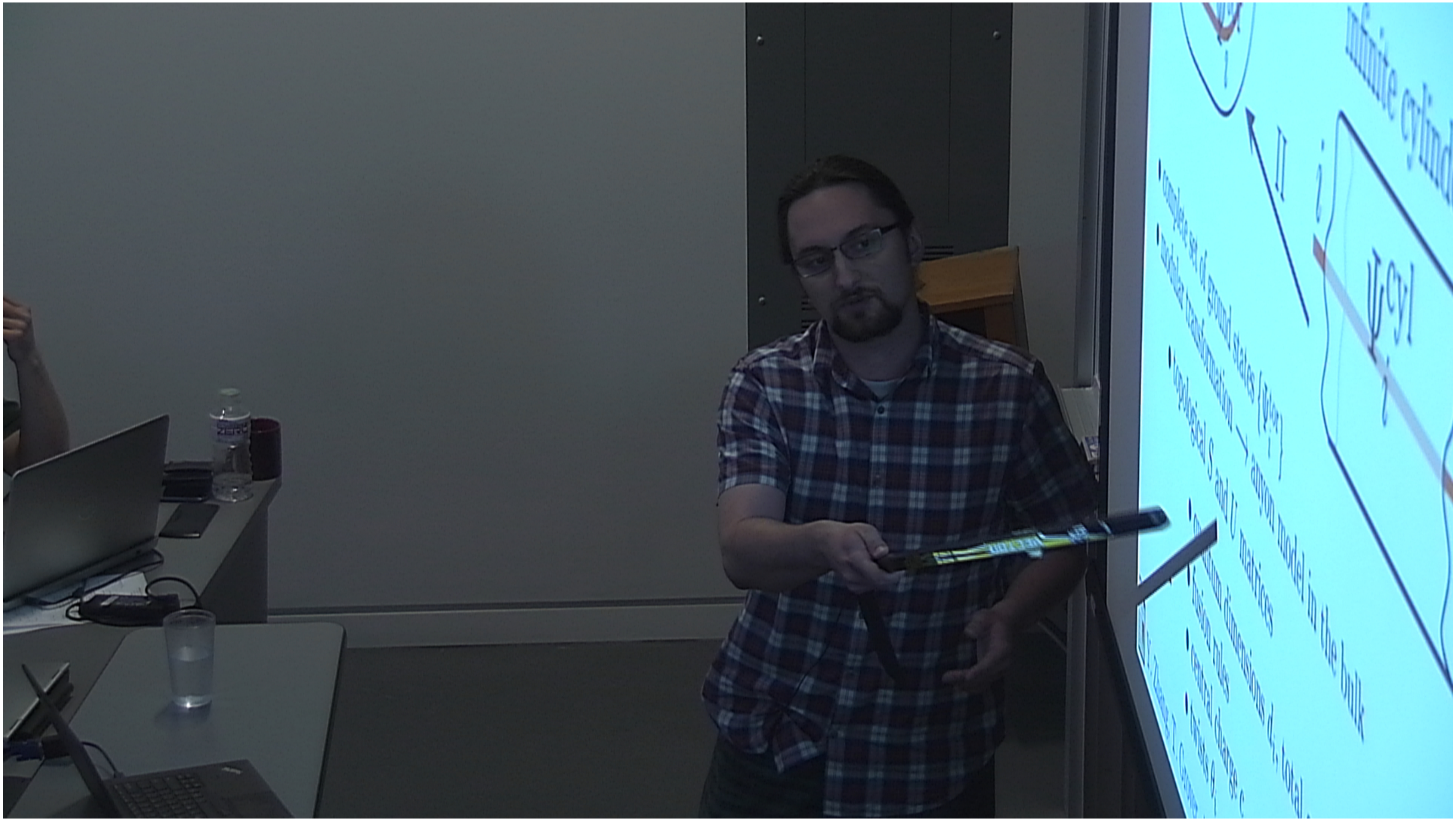
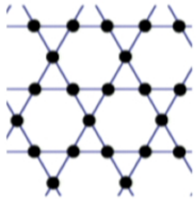
Title: Characterizing topological order form a microscopic lattice Hamiltonian

Date: May 09, 2013 11:45 AM

URL: http://pirsa.org/13050042

Abstract: In this talk I will show how to obtain a detailed characterization of the emergent topological order starting from microscopic Hamiltonian on a two dimensional lattice. A key step is to obtain a tensor network representation for a complete set of ground states of the Hamiltonian, ï¬•rst on an inï¬•nite cylinder and then on a ï¬•nite torus. As an application of the method I will study lattice Hamiltonians that give rise to selected anyon models, namely chiral semion, Ising as well as Z_3 and Z_5 models.

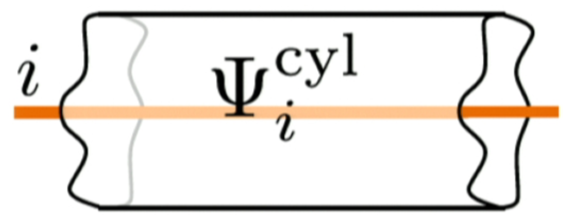

modular transformation → anyon model in the bulk
topological S and U matrices
fusion rules



microscopic lattice
Hamiltonian
$\mathcal{H}$

I

infinite cylinder



- complete set of ground states $\{\Psi_i^{\mathrm{cyl}}\}$
- complete list of anyon types
- entanglement spectrum $\longrightarrow$ edge CFT

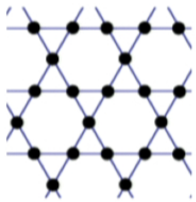
T-code: $i = \mathbb{I}, e, m, \varepsilon$

  - conformal dimensions of primary fields
  - conformal towers in each sector

H. Li, F.D.M. Haldane, **PRL 2008**

X.-L. Qi, H. Katsura, A.W.W. Ludwig , **PRL 2012**

X.-G. Wen, **Int. J. Mod. Phys. 1990**



microscopic lattice
Hamiltonian
$\mathcal{H}$

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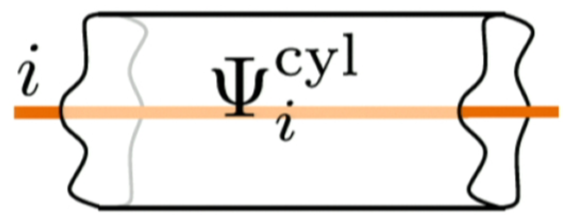

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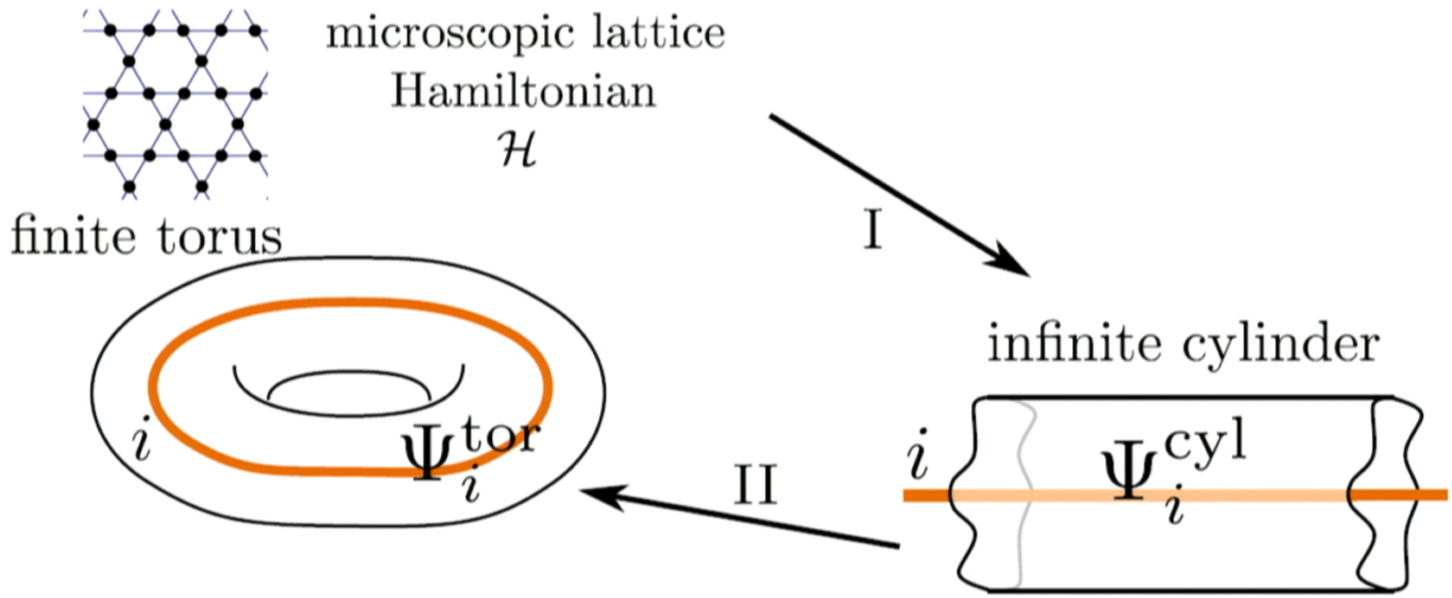
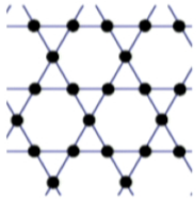
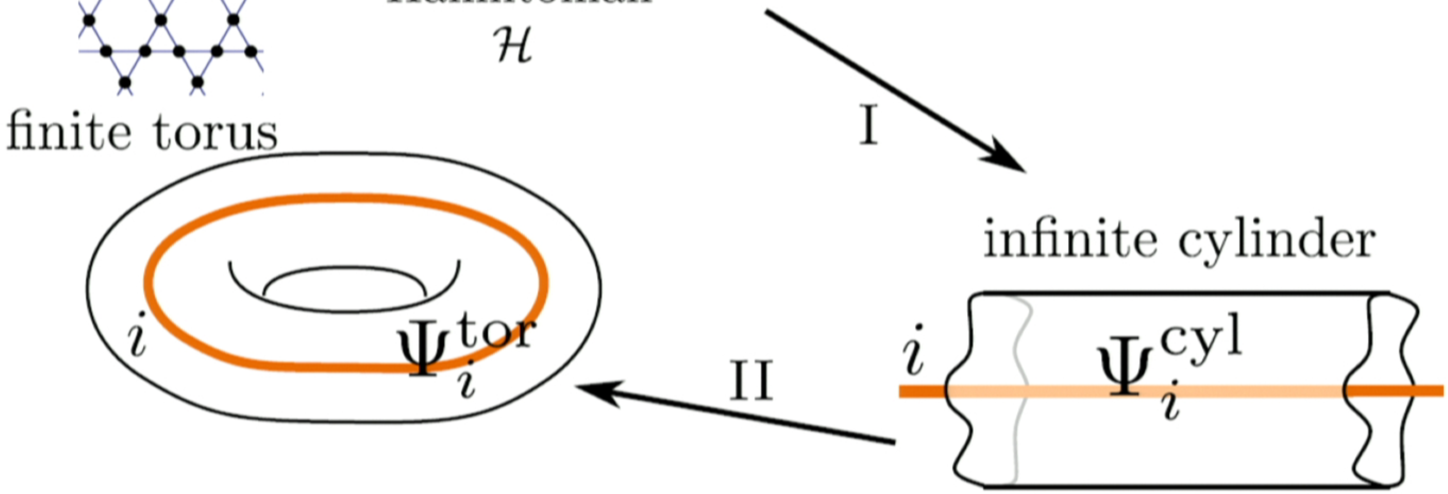


