Title: A 3d Boson Topological Insulator and the â€œStatistical Witten Effectâ€•

Date: May 07, 2013 09:00 AM

URL: http://pirsa.org/13050028

Abstract: <span>Electron topological insulators are members of a broad class of â€œsymmetry protected topologicalâ€• (SPT) phases of fermions and bosons which possess distinctive surface behavior protected by bulk symmetries. For 1d and 2d SPTâ€™s the surfaces are either gapless or symmetry broken, while in 3d, gapped symmetry-respecting surfaces with (intrinsic) 2d topological order are also possible. The electromagnetic response of (some) SPTâ€™s can provide an important characterization, as illustrated by the Witten effect in 3d electron topological insulators. Using a 3d parton-gauge theory construction, we have recently developed a dyon condensation approach to access exotic new phases including some 3d bosonic SPTâ€™s. A bosonic SPT with both time-reversal and charge conservation symmetries, is thereby obtained, a phase which supports a gapped, symmetry-unbroken 2d surface with topological order - a toric code with charge one-half anyons. The 3d electromagnetic response of this bosonic SPT phase is quite remarkable - an external magnetic monopole can remain charge neutral, but is statistically transmuted becoming a fermion - a â€œstatistical Witten effectâ€• that characterizes the phase.</span>

# Non-Abelian Anyons on Fractional Quantum Hall Edges

Erez Berg

מכון ויצמן למדע

Weizmann Institute of Science

**Netanel Lindner, EB, Gil Refael, and Ady Stern,**
PRX 2, 041002 (2012)

**Johannes Motruk, EB, Ari Turner, and Frank Pollmann,**
arXiv:1303.2194

## See also:

Shtengel, and J. Alicea,
Nature Commun. 4, 1348 (2013)

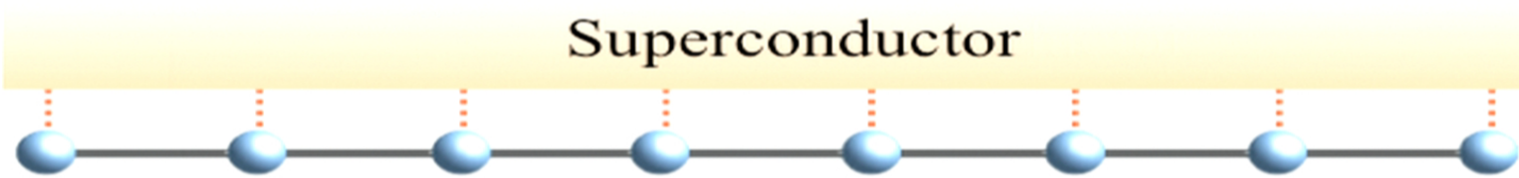
M. Cheng,
Phys. Rev. B 86, 195126 (2012)

A. Vaezi,
Phys. Rev. B 87, 035132 (2013)

M. Barkeshli, C.-M. Jian, and X.-L. Qi,
Phys. Rev. B 87, 045130 (2013)

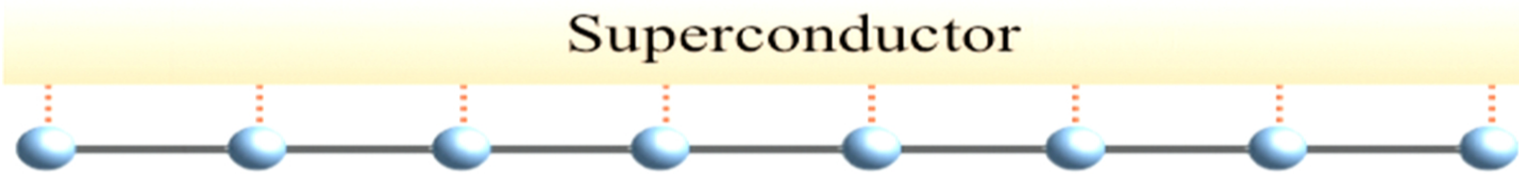
M. Hastings, C. Nayak, and Z. Wang,
arXiv:1210.5477

# Majorana fermions in a superconducting wire




Kitaev (2002), Sau et al. (2010), Oreg et al. (2010), ...

