Title: Propagation of entanglement in strongly coupled systems from gravity

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Abstract:

Propagation of entanglement in strongly coupled systems

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Based on HL and Josephine Suh, to appear
HL, Mark Mezei and Josephine Suh, to appear

Thermalization and Entanglement

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- Equilibration provides to a dynamical setting to study the growth of entanglement

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closely related: What is the maximal rate a Hamiltonian can generate entanglement between subsystems ?

Holographic duality

Many-body system without gravity in d spacetime dimensions

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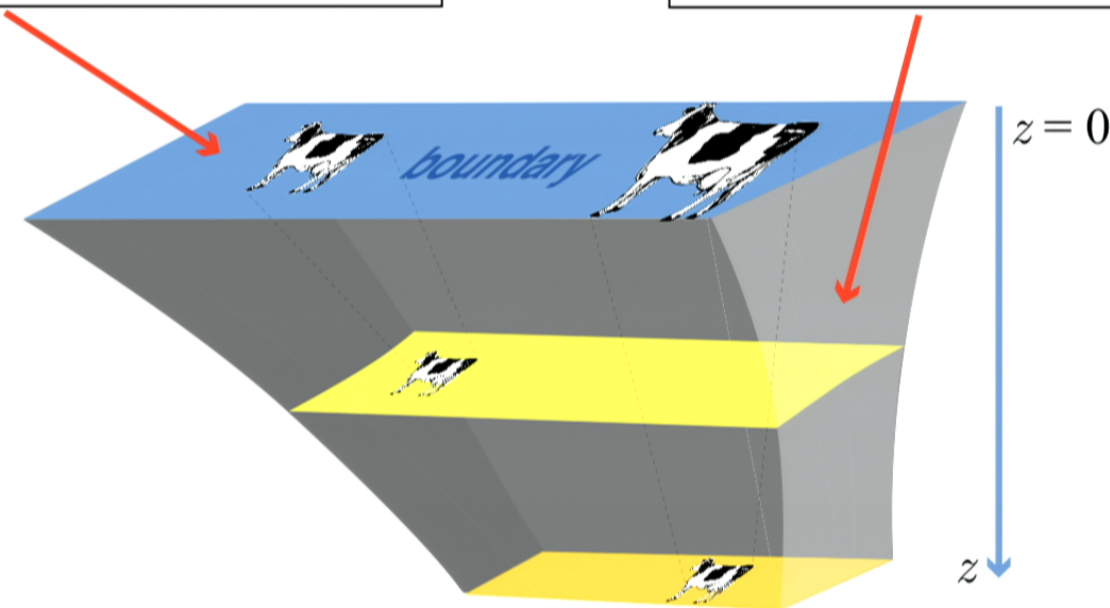
String theory, quantum gravity in d+1 dimensions



Holographic thermalization

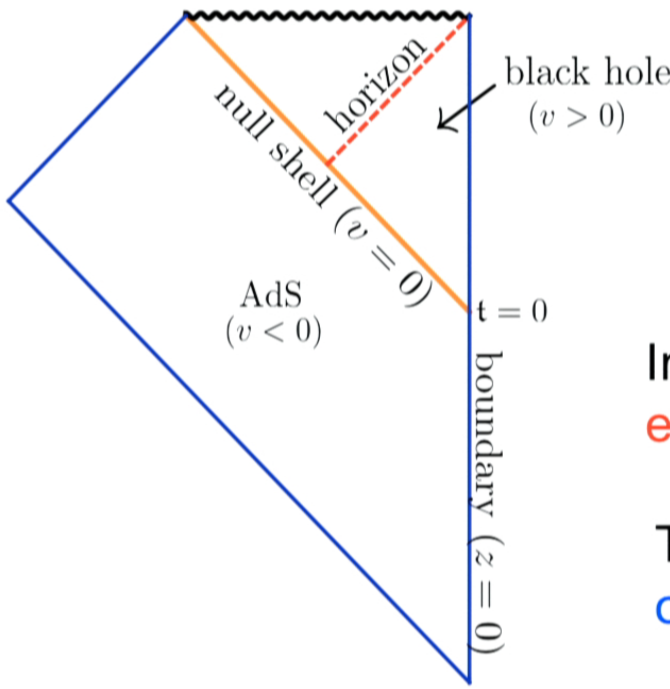
thermalization for a sufficiently excited state ⟺ gravitational collapse of a sufficiently massive object to form a black hole

- Gravity provides powerful tools to study a large class of strongly interacting systems

- New perspectives on puzzling issues of quantum black holes, including the information paradox.

