Title: Directed Influence in the RG Flow

Date: May 09, 2013 11:00 AM

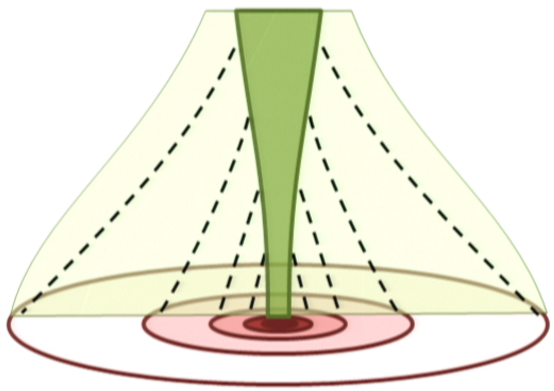
URL: http://pirsa.org/13050041

Abstract: Given two lattice Hamiltonians H_1 and H_2 that are identical everywhere except on a local region R of the lattice, we propose a relationship between their ground states psi_1 and psi_2. Specifically, assuming the states can be represented as multi-scale entanglement renormalization ansatz (MERA), we propose a principle of directed influence which asserts that the tensors in the MERAâ€™s that represent the ground states can be chosen to be identical everywhere except within a specific, localized region of the tensor network. The validity of this principle is justified by demonstrating it to follow from Wilson's renormalization ideas towards systems with manifestly separated energy scales. This result is shown, through numerical examples, to have practical applications towards the efficient simulation of systems with impurities, boundaries and interfaces, and also argued to provide useful insights towards holographic representations of quantum states.

Emergence and Entanglement II
Perimeter Institute, May 2013

Directed Influence in the RG Flow

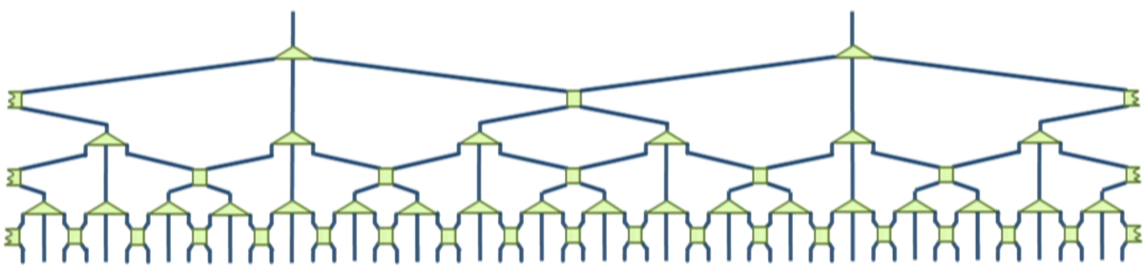
Glen Evenbly



Directed Influence in the RG Flow

To appear soon…

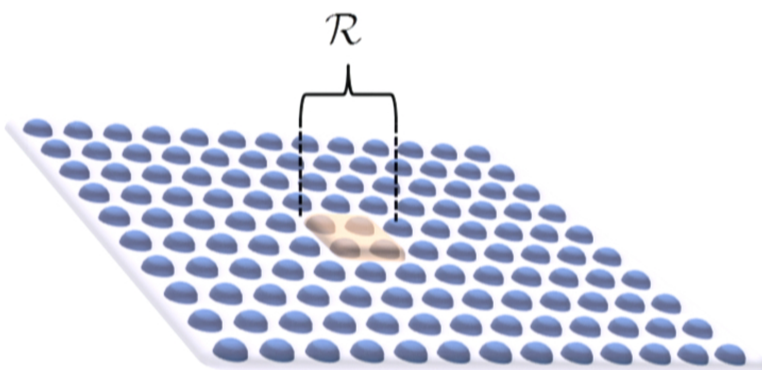
Directed Influence in the Renormalization Group Flow: arXiv:xxxx.xxxx (G.E., G. Vidal)



Motivation

consider two lattice Hamiltonians that differ only on some local region of the lattice:

$$\tilde{H} = H + H_{\mathcal{R}}$$



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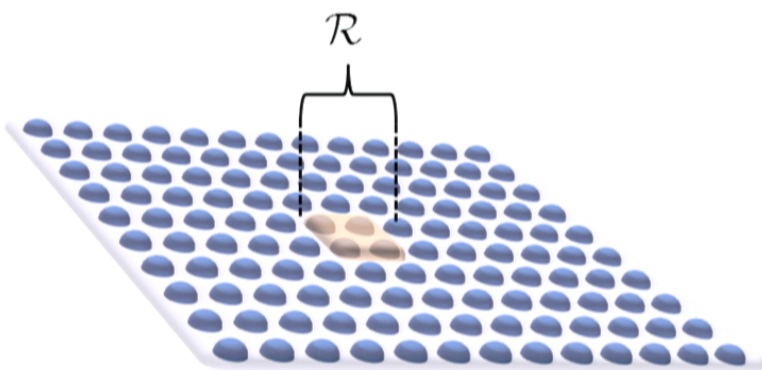
Kondo Impurity problem: 3D bulk of free fermions + local spin impurity



differ locally

$H \longleftrightarrow \tilde{H}$

$\Downarrow$

$|\psi\rangle \longleftrightarrow |\tilde{\psi}\rangle$

is there any relationship between their ground states?

can be difficult!

- arbitrary small impurity coupling can drastically alter low energy

Motivation

consider two lattice Hamiltonians that differ
only on some local region of the lattice:

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e.g. 1D Heisenberg chain
with local impurity

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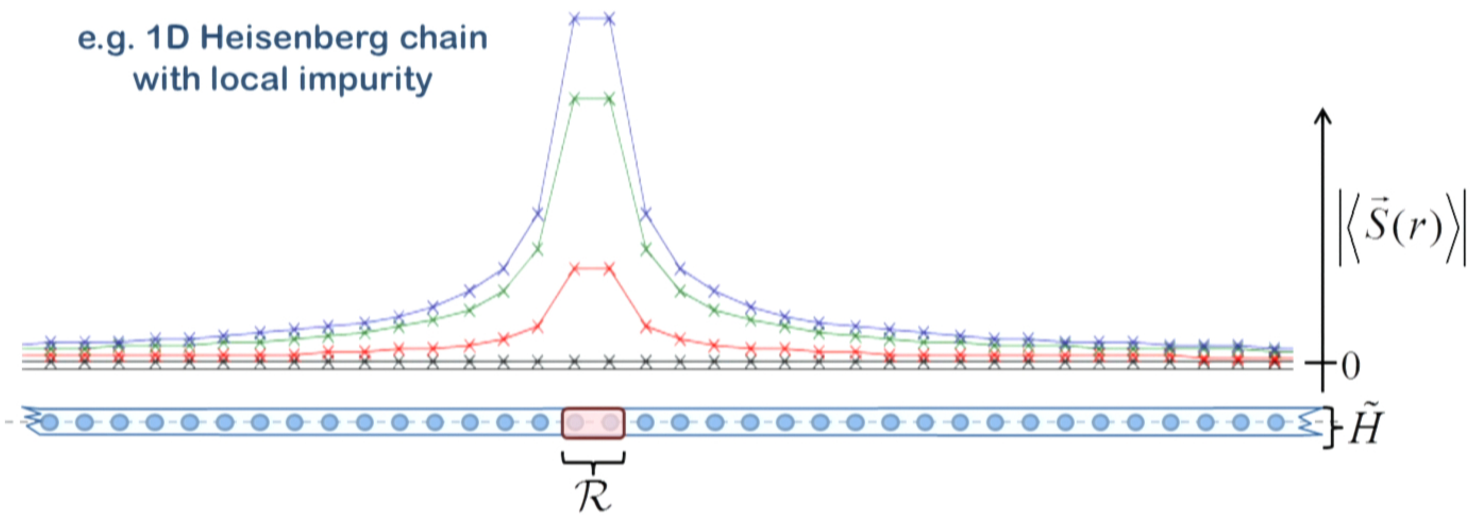


Motivation

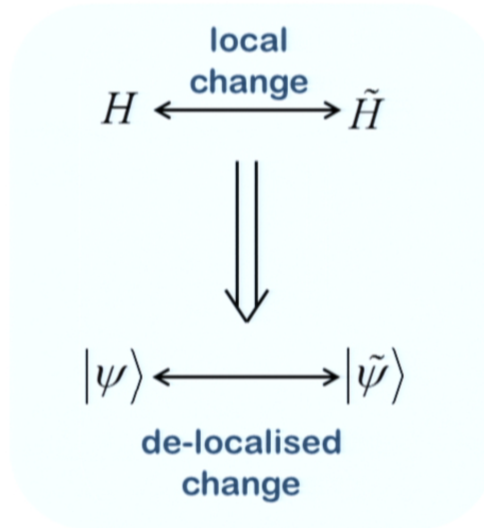
consider two lattice Hamiltonians that differ only on some local region of the lattice:

$$\tilde{H} = H + H_{\mathcal{R}}$$

- want to gain some understanding of how the ground states relate

$$|\psi\rangle \longleftrightarrow |\tilde{\psi}\rangle$$

- want efficient numeric algorithms for simulating homogeneous systems with impurities, boundaries and interfaces



