Title: Entangled states of quantum matter

Date: May 08, 2013 02:00 PM

URL: http://pirsa.org/13050038

Abstract: Theorists have been studying and classifying entanglement in many-particle quantum states for many years. In the past few years, experiments on such states have finally appeared, generating much excitement. I will describe experimental observations on magnetic insulators, ultracold atoms, and high temperature superconductors, and their invigorating influence on our theoretical understanding.

Entangled states of quantum matter

Perimeter Institute
Waterloo, Canada
May 8, 2013

Subir Sachdev

SCIENTIFIC AMERICAN 308, 44 (JANUARY 2013)

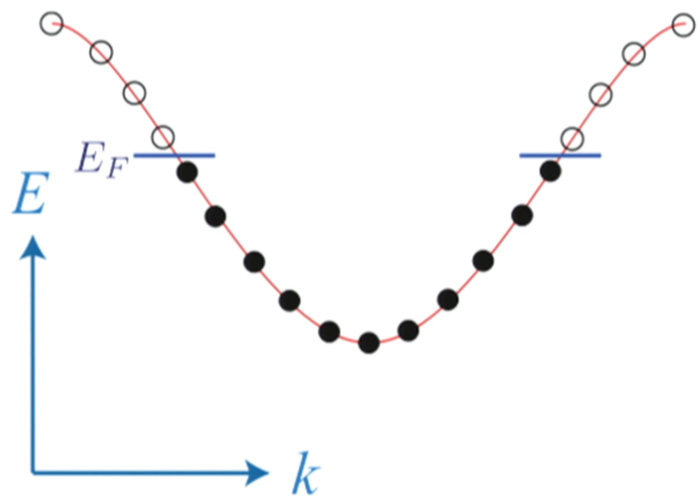
PHYSICS
HARVARD


Rob Myers
Max Metlitski
Ajay Singh
Erez Berg
Matthias Punk
Debanjan Chowdhury
Rolando La Placa
Daniel Podolsky
William Witczak-Krempa
Suvrat Raju

Sommerfeld-Pauli-Bloch theory of metals, insulators, and superconductors:

many-electron quantum states are adiabatically connected to independent electron states

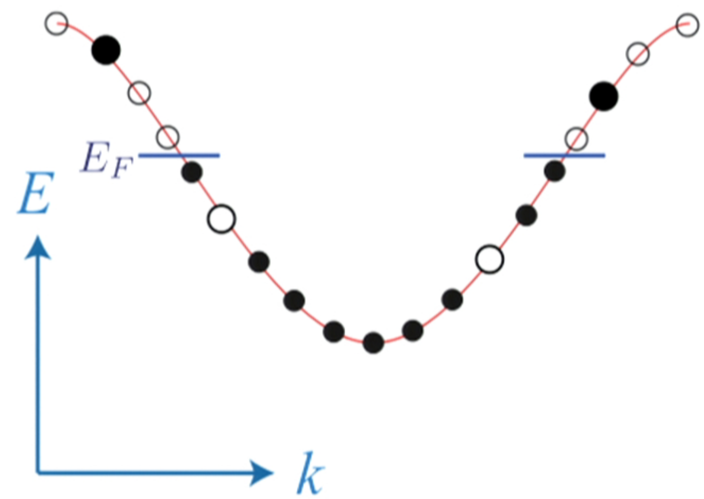
Metals



Boltzmann-Landau theory of dynamics of metals:

Long-lived ***quasiparticles*** (and ***quasiholes***) have weak interactions which can be described by a Boltzmann equation

Metals



Modern phases of quantum matter

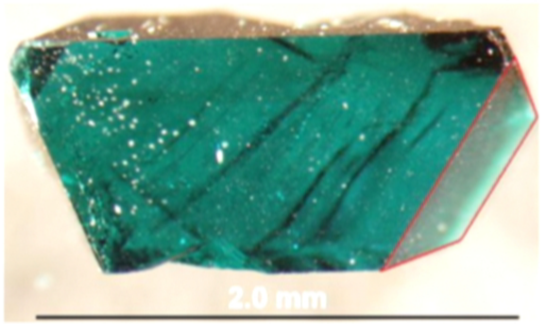
Not adiabatically connected
to independent electron states:

1. Many-particle quantum entanglement

2. (a) Quasiparticles with quantum numbers different from those of the electron

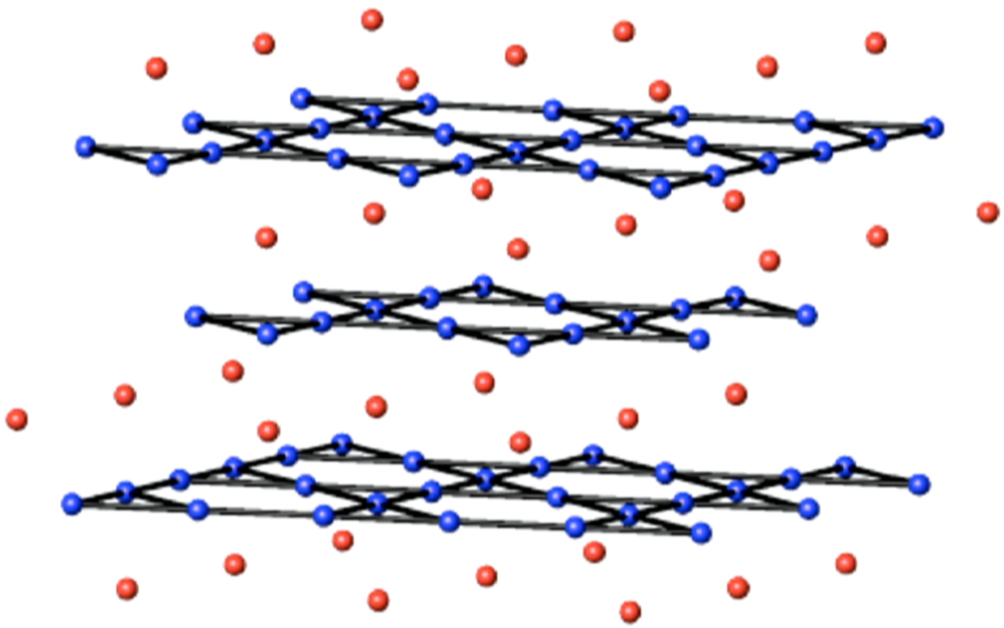
(b) No quasiparticles

Entanglement with quasiparticles



$ZnCu(OH)_6Cl_2$

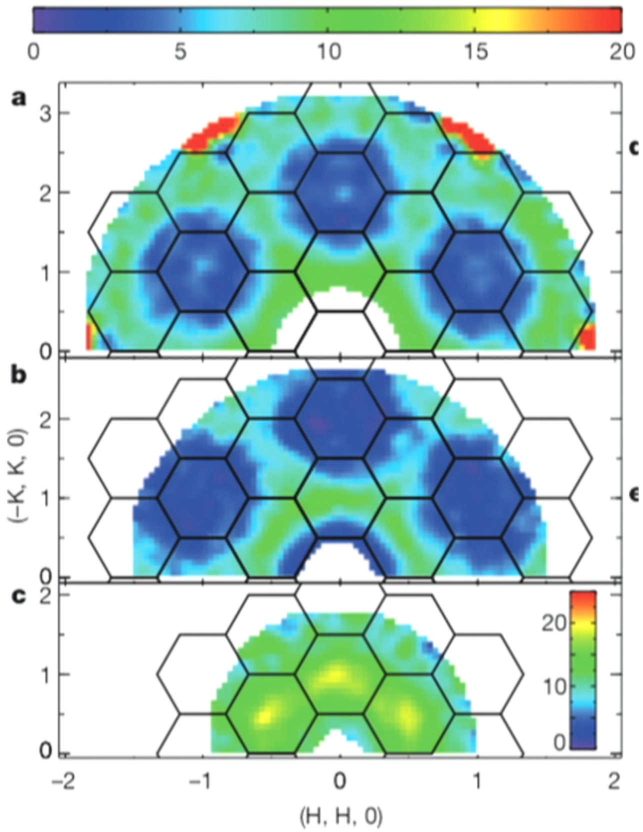
herbertsmithite single crystals



Tian-Heng Han, Young Lee *et al*, Nature **492**, 406 (2012)

Entanglement with quasiparticles

Neutron scattering: excitations over a broad range of momenta and at each energy



Tian-Heng Han, Young Lee *et al*, Nature **492**, 406 (2012)

Entanglement with quasiparticles

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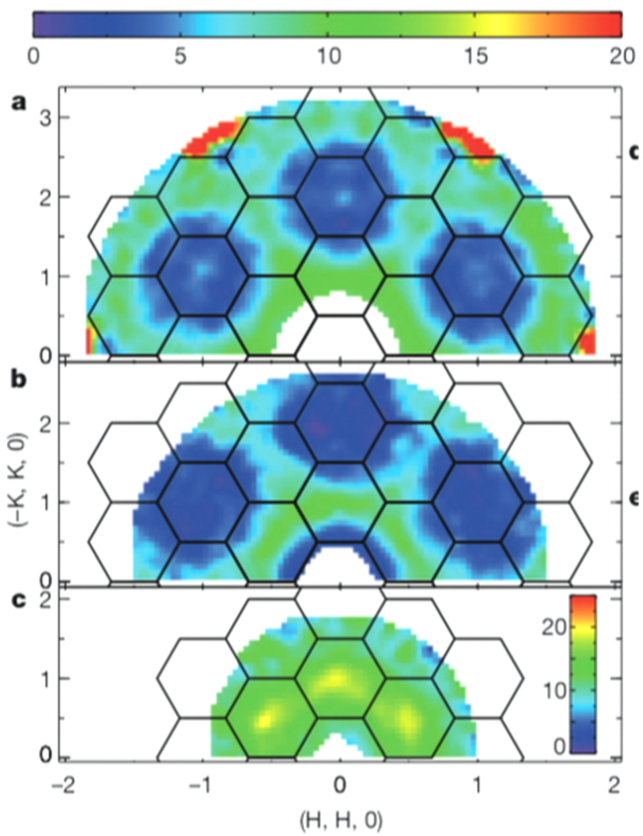
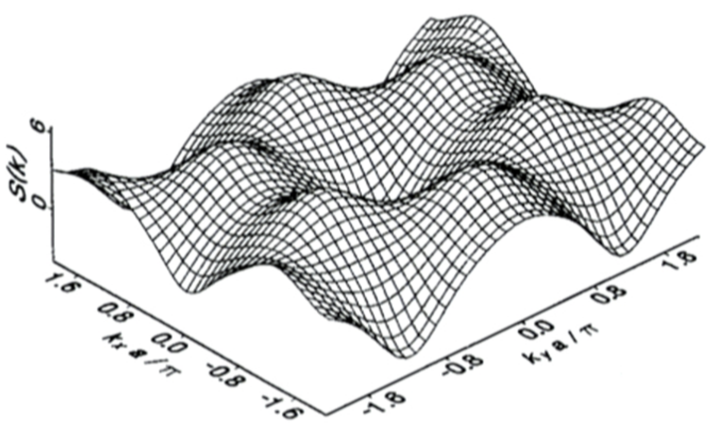


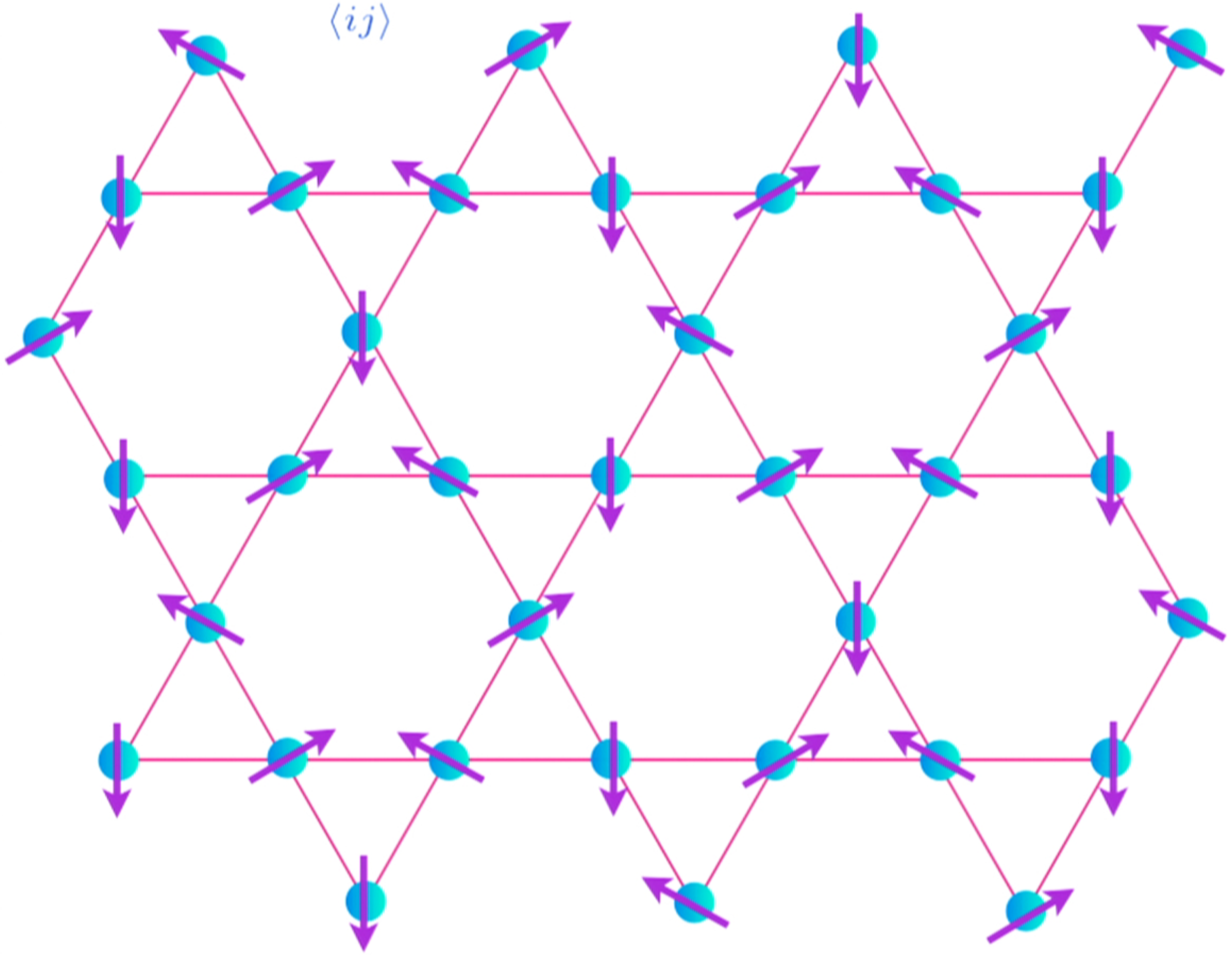
Entangled state: Z_2 spin liquid: There are well-defined emergent quasiparticles, and experiment measures cross-section to create a pair of quasi-particles



S. Sachdev, Phys. Rev. B 45, 12377 (1992)

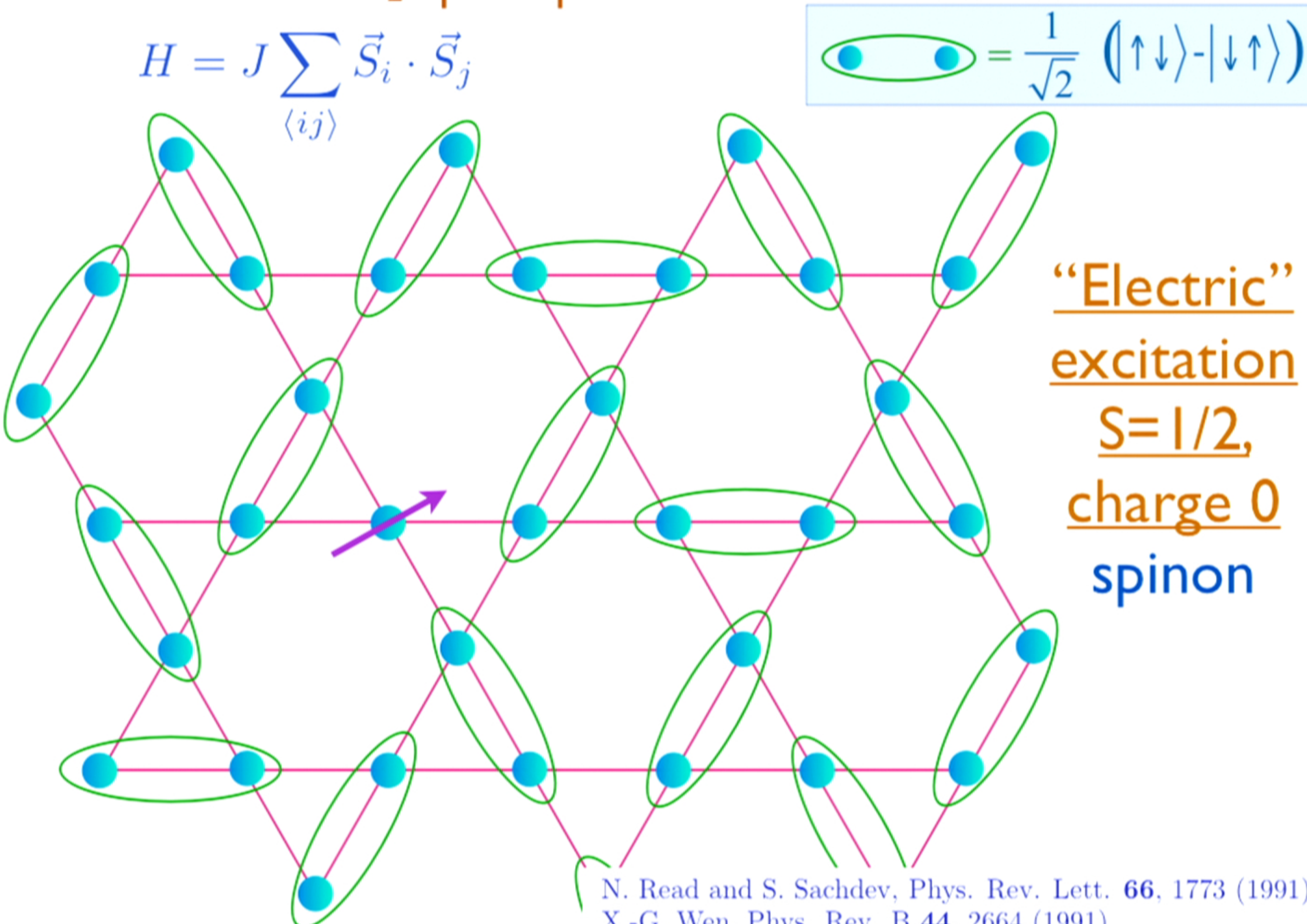
Kagome antiferromagnet

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



Excitations of a Z_2 spin liquid

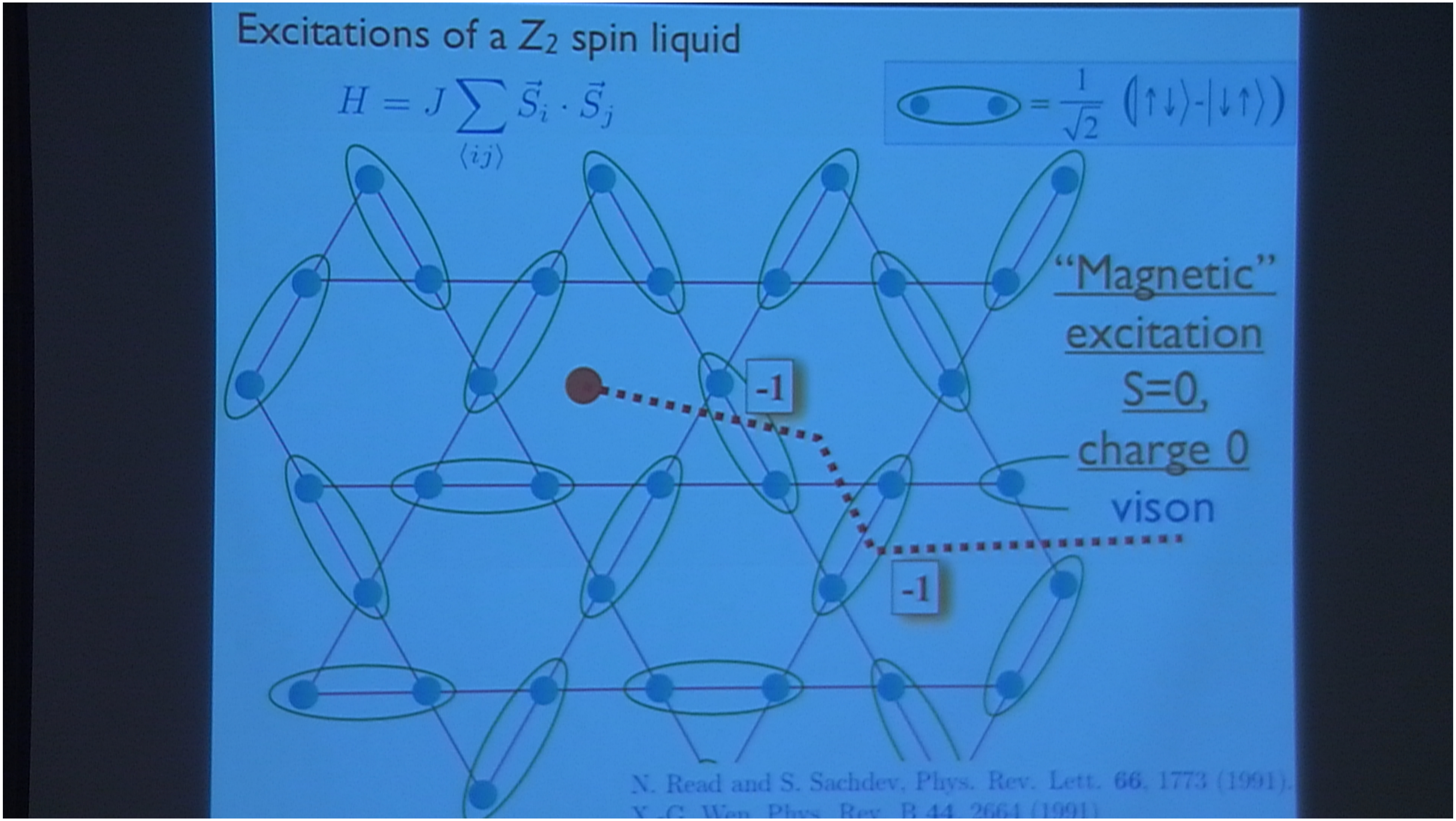
$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