Gravitational collapse: universal phenomenon

Simplest set-up: global quenches



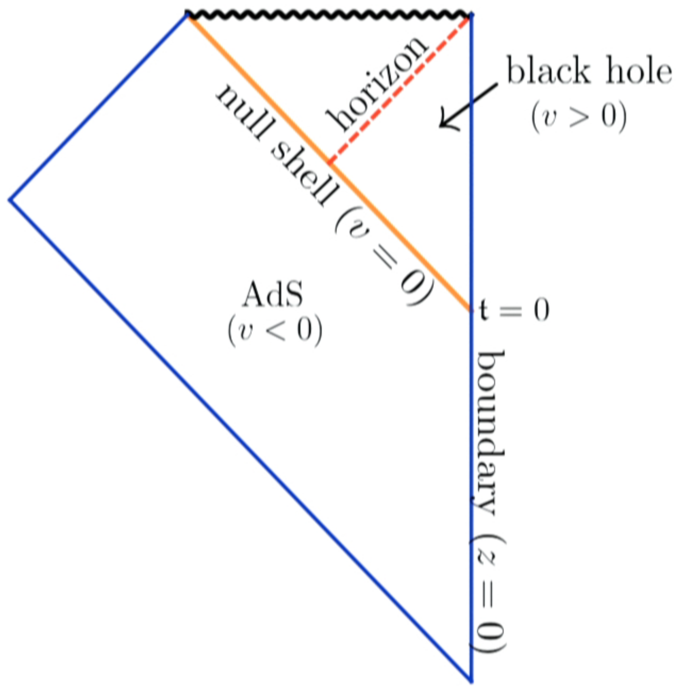
Field theory: start with a gapless system, turn on a source uniformly in space for a short time interval δt at t=0.

Initial state: homogeneous, isotropic energy density: ε

Take δt much smaller than any other time scale

Initial and Equilibrium state

At t = 0_+, essentially all equal-time observables behave as if in the vacuum.



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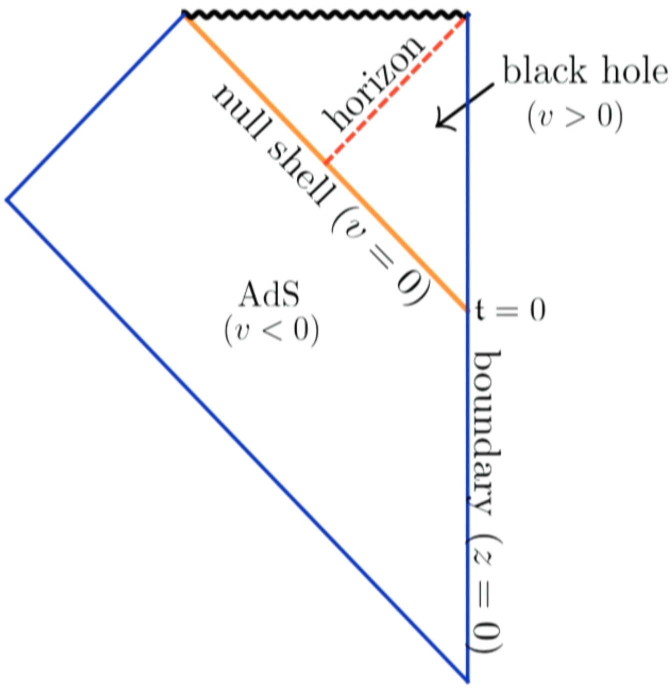

The equilibrium state:

Energy density: ε

Temperature: T

Entropy density: s_{eq}

Chemical potential: μ

....

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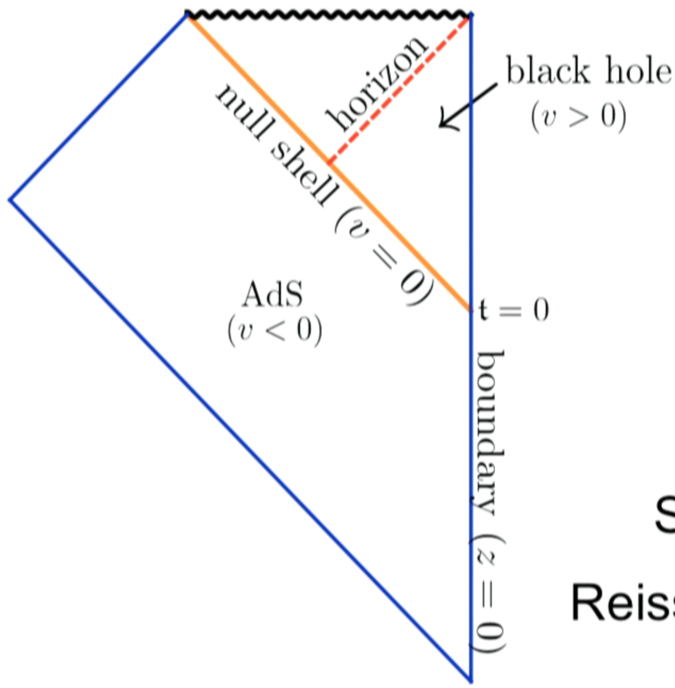

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Energy density: ε

Temperature: T

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Chemical potential: μ

....

