Title: Impossible symmetry enriched topological phases in 2D and their realization on 3D surface

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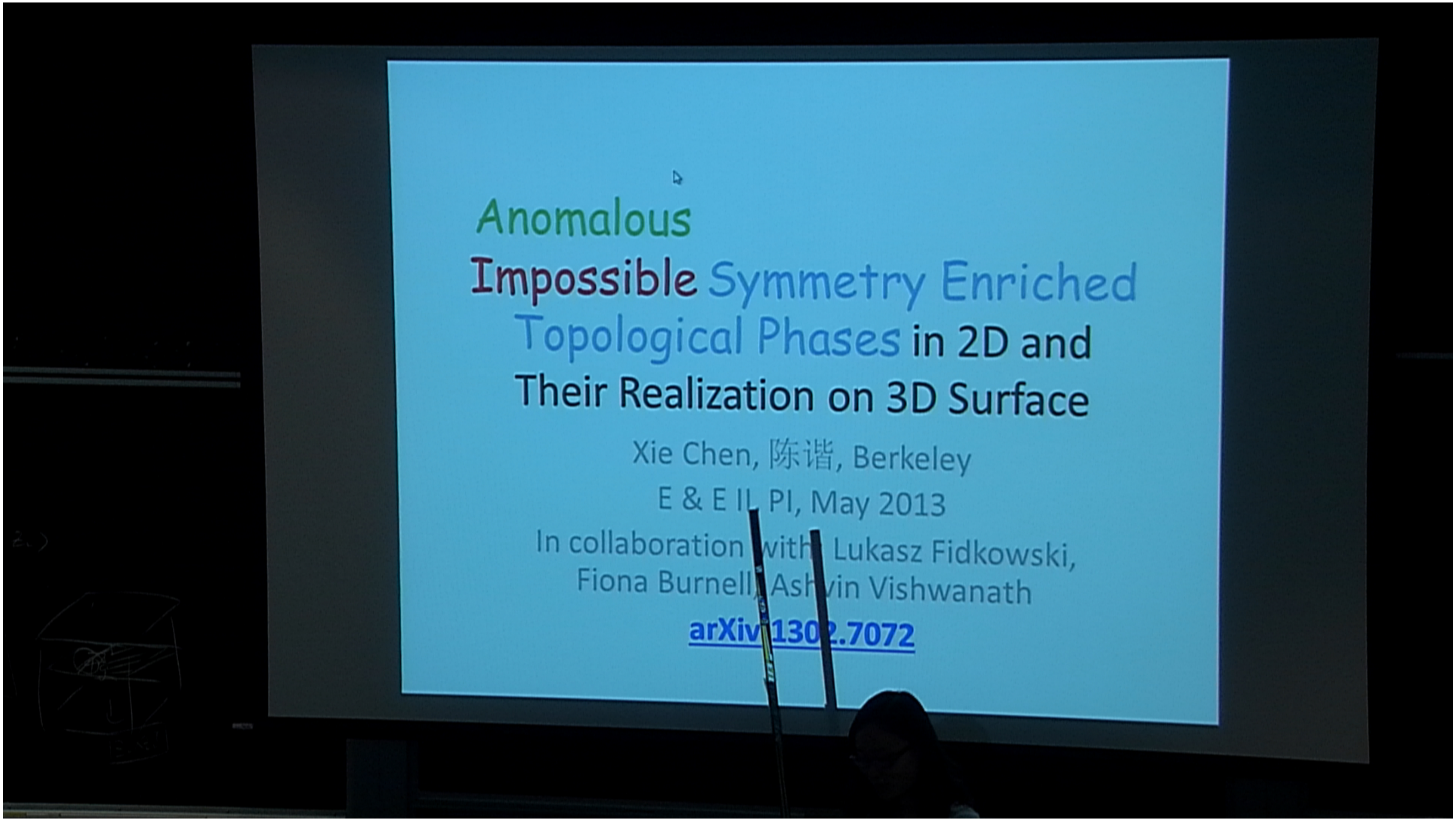
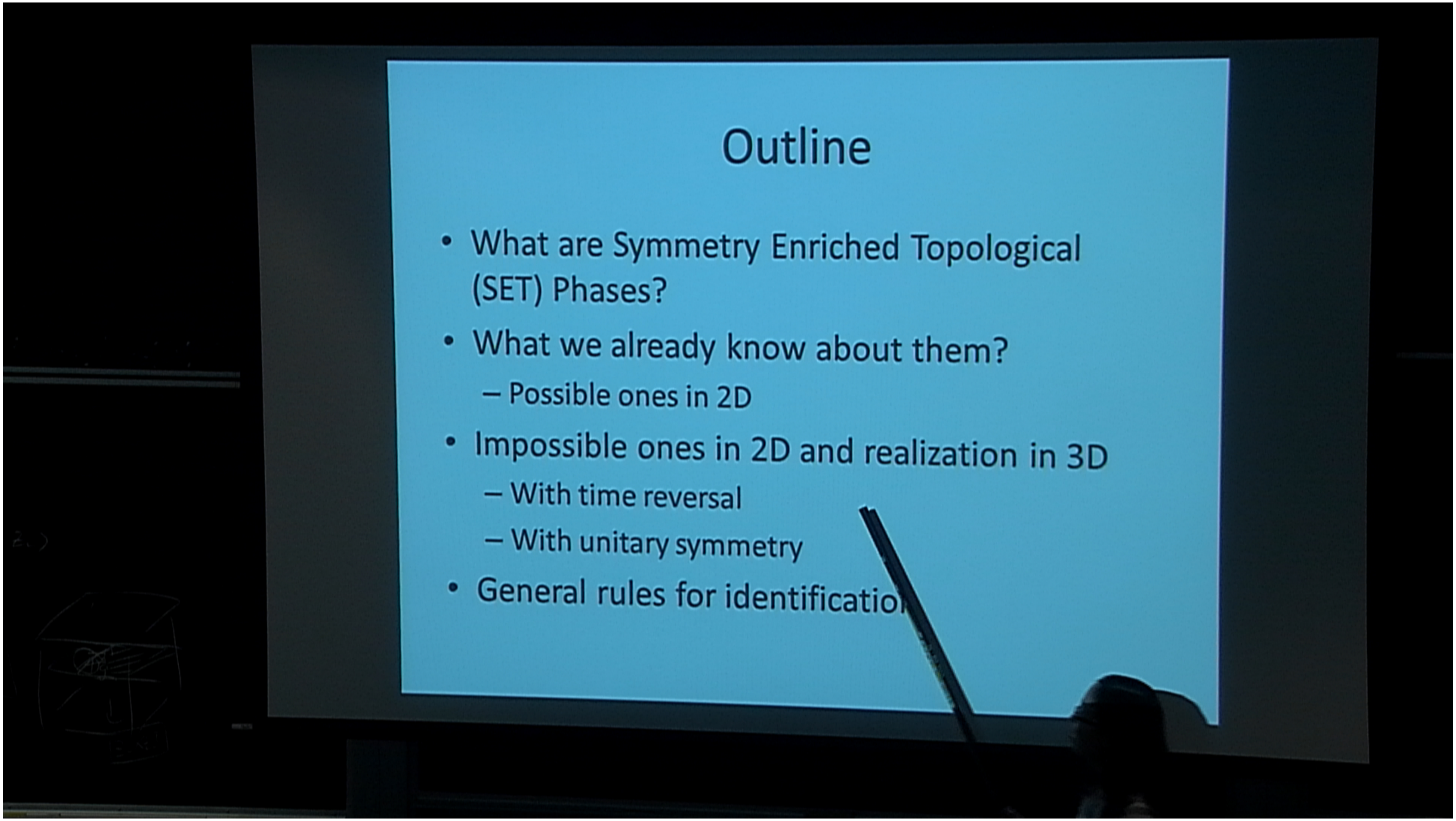
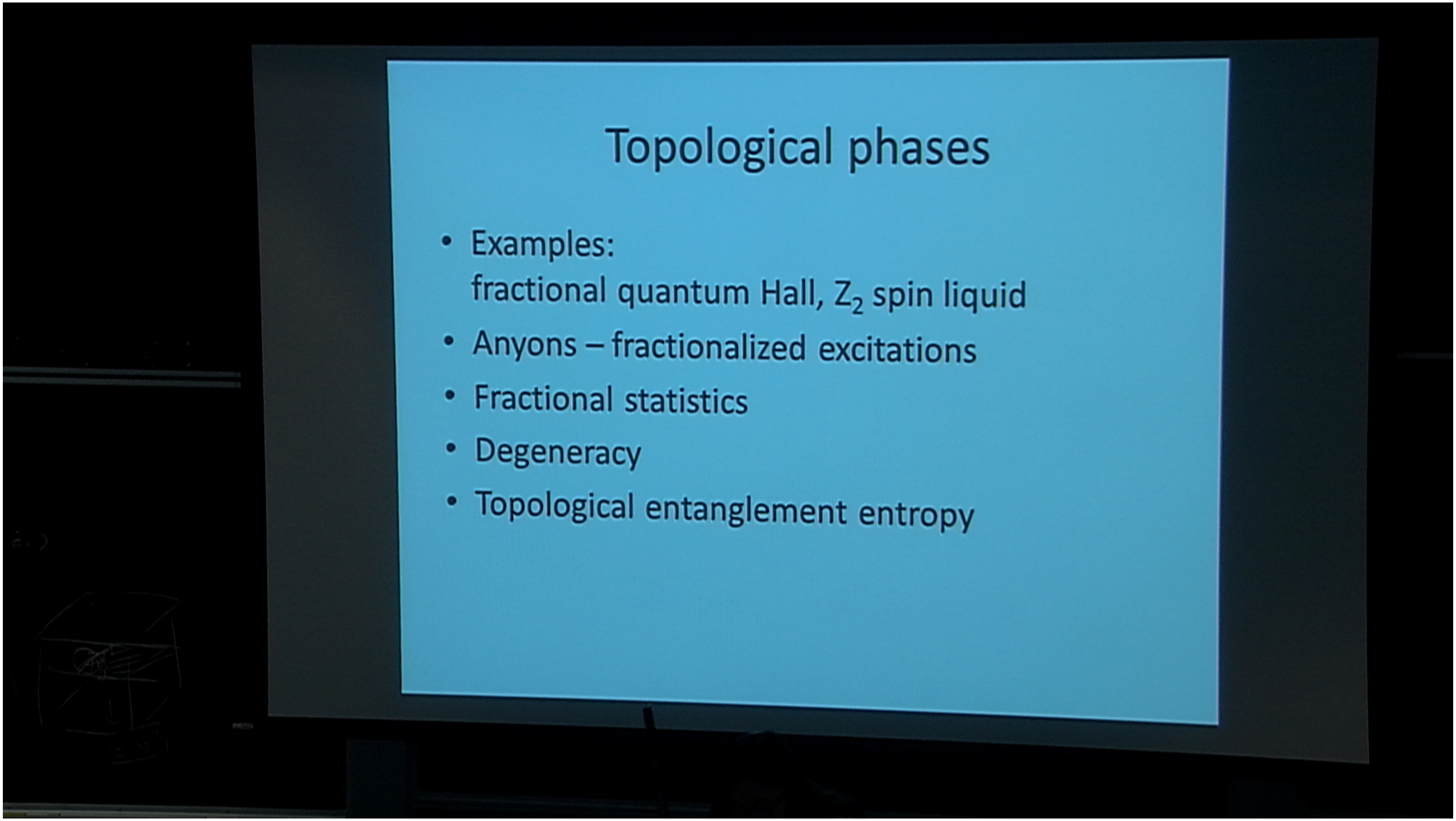
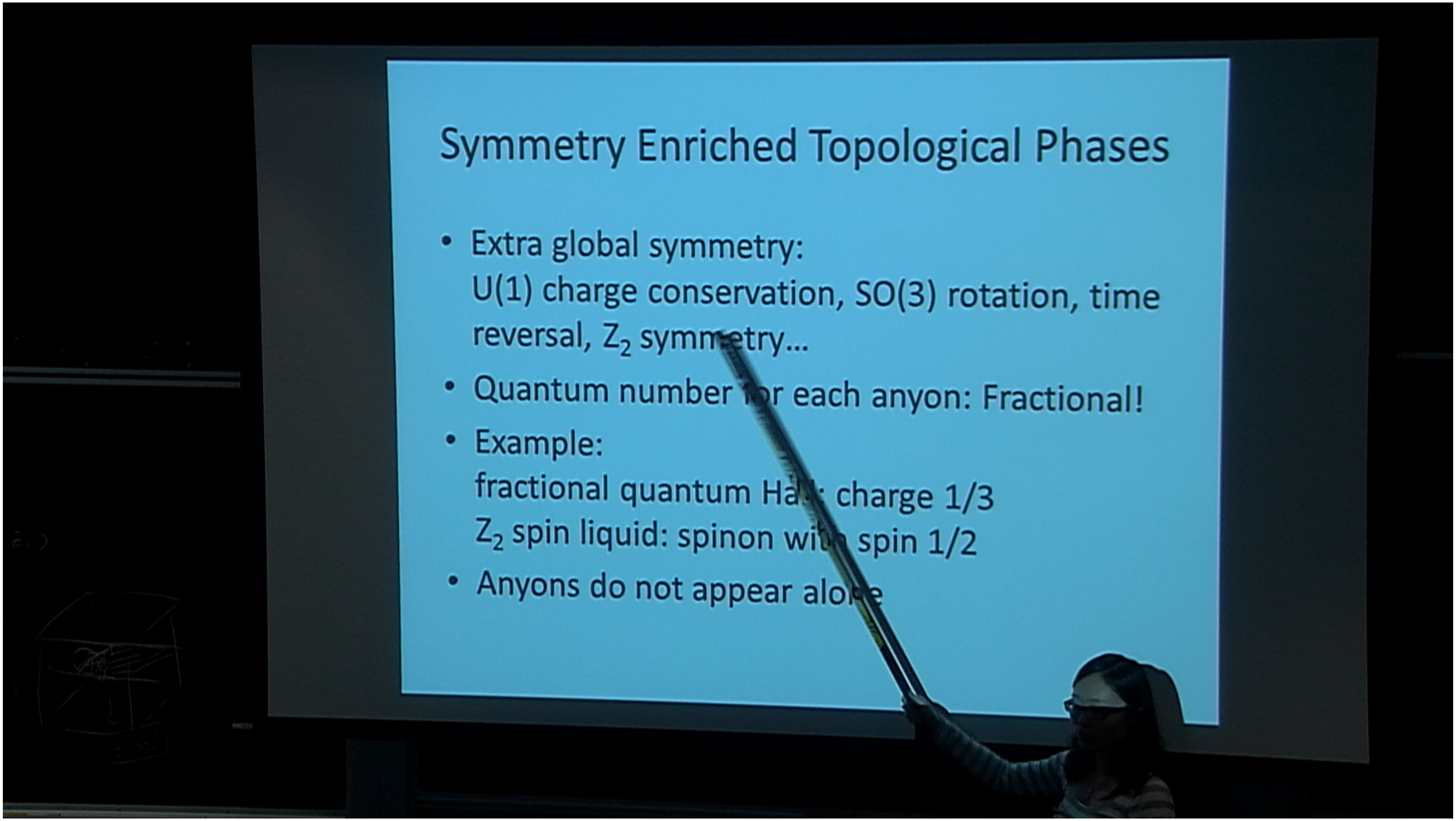
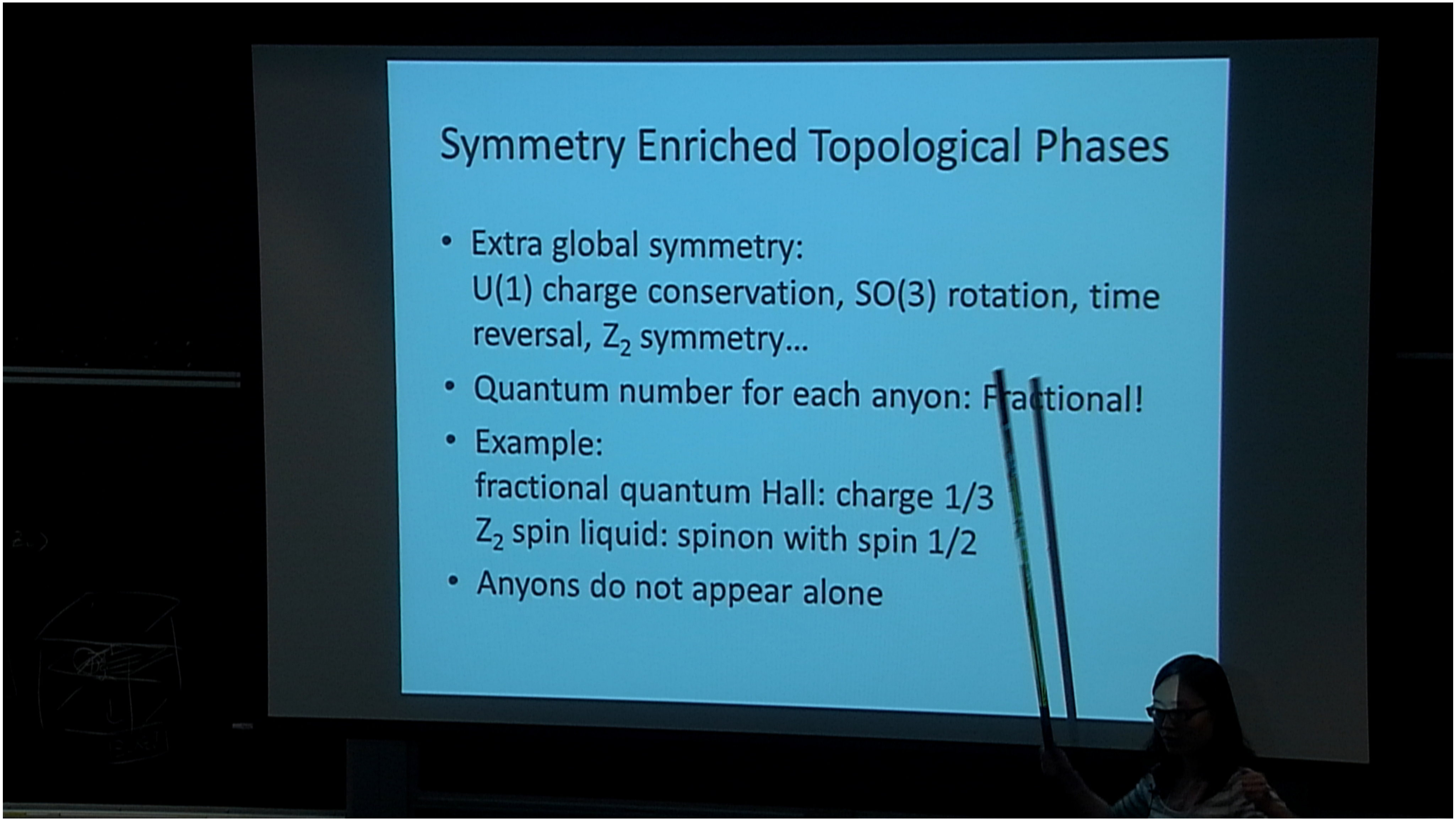
Abstract: In quantum systems with symmetry, the same topological phase can be enriched by symmetry in different ways, resulting in different symmetry transformations of the superselection sectors in the phase. However, not all symmetry transformations are allowed on the superselection sectors in topological phases in purely 2D systems. In this talk, I will discuss some examples of such symmetry enrichment of topological phases, which seem to be consistent with the fusion and braiding rules of the superselection sectors in the theory but are nonetheless impossible to realize in 2D. Interestingly, we show further that they can be realized on the surface of a 3D gapped system with a topologically trivial bulk.


