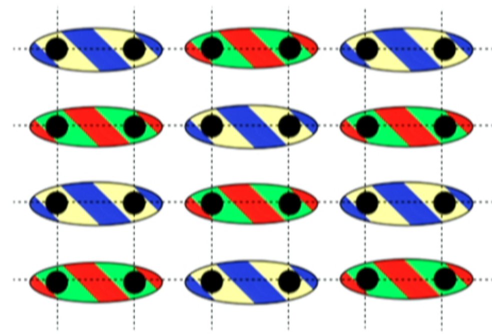

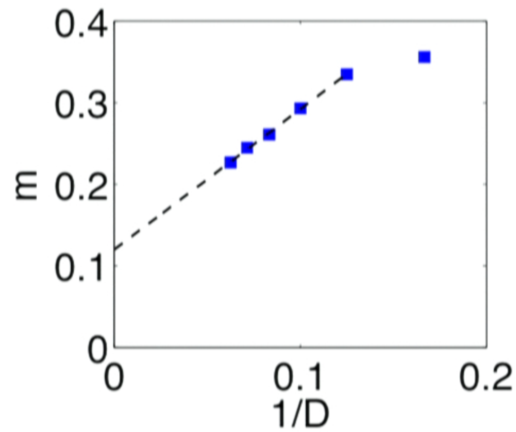

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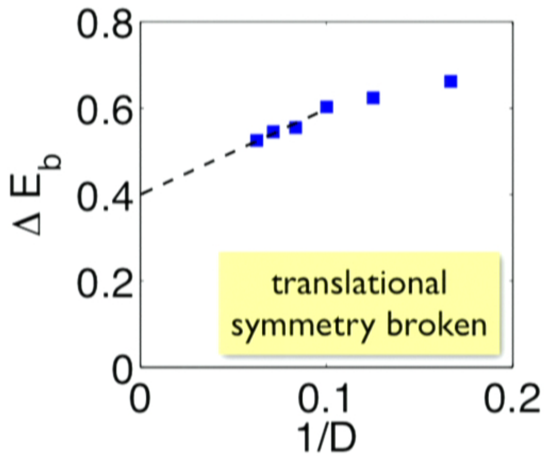
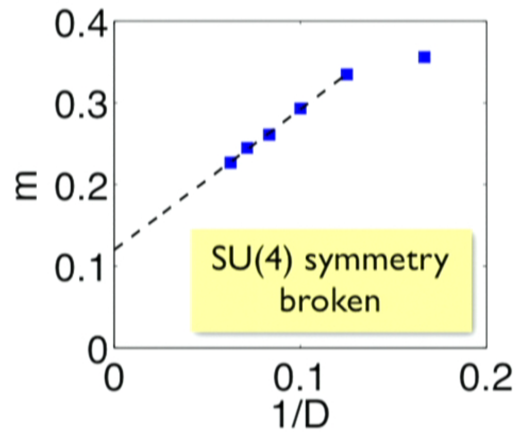
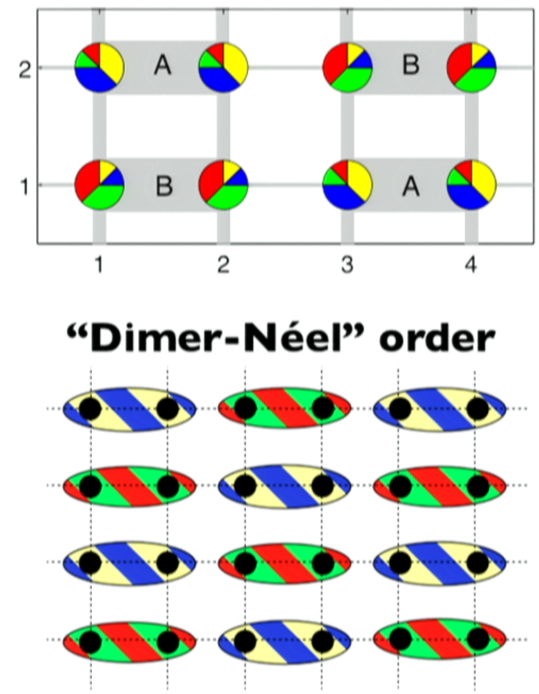




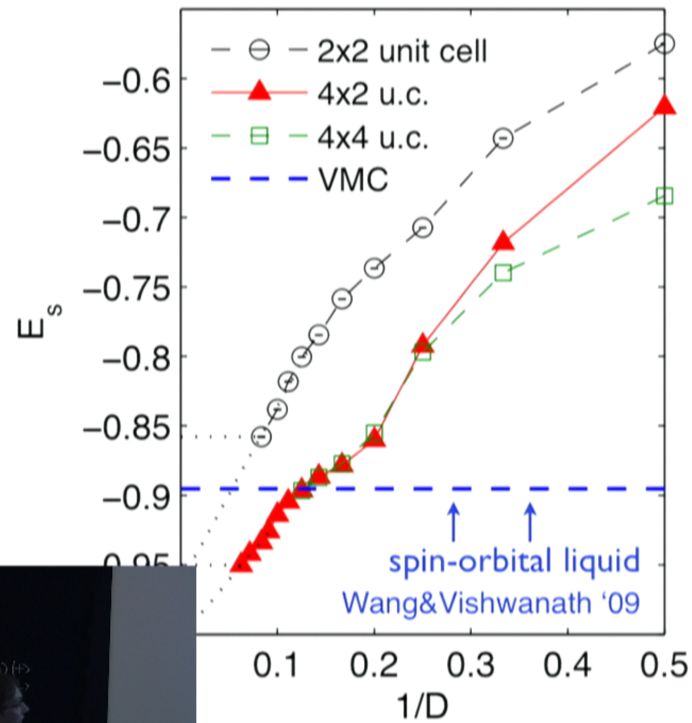


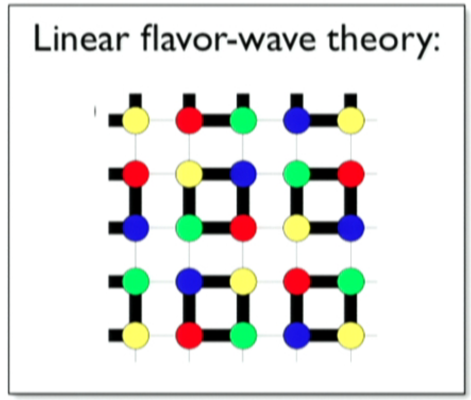
➔ Dimerization **and** SU(4) symmetry breaking

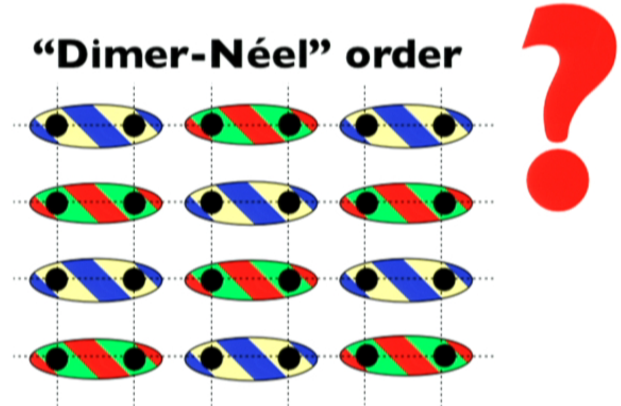
✓ supported by exact diagonalization results

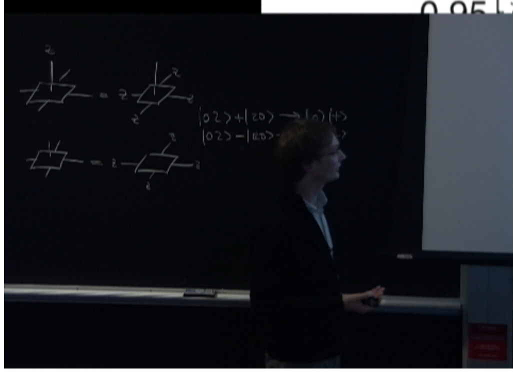
SU(4) Heisenberg model: iPEPS results

Corboz, Läuchli, Mila, Penc, Troyer, PRL **107**, 215301 (2011)