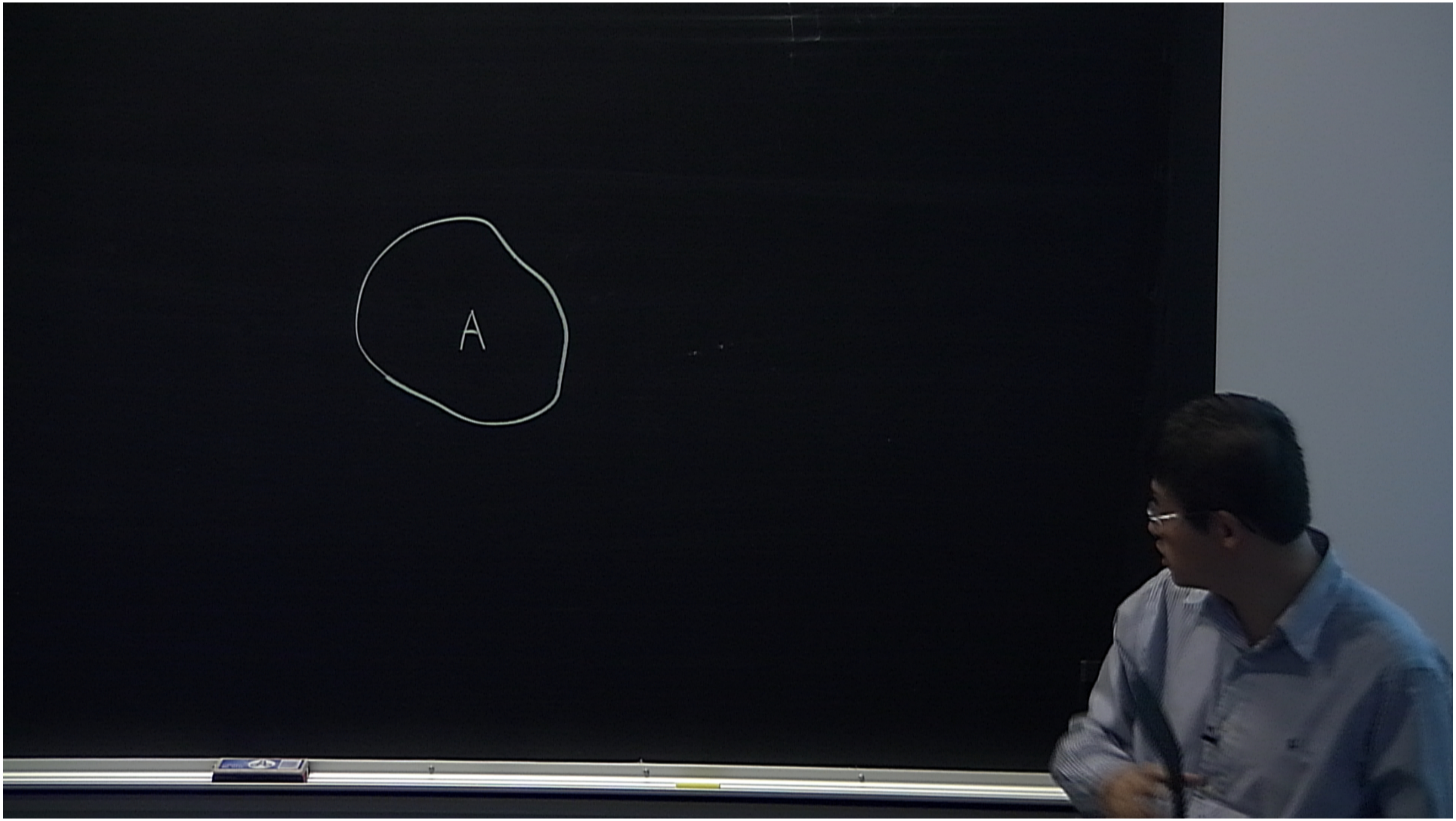
Schwarzschild: $\mu = 0$

Reissner-Norstrom: $\mu \neq 0$

interested in "universal" behavior insensitive to details of energy injection and microscopics of the equilibrium states.

General expectations

The entanglement entropy for any finite region will eventually saturate at the thermal value to leading order in region size.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General expectations

The entanglement entropy for any finite region will eventually saturate at the thermal value to leading order in region size.

So will other non-local observables such as equal-time correlation functions and Wilson loops.

The system should locally equilibrate at first at a time scale $\ell_{\rm eq}$ after which thermodynamics/hydrodynamics should apply locally.

Strongly interacting holographic systems:

$$\ell_{\rm eq} \sim \frac{1}{T} \quad (\mu = 0)$$

More generally:

$$\ell_{\rm eq} \sim \left(\frac{1}{s_{\rm eq}}\right)^{\frac{1}{D}}$$

Holographic entanglement entropy

Ryu, Takayanagi
Hubeny, Rangamani, Takayanagi

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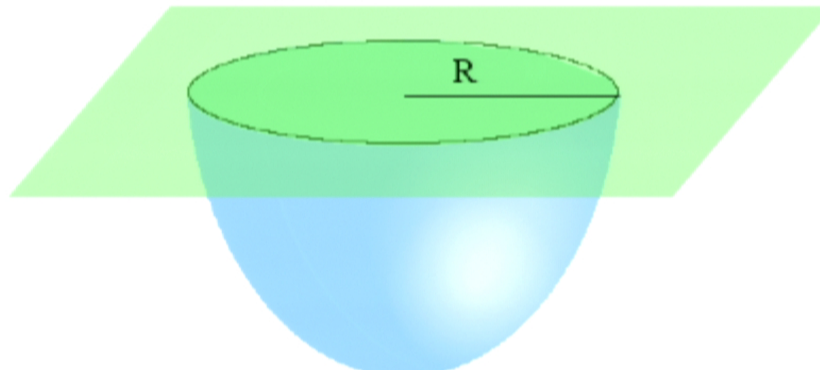