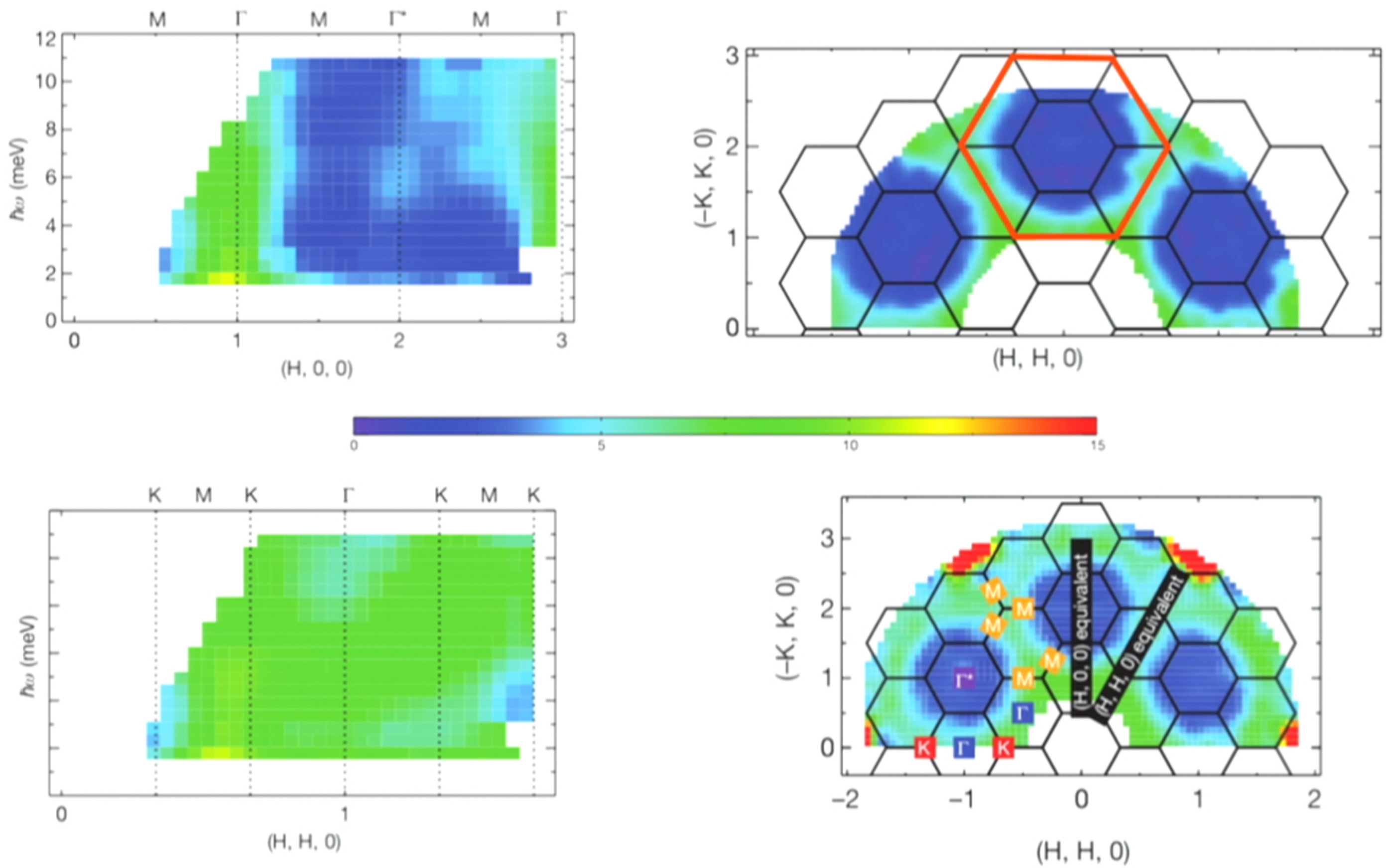
"Electric" excitation S=1/2, charge 0

spinon

N. Read and S. Sachdev, Phys. Rev. Lett. **66**, 1773 (1991).
X.-G. Wen, Phys. Rev. B **44**, 2664 (1991)

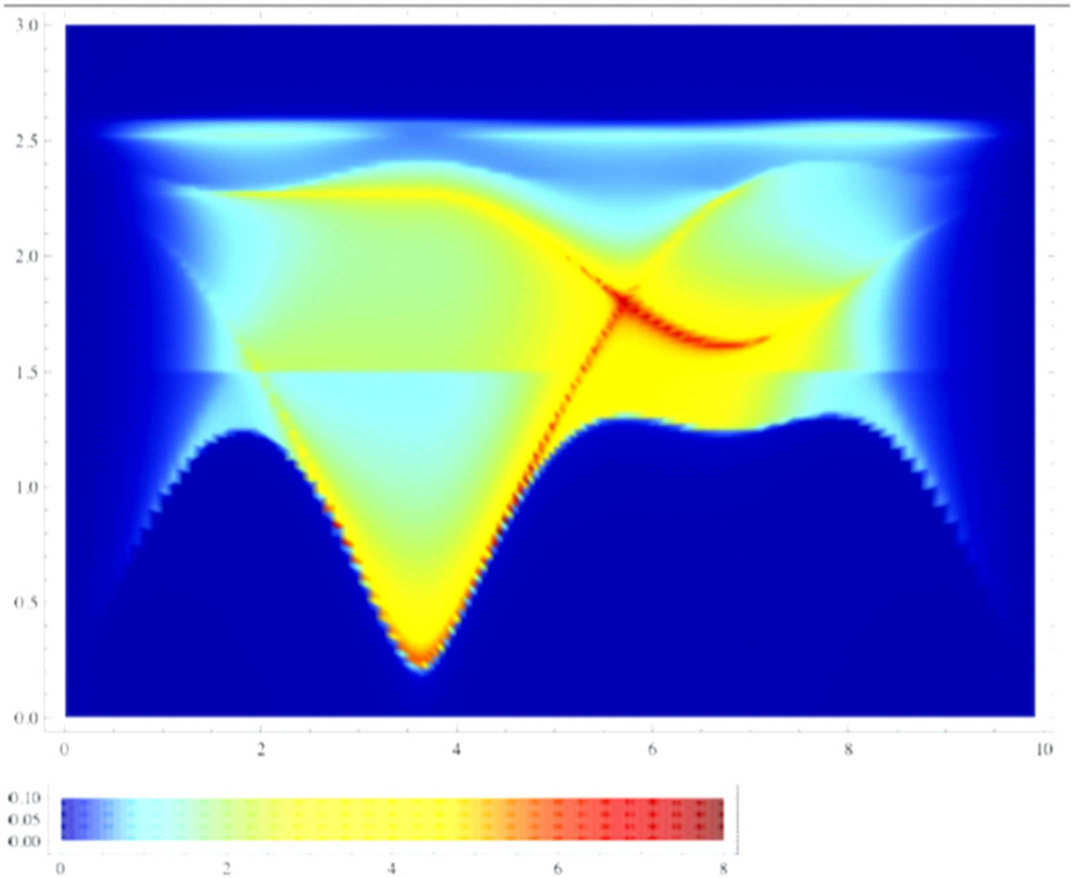

Excitations of a Z_2 spin liquid
$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$
$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$
-1
-1
"Magnetic" excitation S=0, charge 0 vison
N. Read and S. Sachdev, Phys. Rev. Lett. 66, 1773 (1991).

Neutron scattering: a more detailed view as a function of energy



Tian-Heng Han *et. al.*, Nature **492**, 406 (2012)

Neutron scattering: a more detailed view as a function of energy



Contribution of 2 spinons in a Z_2 spin liquid;

In progress: contribution of spinon-induced vison pair-production

M. Punk, D. Chowdhury, S. Gopalakrishnan, and S. Sachdev, to appear

Also: T. Dodds, S. Bhattacharjee, and Yong Baek Kim, arXiv: 1303.1154

Modern phases of quantum matter

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2. (a) Quasiparticles with quantum numbers different from those of the electron

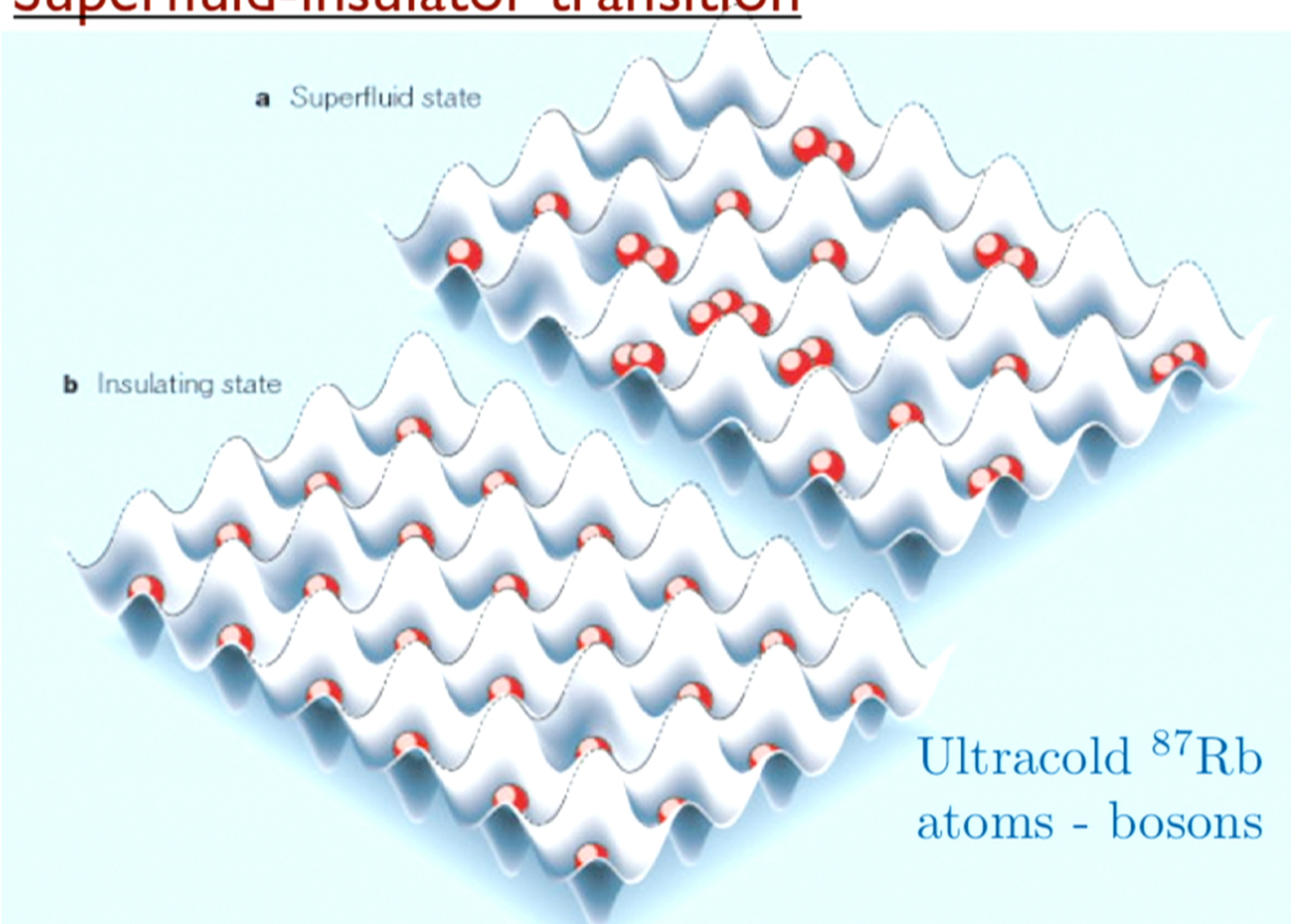
(b) No quasiparticles

Entanglement but no quasiparticles

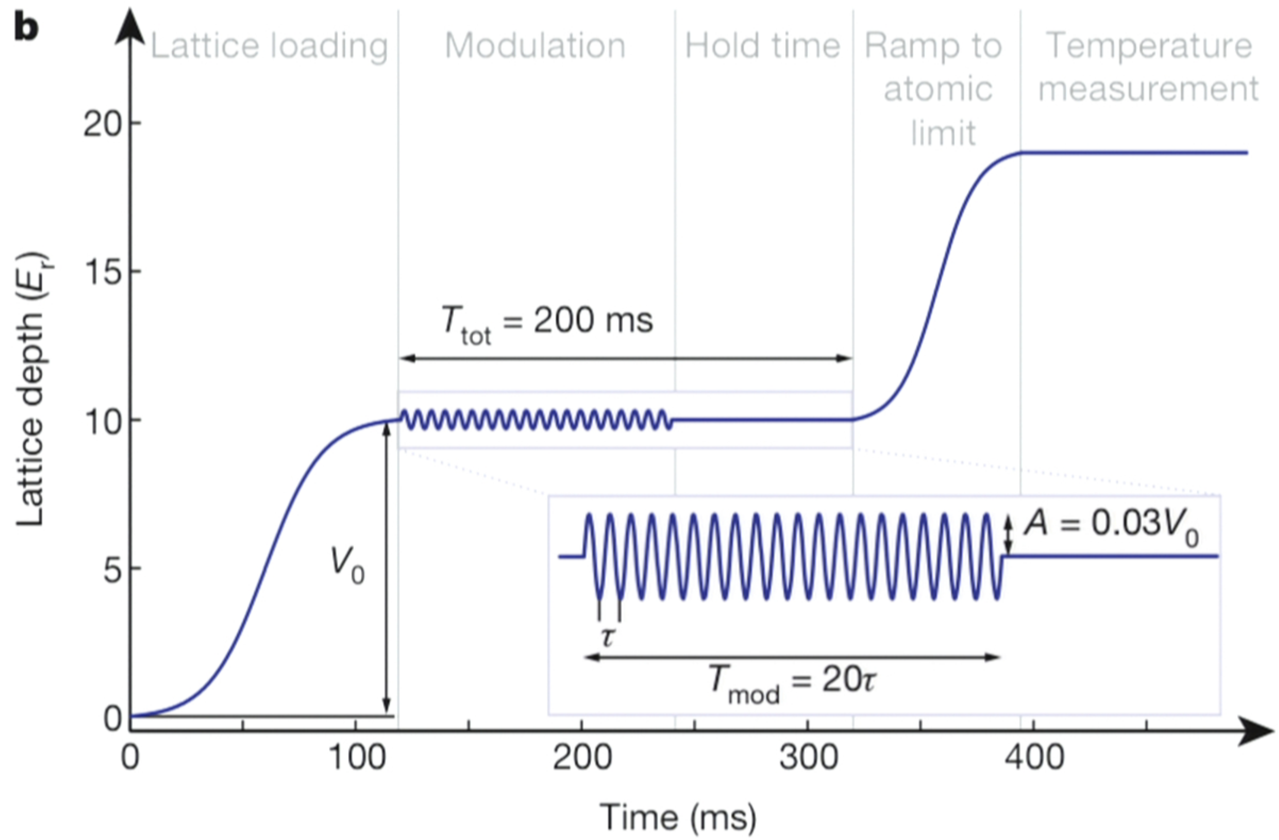
1. Superfluid-insulator transition of ultracold atoms in optical lattices:
   Conformal field theories and gauge-gravity duality

2. Metals with antiferromagnetism, and high temperature superconductivity
   The pnictides and the cuprates

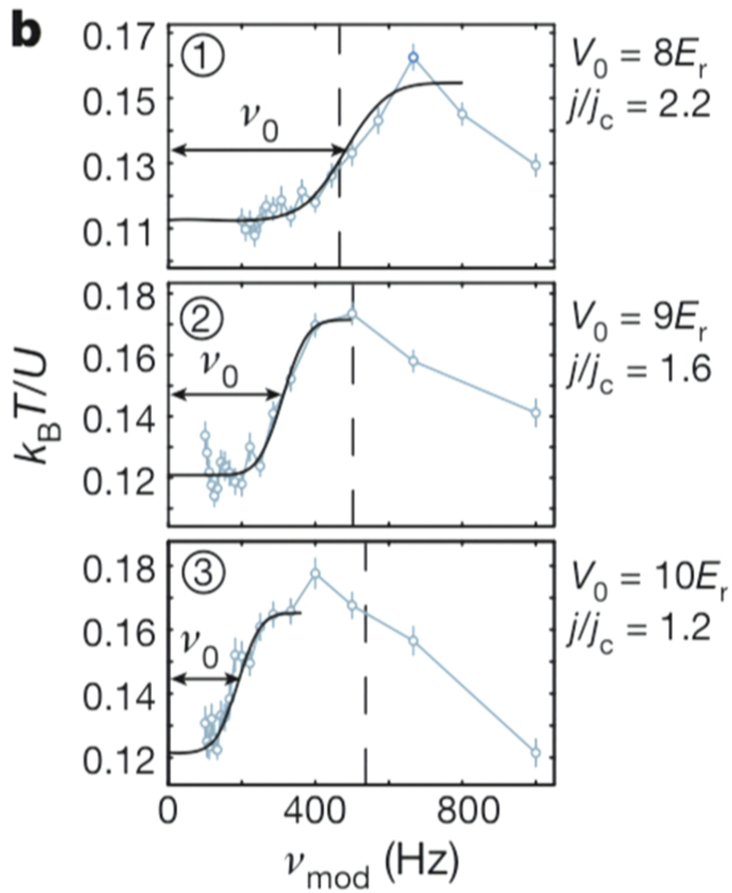
Superfluid-insulator transition



M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).



Manuel Endres, Takeshi Fukuhara, David Pekker, Marc Cheneau, Peter Schaub, Christian Gross, Eugene Demler, Stefan Kuhr, and Immanuel Bloch, *Nature* **487**, 454 (2012).



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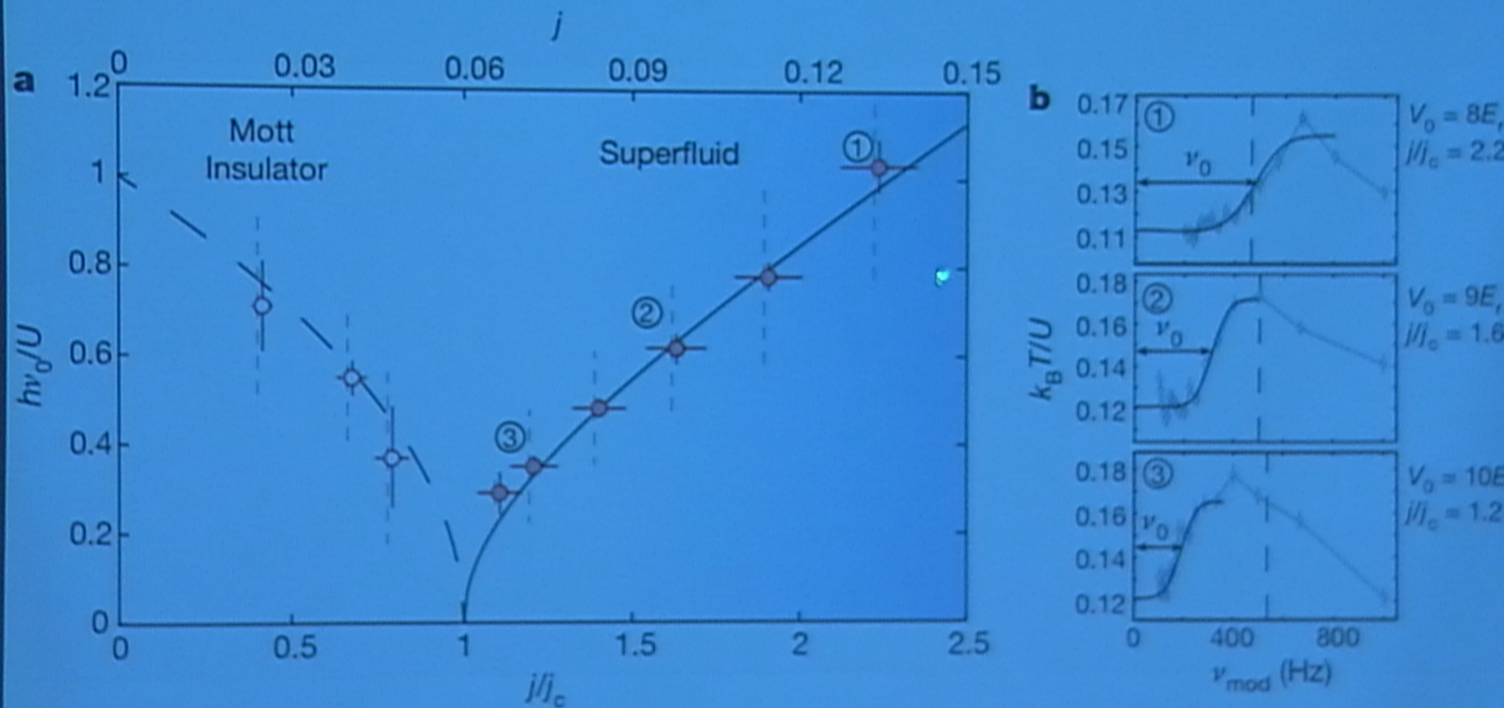

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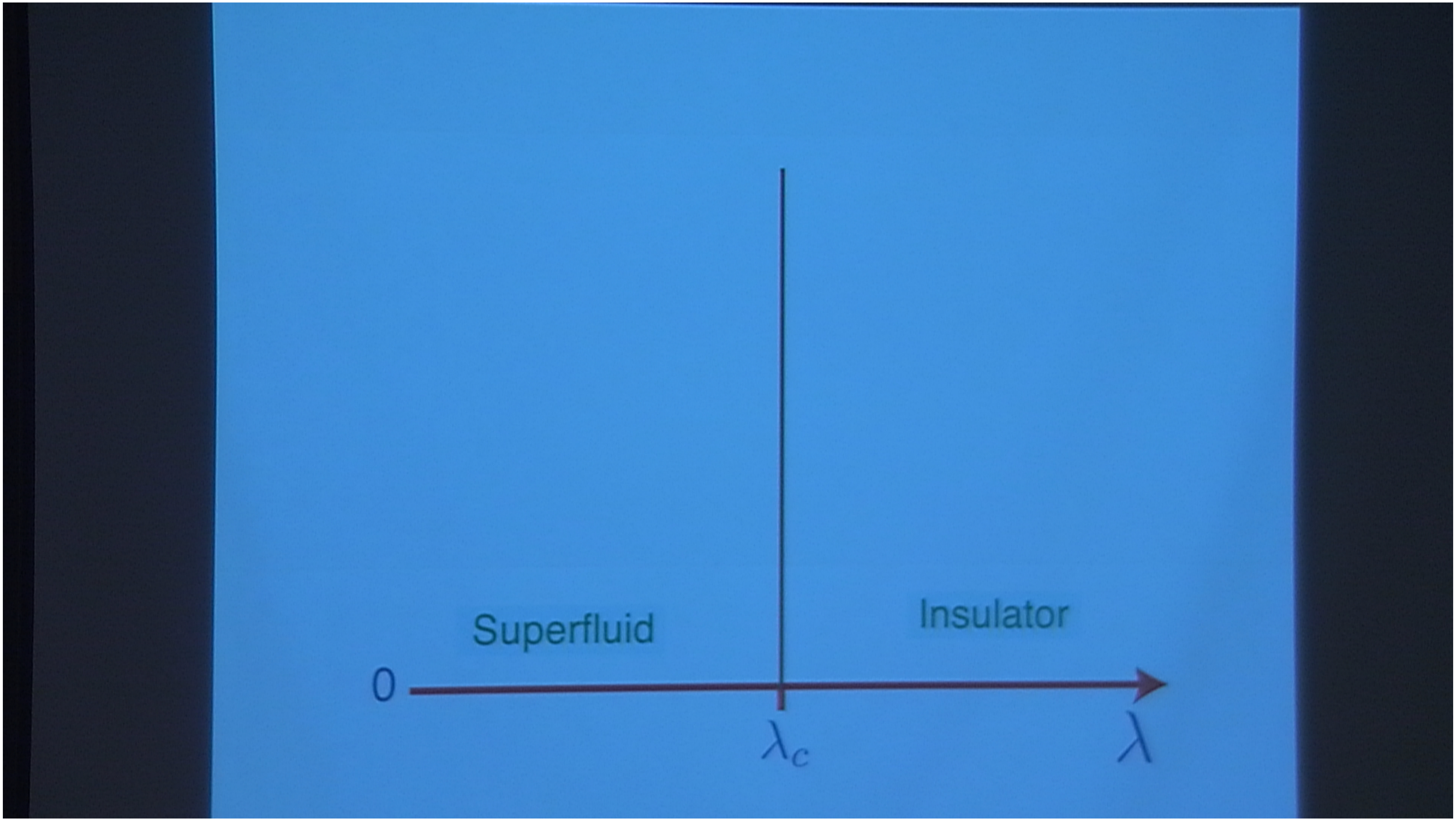
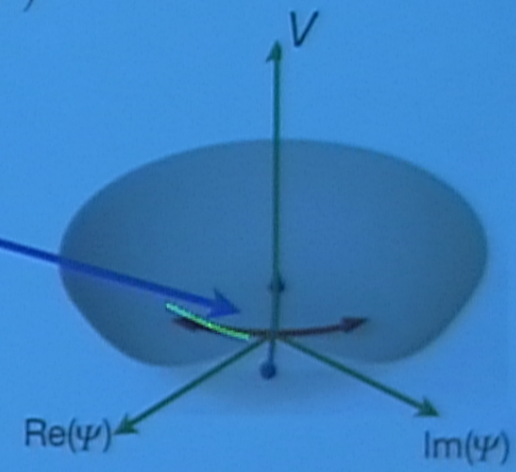

Superfluid
Insulator
0
λ_c
λ

$$\mathcal{S} = \int d^2r dt \left[|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right]$$

$$V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u \left(|\Psi|^2 \right)^2$$

Particles and holes correspond to the 2 normal modes in the oscillation of Ψ about $\Psi = 0$.



