Title: Characterizing topological spin liquids using PEPS

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Abstract: Projected Entangled Pair States (PEPS) provide a local description of correlated many-body states. I will discuss how PEPS can be used to characterize topological spin liquids, in particular Resonating Valence Bond states. On the one hand, I will show how the symmetries in the local PEPS description allow to identify that these states appear as topologically degenerate ground states of local Hamiltonians. On the other hand, I will discuss how from exact diagonalization of the transfer operator one can extract both the topological order and the spin liquid nature of the ground state.

Characterizing topological spin liquids using Projected Entangled Pair States

Norbert Schuch
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joint work with David Pérez-García, Ignacio Cirac, and Didier Poilblanc


using Projected Entangled Pair States
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Introduction

- **Tensor network states**:
  - **efficient description** of strongly correlated quantum states from **local tensors** (local entanglement)
  - powerful ansatz for **numerical simulations**
  - analytical ansatz, **solvable models** (e.g. AKLT, Toric Code, ...)

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  - powerful ansatz for **numerical simulations**
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- This talk: use PEPS to **identify topological systems**, in part. **spin liquids**

- Topological spin liquid:
  - system w/ high symmetry which **does not break any symmetry**
  - **topological ground space** structure
- Difficult: Identify topological order & absence of *any* long-range order

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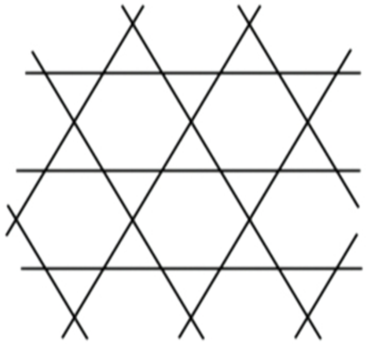
- Apply PEPS to **characterize topological spin liquid** nature of RVB states
- Concepts apply to general (tinv.) PEPS – both to variational ansatzes and to tensors obtained from iPEPS simulations

Resonating valence bond states

- focus on **kagome lattice** (highly frustrated)



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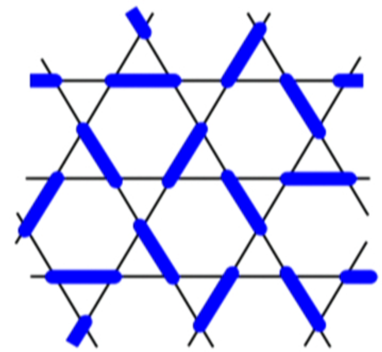

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- **Dimer covering D :** complete covering of lattice with "dimers" (=pairs of adj. vertices)
- dimer config. $D \Leftrightarrow$ vector $|D\rangle$ in some Hilbert space
- associate **spin-$\frac{1}{2}$** to each **vertex**, and **singlet** $|s\rangle = |01\rangle - |10\rangle$ to each **dimer**

  $\Rightarrow$ dimer covering represented by $|\boldsymbol{\sigma(D)}\rangle = \bigotimes |s\rangle$
- **Resonating Valence Bond (RVB) state**:

$$|\mathrm{RVB}\rangle = \sum\nolimits_D |\sigma(D)\rangle$$

- RVB: ansatz for **Heisenberg antiferromagnets** (Pauling, Anderson)
- Is the RVB state a **topological spin liquid?**
  - ground state of some **local Hamiltonian**
  - **topological ground space degeneracy**
  - **no long range order** (symmetry breaking) in ground state

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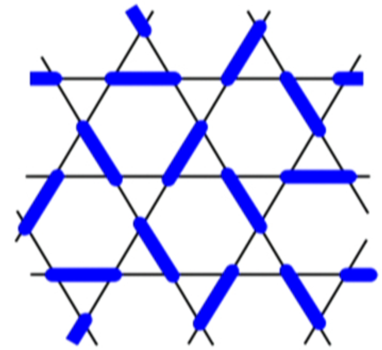

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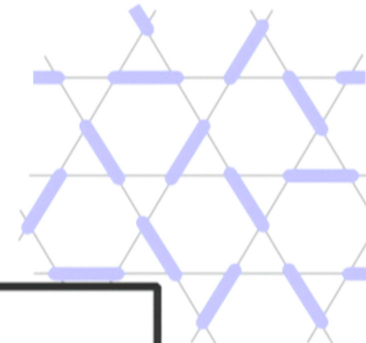

Questions:

- Is the RVB ground state of a local Hamiltonian?
- Does it have a topological ground space structure?
- How does it relate to known topological models?
- Does it have long-range order or not?

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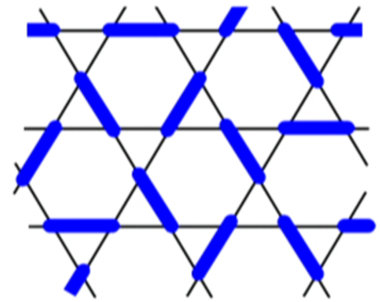
The dimer model

- Difficulty in RVB: Different **singlet configurations not orthogonal**
- "Toy model" for RVB: **Orthogonal dimer state** $\sum_D |D\rangle$ where $\langle D'|D\rangle = 0$.
- Hamiltonian:

$$H = -\sum H_{\triangle} - \sum H_{\text{✡}}$$

valid dimer configurations → $H_{\triangle}$

resonance moves on stars → $H_{\text{✡}}$



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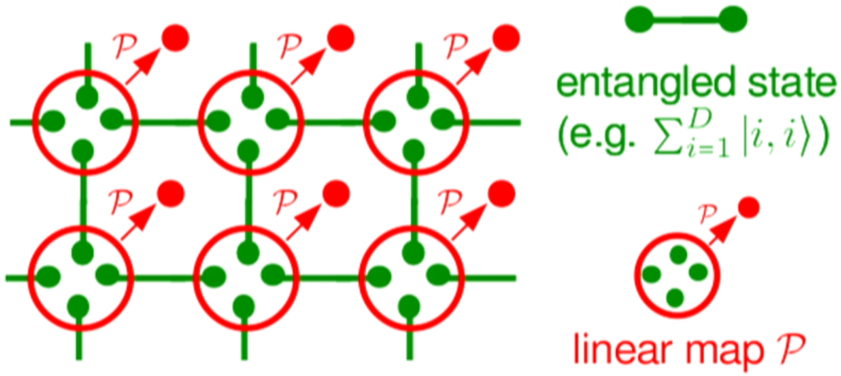

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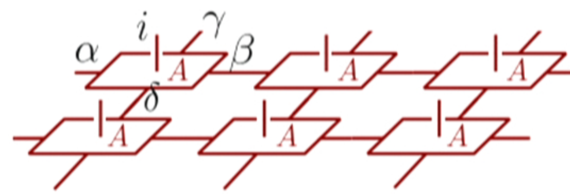

$\mathcal{P} \equiv \sum A^{i}_{\alpha\beta\gamma\delta} |i\rangle\langle\alpha,\beta,\gamma,\delta|$

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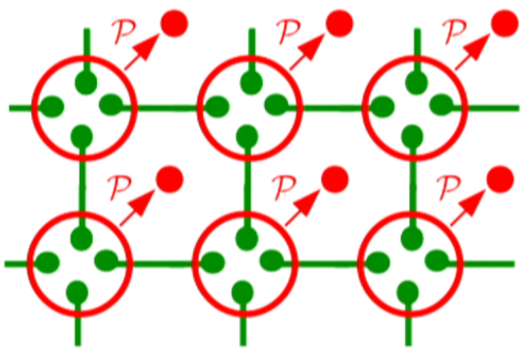

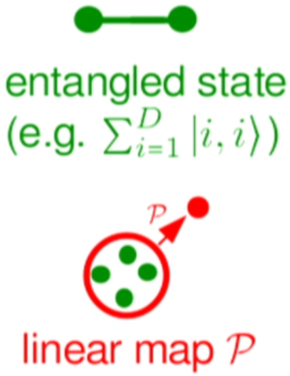

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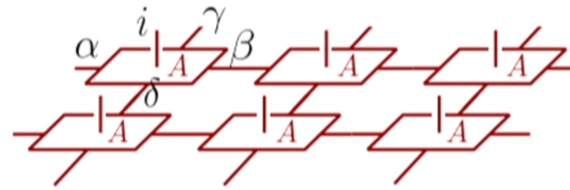

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- many states (Toric Code, RVB state, ...) have an **exact PEPS description**

- flexibility in description: **blocking of sites** (and more ...)