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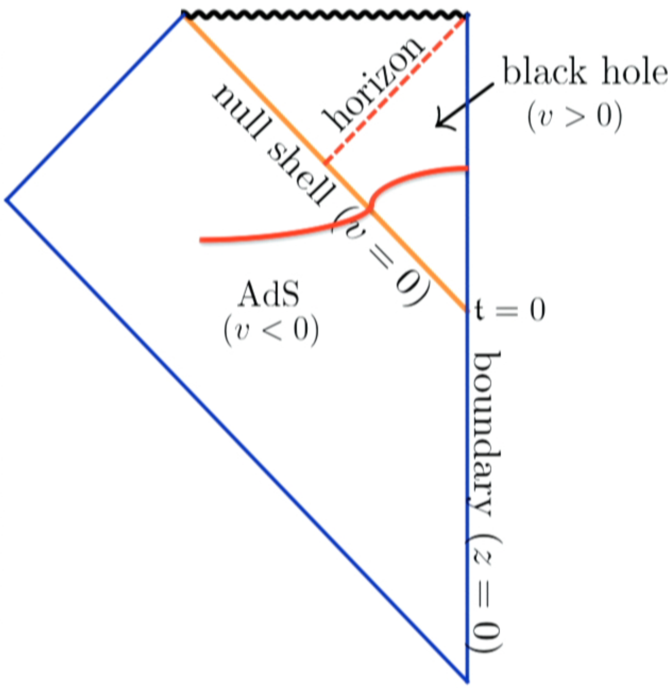

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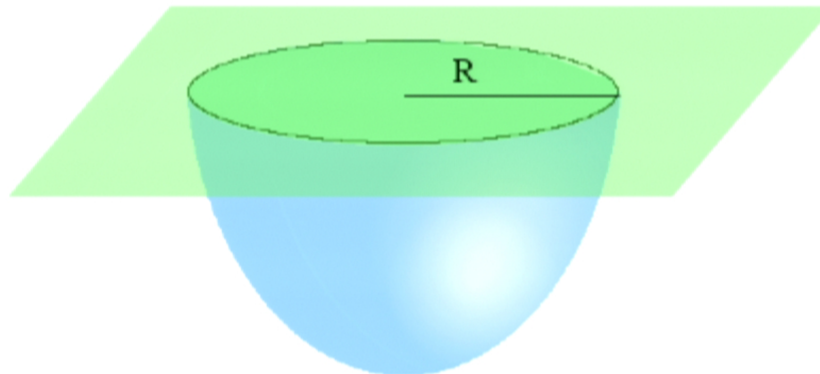

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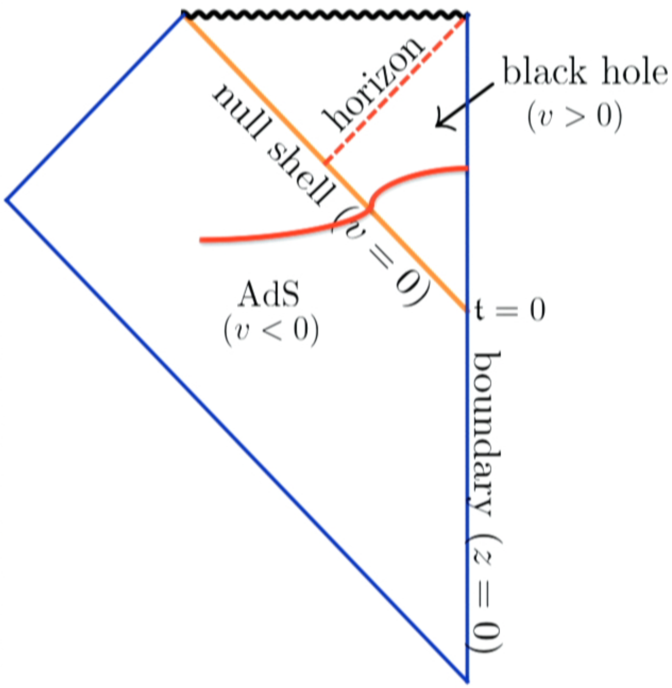

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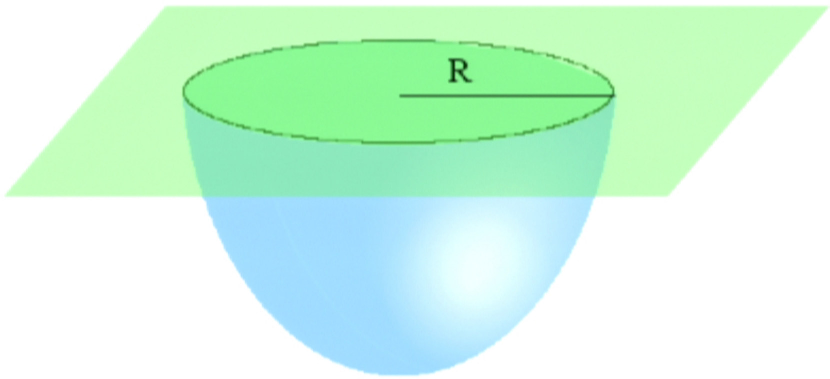