Result

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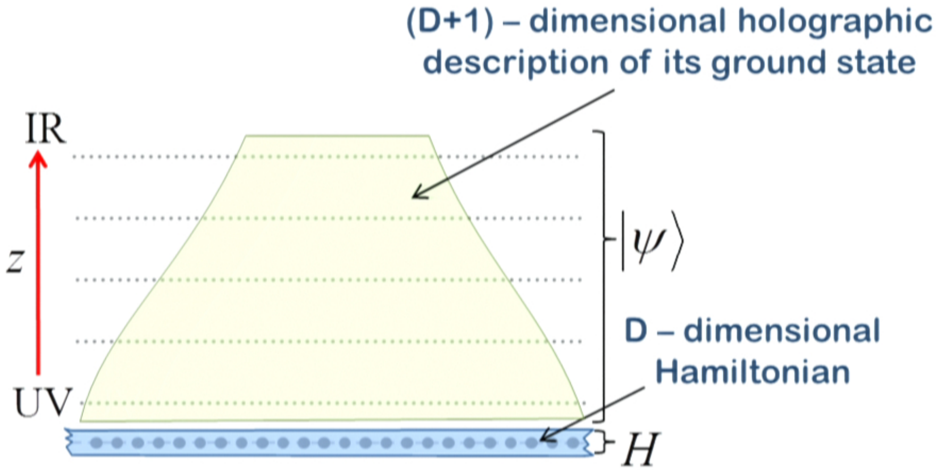
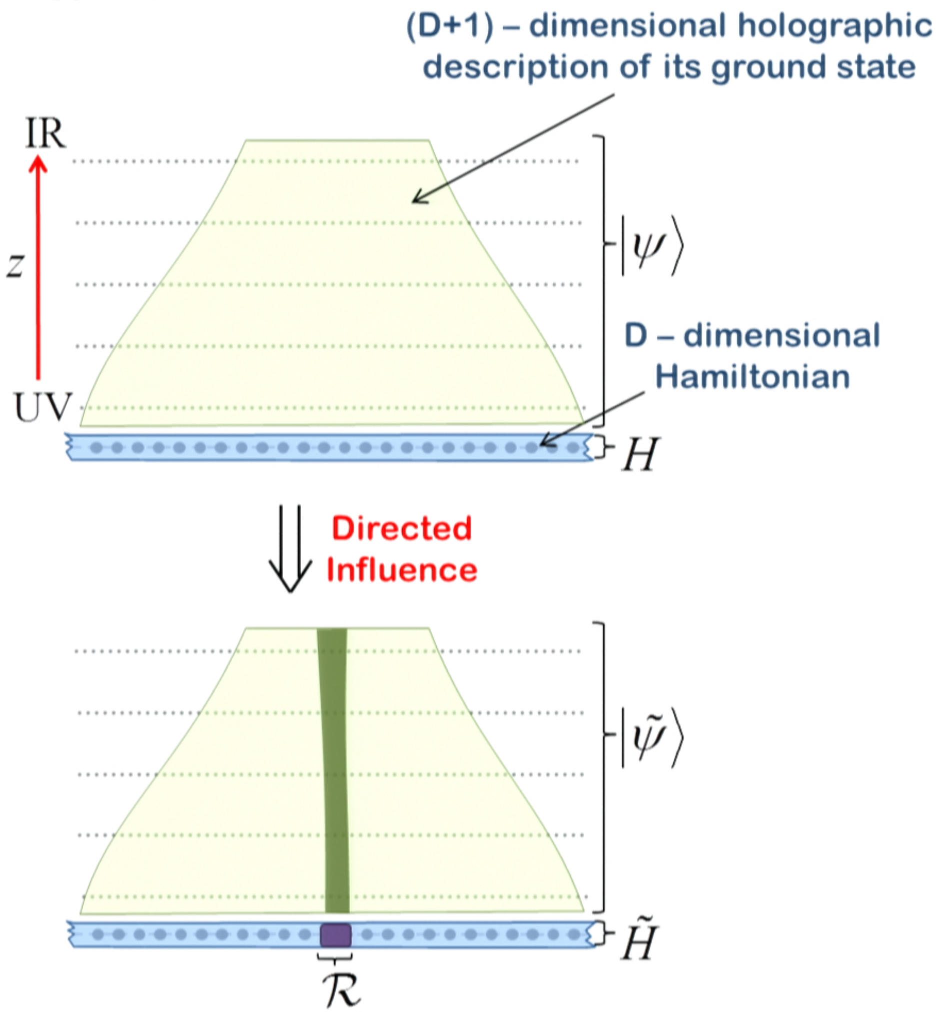


(D+1) – dimensional holographic description of its ground state
IR
z
UV
$|\psi\rangle$
D – dimensional Hamiltonian
H

Result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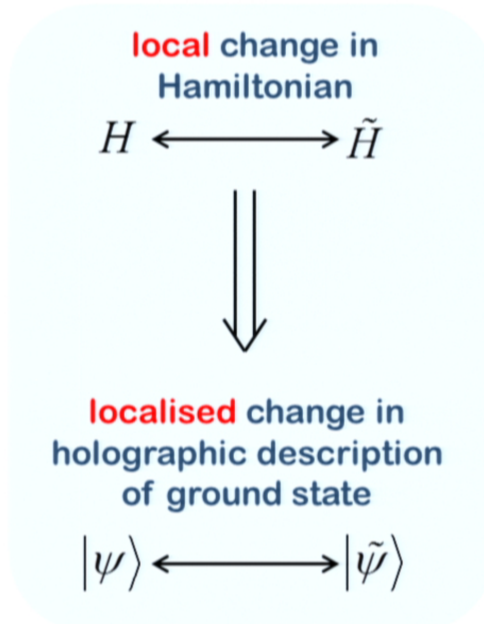
Directed Influence
$|\tilde{\psi}\rangle$
$\tilde{H}$
$\mathcal{R}$

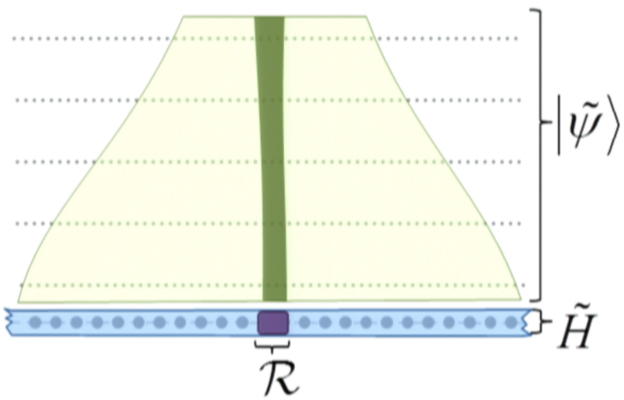

local change in Hamiltonian
$H \longleftrightarrow \tilde{H}$
localised change in holographic description of ground state
$|\psi\rangle \longleftrightarrow |\tilde{\psi}\rangle$

Questions?

- what holographic representation of ground states am I using?

- is directed influence useful for practical purposes?

- why is directed influence valid?



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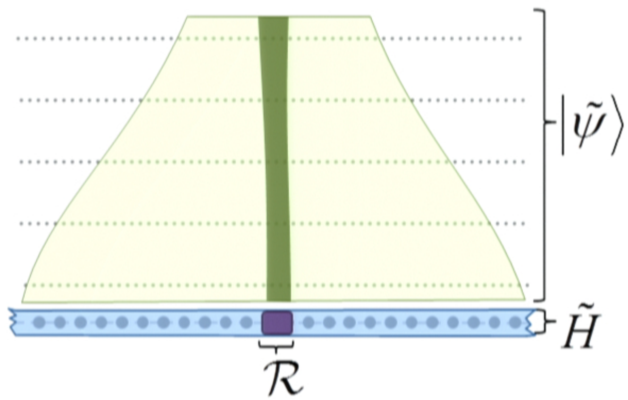


Questions?

- what holographic representation of ground states am I using?

  tensor network representation: multi-scale entanglement renormalization ansatz **(MERA)**

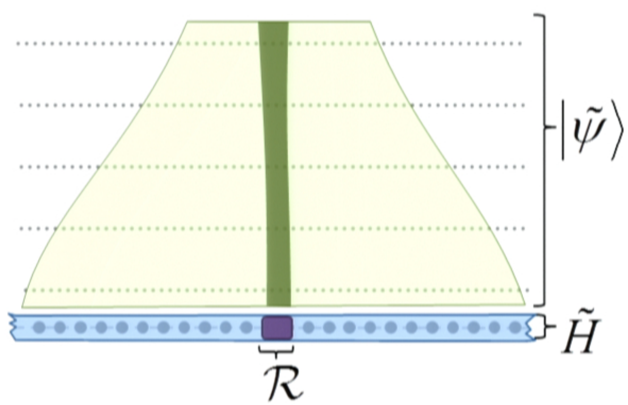
- is directed influence useful for practical purposes?

  allows a more efficient description of ground states of systems with impurities, boundaries and interfaces: **# parameters: O(N) → O(1)**

- why is directed influence valid?

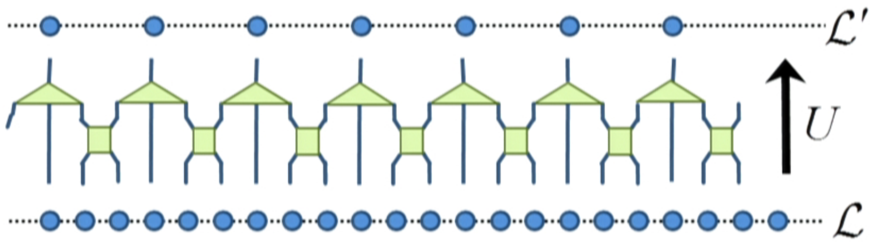
  it can be shown to follow from Wilson's renormalization ideas

Directed Influence



Intro to Entanglement Renormalization

Entanglement Renormalization: a coarse graining transformation for lattice systems

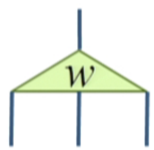


coarser lattice

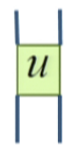
Entanglement Renormalization

initial lattice

Isometries:



Disentanglers:



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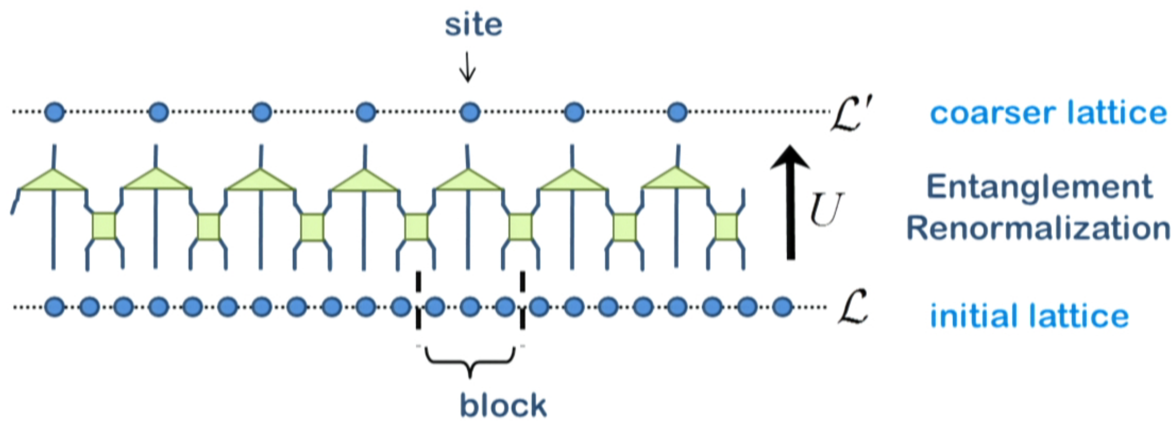

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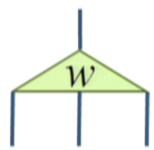