$\langle \Psi \rangle \neq 0$

Superfluid

$\langle \Psi \rangle = 0$

Insulator

0 — λ_c — λ

$$\mathcal{S} = \int d^2r\,dt\,\left[|\partial_t \Psi|^2 - c^2|\nabla_r \Psi|^2 - V(\Psi)\right]$$

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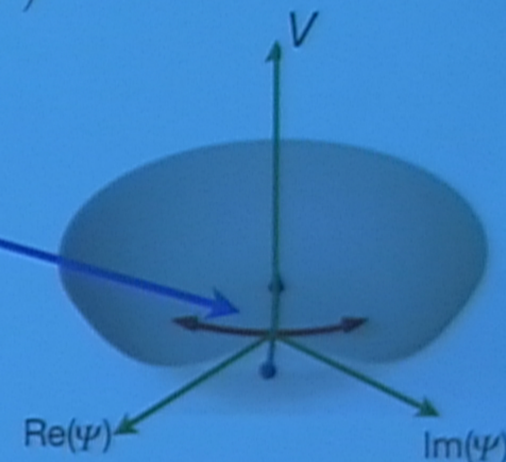
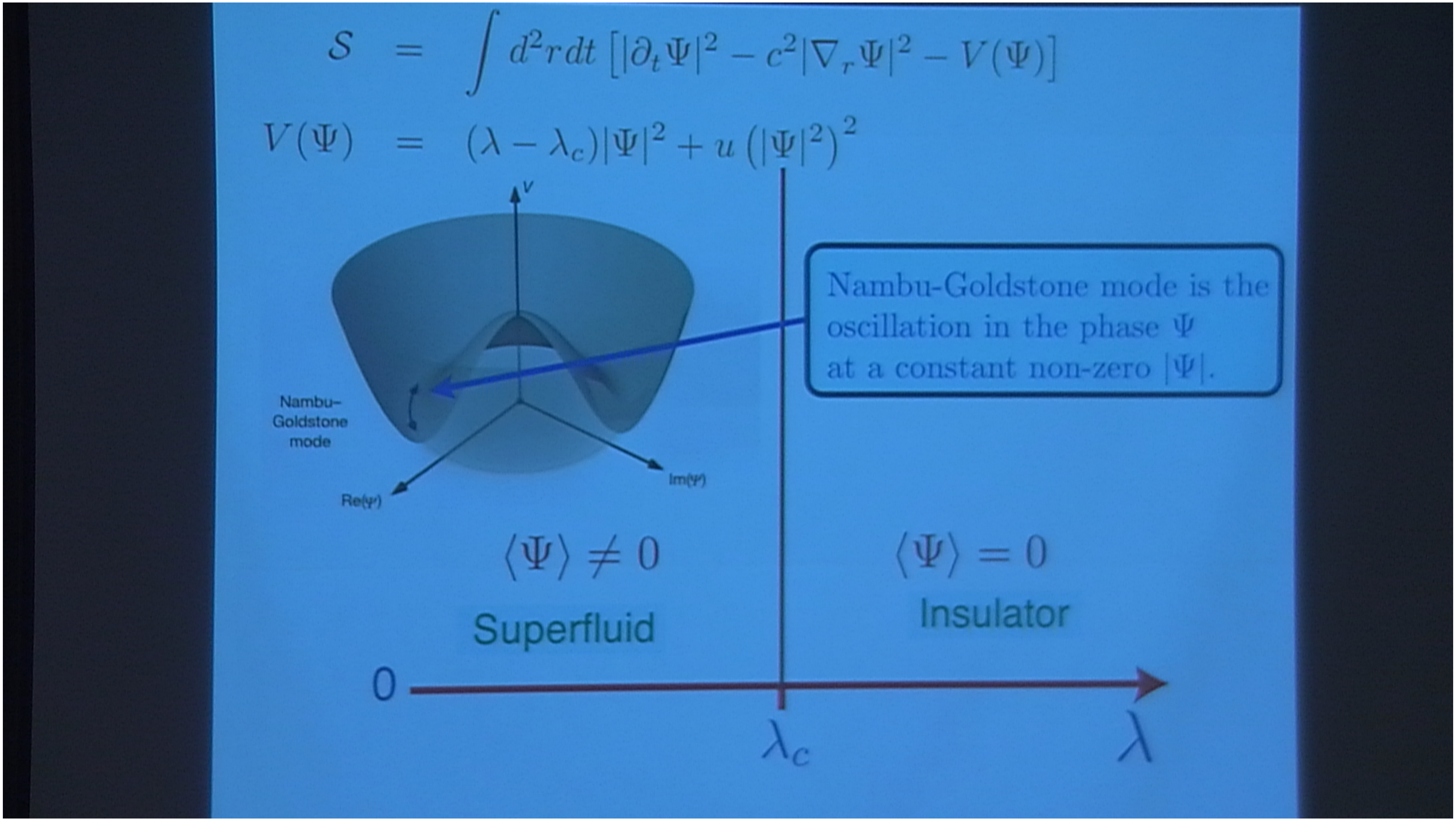


$\langle \Psi \rangle \neq 0$ | $\langle \Psi \rangle = 0$
Superfluid | Insulator

0 — λ_c — λ



$\mathcal{S} = \int d^2r\,dt\,\left[|\partial_t\Psi|^2 - c^2|\nabla_r\Psi|^2 - V(\Psi)\right]$
$V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u\left(|\Psi|^2\right)^2$
V
Nambu–Goldstone mode
Re(Ψ)
Im(Ψ)
Nambu-Goldstone mode is the oscillation in the phase Ψ at a constant non-zero |Ψ|.
$\langle\Psi\rangle \neq 0$
Superfluid
$\langle\Psi\rangle = 0$
Insulator
0
λ_c
λ

$$\mathcal{S} = \int d^2r\,dt \left[|\partial_t \Psi|^2 - c^2|\nabla_r \Psi|^2 - V(\Psi)\right]$$

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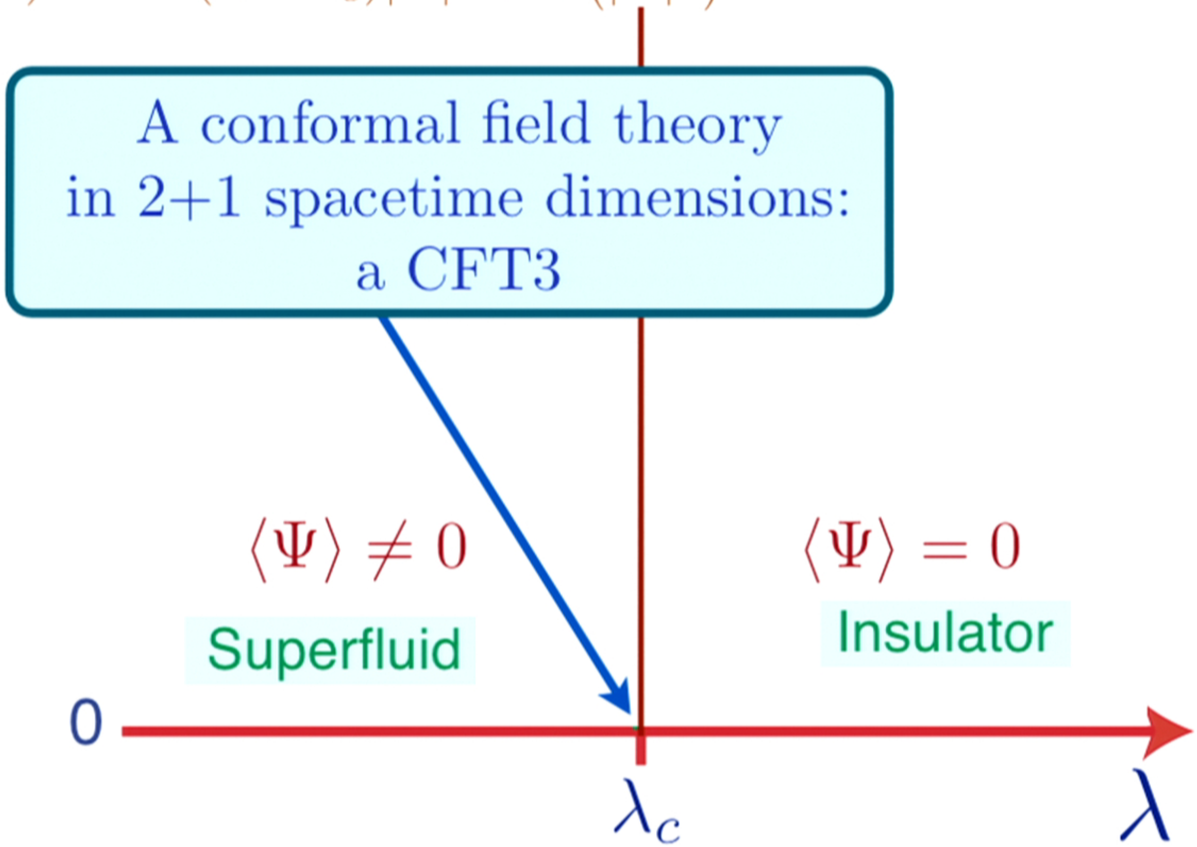
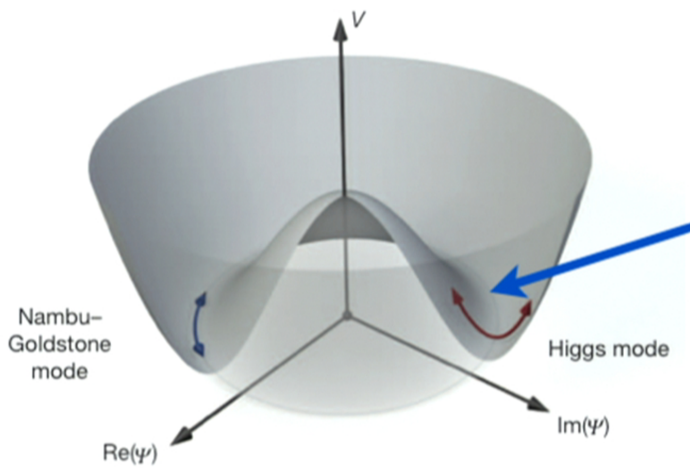


$$\mathcal{S} = \int d^2r dt \left[|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right]$$

$$V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u \left(|\Psi|^2 \right)^2$$



Higgs mode is the oscillation in the amplitude $|\Psi|$. This decays rapidly by emitting pairs of Nambu-Goldstone modes.

$\langle \Psi \rangle \neq 0$

Superfluid

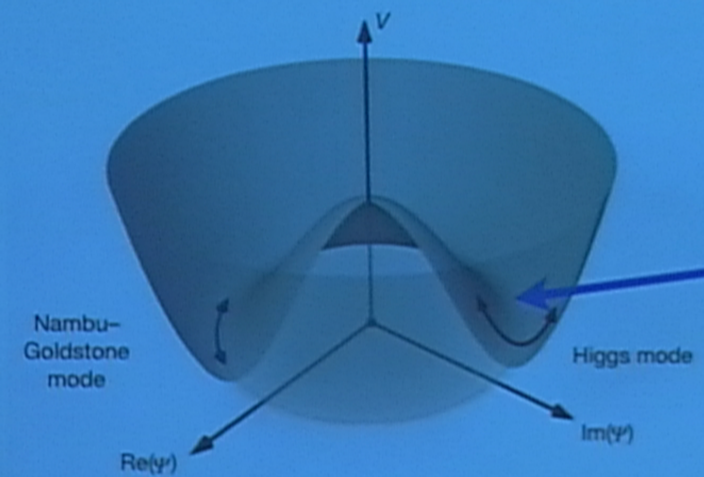
$\langle \Psi \rangle = 0$

Insulator

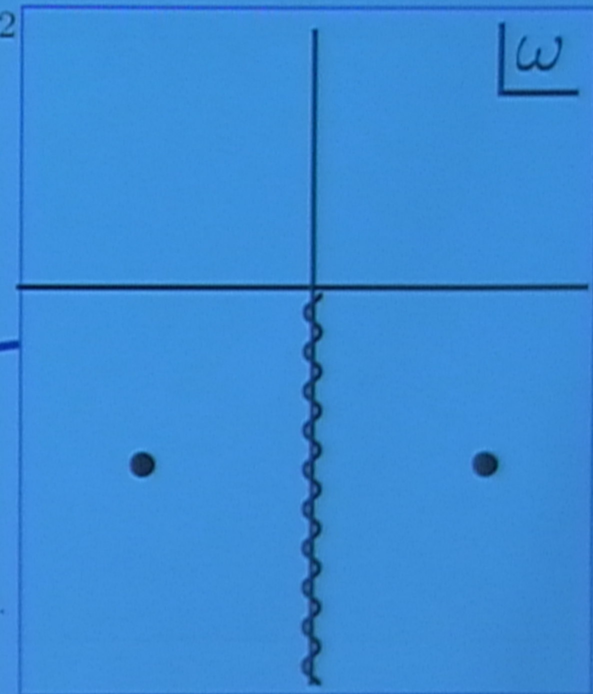
0 λ_c λ

$$\mathcal{S} = \int d^2r\,dt \left[|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right]$$

$$V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u\left(|\Psi|^2\right)^2$$



D. Podolsky and S. Sachdev, Phys. Rev. B **86**, 054508 (2012).



$$\frac{\omega_{\rm pole}}{\Delta} = -i\,\frac{4}{\pi} + \frac{1}{N}\left(\frac{16\left(4+\sqrt{2}\log\left(3-2\sqrt{2}\right)\right)}{\pi^2} + 2.46531203396\, i \right) + \mathcal{O}\left(\frac{1}{N^2}\right)$$

where Δ is the particle gap at the complementary point in the insulator state, and $N = 2$ is the number of vector components of Ψ.

Observation of Higgs quasi-normal mode in experiments

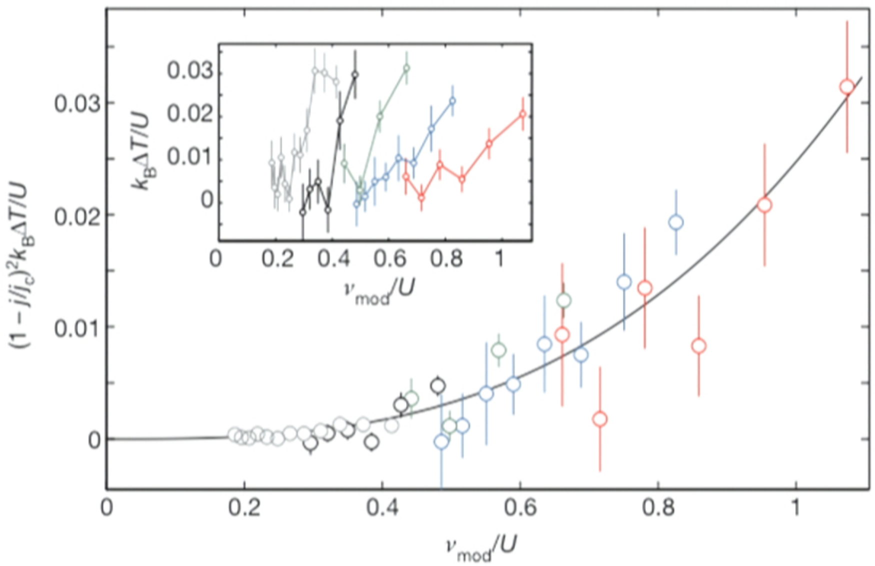
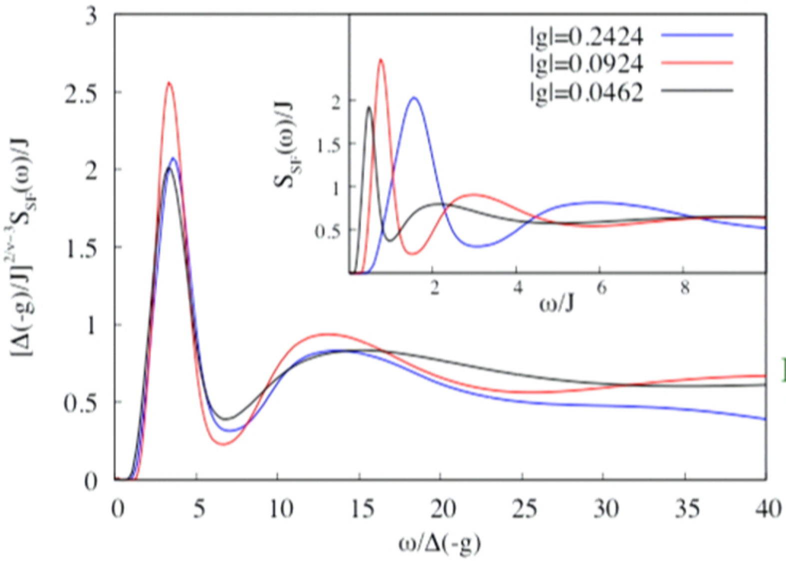


Figure 4 | **Scaling of the low-frequency response.** The low-frequency response in the superfluid regime shows a scaling compatible with the prediction $(1 - j/j_c)^{-2}\nu^3$ (Methods). Shown is the temperature response rescaled with $(1 - j/j_c)^2$ for $V_0 = 10E_r$ (grey), $9.5E_r$ (black), $9E_r$ (green), $8.5E_r$ (blue) and $8E_r$ (red) as a function of the modulation frequency. The black line is a fit of the form $a\nu^b$ with a fitted exponent $b = 2.9(5)$. The inset shows the same data points without rescaling, for comparison. Error bars, s.e.m.

Scaling of spectral response functions predicted in D. Podolsky and S. Sachdev, Phys. Rev. B **86**, 054508 (2012).

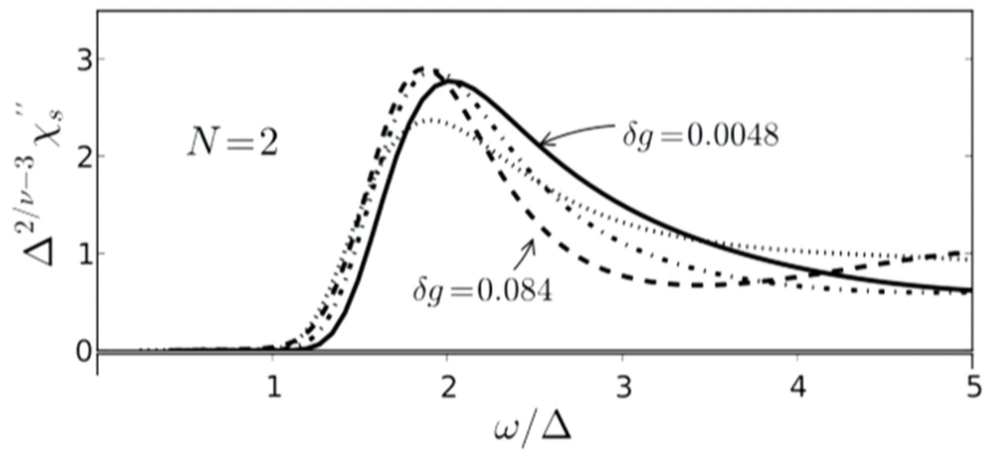
Manuel Endres, Takeshi Fukuhara, David Pekker, Marc Cheneau, Peter Schaub, Christian Gross, Eugene Demler, Stefan Kuhr, and Immanuel Bloch, *Nature* **487**, 454 (2012).