Anomalous
Impossible Symmetry Enriched
Topological Phases in 2D and
Their Realization on 3D Surface
Xie Chen, 陈谐, Berkeley
E & E II, PI, May 2013
In collaboration with Lukasz Fidkowski,
Fiona Burnell, Ashvin Vishwanath
arXiv


Outline
• What are Symmetry Enriched Topological (SET) Phases?
• What we already know about them?
– Possible ones in 2D
• Impossible ones in 2D and realization in 3D
– With time reversal
– With unitary symmetry
• General rules for identificatio


Topological phases
• Examples:
fractional quantum Hall, Z2 spin liquid
• Anyons – fractionalized excitations
• Fractional statistics
• Degeneracy
• Topological entanglement entropy


Symmetry Enriched Topological Phases
• Extra global symmetry:
U(1) charge conservation, SO(3) rotation, time reversal, Z_2 symmetry…
• Quantum number for each anyon: Fractional!
• Example:
fractional quantum Hall: charge 1/3
Z_2 spin liquid: spinon with spin 1/2
• Anyons do not appear alone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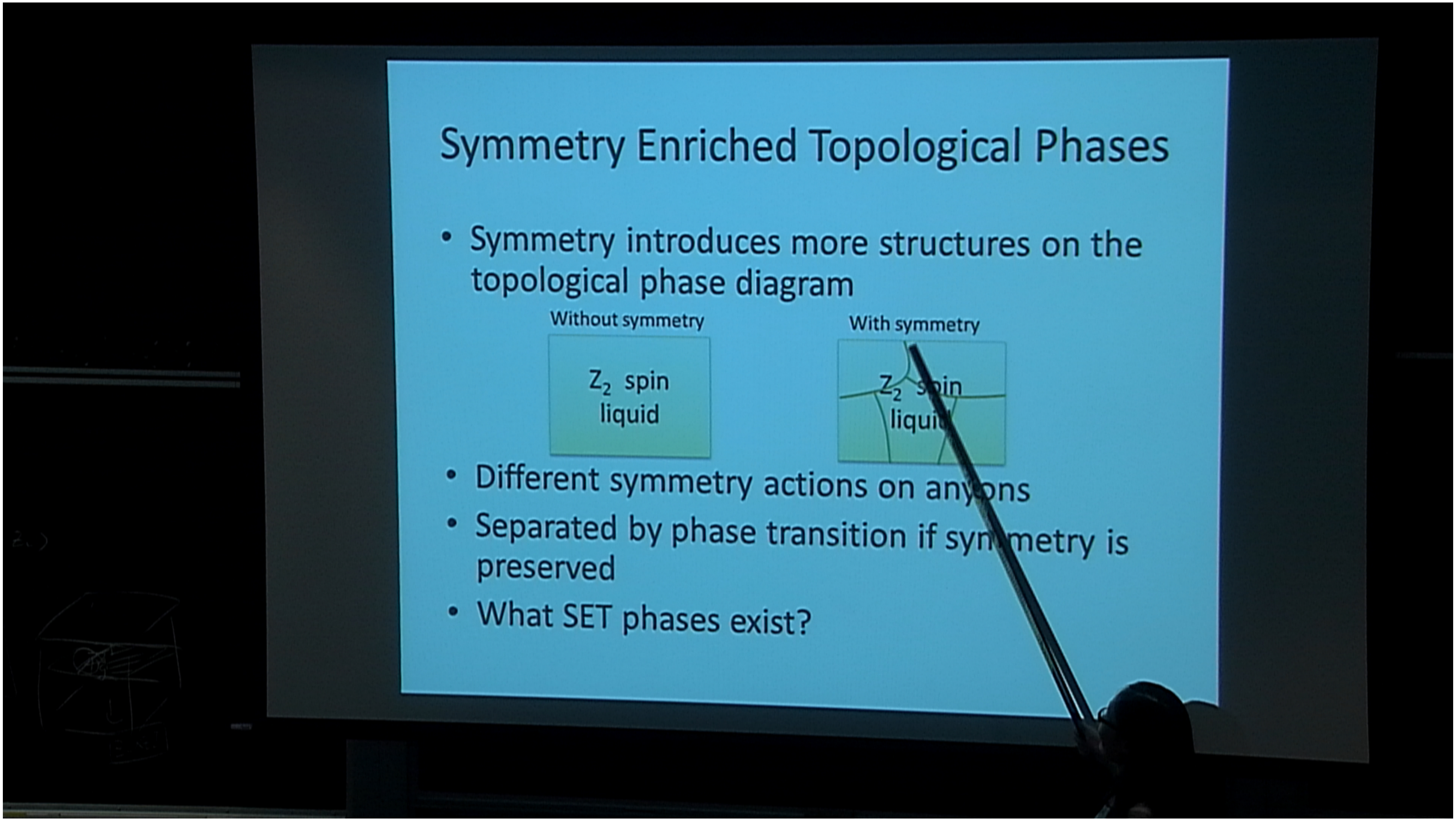
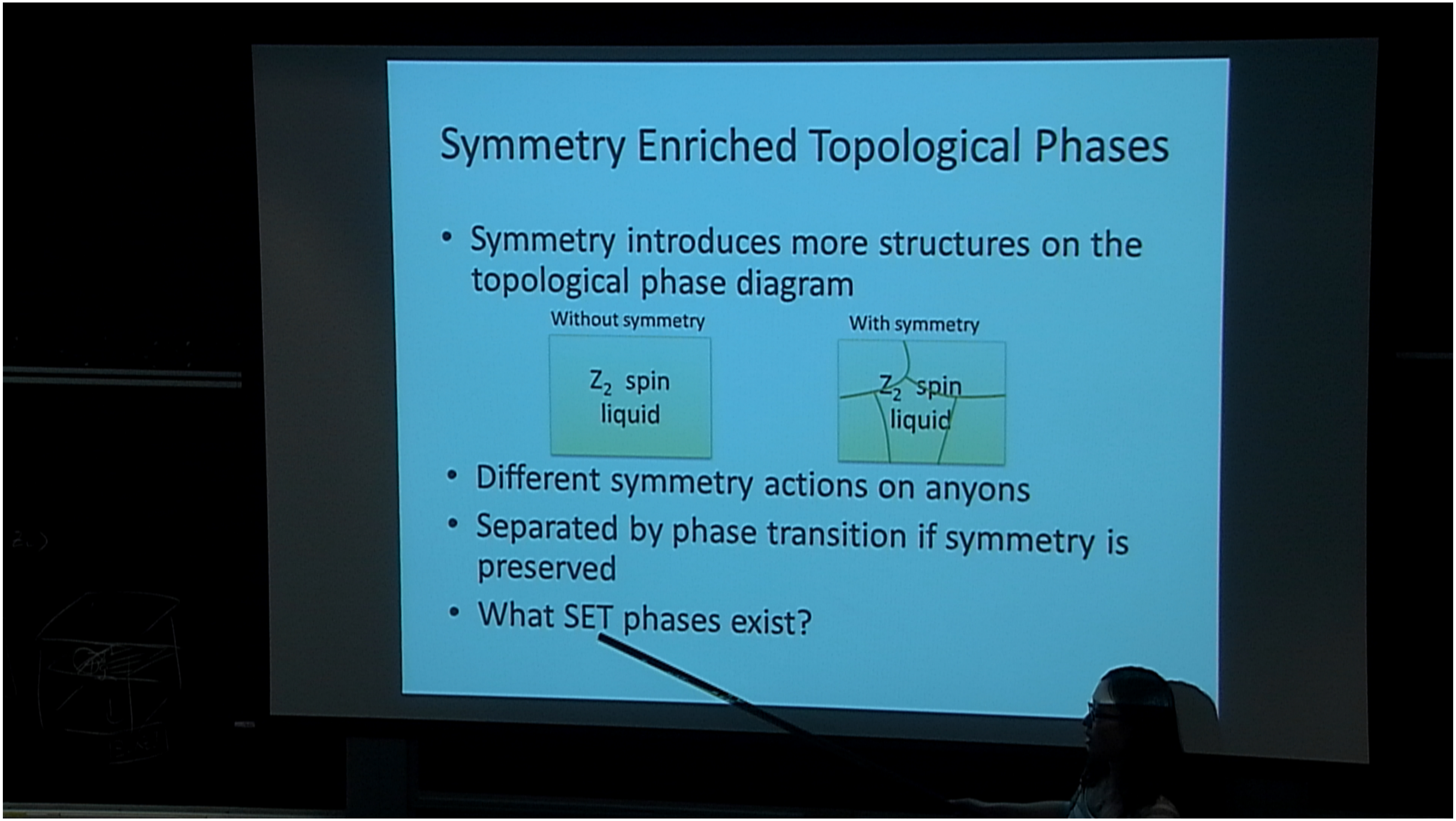

Symmetry Enriched Topological Phases
• Symmetry introduces more structures on the topological phase diagram
Without symmetry
Z_2 spin liquid
With symmetry
Z_2 spin liquid
• Different symmetry actions on anyons
• Separated by phase transition if symmetry is preserved
• What SET phases exist?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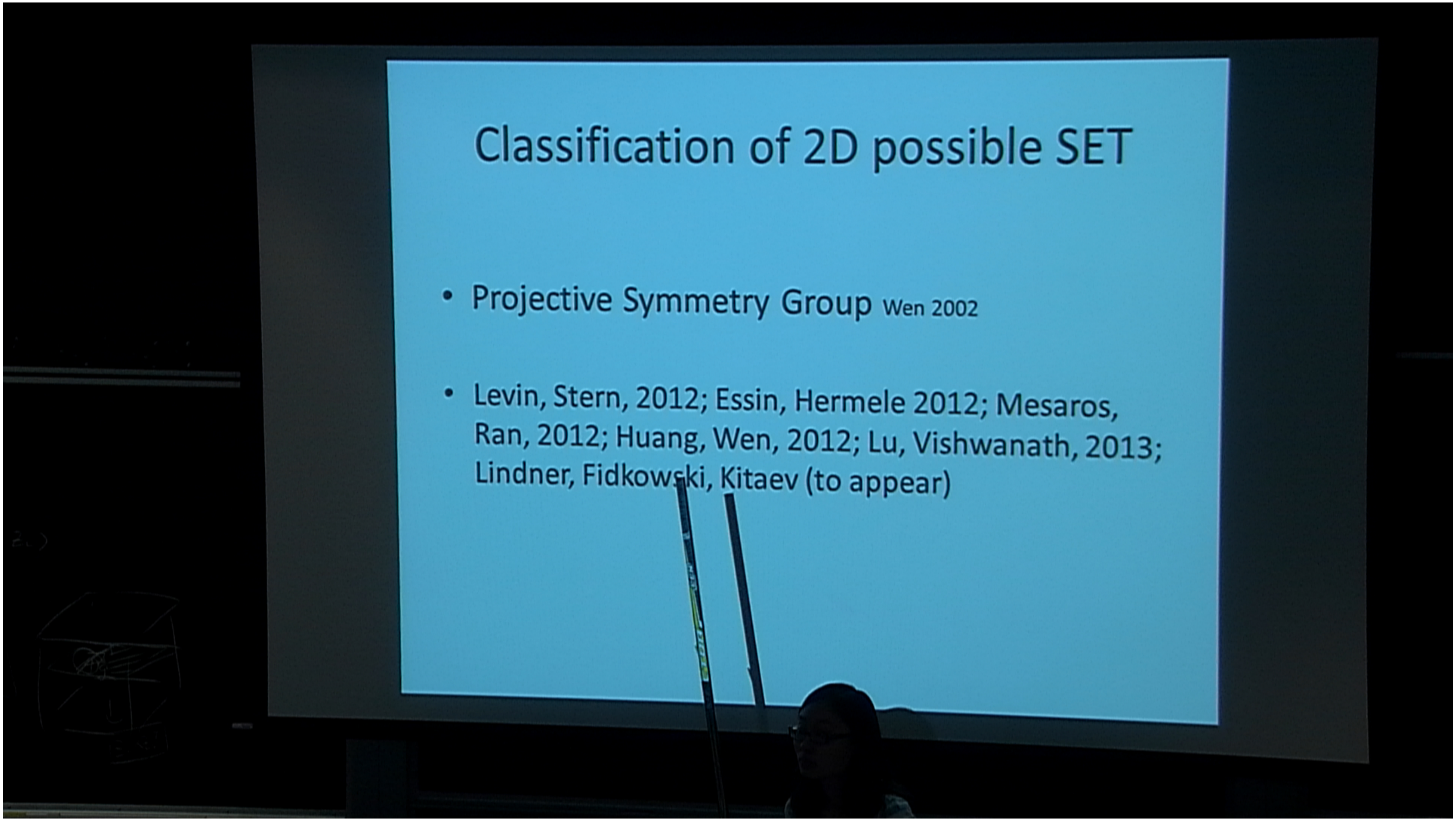
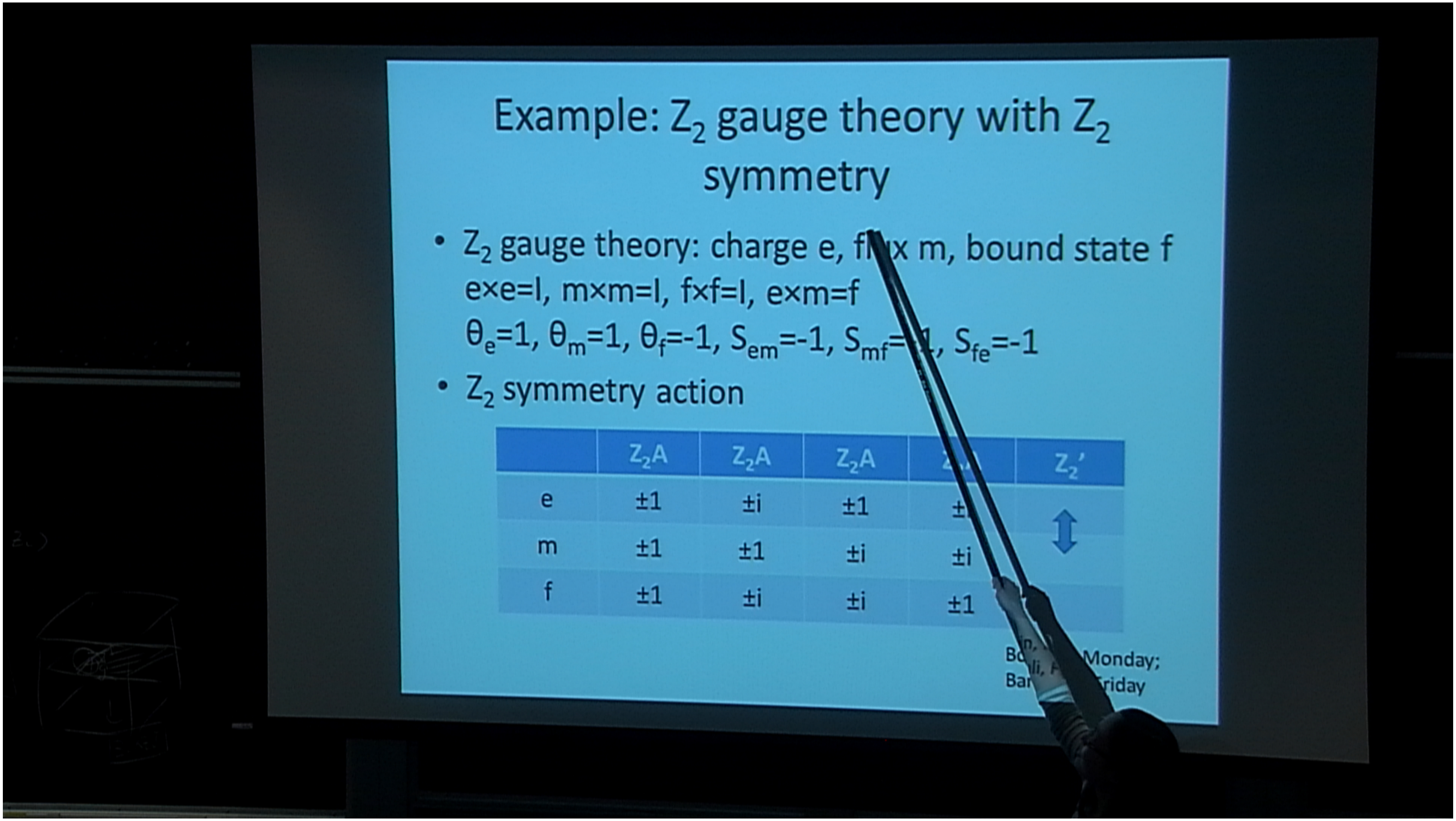
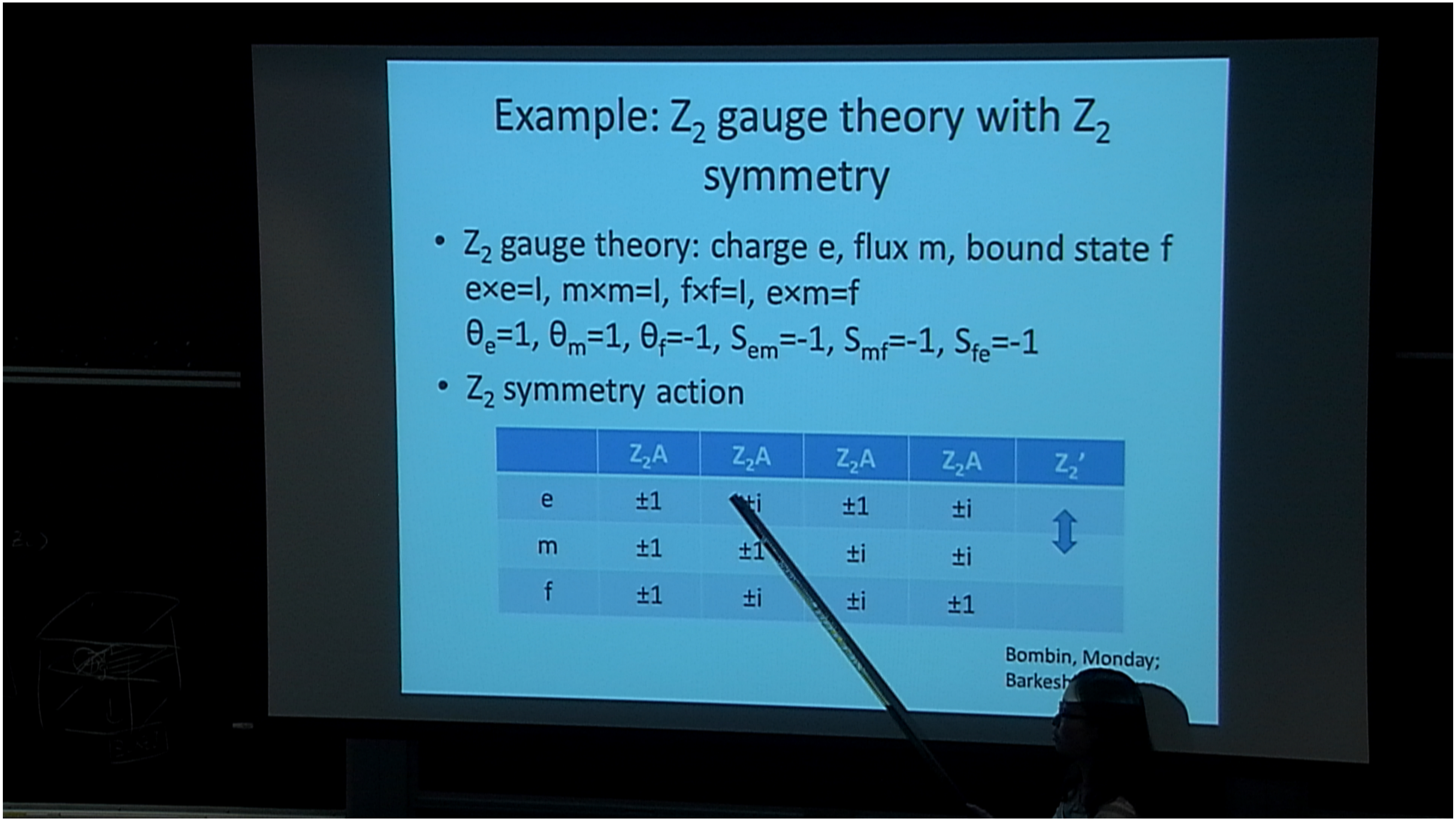