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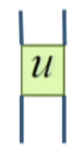
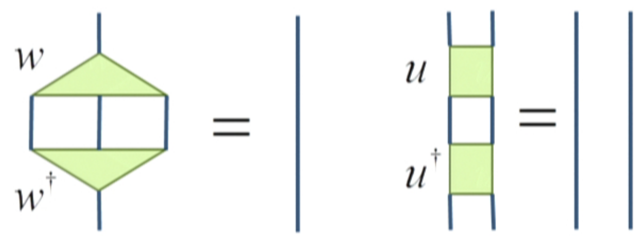
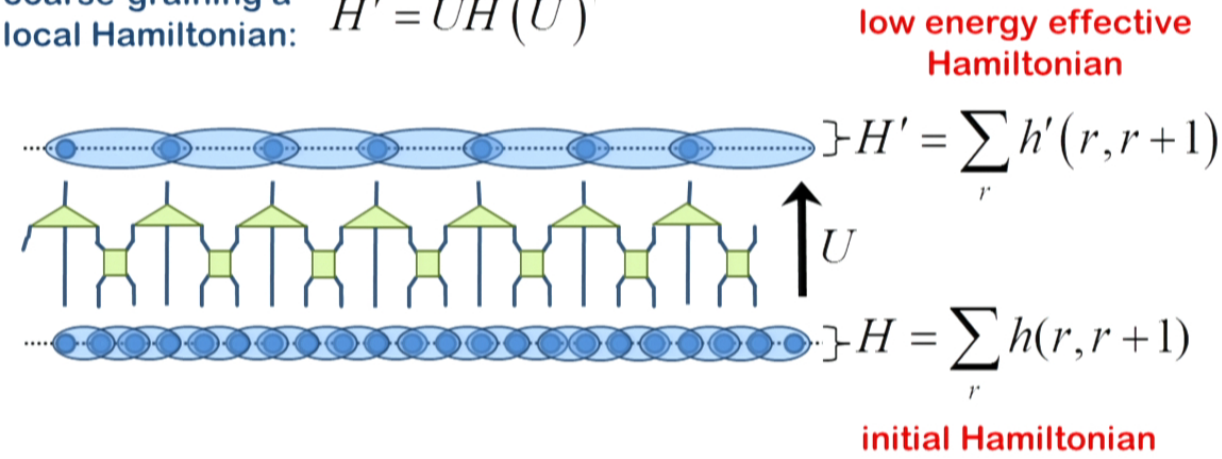


Isometric constraints:



Intro to Entanglement Renormalization

coarse-graining a local Hamiltonian: $H' = UH(U)^{\dagger}$



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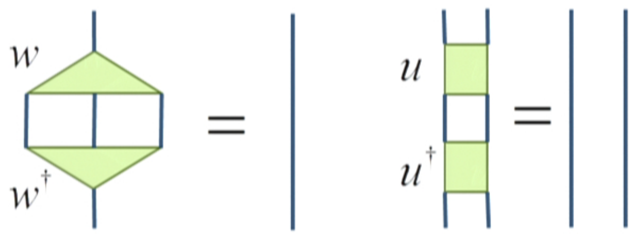
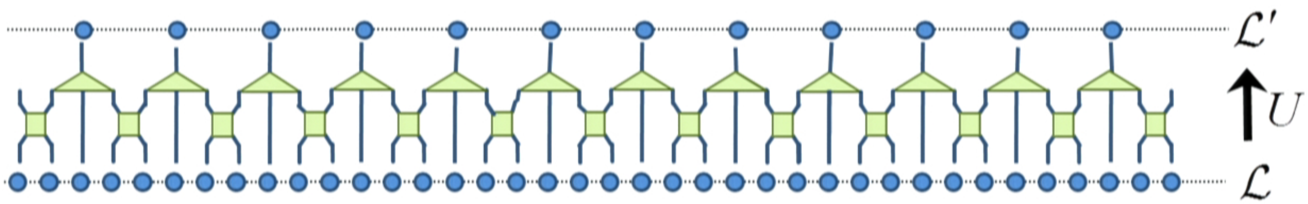
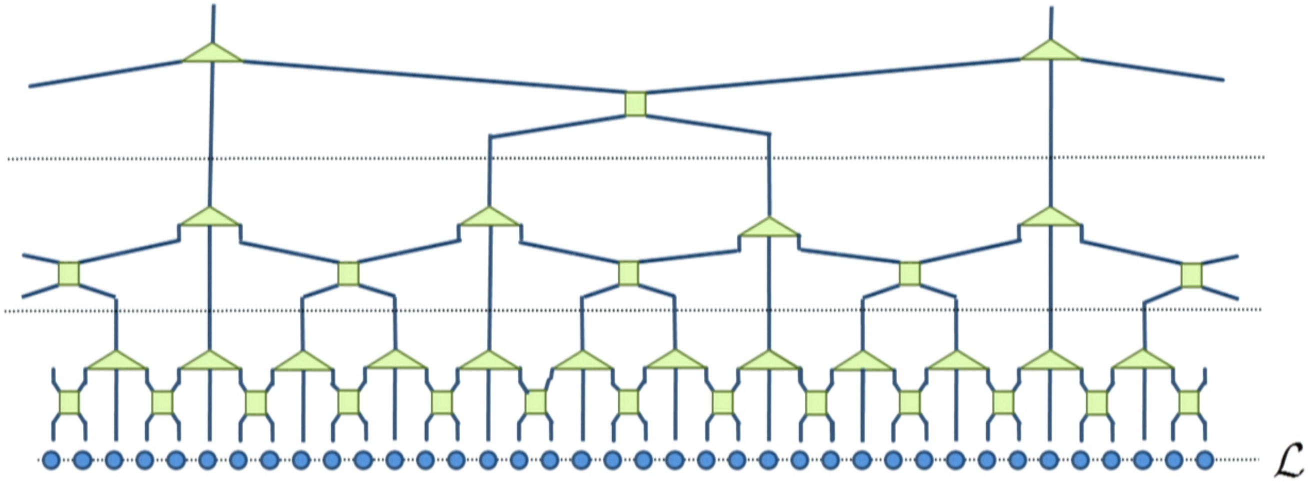


Intro to scale-invariant MERA



Intro to scale-invariant MERA



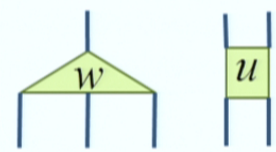
Scale-invariant MERA

- a tensor network ansatz for ground states of scale-invariant (critical) Hamiltonians

- results from a real-space coarse-graining transformation (Entanglement Renormalization)

- holographic interpretation

characterised by a single pair of tensors



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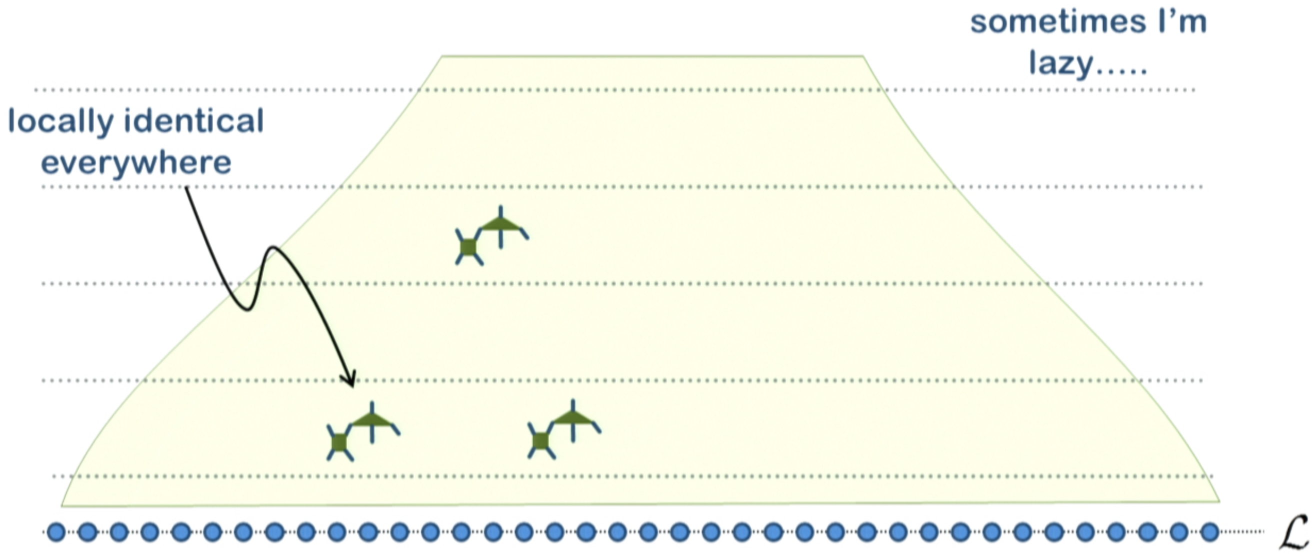

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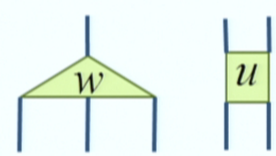

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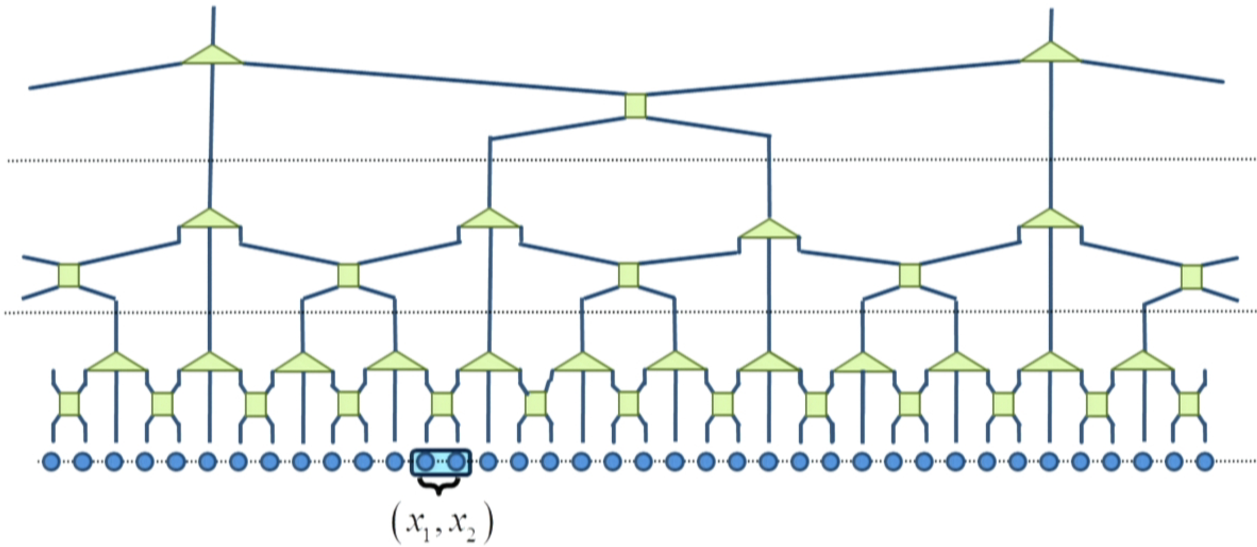