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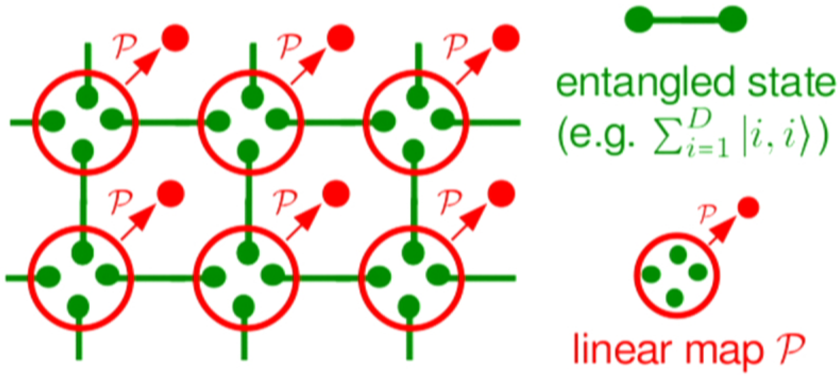

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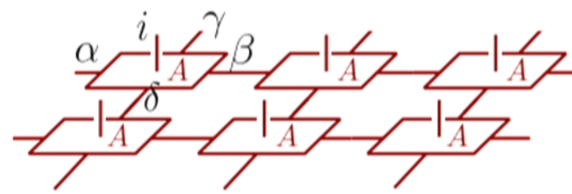

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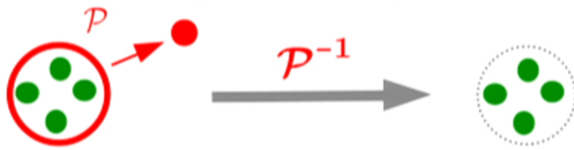
- Relation to **Hamiltonians**? Unique or **topological ground state**?

Injectivity

- **Injectivity:** $\mathcal{P}$ has left-inverse ($\mathcal{P}^{-1}\mathcal{P} = \mathbb{1}$)



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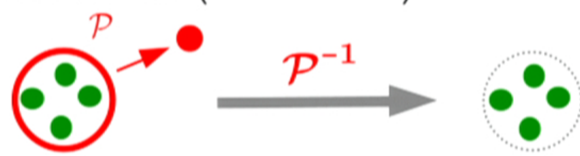

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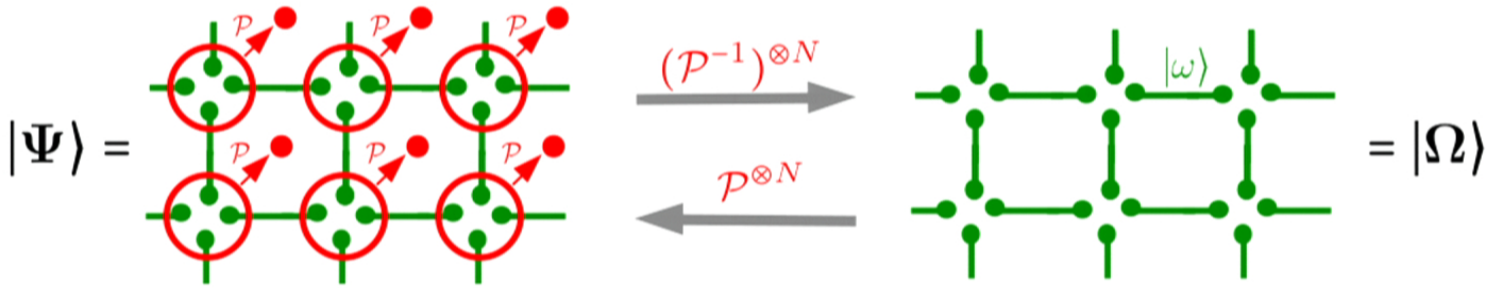

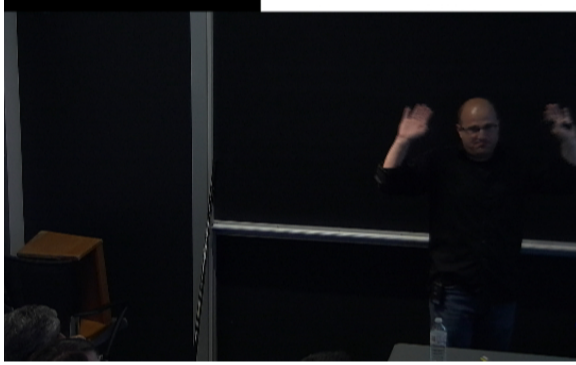

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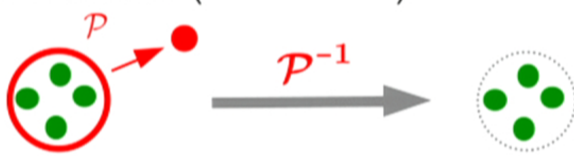

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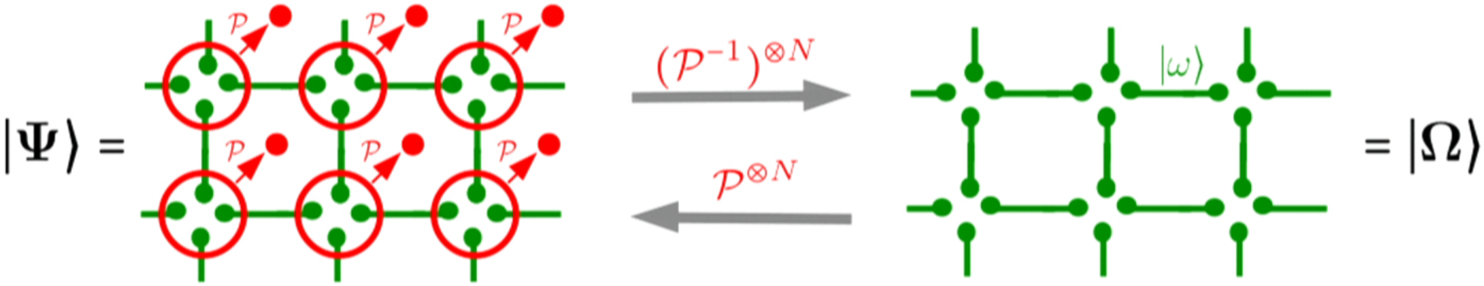


- entangled states $|\Omega\rangle$ unique ground state of local Hamiltonian

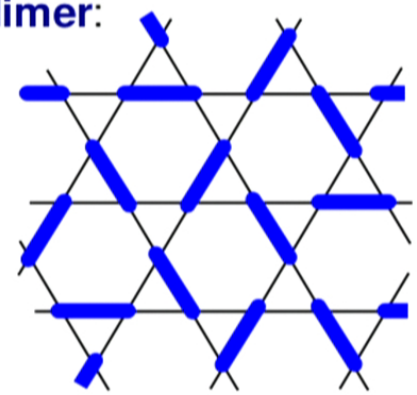
$$H = \sum h \;,\; h = \mathbb{1} - |\omega\rangle\langle\omega| \quad : \quad h \geq 0, \qquad h|\Omega\rangle = 0$$

$\Rightarrow$ PEPS $|\Psi\rangle$ **unique ground state** of local "**parent Hamiltonian**" $H' = \sum h'$,

$$h' = (\mathcal{P}^{-1} \otimes \mathcal{P}^{-1})^\dagger h (\mathcal{P}^{-1} \otimes \mathcal{P}^{-1}) \; : \quad h' \geq 0, \qquad h'|\Psi\rangle = 0$$

