Title: Field theory, Wave function, and Defects of Symmetry Protected Topological Phases

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Abstract:

Field theory, wave function, and Defects of Symmetry Protected Topological Phases

Collaborators:
Prof. T. Senthil

graduate students at UCSB:
Zhen Bi,
Alex Rasmussen

Field theory, wave function, and Defects of Symmetry Protected Topological Phases

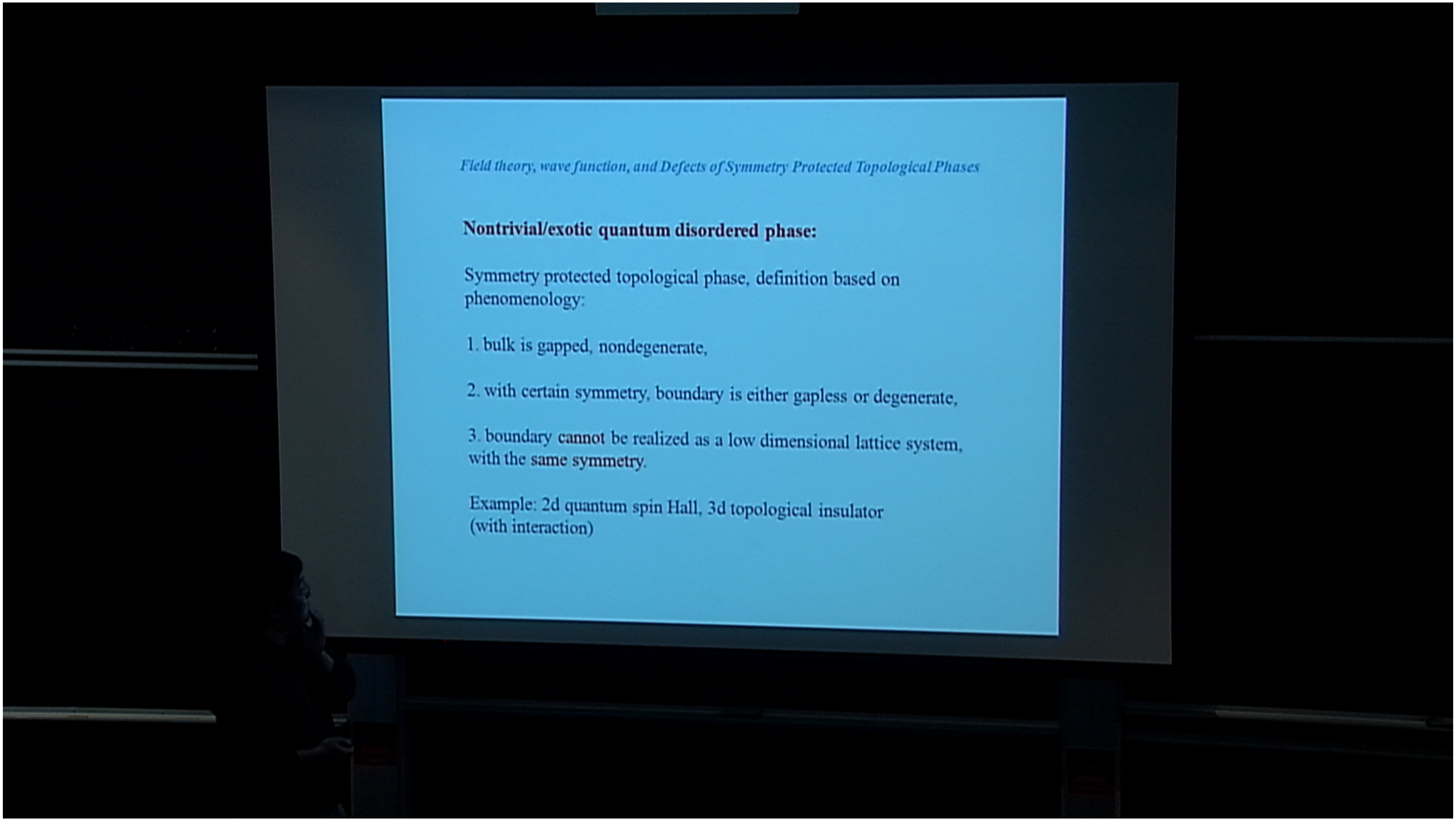
Outline:

(1) Introduction: SPT phases, clarification for field theory description of SPT; Brief review of Haldane phase;

(2) Wave function of SPT phases in 2d and 3d, which can be derived from NLSM with Θ-term. arXiv:1301.6172, Xu, Senthil

(3) Line defects in 3d SPT phases, which in many cases must be either gapless or degenerate. arXiv:1304.7272, Bi, Rasmussen, Xu

(4) 3d SPT with PSU(N) symmetry, generalization of 1d Haldane phase, field theory and possible lattice construction; arXiv:1209.4399, Xu, PRB 2013


Field theory, wave function, and Defects of Symmetry Protected Topological Phases
Nontrivial/exotic quantum disordered phase:
Symmetry protected topological phase, definition based on phenomenology:
1. bulk is gapped, nondegenerate,
2. with certain symmetry, boundary is either gapless or degenerate,
3. boundary cannot be realized as a low dimensional lattice system, with the same symmetry.
Example: 2d quantum spin Hall, 3d topological insulator (with interaction)

Will focus on bosonic SPT phases in this talk

1d SPT with SO(3), Haldane phase

1d spin-1 chain,

Field theory description: O(3) NLSM + Θ-term, for $\pi_2[S^2] = Z$.
Haldane 1988, Ng 1994, Coleman 1976.