Observation of Higgs quasi-normal mode in quantum Monte Carlo

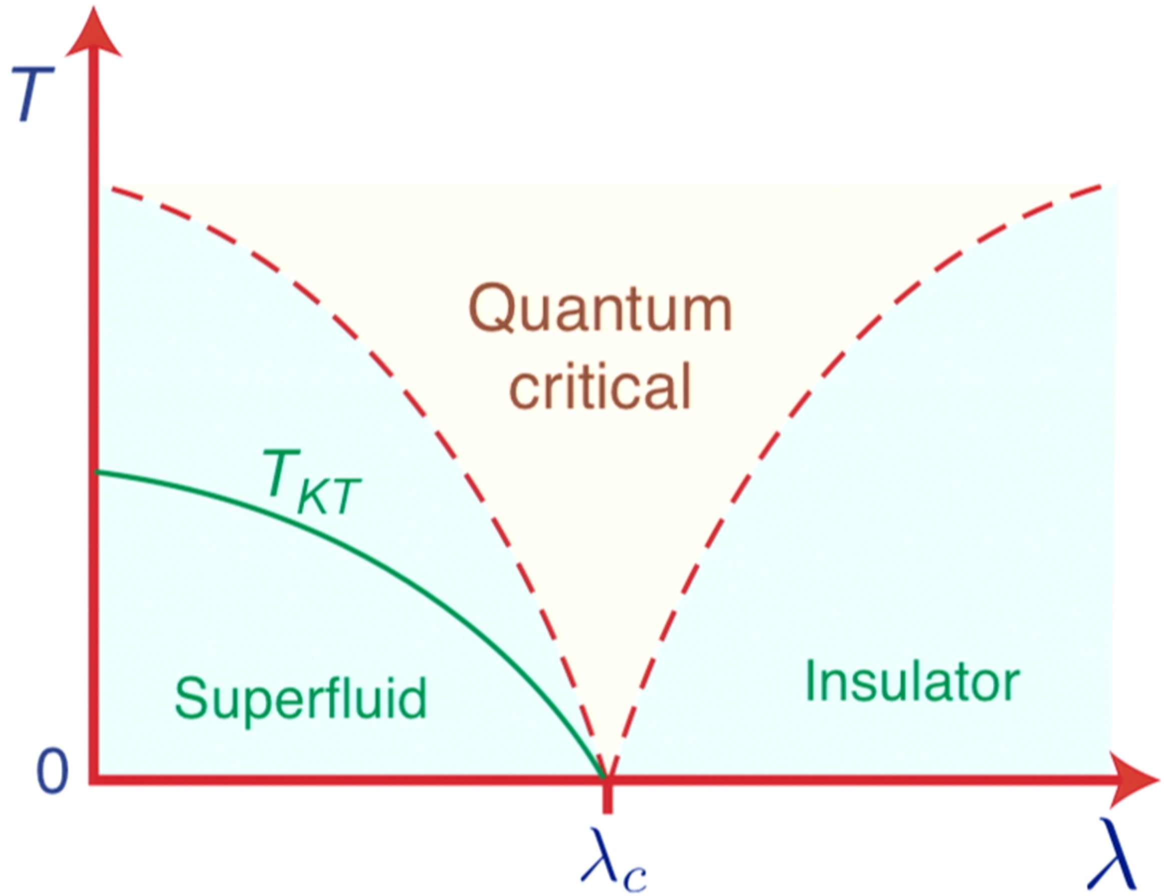


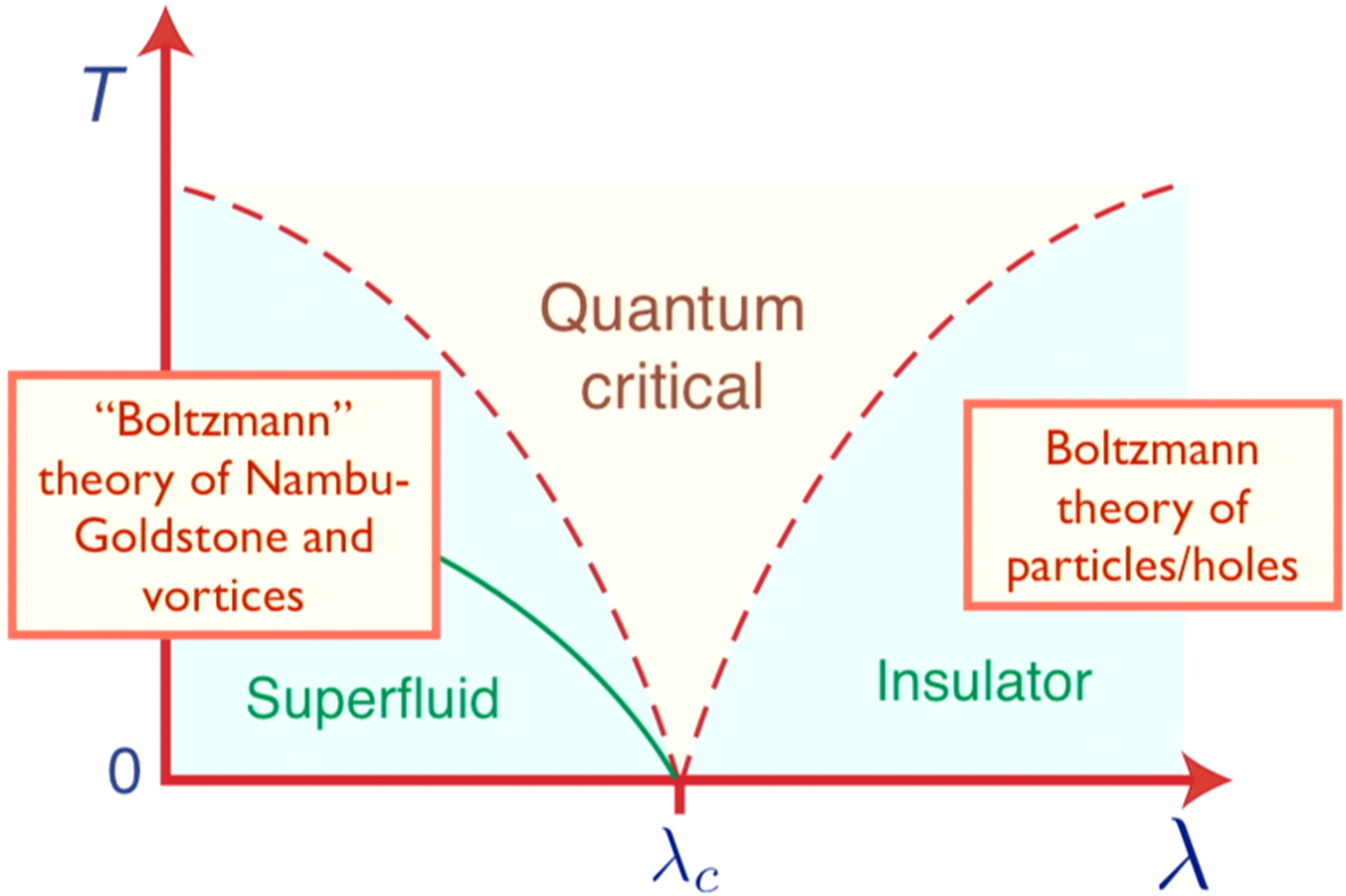
Scaling of spectral response functions predicted in D. Podolsky and S. Sachdev, Phys. Rev. B **86**, 054508 (2012).

Kun Chen, Longxiang Liu, Youjin Deng, Lode Pollet, and Nikolay Prokof'ev, arXiv:1301.3139

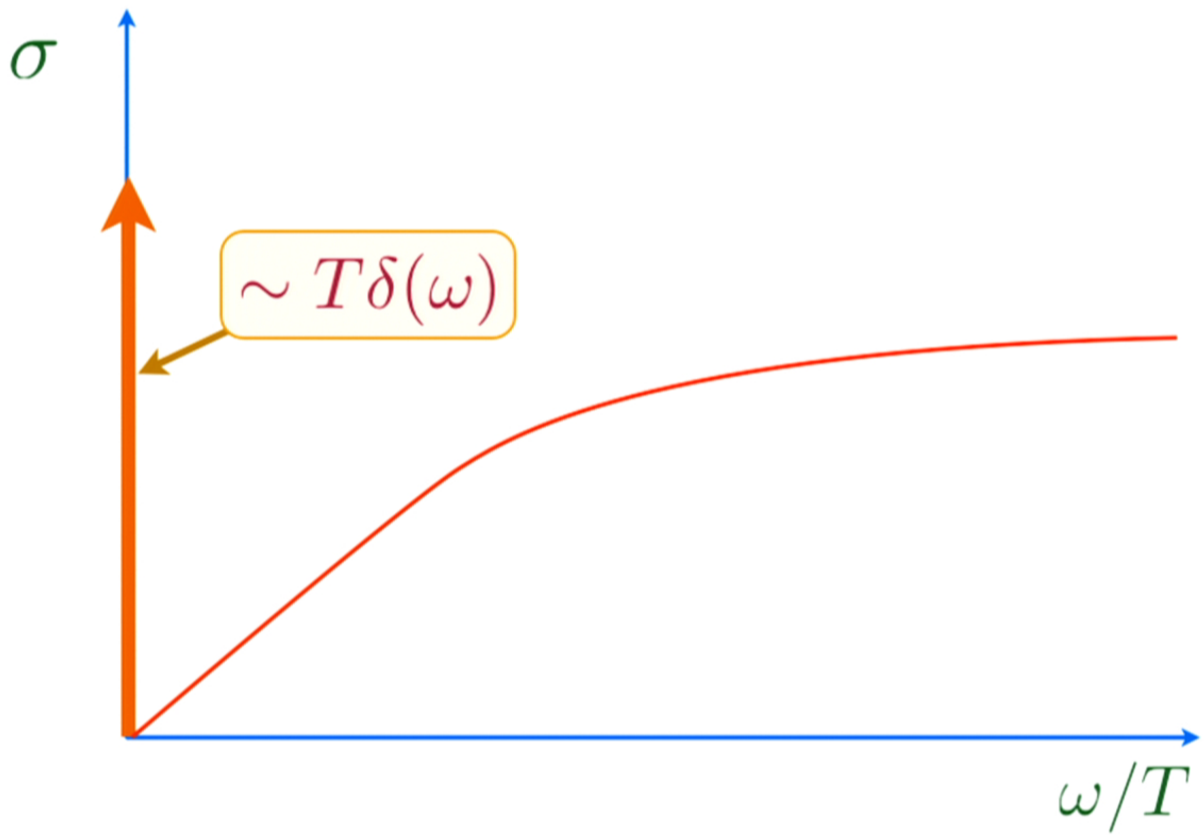


Snir Gazit, Daniel Podolsky, and Assa Auerbach, arXiv:1212.3759


T
Quantum
critical
T_{KT}
Superfluid
Insulator
0
λ_c
λ

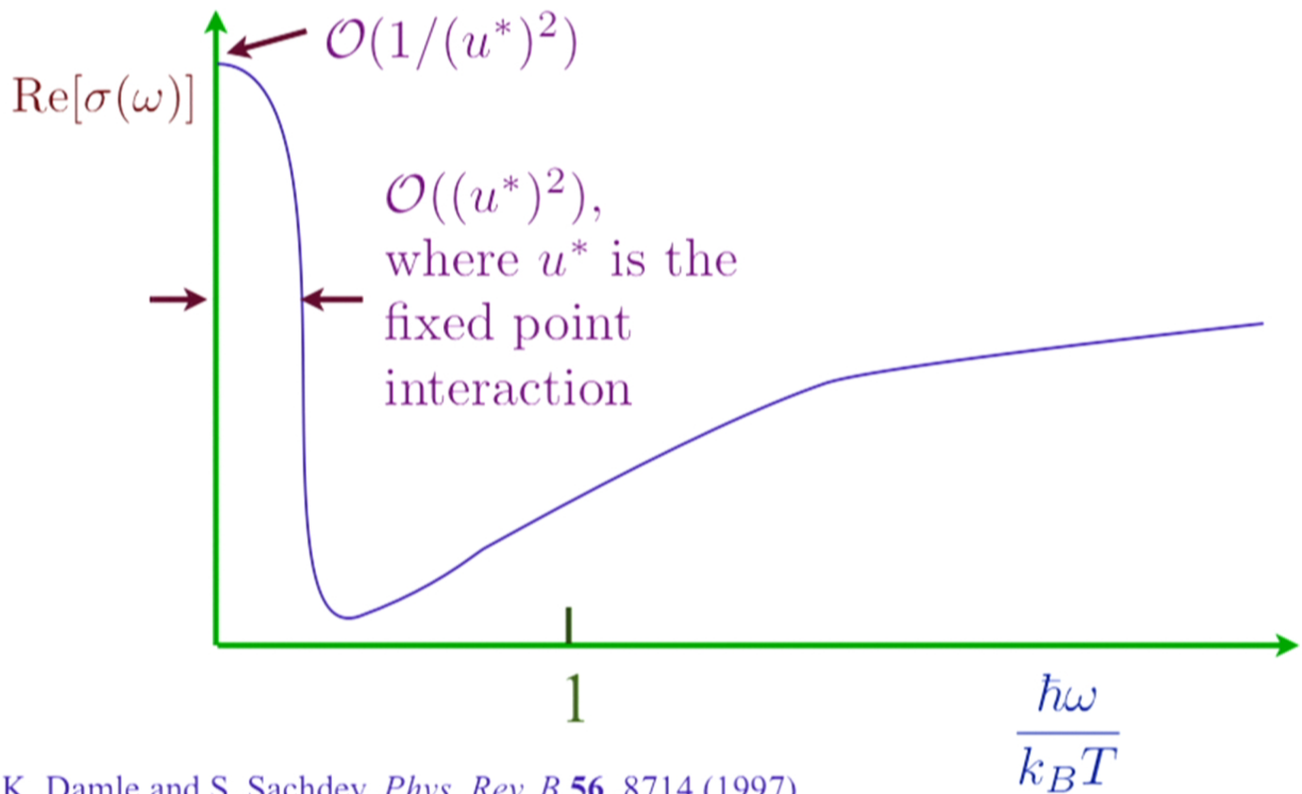

T
Quantum critical
"Boltzmann" theory of Nambu-Goldstone and vortices
Boltzmann theory of particles/holes
Superfluid
Insulator
0
λ_c
λ

Electrical transport in a free CFT3 for *T* > 0



Electrical transport for a CFT3, *assuming* quasiparticles with weak interactions

$$\sigma(\omega, T) = \frac{e^2}{h}\Sigma\left(\frac{\hbar\omega}{k_B T}\right) \quad ; \quad \Sigma \rightarrow \text{a universal function}$$



K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Quantum critical dynamics

Quantum *"nearly perfect fluid"*
with shortest possible *local* equilibration time, $\tau_{\rm eq}$

$$\boxed{\tau_{\rm eq} = \mathcal{C}\,\frac{\hbar}{k_B T}}$$

where $\mathcal{C}$ is a *universal* constant.

Response functions are characterized by poles in LHP
with $\omega \sim k_B T/\hbar$.
These poles (quasi-normal modes) appear naturally in
the holographic theory.
(Analogs of Higgs quasi-normal mode.)

S. Sachdev, *Quantum Phase Transitions,* Cambridge (1999).

Quantum critical dynamics

Transport co-oefficients not determined by collision rate of quasiparticles, but by fundamental constants of nature

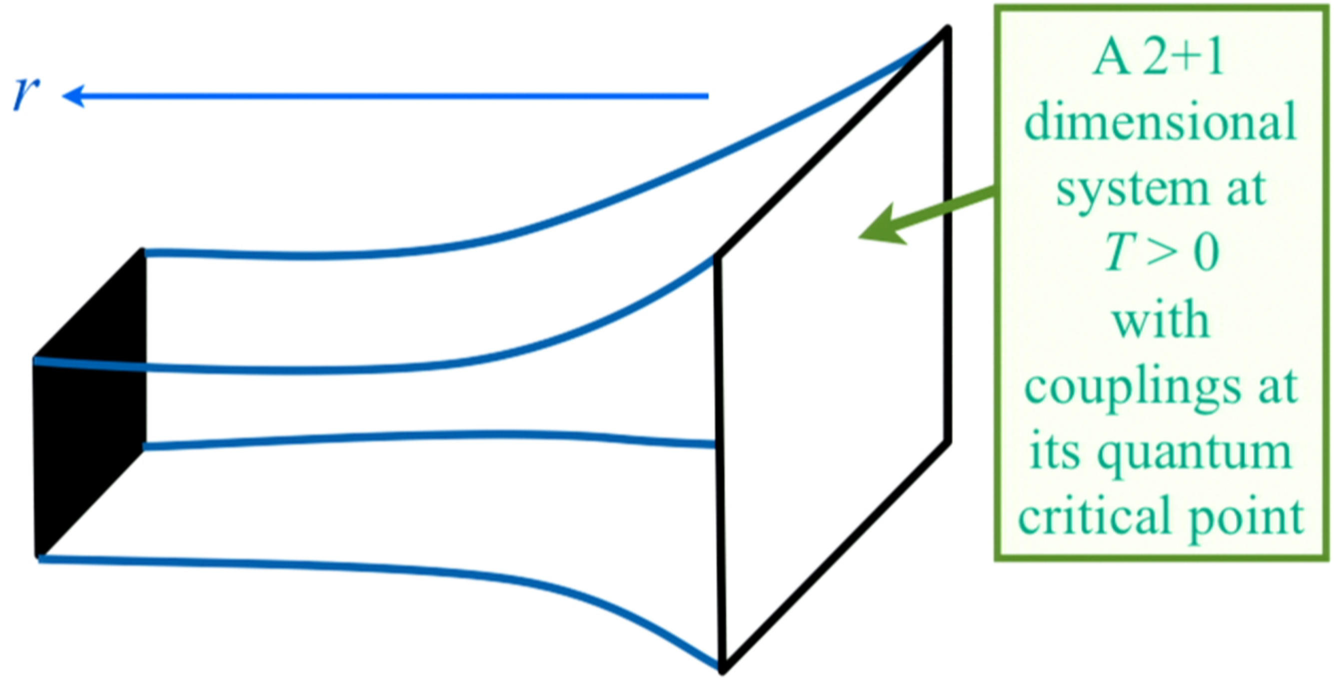
Conductivity

$$\sigma = \frac{Q^2}{h} \times [\text{Universal constant } \mathcal{O}(1) \ ]$$

(Q is the "charge" of one boson)

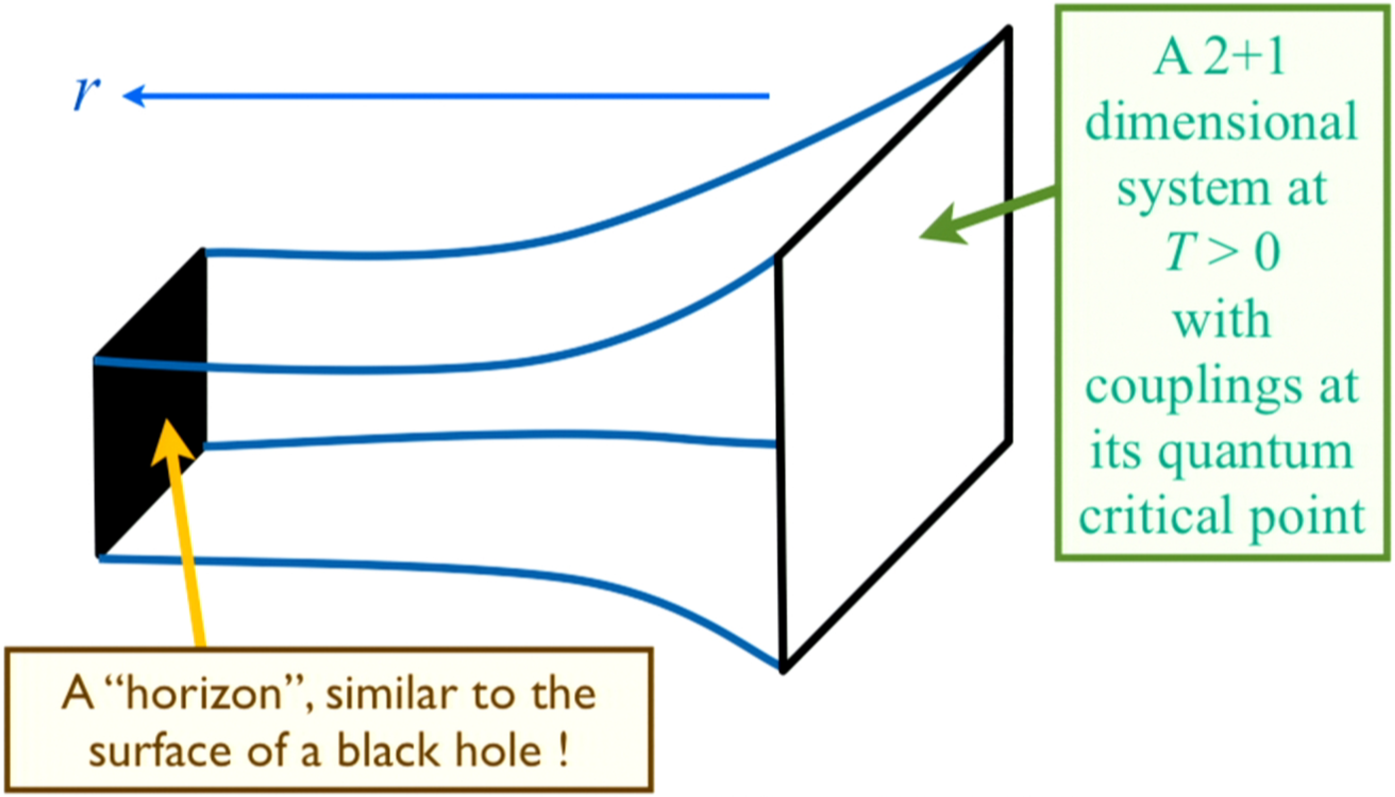
M.P.A. Fisher, G. Grinstein, and S.M. Girvin, *Phys. Rev. Lett.* **64**, 587 (1990)
K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Gauge-gravity duality at non-zero temperatures

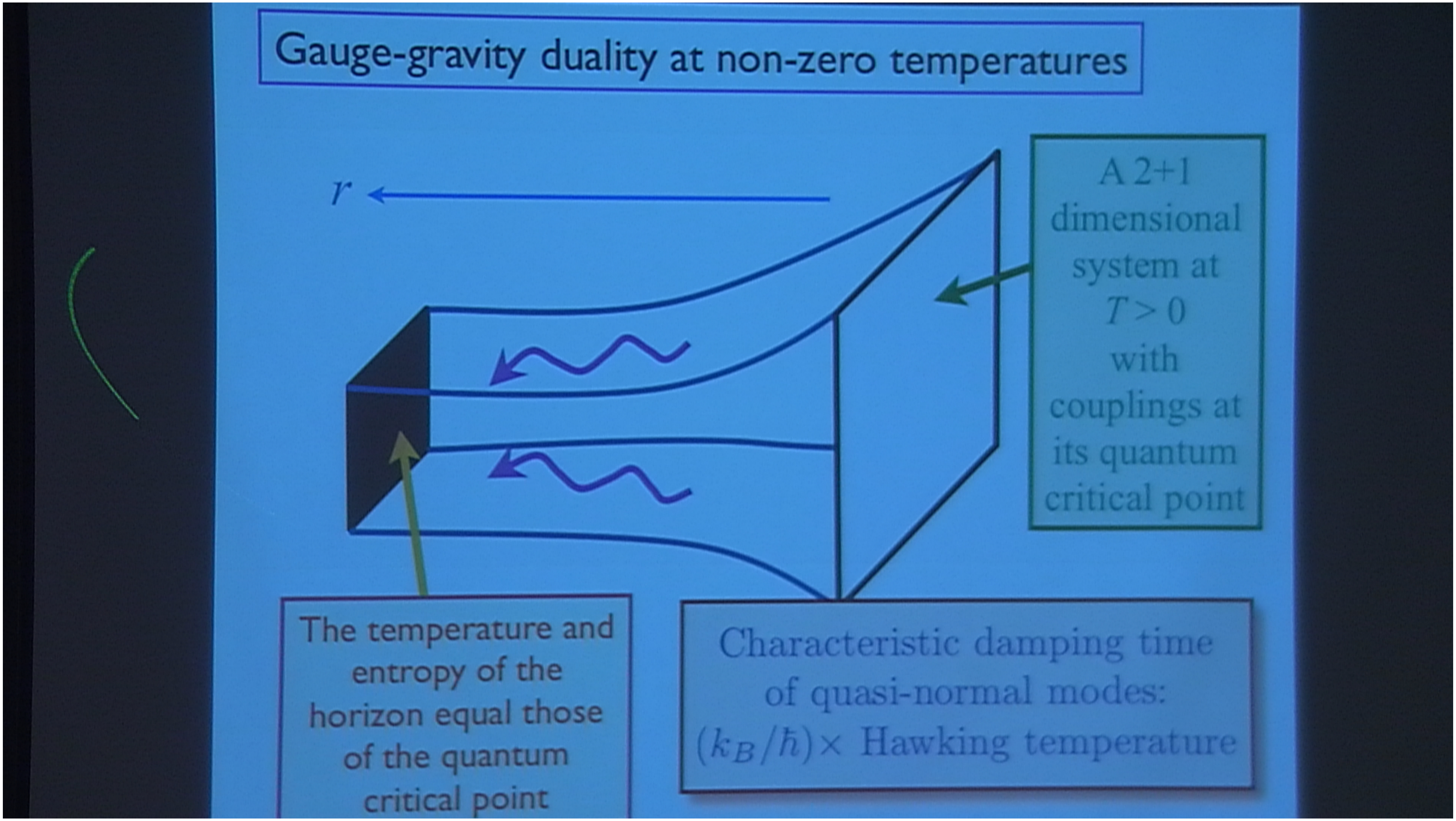


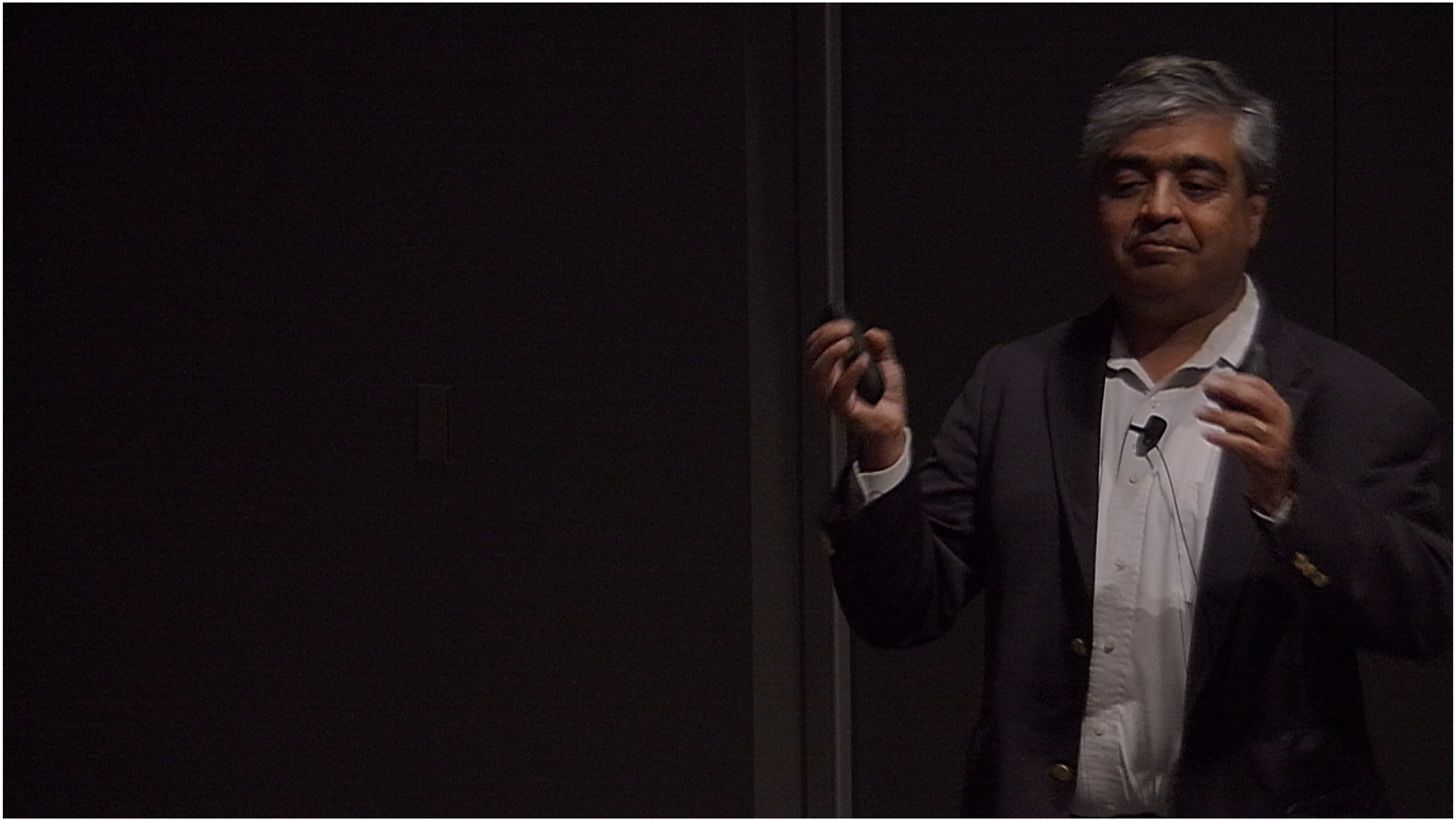
$$\mathcal{S}_E = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right]$$