- complete set of ground states $\{\Psi_i^{\mathrm{tor}}\}$
- modular transformation $\longrightarrow$ anyon model in the bulk
  - topological S and U matrices
    - quantum dimensions d_i, total quantum dimension D
    - fusion rules
    - central charge c
    - twists θ_i

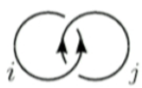
Y. Zhang, T. Grover, A. Turner, M. Oshikawa, A. Vishnavath, **PRB 2012**



microscopic lattice
Hamiltonian
$\mathcal{H}$



- S matrix

$S_{ij} = \frac{1}{D}$ 

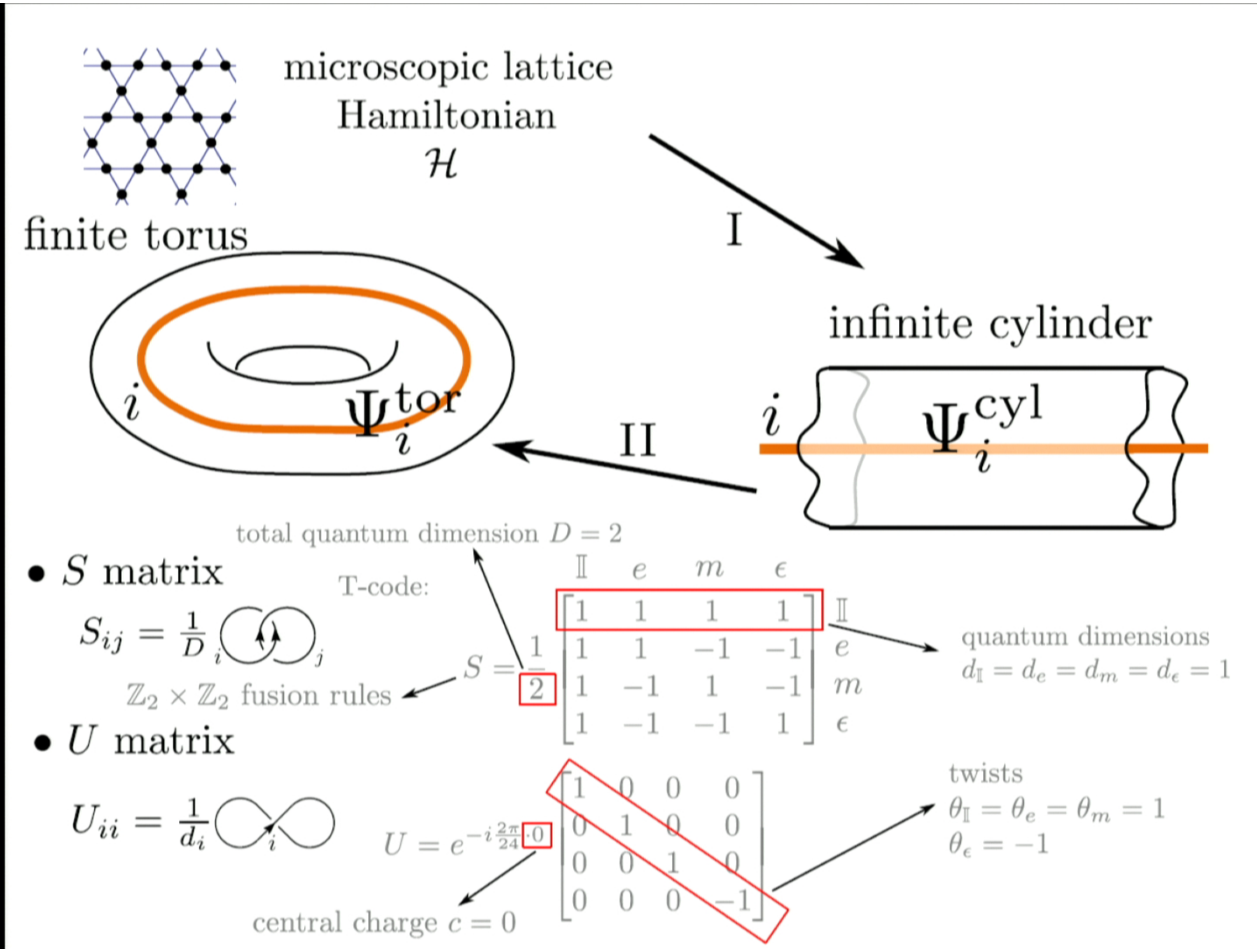
T-code:

$$S = \frac{1}{2}\begin{array}{c|cccc|c} & \mathbb{I} & e & m & \epsilon & \\ \hline & 1 & 1 & 1 & 1 & \mathbb{I} \\ & 1 & 1 & -1 & -1 & e \\ & 1 & -1 & 1 & -1 & m \\ & 1 & -1 & -1 & 1 & \epsilon \end{array}$$

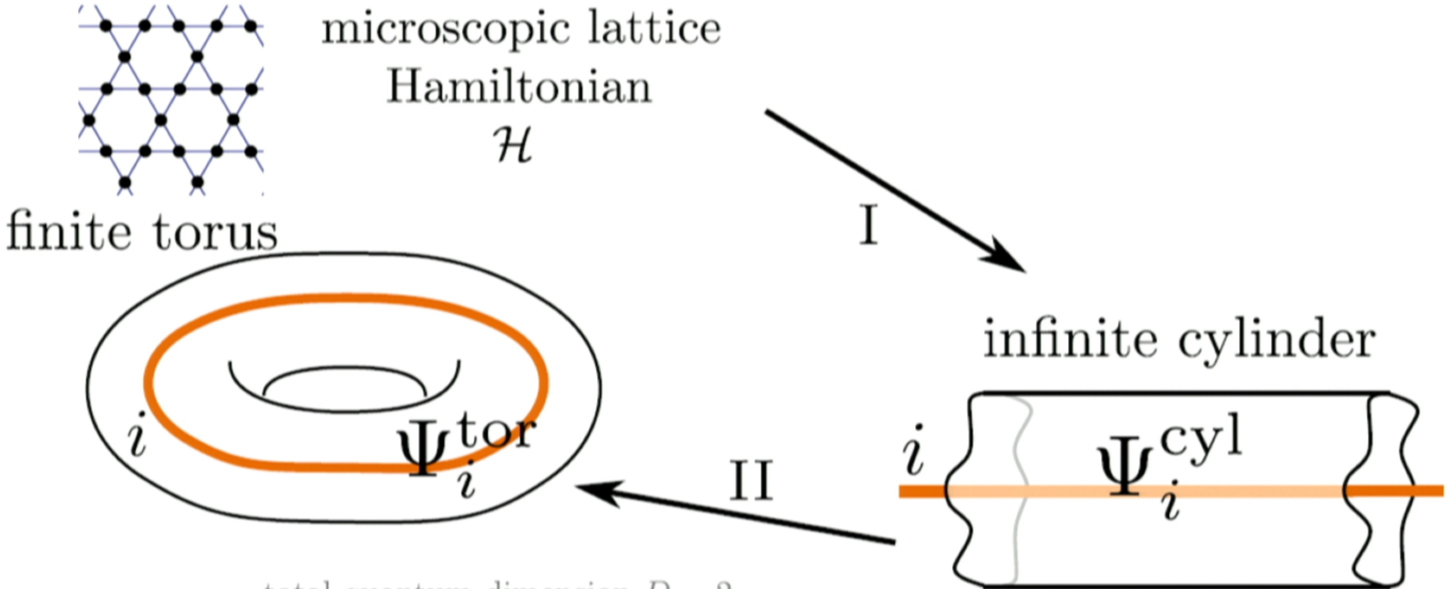
- U matrix

$U_{ii} = \frac{1}{d_i}$ 

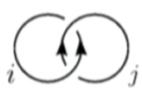
$$U = e^{-i\frac{2\pi}{24}\cdot 0}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$


microscopic lattice
Hamiltonian
$\mathcal{H}$
finite torus
I
infinite cylinder
i
Ψ_i^{tor}
II
i
Ψ_i^{cyl}
total quantum dimension $D = 2$
• S matrix
T-code:
$S_{ij} = \frac{1}{D}$
$\mathbb{Z}_2 \times \mathbb{Z}_2$ fusion rules
$S = \frac{1}{\boxed{2}} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$
$\mathbb{I}$ e m ϵ
quantum dimensions
$d_{\mathbb{I}} = d_e = d_m = d_\epsilon = 1$
• U matrix
$U_{ii} = \frac{1}{d_i}$
$U = e^{-i\frac{2\pi}{24}\boxed{\cdot 0}} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$
twists
$\theta_{\mathbb{I}} = \theta_e = \theta_m = 1$
$\theta_\epsilon = -1$
central charge $c = 0$