$$\mathcal{S} = \int dx d\tau \ \frac{1}{g}(\partial_\mu \vec{n})^2 + \frac{i\Theta}{8\pi}\epsilon_{abc}\epsilon_{\mu\nu}n^a\partial_\mu n^b\partial_\nu n^c \qquad \Theta = 2\pi$$

Some clarification of topological term:

1. Meaning of Θ-term:

Textbook interpretation of Θ-term:

$$\mathcal{Z} = \int D\vec{n}(x,\tau)\exp(-\mathcal{S}) \sim \sum_{\text{instanton number}} \exp(i\Theta n)$$

This interpretation assumes we compactify the space-time without boundary, and it would lead to the wrong conclusion that $\Theta = 0$ and 2π are completely equivalent.

However, partition function does not contain all the information. 0 and 2π have equal partition function, and most likely same spectrum, but they have very different ground state wave functio

Some clarification of topological term:

1. Meaning of Θ-term:

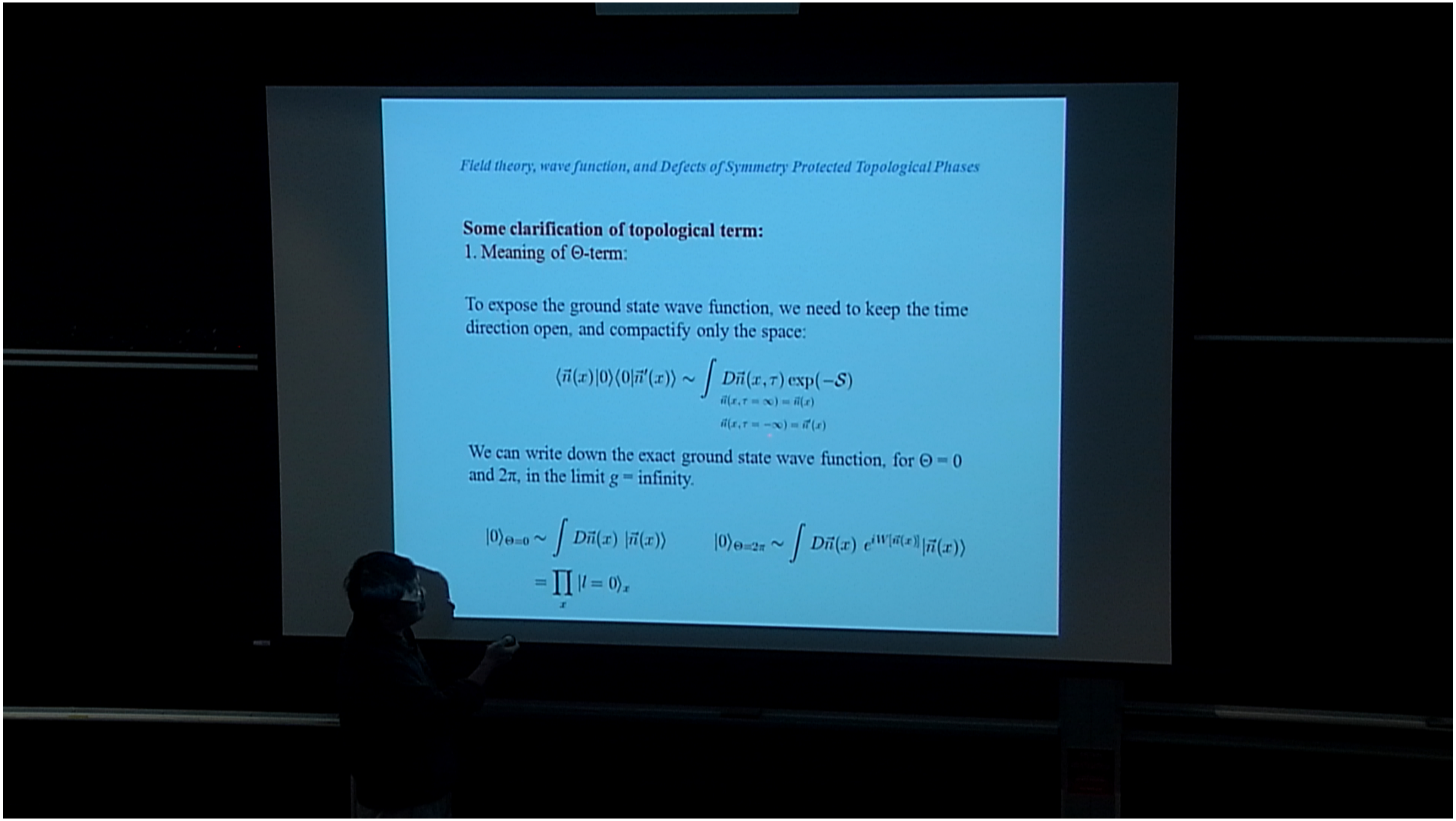
To expose the ground state wave function, we need to keep the time direction open, and compactify only the space:

$$\langle \vec{n}(x)|0\rangle\langle 0|\vec{n}'(x)\rangle \sim \int_{\substack{\vec{n}(x,\tau=\infty)=\vec{n}(x) \\ \vec{n}(x,\tau=-\infty)=\vec{n}'(x)}} D\vec{n}(x,\tau)\exp(-\mathcal{S})$$