Gauge-gravity duality at non-zero temperatures

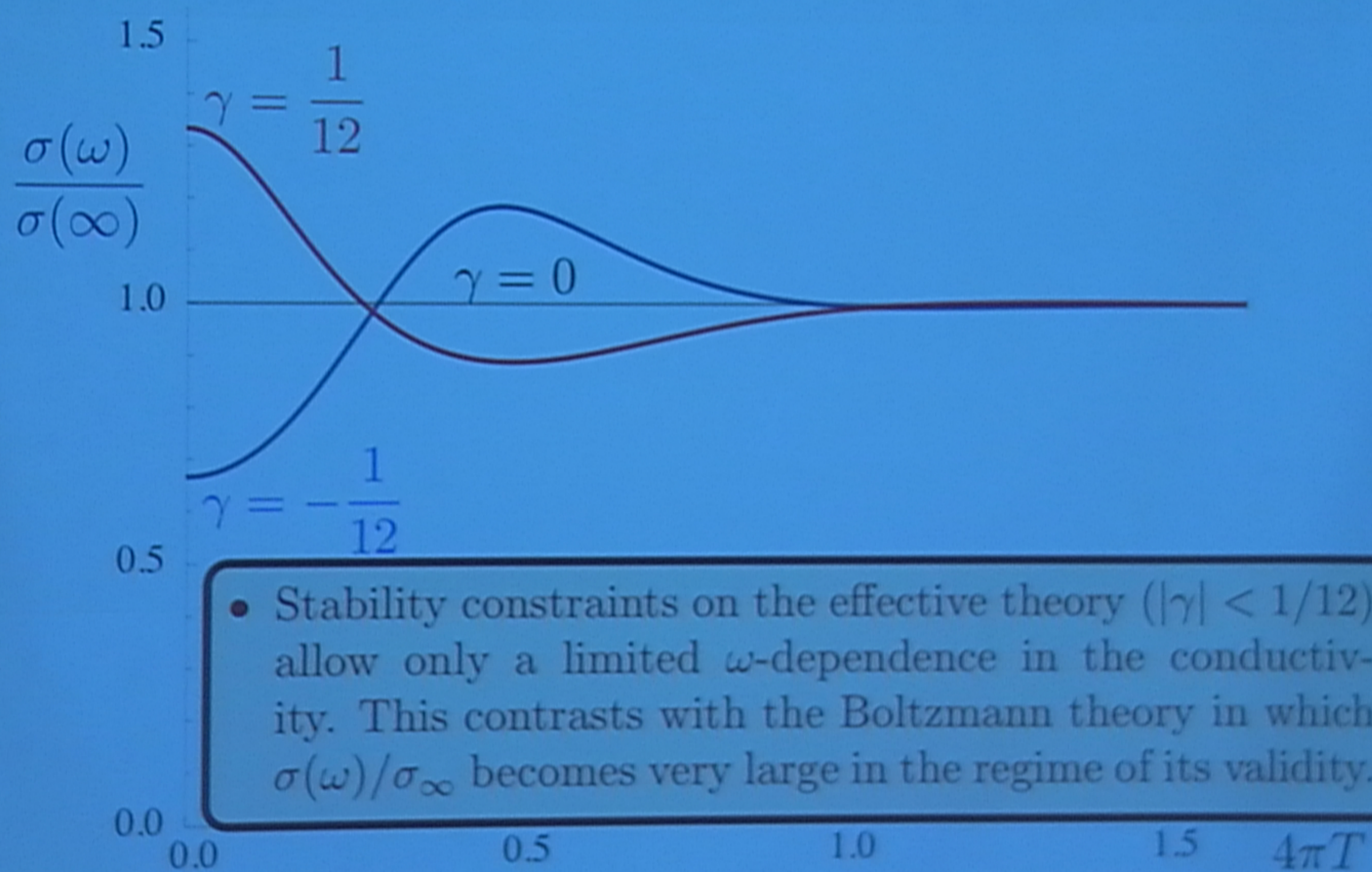


$$S_E = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right]$$


Gauge-gravity duality at non-zero temperatures
r
A 2+1 dimensional system at T > 0 with couplings at its quantum critical point
The temperature and entropy of the horizon equal those of the quantum critical point
Characteristic damping time of quasi-normal modes: (k_B/ℏ)× Hawking temperature



AdS4 theory of quantum criticality



- Stability constraints on the effective theory ($|\gamma| < 1/12$) allow only a limited ω-dependence in the conductivity. This contrasts with the Boltzmann theory in which $\sigma(\omega)/\sigma_\infty$ becomes very large in the regime of its validity.

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

AdS4 theory of quantum criticality

PRL 95, 180603 (2005) PHYSICAL REVIEW LETTERS week ending 28 OCTOBER 2005

Universal Scaling of the Conductivity at the Superfluid-Insulator Phase Transition

Jurij Šmakov and Erik Sørensen
Department of Physics and Astronomy, McMaster University, Hamilton, Ontario L8S 4M1, Canada
(Received 30 May 2005; published 27 October 2005)

The scaling of the conductivity at the superfluid-insulator quantum phase transition in two dimensions is studied by numerical simulations of the Bose-Hubbard model. In contrast to previous studies, we focus on properties of this model in the experimentally relevant thermodynamic limit at finite temperature T. We find clear evidence for *deviations* from ω_k scaling of the conductivity towards ω_k/T scaling at low Matsubara frequencies ω_k. By careful analytic continuation using Padé approximants we show that this behavior carries over to the real frequency axis where the conductivity scales with ω/T at small frequencies and low temperatures. We estimate the universal dc conductivity to be $\sigma^* = 0.45(5)Q^2/h$, distinct from previous estimates in the $T = 0$, $\omega/T \gg 1$ limit.

QMC yields $\sigma(0)/\sigma_\infty \approx 1.36$
Holography yields $\sigma(0)/\sigma_\infty = 1 + 4\gamma$ with $|\gamma| \leq 1/12$.
Maximum possible holographic value $\sigma(0)/\sigma_\infty = 1.33$

Excellent agreement of ω dependence between QMC and holography for $\gamma \approx 1/12$.

W. Witzack-Krempa and S. Sachdev, arXiv:1302.0847; WWK, E. Sorensen, and SS to appear

Traditional CMT

- Identify quasiparticles and their dispersions

- Compute scattering matrix elements of quasiparticles (or of collective modes)

- These parameters are input into a quantum Boltzmann equation

- Deduce dissipative and dynamic properties at non-zero temperatures

Traditional CMT	Holography and black-branes
Identify quasiparticles and their dispersions	Start with strongly interacting CFT without particle- or wave-like excitations
Compute scattering matrix elements of quasiparticles (or of collective modes)	
These parameters are input into a quantum Boltzmann equation	
Deduce dissipative and dynamic properties at non-zero temperatures	

Entanglement but no quasiparticles

1. Superfluid-insulator transition of ultracold atoms in optical lattices:
   Conformal field theories and gauge-gravity duality

2. Metals with antiferromagnetism, and high temperature superconductivity
   The pnictides and the cuprates

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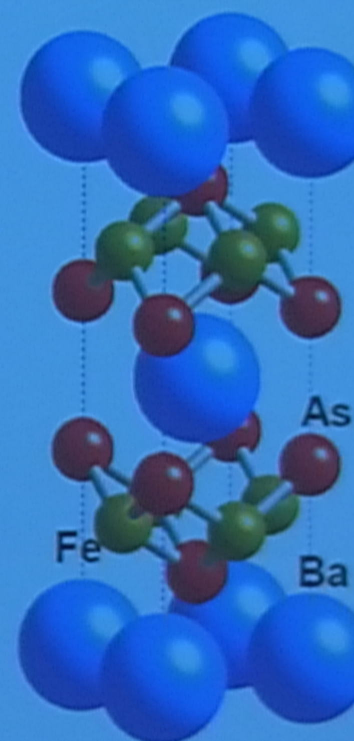
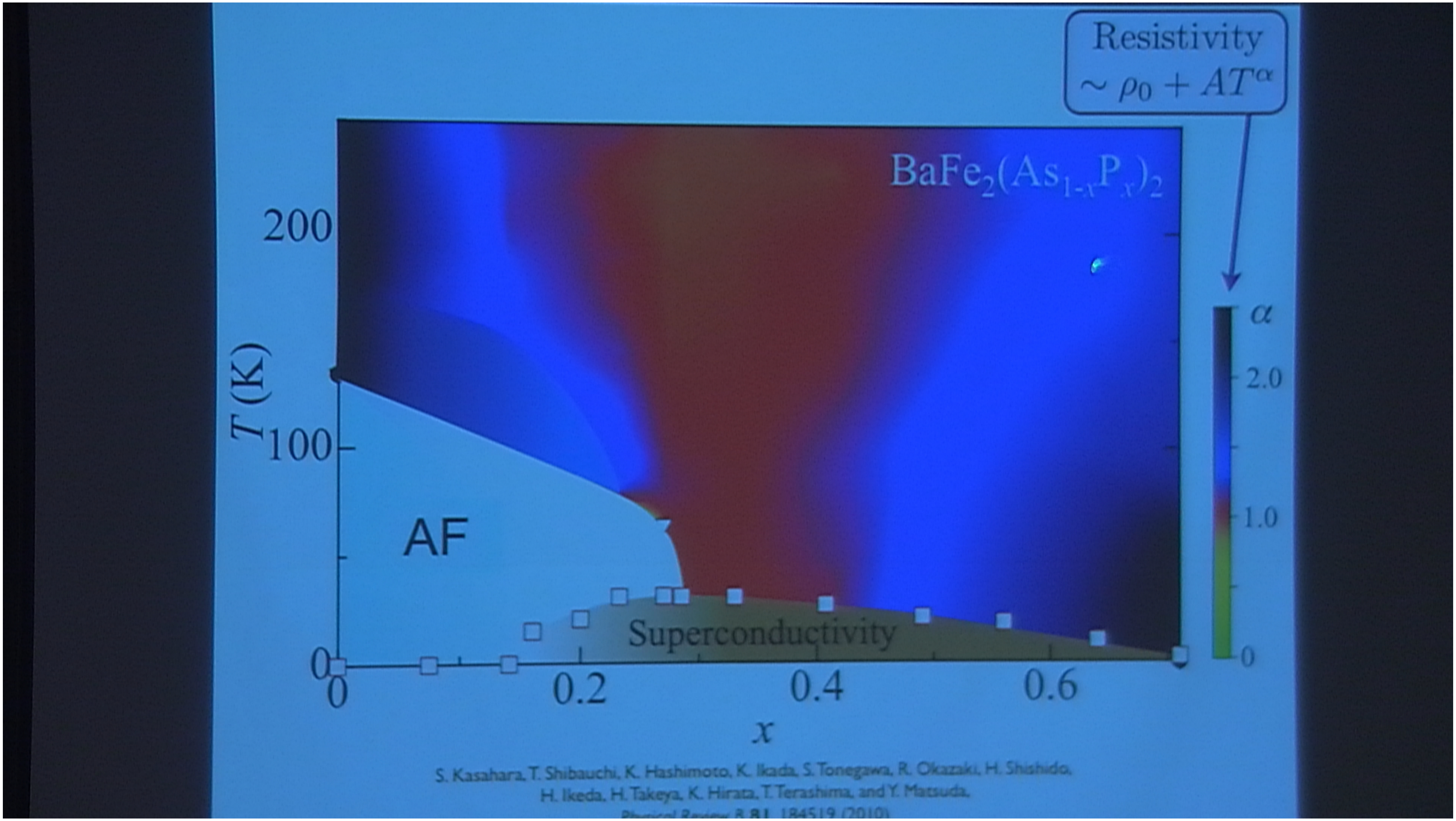
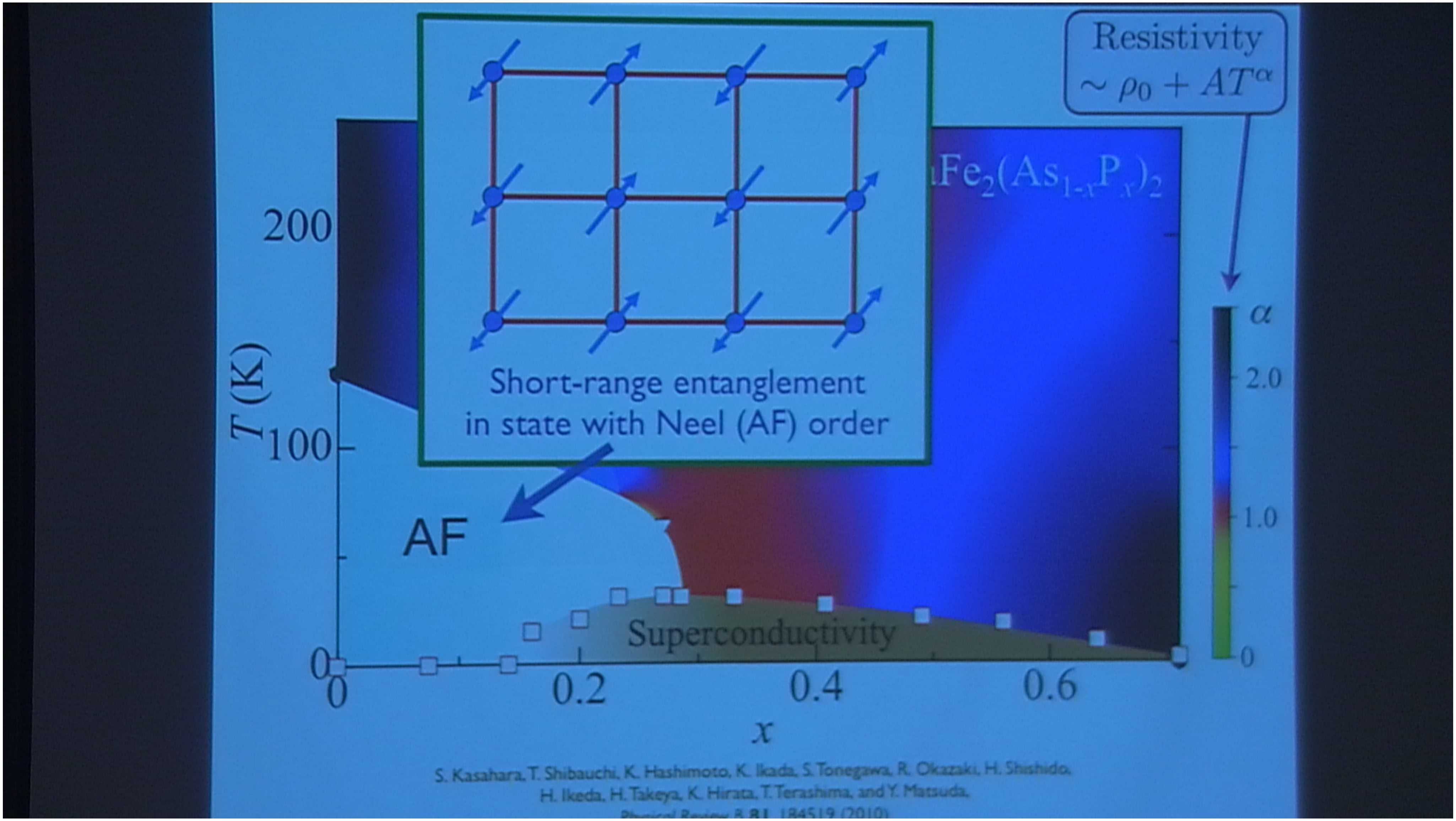
   The pnictides and the cuprates

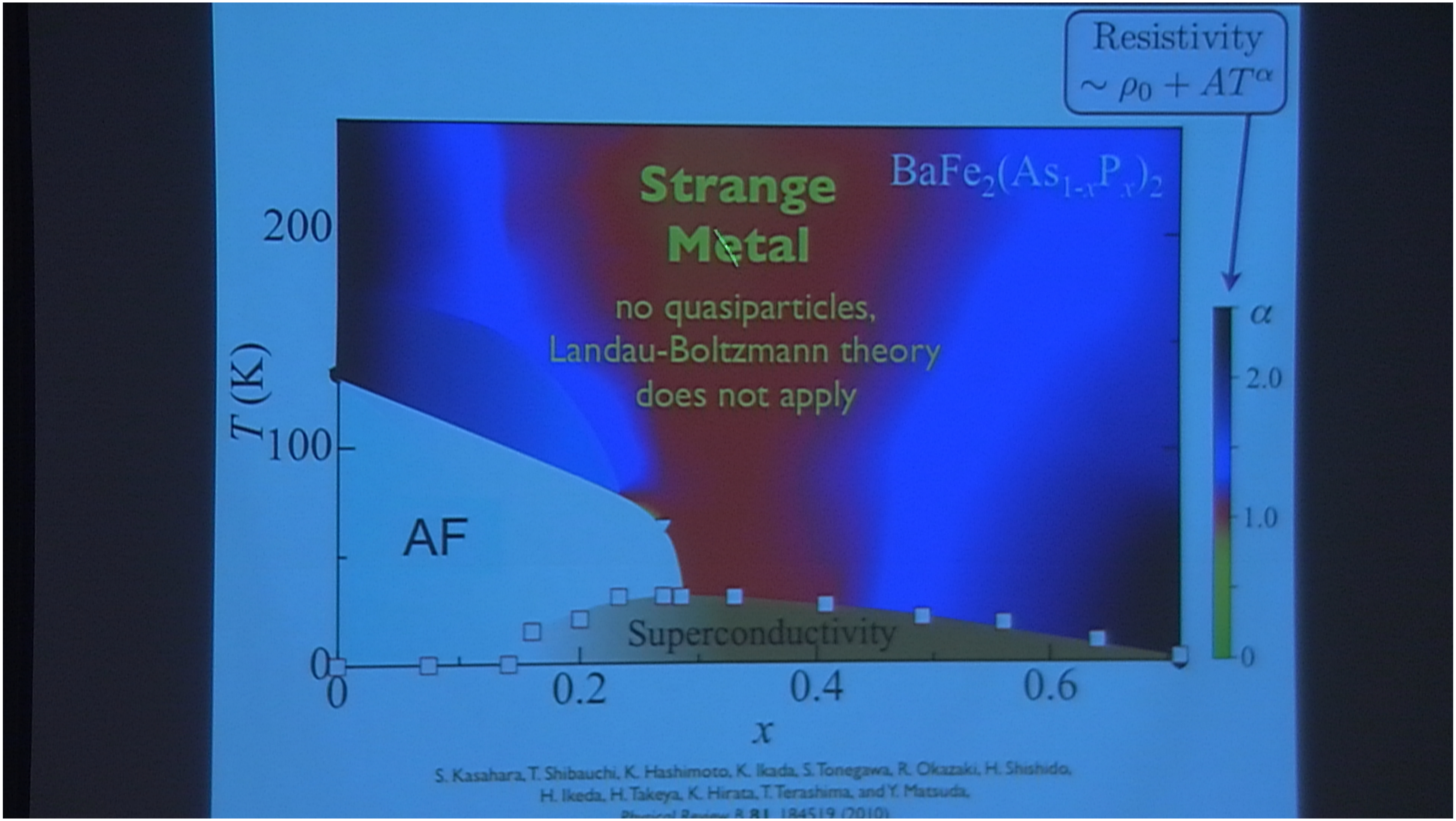
Iron pnictides:

a new class of high temperature superconductors

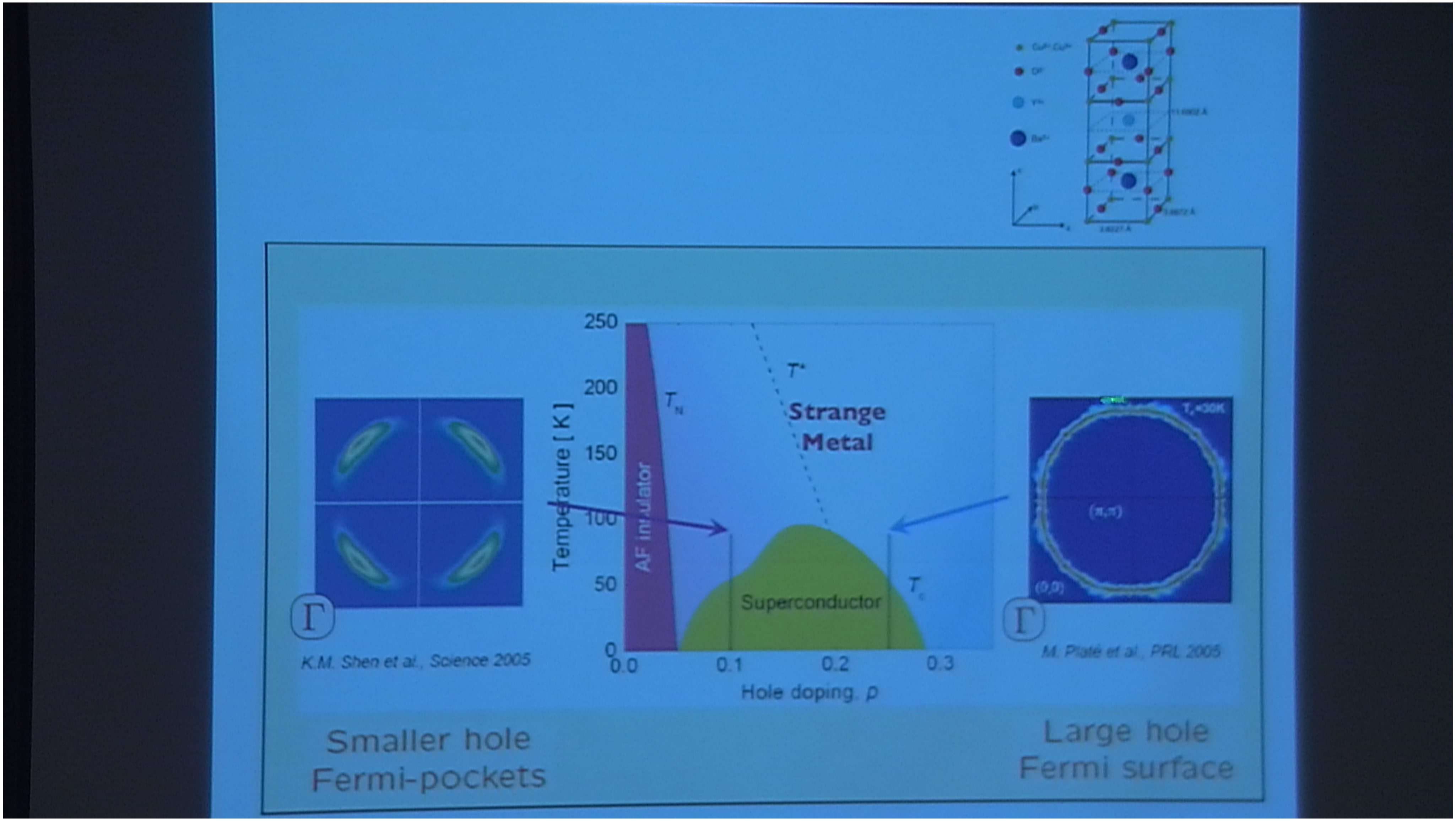



Resistivity
$\sim \rho_0 + AT^\alpha$
$BaFe_2(As_{1-x}P_x)_2$
200
100
0
T(K)
AF
Superconductivity
0
0.2
0.4
0.6
x
α
2.0
1.0
0
S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido,
H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda,