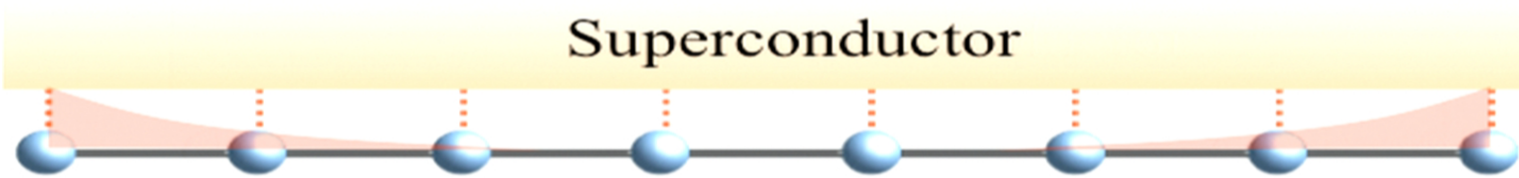
# Majorana fermions in a superconducting wire




$$H = \sum_{i,j} \left[ -t_{ij} \left( c_i^\dagger c_j + H.c. \right) + \Delta_{ij} \left( c_i^\dagger c_j^\dagger + H.c. \right) \right] - \sum_i \mu c_i^\dagger c_i$$

**Kitaev (2002), Sau et al. (2010), Oreg et al. (2010), ...**

# Majorana fermions in a superconducting wire




$$H = \sum_{i,j} \left[ -t_{ij} \left( c_i^\dagger c_j + H.c. \right) + \Delta_{ij} \left( c_i^\dagger c_j^\dagger + H.c. \right) \right] - \sum_i \mu c_i^\dagger c_i$$

**Two degenerate ground states, "Majorana edge modes".**

- **The two states correspond to a total even or odd number of electrons in the system.**
- **Ground state degeneracy is "topological": no local measurement can distinguish between the two states!**

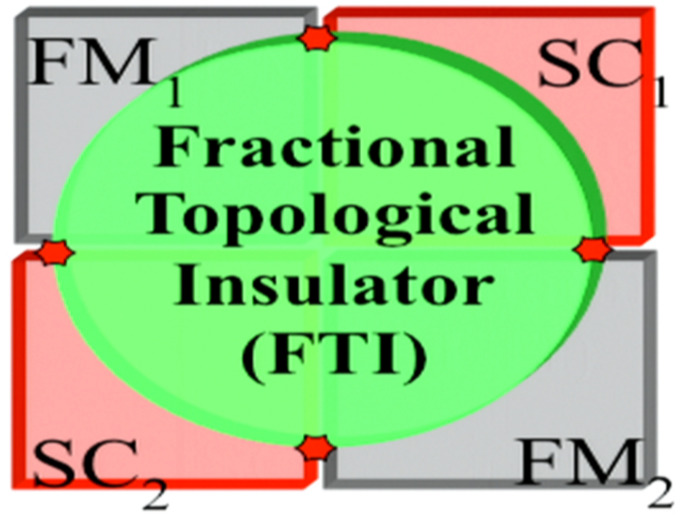
**Kitaev (2002), Sau et al. (2010), Oreg et al. (2010), ...**

# Outline

Consider boundaries of 2D a topological phase which supports (abelian) anyons.

# Outline

Consider boundaries of 2D a topological phase which supports (abelian) anyons.



# Outline

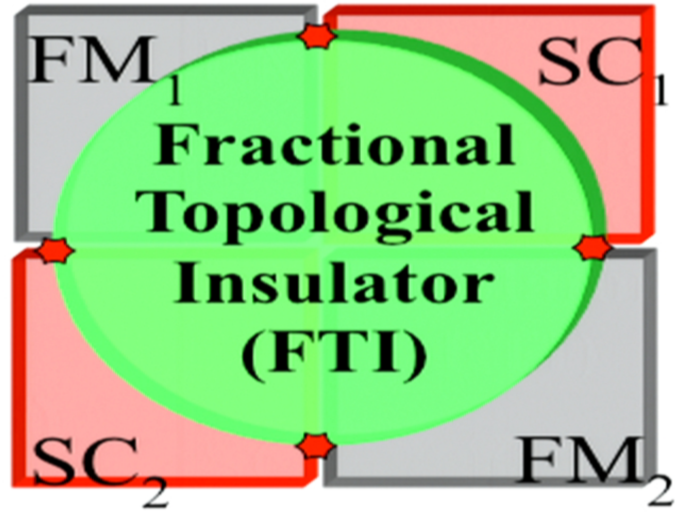
Consider boundaries of 2D a topological phase which supports (abelian) anyons.