PEPS representation of the RVB state

- **PEPS representation** for the **kagome RVB & dimer**:



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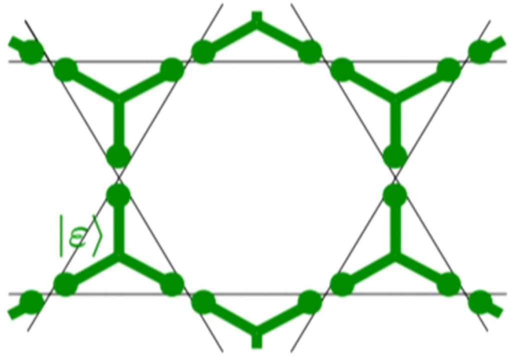

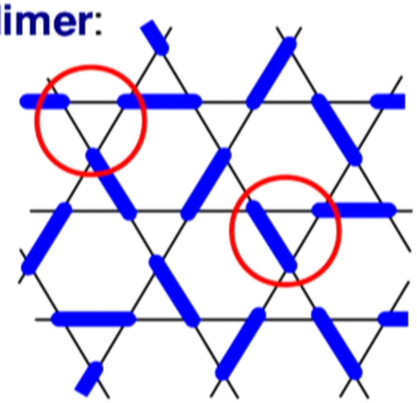

$|0\rangle, |1\rangle$: spin-$\frac{1}{2}$ subspace

$|2\rangle$: "no singlet" tag

- Triangles have **no or one singlet:**

$$|\epsilon\rangle = \frac{1}{\sqrt{2}} \sum \epsilon_{ijk} |ijk\rangle + |222\rangle = \frac{1}{\sqrt{2}} (|01\rangle - |10\rangle)|2\rangle + \text{perm.} + |222\rangle$$

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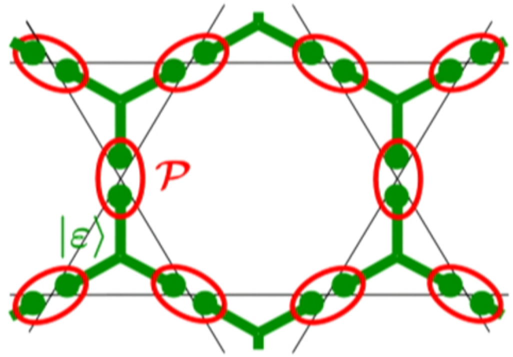

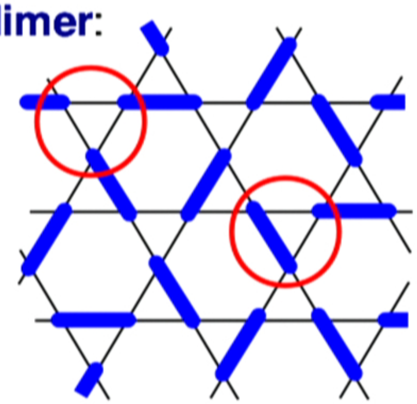

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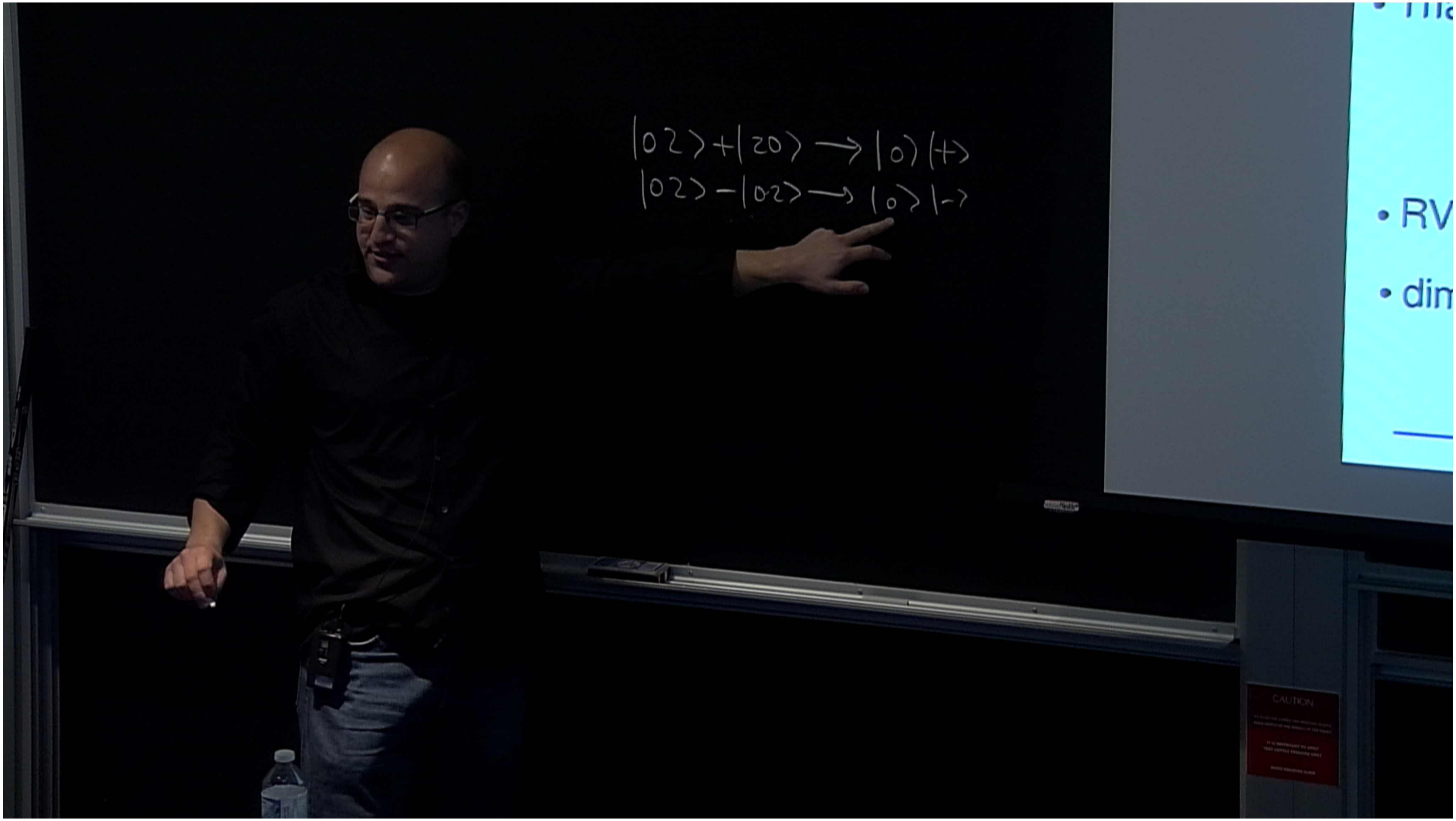
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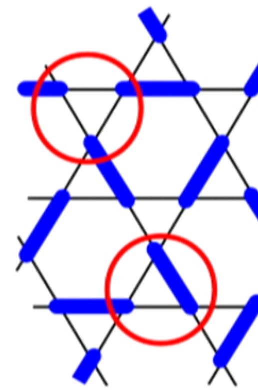
$\mathbb{Z}_2$ symmetry in the RVB PEPS

- PEPS of RVB & dimer state has $\mathbb{Z}_2$ symmetry:

  representation $Z = |0\rangle\langle 0| + |1\rangle\langle 1| - |2\rangle\langle 2| = \mathrm{diag}(1,1,-1)$



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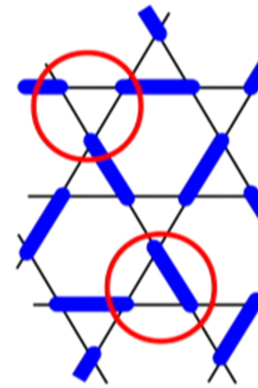

