Title: Quantum renormalization group and AdS/CFT

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Abstract: <span>In this talk, I will discuss about the notion of quantum renormalization group, and explain how (D+1)-dimensional gravitational theories naturally emerge as dual descriptions for D-dimensional quantum field theories. It will be argued that the dynamical gravitational field in the bulk encodes the entanglement between low energy modes and high energy modes of the corresponding quantum field theory.</span>

# Quantum Renormalization Group and AdS/CFT

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# AdS/CFT correspondence

[Maldacena]

- **Conjecture :**

  D-dim QFT = (D+1)-dim quantum gravity

  – One can learn about QFT from Gravity
    - In large N limit, the gravity becomes classical
    - Classical gravity may capture non-trivial quantum fluctuations in QFT
    - Potential application to QCD, condensed matter systems

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    - QFT as a non-perturbative definition of quantum gravity
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# AdS/CFT Dictionary

[Gubser, Klebanov, Polyakov; Witten]

Low energy

(D+1)-dim space

$$O_n(x) \leftrightarrow j_n(x, z)$$

$$T_{\mu\nu}(x) \leftrightarrow g_{\mu\nu}(x, z)$$

z

High energy

x

D –dim flat space

$$\int D\phi(x) e^{iS_D[\phi(x)] + i\int J_n(x) O_n} = \int Dj(x, z) e^{iS_{D+1}[j(x,z)]}\Big|_{j_n(x,z=0)=J_n(x)}$$

$$\rightarrow e^{iS_{D+1}[\bar{j}(x,z)]}\Big|_{\bar{j}_n(x,z=0)=J_n(x)}$$

# What is behind the correspondence?

## RG ≈ GR

- Radial direction in the bulk = length scale of QFT
- Bulk variables : scale dependent coupling functions
- Equations of motion in the bulk corresponds to the beta functions of QFT
- Radial evolution of the bulk fields correspond to the RG flow

# However, the connection between RG and GR is incomplete

| RG | GR |
|---|---|
| Non-dynamical coupling functions : Obey first-order beta functions | Bulk variables are dynamical : Bulk action has two-derivative term |
| RG flow is classical : Given initial condition, coupling functions are deterministic without uncertainty | Bulk variables have quantum fluctuations |

# Matrix model

$$Z[J(x)] = \int D\phi \; e^{i \int dx \mathcal{L}}$$

$$\mathcal{L} = J^n(x) O_n$$

- $O_n$ : complete set of single-trace operators

e.g. $\mathrm{tr}[\phi^n],\ \mathrm{tr}[\phi \partial_\mu \partial_\nu \phi],\ \mathrm{tr}[\phi(\partial_{\mu_1}\partial_{\mu_2}...\partial_{\mu_i}\phi)...(\partial_{\nu_1}\partial_{\nu_2}...\partial_{\nu_i}\phi)], ..$