Def: Causal Cone of sites (x_1, x_2) = set of tensors that could affect the reduced density matrix $\rho(x_1, x_2)$

$$\rho(x_1, x_2) \equiv tr_{\overline{(x_1, x_2)}} |\Psi\rangle\langle\Psi|$$

Outline

- what holographic representation of ground states am I using?

  introduction to the MERA

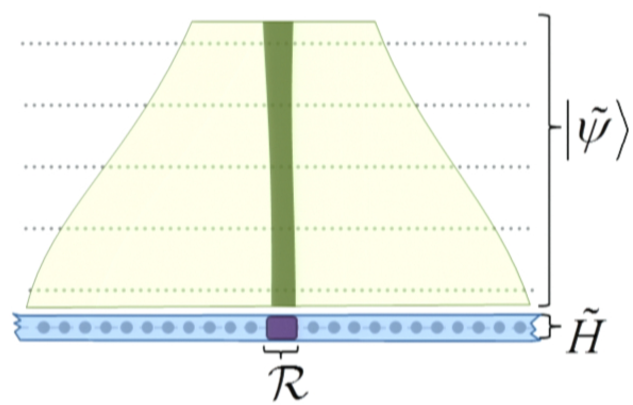
- why is directed influence useful?

  applications of directed influence: impurities, boundaries and interfaces

- why is directed influence valid?

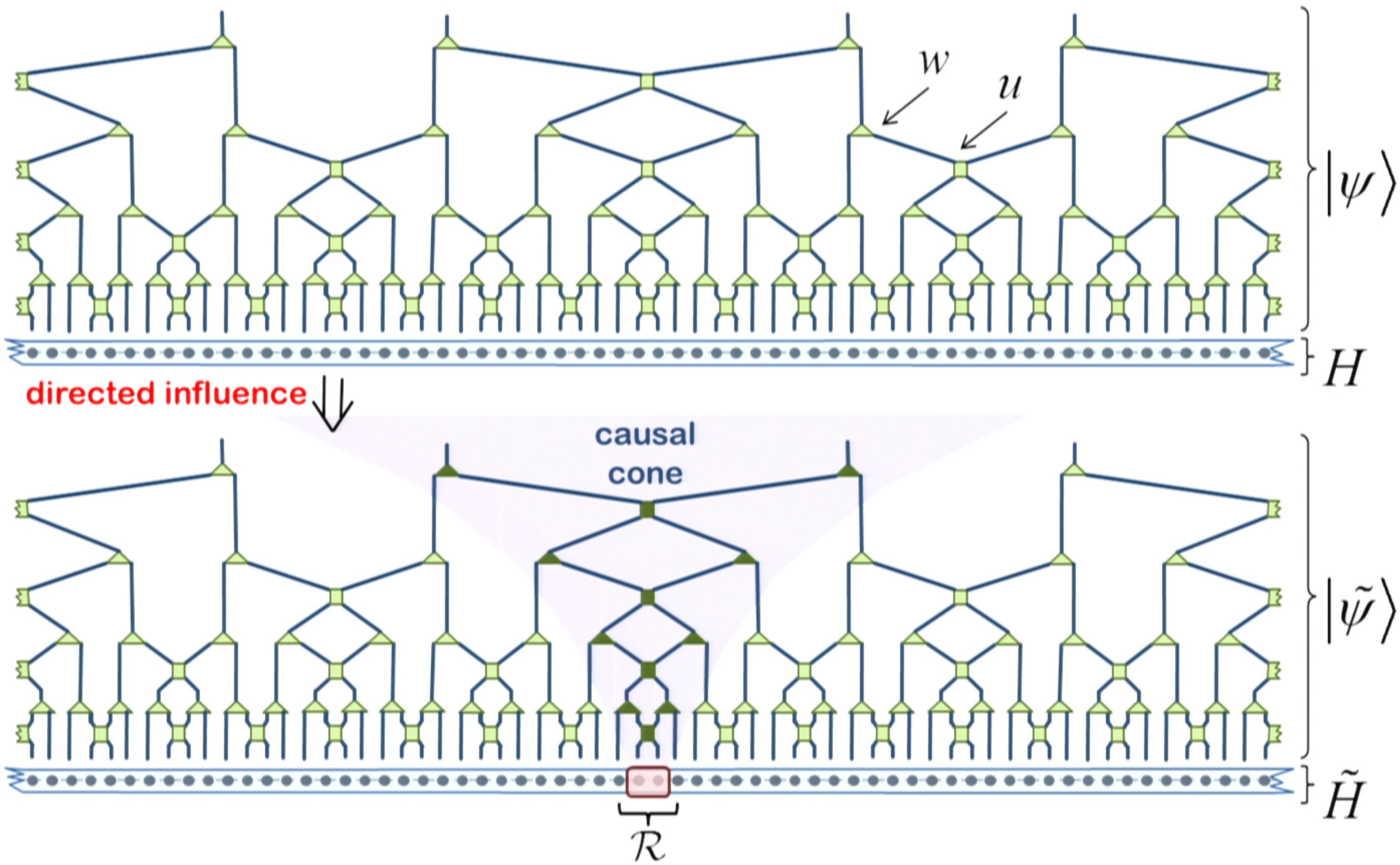
  directed influence arising from
  Wilson's renormalization group

Directed Influence

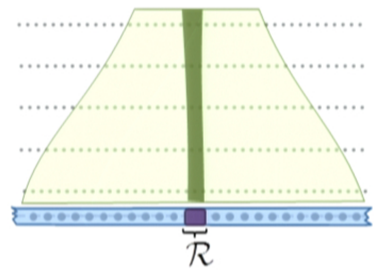


Directed Influence in MERA

modified binary MERA:



Directed Influence in MERA



Practical applications???

- directed influence is useful when the problem under consideration can be written:

$$\tilde{H} = \underbrace{H^{\text{sym}}} + \underbrace{H_{\mathcal{R}}^{\text{local}}}$$

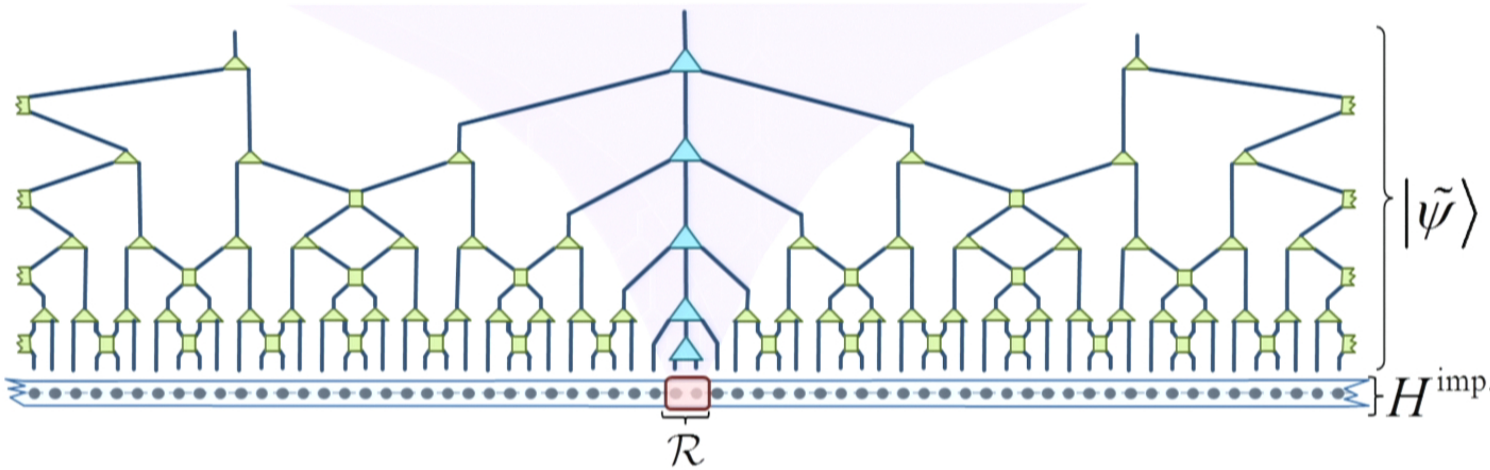
extensive part some with symmetry

local part that breaks symmetry

spatial symmetries: translation invariance, scale invariance, reflection invariance

global internal symmetries: i.e SU(2), U(1)

Impurity MERA



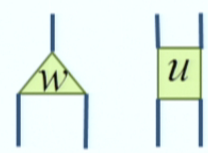
-impurity Hamiltonian: $H^{\text{imp.}} = H^{\text{bulk}} + H_{\mathcal{R}}$

-directed influence → impurity MERA

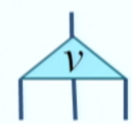
-$O(1)$ unique tensors (independent of system size!)

…even though we don't have translation invariance anymore!