"Fractional topological insulator":

Laughlin Quantum Hall state with:

$\nu=1/m$ for spin up

$\nu=-1/m$ for spin down (m odd)

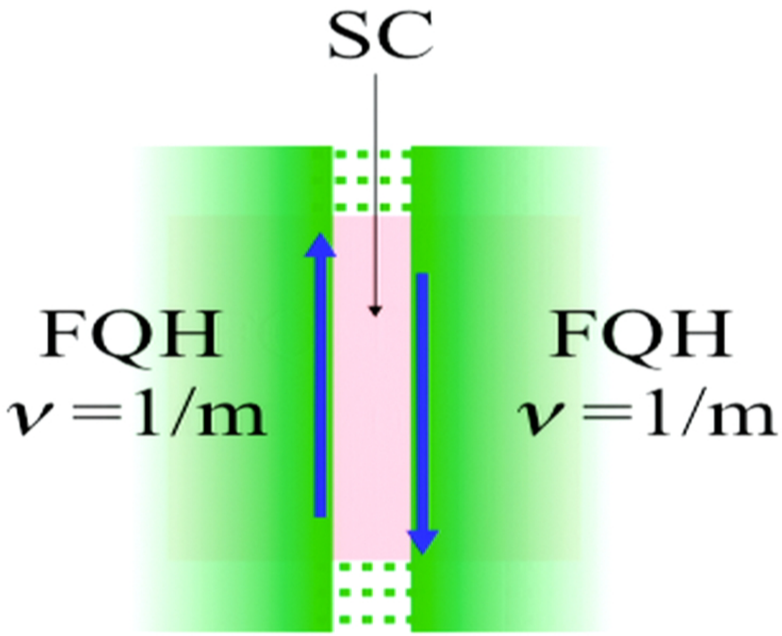



Stable phase: Levin and Stern (2010)

Majorana fermions at SC/FM interfaces: Fu and Kane (2009)