Old Navy



- S matrix

$S_{ij} = \frac{1}{D}$ 

- U matrix

$U_{ii} = \frac{1}{d_i}$ 

T-code:

total quantum dimension $D = 2$

$$S = \frac{1}{\boxed{2}} \begin{array}{c} \\ \\ \\ \\ \\ \end{array}\begin{array}{cc} \begin{array}{cccc} \mathbb{I} & e & m & \epsilon \end{array} & \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} & \begin{array}{c} \mathbb{I} \\ e \\ m \\ \epsilon \end{array} \end{array}$$

$\mathbb{Z}_2 \times \mathbb{Z}_2$ fusion rules

quantum dimensions
$d_{\mathbb{I}} = d_e = d_m = d_\epsilon = 1$

$$U = e^{-i\frac{2\pi}{24}\boxed{\cdot 0}} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

twists
$\theta_{\mathbb{I}} = \theta_e = \theta_m = 1$
$\theta_\epsilon = -1$

central charge $c = 0$

Lattice models

- Wen
- Haldane HC
- fermA
- fermB
- Kitaev HC

- Heisenberg KG

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- Haldane HC
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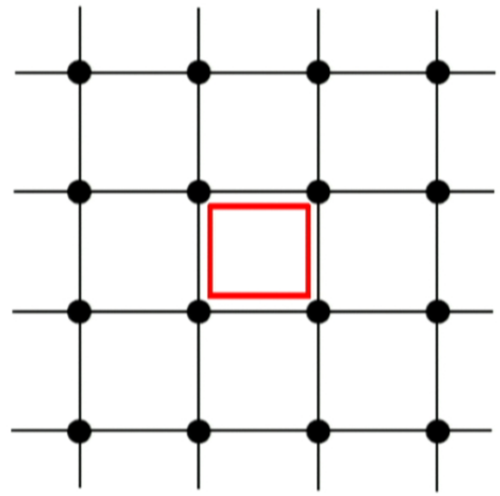
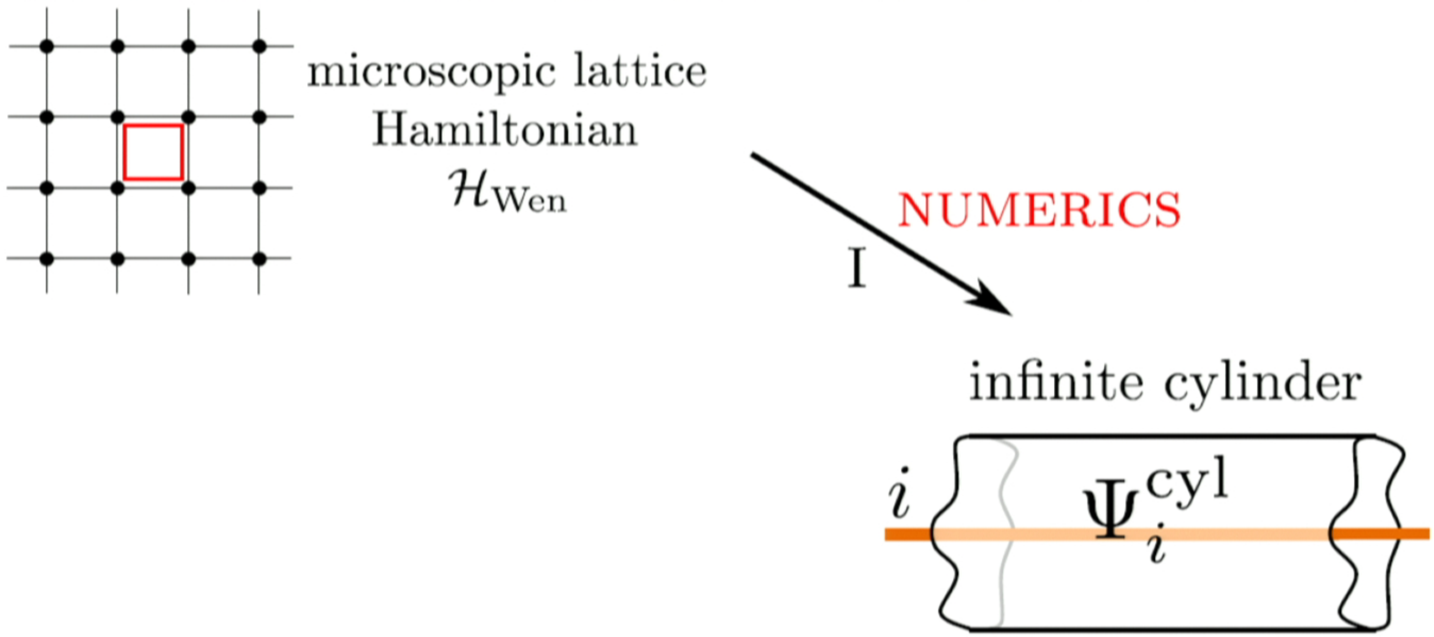
Wen's plaquette model



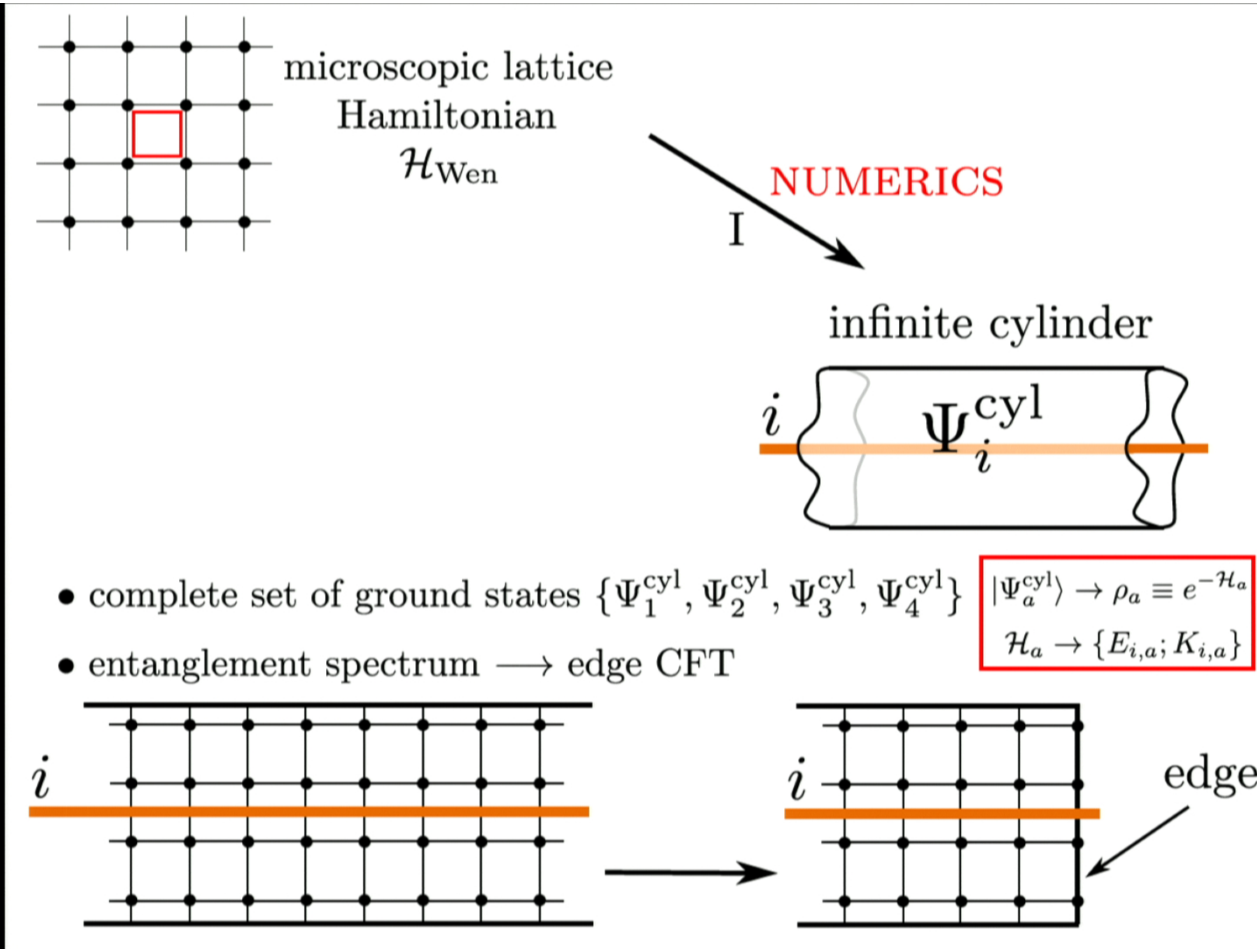
$$\mathcal{H}_{\text{Wen}} = -\sum_p \begin{array}{ccc} Z & & X \\ & \square & \\ X & & Z \end{array} - \sum_r (h_x X_r + h_y Y_r + h_z Z_r)$$