Classification of 2D possible SET
Projective Symmetry Group Wen 2002
Levin, Stern, 2012; Essin, Hermele 2012; Mesaros, Ran, 2012; Huang, Wen, 2012; Lu, Vishwanath, 2013; Lindner, Fidkowski, Kitaev (to appear)


Example: Z2 gauge theory with Z2 symmetry
Z2 gauge theory: charge e, x m, bound state f
e×e=I, m×m=I, f×f=I, e×m=f
θe=1, θm=1, θf=-1, Sem=-1, Smf=, Sfe=-1
Z2 symmetry action
Z2A | Z2A | Z2A | Z2'
e | ±1 | ±i | ±1
m | ±1 | ±1 | ±i | ±i
f | ±1 | ±i | ±i | ±1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Z2A Z2A Z2A Z2A Z2'
e ±1 ±i ±1 ±i
m ±1 ±1 ±i ±i
f ±1 ±i ±i ±1
Bombin, Monday;
Barkesh

Example: Z_2 gauge theory with Z_2 symmetry

- Z_2 gauge theory: charge e, flux m, bound state f
  $e \times e = I$, $m \times m = I$, $f \times f = I$, $e \times m = f$
  $\theta_e = 1$, $\theta_m = 1$, $\theta_f = -1$, $S_{em} = -1$, $S_{mf} = -1$, $S_{fe} = -1$
- Z_2 symmetry action

	Z_2A	Z_2A	Z_2A	Z_2A	Z_2'
e	±1	±i	±1	±i	
m	±1	±1	±i	±i	
f	±1	±i	±i	±1	

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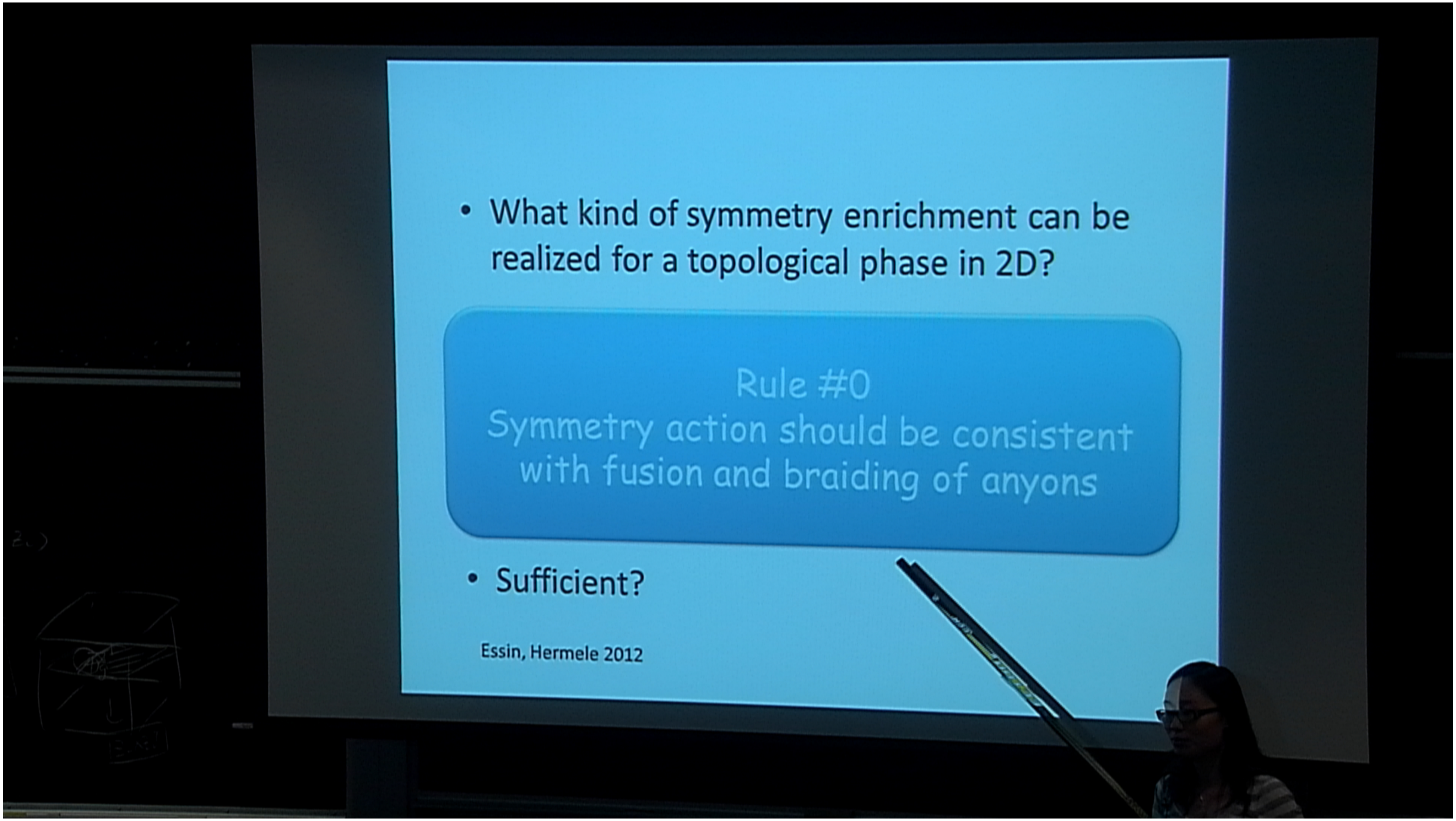
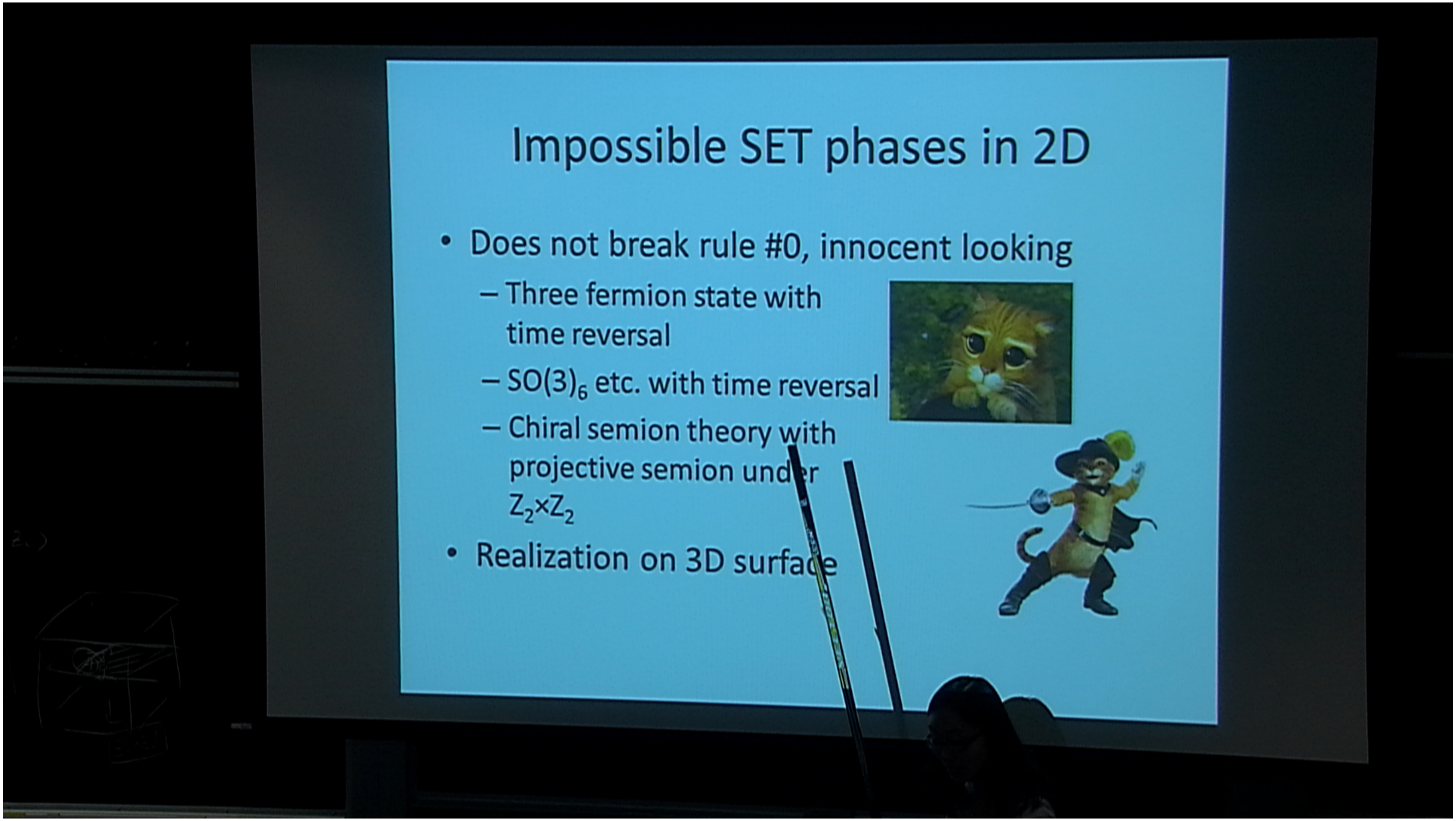
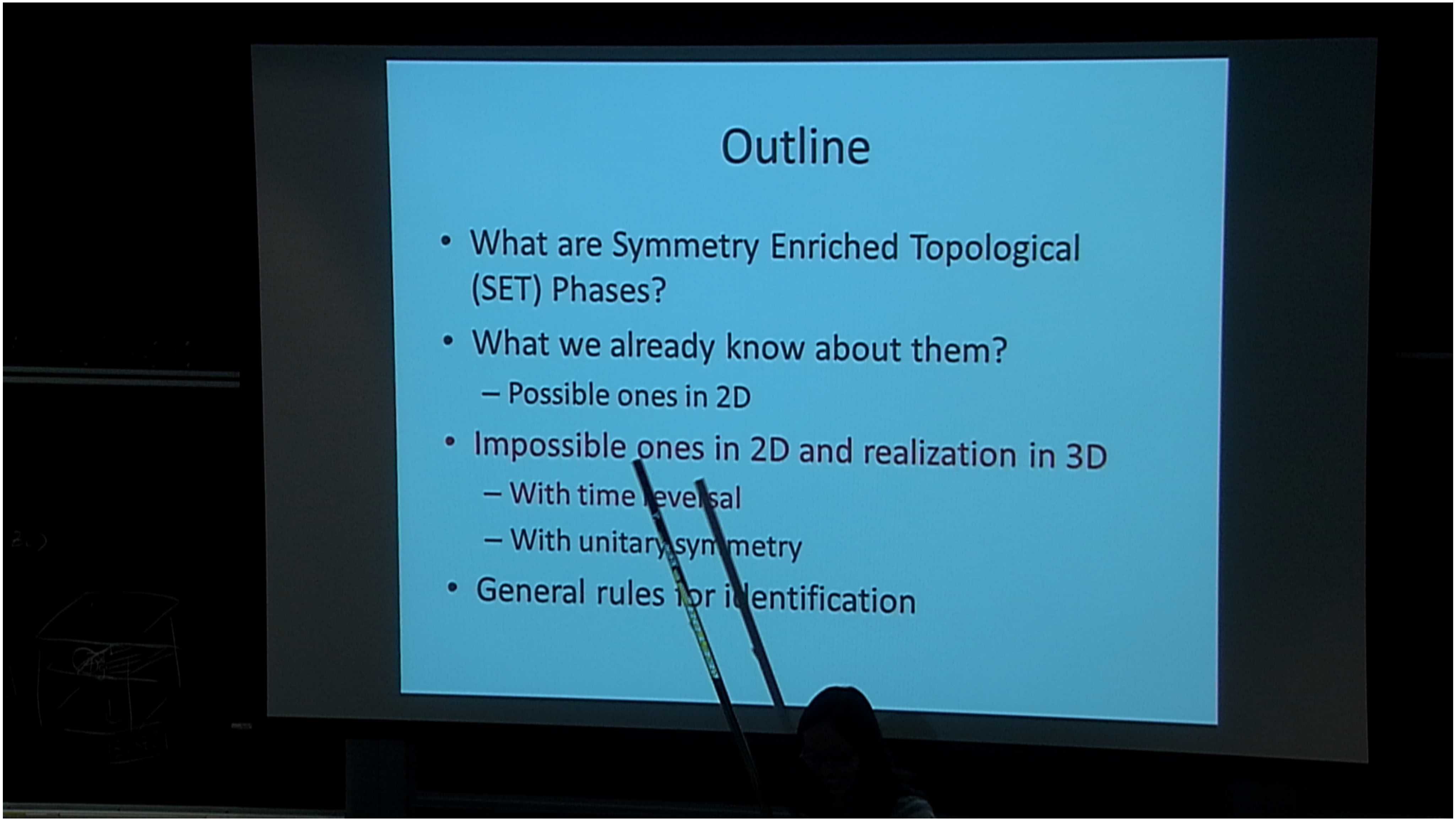
Barkeshli, Friday



• What kind of symmetry enrichment can be realized for a topological phase in 2D?
Rule #0
Symmetry action should be consistent with fusion and braiding of anyons
• Sufficient?
Essin, Hermele 2012


Impossible SET phases in 2D
• Does not break rule #0, innocent looking
– Three fermion state with time reversal
– $SO(3)_6$ etc. with time reversal
– Chiral semion theory with projective semion under $Z_2 \times Z_2$
• Realization on 3D surface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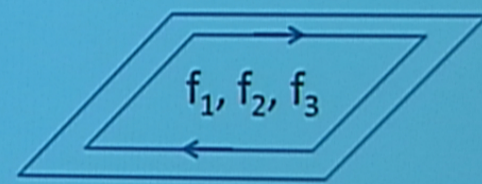
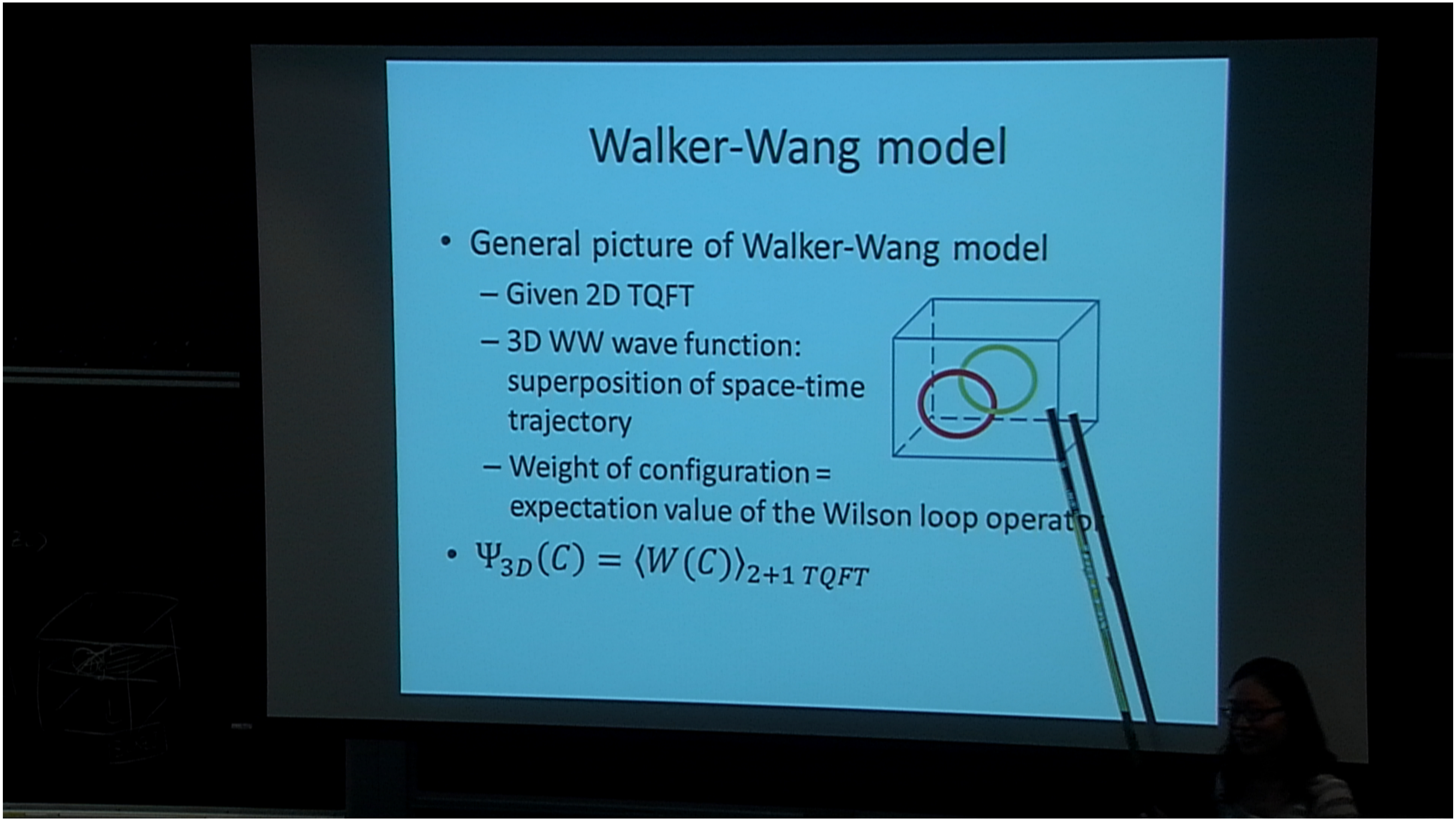
General rules for identification