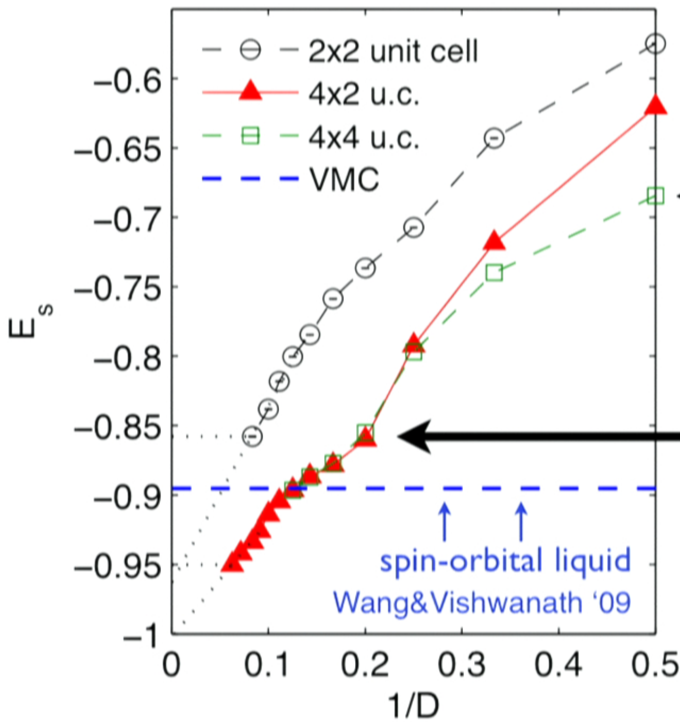




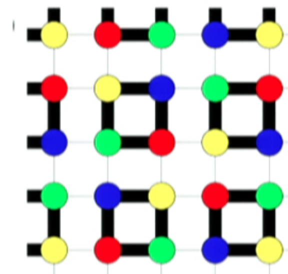


SU(4) Heisenberg model: iPEPS results

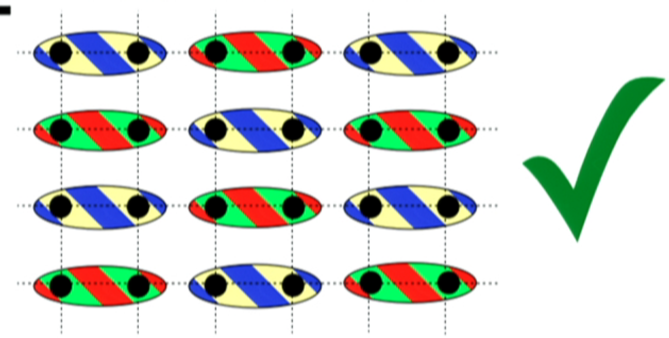
Corboz, Läuchli, Mila, Penc, Troyer, PRL **107**, 215301 (2011)



Linear flavor-wave theory:



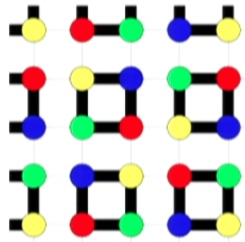
"Dimer-Néel" order



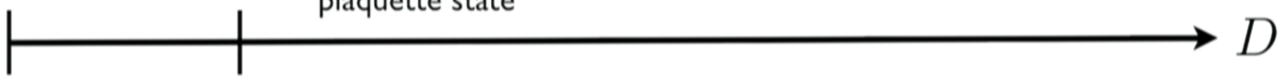
Study as a function of bond dimension

Study as a function of bond dimension

infinite
ground
state
degeneracy



plaquette state

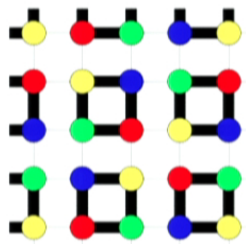


D=1
product state
(mean-field)

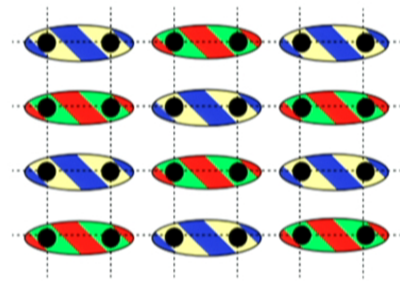
D=2
add quantum fluctuations
on top of product state
same solution as
linear flavor-wave theory

Study as a function of bond dimension

infinite
ground
state
degeneracy



plaquette state



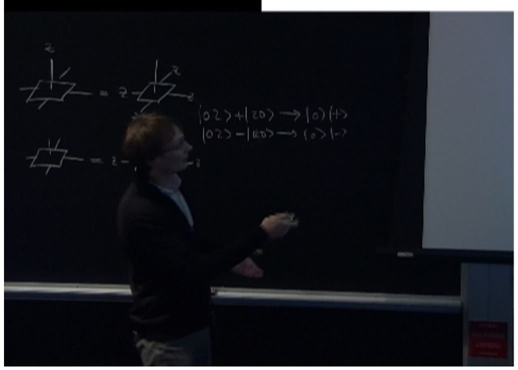
Dimer-Néel ordered state

→ D

D=1
product state
(mean-field)

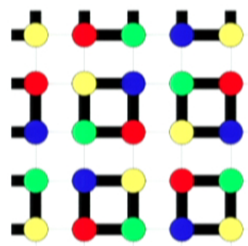
D=2
add quantum fluctuations
on top of product state
same solution as
linear flavor-wave theory

D=5

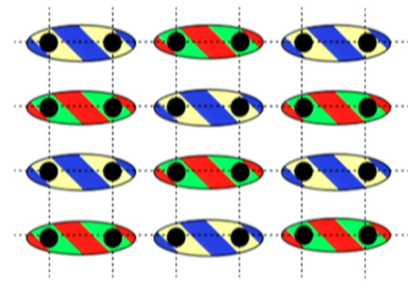


Study as a function of bond dimension

infinite
ground
state
degeneracy

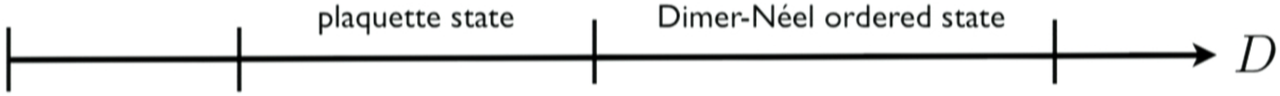


plaquette state



Dimer-Néel ordered state

studying
extrapolations
of the data for
D=5-16
suggests that
state is stable

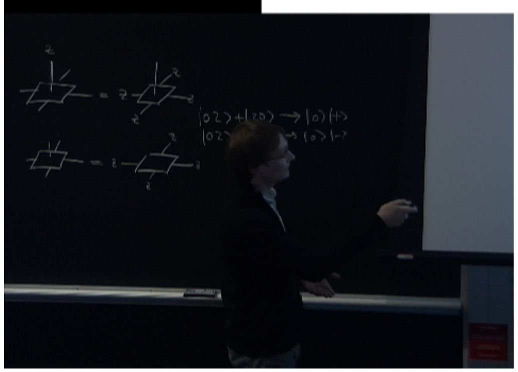


D=1
product state
(mean-field)

D=2
add quantum fluctuations
on top of product state
same solution as
linear flavor-wave theory

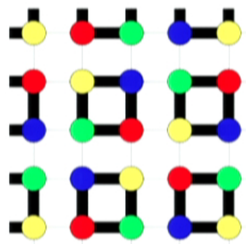
D=5

D=16
largest D tested
in this study

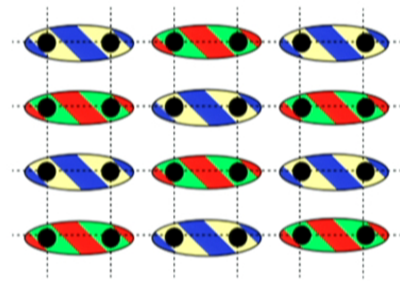


Study as a function of bond dimension

infinite ground state degeneracy



plaquette state



Dimer-Néel ordered state

studying extrapolations of the data for D=5-16 suggests that state is stable

$\longrightarrow D$

D=1
product state (mean-field)

D=2
add quantum fluctuations on top of product state
same solution as
linear flavor-wave theory

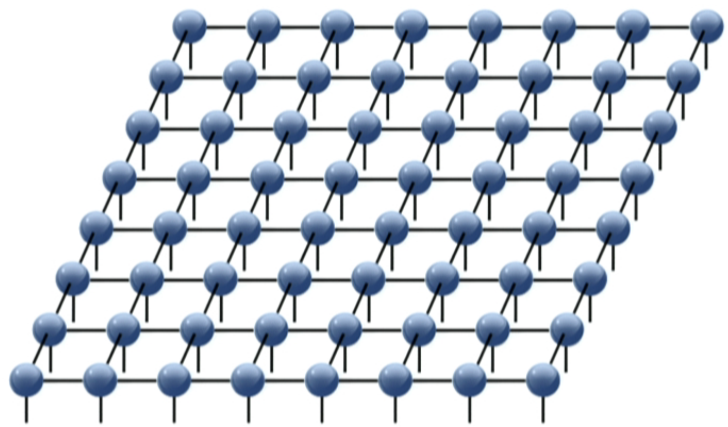
D=5

D=16
largest D tested in this study

increase **quantum fluctuations** / increase **entanglement** $\Rightarrow D$

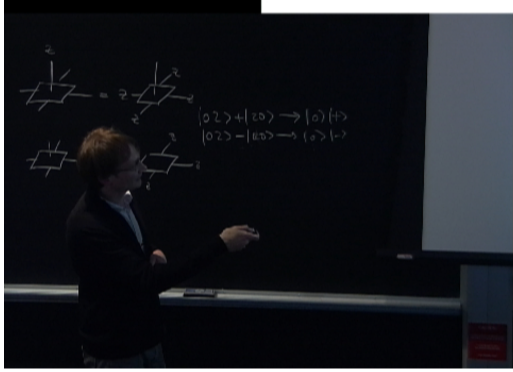
Adding quantum fluctuations systematically...