$h_x = 0.01,\ h_y = 0.015,\ h_z = 0.02$

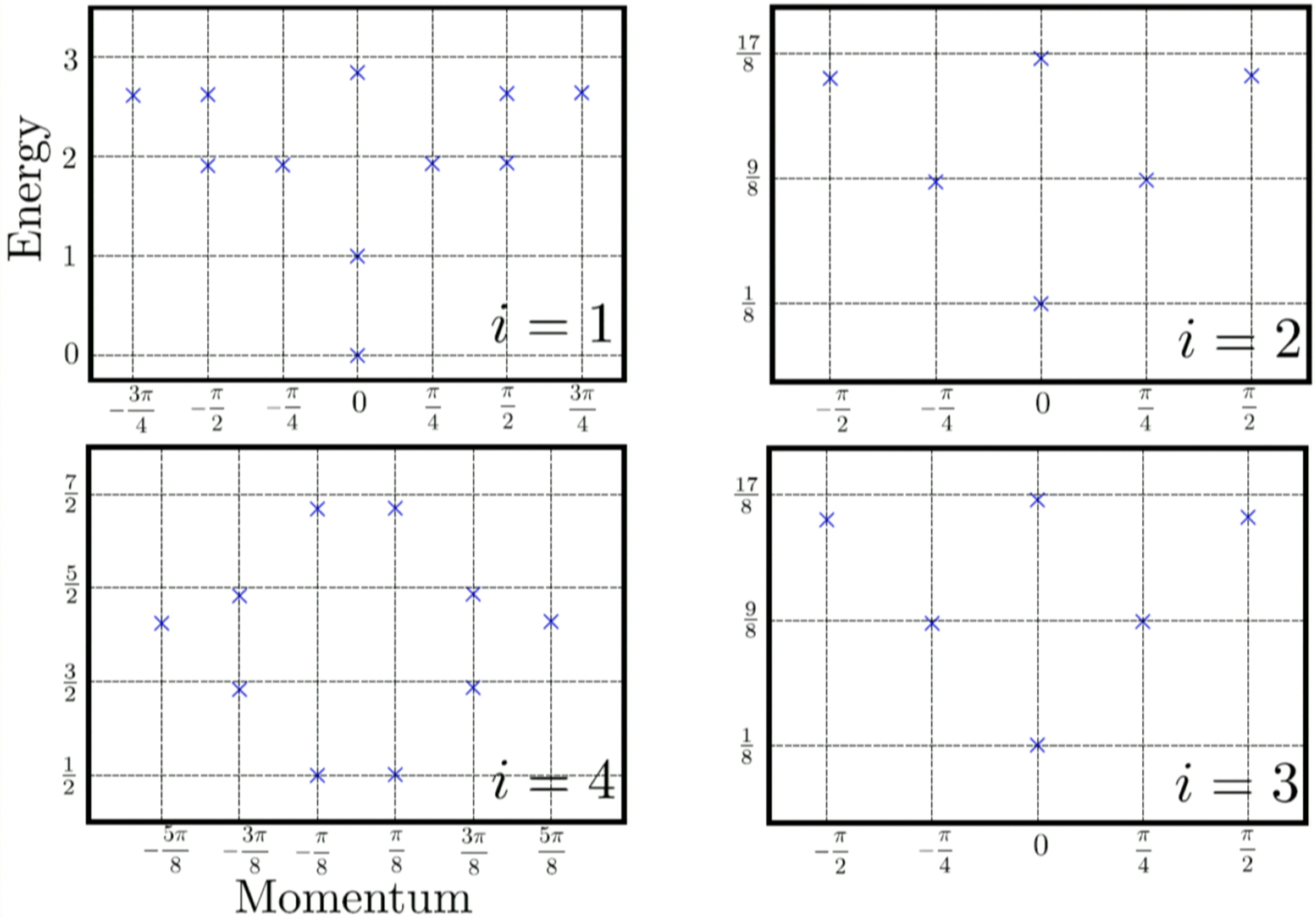
X.-G. Wen, **PRL 2003**



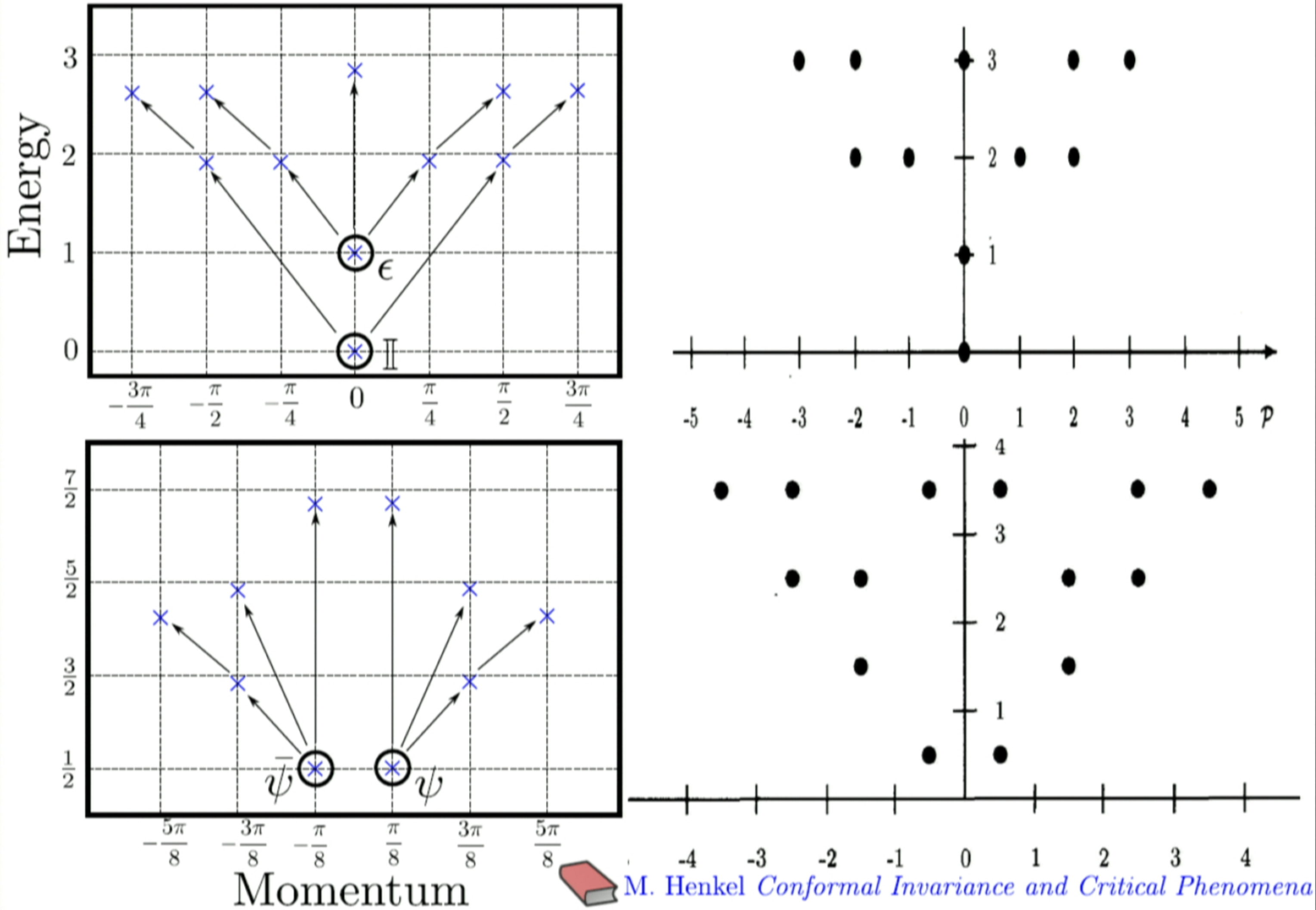
- complete set of ground states $\{\Psi_1^{\mathrm{cyl}}, \Psi_2^{\mathrm{cyl}}, \Psi_3^{\mathrm{cyl}}, \Psi_4^{\mathrm{cyl}}\}$


microscopic lattice
Hamiltonian
$\mathcal{H}_{\text{Wen}}$
NUMERICS
I
infinite cylinder
i
Ψ_i^{cyl}
• complete set of ground states $\{\Psi_1^{\text{cyl}}, \Psi_2^{\text{cyl}}, \Psi_3^{\text{cyl}}, \Psi_4^{\text{cyl}}\}$
• entanglement spectrum $\longrightarrow$ edge CFT
$|\Psi_a^{\text{cyl}}\rangle \to \rho_a \equiv e^{-\mathcal{H}_a}$
$\mathcal{H}_a \to \{E_{i,a}; K_{i,a}\}$
i
i
edge