We can write down the exact ground state wave function, for $\Theta = 0$ and 2π, in the limit g = infinity.

$$|0\rangle_{\Theta=0} \sim \int D\vec{n}(x)\ |\vec{n}(x)\rangle \qquad |0\rangle_{\Theta=2\pi} \sim \int D\vec{n}(x)\ e^{iW[\vec{n}(x)]}|\vec{n}(x)\rangle$$

$$= \prod_x |l=0\rangle_x$$



Field theory, wave function, and Defects of Symmetry Protected Topological Phases
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Bulk ground state wave function of SPT phases;

Example: 2d SPT phase Z2 symmetry (Levin, Gu, 2012)

The wave function is a superposition of Ising configurations, with (-1) to every domain wall:

$$\Psi_1(\{\alpha_p\}) = (-1)^{N_{dw}}$$

We propose that the effective field theory for this SPT phase is a 2+1d SO(4) NLSM with $\Theta = 2\pi$. Xu, Senthil, 2013

$$S = \int d^2x d\tau \frac{1}{g}(\partial_\mu \vec{\phi})^2 + \frac{i\Theta}{12\pi^2}\epsilon_{abcd}\epsilon_{\mu\nu\rho}\phi^a\partial_\mu\phi^b\partial_\nu\phi^c\partial_\rho\phi^d$$

Strategy: reduce symmetry to Z2, “integrate out” high energy components one by one.

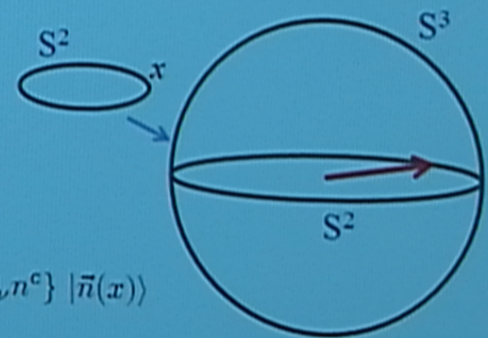
Wave function of Symmetry Protected Topological Phases

Use the same strategy, we can write down the ground state wave function of this field theory: Xu, Senthil, 2013

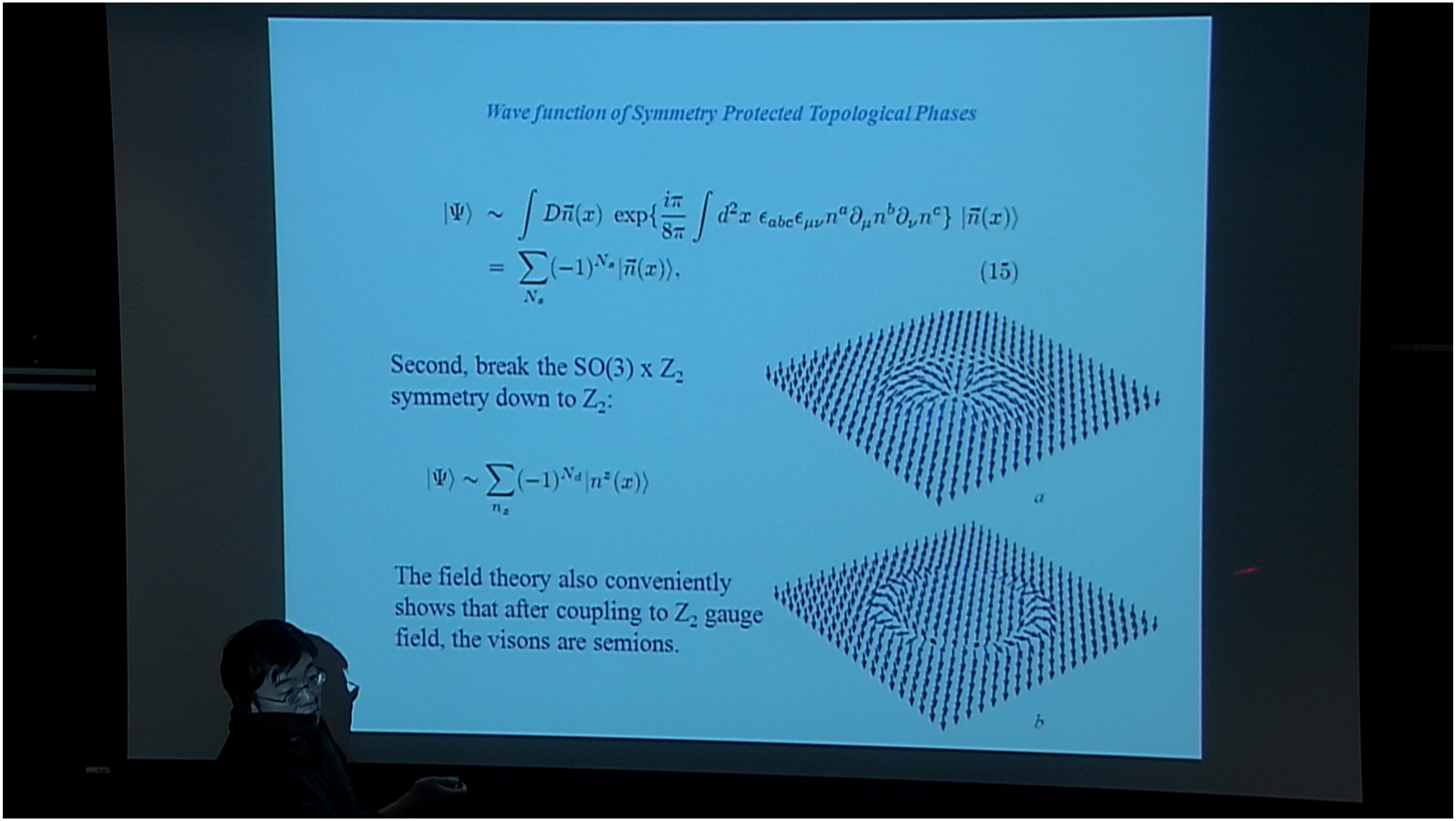
$$|\Psi\rangle \sim \int D\vec{\phi}(x)\ \exp\{\frac{i2\pi}{12\pi^2} \times$$
$$\int d^2x \int_0^1 du\ \epsilon_{abcd}\epsilon_{\mu\nu\rho}\phi^a\partial_\mu\phi^b\partial_\nu\phi^c\partial_\rho\phi^d\}\ |\vec{\phi}(x)\rangle$$