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# Renormalization Group

$$\Lambda \to \Lambda e^{-dz}$$

$$\mathcal{L} \to \mathcal{L}' = \mathcal{L} + \delta\mathcal{L},$$

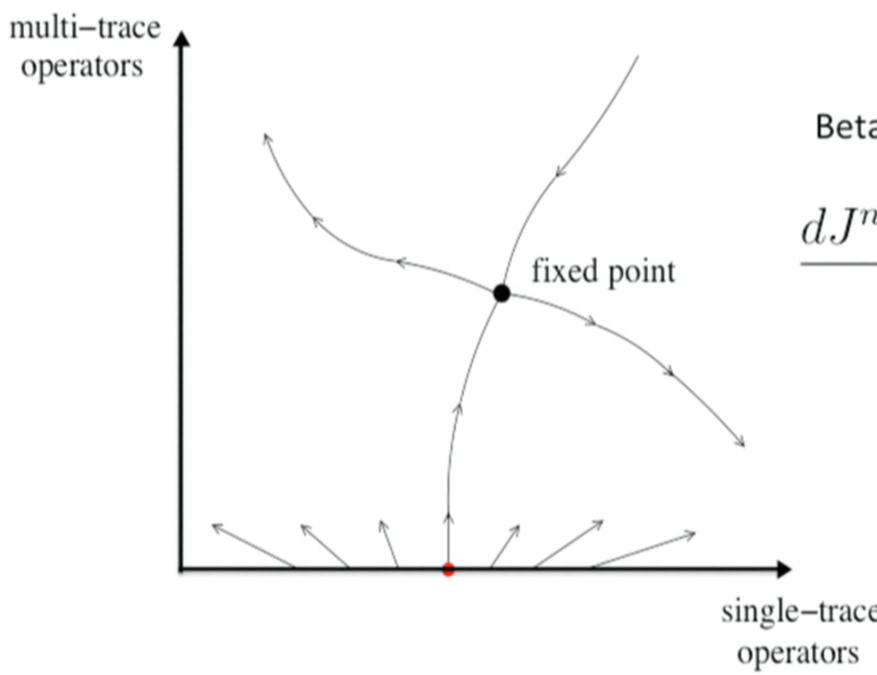
$$\delta\mathcal{L} = dz\left\{ \mathcal{L}_c[J,x) - \beta^n[J,x)O_n + G^{mn}[J,x)O_mO_n \right\}$$

- Under coarse graining, the original theory is mapped into another theory
- Although only a subset of operators are turned on at UV, all other symmetry allowed operators are generated at low energy
- Specifically, double-trace operators are generated out of single-trace operators to the linear order of dz

[ I. Heemskerk and J. Polchinski, arXiv:1010.1264; T. Faulkner, H. Liu and M. Rangamani, arXiv:1010.4036. ]

# Conventional (Classical) RG

- One needs to keep track of the flow of all operators
- RG flow is deterministic
- Intractable for strongly coupled field theories




Beta function

$$\frac{dJ^{nm\ldots}(x,z)}{d\;z} = -\beta[J^n, J^{nm}, \ldots]$$

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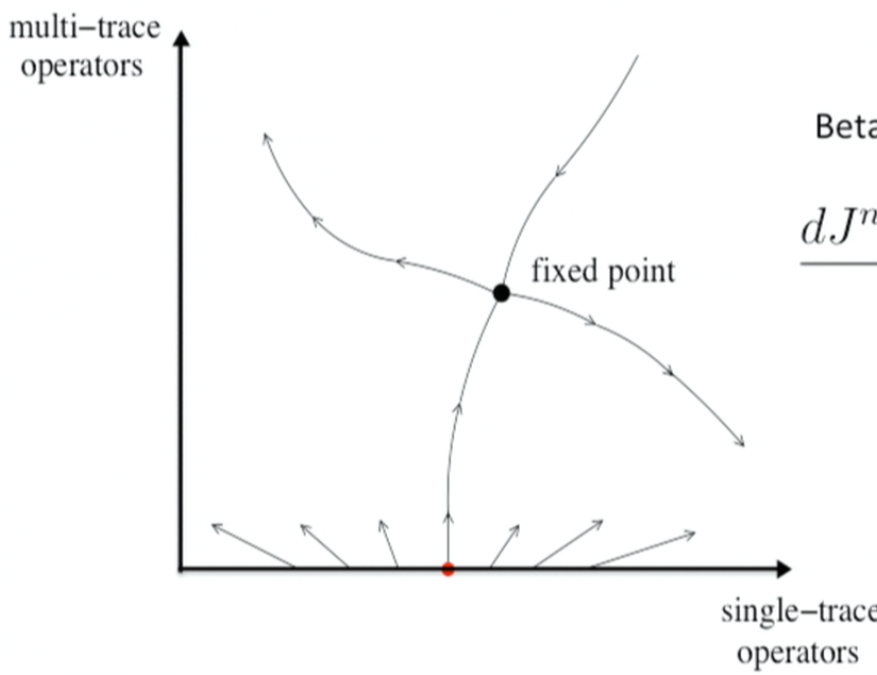