$\mathcal{H}_{\mathrm{Wen}} \longrightarrow$ edge CFT

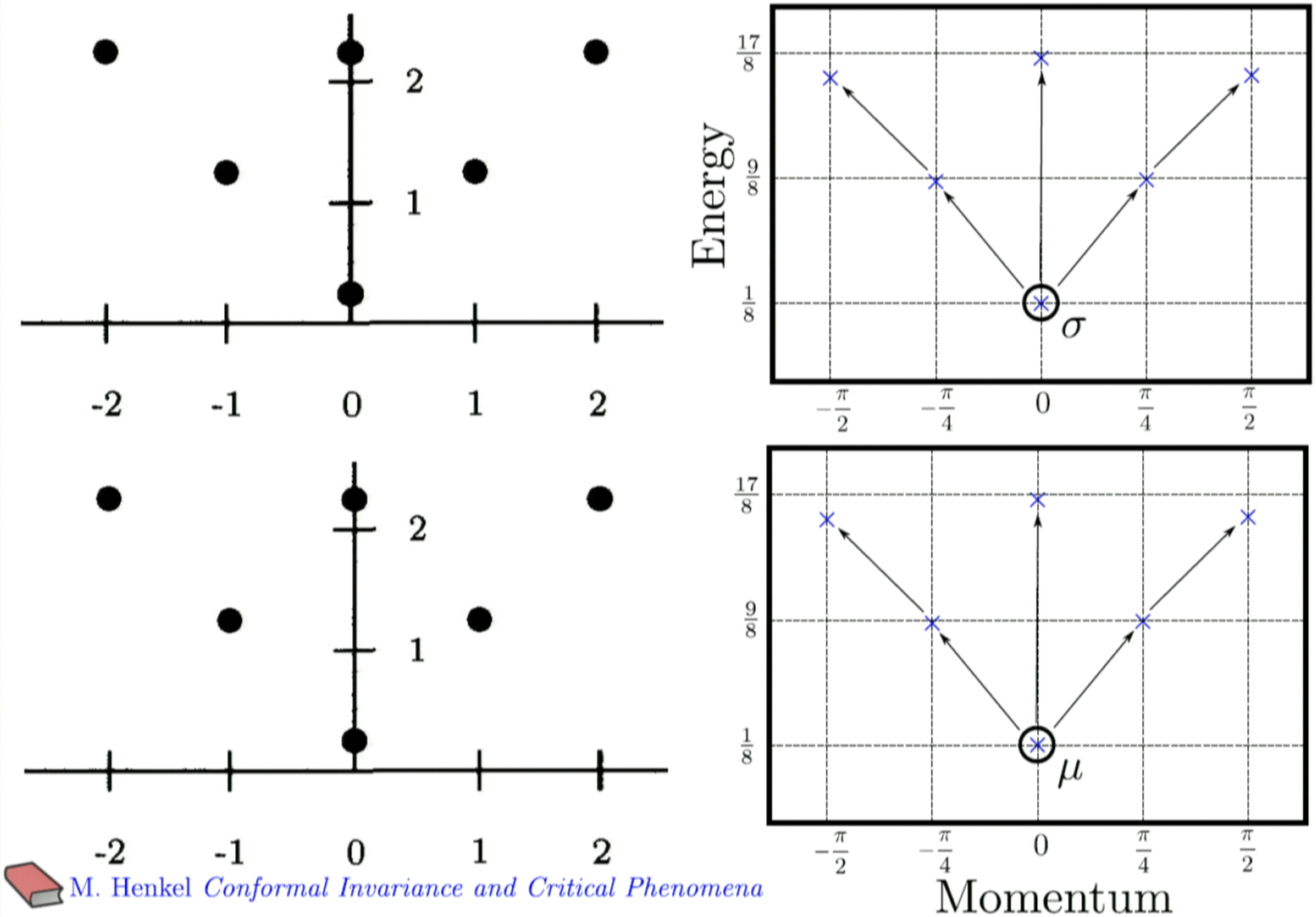


$\mathcal{H}_{\mathrm{Wen}} \longrightarrow$ edge CFT $\longrightarrow$ Ising



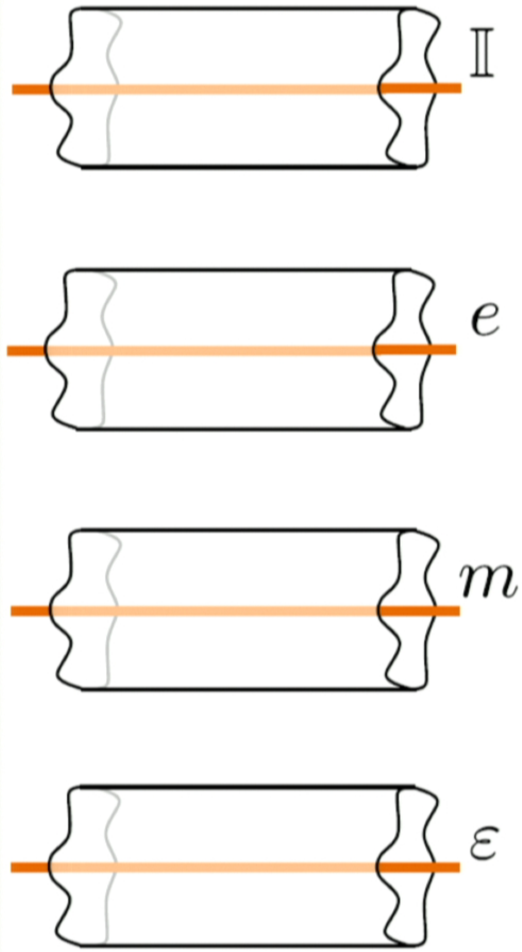
M. Henkel *Conformal Invariance and Critical Phenomena*

$\mathcal{H}_{\text{Wen}} \longrightarrow$ edge CFT $\longrightarrow$ Ising



M. Henkel *Conformal Invariance and Critical Phenomena*

Wen's plaquatte model



Ising CFT

$\mathbb{I} = (0, 0)$

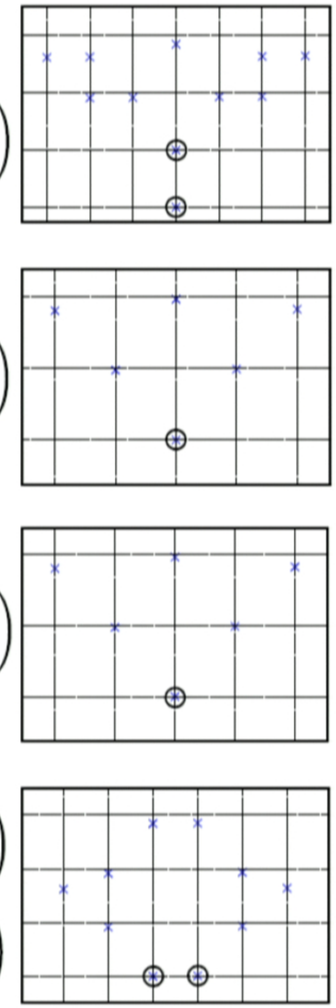
$\epsilon = \left(\frac{1}{2}, \frac{1}{2}\right)$

$\sigma = \left(\frac{1}{16}, \frac{1}{16}\right)$

$\mu = \left(\frac{1}{16}, \frac{1}{16}\right)$

$\psi = \left(\frac{1}{2}, 0\right)$

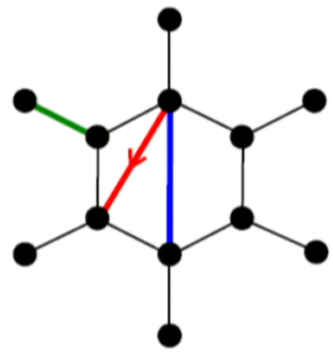
$\bar{\psi} = \left(0, \frac{1}{2}\right)$



Lattice models

- Wen
- Haldane HC
- fermA
- fermB
- Kitaev HC
- Heisenberg KG

Haldane model, honeycomb



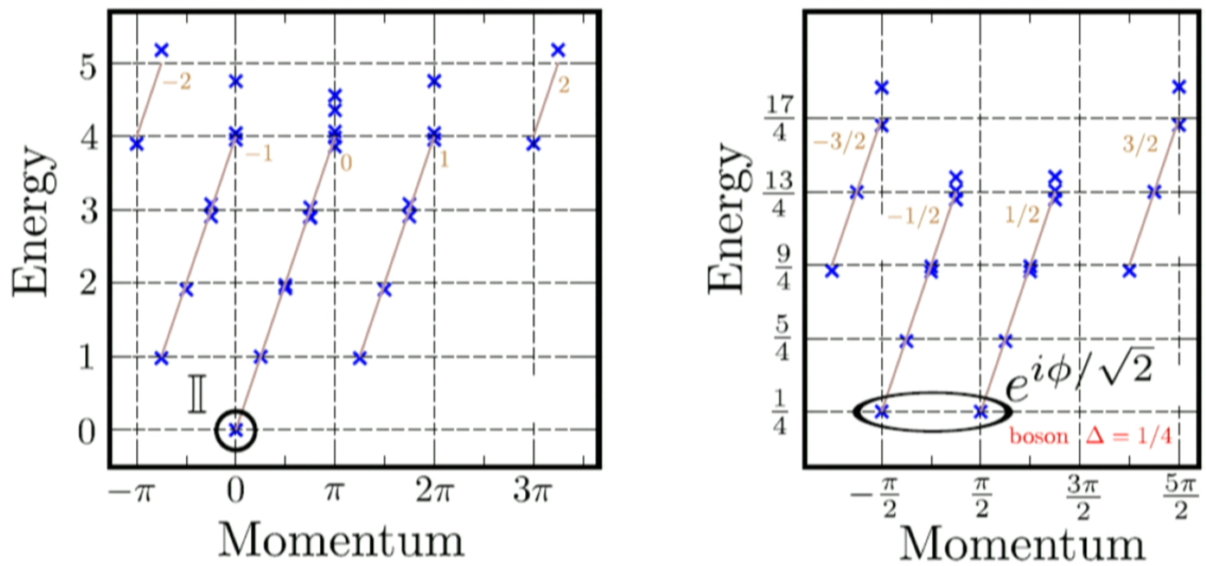
$$\mathcal{H}_{\mathrm{HaldaneHC}} = -\,t \sum_{\langle rr'\rangle} b_r^\dagger b_{r'} - t' \sum_{\langle\langle rr'\rangle\rangle} e^{i\phi_{rr'}} b_r^\dagger b_{r'} - t'' \sum_{\langle\langle\langle rr'\rangle\rangle\rangle} b_r^\dagger b_{r'} + \mathrm{H.c.}$$

$t = 1 \quad t' = 0.6 \quad \phi = 0.4\pi \quad t'' = -0.58$

F.D.M. Haldane, **PRL 1988**

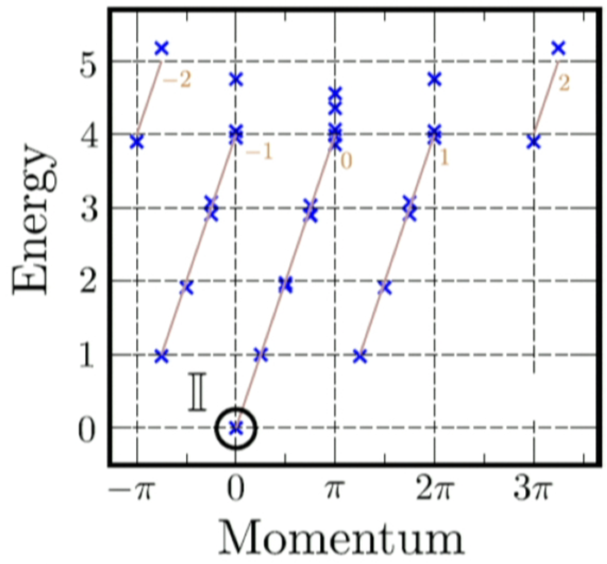
Y.-F. Wang, Z.-C. Gu, C.-D. Gong, D.N. Sheng, **PRL 2011**

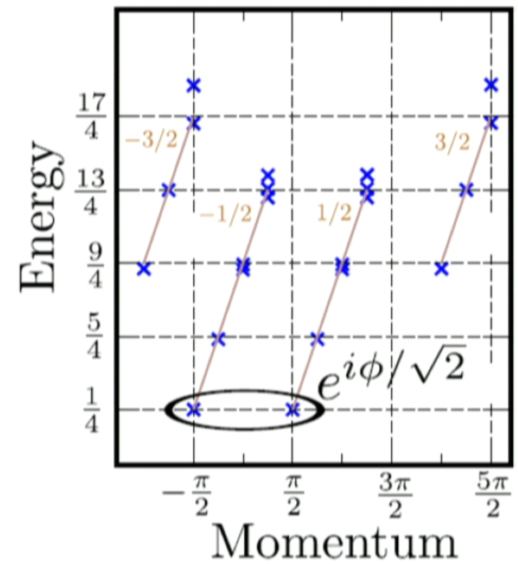
$\mathcal{H}_{\mathrm{HaldaneHC}} \longrightarrow$ edge CFT $\longrightarrow SU(2)_1$ WZW