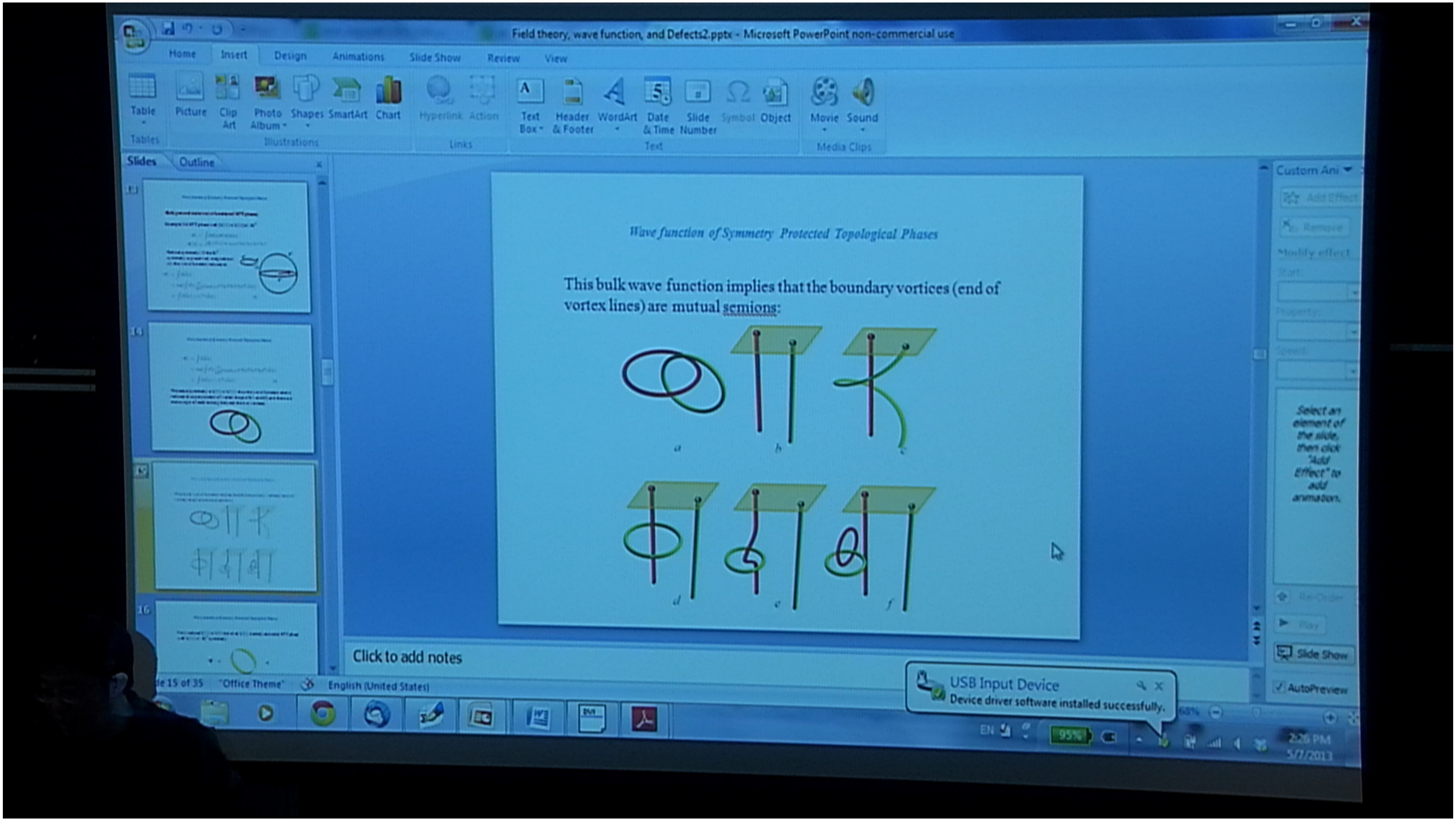
First reduce the SO(4) symmetry down to SO(3) x Z_2: Integrate out the ϕ_0 component:

$$\vec{\phi} \sim (\phi_0,\ \vec{n})$$



$$|\Psi\rangle \sim \int D\vec{n}(x)\ \exp\{\frac{i\pi}{8\pi}\int d^2x\ \epsilon_{abc}\epsilon_{\mu\nu}n^a\partial_\mu n^b\partial_\nu n^c\}\ |\vec{n}(x)\rangle$$
$$= \sum_{N_s}(-1)^{N_s}|\vec{n}(x)\rangle, \qquad (15)$$