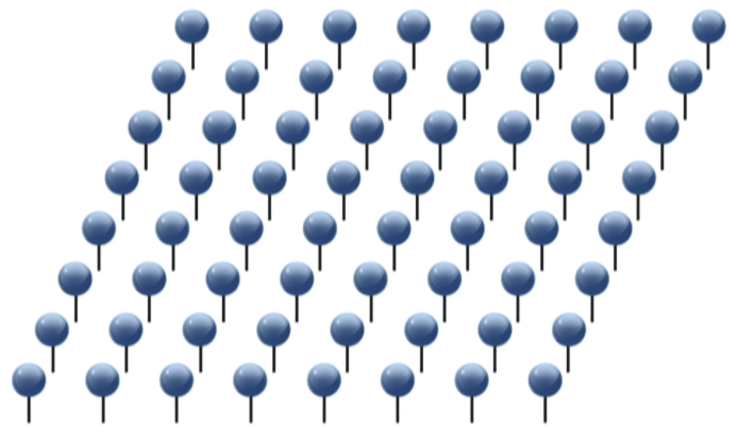
control
quantum fluctuations
(entanglement)



Adding quantum fluctuations systematically...

control
ıtum fluctuations
ɛntanglement)

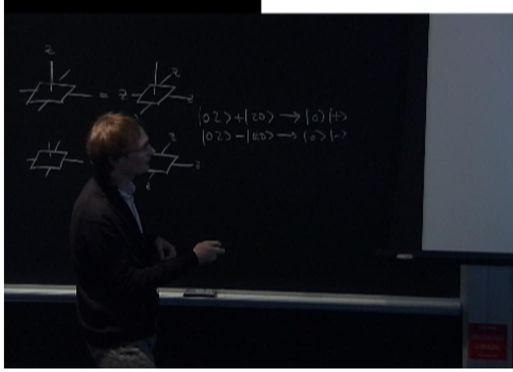


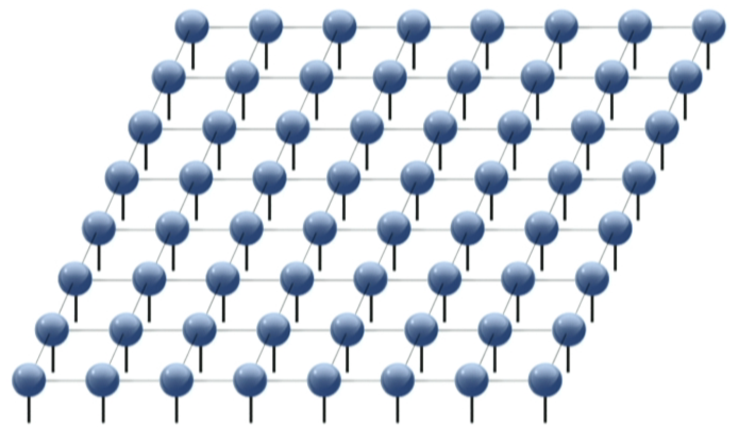


product state

Adding quantum fluctuations systematically...

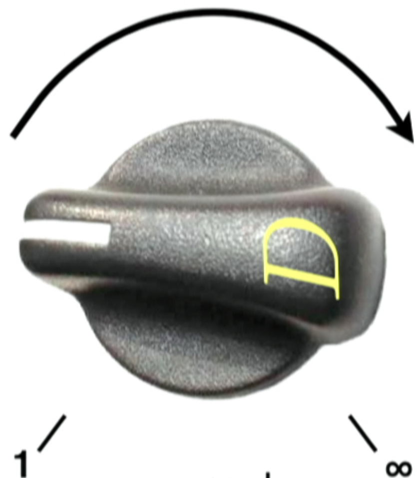
control
ıtum fluctuations
entanglement)



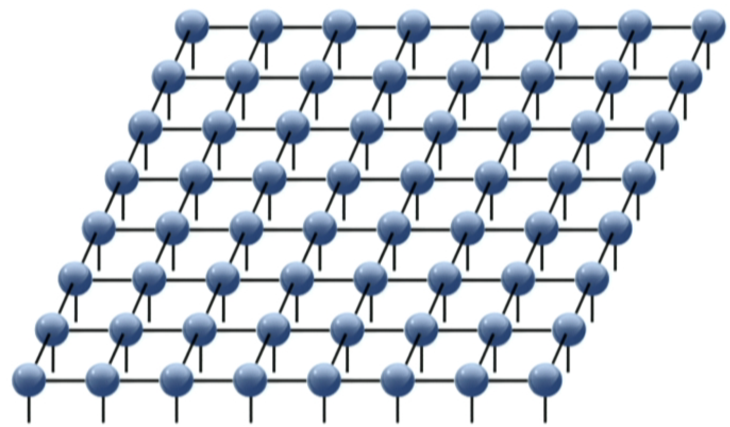


slightly entangled state

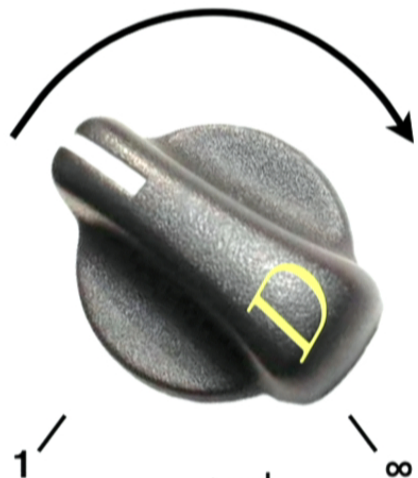
Adding quantum fluctuations systematically...



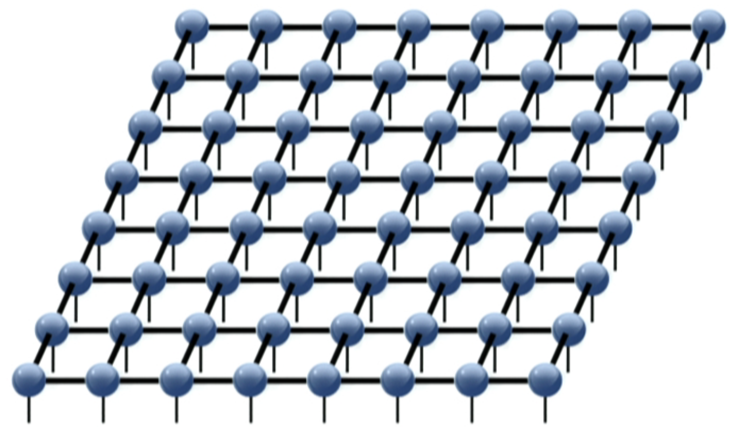
control
tum fluctuations
entanglement)

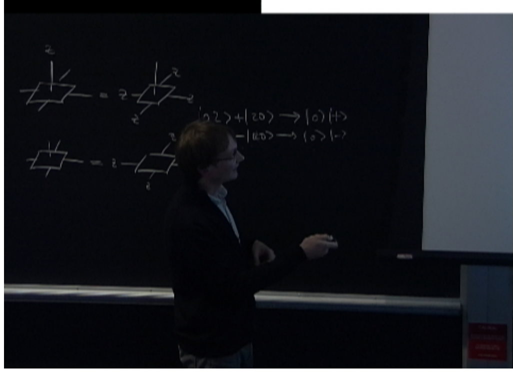


Adding quantum fluctuations systematically...