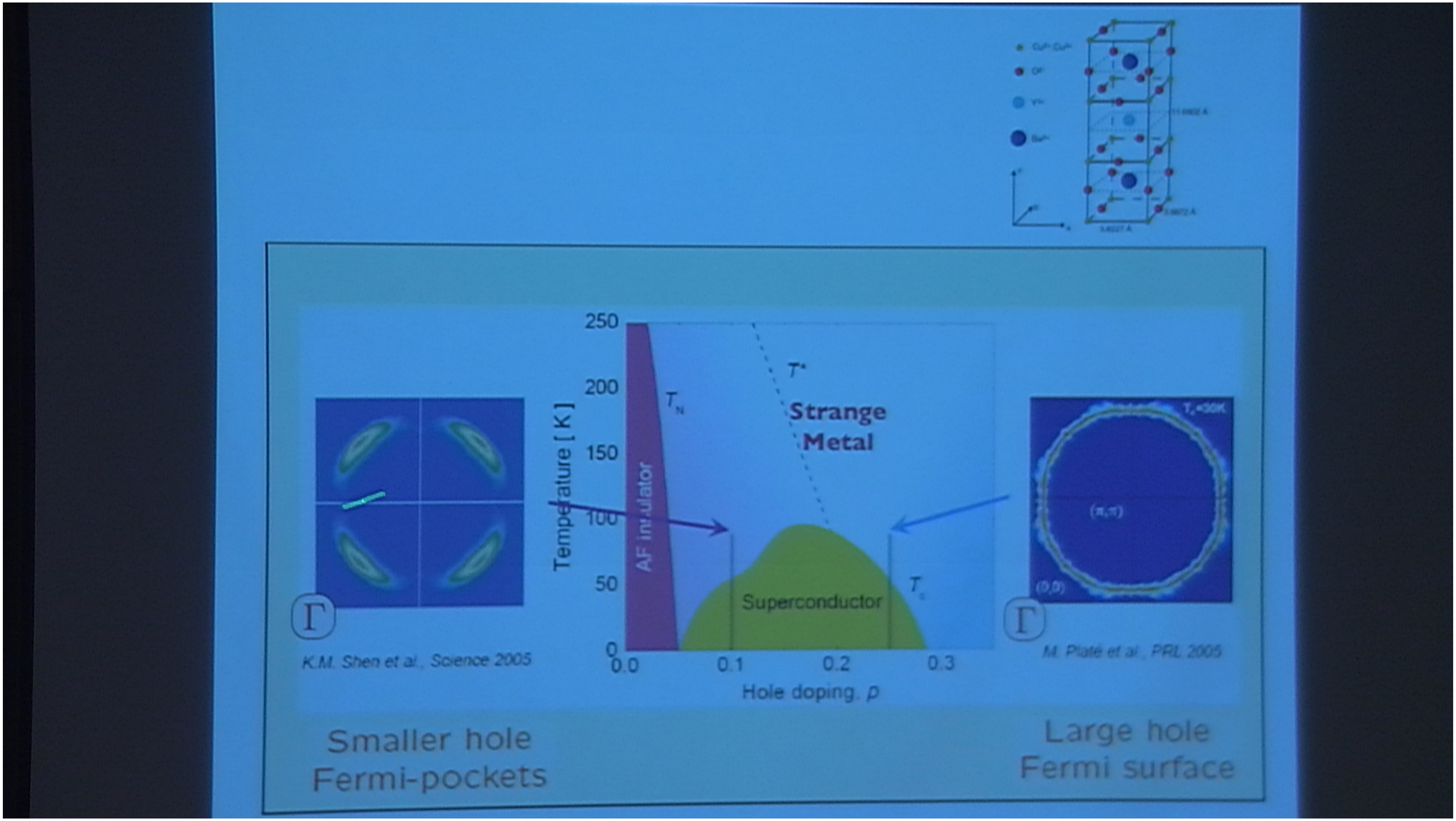

Resistivity
$\sim \rho_0 + AT^\alpha$
Short-range entanglement
in state with Neel (AF) order
$Fe_2(As_{1-x}P_x)_2$
AF
Superconductivity
T (K)
200
100
0
0
0.2
0.4
0.6
x
α
2.0
1.0
0
S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido,
H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda,



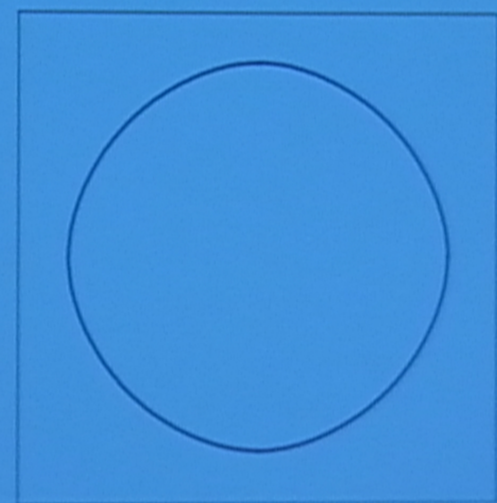
Resistivity
$\sim \rho_0 + AT^\alpha$
$BaFe_2(As_{1-x}P_x)_2$
Strange Metal
no quasiparticles,
Landau-Boltzmann theory
does not apply
AF
Superconductivity
T (K)
200
100
0
0
0.2
0.4
0.6
x
α
2.0
1.0
0
S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido,
H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda,
Physical Review B 81, 184519 (2010)



250
200
150
100
50
0
Temperature [K]
AF insulator
T_N
T^*
Strange Metal
Superconductor
T_c
0.0
0.1
0.2
0.3
Hole doping, p
Γ
K.M. Shen et al., Science 2005
(π,π)
(0,0)
M. Platé et al., PRL 2005
Smaller hole Fermi-pockets
Large hole Fermi surface

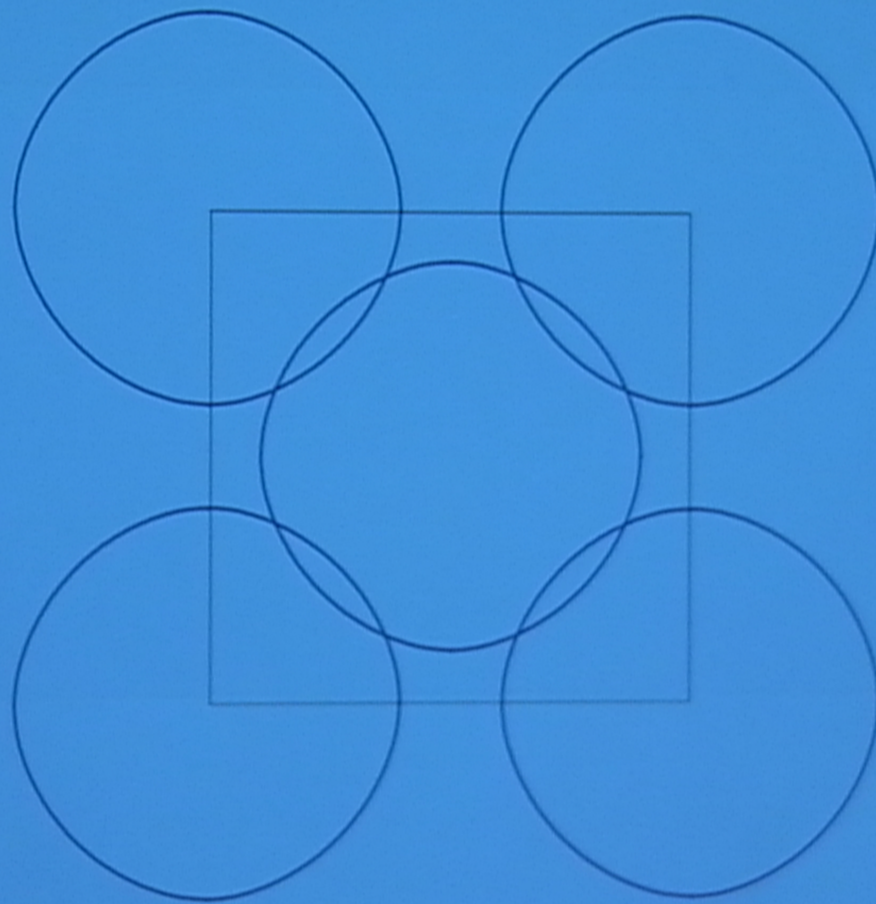

250
200
150
100
50
0
Temperature [K]
AF insulator
T_N
T*
Strange Metal
Superconductor
T_c
0.0
0.1
0.2
0.3
Hole doping, p
Γ
K.M. Shen et al., Science 2005
(π,π)
(0,0)
Γ
M. Platé et al., PRL 2005
Smaller hole Fermi-pockets
Large hole Fermi surface

Fermi surface+antiferromagnetism

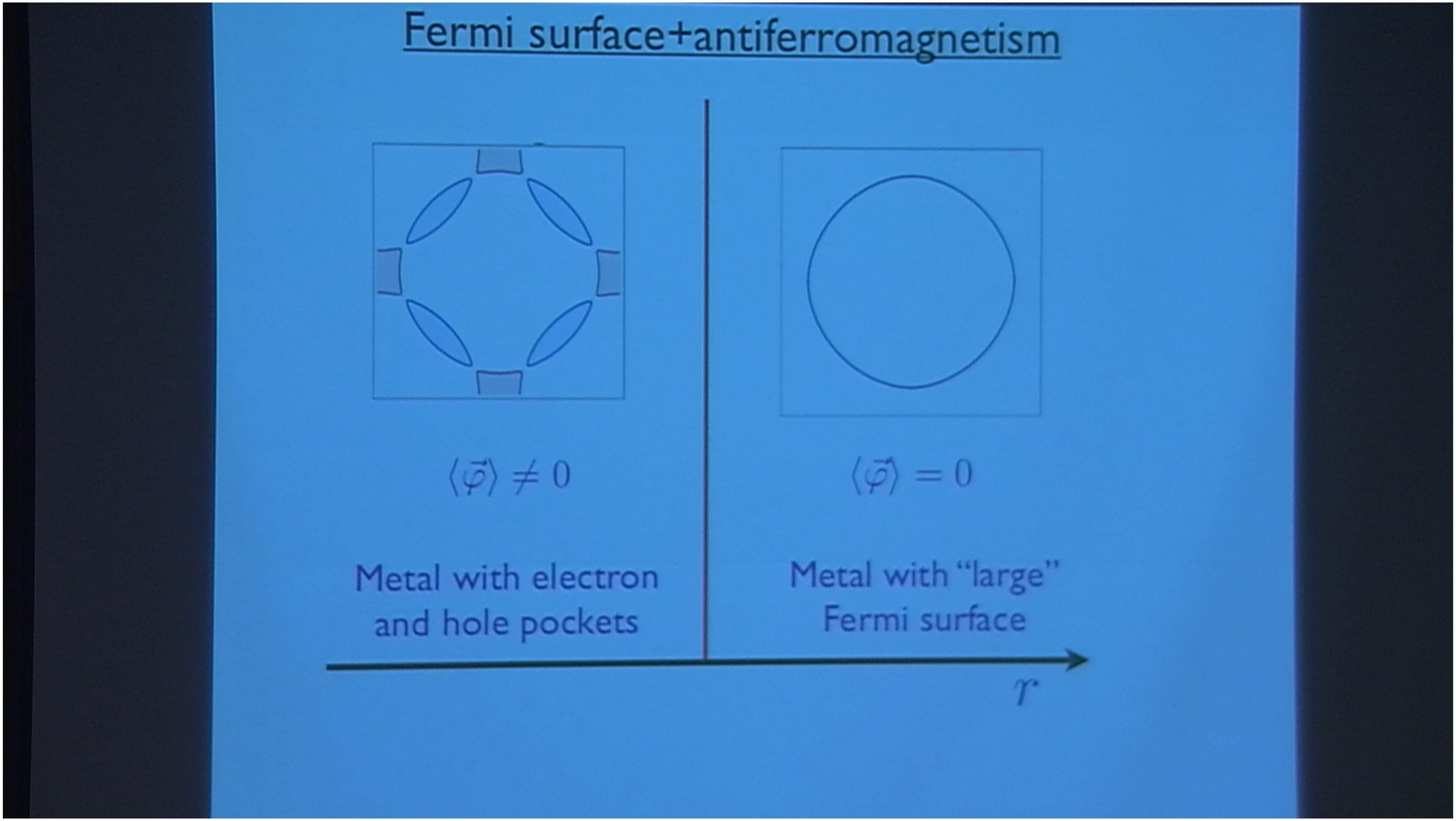


Metal with "large" Fermi surface

Fermi surface+antiferromagnetism

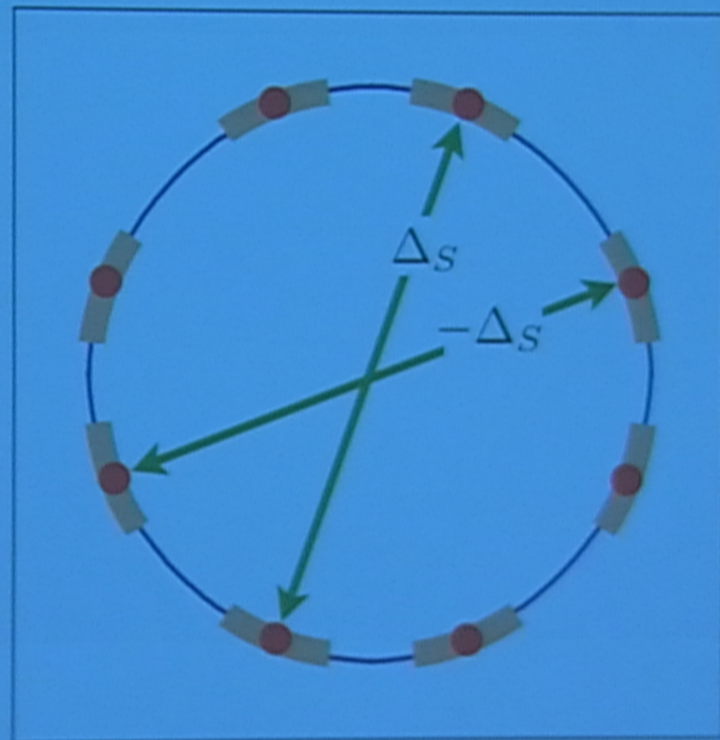


Fermi surfaces translated by $\mathbf{K} = (\pi, \pi)$.


Fermi surface+antiferromagnetism
$\langle \vec{\varphi} \rangle \neq 0$
Metal with electron and hole pockets
$\langle \vec{\varphi} \rangle = 0$
Metal with "large" Fermi surface
r

$$\left\langle c^{\dagger}_{\mathbf{k}\alpha} c^{\dagger}_{-\mathbf{k}\beta} \right\rangle = \varepsilon_{\alpha\beta} \Delta_S (\cos k_x - \cos k_y)$$

V. J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)
D.J. Scalapino, E. Loh, and J.E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
K. Miyake, S. Schmitt-Rink, and C. M.Varma, *Phys. Rev. B* **34**, 6554 (1986)
S. Raghu, S.A. Kivelson, and D.J. Scalapino, *Phys. Rev. B* **81**, 224505 (2010)



d-wave superconductor: particle-particle pairing at *and near* hot spots, with sign-changing pairing amplitude

$$\left\langle c^{\dagger}_{\mathbf{k}\alpha} c^{\dagger}_{-\mathbf{k}\beta} \right\rangle = \varepsilon_{\alpha\beta} \Delta_S (\cos k_x - \cos k_y)$$

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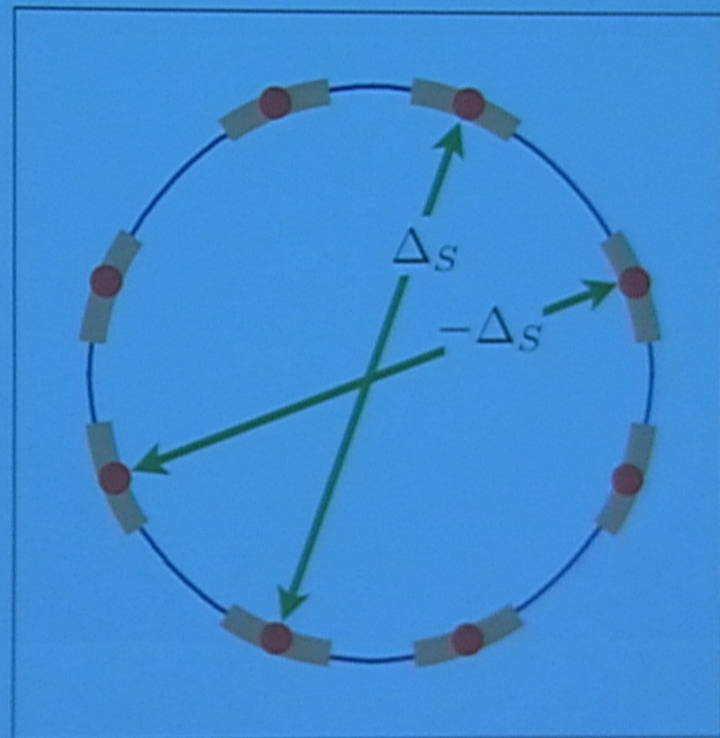
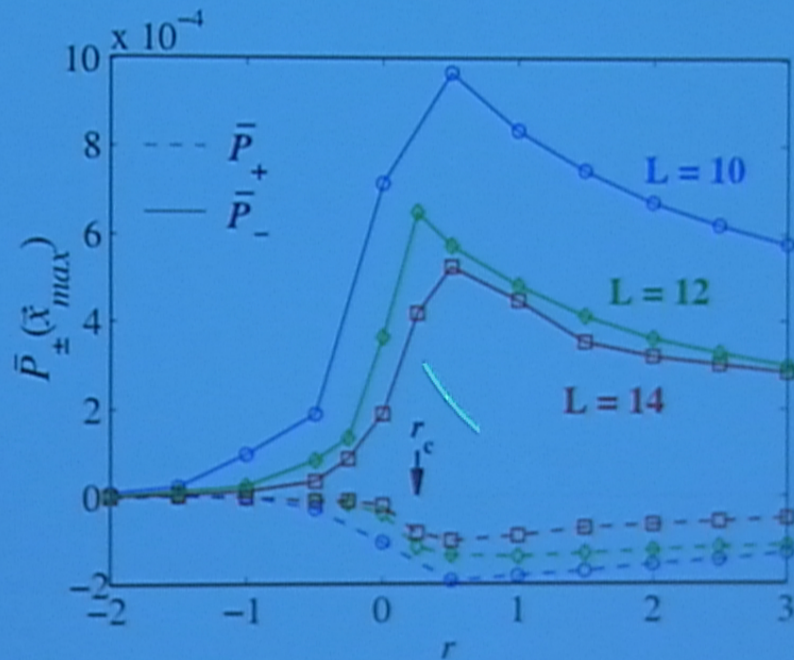


d-wave superconductor: particle-particle pairing at *and near* hot spots, with sign-changing pairing amplitude