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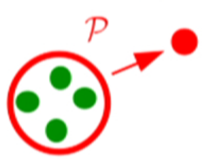
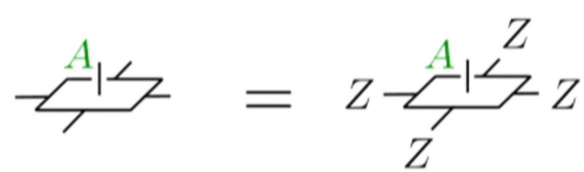
- composition of $\mathbb{Z}_2$- invariant objects $\Rightarrow$ $\mathbb{Z}_2$- invariant object

$\Rightarrow$ $\mathcal{P}$ is **supported on invariant subspace** / tensor A **has symmetry**:

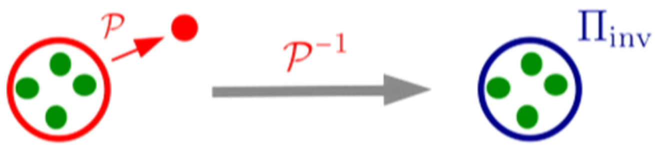


$$\mathcal{P} = \mathcal{P}\,(Z \otimes Z \otimes Z \otimes Z)$$



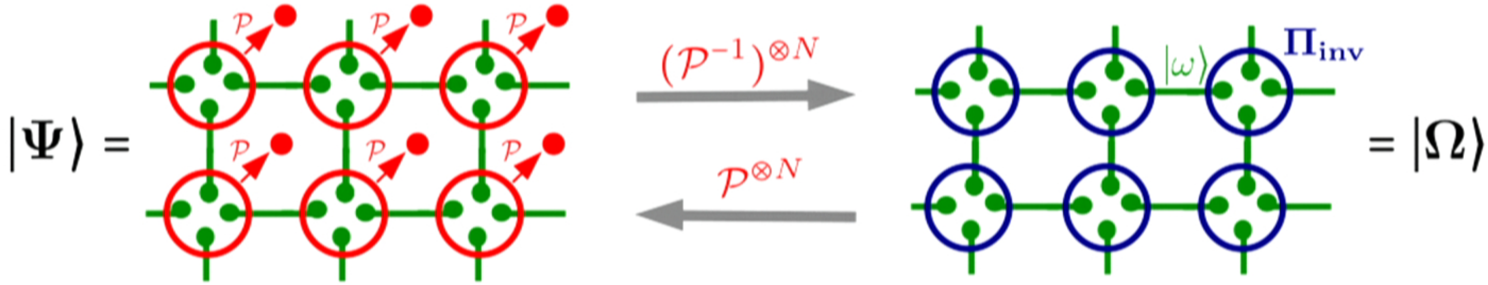
Hamiltonians for topological tensor networks

- ***G*-injectivity**: $\mathcal{P}$ invertible on **invariant subspace** of symmetry, $\mathcal{P}^{-1}\mathcal{P} = \Pi_{\text{inv}}$:



where $\Pi_{\text{inv}} = \frac{1}{|G|}\sum_g U_g^{\otimes 4}$

- state **mapped reversibly** to **known topol. model** (e.g. Toric Code: $\mathcal{P} = \Pi_{\text{inv}}$)



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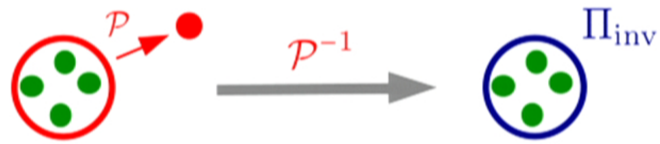

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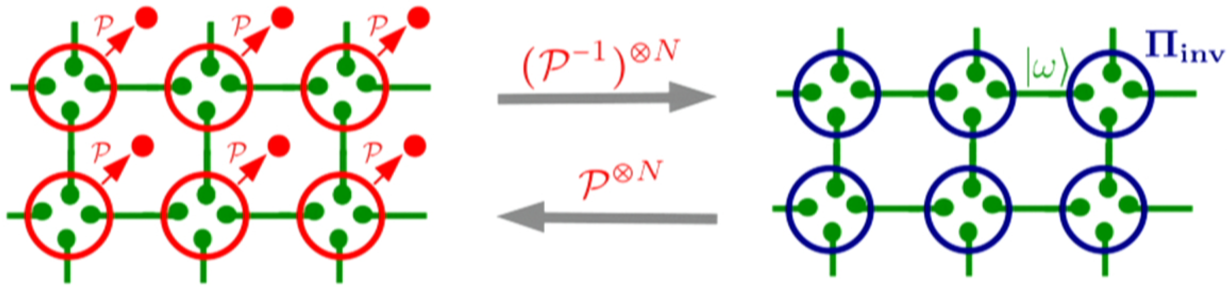


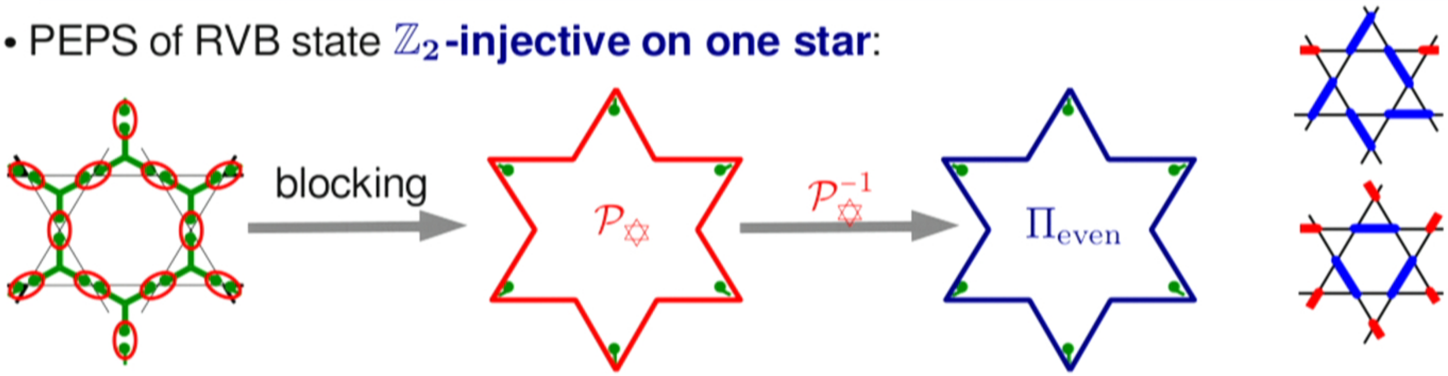
$= |\Omega\rangle$

→ **parent Hamiltonian** $h' = ((\mathcal{P}^{-1})^{\otimes 4})^\dagger h_{\rm TCode}(\mathcal{P}^{-1})^{\otimes 4}$

- $|\Psi\rangle$ ground state of **local Hamiltonian** with **topo. ground space degeneracy**
- one-to-one mapping between ground states $\leftrightarrow$ same **ground space structure**
- h' depends smoothly on $\mathcal{P}$: interpolation $\mathcal{P}(\theta) \rightarrow$ interpolation $H'(\theta) = \sum h'(\theta)$

Relating RVB and Toric Code

- PEPS of RVB state $\mathbb{Z}_2$**-injective on one star**:



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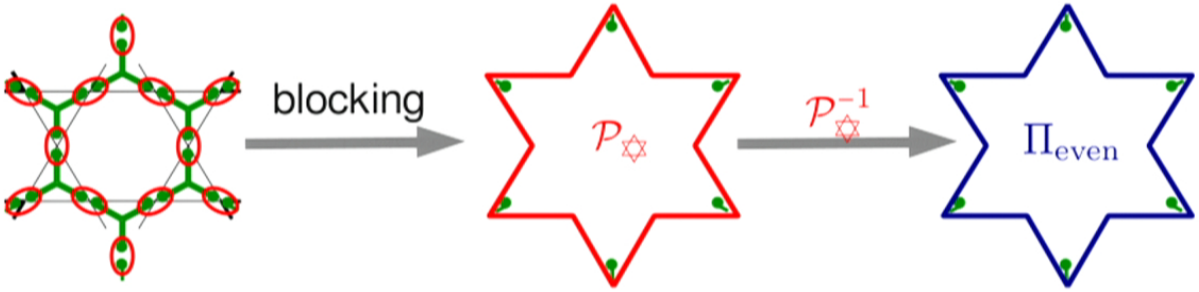
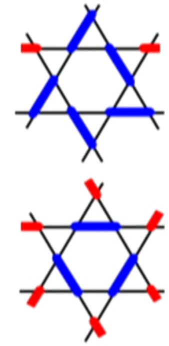
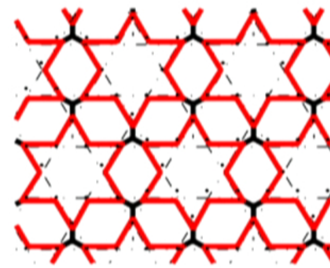




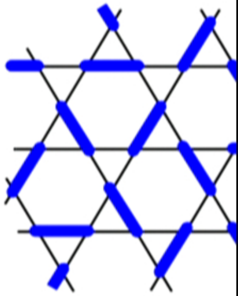


- **lattice** can be **covered** with stars
- **singlet configurations** can be **locally distinguished**
  $\Rightarrow$ reversible transformation to **orthogonal dimer state**

- Hamiltonian for dimer state $\Rightarrow$ **Hamiltonian for RVB**
- RVB has **parent Hamiltonian** with **topological** ground space structure!

Summary of analytical results

- RVB: ground state of **local parent Hamiltonian** with **topologically degenerate ground space** (4-fold on torus)



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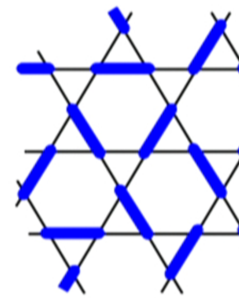
  $\Leftrightarrow$ invertible **mapping to Toric Code**



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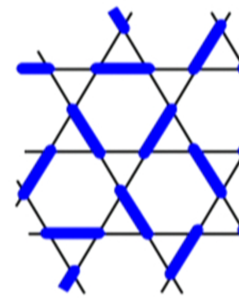

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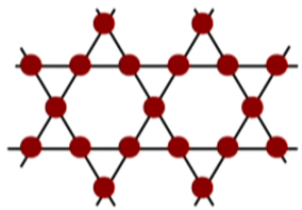
  $\Leftrightarrow$ invertible **mapping to Toric Code**
- **Dimer** $\leftrightarrow$ **RVB interpolation** by "continuously removing tagging":
  $\Rightarrow$ continuous **path of parent Hamiltonians** $H(\theta) = \sum h(\theta)$
- Caveat: This does not tell us whether Hamiltonian has **spectral gap**!
  (but this is checkable in principle)

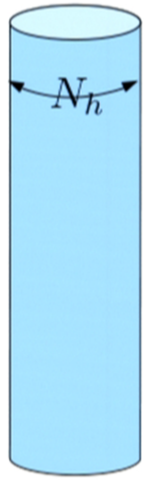


- smallest (provable) Hamiltonian acts on **two stars**
  (proof uses more low-level techniques)

- Is the RVB a **spin liquid** – do all **correlations decay exponentially**?
- Is the system really **topologically ordered as** $N \to \infty$?
- Are **RVB and dimer** in the **same phase**,
  or is there a **phase transition** along the dimer–RVB interpolation?

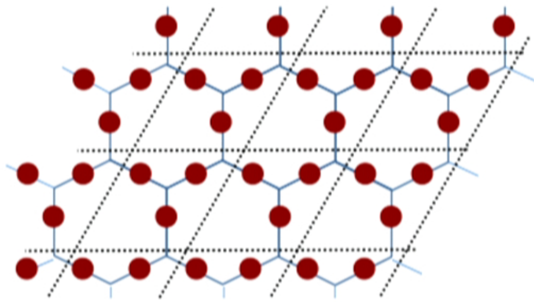
Numerical study of RVB states

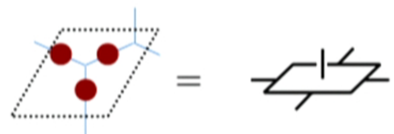
- **numerical study** on **infinite cylinders**

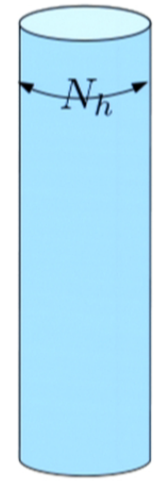


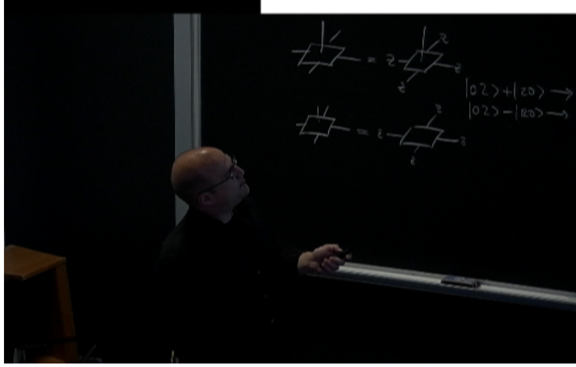
Numerical study of RVB states

- **numerical study** on **infinite cylinders**
- blocking $\Rightarrow$ square lattice









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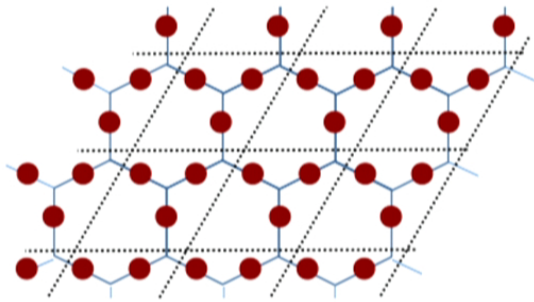
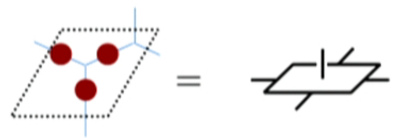
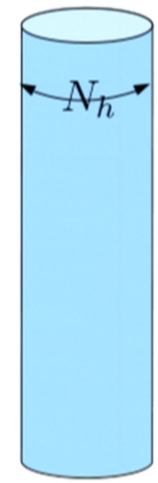
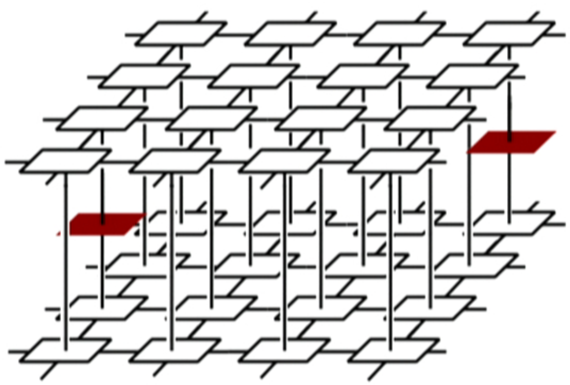