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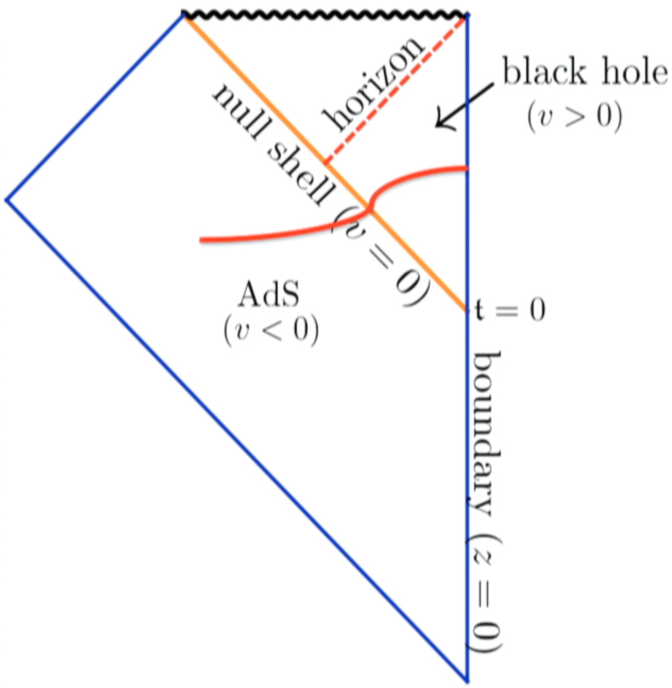
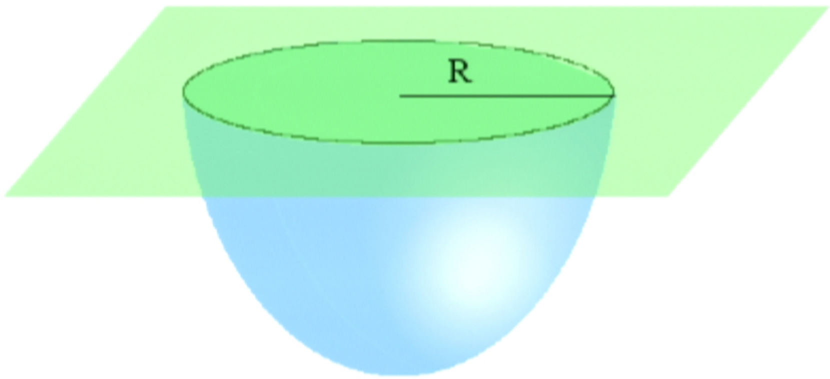
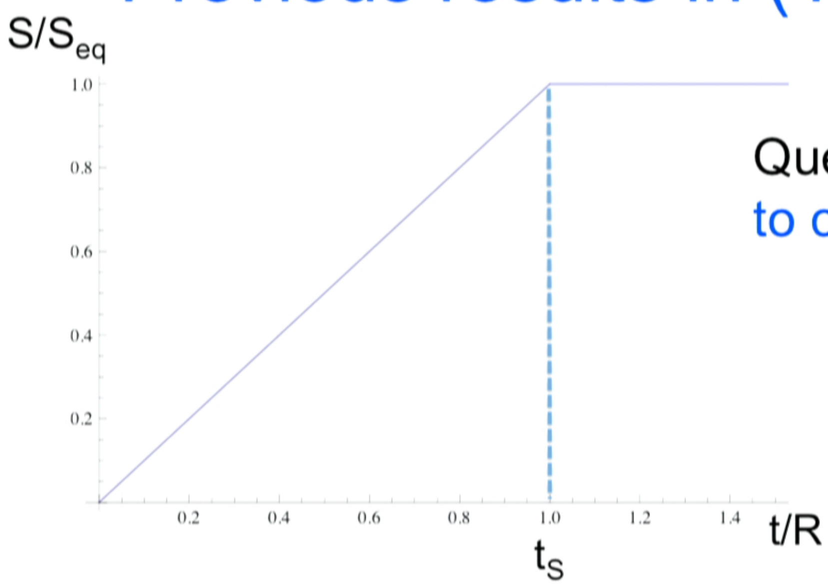




Previous results in (1+1)-dimension



Calabrese and Cardy

Quench: tune a gapped system to criticality

1. Linear growth with time

(not too early time)