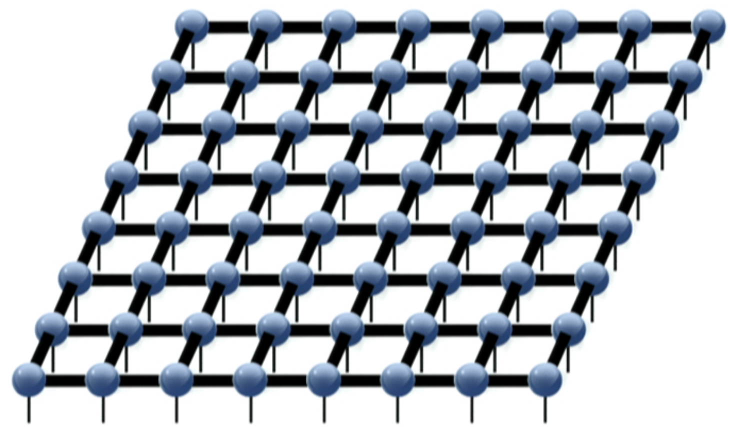
control
tum fluctuations
entanglement)

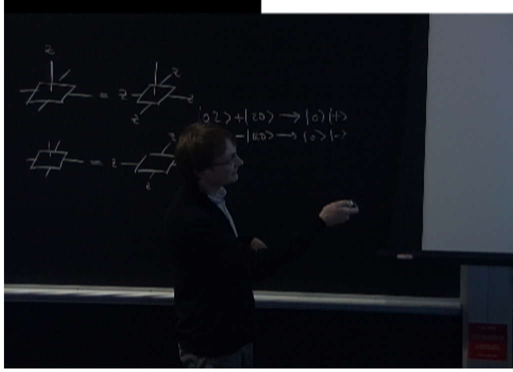




Adding quantum fluctuations systematically...

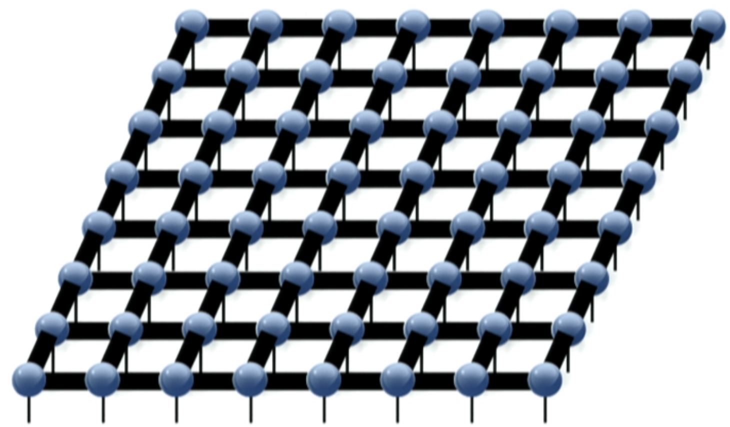
control
tum fluctuations
entanglement)



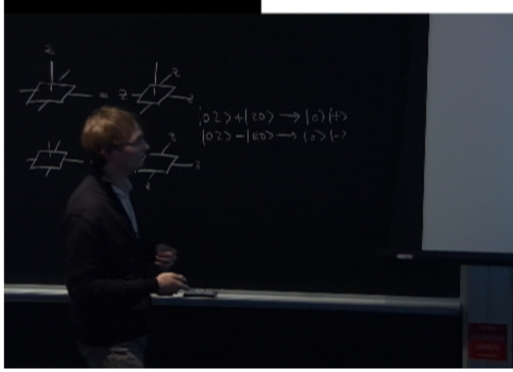


Adding quantum fluctuations systematically...

control
tum fluctuations
ntanglement)



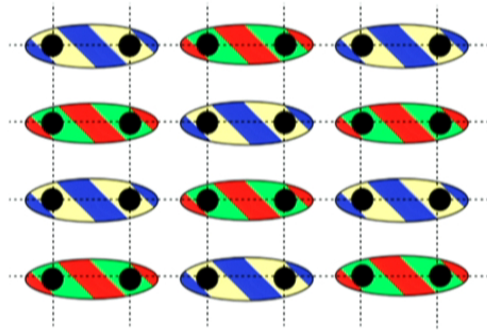
strongly entangled state



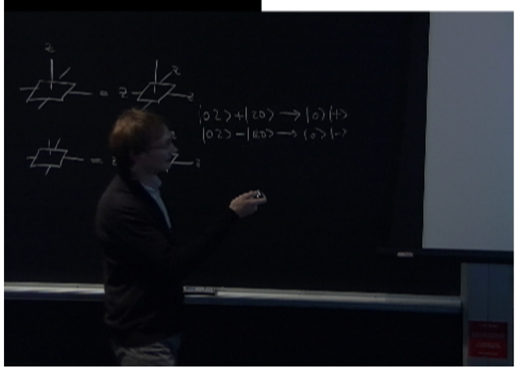
Other lattices: Checkerboard and honeycomb

square lattice:

Dimer-Néel order



Corboz, Läuchli, Penc, Troyer, Mila, PRL **107**, 215301 (2011)



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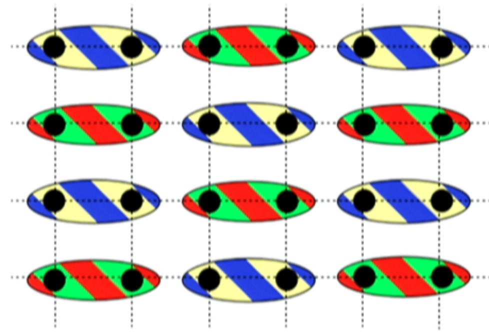


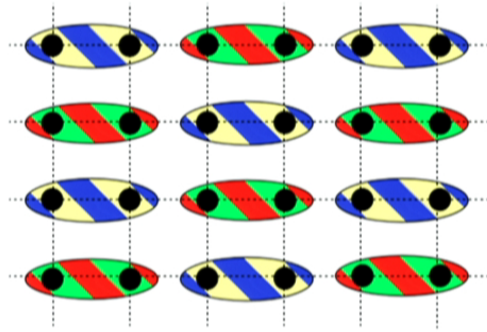
Corboz, Läuchli, Penc, Troyer, Mila, PRL **107**, 215301 (2011)

stabilize SU(4) singlets?

Other lattices: Checkerboard and honeycomb

square lattice:

Dimer-Néel order



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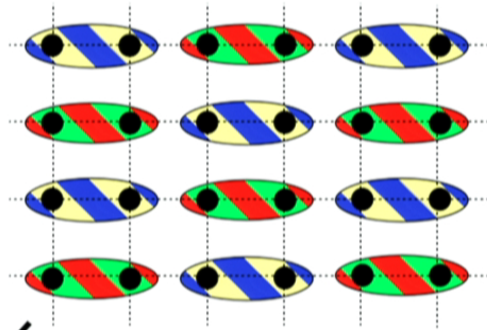
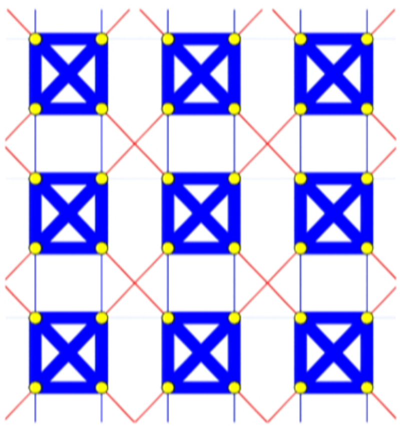


Corboz, Läuchli, Penc, Troyer, Mila, PRL **107**, 215301 (2011)

Checkerboard lattice:

Quadrumerized!

stabilize SU(4) singlets?

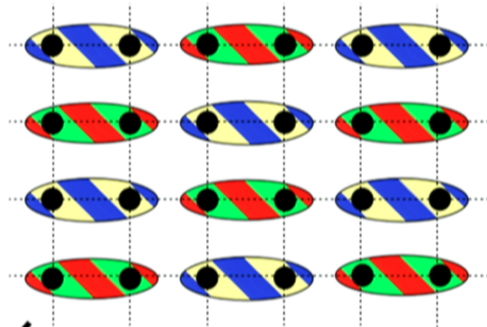


Corboz, Penc, Mila, Läuchli, PRB **86** (2012)

Other lattices: Checkerboard and honeycomb

square lattice:

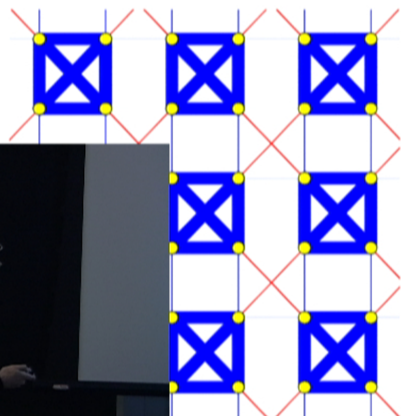
Dimer-Néel order



Corboz, Läuchli, Penc, Troyer, Mila, PRL **107**, 215301 (2011)