- emergent $SU(2)$ symmetry
- primary fields
  - $\mathbb{I}$, identity
  - $e^{i\phi/\sqrt{2}}$, chiral boson
- Kac-Moody-Virasoro descendants
  - degeneracy pattern: $1, 1, 2, 3, 5, 7, \ldots$

$\mathcal{H}_{\text{HaldaneHC}} \longrightarrow$ edge CFT $\longrightarrow SU(2)_1$ WZW

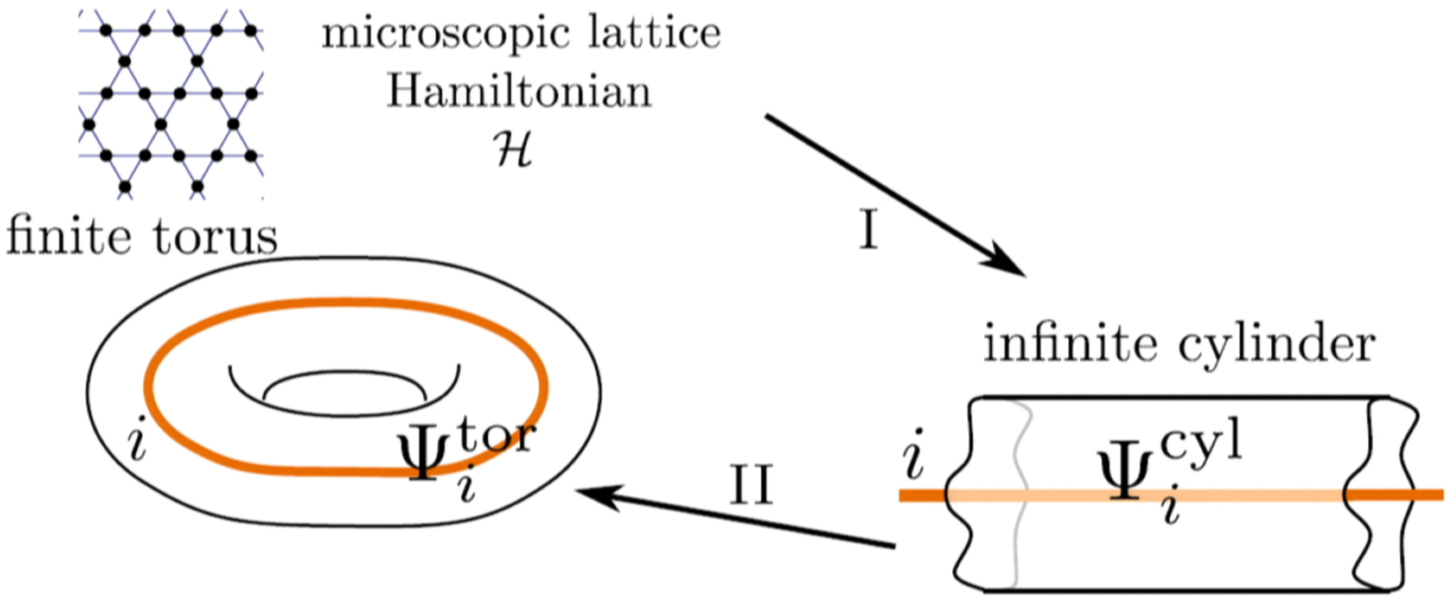




L_0	m: -2	-1	0	1	2	$su(2)$ decomposition
0			1			(0)
1		1	1	1		(2)
2		1	2	1		(2)+(0)
3		2	3	2		2(2)+(0)
4	1	3	5	3	1	(4)+2(2)+2(0)
5	1	5	7	5	1	(4)+4(2)+2(0)
6	2	7	11	7	2	2(4)+5(2)+4(0)

L_0	m: -2	-1	0	1	2	3	$su(2)$ decomposition
$\frac{1}{4}$			1	1			(1)
$\frac{5}{4}$			1	1			(1)
$\frac{9}{4}$		1	2	2	1		(3)+(1)
$\frac{13}{4}$		1	3	3	1		(3)+2(1)
$\frac{17}{4}$		2	5	5	2		2(3)+3(1)
$\frac{21}{4}$		3	7	7	3		3(3)+4(1)
$\frac{25}{4}$	1	5	11	11	5	1	(5)+4(3)+6(1)

P. Di Francesco, P. Mathieu, D. Sénéchal, *Conformal Field Theory*



- complete set of ground states $\{\Psi_i^{\text{tor}}\}$
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Y. Zhang, T. Grover, A. Turner, M. Oshikawa, A. Vishnavath, **PRB 2012**


• fermB
• Kitaev HC
• Heisenberg KG

Lattice models

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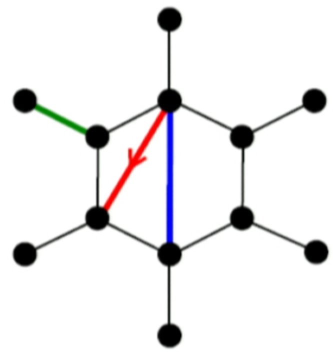
Haldane model, honeycomb

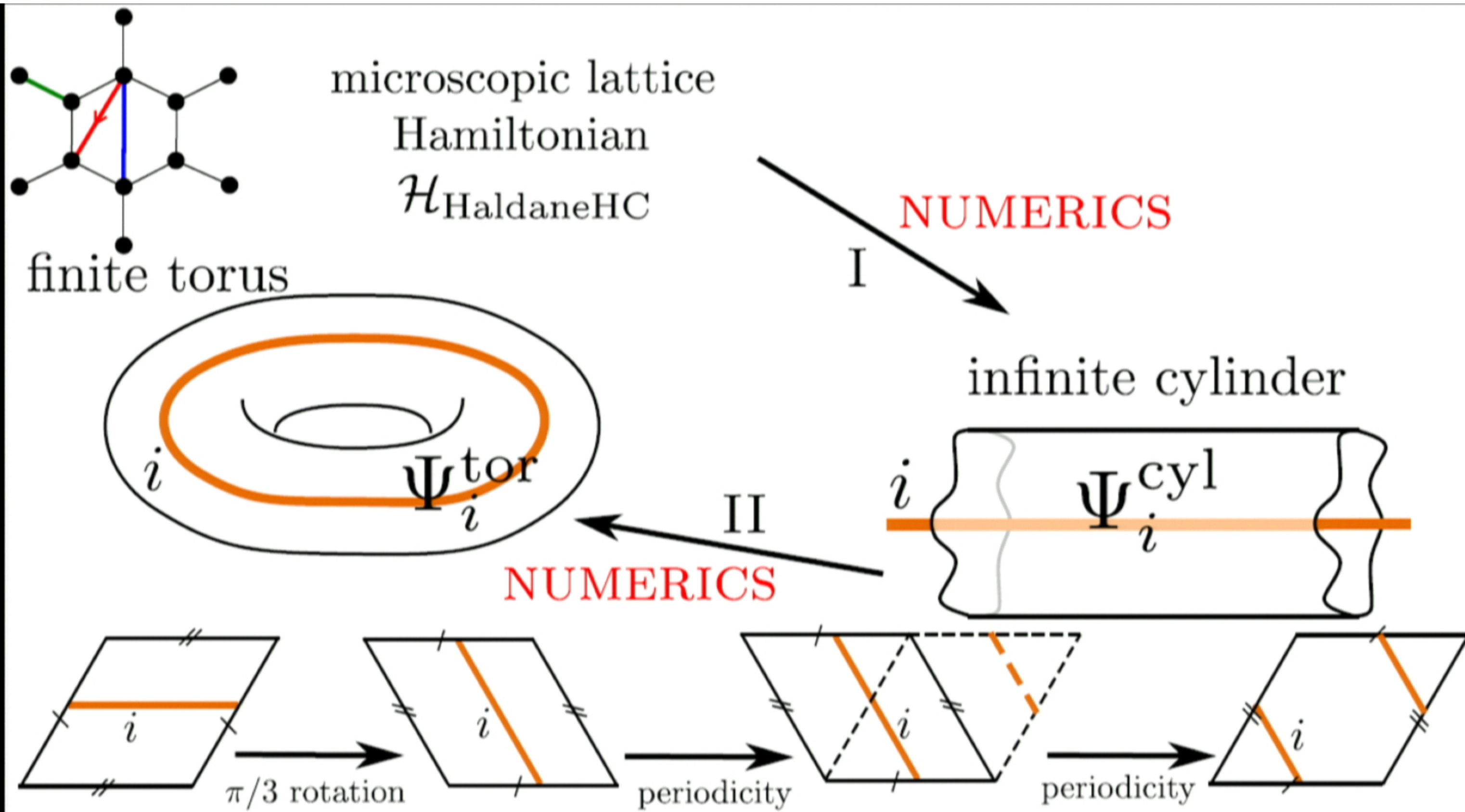


$$\mathcal{H}_{\mathrm{HaldaneHC}} = -\,t \sum_{\langle rr' \rangle} b_r^\dagger b_{r'} - t' \sum_{\langle\langle rr' \rangle\rangle} e^{i\phi_{rr'}} b_r^\dagger b_{r'} - t'' \sum_{\langle\langle\langle rr' \rangle\rangle\rangle} b_r^\dagger b_{r'} + \mathrm{H.c.}$$