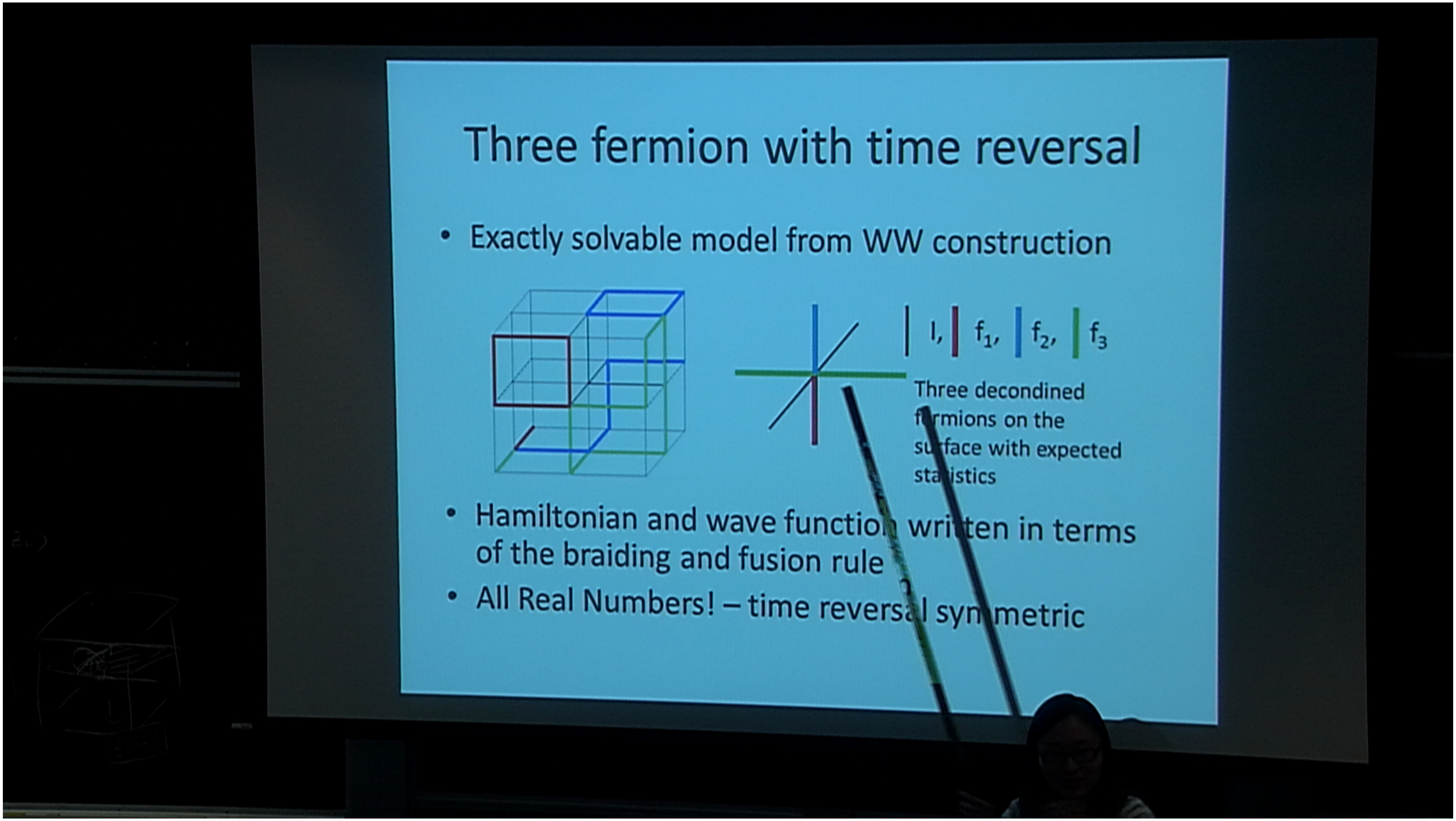
Three fermion with time reversal

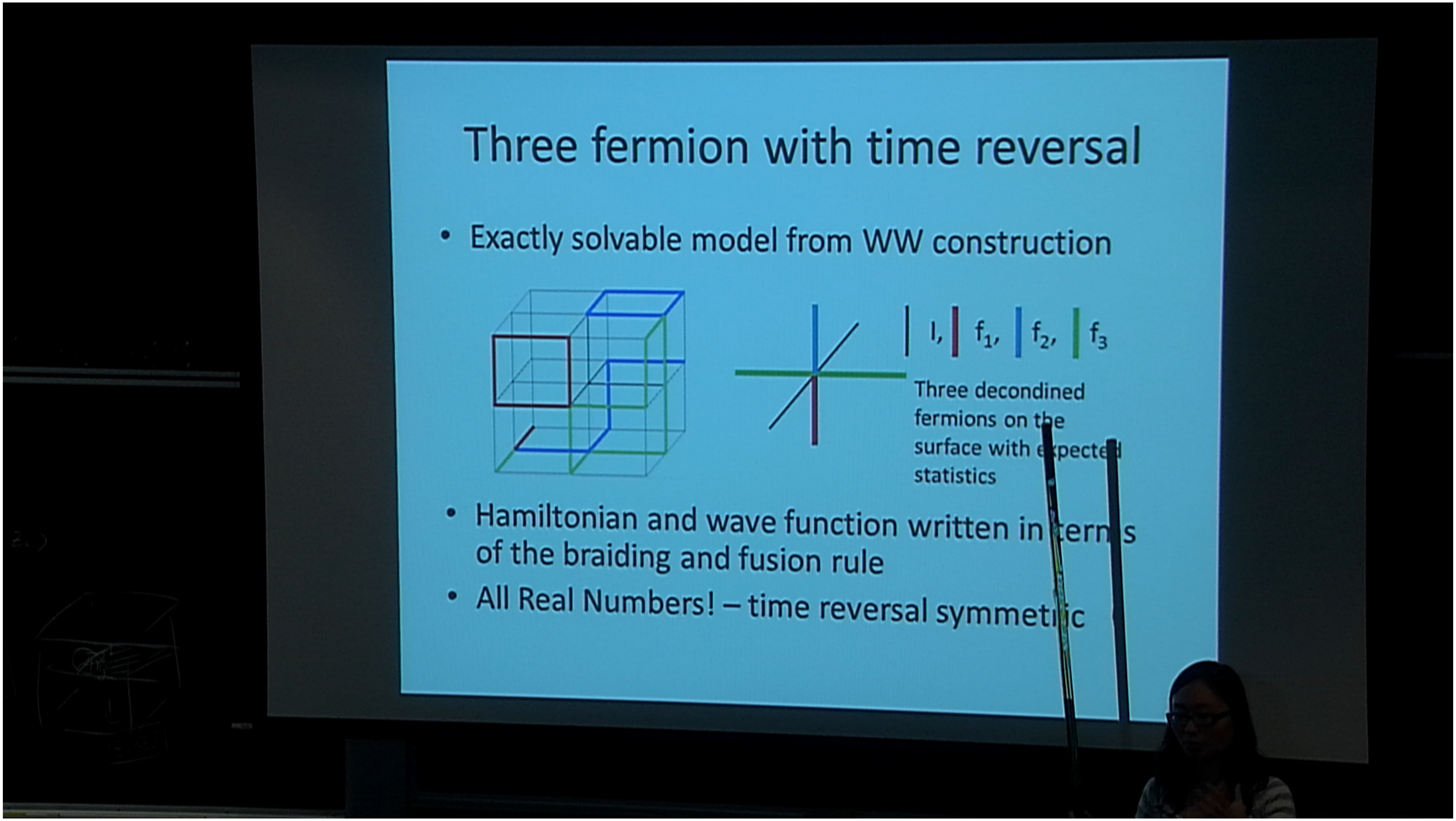
- No
- chiral edge state



- $\frac{\sum_a {d_a}^2 \theta_a}{\sqrt{\sum_a {d_a}^2}} = e^{-2\pi i c_-/8}$ Kitaev 2006
- $c_- = 4 \ mod \ 8$
- The three fermion state impossible in 2D with time reversal symmetry


Walker-Wang model
• General picture of Walker-Wang model
– Given 2D TQFT
– 3D WW wave function: superposition of space-time trajectory
– Weight of configuration = expectation value of the Wilson loop operat
• $\Psi_{3D}(C) = \langle W(C) \rangle_{2+1\ TQFT}$


Three fermion with time reversal
• Exactly solvable model from WW construction
1, f1, f2, f3
Three deconfined fermions on the surface with expected statistics
• Hamiltonian and wave function written in terms of the braiding and fusion rule
• All Real Numbers! – time reversal symmetric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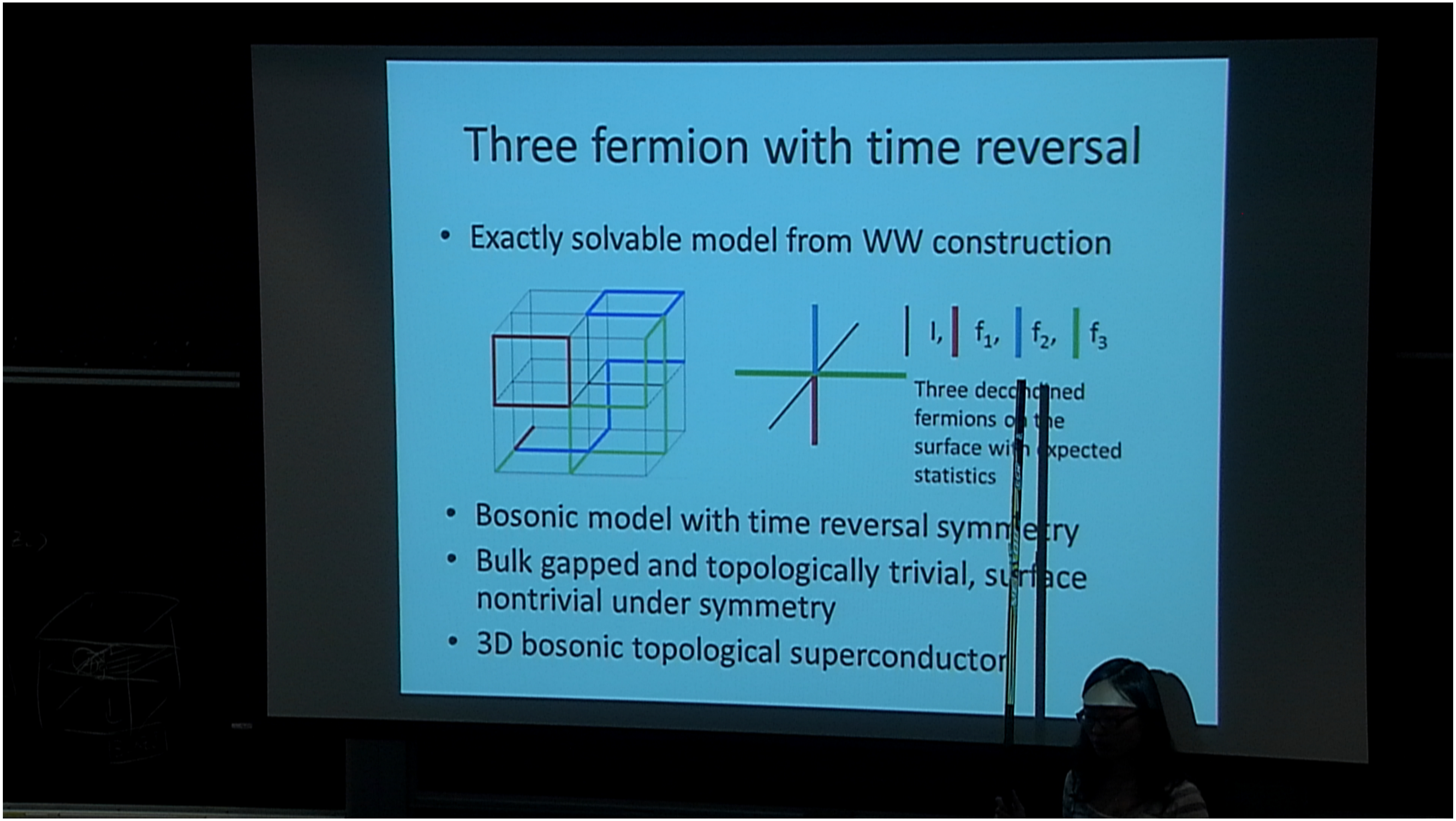
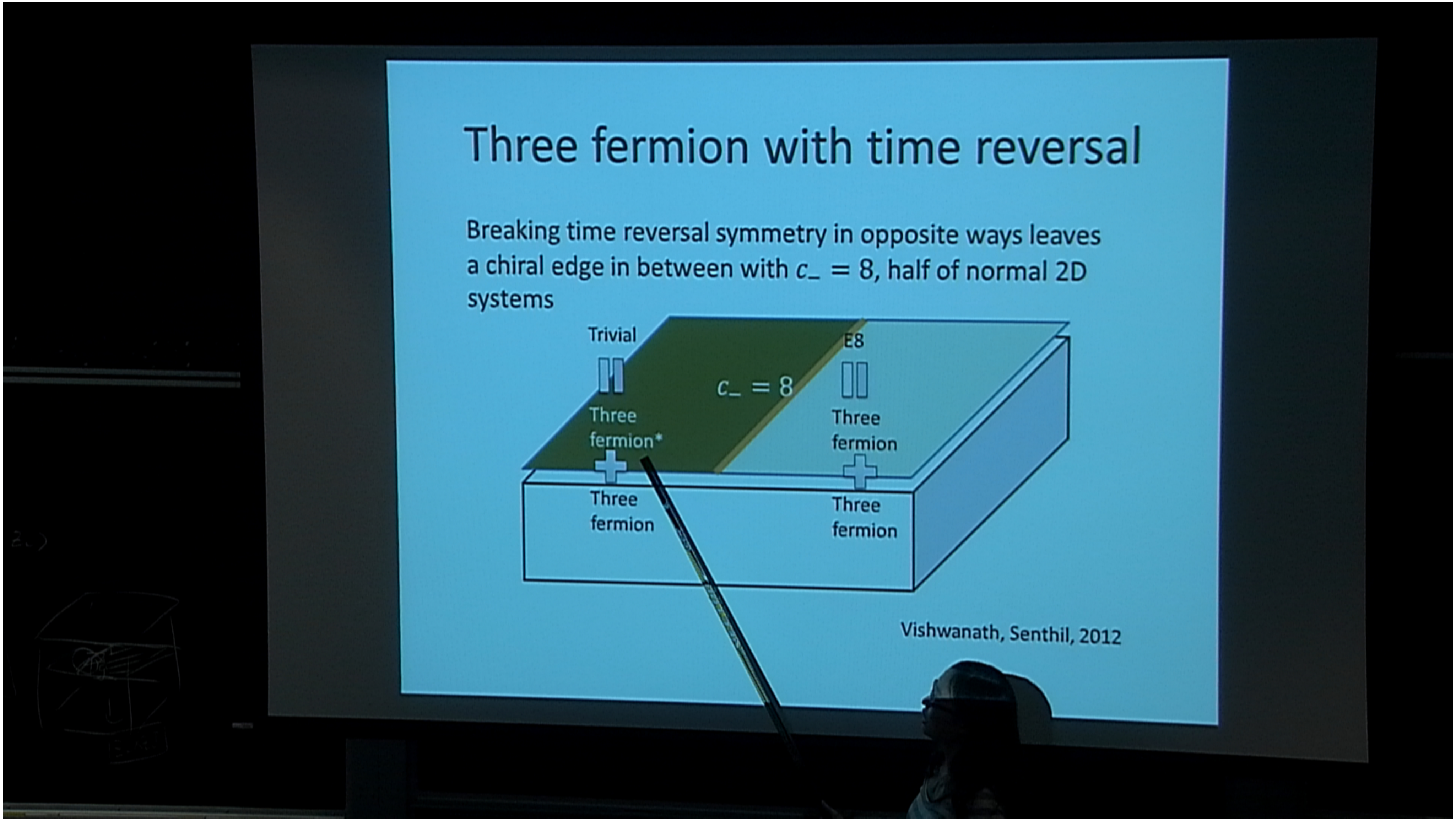
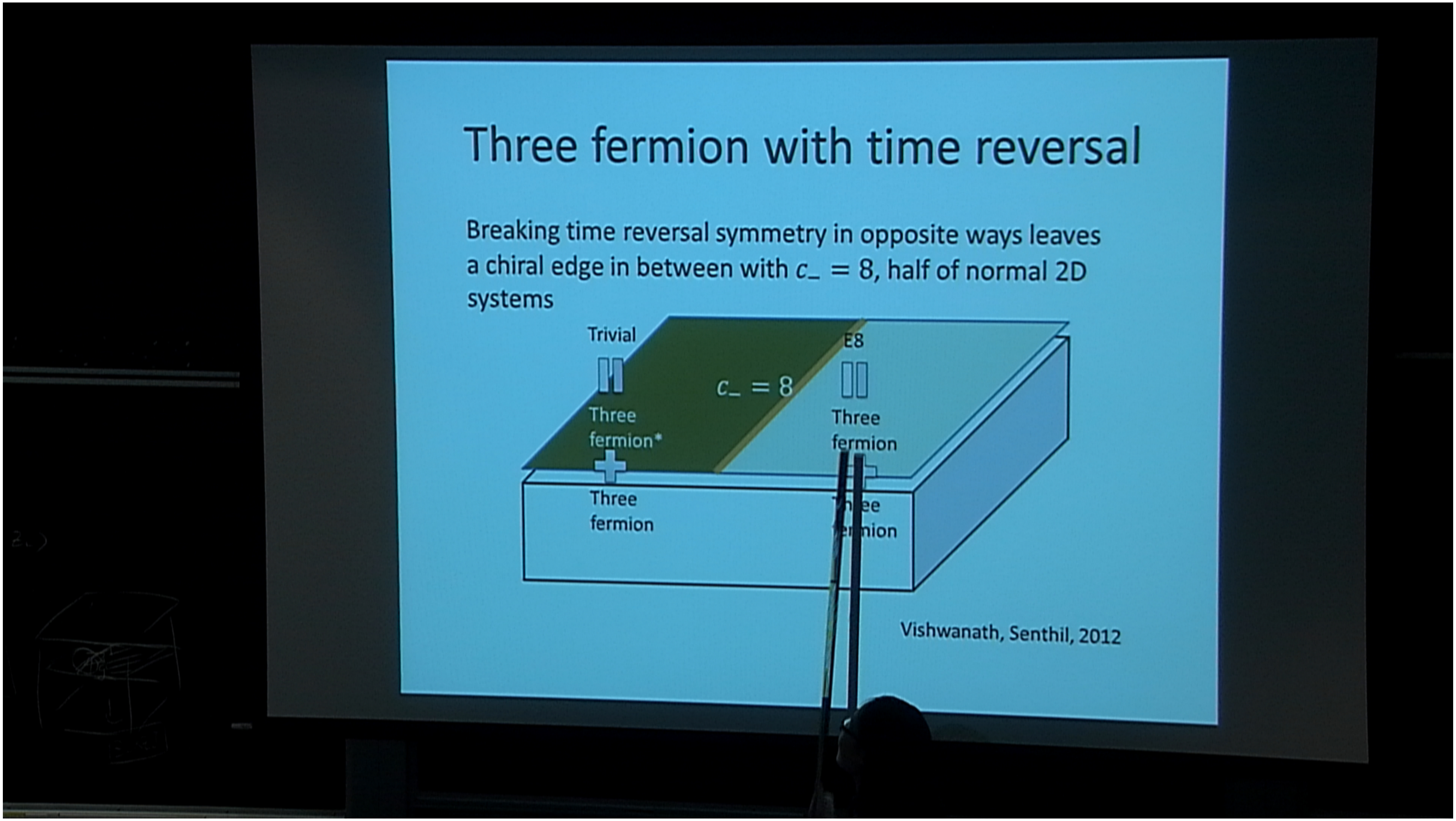

Three fermion with time reversal
• Exactly solvable model from WW construction
I, f1, f2, f3
Three deconfined fermions on the surface with expected statistics
• Bosonic model with time reversal symmetry
• Bulk gapped and topologically trivial, surface nontrivial under symmetry
• 3D bosonic topological superconductor