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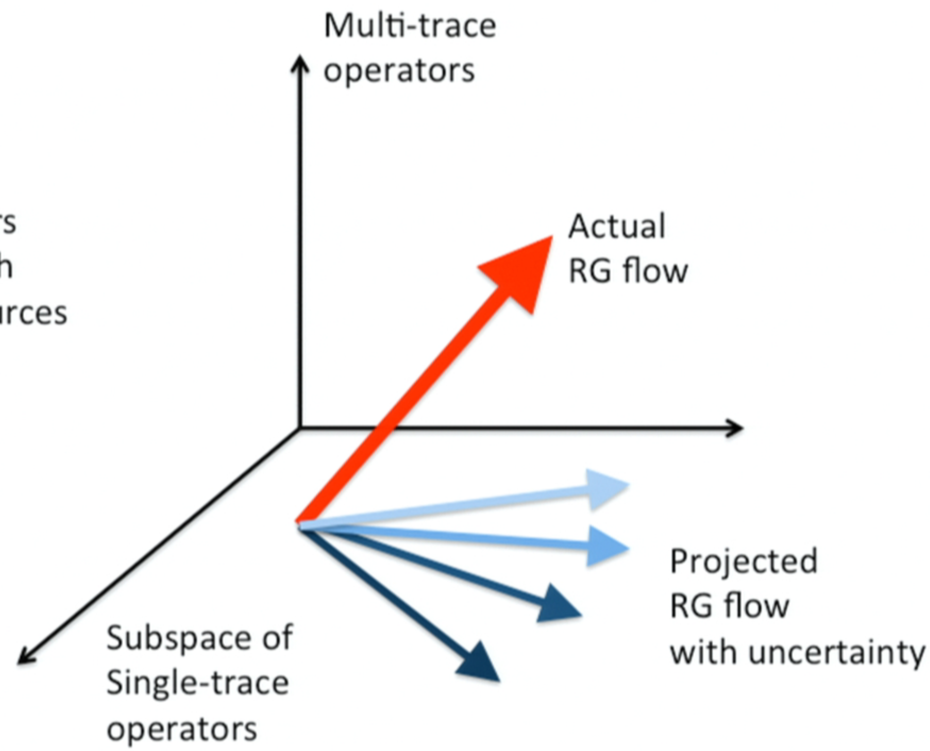
# Quantum RG :
# projection creates uncertainty

- Theory with multi-trace operators can be mapped into a theory with single-trace operators whose sources are dynamical



# Dynamical source and operator fields

$$Z = \int D\Phi e^{i\int \mathcal{L}'}$$

$$\mathcal{L}' = dz\mathcal{L}_c[J,x) + (J^n - dz\beta^n[J,x))\,O_n + dzG^{mn}[J,x)O_mO_n$$

$$Z = \int D\Phi Dj^{(1)n} Dp_n^{(1)} e^{i\int \mathcal{L}''}$$

$$\mathcal{L}'' = dz\mathcal{L}_c[J,x) + j^{(1)n}O_n + p_n^{(1)}(j^{(1)n} - J^n) + dz\beta^n[J,x)p_n^{(1)} + dzG^{mn}[J,x)p_m^{(1)}p_n^{(1)}$$