Checkerboard lattice: **stabilize SU(4) singlets?**

Quadrumerized!



consistent with ED results up to N=20 (by Andreas Läuchli)

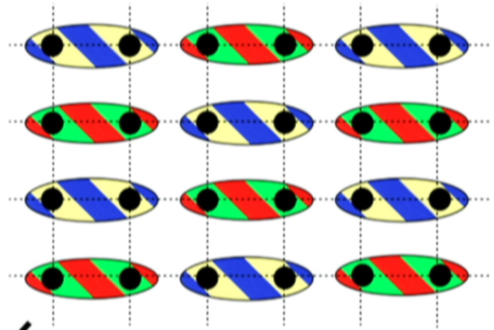
also called "*simplex solid state*", see Arovas, PRB **77** (2008)
or "*valence cluster state*", see Hermele&Gurarie, PRB **84** (2011)

a, Läuchli, PRB **86** (2012)

Other lattices: Checkerboard and honeycomb

square lattice:

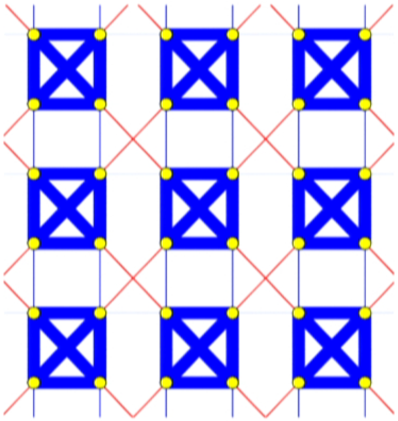
Dimer-Néel order



Corboz, Läuchli, Penc, Troyer, Mila, PRL **107**, 215301 (2011)

Checkerboard lattice: **stabilize SU(4) singlets?**

Quadrumerized!



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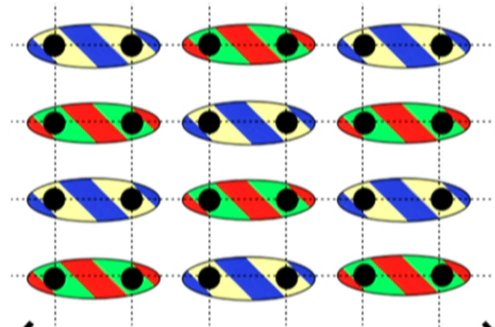

Corboz, Läuchli, Penc, Troyer, Mila, PRL **107**, 215301 (2011)

Checkerboard lattice:

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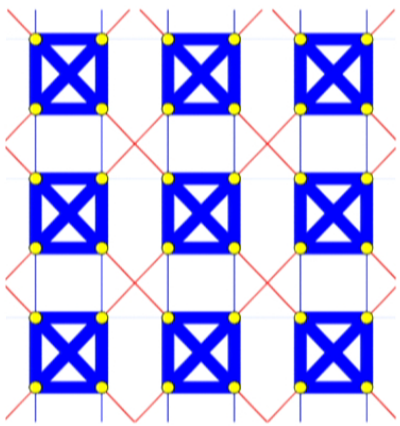
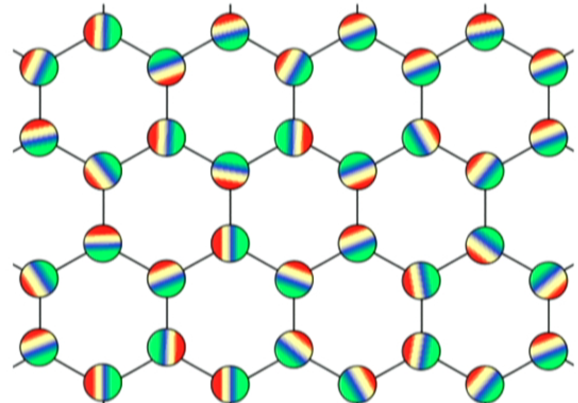


Corboz, Penc, Mila, Läuchli, PRB **86** (2012)

Honeycomb lattice:

spin-orbital (4-color) liquid



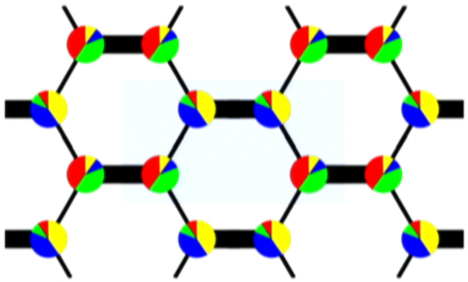
Corboz, Lajkó, Läuchli, Penc, Mila, PRX **2** (2012)

Honeycomb results

Corboz, Lajko, Läuchli, Penc, Mila, PRX **2** (2012)

Low-D iPEPS solutions

2x2 unit cell:



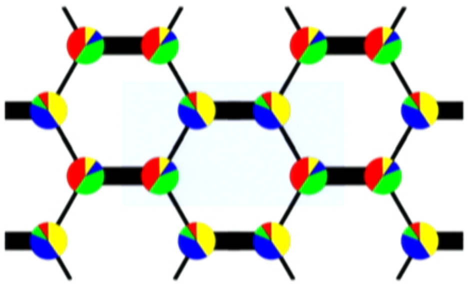
Dimer-Néel ordered?

Honeycomb results

Corboz, Lajko, Läuchli, Penc, Mila, PRX **2** (2012)

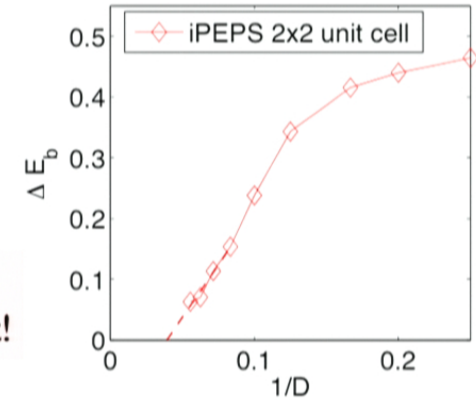
Low-D iPEPS solutions

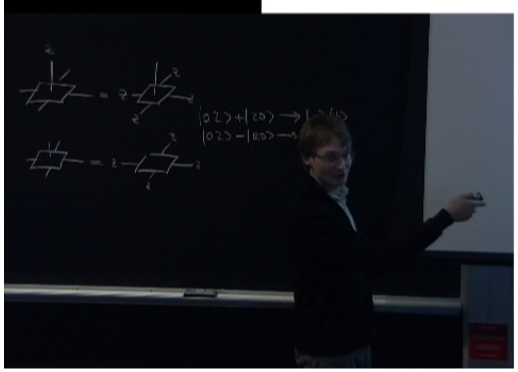
2x2 unit cell:



Dimer-Néel ordered?

NO! Order parameter vanishes in the large D limit!





Honeycomb results

Corboz, Lajko, Läuchli, Penc, Mila, PRX **2** (2012)

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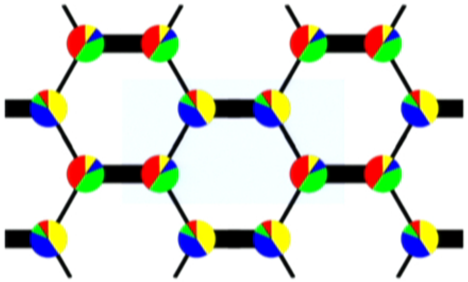

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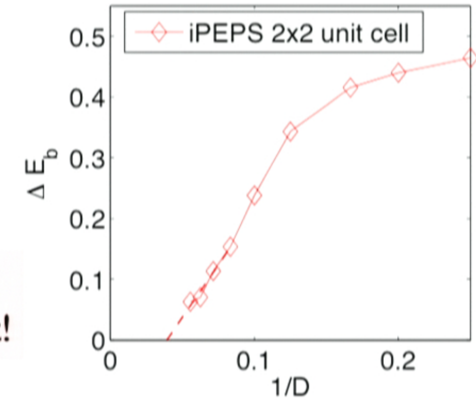
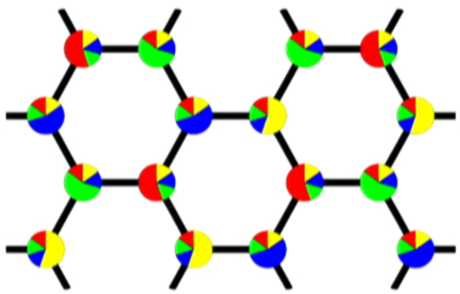


4x4 unit cell:



Color ordered?