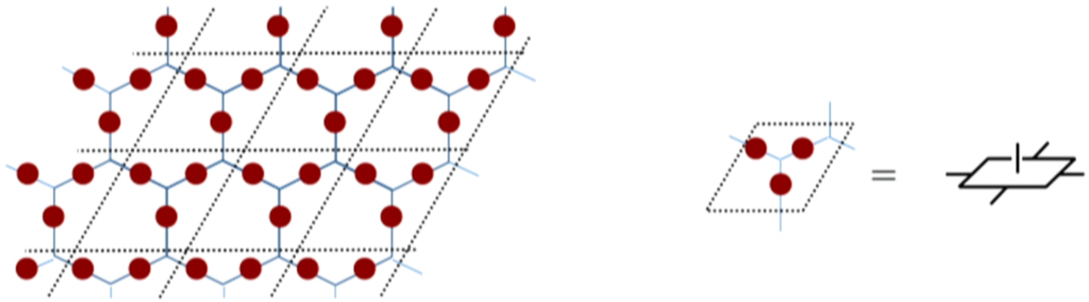


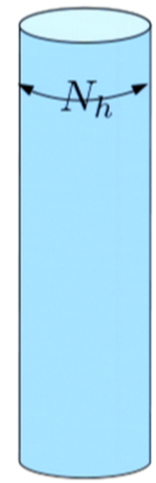
- **expectation values** (e.g. correlation functions):



Numerical study of RVB states

- **numerical study** on **infinite cylinders**
- blocking ⇒ square lattice





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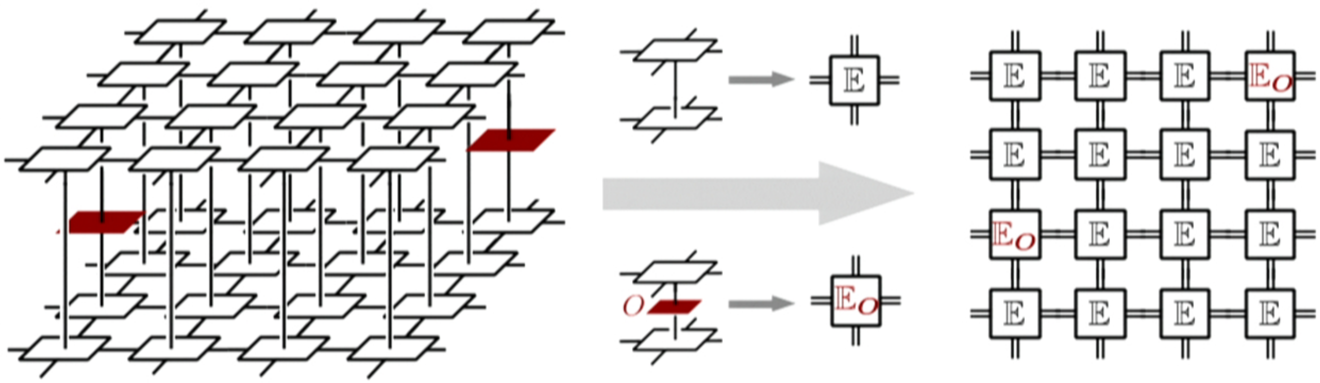
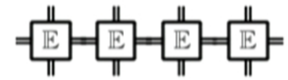
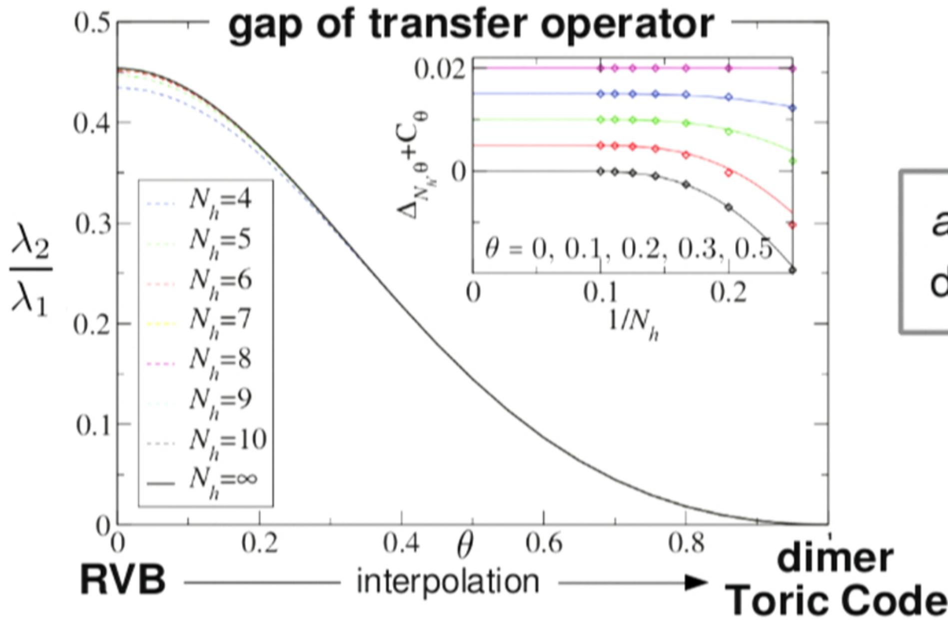
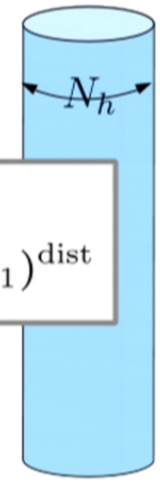


Numerical result: RVB is topological spin liquid

- All correlations are bounded by **gap of transfer operator**!
- Lanczos **exact diagonalization** of transfer operator:







all correlations decays as $(\lambda_2/\lambda_1)^{\text{dist}}$

Numerical result: RVB is topological spin liquid

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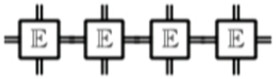
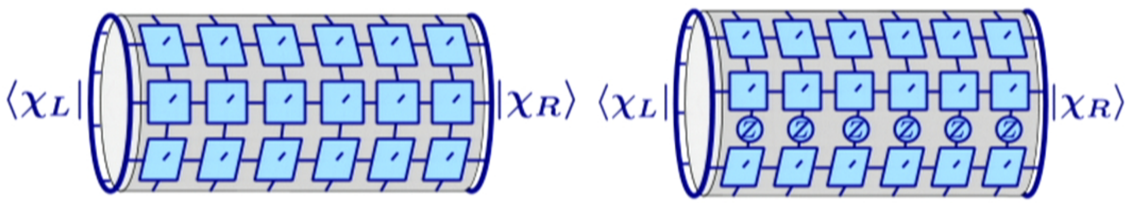
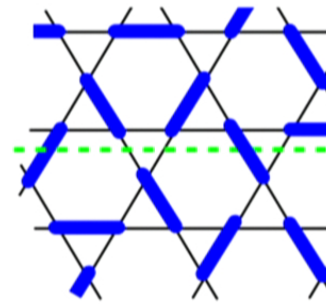


Ground state manifold with PEPS

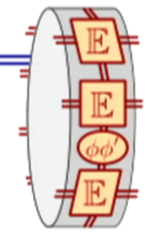
- Different topological ground states distinguished by:





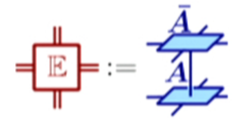
  - **parity** of left/right **boundary condition** (# of dimers)
  - **string of** Z **'s** when closing boundaries ($\leftrightarrow$ flux inside torus)

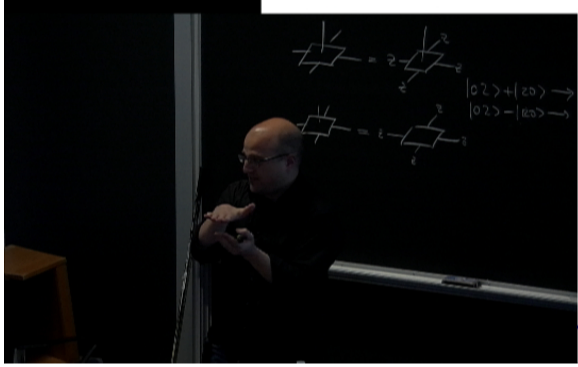
Transfer operator for all sectors



- Symmetry of PEPS tensor / different ground states $|\Psi_\alpha\rangle$:

  Transfer operator has **16 blocks** (corresp. to $\langle\Psi_\alpha|\Psi_\beta\rangle$)





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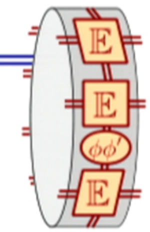