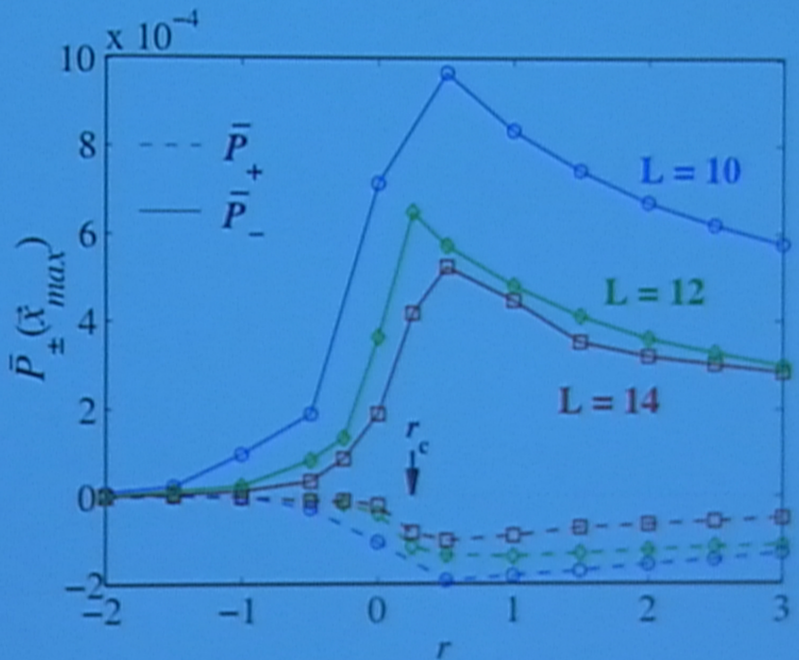
Sign-problem-free Quantum Monte Carlo for antiferromagnetism in metals



s/d pairing amplitudes P_+/P_-
as a function of the tuning parameter r

E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

Sign-problem-free Quantum Monte Carlo for antiferromagnetism in metals



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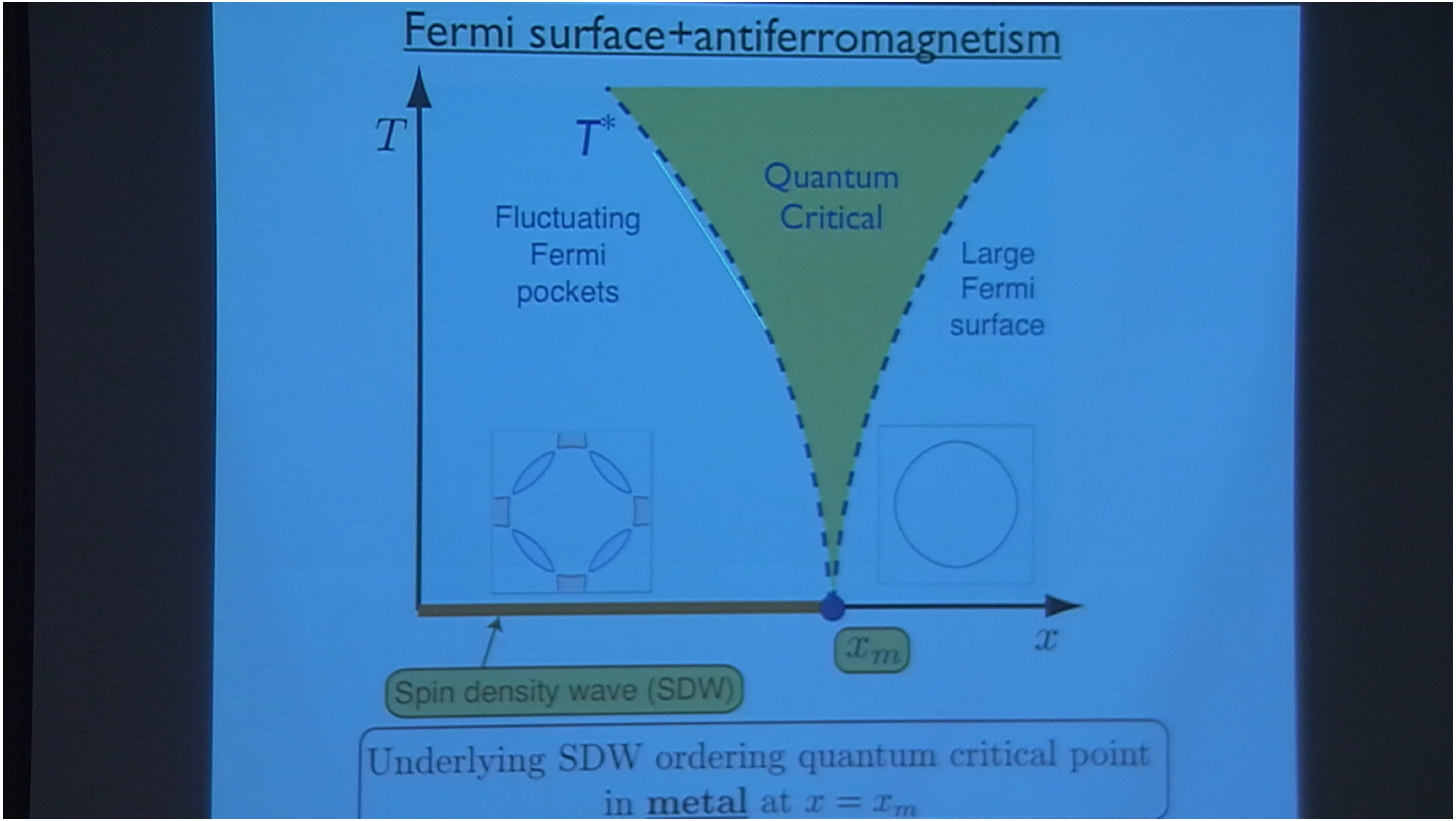
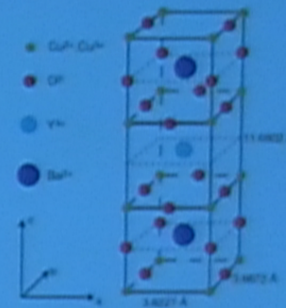
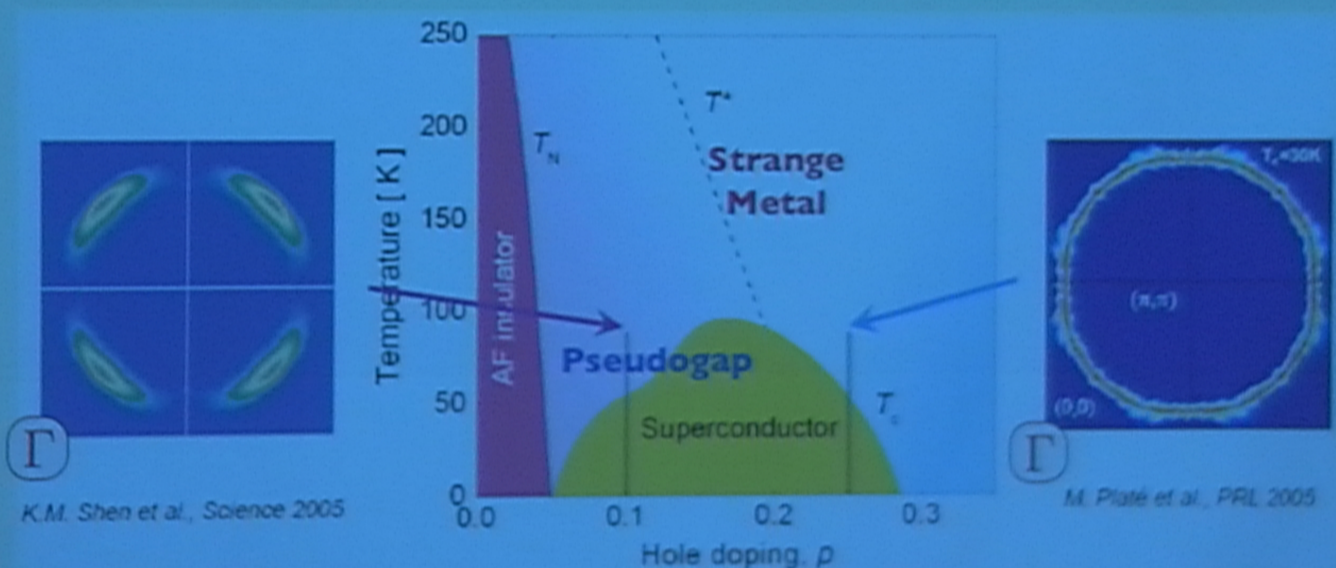


Fermi surface+antiferromagnetism
T
T^*
Quantum
Critical
Fluctuating
Fermi
pockets
Large
Fermi
surface
x_m
x
Spin density wave (SDW)
Underlying SDW ordering quantum critical point
in **metal** at $x = x_m$

What about the pseudogap ?





Smaller hole Fermi-pockets

Large hole Fermi surface

- There is an approximate pseudospin symmetry in metals with antiferromagnetic spin correlations.

- The pseudospin partner of *d*-wave superconductivity is an incommensurate *d*-wave bond order

- These orders form a pseudospin doublet, which is responsible for the "pseudogap" phase.

M. A. Metlitski and S. Sachdev, Phys. Rev. B **85**, 075127 (2010)
T. Holder and W. Metzner, Phys. Rev. B **85**, 165130 (2012)
C. Husemann and W. Metzner, Phys. Rev. B **86**, 085113 (2012)
M. Bejas, A. Greco, and H. Yamase, Phys. Rev. B **86**, 224509 (2012)
K. B. Efetov, H. Meier, and C. Pépin, Nature Physics, to appear, arXiv:1210.3276.
S. Sachdev and R. La Placa, arXiv:1303.2114

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S. Sachdev and R. La Placa, arXiv:1303.2114

Pseudospin symmetry of the exchange interaction

$$H_J = \sum_{i<j} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

with $\vec{S}_i = \frac{1}{2} c^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} c_{i\beta}$ is the antiferromagnetic exchange interaction. Introduce the Nambu spinor

$$\Psi_{i\uparrow} = \begin{pmatrix} c_{i\uparrow} \\ c^\dagger_{i\downarrow} \end{pmatrix} \quad , \quad \Psi_{i\downarrow} = \begin{pmatrix} c_{i\downarrow} \\ -c^\dagger_{i\uparrow} \end{pmatrix}$$

Then we can write

$$H_J = \frac{1}{8} \sum_{i<j} J_{ij} \left(\Psi^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} \Psi_{i\beta} \right) \cdot \left(\Psi^\dagger_{j\gamma} \vec{\sigma}_{\gamma\delta} \Psi_{i\delta} \right)$$

which is invariant under independent SU(2) pseudospin transformations on each site

$$\Psi_{i\alpha} \to U_i \Psi_{i\alpha}$$

This pseudospin (gauge) symmetry is important in classifying spin liquid ground states of H_J.

I. Affleck, Z. Zou, T. Hsu, and P. W. Anderson, Phys. Rev. B **38**, 745 (1988)
E. Dagotto, E. Fradkin, and A. Moreo, Phys. Rev. B **38**, 2926 (1988)
P. A. Lee, N. Nagaosa, and X.-G. Wen, Rev. Mod. Phys. **78**, 17 (2006)

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Pseudospin symmetry of the exchange interaction

$$H_{tJ} = -\sum_{i,j} t_{ij} c^\dagger_{i\alpha} c_{j\alpha} + \sum_{i<j} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

with $\vec{S}_i = \frac{1}{2} c^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} c_{i\beta}$ is the antiferromagnetic exchange interaction. Introduce the Nambu spinor

$$\Psi_{i\uparrow} = \begin{pmatrix} c_{i\uparrow} \\ c^\dagger_{i\downarrow} \end{pmatrix} \quad , \quad \Psi_{i\downarrow} = \begin{pmatrix} c_{i\downarrow} \\ -c^\dagger_{i\uparrow} \end{pmatrix}$$

Then we can write

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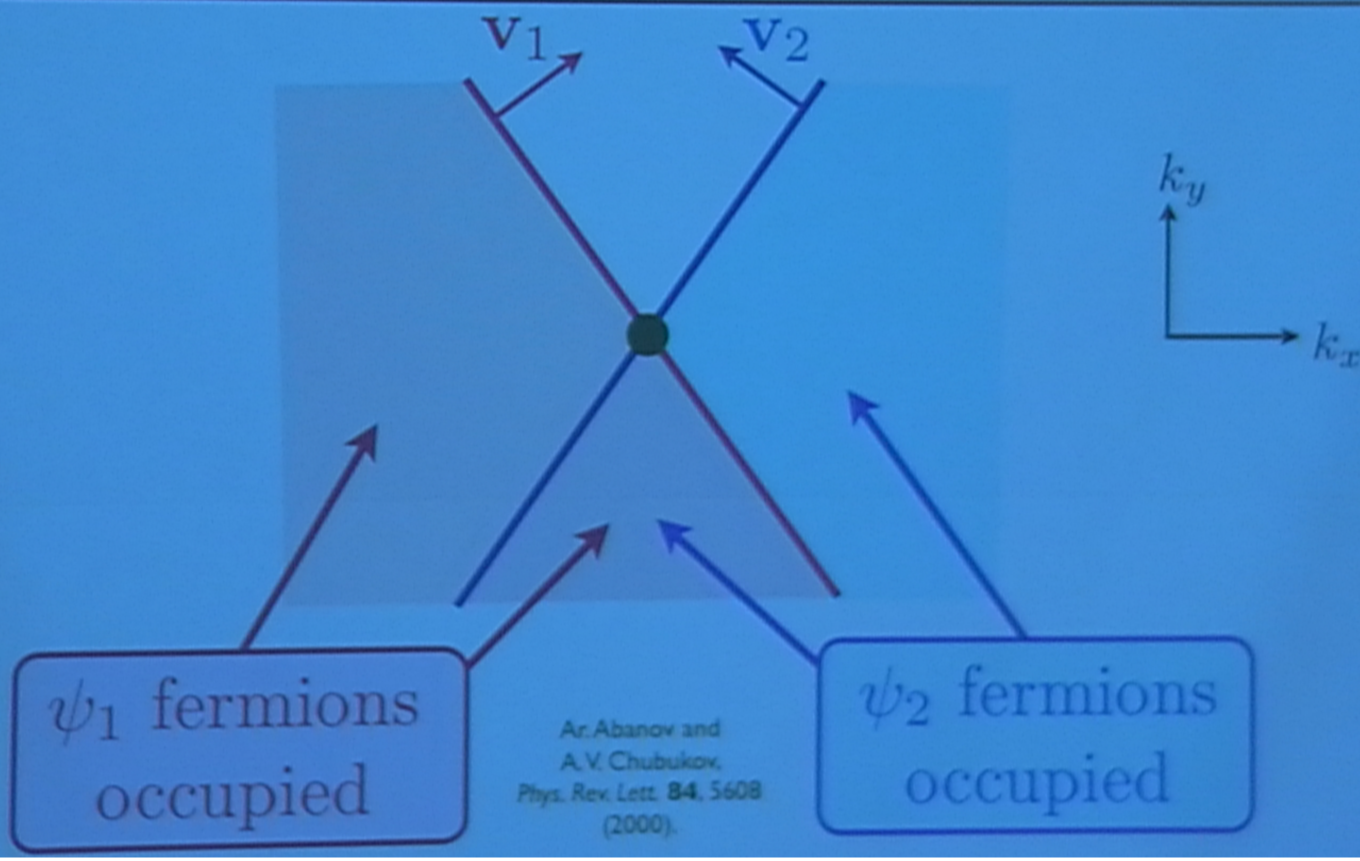
$$\Psi_{i\alpha} \to U_i \Psi_{i\alpha}$$

This pseudospin (gauge) symmetry is important in classifying spin liquid ground states of H_J. It is fully broken by the electron hopping t_{ij} but does have remnant consequences in doped spin liquid states.

I. Affleck, Z. Zou, T. Hsu, and P. W. Anderson, Phys. Rev. B **38**, 745 (1988)
E. Dagotto, E. Fradkin, and A. Moreo, Phys. Rev. B **38**, 2926 (1988)
P. A. Lee, N. Nagaosa, and X.-G. Wen, Rev. Mod. Phys. **78**, 17 (2006)

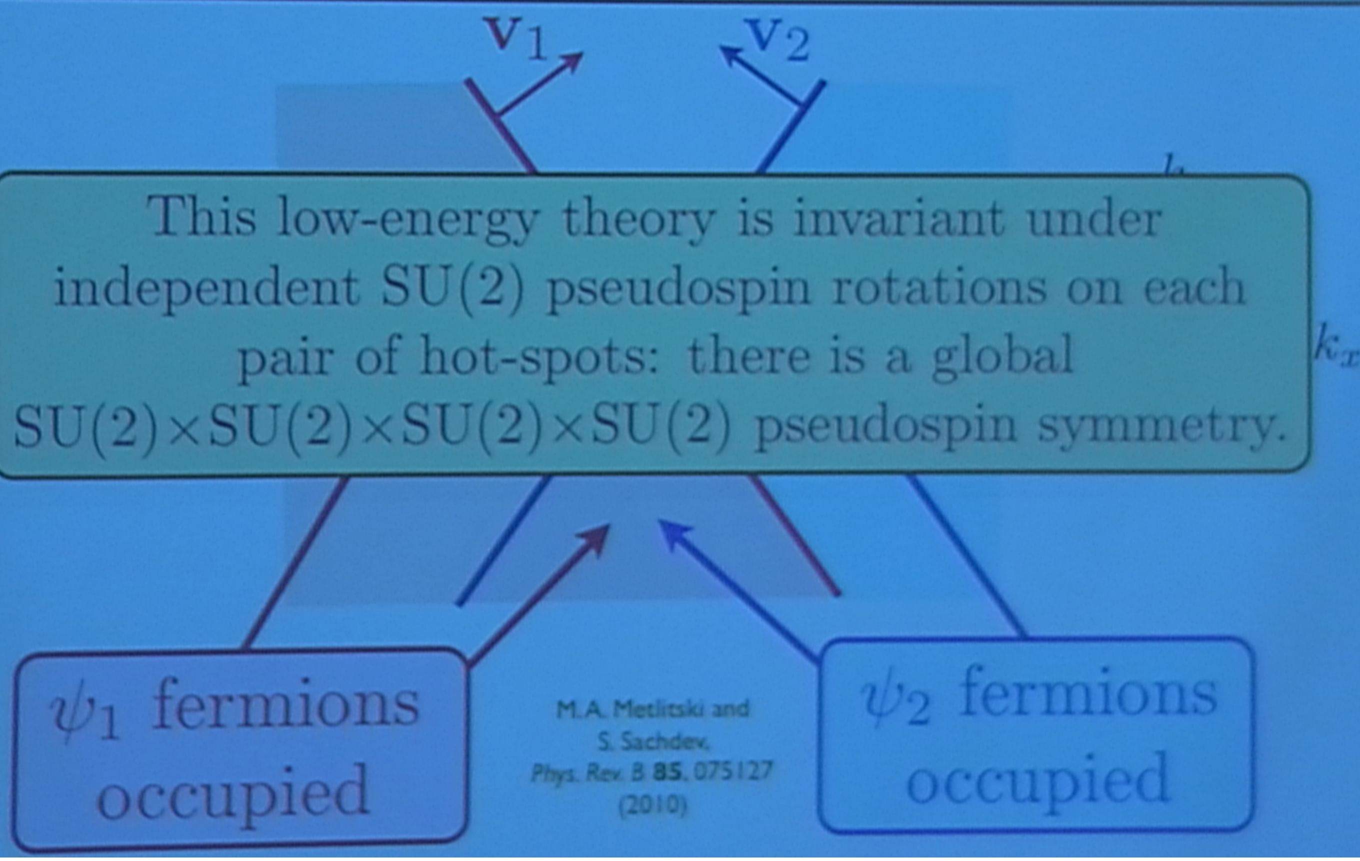
$$S = \int d^2r d\tau \Big[\psi_{1\alpha}^\dagger \left(\partial_\tau - i\mathbf{v}_1 \cdot \boldsymbol{\nabla}_r\right) \psi_{1\alpha} + \psi_{2\alpha}^\dagger \left(\partial_\tau - i\mathbf{v}_2 \cdot \boldsymbol{\nabla}_r\right) \psi_{2\alpha}$$
$$+ \frac{1}{2} \left(\boldsymbol{\nabla}_r \vec{\varphi}\right)^2 + \frac{s}{2}\vec{\varphi}^2 + \frac{u}{4}\vec{\varphi}^4 - \lambda \vec{\varphi} \cdot \left(\psi_{1\alpha}^\dagger \vec{\sigma}_{\alpha\beta} \psi_{2\beta} + \psi_{2\alpha}^\dagger \vec{\sigma}_{\alpha\beta} \psi_{1\beta}\right) \Big]$$



Ar. Abanov and A. V. Chubukov, Phys. Rev. Lett. **84**, 5608 (2000).

$$\mathcal{S} = \int d^2r d\tau \Big[\psi_{1\alpha}^\dagger \left(\partial_\tau - i\mathbf{v}_1 \cdot \boldsymbol{\nabla}_r\right) \psi_{1\alpha} + \psi_{2\alpha}^\dagger \left(\partial_\tau - i\mathbf{v}_2 \cdot \boldsymbol{\nabla}_r\right) \psi_{2\alpha} + \frac{1}{2}\left(\boldsymbol{\nabla}_r \vec{\varphi}\right)^2 + \frac{s}{2}\vec{\varphi}^2 + \frac{u}{4}\vec{\varphi}^4 - \lambda \vec{\varphi} \cdot \left(\psi_{1\alpha}^\dagger \vec{\sigma}_{\alpha\beta} \psi_{2\beta} + \psi_{2\alpha}^\dagger \vec{\sigma}_{\alpha\beta} \psi_{1\beta}\right)\Big]$$

This low-energy theory is invariant under independent SU(2) pseudospin rotations on each pair of hot-spots: there is a global SU(2)×SU(2)×SU(2)×SU(2) pseudospin symmetry.



M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

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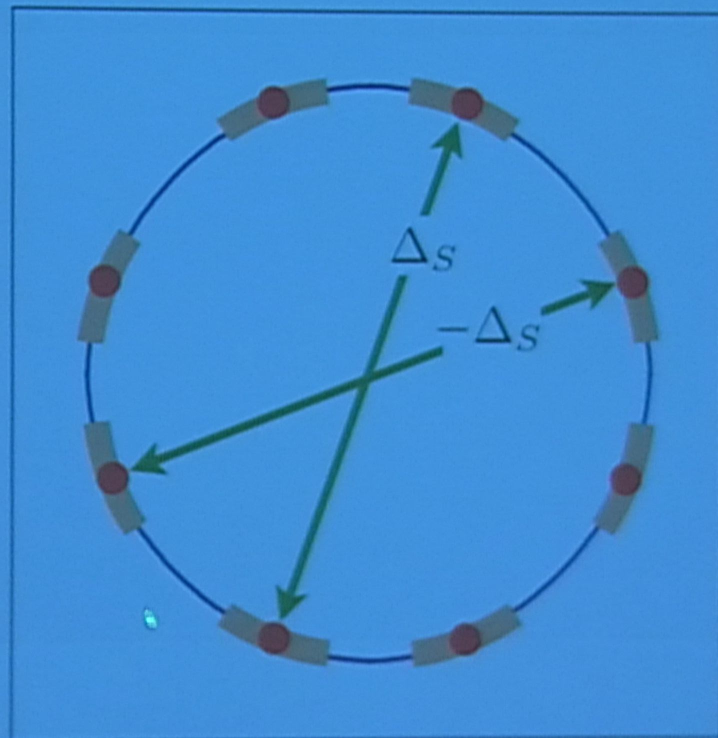
K. Miyake, S. Schmitt-Rink, and C. M.Varma, *Phys. Rev. B* **34**, 6554 (1986)
S. Raghu, S.A. Kivelson, and D.J. Scalapino, *Phys. Rev. B* **81**, 224505 (2010)



d-wave superconductor: particle-particle pairing at <u>*and near*</u> hot spots, with sign-changing pairing amplitude

Incommensurate *d*-wave bond order

Consider modulation in an off-site "density" like variable at sites $\mathbf{r}_i$ and $\mathbf{r}_j$

$$\left\langle c_{i\alpha}^{\dagger} c_{j\alpha} \right\rangle \sim \left[\sum_{\mathbf{k}} \Delta_{\mathbf{Q}}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}\cdot(\mathbf{r}_i + \mathbf{r}_j)/2}$$

relative co-ord. average co-ord.

The wavevector $\mathbf{Q}$ is associated with a modulation in the *average* co-ordinate $(\mathbf{r}_i - \mathbf{r}_j)/2$: this determines the wavevector of the neutron/X-ray scattering peak.