Wave function of Symmetry Protected Topological Phases
$|\Psi\rangle \sim \int D\vec{n}(x)\ \exp\{\frac{i\pi}{8\pi}\int d^2x\ \epsilon_{abc}\epsilon_{\mu\nu}n^a\partial_\mu n^b\partial_\nu n^c\}\ |\vec{n}(x)\rangle$
$= \sum_{N_s}(-1)^{N_s}|\vec{n}(x)\rangle,$ (15)
Second, break the SO(3) x Z_2 symmetry down to Z_2:
$|\Psi\rangle \sim \sum_{n_z}(-1)^{N_d}|n^z(x)\rangle$
The field theory also conveniently shows that after coupling to Z_2 gauge field, the visons are semions.
a
b



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Wave function of Symmetry Protected Topological Phases
This bulk wave function implies that the boundary vortices (end of vortex lines) are mutual semions:
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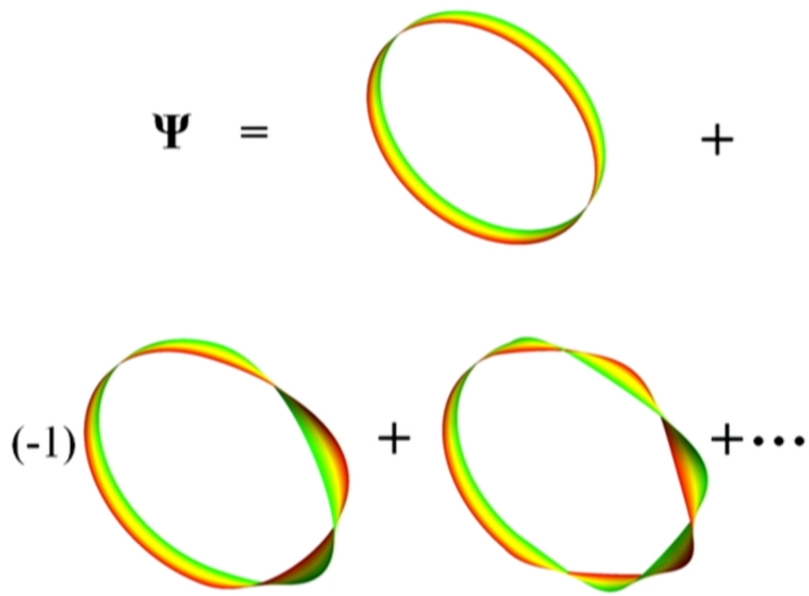
Wave function of Symmetry Protected Topological Phases

Now reduce U(1) x U(1) down to U(1), namely, consider SPT phase with U(1) x! Z_2^T symmetry:



Defects of Symmetry Protected Topological Phases

Point and line defects in topological insulator:

If we couple a 2d quantum spin Hall insulator to a Z_2 gauge field, then the vison must be a Kramers doublet. (Ran, Vishwanath, Lee, 2008, Qi, Zhang, 2008)

A vison loop in 3d TI must be gapless (Zhang, Ran, Vishwanath, 2009)

A dislocation in 3d TI must be gapless (Ran, Zhang, Vishwanath 2009)

Defects of Symmetry Protected Topological Phases

Let us now consider vison loop in bosonic SPT.
As long as the symmetry has a Z_2 center, we can always couple the lattice model to a Z_2 gauge field.

Example 1: 3d SPT phase with [U(1) x U(1)] x! Z_2^T

$$S = \int d^3x d\tau \, \frac{1}{g}(\partial_\mu \vec{n})^2 + \frac{i\Theta}{\Omega_4}\epsilon_{abcde} n^a \partial_x n^b \partial_y n^c \partial_z n^d \partial_\tau n^e$$

$$\vec{n} = (\phi_0, \mathrm{Re}[b_1], \mathrm{Im}[b_1], \mathrm{Re}[b_2], \mathrm{Im}[b_2])$$

Let us couple b_1 to a Z_2 gauge field.
A vison line of the Z_2 gauge field is bound with a half-vortex line of b_1.

Defects of Symmetry Protected Topological Phases

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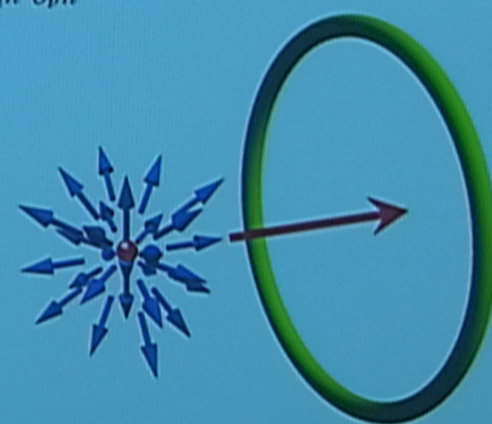
$$\mathcal{S}_v = \int dz d\tau \frac{1}{g'}(\partial_\mu \vec{n})^2 + \frac{i\Theta_{1d}}{8\pi}\epsilon_{\mu\nu}\epsilon_{abc} n^a \partial_\mu n^b \partial_\nu n^c$$

$$\Theta_{1d} = \oint d\vec{l} \ \epsilon_{ef} n^e \partial_l n^f = \pi, \quad e, f = 1, 2.$$

An instanton in the 1+1d vison loop space-time corresponds to moving a hedgehog monopole of (n^3, n^4, n^5) through the vison loop.