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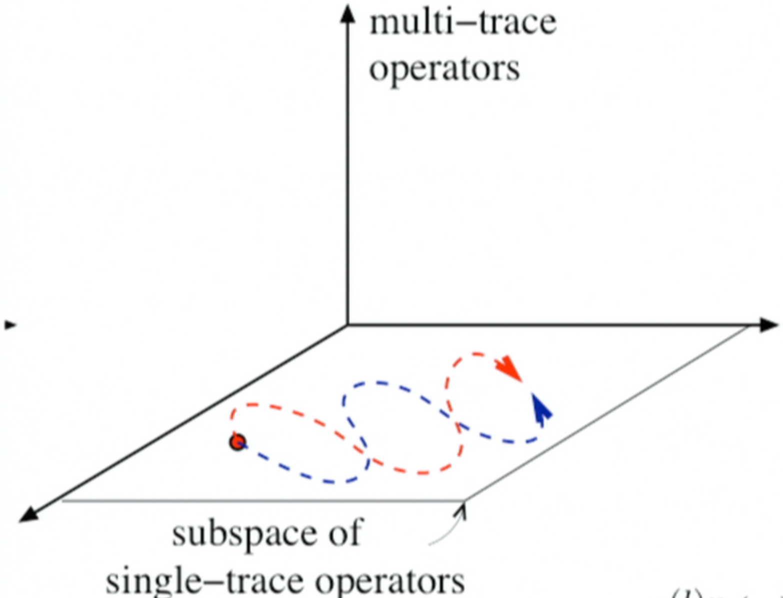
# Quantum fluctuations in RG path




- Only single-trace operators are included
- Generating function is given by a sum over all RG paths
- The weight of each path is determined by a (D +1)-dimensional action

$$j^{(l)n}(x),\ p_n^{(l)}(x) \to j^n(x,z),\ p_n(x,z)$$

$$Z = \int Dj(x,z)Dp(x,z) \quad e^{iS^{D+1}[j,p]}$$

SL, NPB (2011); JHEP (2012)

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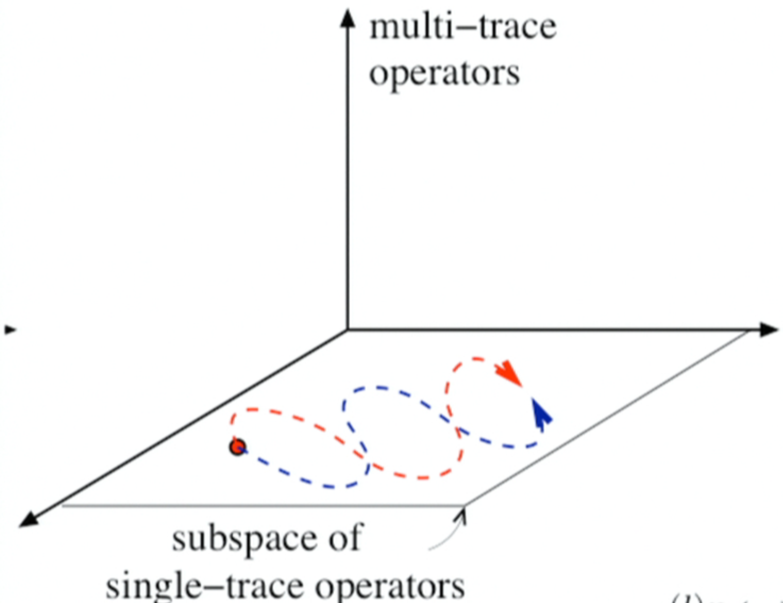


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# Holographic Action

$$S^{D+1} = N^2 \int dz \int d^D x \left\{ p_n(\partial_z j^n) + \mathcal{L}_c(x;j] + \beta^m(x;j]p_m + \frac{G^{mn}(x;j]}{2} p_m p_n \right\}$$

- Casimir energy, beta functions of single-trace operators and double-trace operators on the subspace of single-trace operators completely specify the (D+1)-dimensional action
- j and p are canonical conjugate to each other with respect to the RG `time' evolution
  - Casimir energy : potential energy for j
  - Double-trace operator : kinetic energy for p

# Scale-Reversal Symmetry

$$S^{D+1} = N^2 \int dz \int d^D x \left\{ p_n(\partial_z j^n) + \mathcal{L}_c(x;j] + \beta^m(x;j] p_m + \frac{G^{mn}(x;j]}{2} p_m p_n \right\}$$