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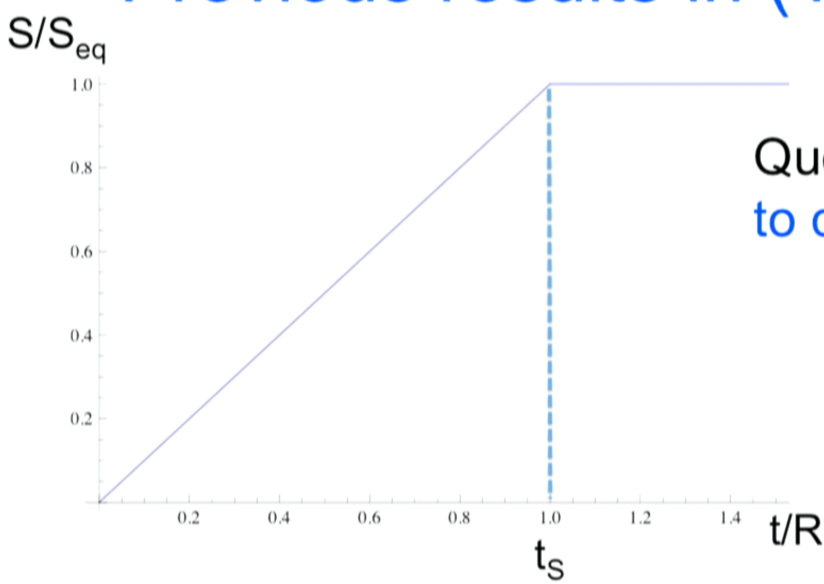

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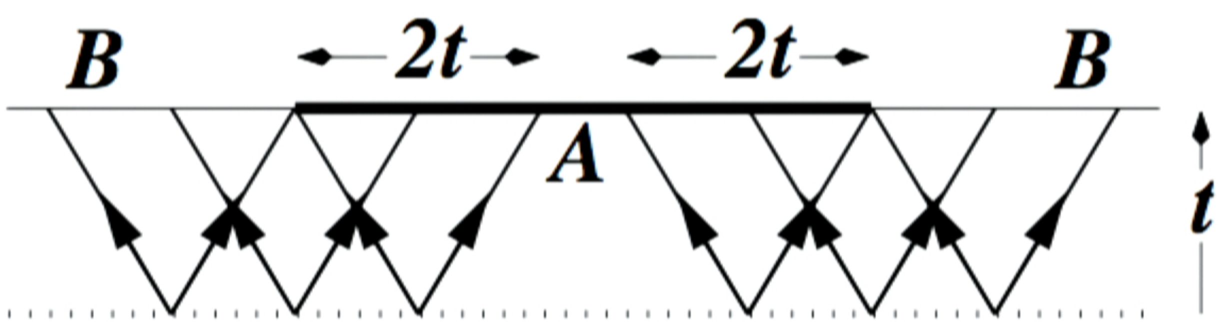
2. Saturation: $t_S = R$



Both features found in holographic systems with final state given by BTZ black hole.

Abajo-Arrastia, Aparicio, Lopez

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In higher dimensions, there are only holographic (numerical) studies:

T. Albash, Johnson;
Balasubramanian et al;
Caceres and Kundu;
……

Both features found in holographic systems with final state given by BTZ black hole.

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In higher dimensions, there are only holographic (numerical) studies:

There is a sharp saturation time, which is proportional to system size.