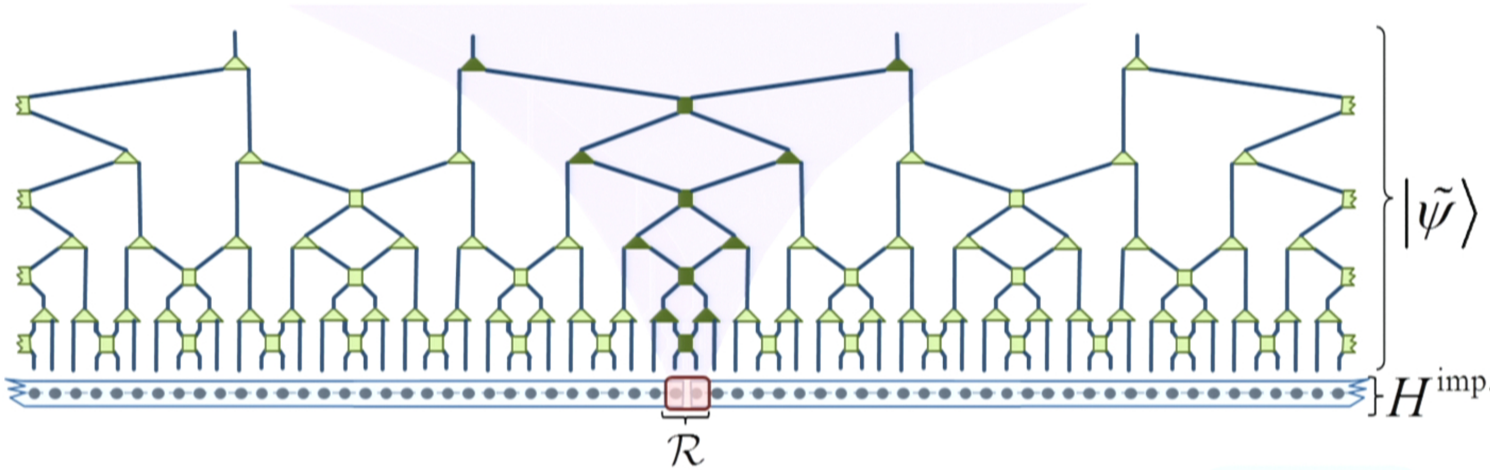
single pair of unique 'bulk' tensors



single (or a few) bulk tensors



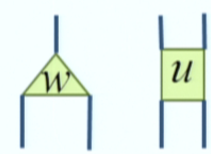
Boundary MERA



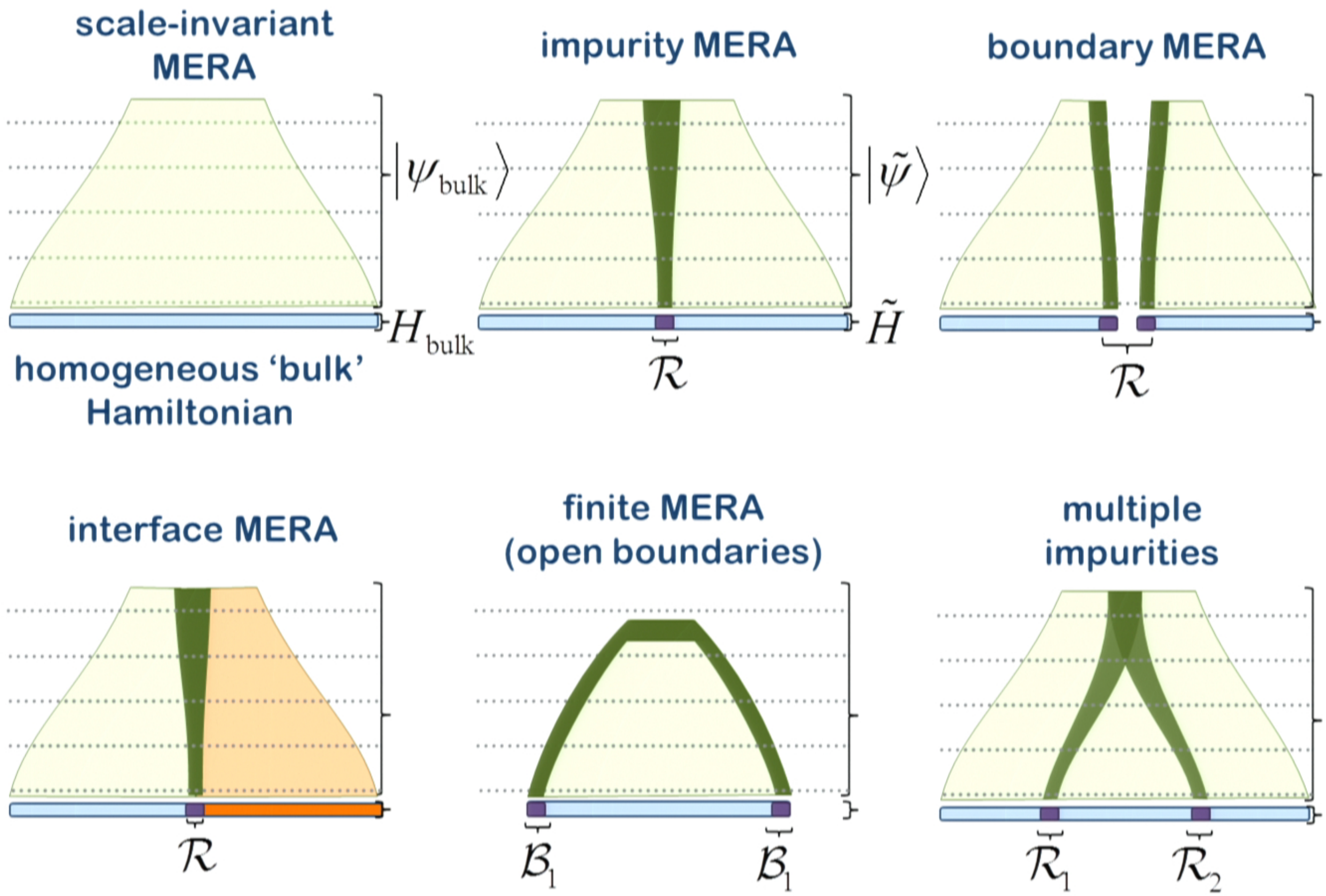
-consider a homogeneous 'bulk' Hamiltonian:
(assumed to be scale-invariant)

$$H^{\text{bulk}} = \sum_r h^{\text{bulk}}_{[r,r+1]}$$

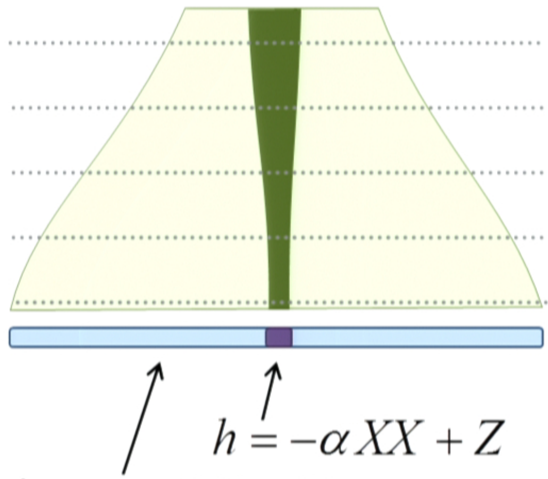
single pair of unique 'bulk' tensors



Directed Influence in MERA



Numerical Example: Impurity MERA

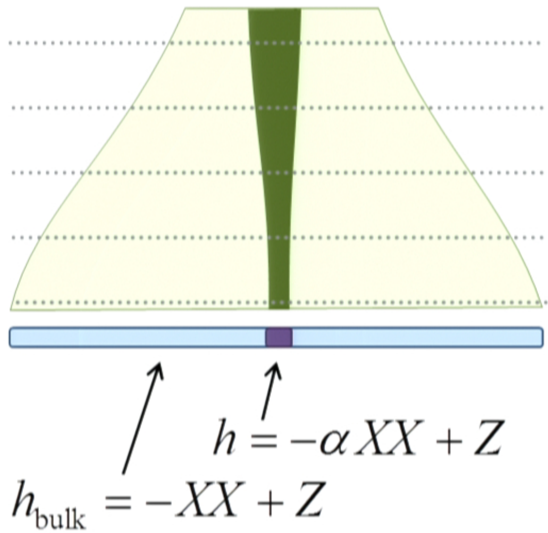


$$h = -\alpha XX + Z$$

$$h_{\text{bulk}} = -XX + Z$$

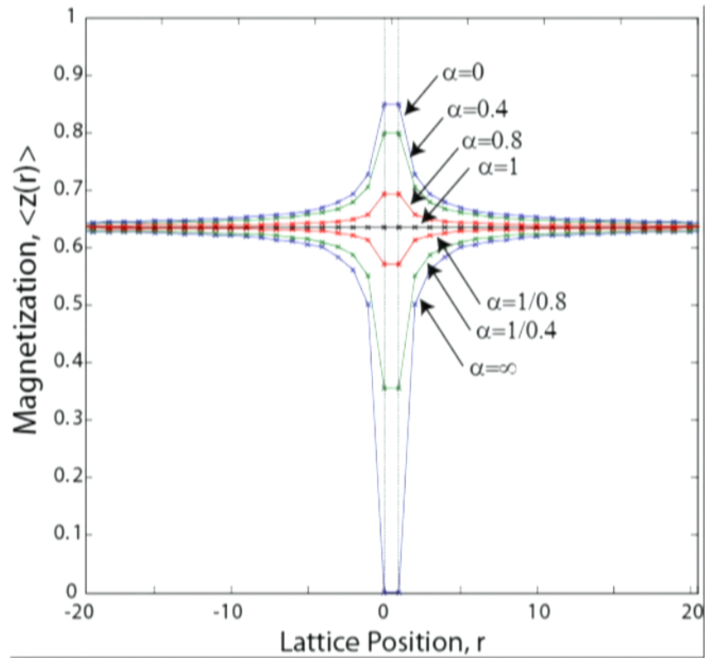
critical 1D Ising chain
with impurity

Numerical Example: Impurity MERA



critical 1D Ising chain
with impurity

ground state magnetization:

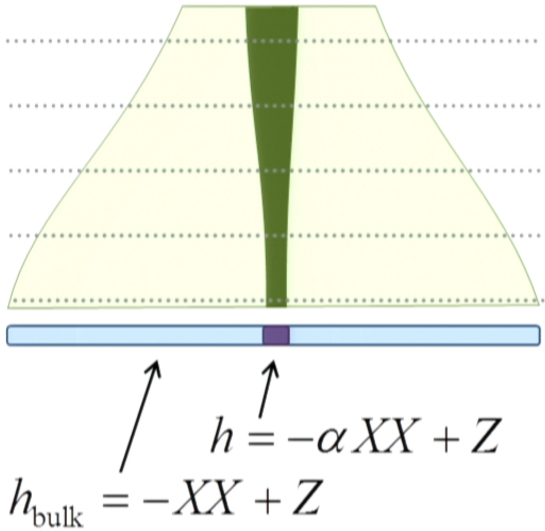


impurity energy:

$$\Delta E_{\text{MERA}} = 0.18169023$$

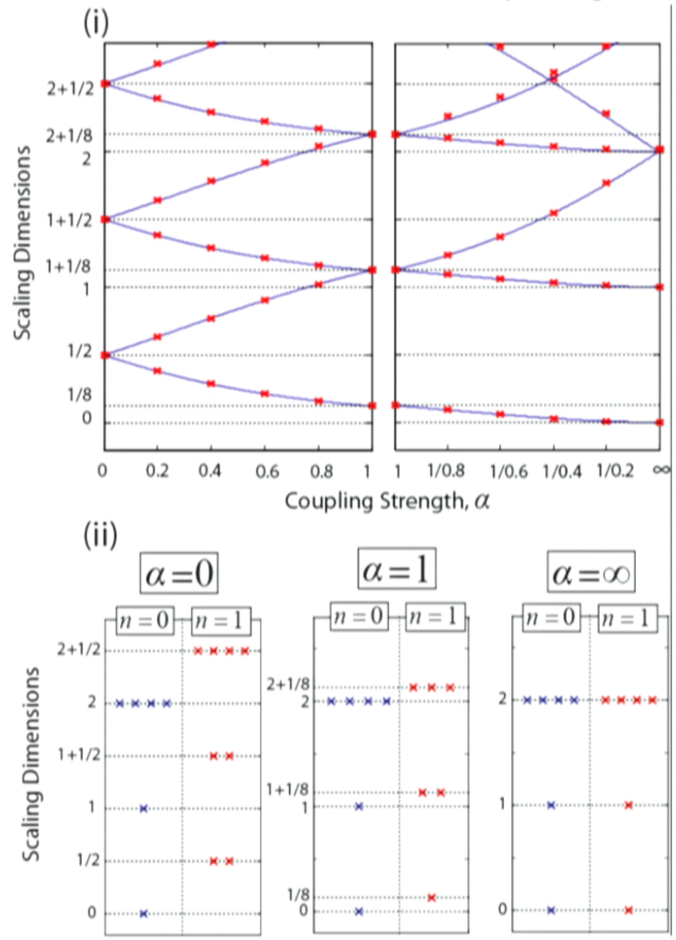
$$\Delta E_{\text{exact}} = 0.181690113\ldots$$