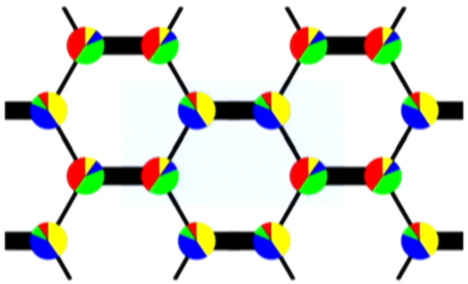
(same as LFWT)

Honeycomb results

Corboz, Lajko, Läuchli, Penc, Mila, PRX **2** (2012)

Low-D iPEPS solutions

2x2 unit cell:



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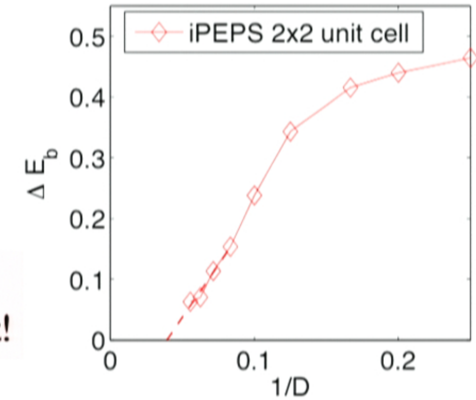
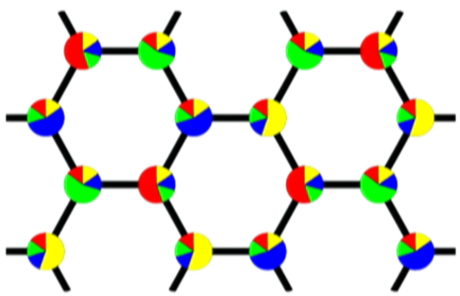


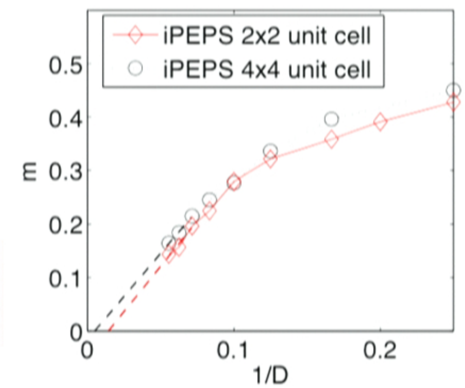
4x4 unit cell:



Color ordered?

(same as LFWT)

NO! Order parameter vanishes in the large D limit!



Honeycomb results

Corboz, Lajko, Läuchli, Penc, Mila, PRX **2** (2012)

Low-D iPEPS solutions

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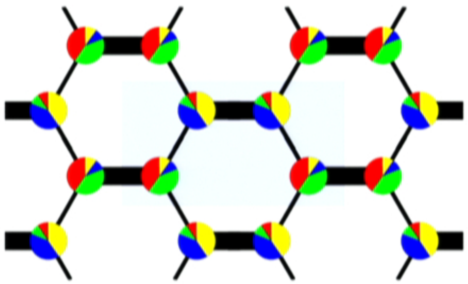

Dimer-Néel ordered?

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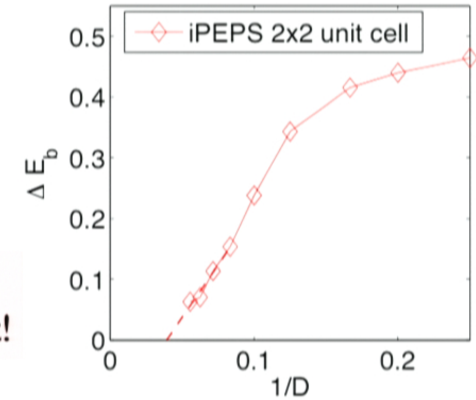
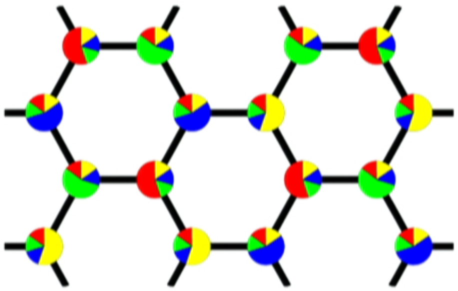


4x4 unit cell:



Color ordered?

(same as LFWT)

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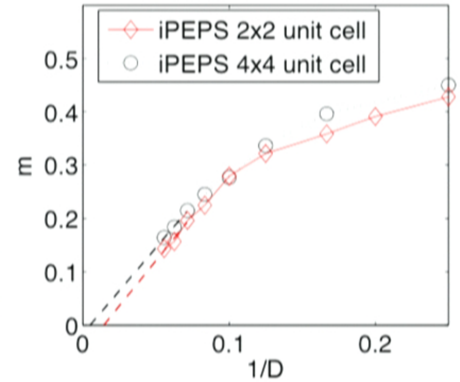


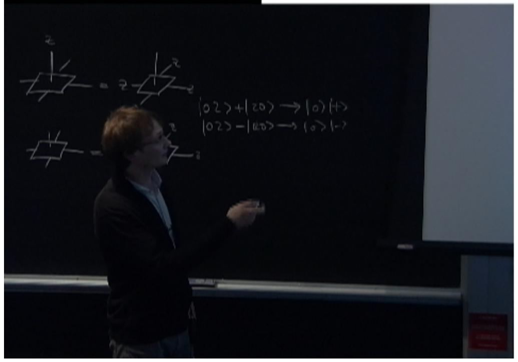
pictures only reflect short range physics!

Honeycomb results

Corboz, Lajko, Läuchli, Penc, Mila, PRX **2** (2012)

Variational Monte Carlo study (Miklos Lajko, Karlo Penc)

- Compare variational energies of different projected fermionic ansatz wave functions



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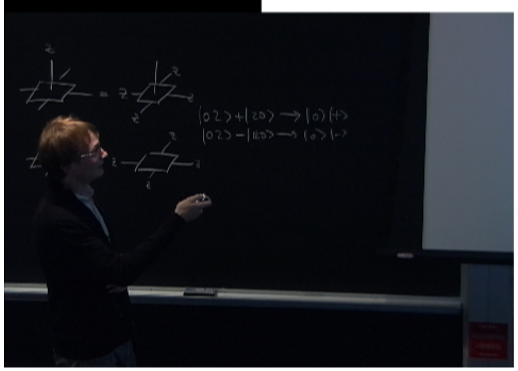
$$\mathcal{H}_f = -\sum_{\langle i,j \rangle} \sum_{\alpha=1}^{4} \left(\chi_{i,j} f^\dagger_{j,\alpha} f_{i,\alpha} + \text{h.c.} \right)$$

↓

Gutzwiller projection (1 particle per site)

↓

Evaluate with Monte Carlo sampling



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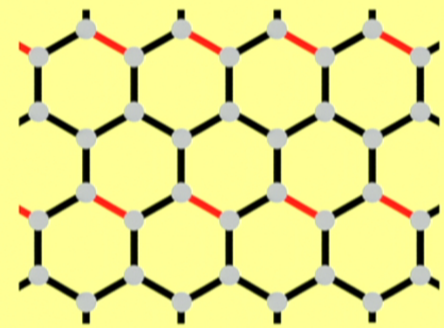
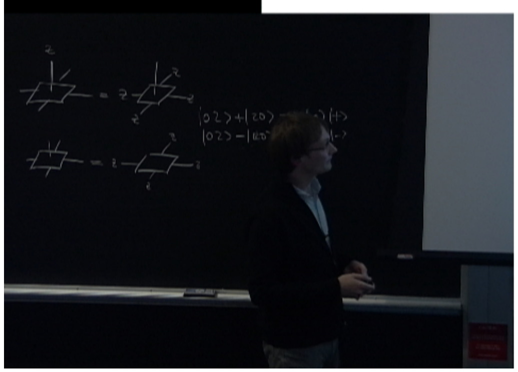
Evaluate with Monte Carlo sampling