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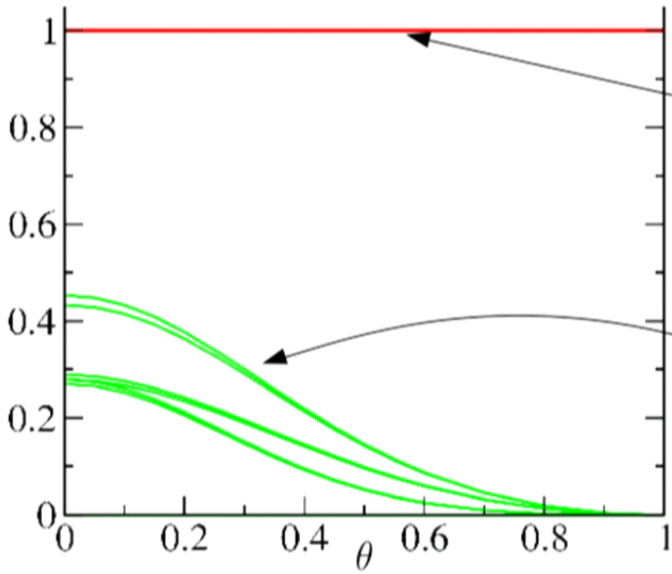
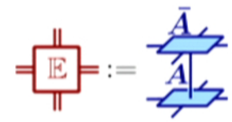
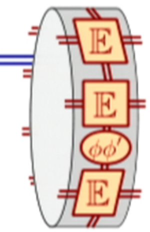


largest eigenvalue $\gamma_{\alpha\beta}$ of diagonal blocks ($\alpha = \beta$)
$\rightarrow$ splitting vanishes exponentially in N_v



largest eigenvalue of $\gamma_{\alpha\beta}$ off-diagonal blocks ($\alpha \neq \beta$)

Transfer operator for all sectors



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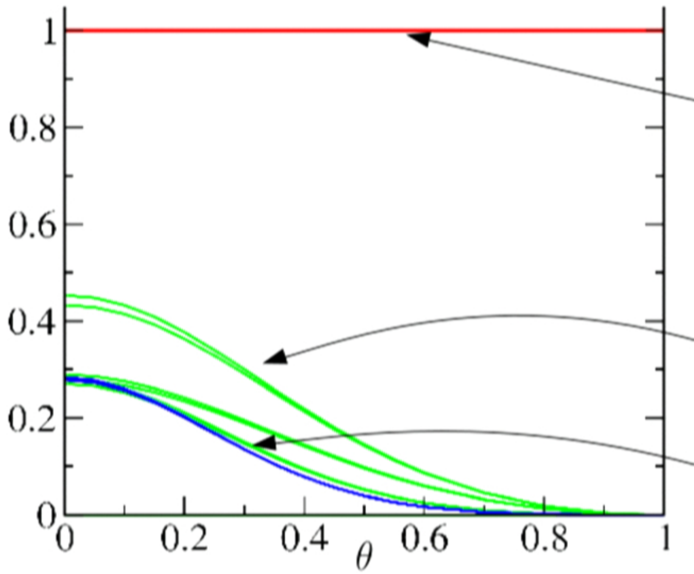

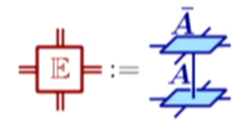

largest eigenvalue $\gamma_{\alpha\beta}$ of diagonal blocks ($\alpha = \beta$)
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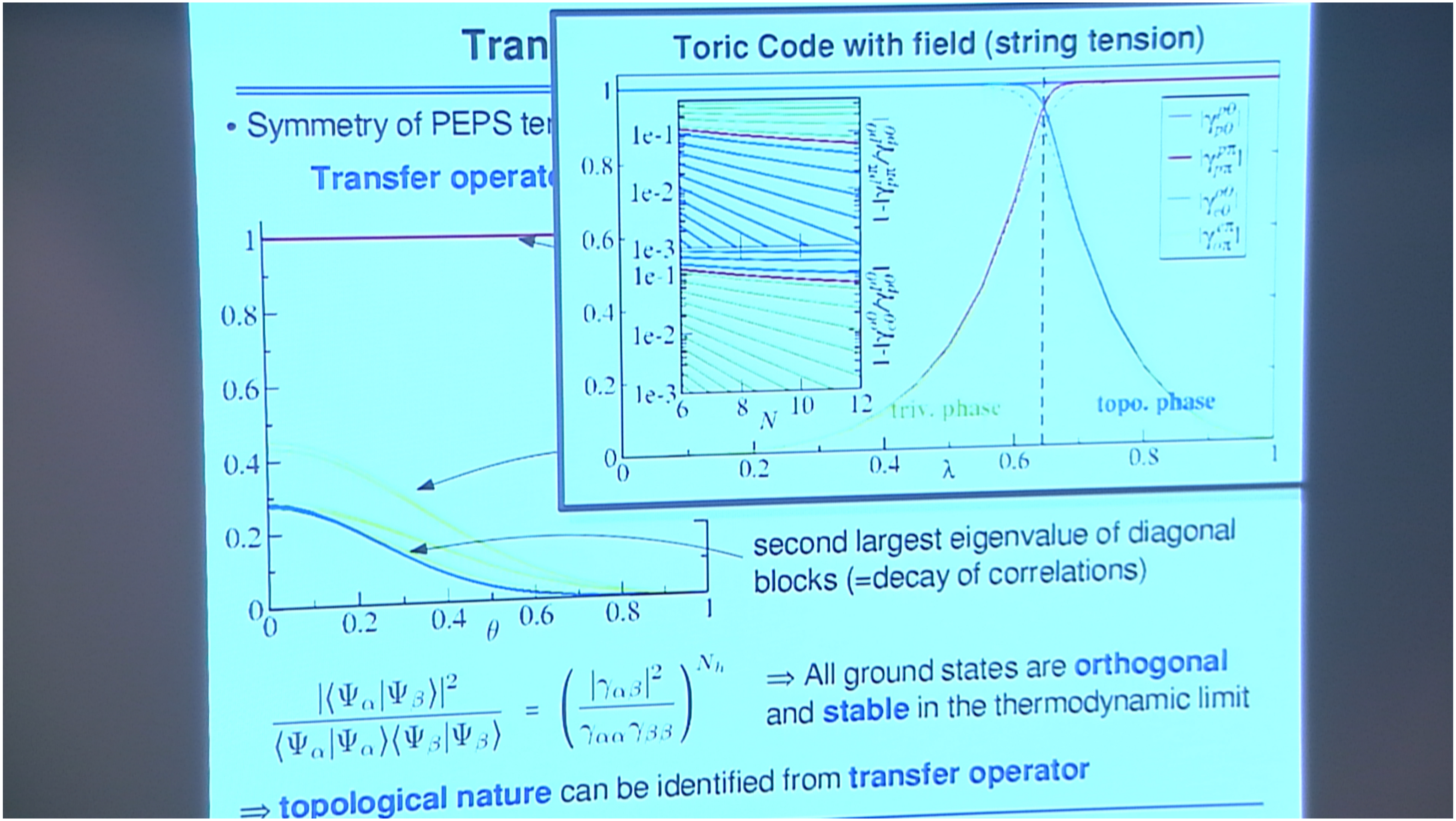
largest eigenvalue of $\gamma_{\alpha\beta}$ off-diagonal blocks ($\alpha \neq \beta$)

second largest eigenvalue of diagonal blocks (=decay of correlations)

$$\frac{|\langle\Psi_\alpha|\Psi_\beta\rangle|^2}{\langle\Psi_\alpha|\Psi_\alpha\rangle\langle\Psi_\beta|\Psi_\beta\rangle} = \left(\frac{|\gamma_{\alpha\beta}|^2}{\gamma_{\alpha\alpha}\gamma_{\beta\beta}}\right)^{N_h}$$

$\Rightarrow$ All ground states are **orthogonal** and **stable** in the thermodynamic limit