Numerical Example: Impurity MERA

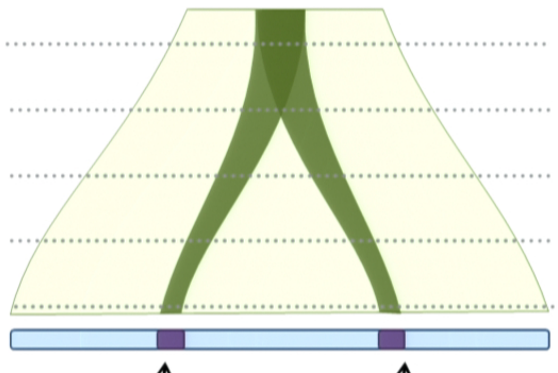


critical 1D Ising chain
with impurity

conformal data of impurity



Numerical Example: Double Impurity MERA



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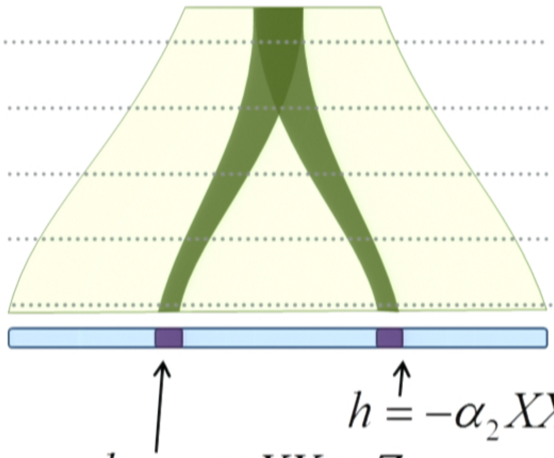
- critical 1D Ising chain two impurities

Numerical Example: Double Impurity MERA



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- critical 1D Ising chain two impurities

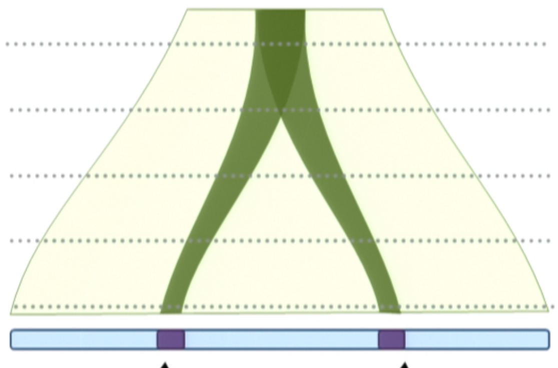
- impurities fuse to a single effective impurity under coarse graining

CFT result: 'opposite' impurities, i.e.

$$\alpha_2 = 1/\alpha_1$$

fuse to identity (no impurity)

Numerical Example: Double Impurity MERA



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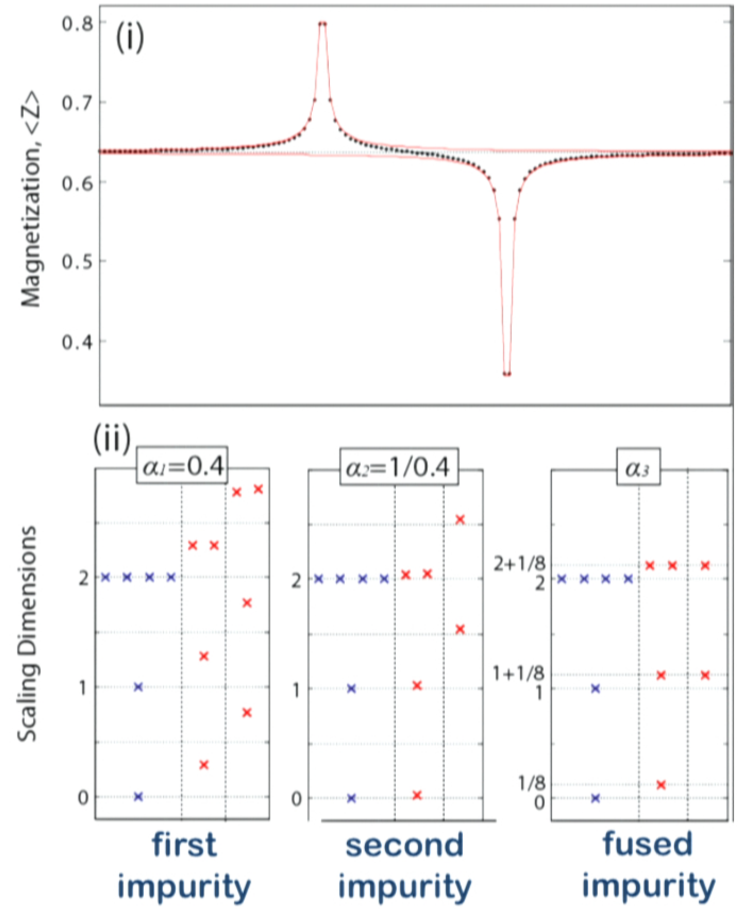

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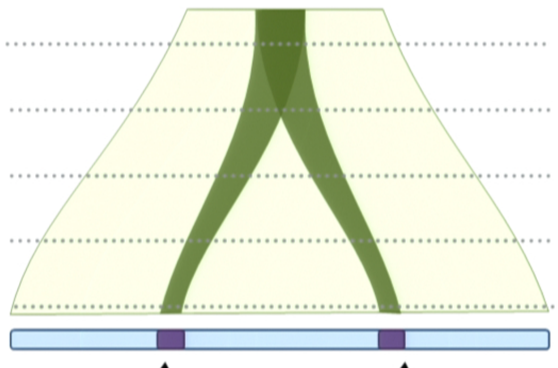

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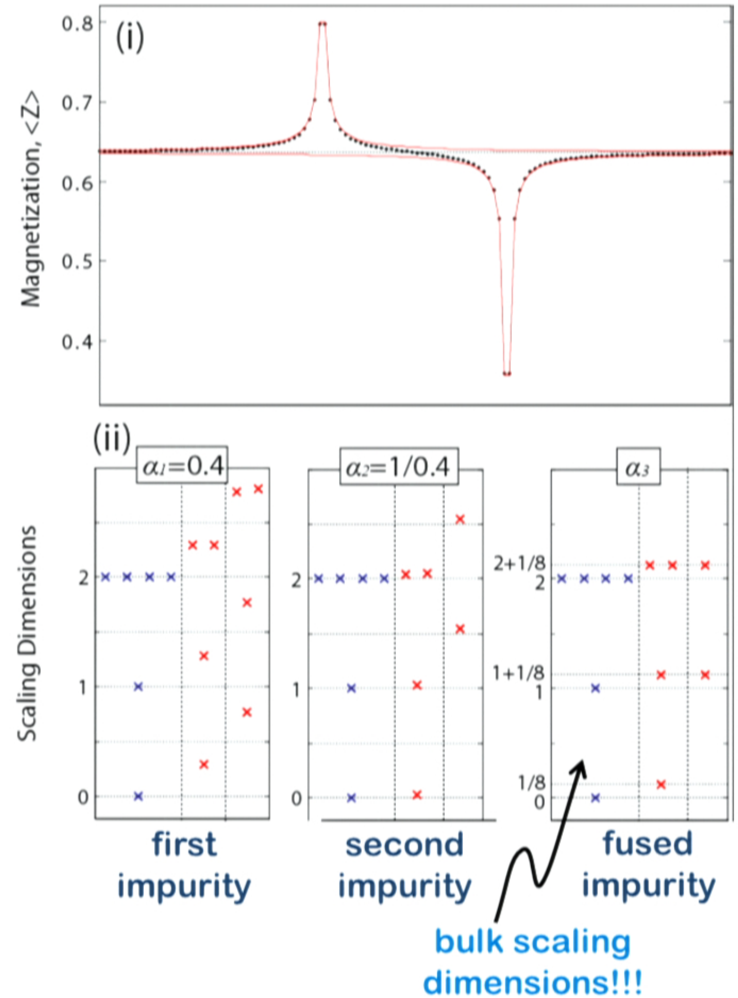

Applications of Directed Influence

- useful for study of homogeneous many-body systems that have local ‘defects’ (e.g. impurities, boundaries, interfaces)

- less parameters required to describe ground states

 # parameters: $O(N) \mapsto O(1)$

- study thermodynamic limit directly (despite broken translation invariance)

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- accurate long range properties of ground states (correlators, conformal data)

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- study thermodynamic limit directly (despite broken translation invariance)

- accurate short range properties of ground states (energy, local observables)

- accurate long range properties of ground states (correlators, conformal data)

- can introduce strong defects, i.e. not limited to perturbatively weak defects

…also have similar numeric results for boundaries, interfaces, Y-junctions…

To appear soon…

Algorithms for Entanglement Renormalization: impurities, boundaries and interfaces
arXiv:xxxx.xxxx (G.E., G. Vidal)

Outline

- what holographic representation of ground states am I using?