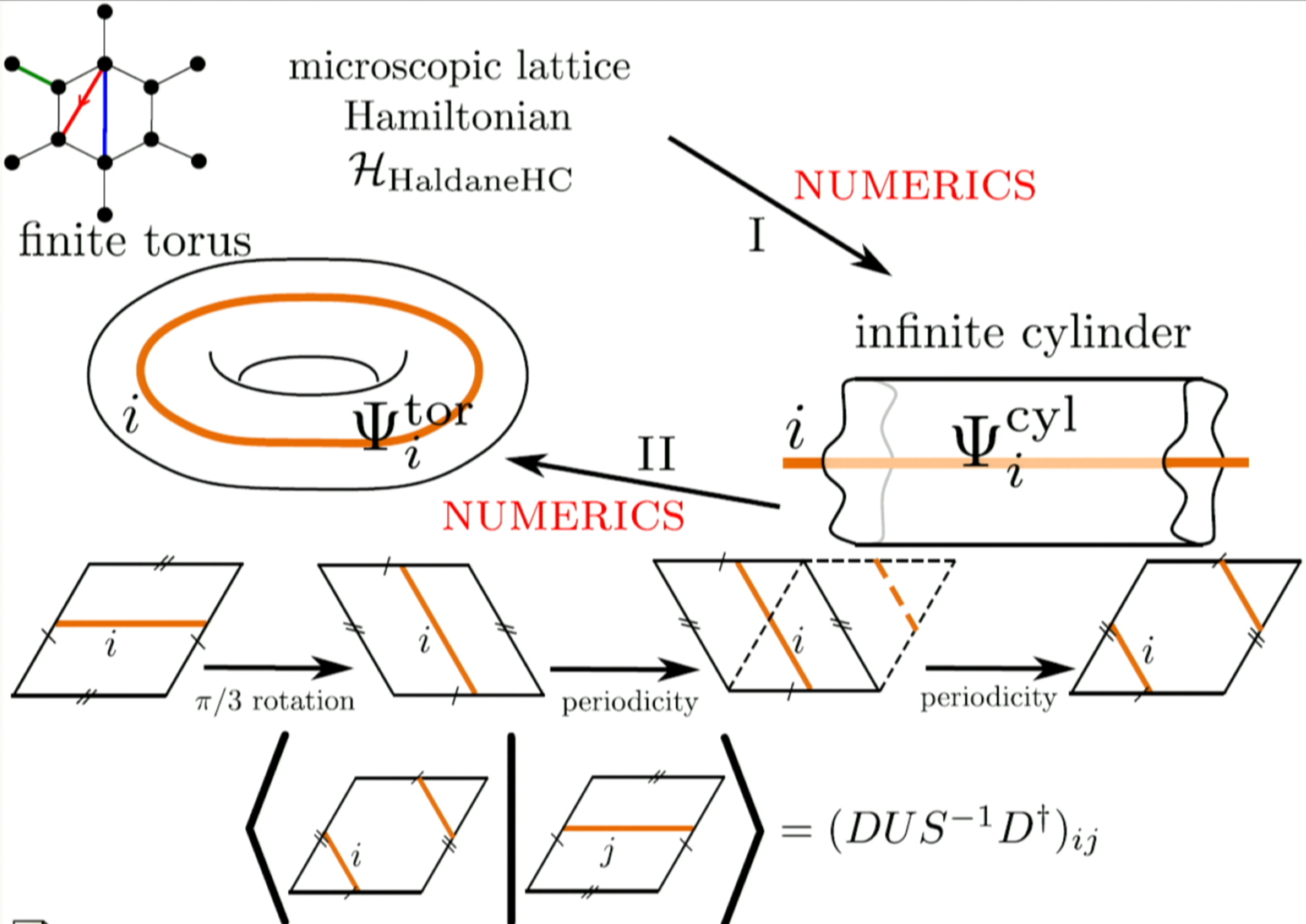
$t = 1 \quad t' = 0.6 \quad \phi = 0.4\pi \quad t'' = -0.58$

F.D.M. Haldane, **PRL 1988**

Y.-F. Wang, Z.-C. Gu, C.-D. Gong, D.N. Sheng, **PRL 2011**



microscopic lattice Hamiltonian $\mathcal{H}_{\text{HaldaneHC}}$
finite torus
NUMERICS
I
infinite cylinder
i
Ψ_i^{tor}
II
Ψ_i^{cyl}
NUMERICS
$\pi/3$ rotation
periodicity
periodicity



Y. Zhang, T. Grover, A. Turner, M. Oshikawa, A. Vishnavath, **PRB 2012**

$\mathcal{H}_{\text{HaldaneHC}} \longrightarrow$ anyon model

$$S = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad + \frac{10^{-3}}{\sqrt{2}} \begin{bmatrix} -1.4 & 0.2 \\ -1.4 & 4+4i \end{bmatrix}$$

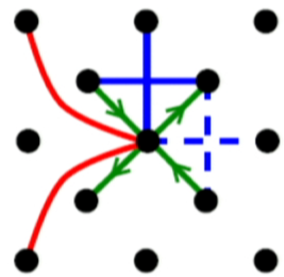
$$U = e^{-i\frac{2\pi}{24}\cdot 1} \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \qquad \times \left(e^{i\frac{2\pi}{24}0.01} \begin{bmatrix} 1 & 0 \\ 0 & e^{-i0.007} \end{bmatrix} \right)$$