π-flux state has lowest energy (close to the iPEPS and ED energy)

$$|\chi_{i,j}| = 1, \quad \prod_{k=1}^{6} \chi_{i_k, i_{k+1}} = -1$$





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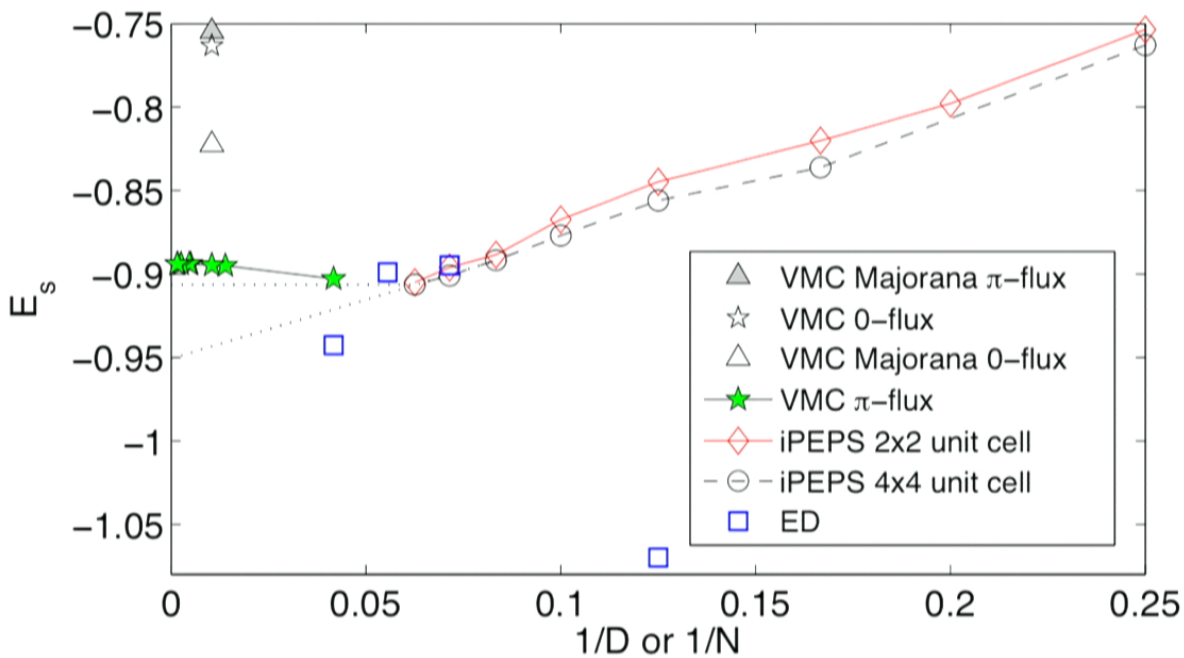

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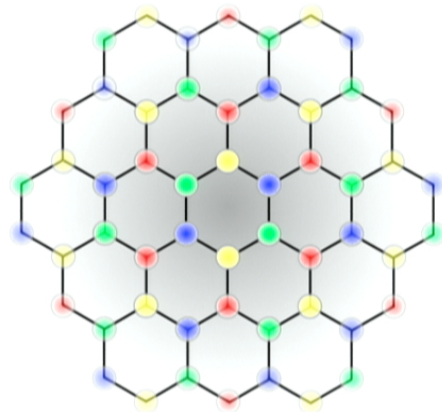

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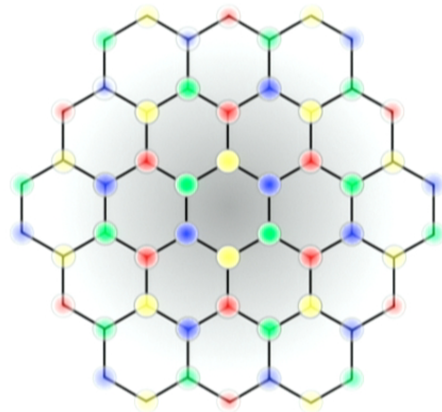

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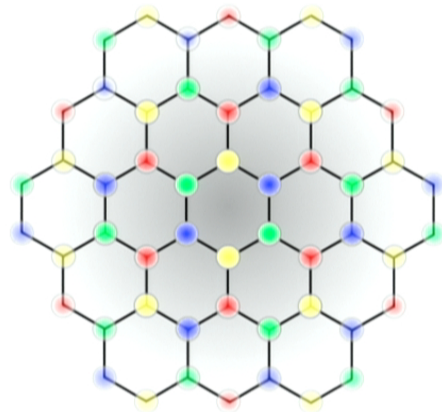

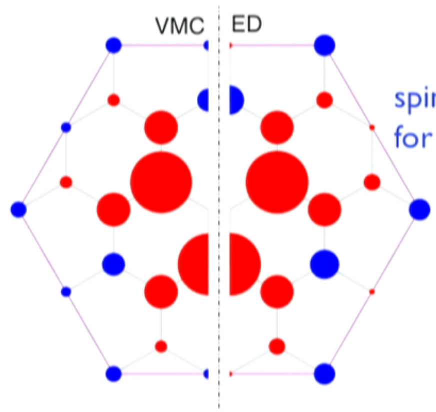

spin-spin correlations for N=24 cluster

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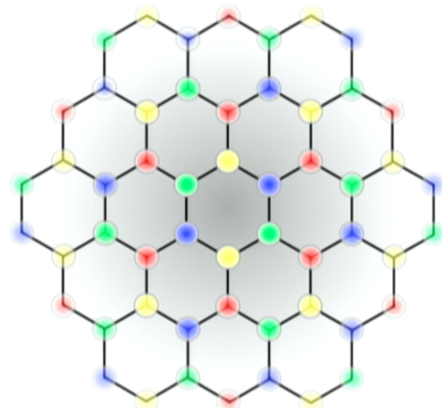
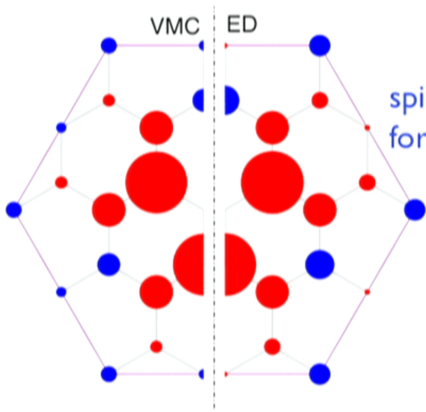




spin-spin correlations for N=24 cluster

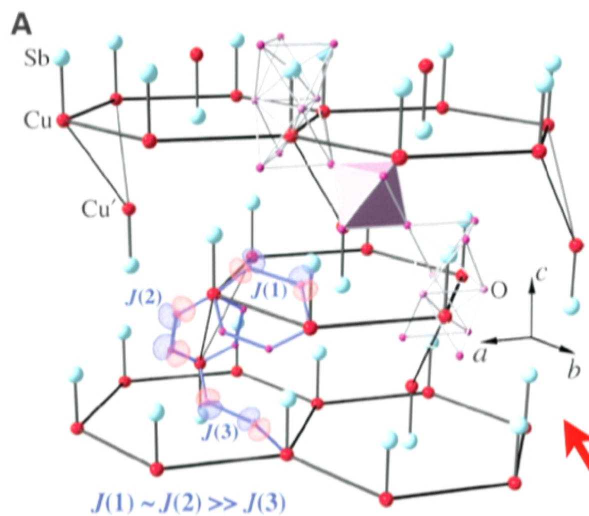
➜ Strong support for an algebraic spin-orbital liquid

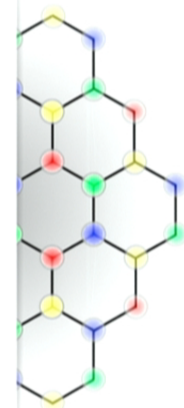
SCIENCE VOL 336 4 MAY 2012

c, Mila, PRX **2** (2012)

Spin-Orbital Short-Range Order on a Honeycomb-Based Lattice

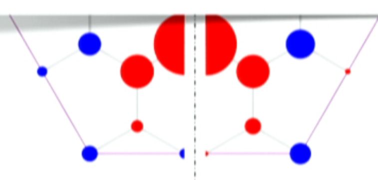
S. Nakatsuji,[1*] K. Kuga,[1] K. Kimura,[1] R. Satake,[2] N. Katayama,[2] E. Nishibori,[2] H. Sawa,[2] R. Ishii,[3] M. Hagiwara,[3] F. Bridges,[4] T. U. Ito,[5] W. Higemoto,[5] Y. Karaki,[6] M. Halim,[7] A. A. Nugroho,[7] J. A. Rodriguez-Rivera,[8,9] M. A. Green,[8,9] C. Broholm[8,10]





$Ba_3CuSb_2O_9$

Connection?