- Generically, one expects that the bulk action breaks the Scale-Reversal (SR) symmetry under which z goes to -z because RG flow is irreversible
- However, irreversible RG flow can be still described by SR symmetric bulk action because of the boundary at UV cut-off, which explicitly breaks the SR symmetry
- On the other hand, all known holographic duals respect the SR symmetry in the bulk
- If this is indeed the case for all QFTs, what is implications ?
  - It turns out that the SR symmetry in the bulk can be maintained if the beta function for the single-trace operators is a gradient flow with respect to the metric given by the beta function for the double-trace operators

$$\beta^m(x;j] = G^{mn}(x;j] \frac{\delta c[j]}{\delta j^n(x)}$$

  - In this case, one can shift p to trade SR-odd term with boundary terms and a mass term that is proportional to the scaling dimension [SL, to appear]

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# What is the nature of the (D+1)-dimensional holographic action ?

- In general, it is a strongly coupled theory for an infinitely many fields for each primary single-trace operators
  - In large N limit, it becomes classical
  - If there are only a few single-trace operators with small scaling dimension, one can keep only those fields in the bulk
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# How does gravity arise ?

$$Z[g^{(0)}] = \int D\Phi \;\; e^{iS_1[\Phi; g^{(0)}(x)]}$$

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# Coarse graining

spacetime dependent speed of RG

$$\delta S = dz N^2 \int d^D x \; N(x) \Big\{ \sqrt{|g^{(0)}|} \left( C_0 + C_1^D \mathcal{R}(x; g^{(0)}] \right)$$

$$-\beta_{\mu\nu} T^{\mu\nu} + \frac{B_{\mu\nu;\rho\sigma}}{2} T^{\mu\nu} T^{\rho\sigma} + .. \Big\}$$

Casimir energy
[Sakharov]

Change of scale :
Warping factor

Double-trace operators

Higher derivative terms

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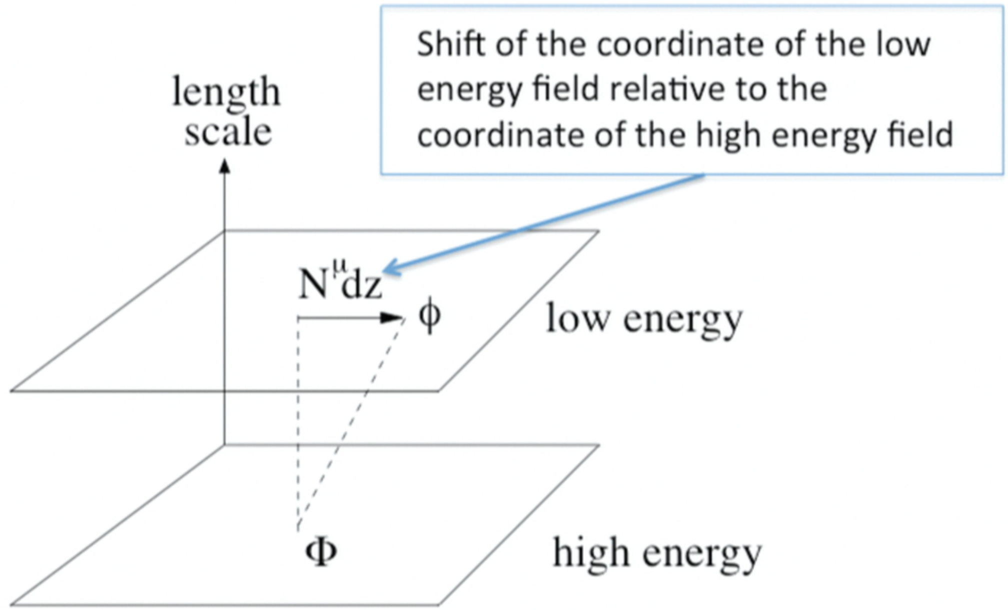
# Shift