Lattice models

- Wen
- Haldane HC
- fermA
- fermB
- Kitaev HC

- Heisenberg KG

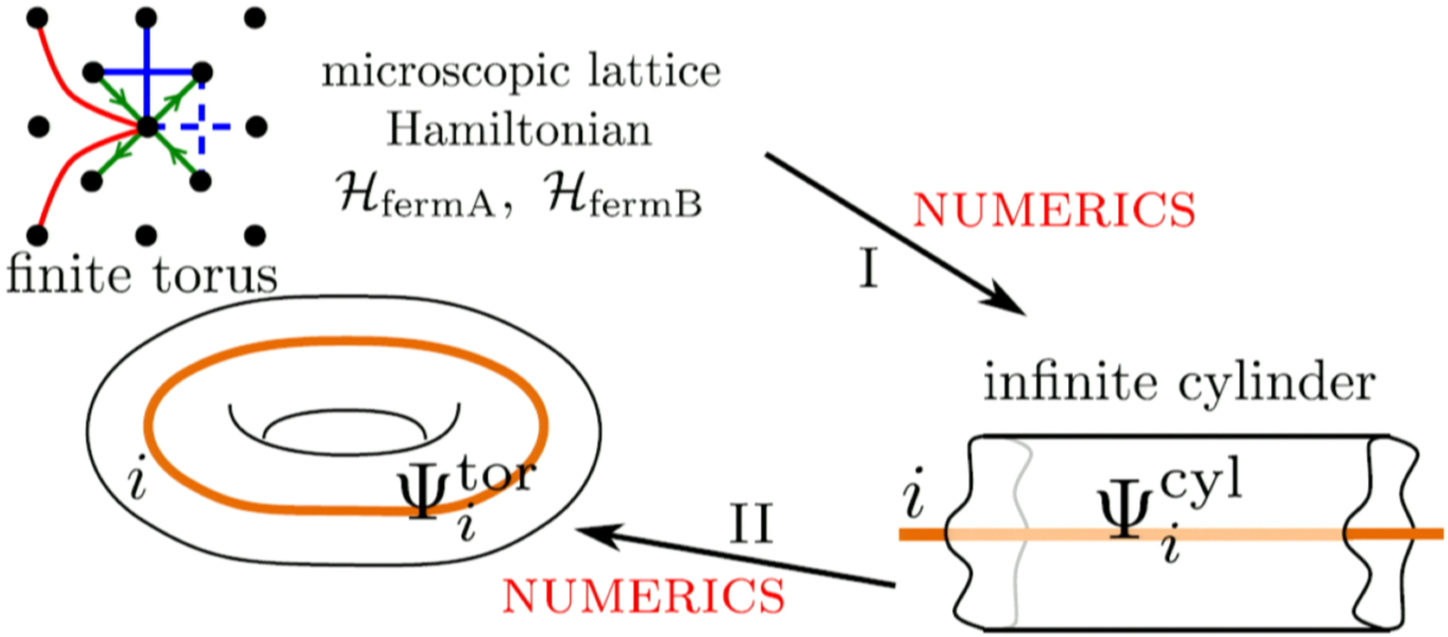
interacting fermions, checkerboard



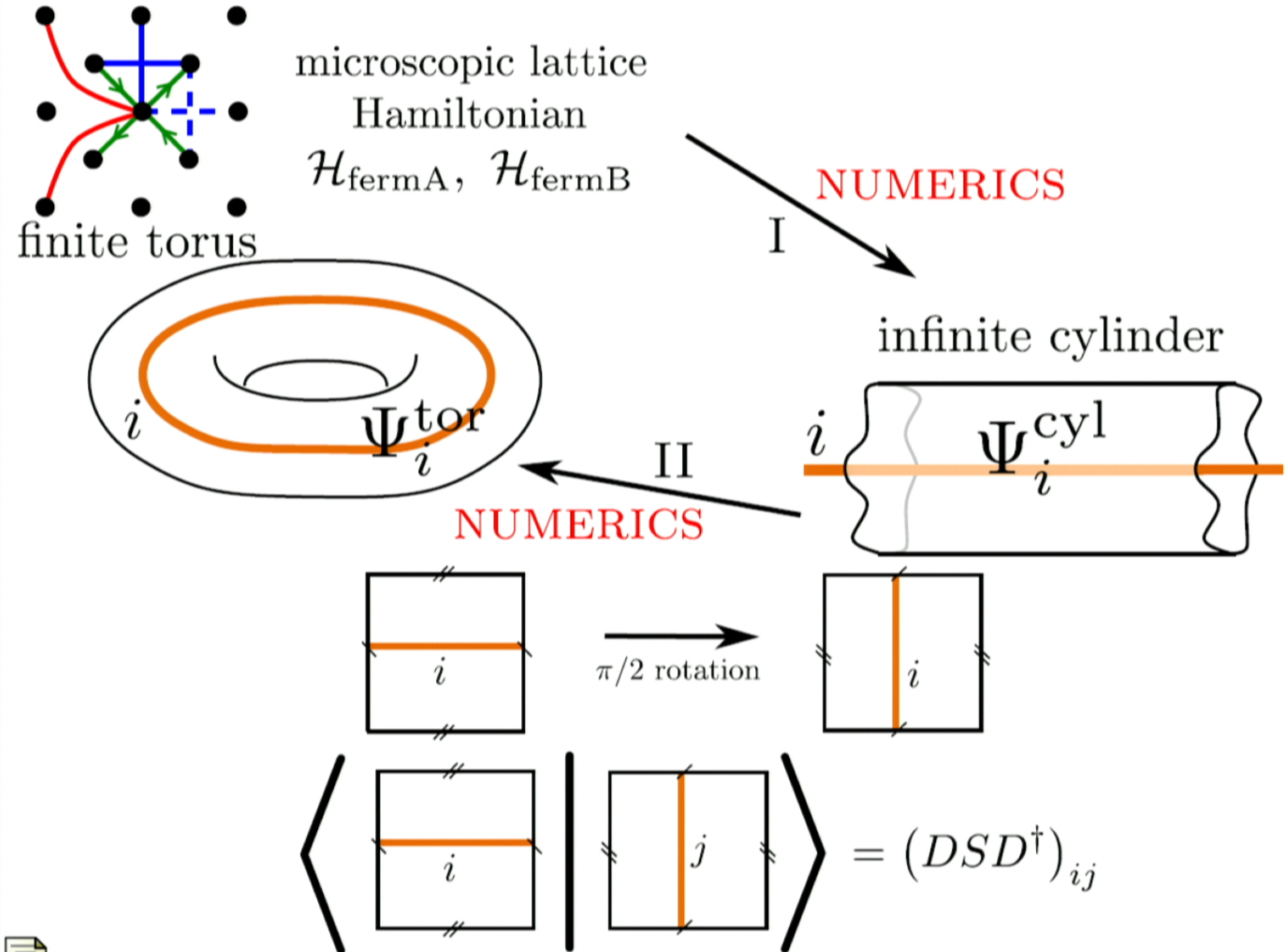
$$\mathcal{H}_{\text{fermB}} = \sum_{\langle rr' \rangle} e^{i\phi_{rr'}} c_r^\dagger c_{r'} + U \sum_{\langle rr' \rangle} n_r n_{r'} + \sum_{\langle\langle rr' \rangle\rangle} t_{rr'} c_r^\dagger c_{r'} + V \sum_{\langle\langle rr' \rangle\rangle} n_r n_{r'} + t' \sum_{\langle\langle\langle rr' \rangle\rangle\rangle} c_r^\dagger c_{r'} + \text{H.c.}$$

$U = 1 \quad V_A = 0 \; V_B = 1 \quad t_{rr'} = \pm 1/(2+\sqrt{2}) \quad t' = 1/(2+2\sqrt{2})$

D.N. Sheng, Z.-C. Gu, K. Sun, L. Sheng **Nat Commun 2011**



- complete set of ground states for $\mathcal{H}_{\text{fermA}}$: $\{\Psi_1^{\text{tor}}, \Psi_2^{\text{tor}}, \Psi_3^{\text{tor}}\}$
- complete set of ground states for $\mathcal{H}_{\text{fermB}}$: $\{\Psi_1^{\text{tor}}, \Psi_2^{\text{tor}}, \Psi_3^{\text{tor}}, \Psi_4^{\text{tor}}, \Psi_5^{\text{tor}}\}$



Y. Zhang, T. Grover, A. Turner, M. Oshikawa, A. Vishnavath, **PRB 2012**

$$\mathcal{H}_{\text{fermA}} \longrightarrow \text{anyon model} \longrightarrow \mathbb{Z}_3^{(1)}$$

quantum dimensions: $d_i = 1$

$$S = \frac{1}{\boxed{\sqrt{3}}} \begin{array}{c} \begin{array}{ccc} \mathbb{I} & \omega & \omega^2 \end{array} \\ \begin{bmatrix} \boxed{1 \quad 1 \quad 1} \\ 1 & e^{-i2\pi/3} & e^{2i\pi/3} \\ 1 & e^{2i\pi/3} & e^{-2i\pi/3} \end{bmatrix} \end{array} \begin{array}{c} \mathbb{I} \\ \omega \\ \omega^2 \end{array} + 2 \cdot 10^{-2} \times \Delta S$$

$\max |\Delta S| < 1$

total quantum dimension: $D = \sqrt{3}$

$$\mathcal{H}_{\text{fermB}} \longrightarrow \text{anyon model} \longrightarrow \mathbb{Z}_5^{(2)}$$

quantum dimensions: $d_i = 1$

$$S = \frac{1}{\boxed{\sqrt{5}}} \begin{array}{c} \begin{array}{ccccc} \mathbb{I} & \omega & \omega^2 & \omega^3 & \omega^4 \end{array} \\ \begin{bmatrix} \boxed{1 \quad 1 \quad 1 \quad 1 \quad 1} \\ 1 & e^{-2i\pi/5} & e^{-4i\pi/5} & e^{4i\pi/5} & e^{2i\pi/5} \\ 1 & e^{-4i\pi/5} & e^{2i\pi/5} & e^{-2i\pi/5} & e^{4i\pi/5} \\ 1 & e^{4i\pi/5} & e^{-2i\pi/5} & e^{2i\pi/5} & e^{-4i\pi/5} \\ 1 & e^{2i\pi/5} & e^{4i\pi/5} & e^{-4i\pi/5} & e^{-2i\pi/5} \end{bmatrix} \end{array} \begin{array}{c} \mathbb{I} \\ \omega \\ \omega^2 \\ \omega^3 \\ \omega^4 \end{array} + 4 \cdot 10^{-2} \times \Delta S$$

$\max |\Delta S| < 1$

total quantum dimension: $D = \sqrt{5}$

Lattice models

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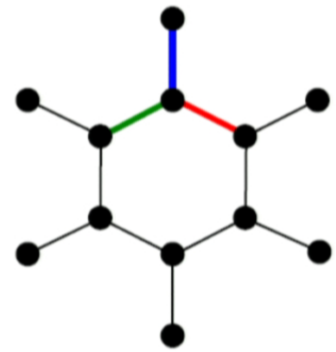
Kitaev model, honeycomb



$$\mathcal{H}_{\mathrm{KitaevHC}} = -\sum_{\langle rr' \rangle_x} X_r X_{r'} - \sum_{\langle rr' \rangle_y} Y_r Y_{r'} - \sum_{\langle rr' \rangle_z} Z_r Z_{r'} - h \sum_r (X_r + Y_r + Z_r)$$

$$h = 0.01$$

A. Kitaev, **Annals of Physics 2006**

$\mathcal{H}_{\text{KitaevHC}} \longrightarrow$ anyon model

$$S = \frac{1}{2}\begin{bmatrix} 1 & \sqrt{2} & 1 \\ 1 & 0 & -\sqrt{2} \\ 1 & -\sqrt{2} & 1 \end{bmatrix} + 5 \cdot 10^{-2} \times \begin{bmatrix} -0.4 & 0.3 & -0.2 \\ 0.2 & 0.6 & 0.2 \\ -0.8 & 1 & -0.8 \end{bmatrix}$$

$$U \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{0.487i\pi} & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$\mathcal{H}_{\text{KitaevHC}} \longrightarrow$ anyon model $\longrightarrow$ Ising

quantum dimensions: $d_{\mathbb{I}} = 1$, $d_\sigma = \sqrt{2}$, $d_\psi = 1$

$$S = \frac{1}{\boxed{2}} \begin{array}{c} \begin{array}{ccc} \mathbb{I} & \sigma & \psi \end{array} \\ \left[\begin{array}{ccc} 1 & \sqrt{2} & 1 \\ 1 & 0 & -\sqrt{2} \\ 1 & -\sqrt{2} & 1 \end{array}\right] \end{array} \begin{array}{c} \\ \mathbb{I} \\ \sigma \\ \psi \end{array} + 5 \cdot 10^{-2} \times \left[\begin{array}{ccc} -0.4 & 0.3 & -0.2 \\ 0.2 & 0.6 & 0.2 \\ -0.8 & 1 & -0.8 \end{array}\right]$$

total quantum dimension: $D = 2$

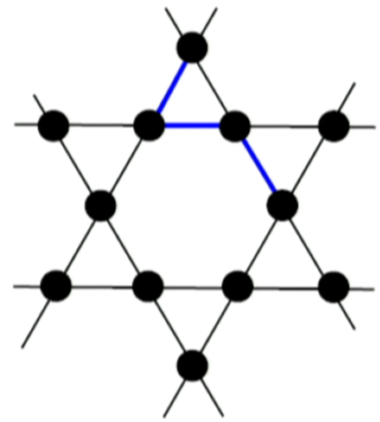
should be $e^{i3\pi/8}$?

$$U \sim \begin{array}{c} \begin{array}{ccc} \mathbb{I} & \sigma & \psi \end{array} \\ \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & \boxed{e^{0.487 i\pi}} & 0 \\ 0 & 0 & -1 \end{array}\right] \end{array} \begin{array}{c} \\ \mathbb{I} \\ \sigma \\ \psi \end{array}$$

Lattice models

- Wen
- Haldane HC
- fermA
- fermB
- Kitaev HC
- Heisenberg KG

Heisenberg model, kagome



$$\mathcal{H}_{\text{HeisenbergKG}} = \sum_{\langle rr' \rangle} \vec{S}_r \cdot \vec{S}_{r'}$$

S. Yan, D.A. Huse, S.R. White, **Science 2011**

$\mathcal{H}_{\text{HeisenbergKG}} \longrightarrow$ anyon model $\longrightarrow$?

$$S = \begin{bmatrix} 0.72(1) & 0.24(1) & \cdots \\ 0.22(1) & -0.21(1) & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & 0 & \cdots \\ 0 & -1 & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}$$

conclusions

microscopic lattice
Hamiltonian
$\mathcal{H}$

NUMERICS

NUMERICS

anyon model in the bulk

- topological S and U matrices
  - quantum dimensions d_i
  - total quantum dimension D
  - fusion rules
  - central charge c
  - twists θ_i

edge CFT

- conformal dimensions of primary fields
- conformal towers in each sector

related works

H.-C. Jiang, Z. Wang, L. Balents, **Nat Phys 2012**
H.-C. Jiang, H. Yao, L. Balents, **PRB 2012**
M.P. Zaletel, R.S.K. Mong, F. Pollmann, **1211.3733**

THANKS!