Tran
Toric Code with field (string tension)
• Symmetry of PEPS te
Transfer operat
triv. phase
topo. phase
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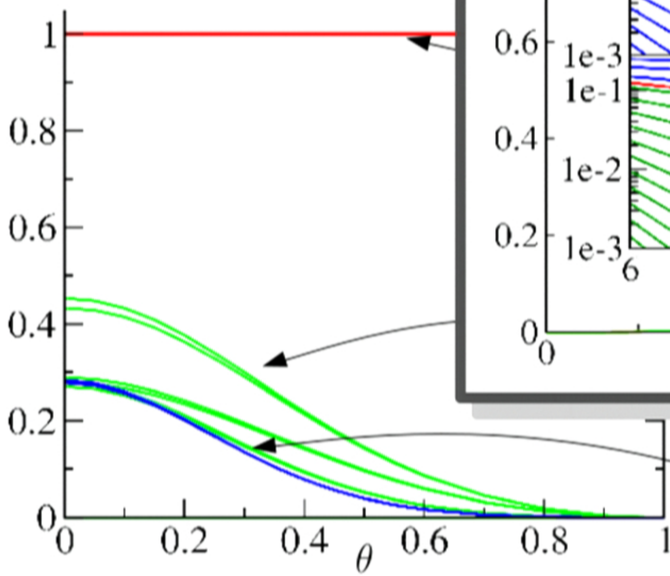
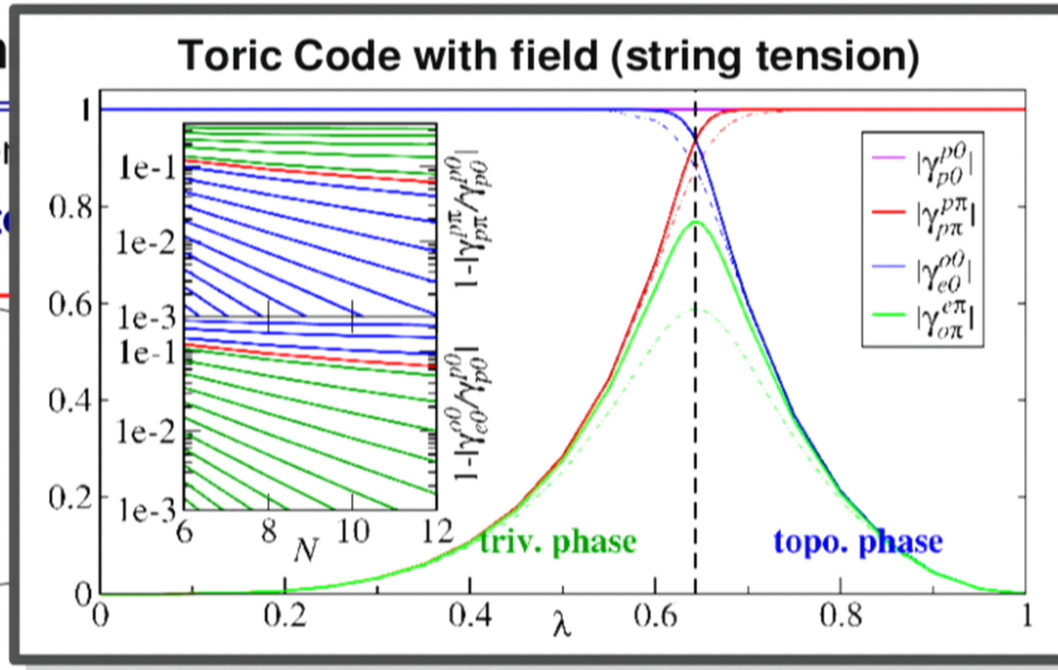
⇒ All ground states are **orthogonal** and **stable** in the thermodynamic limit
⇒ **topological nature** can be identified from **transfer operator**

Tran

- Symmetry of PEPS ter

Transfer operat





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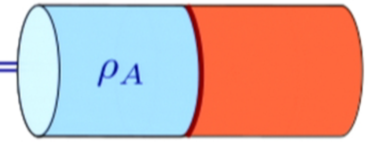
Entanglement spectrum



- Bipartition $\rightarrow$ entanglement spectrum ρ_A

  $\Rightarrow$ **entanglement Hamiltonian** H_{ent}: $\mathrm{spec}(e^{-H_{\mathrm{ent}}}) = \mathrm{spec}(\rho_{\mathrm{A}})$

- PEPS $\rightarrow$ **natural 1D structure** of entanglemer

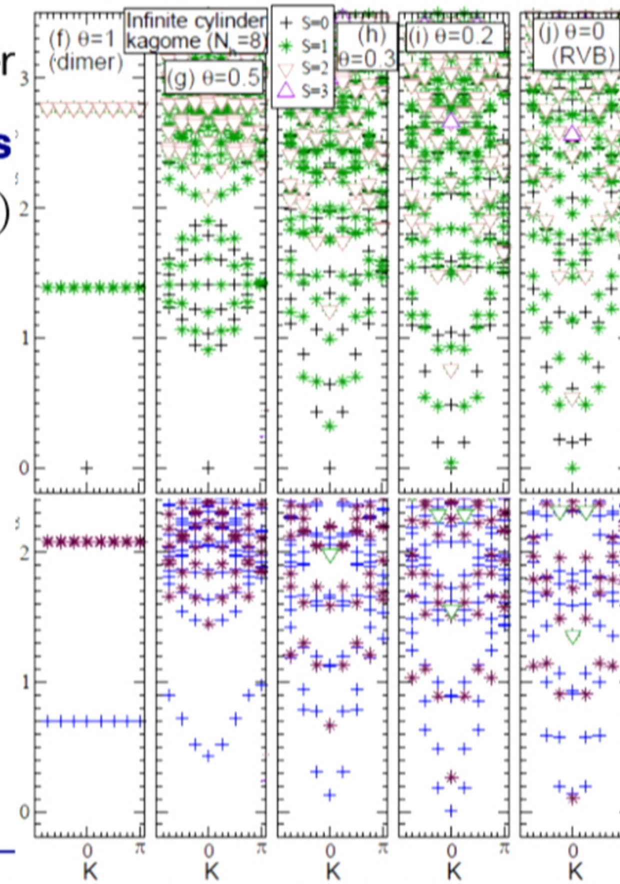
- H_{ent} inherits topological & on-site **symmetries**

  - topological symmetry: $Z = \mathrm{diag}(1, 1, -1)$
  - local SU(2) symmetry: $\frac{1}{2} \oplus 0$

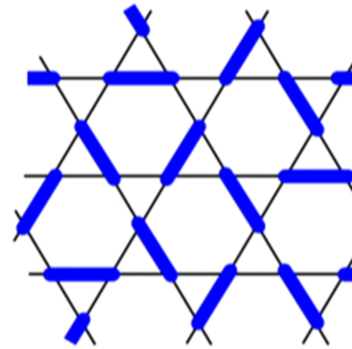
  $\rightarrow H_{\mathrm{ent}}$ has **t-J-model type structure**

- numerical study:

  - ***H_{ent}*** is **(bosonic) t-J-model**:
    dominant NN hopping,
    smaller Heisenberg & repulsion
  - Dimer $\leftrightarrow$ RVB interpolation:
    entanglement Hamiltonian
    becomes gapless at RVB point (?)



Summary



- kagome RVB: ground state of **parent Hamiltonian**
  with **topologically degenerate ground space**
- tool: $\mathbb{Z}_2$**-injectivity** $\Leftrightarrow$ invertible **mapping to Toric Code**

- **Numerical study**: All correlations decay exponentially
  $\Rightarrow$ **RVB is spin liquid** (= has no long-range order),
  in the same phase as the Toric Code
- **Symmetries** in PEPS tensor $\Rightarrow$ block-diagonal transfer operator
  Spectral structure allows to identify **topological nature**

- Tensor networks: **Symmetry structure** of tensor &
  numerical study of **transfer operator**
  allows to **identify topological order**