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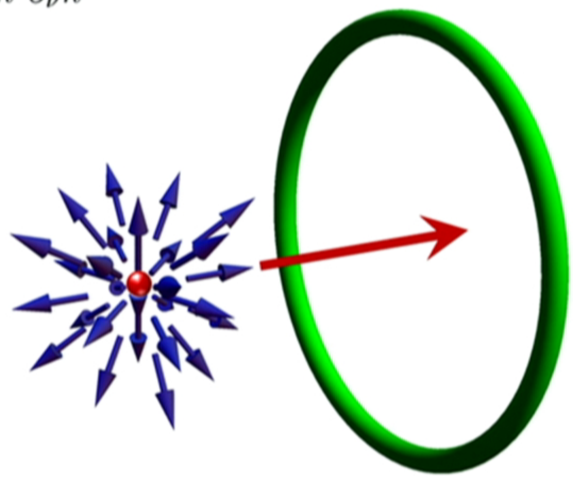


Defects of Symmetry Protected Topological Phases

Example 3: 3d SPT phase with Z_2 x Z_2^T

$$S = \int d^3x d\tau \ \frac{1}{g}(\partial_\mu \vec{n})^2 + \frac{i\Theta}{\Omega_4}\epsilon_{abcde}n^a\partial_x n^b \partial_y n^c \partial_z n^d \partial_\tau n^e$$

$$Z_2 \ : \ n^a \to n^a, a = 1-3, \quad n^b \to -n^b, b = 4, 5.$$

$$Z_2^T \ : \ n^a \to -n^a, a = 1 \cdots 5.$$

Can be realized as local spin model. Z_2 is the π -rotation around S^z.

We can couple n^4, n^5 (S^x, S^y) to a Z_2 gauge field on the lattice:

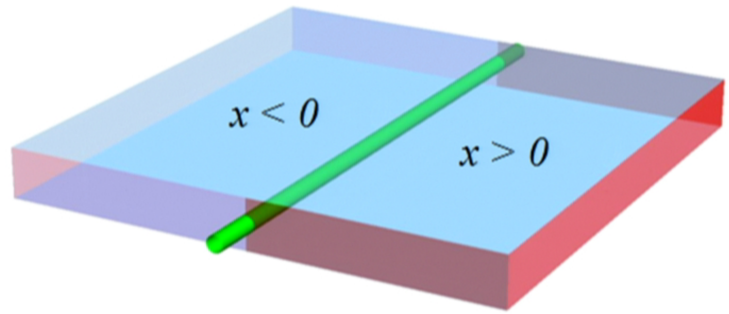
Defects of Symmetry Protected Topological Phases

Example 3: 3d SPT phase with Z_2 x Z_2^T

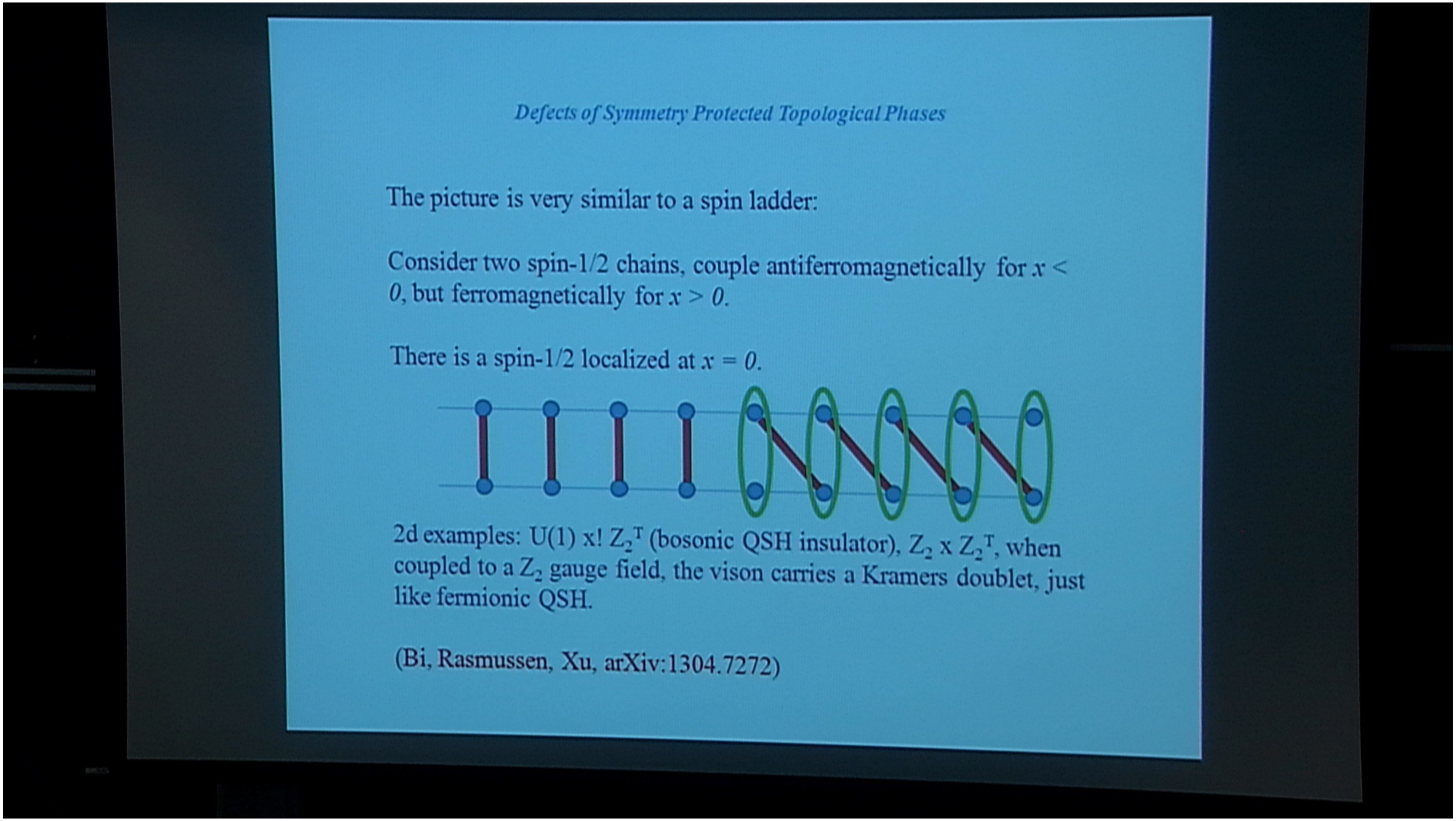
$$S = \int d^3x d\tau \ \frac{1}{g}(\partial_\mu \vec{n})^2 + \frac{i\Theta}{\Omega_4}\epsilon_{abcde} n^a \partial_x n^b \partial_y n^c \partial_z n^d \partial_\tau n^e$$