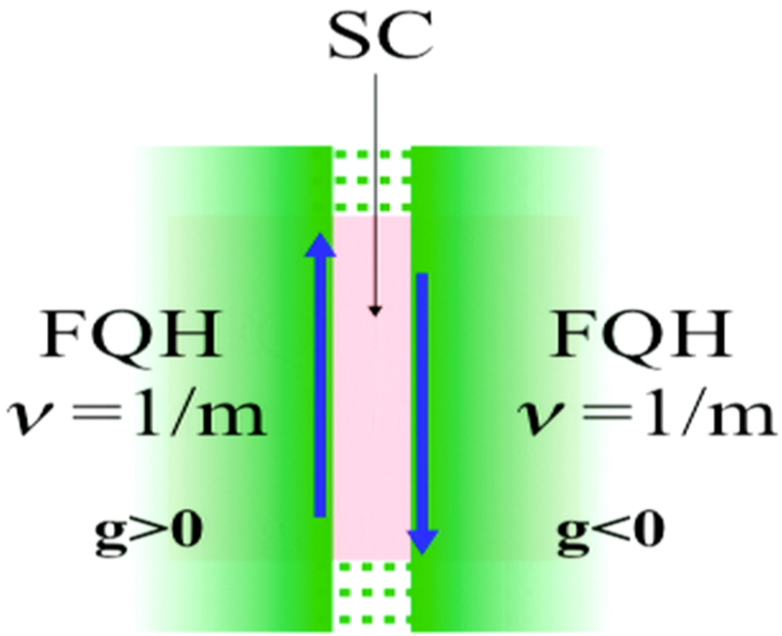
# Fractional Quantum Hall realization

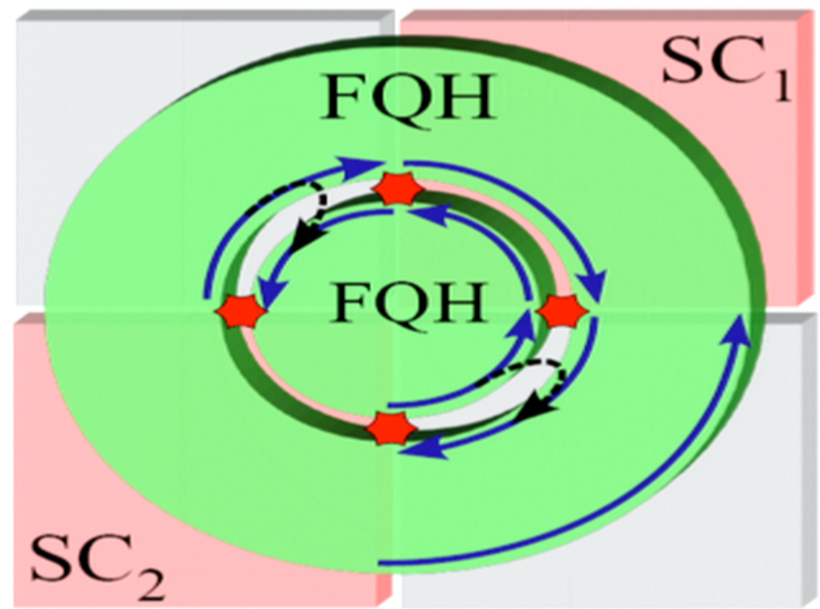
Conceptually, this can be realized without a fractional topological insulator.



# Fractional Quantum Hall realization

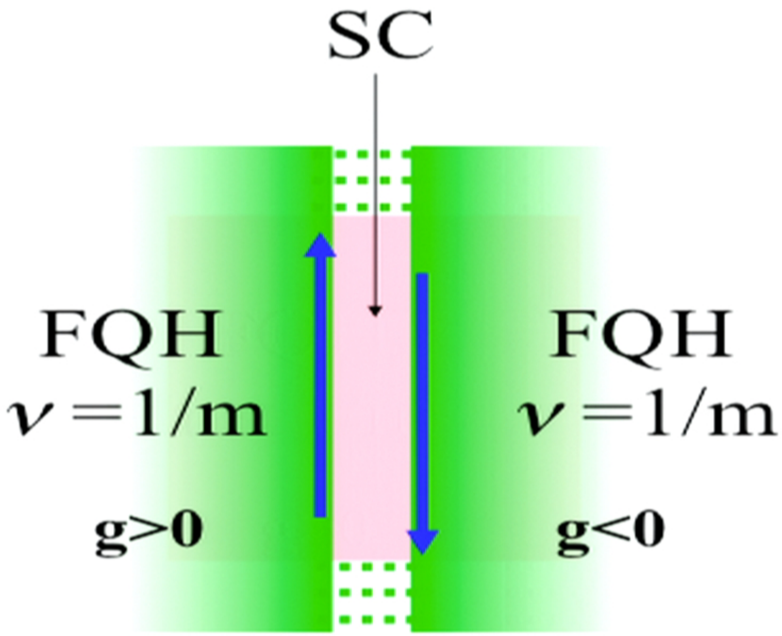
Conceptually, this can be realized without a fractional topological insulator.