Three fermion with time reversal
Breaking time reversal symmetry in opposite ways leaves a chiral edge in between with $c_- = 8$, half of normal 2D systems
Trivial
E8
$c_- = 8$
Three fermion*
Three fermion
Three fermion
Three fermion
Vishwanath, Senthil, 2012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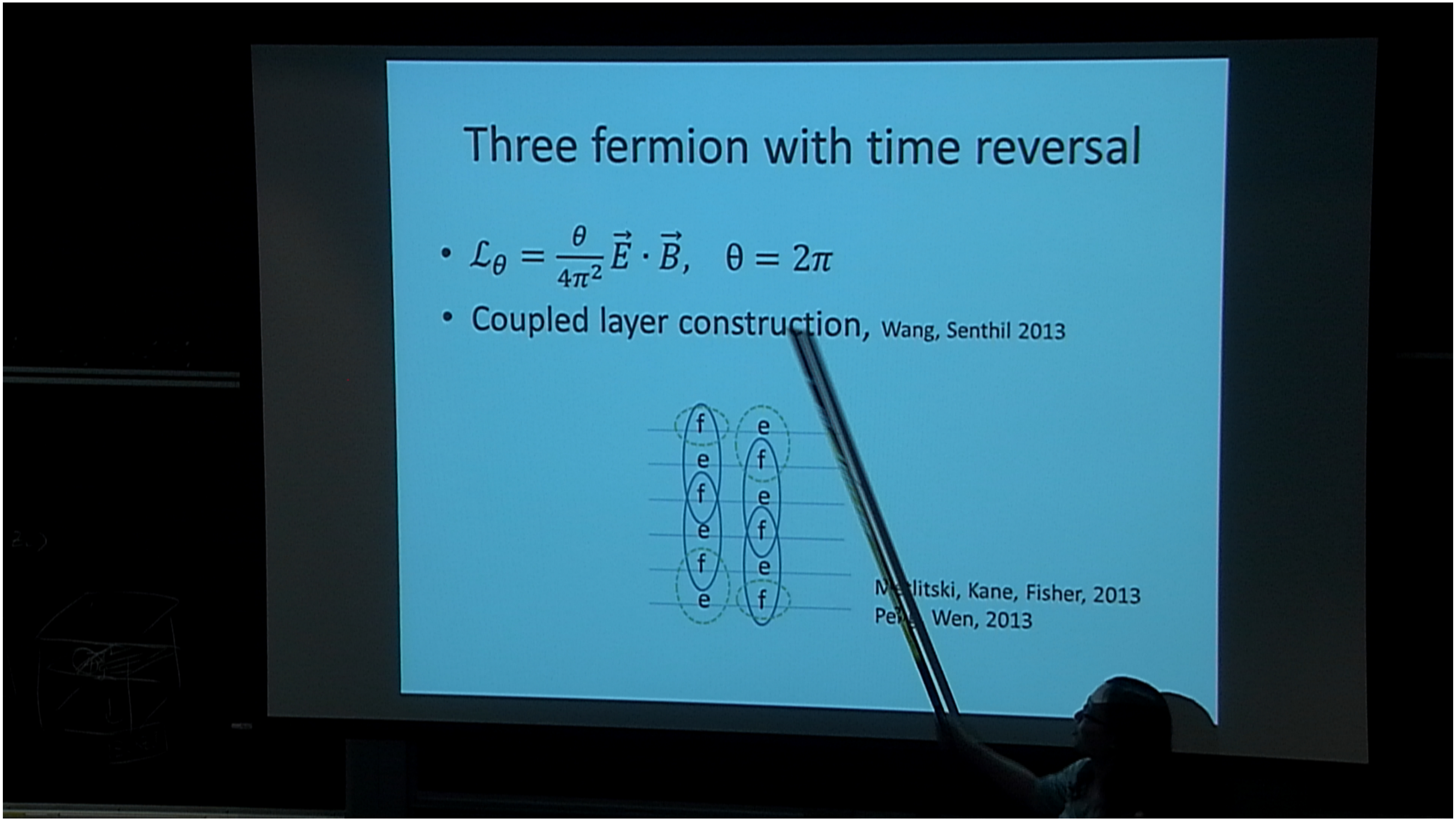
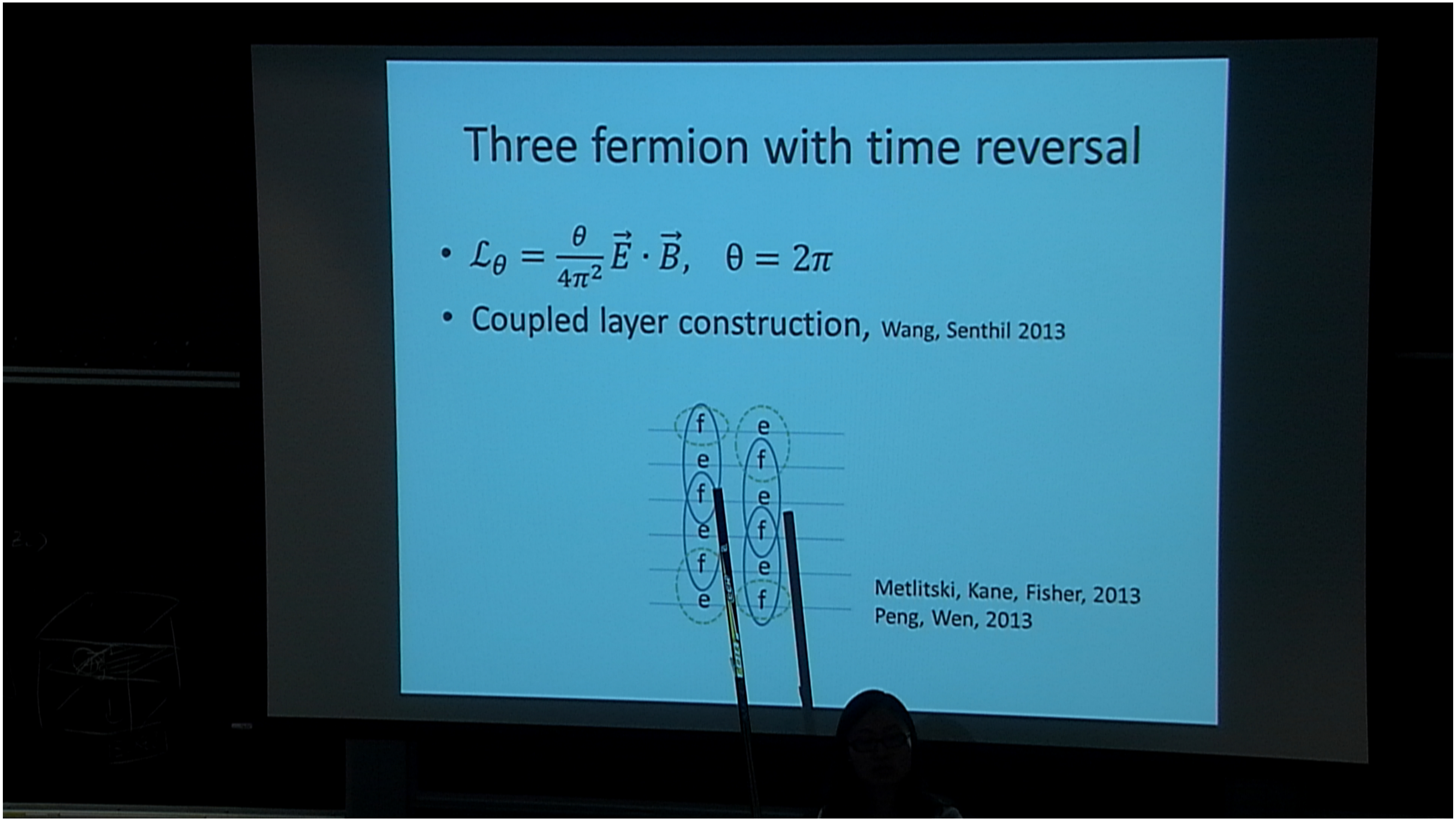

Three fermion with time reversal
• $\mathcal{L}_\theta = \frac{\theta}{4\pi^2}\vec{E}\cdot\vec{B}$, $\theta = 2\pi$
• Coupled layer construction, Wang, Senthil 2013
litski, Kane, Fisher, 2013
Wen, 2013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f
e
f
e
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e
f
e
f
e
f
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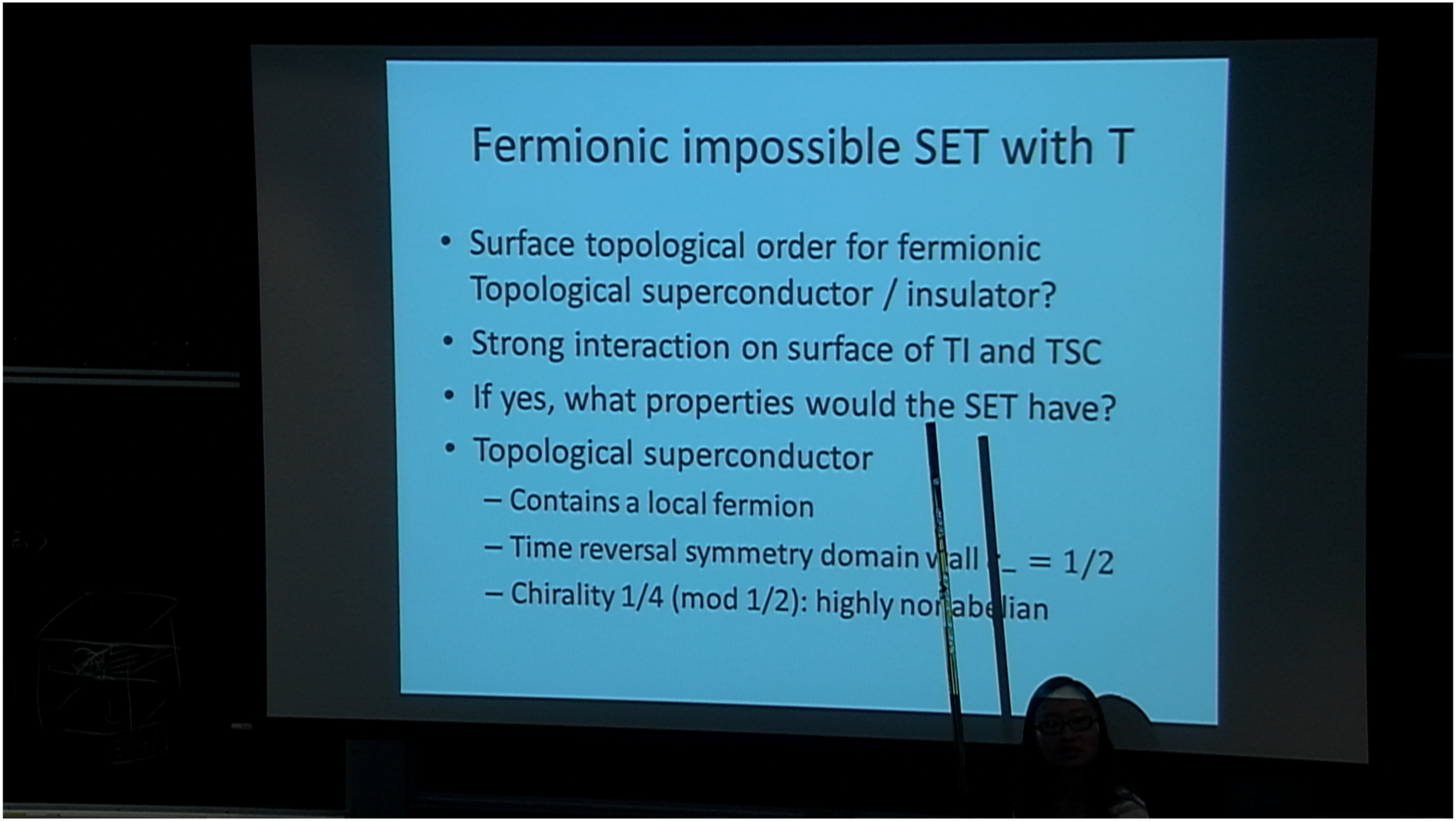
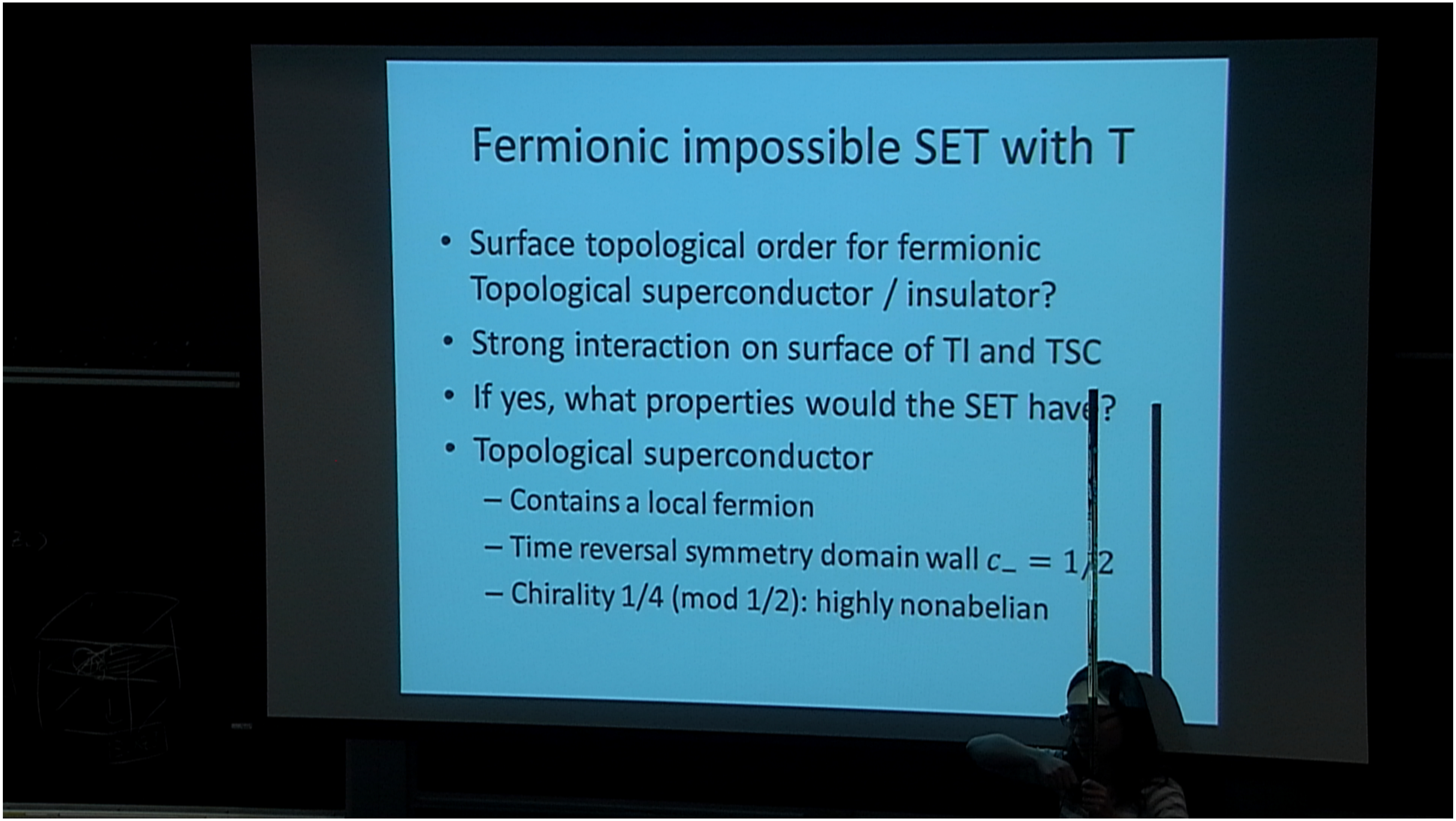
Peng, Wen, 2013