T. Albash, Johnson;
Balasubramanian et al;
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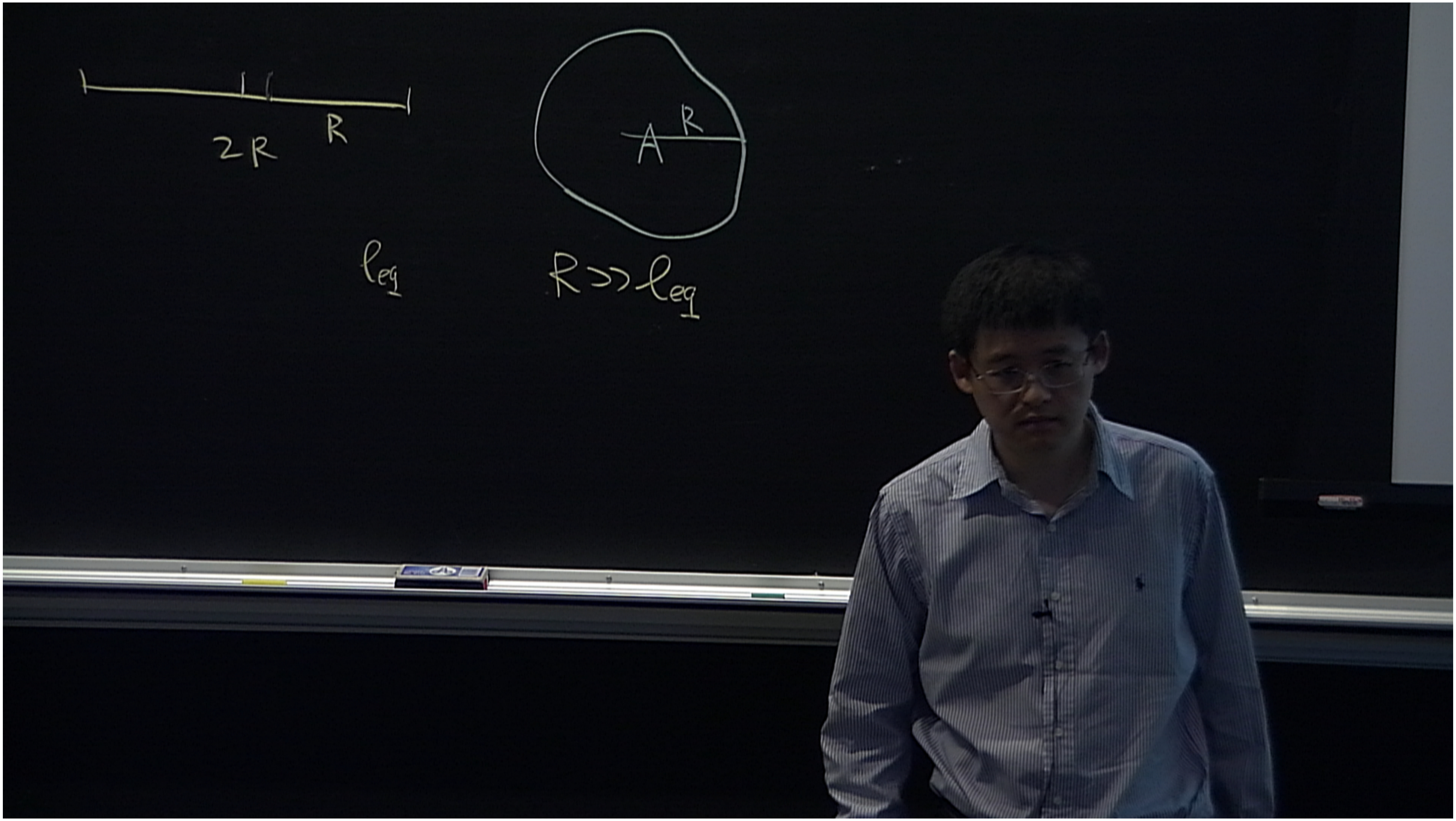
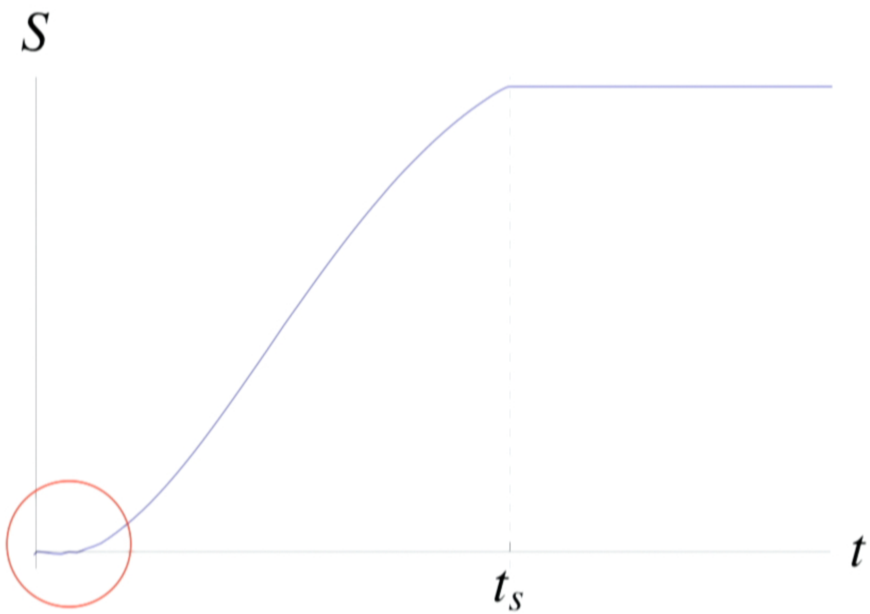
non-analytic behavior at saturation


2R
R
A
R
ℓ_{eq}
$R \gg \ell_{eq}$

Four stages of the evolution in general dimensions

HL and Suh, to appear



Pre-local-equilibration evolution

Post-local-equilibration linear growth

For $R \gg t \gg \ell_{\rm eq}$

$$S(t) - S(0) = v\, s_{\rm eq}\, A_\Sigma\, t + \cdots$$

$$s_{\rm eq}: \text{ equilibrium entropy density}$$

See also
Hartman, Maldacena

Again independent of shape, independent of theories under consideration, and independent of the nature of equilibrium state.

v: a dimensionless number who value depends on the nature of equilibrium state

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Schwarzschild $\quad v_S = \dfrac{(\eta-1)^{\frac{1}{2}(\eta-1)}}{\eta^{\frac{1}{2}\eta}} \qquad \eta \equiv \dfrac{2D}{D+1}$

Turning on chemical potential reduces v from this value.