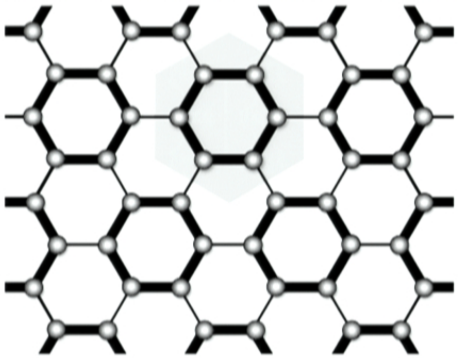


Strong support for an algebraic spin-orbital liquid

SU(N) Heisenberg models

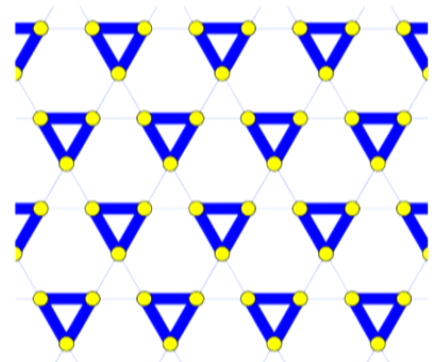
SU(3) honeycomb: *Plaquette state*

Corboz, Läuchli, Penc, Mila, arxiv:1302.1108



SU(3) kagome: *Simplex solid state*

Corboz, Penc, Mila, Läuchli, PRB **86** (2012)

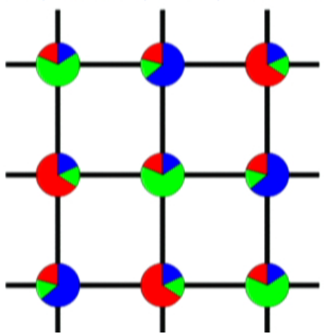


Generalizations

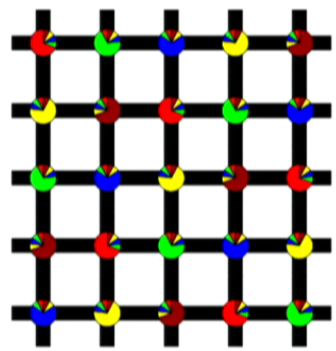
- other lattices
- other values of N
- other representations
- XXZ-type models

SU(3) square/triangular:
3-sublattice Néel order

Bauer, Corboz, et al., PRB **85** (2012)

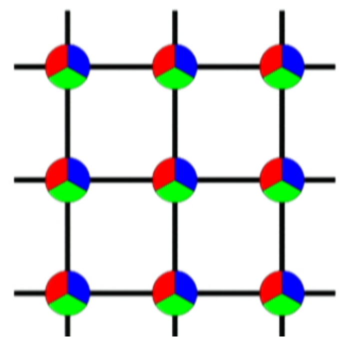


SU(5) square: *Color order*



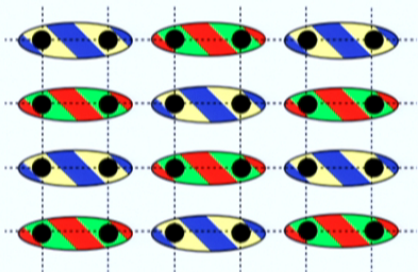
3-color quantum Potts:
superfluid phases

Messio, Corboz, Mila, arXiv:1304.7676

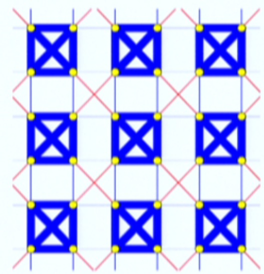


Summary: symmetric Kugel-Khomskii model in 2D

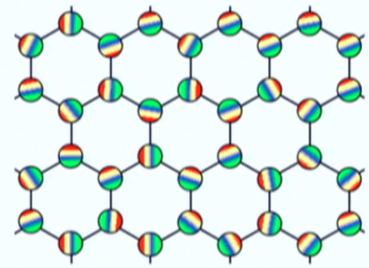
- The symmetric Kugel-Khomskii model exhibits a rich variety of states:



Dimer-Neel ordered state



quadrumerized state



algebraic spin-orbital liquid

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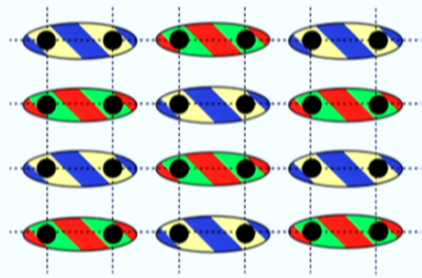

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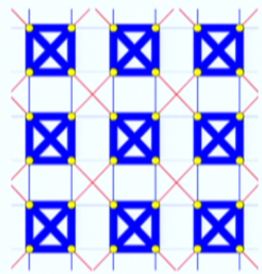

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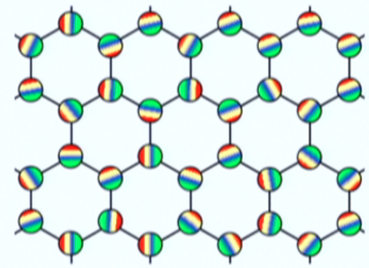

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Progress in the simulation of strongly-correlated systems

✓ Tensor networks (**iPEPS**): Efficient variational ansatz where the accuracy can be systematically controlled

✓ Combination of analytical (LFWT) and numerical methods (ED, iPEPS, VMC, ...) to get a consistent picture

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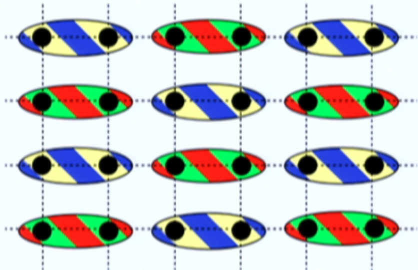

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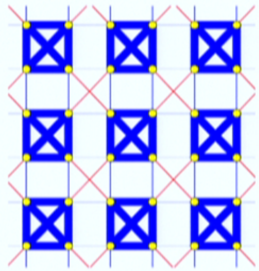

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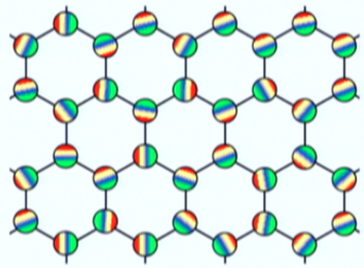

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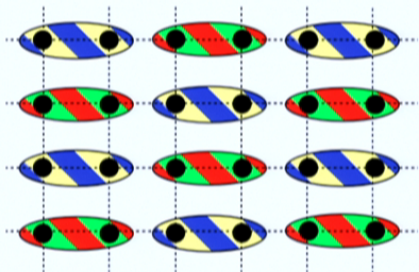

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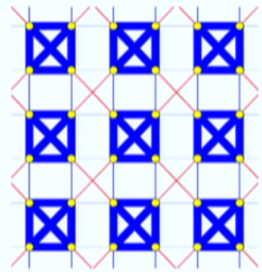

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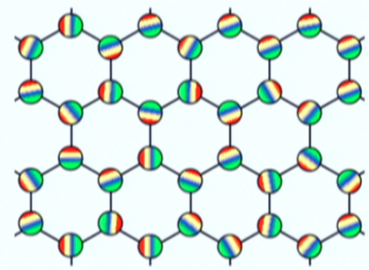

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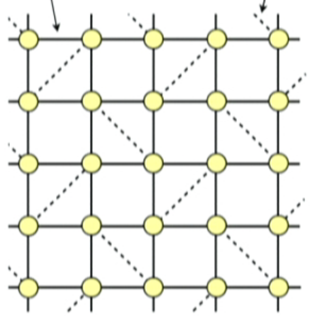
Thank you for your attention!

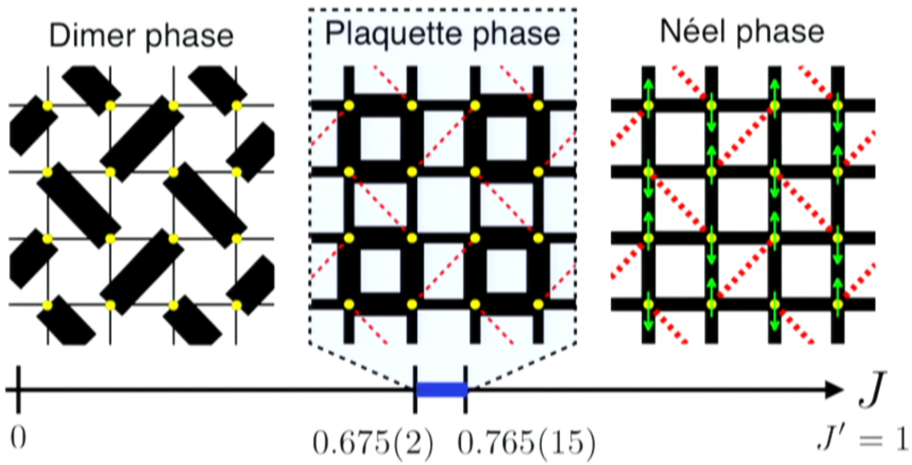
The Shastry-Sutherland model

Corboz, Mila, PRB **87** (2013)

$$\hat{H} = J \sum_{\langle i,j \rangle} S_i \cdot S_j + J' \sum_{\langle\langle i,j \rangle\rangle_{\text{dimer}}} S_i \cdot S_j$$



model for $SrCu_2(BO_3)_2$



Plaquette phase with an unprecedented accuracy of the phase boundaries!