Fermionic impossible SET with T
• Surface topological order for fermionic Topological superconductor / insulator?
• Strong interaction on surface of TI and TSC
• If yes, what properties would the SET have?
• Topological superconductor
– Contains a local fermion
– Time reversal symmetry domain wall $c_- = 1/2$
– Chirality 1/4 (mod 1/2): highly nonabelian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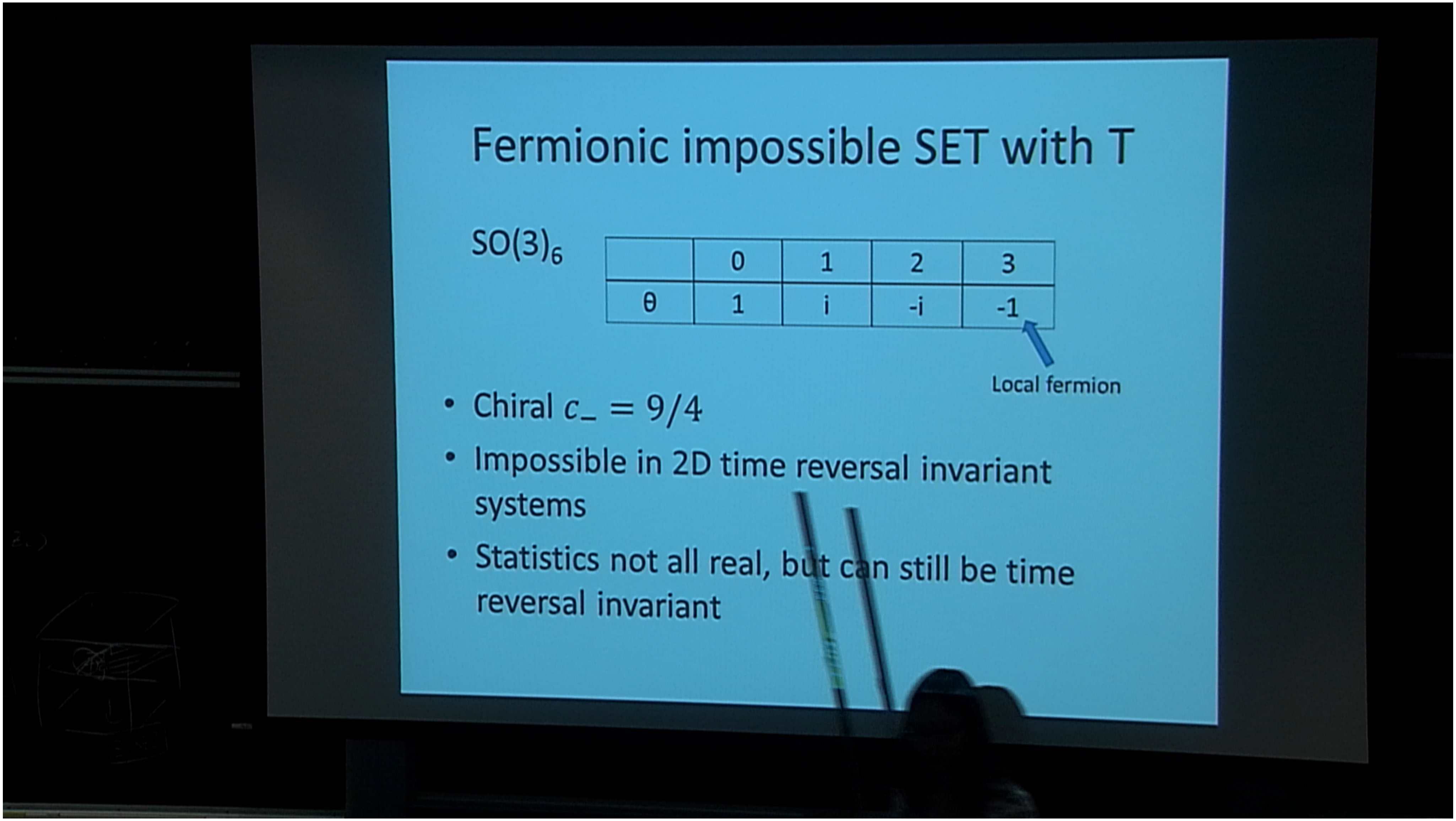
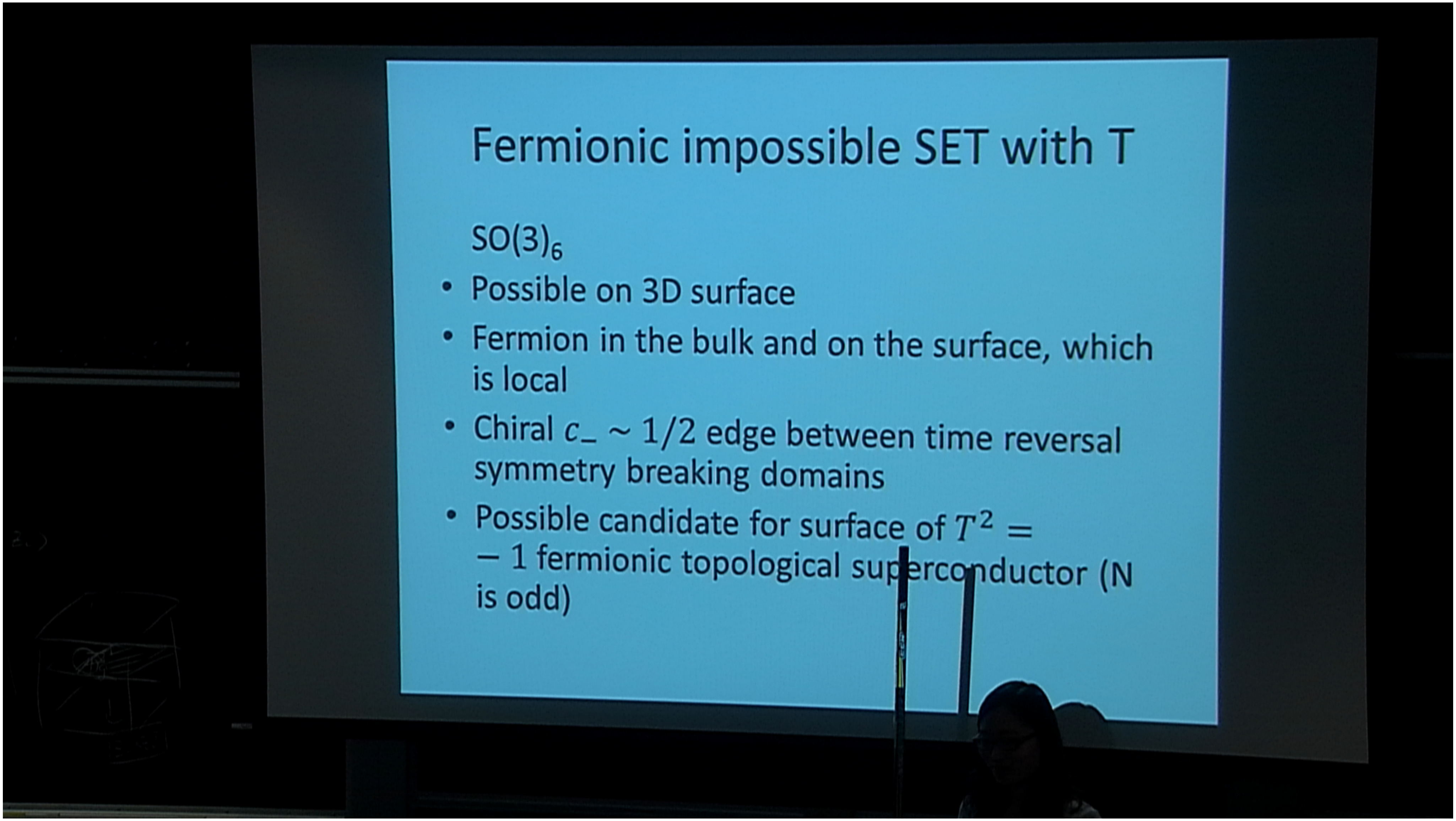
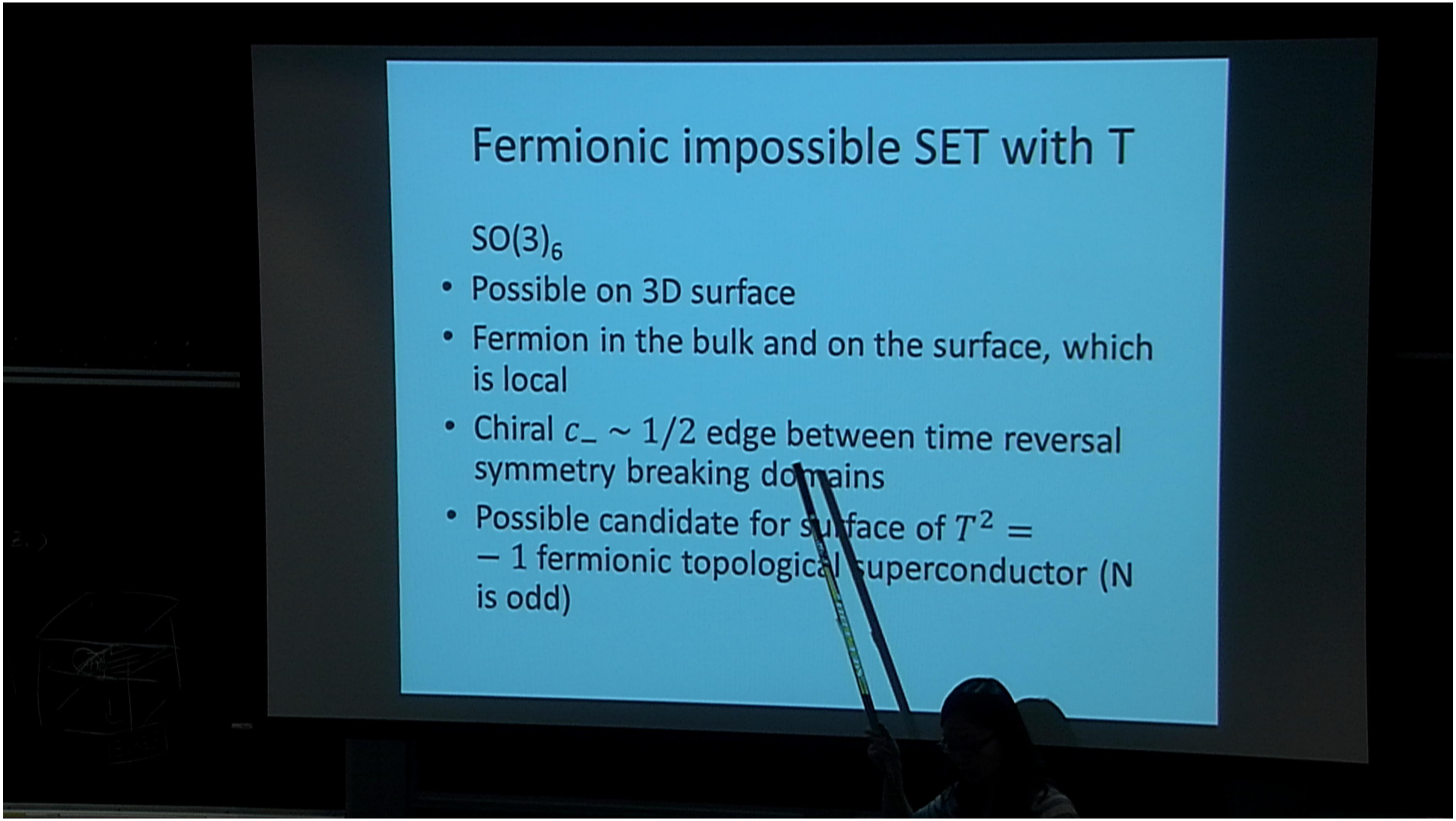

Fermionic impossible SET with T
SO(3)$_6$
θ
0 1 2 3
1 i -i -1
Local fermion
• Chiral $c_- = 9/4$
• Impossible in 2D time reversal invariant systems
• Statistics not all real, but can still be time reversal invariant


Fermionic impossible SET with T
SO(3)$_6$
• Possible on 3D surface
• Fermion in the bulk and on the surface, which is local
• Chiral $c_- \sim 1/2$ edge between time reversal symmetry breaking domains
• Possible candidate for surface of $T^2 = -1$ fermionic topological superconductor (N is odd)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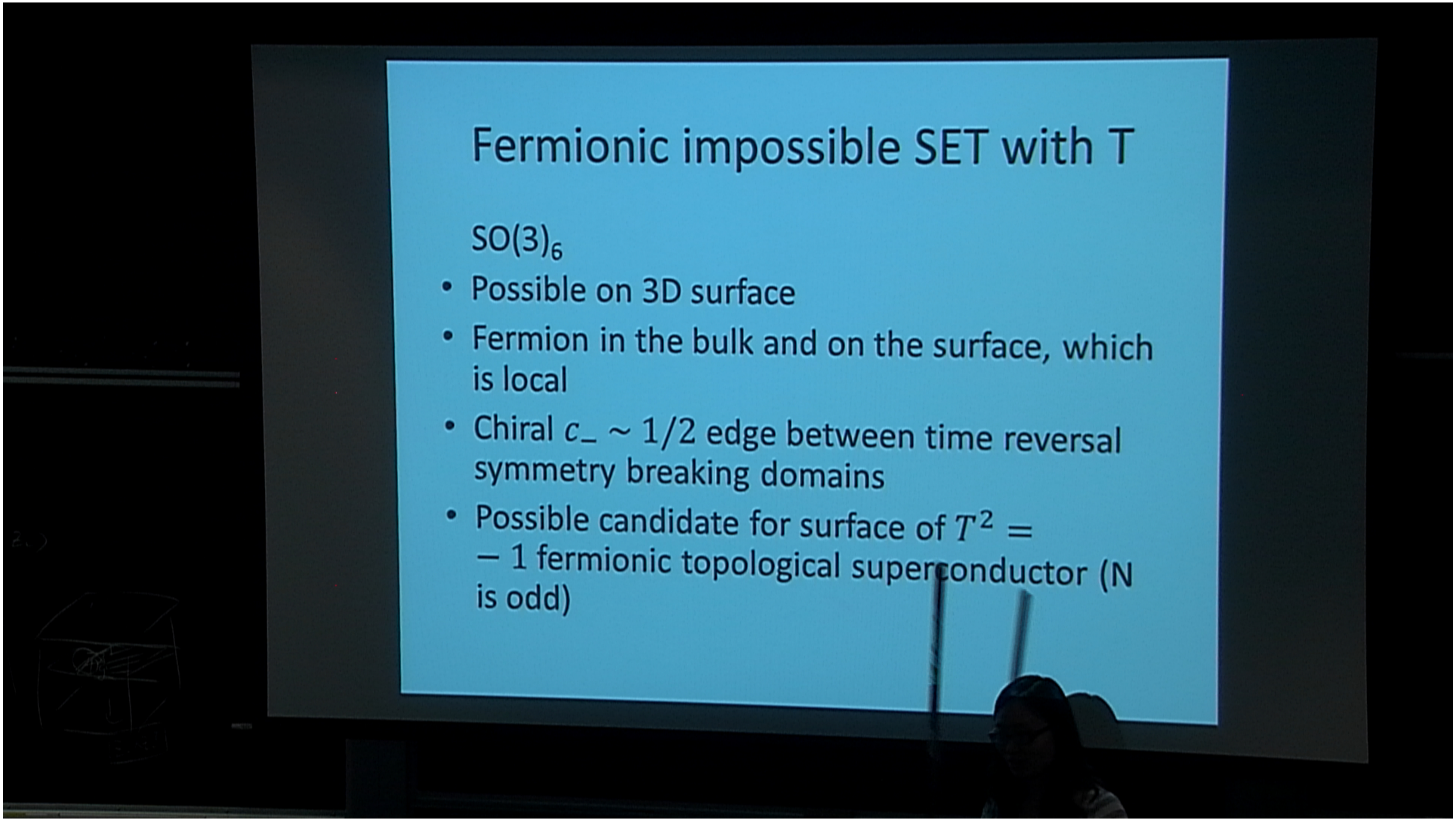
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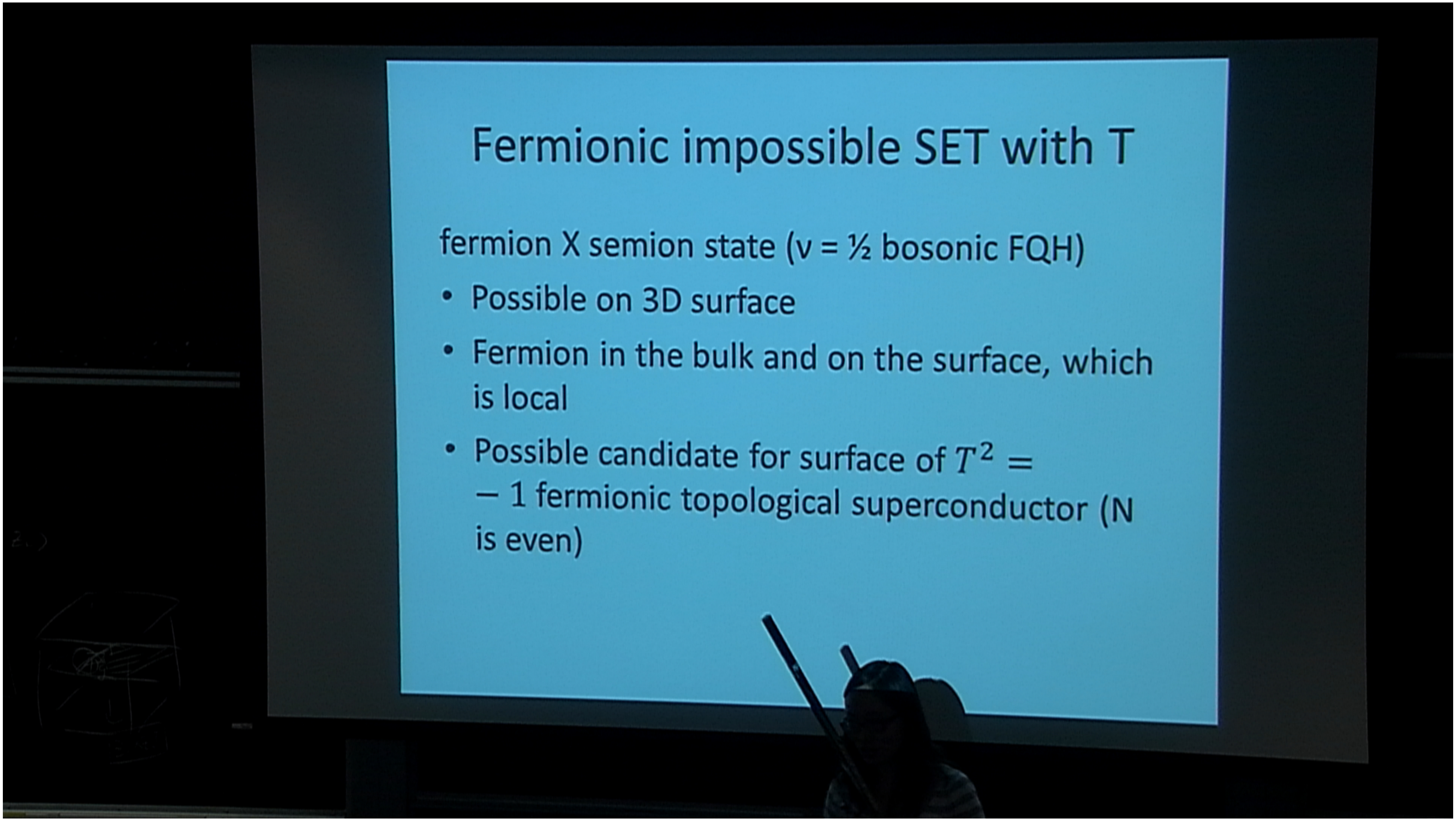
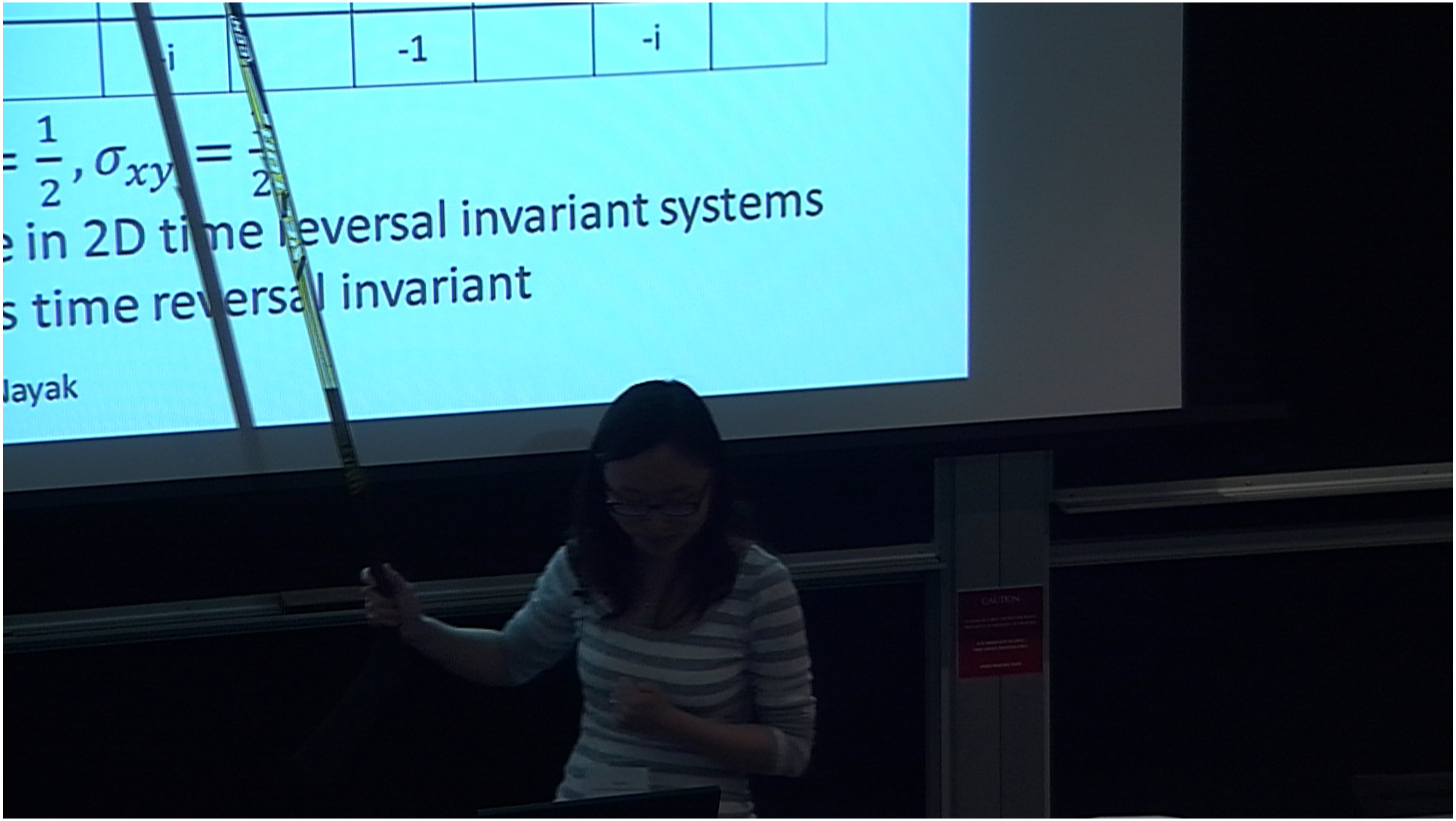
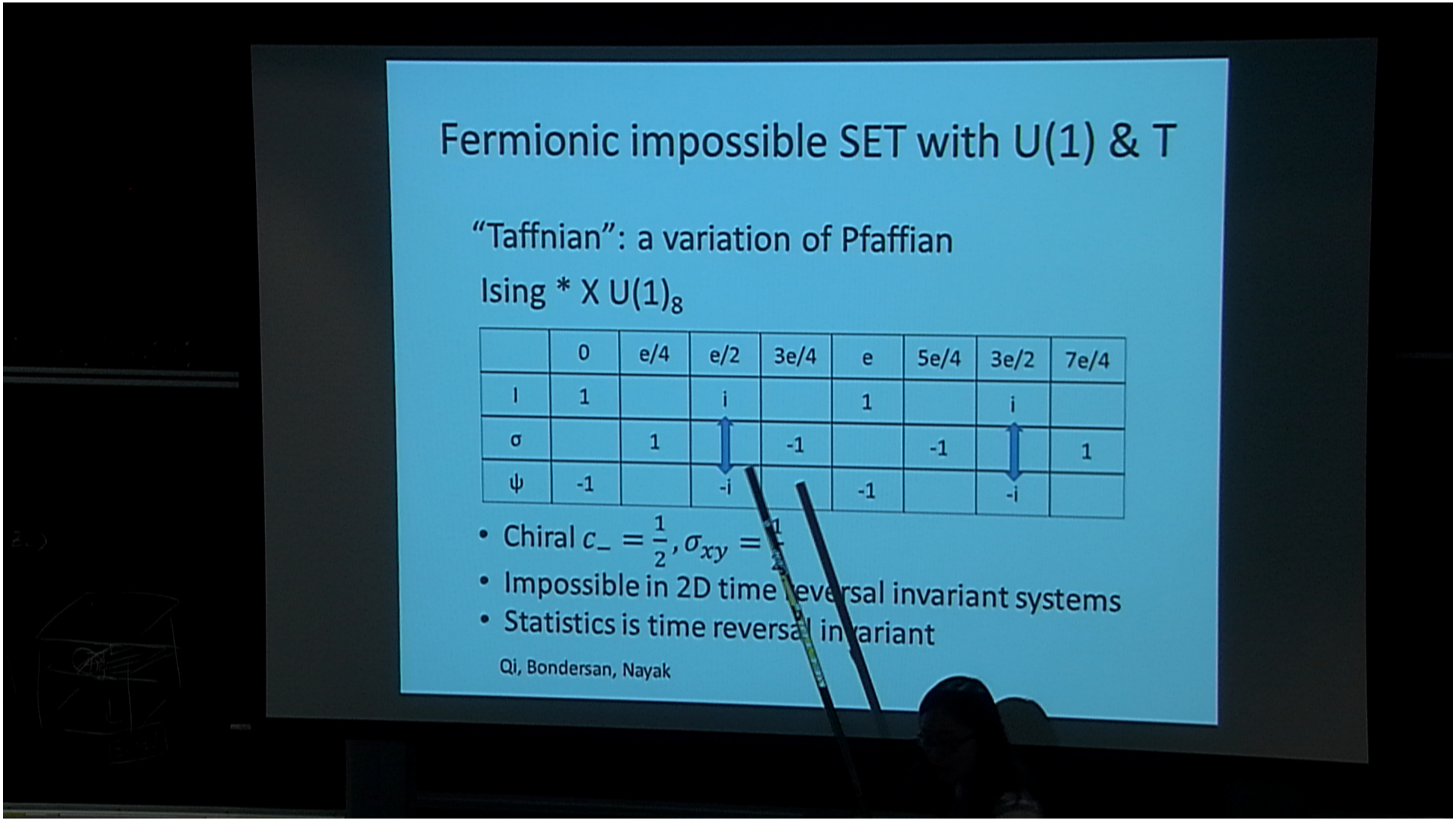
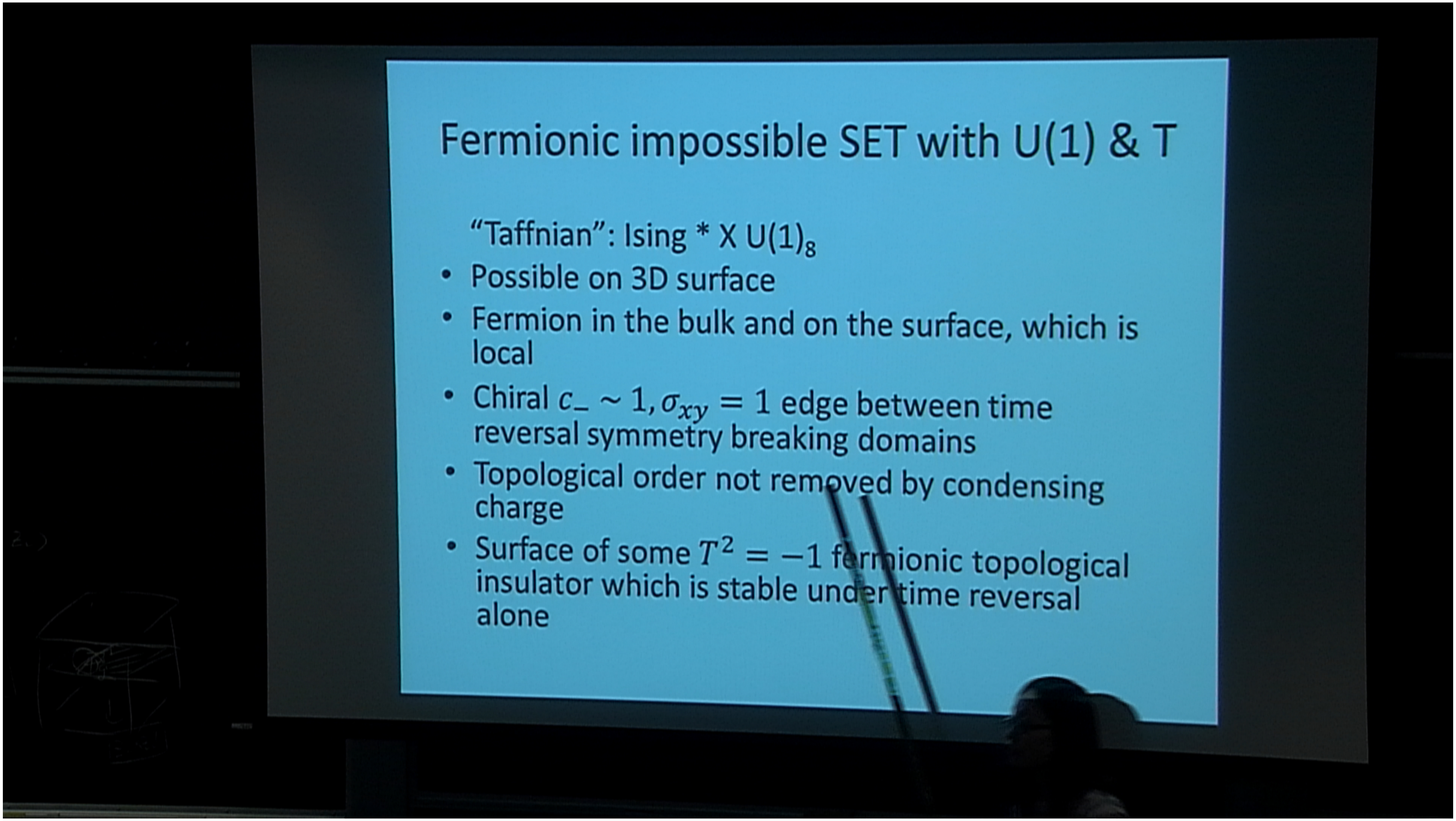