  introduction to the MERA

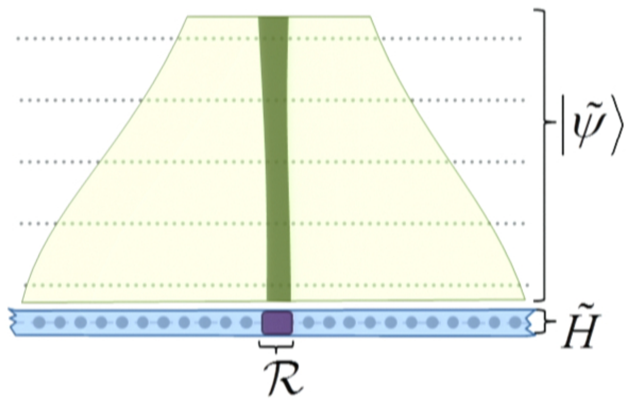
- why is directed influence useful?

  applications of directed influence: impurities, boundaries and interfaces

- why is directed influence valid?

  directed influence arising from Wilson's renormalization group

Directed Influence



Wilson's Renormalization Group

- Kondo impurity problem: 3D bulk of free fermions with local magnetic impurity

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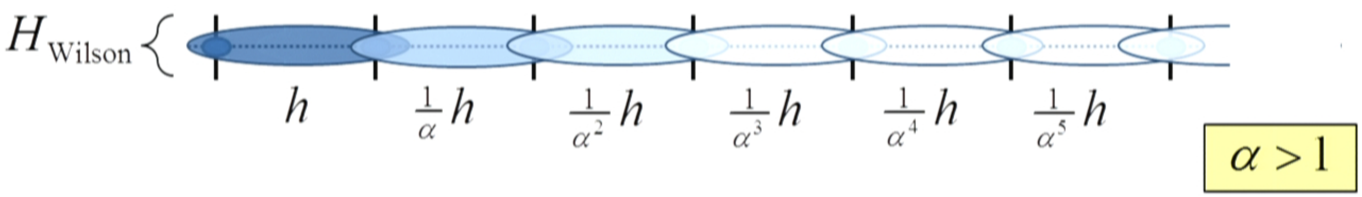
- Wilson derived an effective 1D Hamiltonian for this problem:



- solve with NRG (numerical renormalization group):
iterative block diagonalization based on perturbation theory

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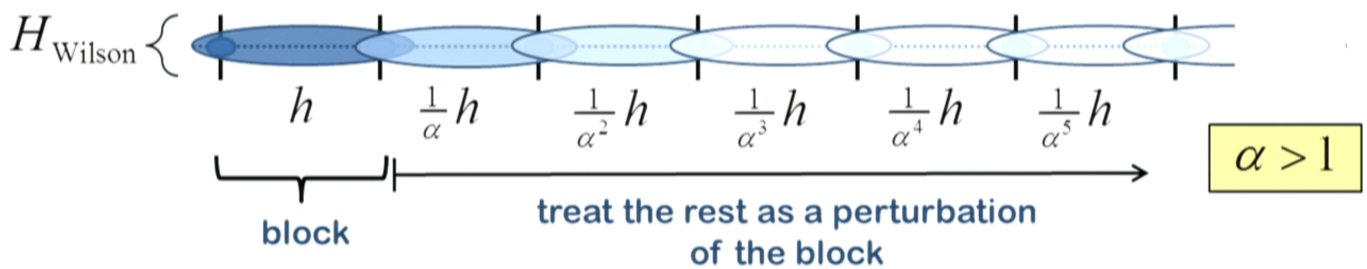
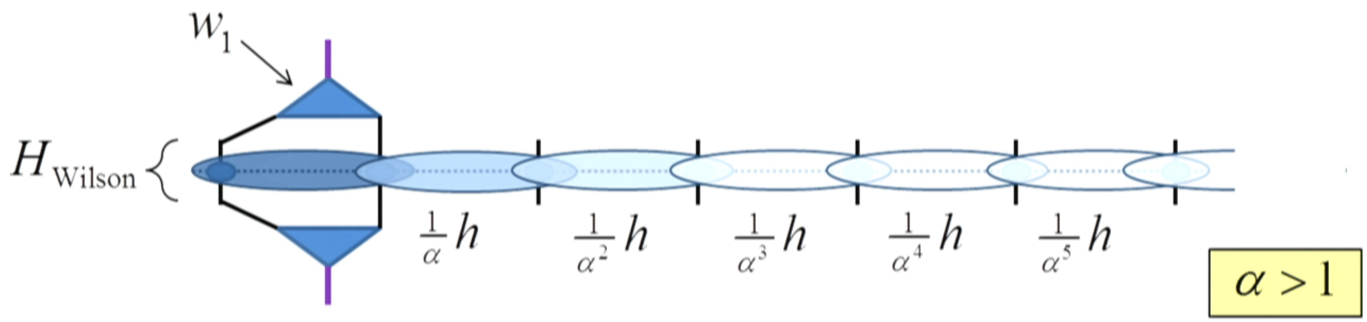


Wilson's Renormalization Group

- Kondo impurity problem: 3D bulk of free fermions with local magnetic impurity

- Wilson derived an effective 1D Hamiltonian for this problem:



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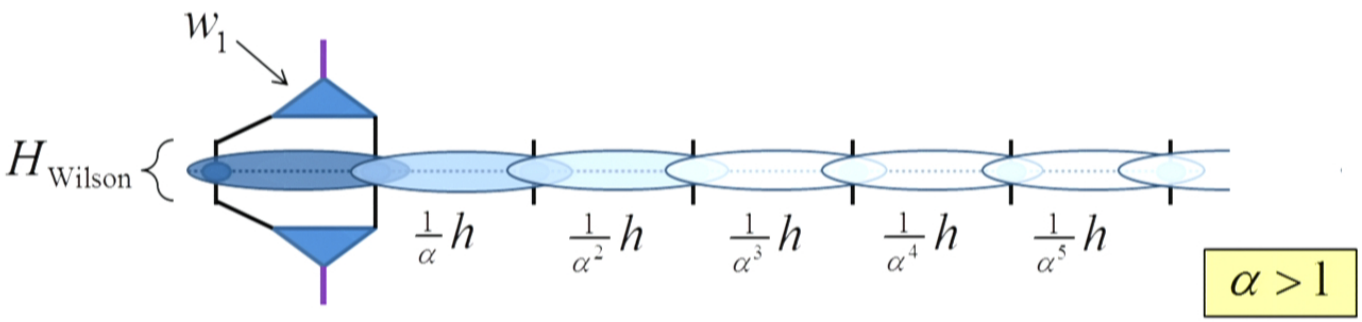

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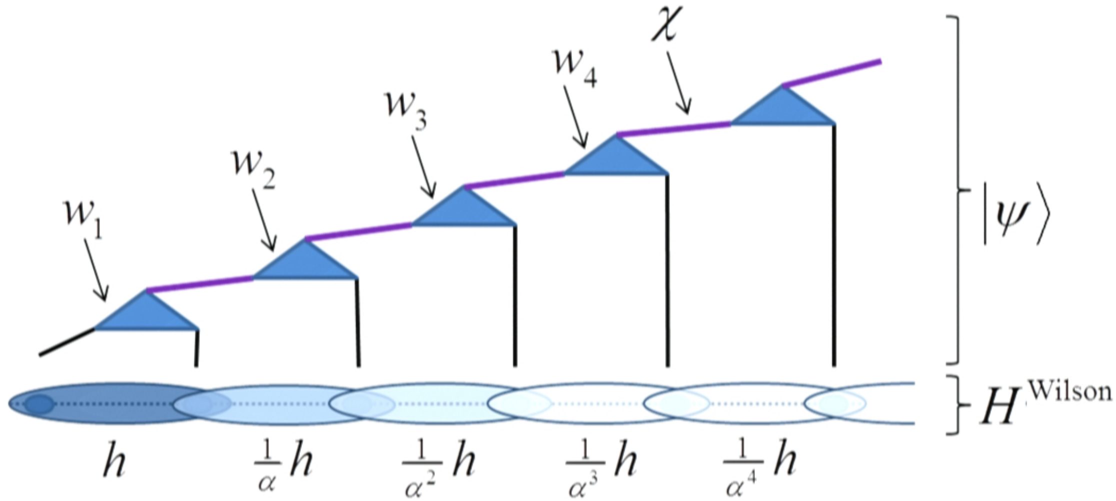
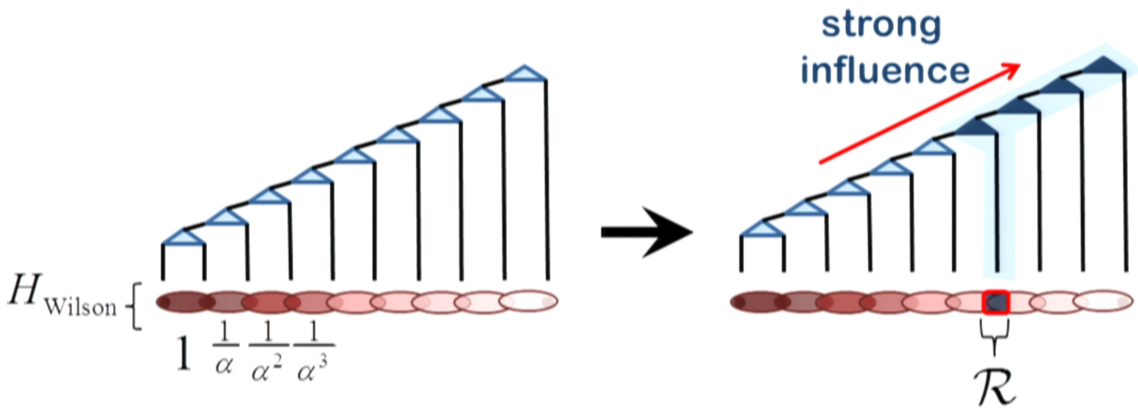


- NRG: iterative block diagonalization, justified by the separation of energy scales in the Hamiltonian
- ground state approximated by matrix product state (MPS)