The interesting part is the dependence on the *relative* co-ordinate $\mathbf{r}_i - \mathbf{r}_j$. Assuming time-reversal, the order parameter $\Delta_{\mathbf{Q}}(\mathbf{k})$ can always be expanded as

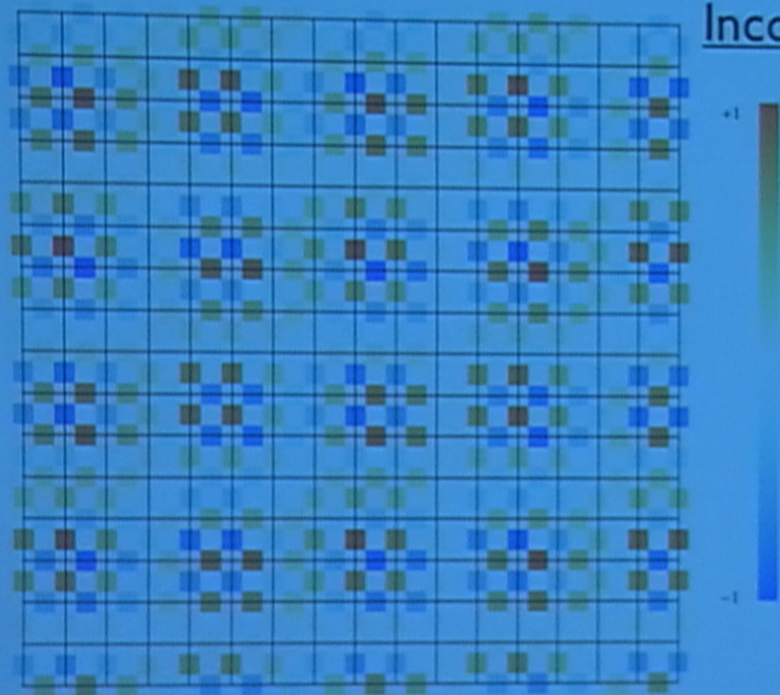
$$\Delta_{\mathbf{Q}}(\mathbf{k}) = c_s + c_{s'}(\cos k_x + \cos k_y) + c_d(\cos k_x - \cos k_y) + \ldots$$

The usual charge-density-wave has only $c_s \neq 0$.

The bond-ordered state we find has

$$|c_d| \gg c_s, c_{s'}, \ldots$$

Incommensurate *d*-wave bond order



"Bond density" measures amplitude for electrons to be in spin-singlet valence bond.

M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

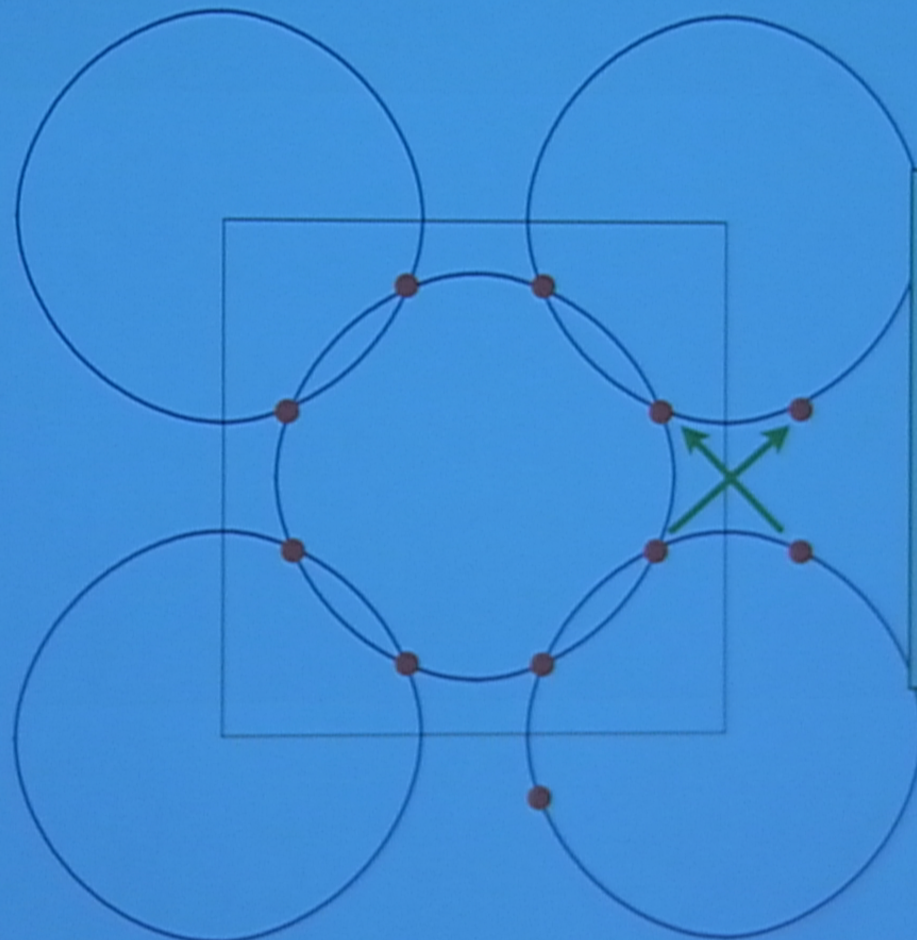
$$\left\langle c^\dagger_{\mathbf{r}\alpha} c_{\mathbf{s}\alpha} \right\rangle = \sum_{\mathbf{Q}} \sum_{\mathbf{k}} e^{i\mathbf{Q}\cdot(\mathbf{r}+\mathbf{s})/2} e^{-i\mathbf{k}\cdot(\mathbf{r}-\mathbf{s})} \left\langle c^\dagger_{\mathbf{k}-\mathbf{Q}/2,\alpha} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle$$

where $\mathbf{Q}$ extends over $\mathbf{Q} = (\pm Q_0, \pm Q_0)$ with $Q_0 = 2\pi/(7.3)$ and

$$\left\langle c^\dagger_{\mathbf{k}-\mathbf{Q}/2,\alpha} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = \Delta_{\mathbf{Q}}(\cos k_x - \cos k_y)$$

Note $\left\langle c^\dagger_{\mathbf{r}\alpha} c_{\mathbf{s}\alpha} \right\rangle$ is non-zero *only* when $\mathbf{r}, \mathbf{s}$ are nearest neighbors.

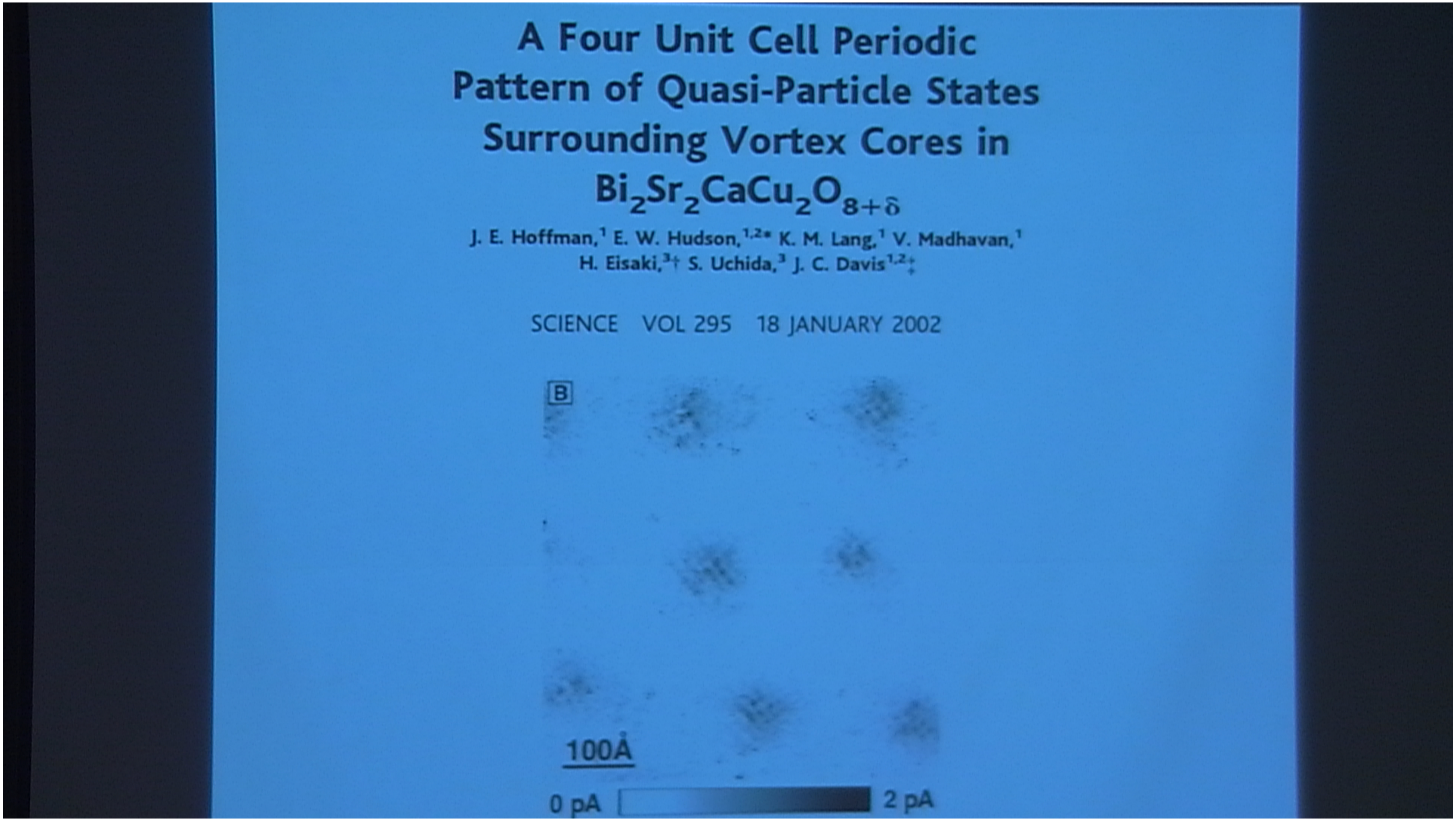
Incommensurate *d*-wave bond order



High T pseudogap: Fluctuating composite order parameter of nearly degenerate d-wave pairing and incommensurate d-wave bond order. (Approximate) SU(2) symmetry of composite order prevents long-range order $T > 0$.

K. B. Efetov, H. Meier, and C. Pepin, Nature Physics, to appear, arXiv:1210.3276

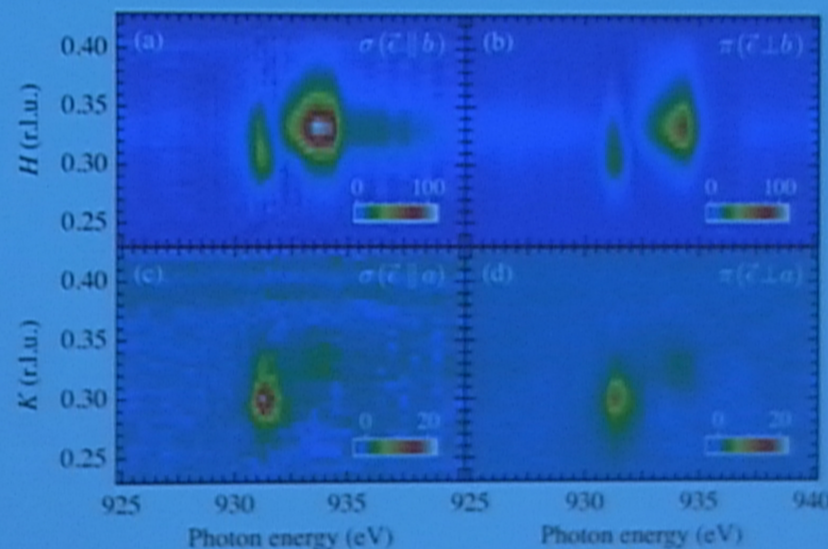
$$\left\langle c^\dagger_{\mathbf{k}-\mathbf{Q}/2,\alpha} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = \Delta_{\mathbf{Q}} (\cos k_x - \cos k_y)$$


A Four Unit Cell Periodic Pattern of Quasi-Particle States Surrounding Vortex Cores in $Bi_2Sr_2CaCu_2O_{8+\delta}$
J. E. Hoffman,[1] E. W. Hudson,[1,2*] K. M. Lang,[1] V. Madhavan,[1]
H. Eisaki,[3†] S. Uchida,[3] J. C. Davis[1,2‡]
SCIENCE VOL 295 18 JANUARY 2002
B
100Å
0 pA
2 pA

Distinct Charge Orders in the Planes and Chains of Ortho-III-Ordered $YBa_2Cu_3O_{6+\delta}$ Superconductors Identified by Resonant Elastic X-ray Scattering

A. J. Achkar,[1] R. Sutarto,[2,3] X. Mao,[1] F. He,[3] A. Frano,[4,5] S. Blanco-Canosa,[4] M. Le Tacon,[4] G. Ghiringhelli,[6] L. Braicovich,[6] M. Minola,[6] M. Moretti Sala,[7] C. Mazzoli,[6] Ruixing Liang,[2] D. A. Bonn,[2] W. N. Hardy,[2] B. Keimer,[4] G. A. Sawatzky,[2] and D. G. Hawthorn[1,*]

PRL **109**, 167001 (2012)



Moreover, the energy dependence of the CDW order in the planes is shown to result from a spatial modulation of energies of the Cu $2p$ to $3d_{x^2-y^2}$ transition, similar to stripe-ordered 214 cuprates.

These energy shifts are interpreted as a spatial modulation of the electronic structure and may point to a valence-bond-solid interpretation of the stripe phase.

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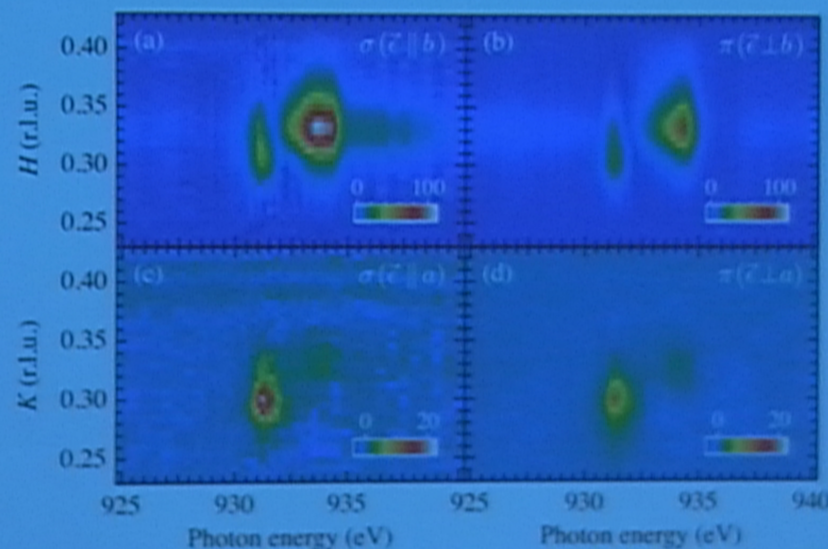
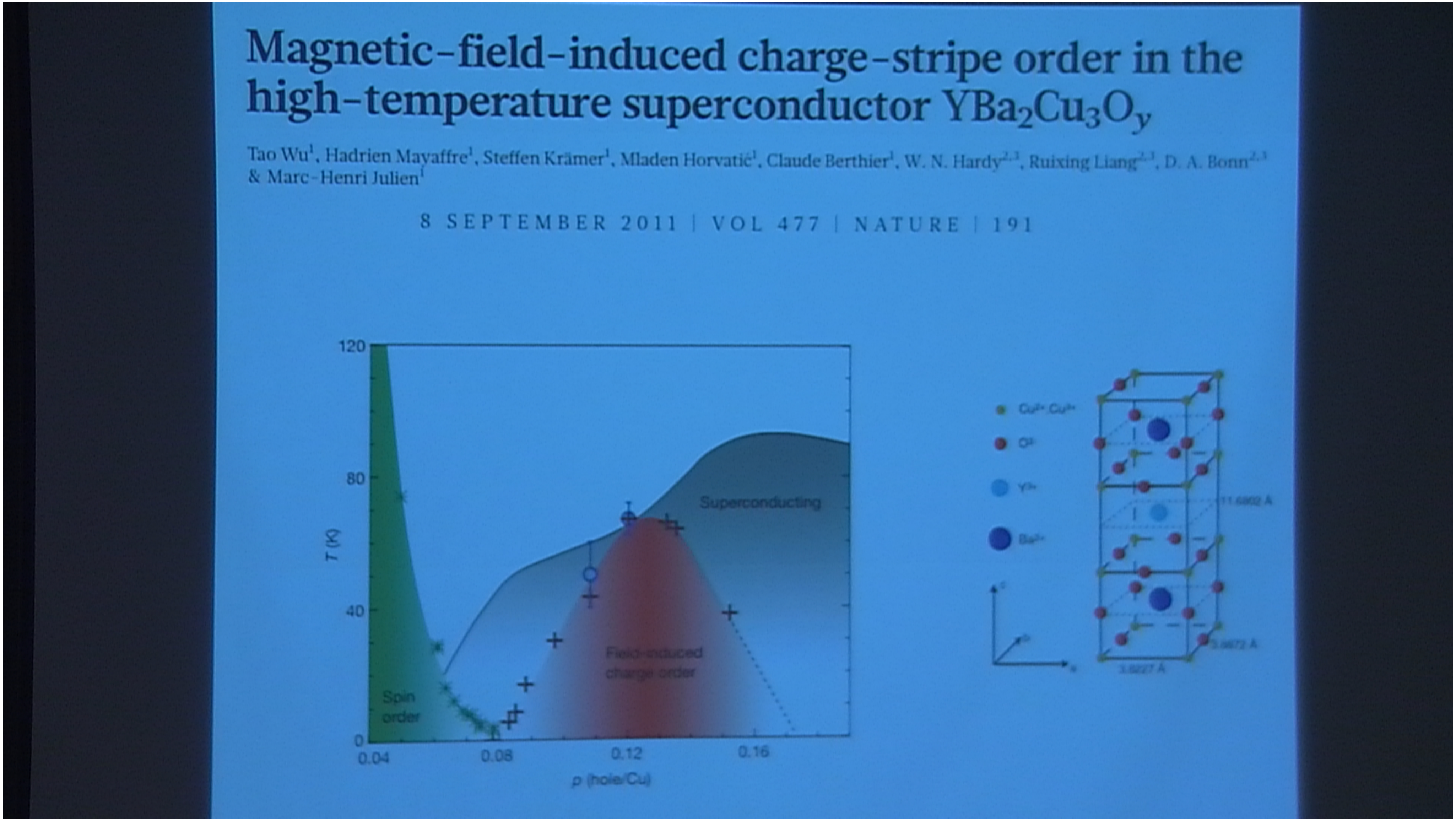
PRL **109**, 167001 (2012)



Moreover, the energy dependence of the CDW order in the planes is shown to result from a spatial modulation of energies of the Cu $2p$ to $3d_{x^2-y^2}$ transition, similar to stripe-ordered 214 cuprates.

These energy shifts are interpreted as a spatial modulation of the electronic structure and may point to a valence-bond-solid interpretation of the stripe phase.


Magnetic-field-induced charge-stripe order in the high-temperature superconductor $YBa_2Cu_3O_y$
Tao Wu[1], Hadrien Mayaffre[1], Steffen Krämer[1], Mladen Horvatić[1], Claude Berthier[1], W. N. Hardy[2,3], Ruixing Liang[2,3], D. A. Bonn[2,3] & Marc-Henri Julien[1]
8 SEPTEMBER 2011 | VOL 477 | NATURE | 191
120
80
40
0
T (K)
0.04
0.08
0.12
0.16
p (hole/Cu)
Superconducting
Field-induced charge order
Spin order

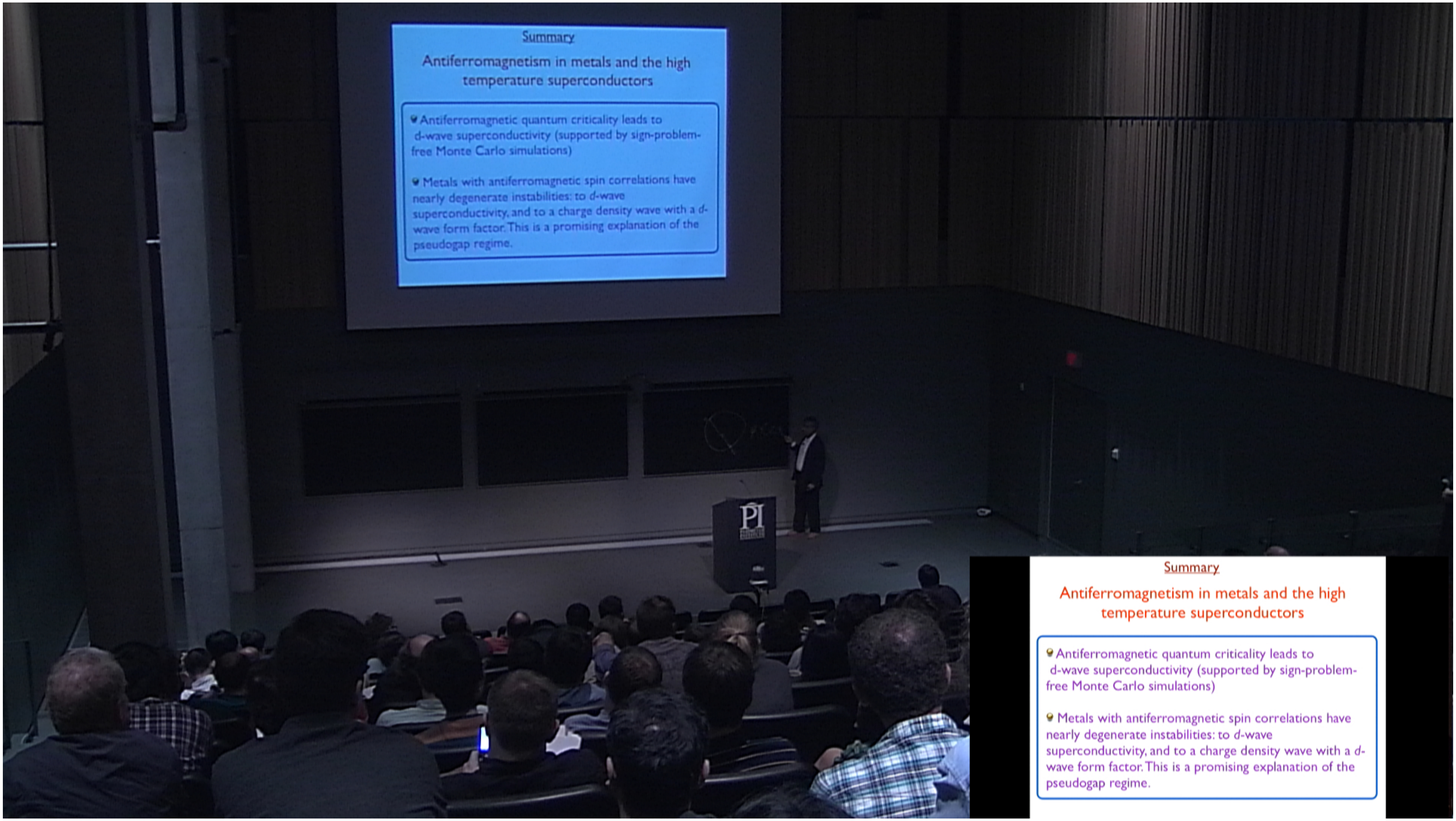
Summary

Conformal quantum matter

- New insights and solvable models for diffusion and transport of strongly interacting systems near quantum critical points using the methods of gauge-gravity duality.

- The description is far removed from, and complementary to, that of the quantum Boltzmann equation which builds on the quasiparticle/vortex picture.

- Good prospects for experimental tests of frequency-dependent, non-linear, and non-equilibrium transport


Summary
Antiferromagnetism in metals and the high temperature superconductors
Antiferromagnetic quantum criticality leads to d-wave superconductivity (supported by sign-problem-free Monte Carlo simulations)
Metals with antiferromagnetic spin correlations have nearly degenerate instabilities: to d-wave superconductivity, and to a charge density wave with a d-wave form factor. This is a promising explanation of the pseudogap regime.
PI