Fermionic impossible SET with T
fermion X semion state (ν = ½ bosonic FQH)
• Possible on 3D surface
• Fermion in the bulk and on the surface, which is local
• Possible candidate for surface of $T^2 = -1$ fermionic topological superconductor (N is even)


-1
-i
invariant systems
invariant



Fermionic impossible SET with U(1) & T
"Taffnian": a variation of Pfaffian
Ising * X $U(1)_8$
| 0 | e/4 | e/2 | 3e/4 | e | 5e/4 | 3e/2 | 7e/4
I | 1 | | i | | 1 | | i |
σ | | 1 | | -1 | | -1 | | 1
ψ | -1 | | -i | | -1 | | -i |
• Chiral $c_- = \frac{1}{2}, \sigma_{xy} = \frac{1}{2}$
• Impossible in 2D time reversal invariant systems
• Statistics is time reversal invariant
Qi, Bondersan, Nayak


Fermionic impossible SET with U(1) & T
"Taffnian": Ising * X U(1)$_8$
• Possible on 3D surface
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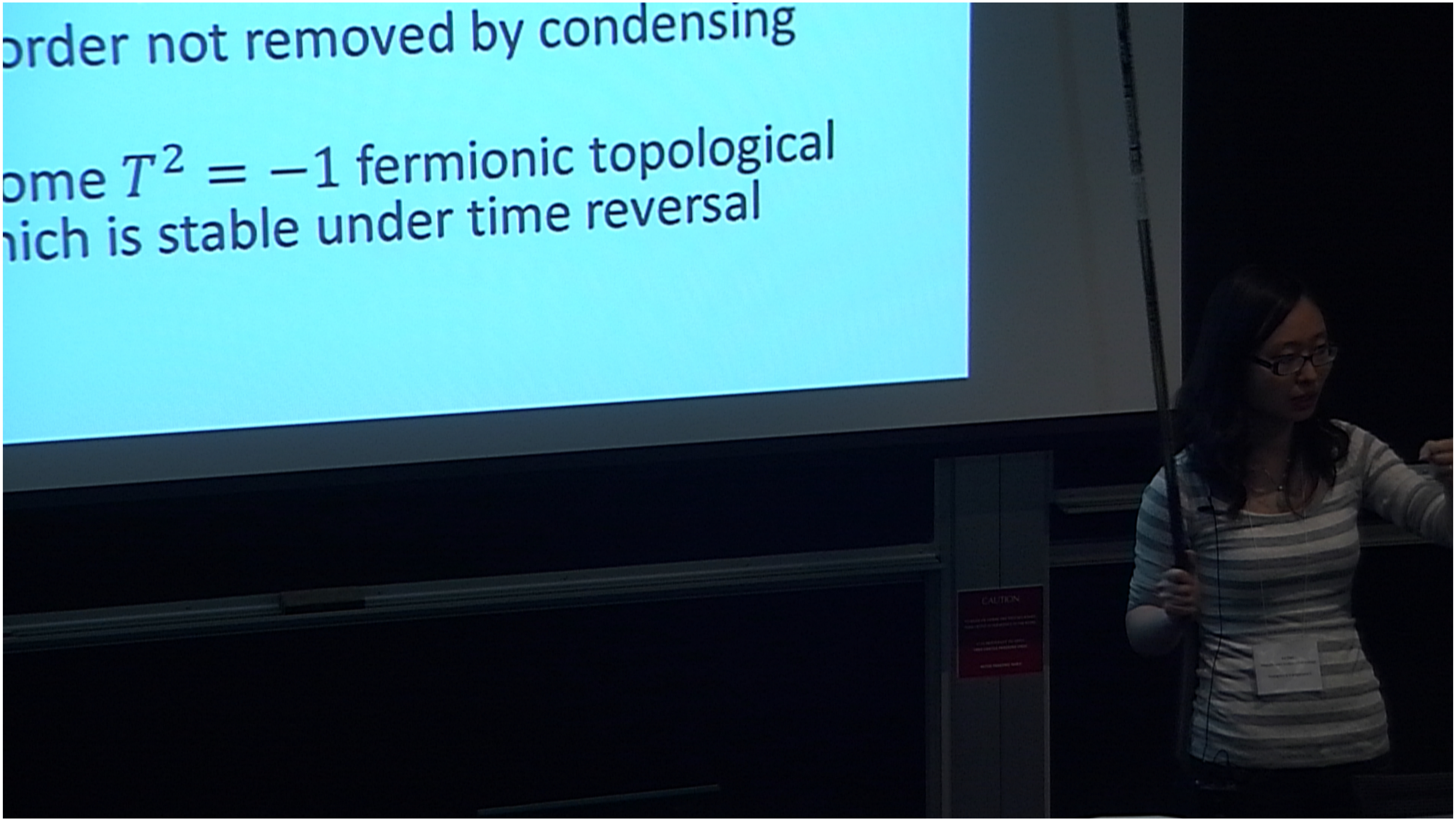
• Chiral $c_- \sim 1, \sigma_{xy} = 1$ edge between time reversal symmetry breaking domains
• Topological order not removed by condensing charge
• Surface of some $T^2 = -1$ fermionic topological insulator which is stable under time reversal alone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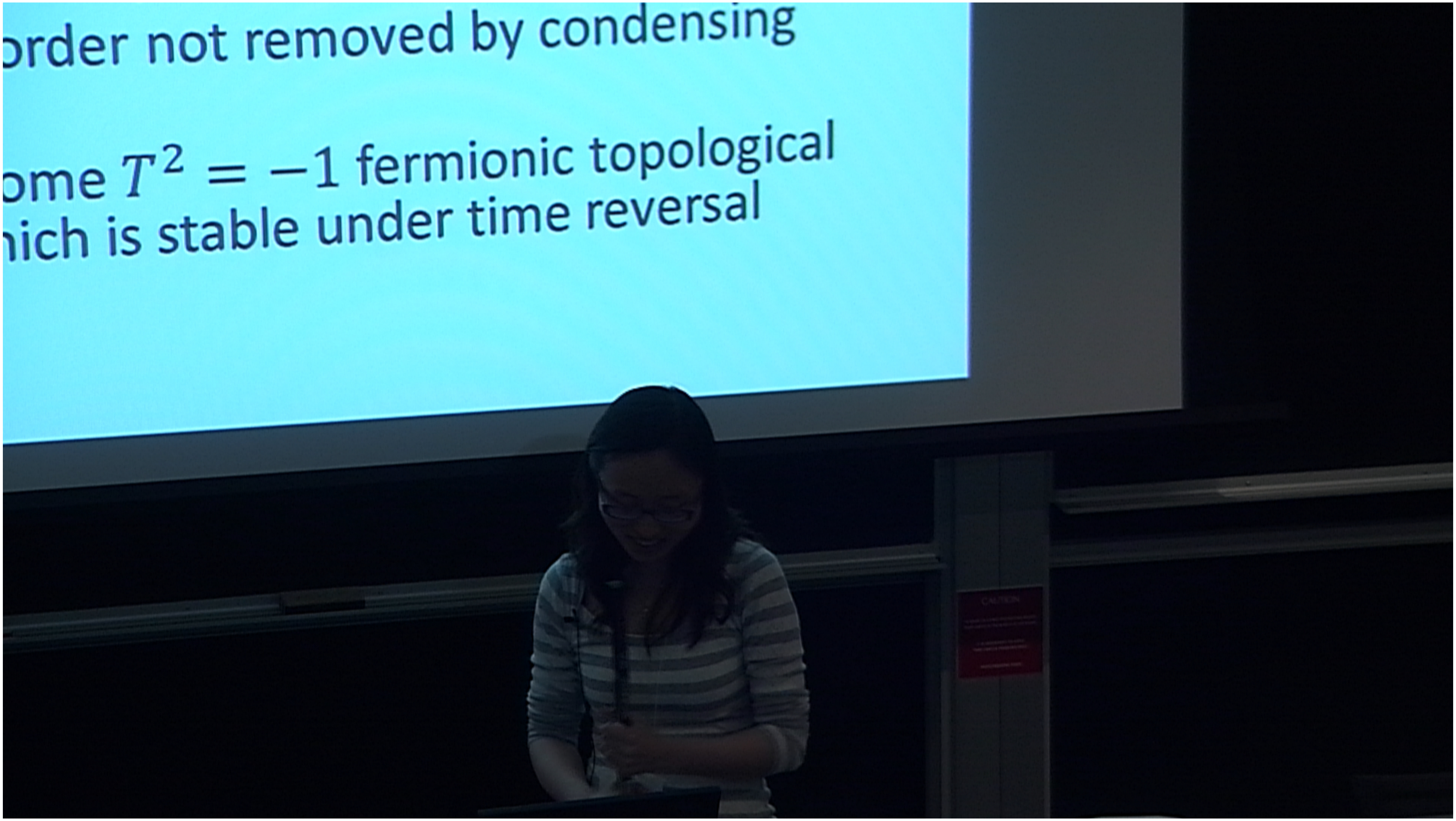
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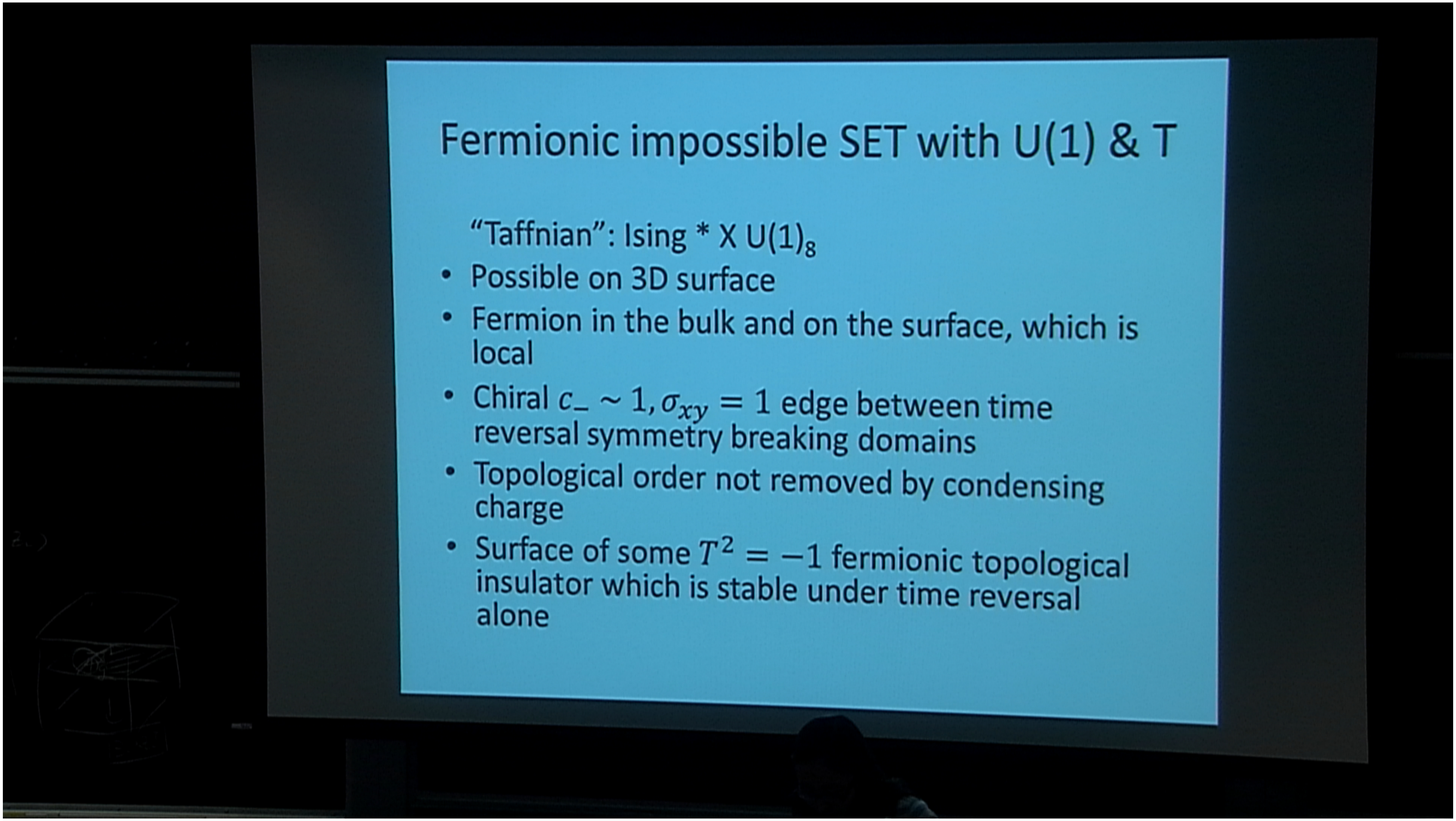
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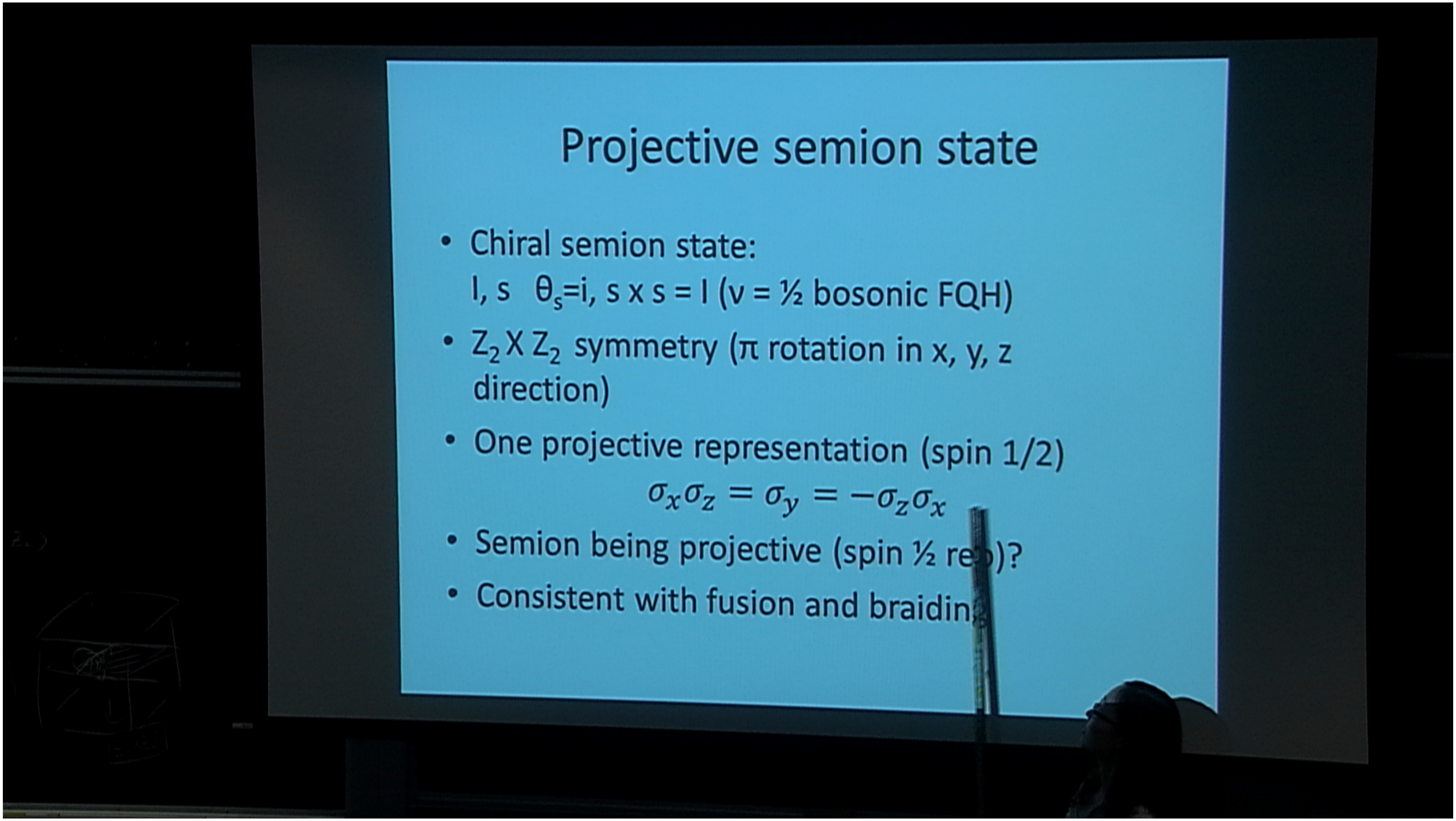
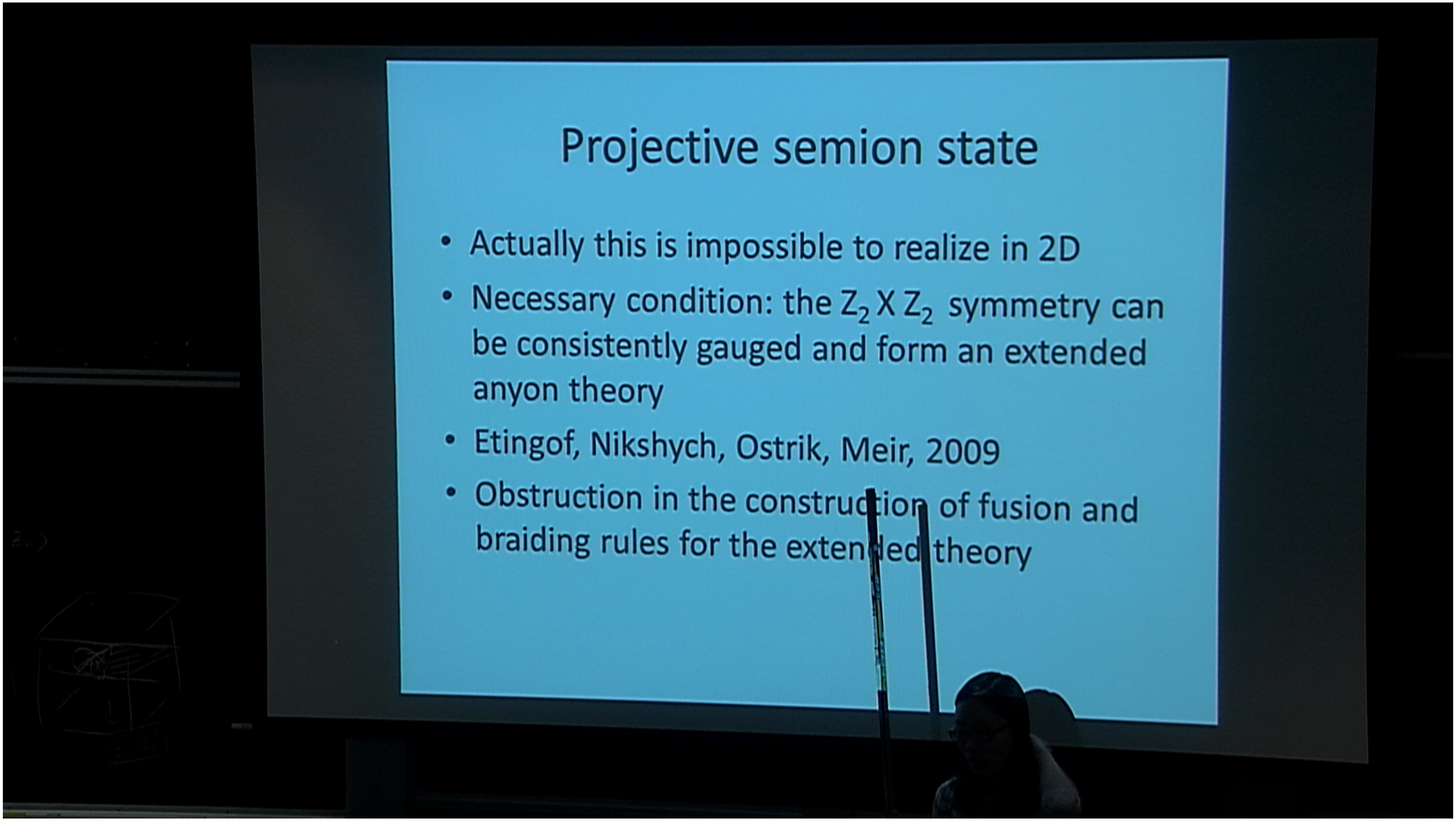
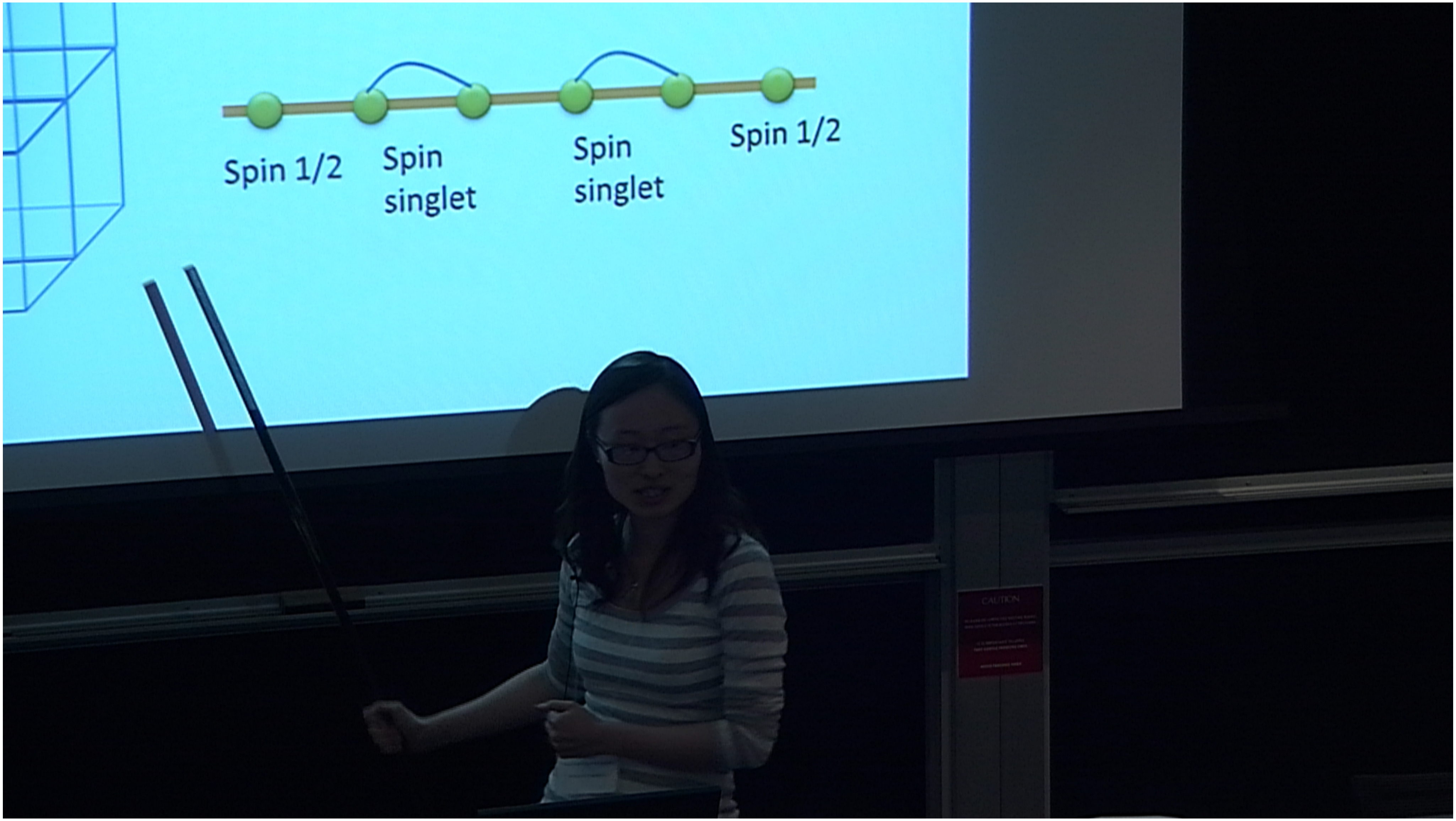