Directed Influence

NRG: (0+1) dimensions



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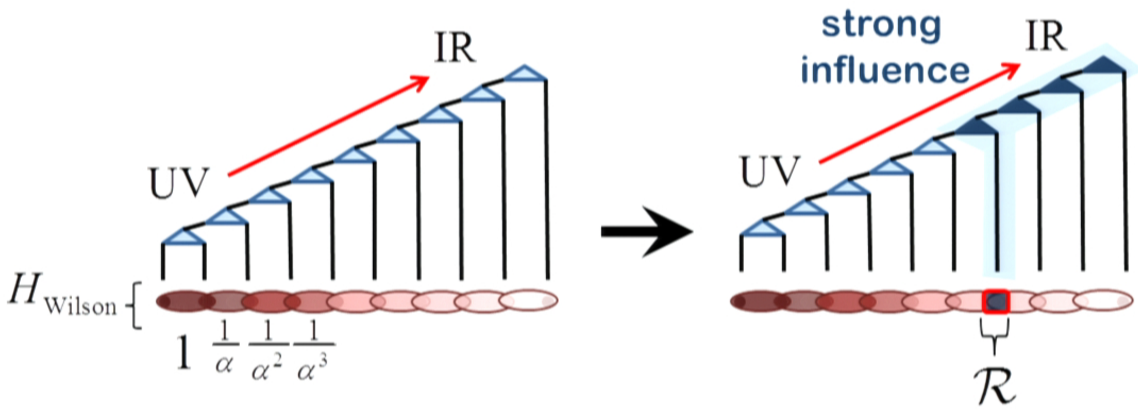
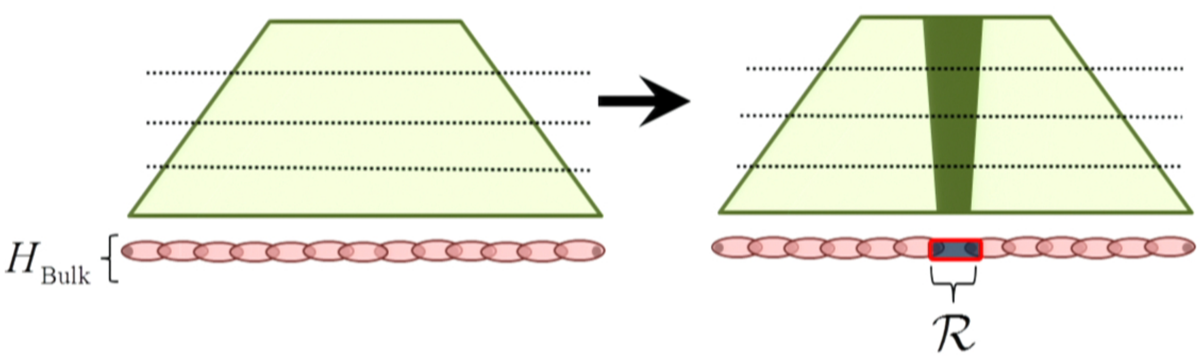
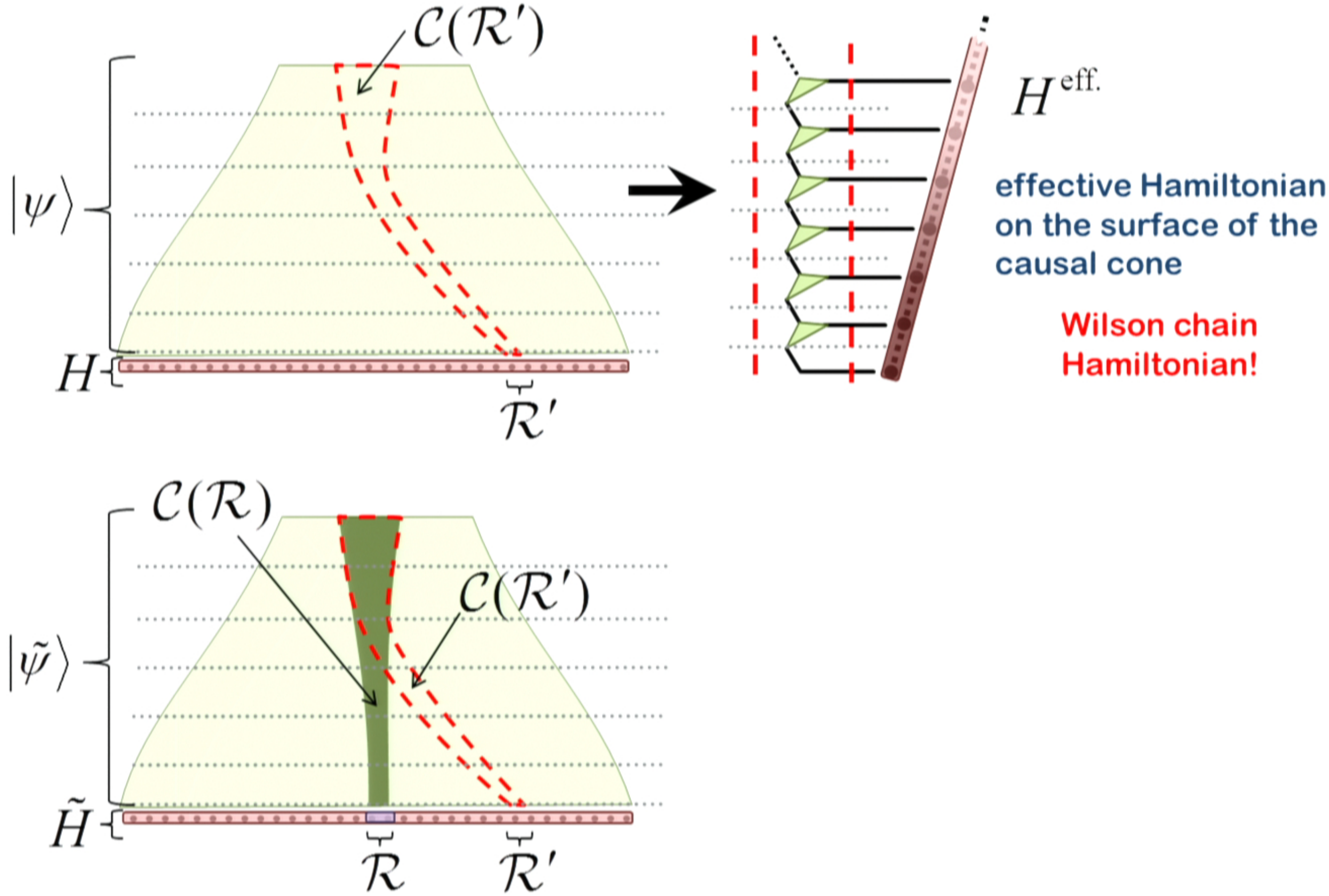


MERA: (D+1) dimensions



Justification of Directed Influence in MERA



Directed Influence

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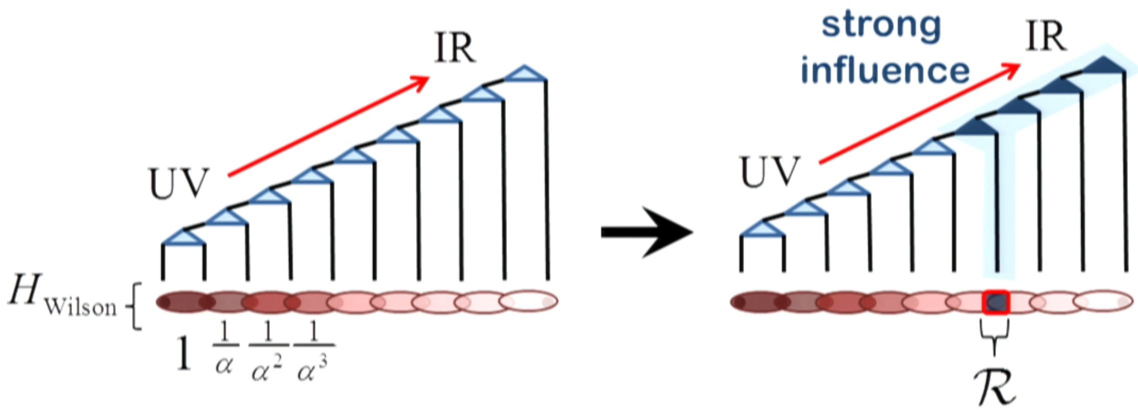

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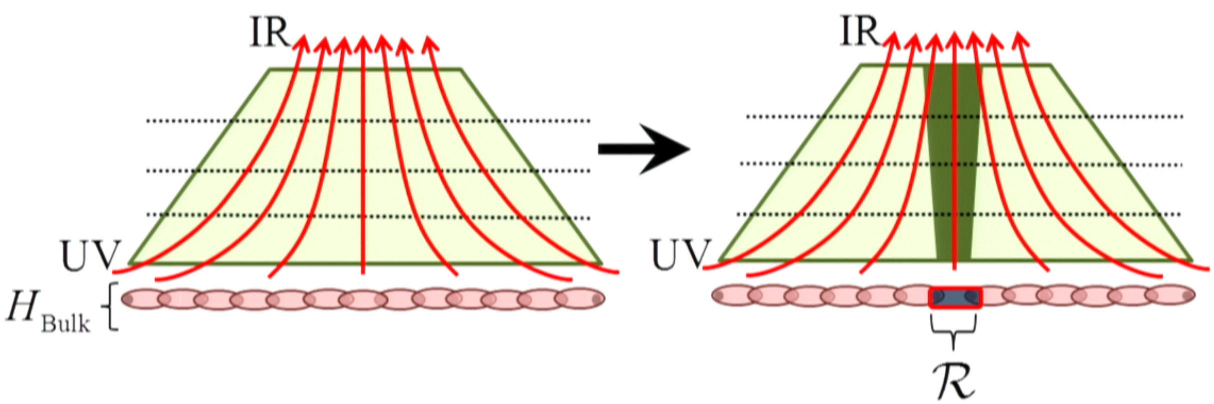
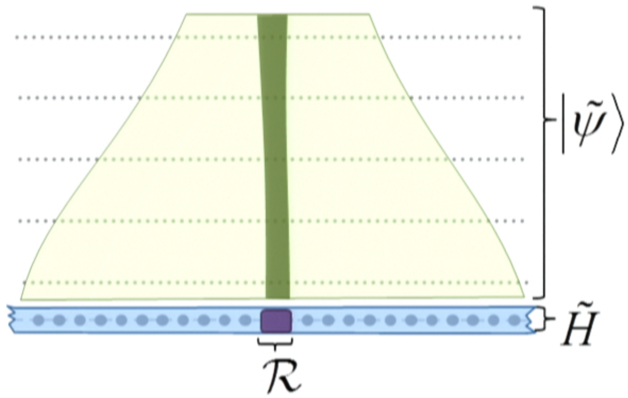


Summary: Directed Influence



local change in Hamiltonian

$$H \longleftrightarrow \tilde{H}$$

$$\Downarrow$$

localised change in holographic description of ground state

$$|\psi\rangle \longleftrightarrow |\tilde{\psi}\rangle$$

- conceptually appealing relationship between ground states of Hamiltonians that differ only on a local region

- exposed some causal structure in the holographic geometry

- useful for numerical simulation of many body systems with impurities, boundaries and interfaces

- justified from Wilsons renormalization ideas