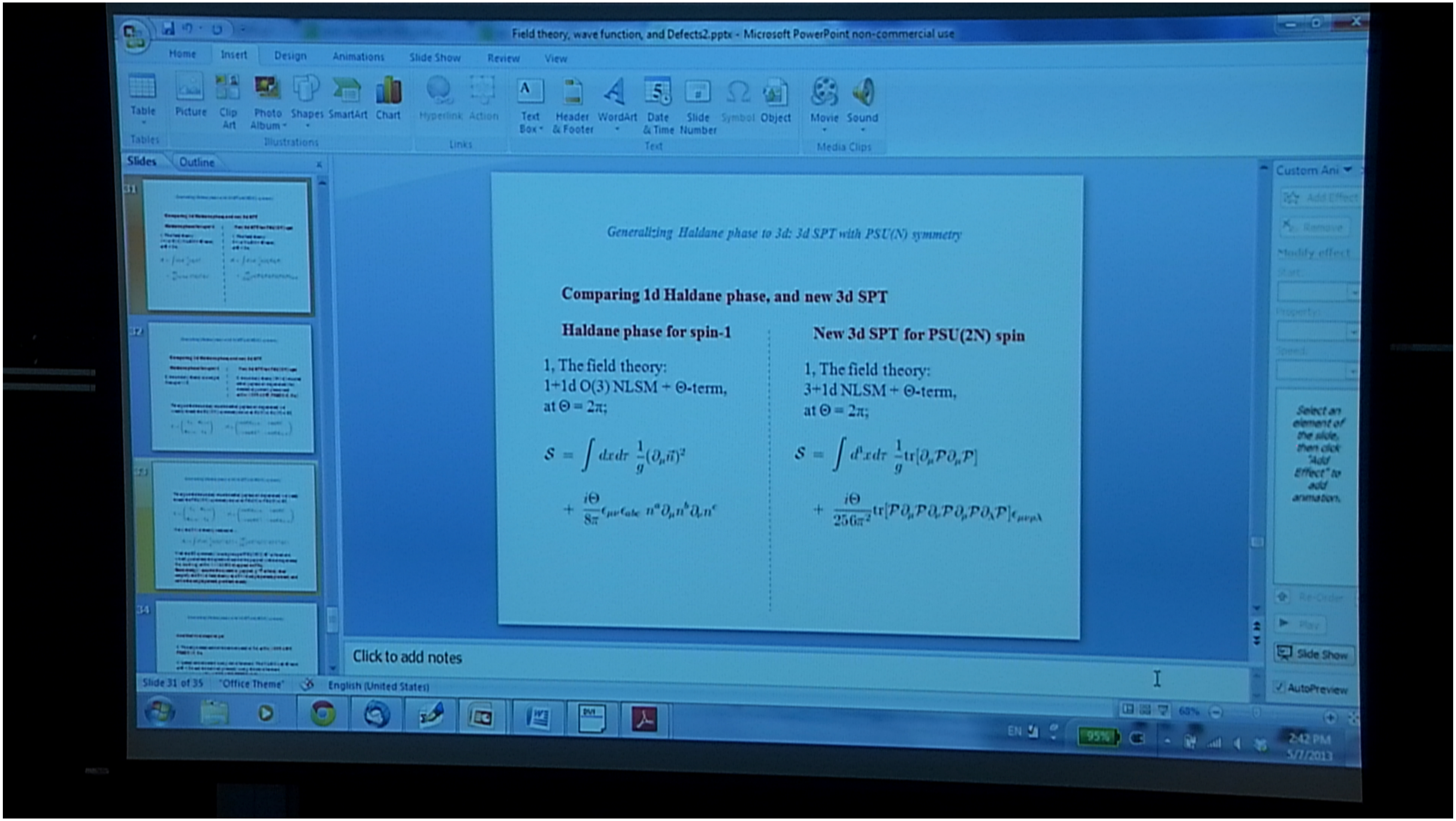
Consider the following structure: Cut the 3d SPT open at z =0, which exposes two boundaries:

Each boundary is a 2+1d O(5) NLSM with WZW k = ± 1:



$$\mathcal{S}_\alpha = \int d^2x d\tau \frac{1}{g}(\partial_\mu \vec{n}_\alpha)^2 \pm \int d^3x \int_0^1 du \frac{2\pi i}{\Omega_4}\epsilon_{abcde} n^a_\alpha \partial_x n^b_\alpha \partial_y n^c_\alpha \partial_u n^d_\alpha \partial_\tau n^e_\alpha,$$


Defects of Symmetry Protected Topological Phases
The picture is very similar to a spin ladder:
Consider two spin-1/2 chains, couple antiferromagnetically for $x < 0$, but ferromagnetically for $x > 0$.
There is a spin-1/2 localized at $x = 0$.
2d examples: U(1) x! Z_2^T (bosonic QSH insulator), Z_2 x Z_2^T, when coupled to a Z_2 gauge field, the vison carries a Kramers doublet, just like fermionic QSH.
(Bi, Rasmussen, Xu, arXiv:1304.7272)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Generalizing Haldane phase to 3d: 3d SPT with PSU(N) symmetry
Comparing 1d Haldane phase, and new 3d SPT
Haldane phase for spin-1
1, The field theory:
1+1d O(3) NLSM + Θ-term,
at Θ = 2π;
New 3d SPT for PSU(2N) spin
1, The field theory:
3+1d NLSM + Θ-term,
at Θ = 2π;
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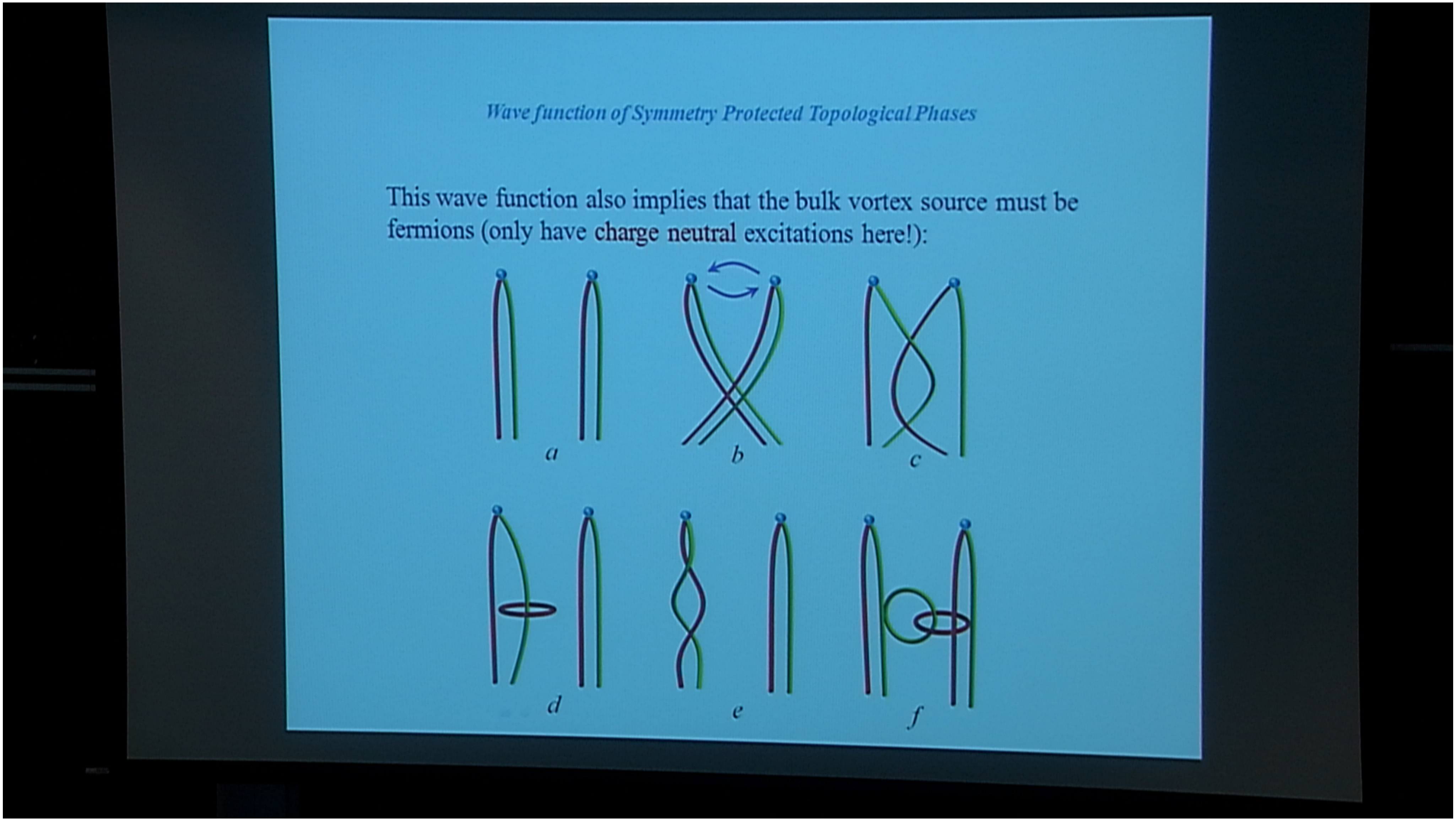
Wave function of Symmetry Protected Topological Phases
This wave function also implies that the bulk vortex source must be fermions (only have charge neutral excitations here!):
a
b
c
d
e
f