Projective semion state
• Chiral semion state:
I, s θ_s=i, s x s = I (ν = ½ bosonic FQH)
• Z_2 X Z_2 symmetry (π rotation in x, y, z direction)
• One projective representation (spin 1/2)
$\sigma_x\sigma_z = \sigma_y = -\sigma_z\sigma_x$
• Semion being projective (spin ½ re)?
• Consistent with fusion and braidin


Projective semion state
• Actually this is impossible to realize in 2D
• Necessary condition: the Z_2 X Z_2 symmetry can be consistently gauged and form an extended anyon theory
• Etingof, Nikshych, Ostrik, Meir, 2009
• Obstruction in the construction of fusion and braiding rules for the extended theory


Spin 1/2
Spin singlet
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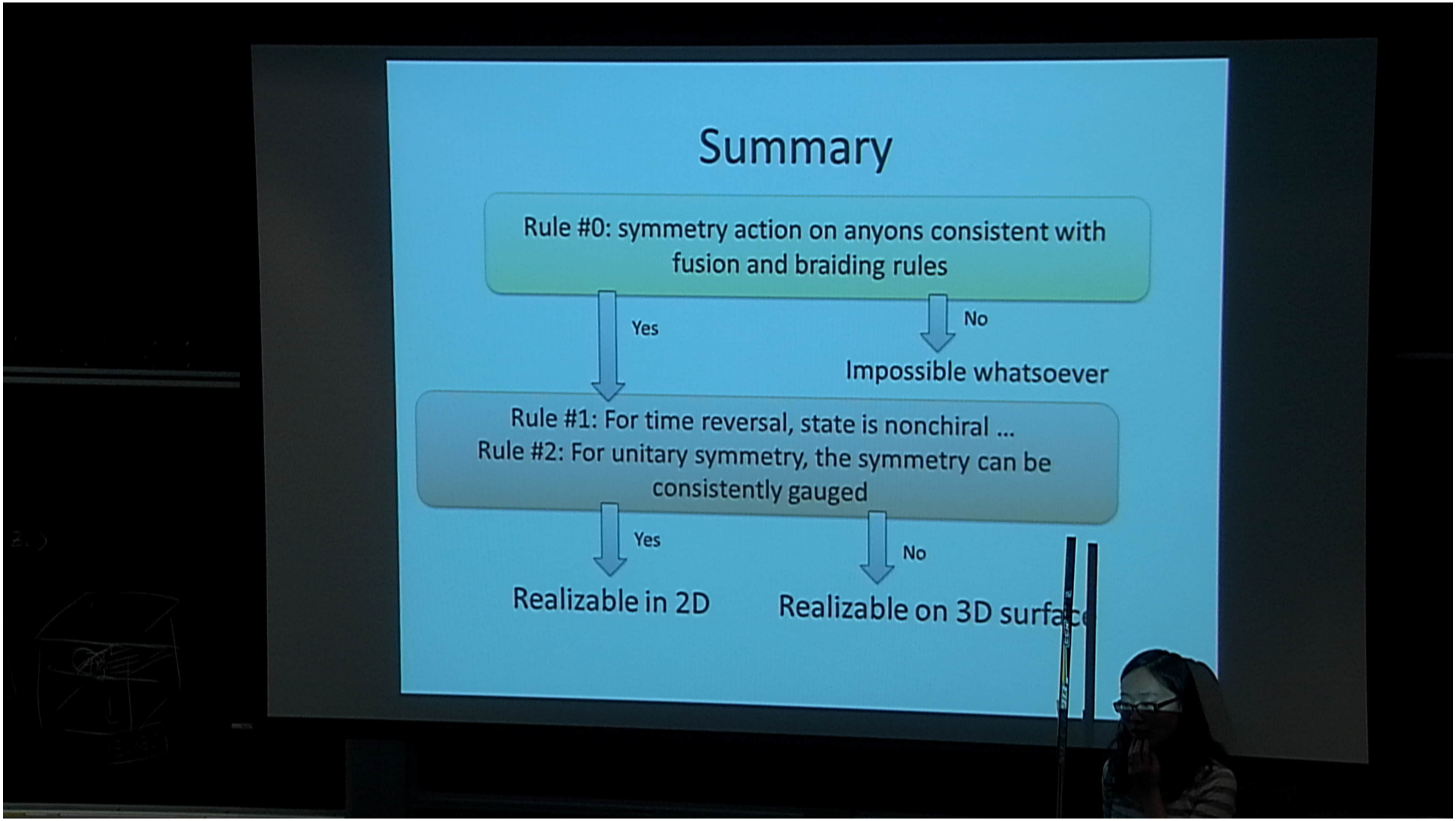
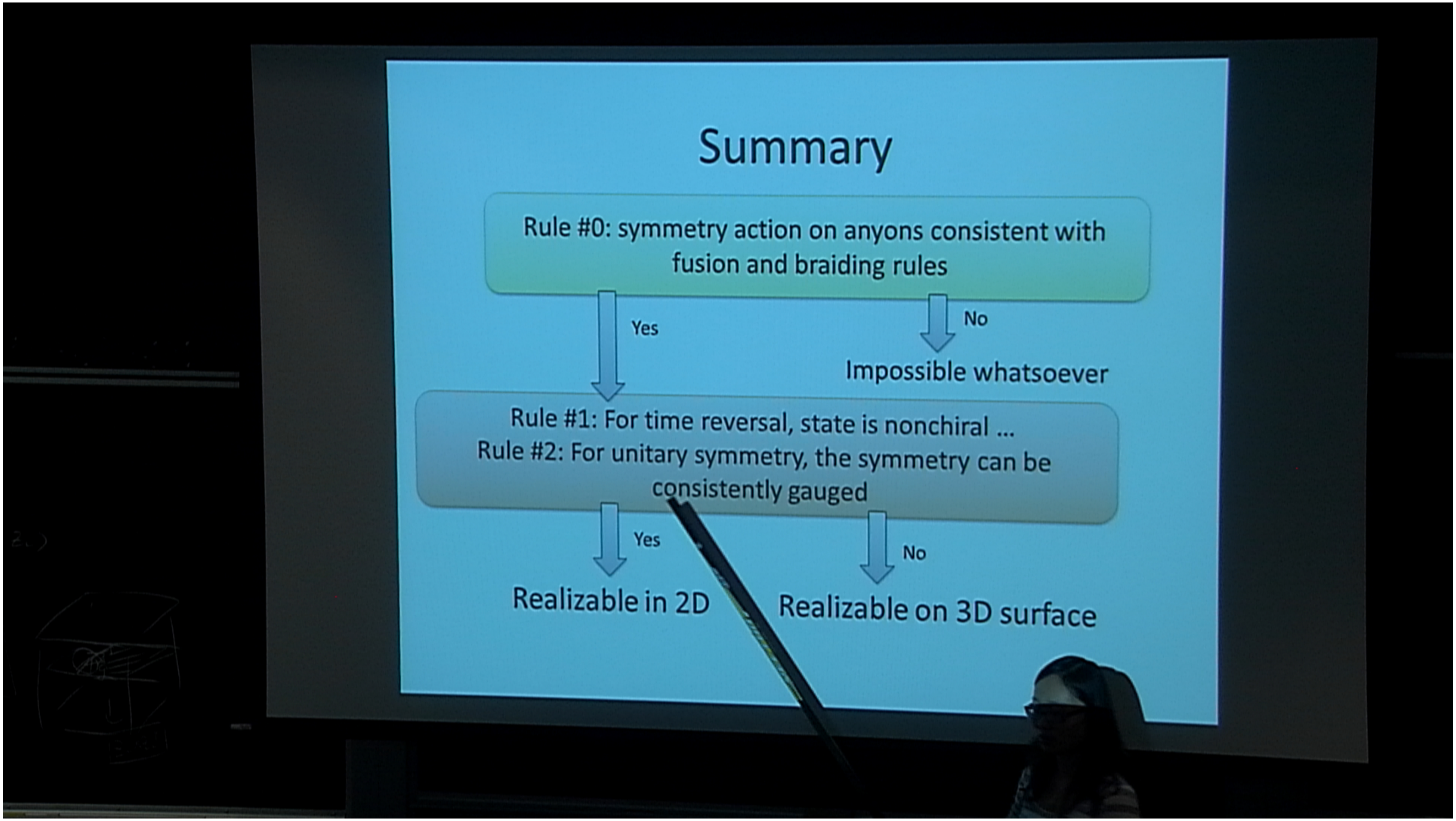

Summary
Rule #0: symmetry action on anyons consistent with fusion and braiding rules
Yes
No
Impossible whatsoever
Rule #1: For time reversal, state is nonchiral ...
Rule #2: For unitary symmetry, the symmetry can be consistently gauged
Yes
No
Realizable in 2D
Realizable on 3D surf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