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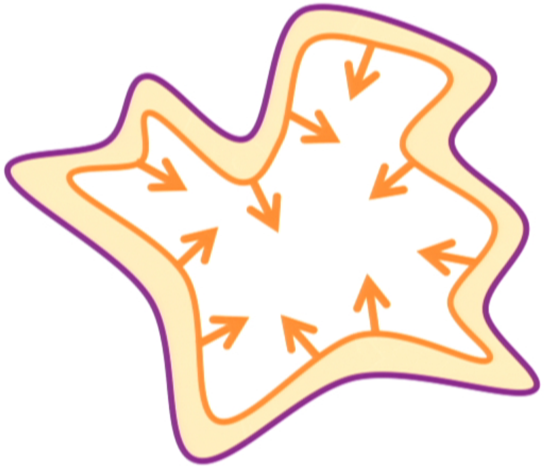
Entanglement Tsunami

The early time quadratic and linear growth suggests a picture of tsunami wave of entanglement, with a sharp wave front.



The wave velocity increases linearly during pre-local-equilibration stage, but stabilizes to a constant in the post-local-equilibration stage.

Shape independence and area laws in both regimes indicate the propagation is local, consistent with evolution from a local Hamiltonian.

Comparing with free particle streaming

Suh, HL, Mezei, to appear

Assume:

- At t=0, there is a uniform "quasiparticle" density
- "quasiparticles" starts propagating at speed of light at t=0.

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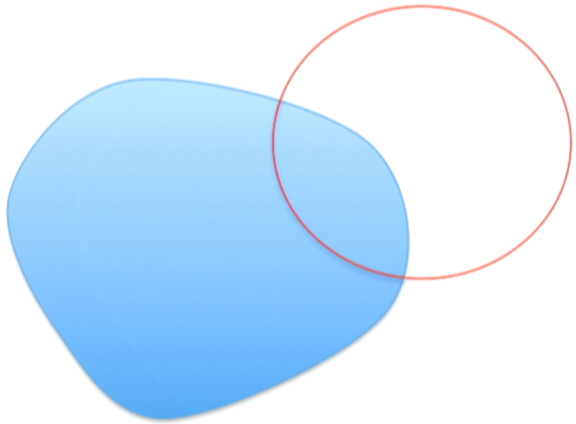

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Leading to shape independent linear growth at early times:

$D \geq 2$

$$v_{\text{streaming}} = \frac{\Gamma(\frac{D}{2})}{\sqrt{\pi}\Gamma(\frac{D+1}{2})} < \quad v_S \quad < 1$$

In strongly coupled systems, entanglement tsunami propagates faster than those from free particles traveling at speed of light !

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Free quasiparticle model also be generalized to capture the quadratic growth, when taking into account of the building up of ``quasiparticle" density during pre-local-equilibration stage.

Comparing with small incremental entangling conjecture/theorem

$$S(t) - S(0) = v \, s_{\rm eq} \, A_\Sigma \, t + \cdots$$

For spin systems:

$$\frac{dS}{dt} \leq c||H|| \log d$$

Dur, Vidal et al
Bravyi
Kitaev
Bennett et al
Van Acoleyen, Marien, Verstraete

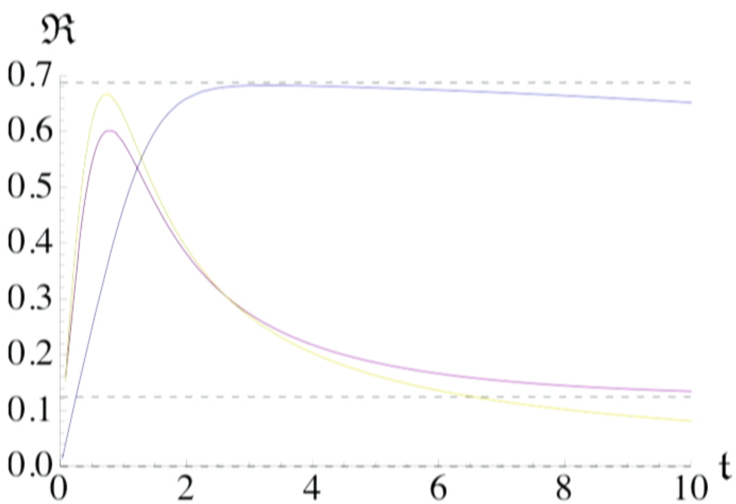
Could Schwarzschild velocity provide an upper bound on the rate of growth in the linear regime?

Entanglement growth

Introduce a dimensionless number:

$$\mathfrak{R}_\Sigma(t) \equiv \frac{1}{s_{\rm eq} A_\Sigma} \frac{dS_\Sigma}{dt}$$

Σ: sphere



Strong coupling

An upper bound on the rate?

$$\mathfrak{R}_\Sigma(t) \equiv \frac{1}{s_{\rm eq} A_\Sigma} \frac{dS_\Sigma}{dt}$$

Natural to speculate for relativistic systems:

$$\mathfrak{R}_\Sigma(t) \leq \frac{(\eta-1)^{\frac{1}{2}(\eta-1)}}{\eta^{\frac{1}{2}\eta}} = \begin{cases} 1 & D=1 \\ \frac{\sqrt{3}}{2^{\frac{4}{3}}} = 0.687 & D=2 \\ \frac{\sqrt{2}}{3^{\frac{3}{4}}} = 0.620 & D=3 \\ \frac{1}{2} & D=\infty \end{cases}$$

$$\eta \equiv \frac{2D}{D+1}$$

Geometric origin