- One does not have to choose the coordinate of the low energy field as the coordinate of the high energy mode



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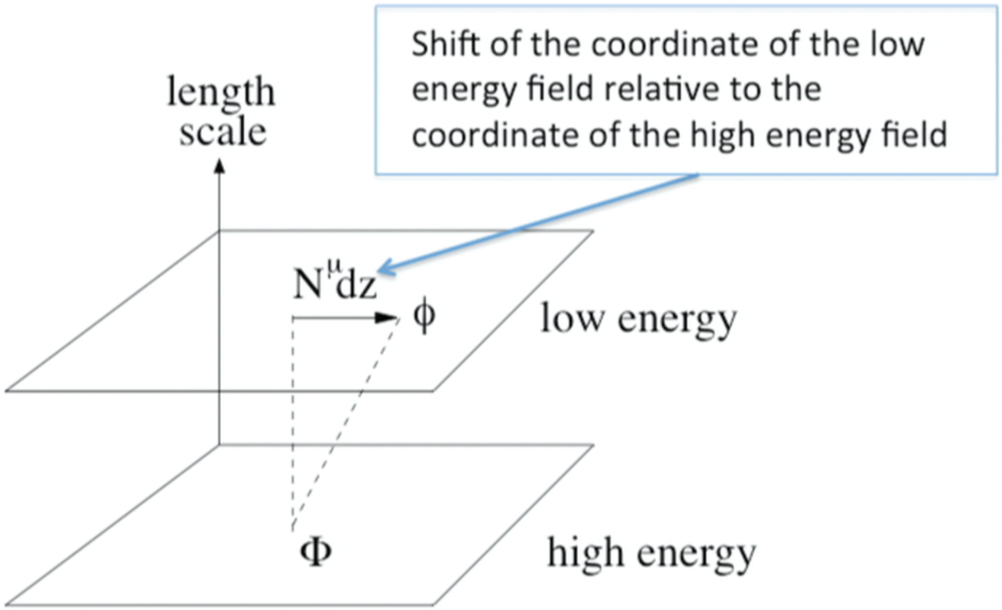

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# Einstein Gravity

$$S_{D+1} = \int d^{D+1}x\sqrt{G}[C_0 + R^{D+1} + ...]$$
$$= \int dz \int d^D x\,[\pi_{\mu\nu}\partial_z g^{\mu\nu} - N\mathcal{H} - N^\mu \mathcal{H}_\mu]$$

Casimir energy

Beta function of $T^{\mu\nu}T^{\rho\sigma}$

$$\mathcal{H} = -\sqrt{g}\left[C_0 + R^D + \frac{g^{-1}}{2}\left(\pi^2/2 - \pi^{\mu\nu}\pi_{\mu\nu}\right) + ..\right]$$
$$\mathcal{H}^\mu = -2\nabla_\nu \pi^{\mu\nu} + ..$$

This form is fixed by D-dimensional diff. inv. and the gauge invariance associated with the choice of local RG scheme

# First-class constraints

- Independence of partition function on RG schemes (speed of RG and shifts) ➔ (D+1)-constraints

$$< \mathcal{H}_M(x,z) >= \frac{1}{Z}\frac{\delta Z}{\delta N^M(x,z)} = 0 \qquad \mathcal{H} = 0, \quad \mathcal{H}_\mu = 0$$

M=0, 1, 2, ..., (D-1), D $\quad N^D(x,z) \equiv \alpha(x,z)$ and $\mathcal{H}_D \equiv \mathcal{H}$

- The (D+1)-constraints are (classically) first-class

$$\frac{\partial}{\partial z} < \mathcal{H}_M(x,z) >= \int d^D y\, N^{M'}(y,z) \langle\{\mathcal{H}_M(x,z), \mathcal{H}_{M'}(y,z)\}\rangle = 0$$

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