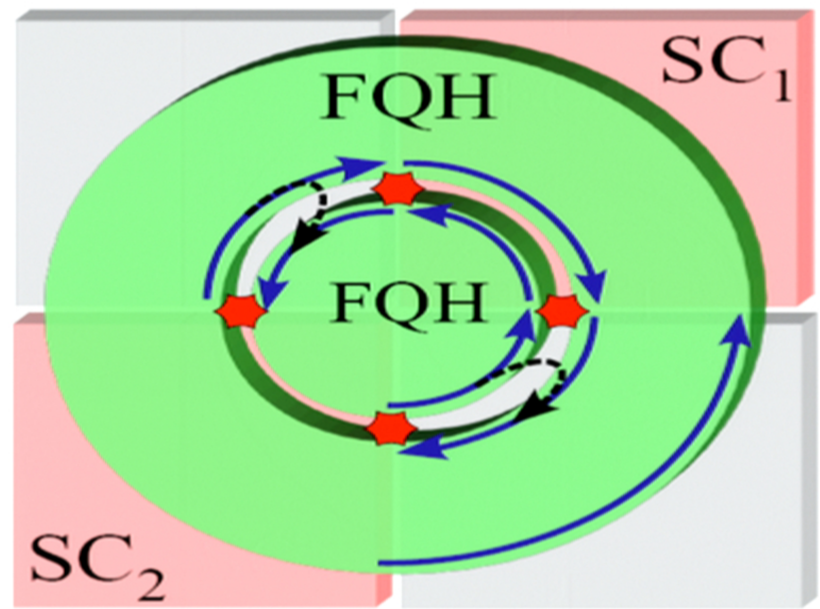


# Fractional Quantum Hall realization

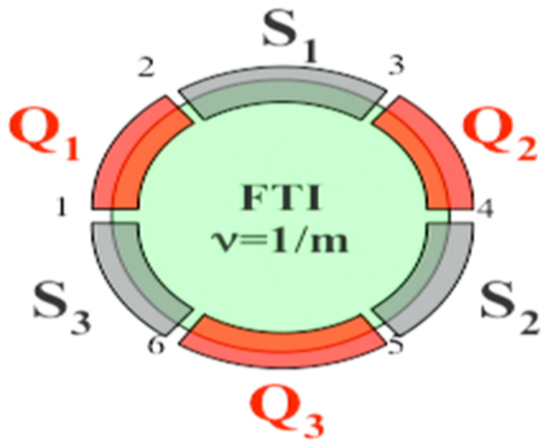
Conceptually, this can be realized without a fractional topological insulator.



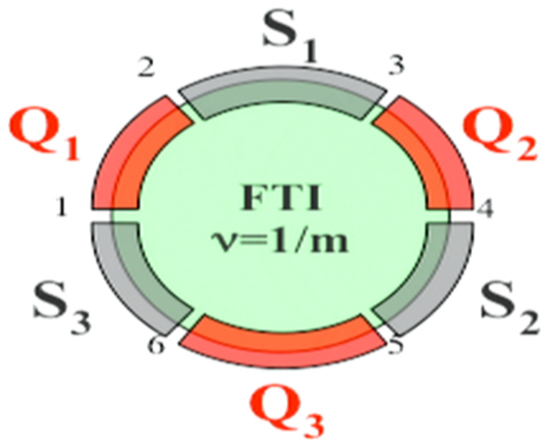
# Ground state degeneracy

**Physical picture:**



# Ground state degeneracy

**Physical picture:**

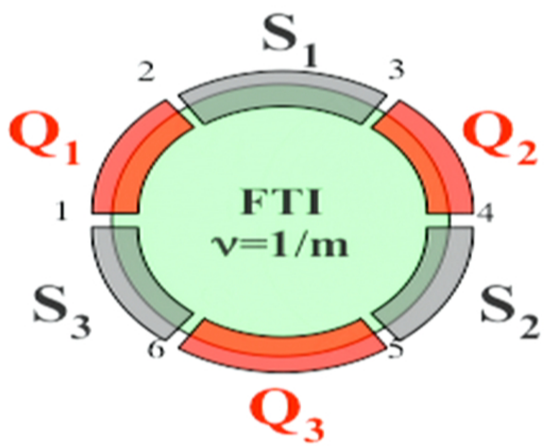



Charges in SC conserved mod(2)

$Q_j$ = n/m, n = 0,....,2m-1

# Ground state degeneracy

Physical picture:




Charges in SC conserved mod(2)

$Q_j = n/m,\ n = 0,...,2m-1$

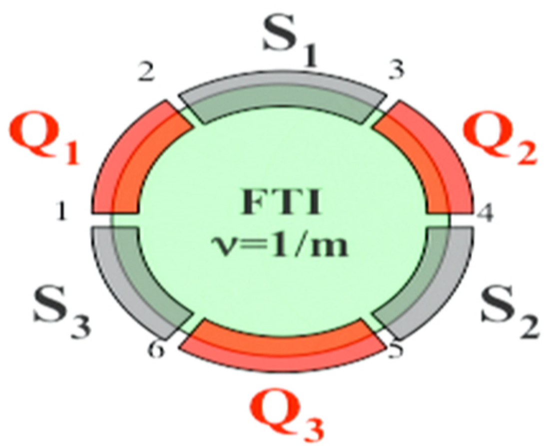
Spins in FM conserved mod(2)

(el. spin=1)

$S_j = n/m,\ n = 0,...,2m-1$

# Ground state degeneracy

Physical picture:




Charges in SC conserved mod(2)

$Q_j$ = n/m, n = 0,...,2m-1

Spins in FM conserved mod(2)

(el. spin=1)

$S_j$ = n/m, n = 0,...,2m-1

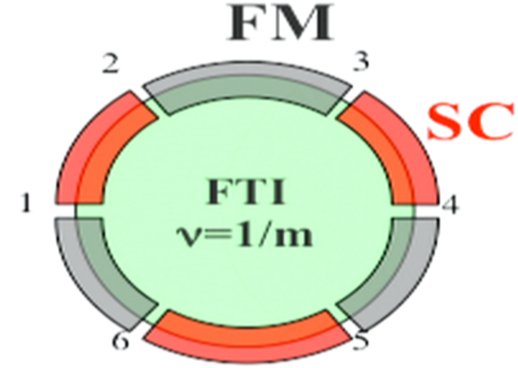
Spin and charge are conjugate variables:

$$e^{i\pi S_i} e^{i\pi Q_j} = e^{\frac{i\pi}{m}(\delta_{i,j+1}-\delta_{i,j})} e^{i\pi Q_j} e^{i\pi S_i}$$

2N domains, fixed total Q, S: $(2m)^{N-1}$ approximately degenerate ground states

# Effective Model for Fractional Topological Insulator Edge States

$$H = \frac{u}{2\pi\nu}\int dx\left[K(x)(\partial_x\phi)^2 + \frac{1}{K(x)}(\partial_x\theta)^2\right] - \int dx\left[g_S(x)\cos(2m\phi) + g_F(x)\cos(2m\theta)\right]$$




2N domains

**Electron:** $\psi_\pm \propto e^{im(\phi\pm\theta)}$

**Laughlin q.p.:** $\chi_\pm \propto e^{i(\phi\pm\theta)}$

# Ground state degeneracy

**Large cosine terms (strong coupling to SC/FM)**

$$-\int dx\,[g_S(x)\cos(2m\phi) + g_F(x)\cos(2m\theta)]$$

# Ground state degeneracy

**Large cosine terms (strong coupling to SC/FM)**

$$-\int dx\,[g_S(x)\cos(2m\phi)+g_F(x)\cos(2m\theta)]$$

**φ, θ pinned near the minima of the cosines:**

$$\phi_n=\frac{\pi}{m}n,\ n\in 0,1,\ldots,2m-1$$

$$\theta_k=\frac{\pi}{m}n,\ k\in 0,1,\ldots,2m-1$$

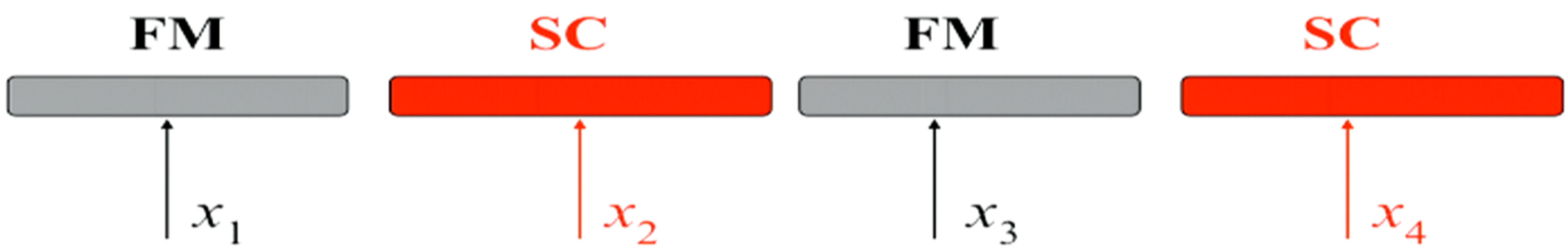
**But... φ, θ are dual vararables: cannot be "localized" simultaneously**

$$e^{i\theta(x)}e^{i\phi(x')}=e^{i\frac{\pi}{m}\Theta(x-x')}e^{i\phi(x)}e^{i\theta(x)}$$

**2N domains: ~$(2m)^N$ approximately degenerate ground states**

# Q and S operators

In terms of the $\phi$, $\theta$ fields, one can define the Q, S operators:

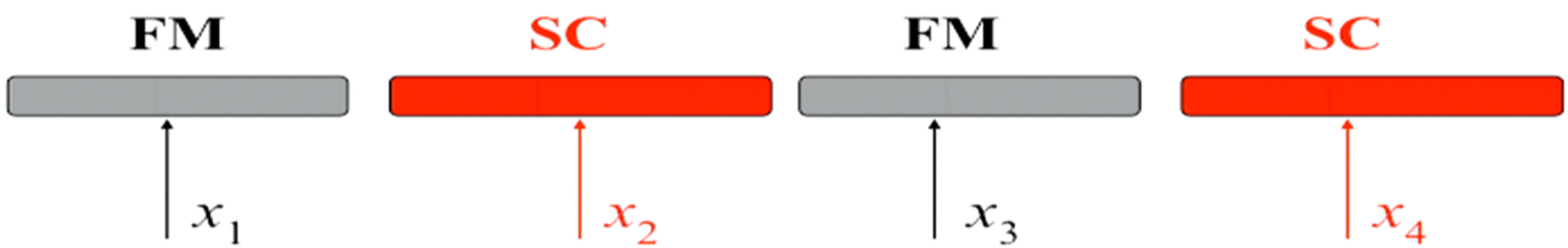



$$e^{i\pi Q_2} = e^{i\int_{x_1}^{x_3} dx\,\partial_x \theta} = e^{i[\theta(x_3)-\theta(x_1)]}$$

$$e^{i\pi S_3} = e^{i\int_{x_2}^{x_4} dx\,\partial_x \phi} = e^{i[\phi(x_4)-\phi(x_2)]}$$

# Q and S operators

In terms of the $\phi$, $\theta$ fields, one can define the Q, S operators:

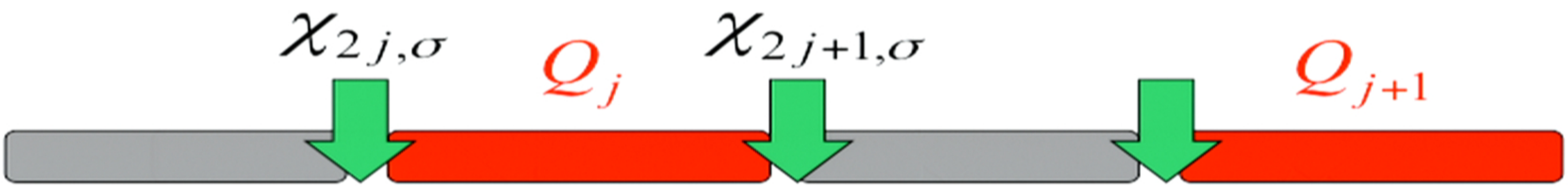



$$e^{i\pi Q_2} = e^{i\int_{x_1}^{x_3} dx \partial_x \theta} = e^{i[\theta(x_3)-\theta(x_1)]}$$

$$e^{i\pi S_3} = e^{i\int_{x_2}^{x_4} dx \partial_x \phi} = e^{i[\phi(x_4)-\phi(x_2)]}$$

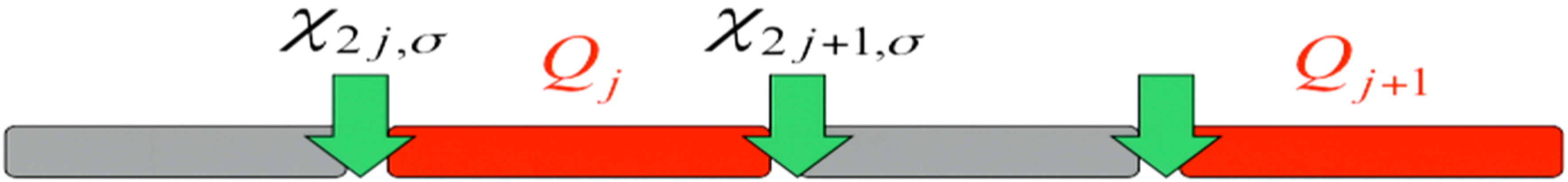
$$e^{i\pi S_i} e^{i\pi Q_j} = e^{i\frac{\pi}{m}(\delta_{i,j+1}-\delta_{i,j-1})} e^{i\pi Q_j} e^{i\pi S_i}$$

# "Fractionalized Majorana operators"



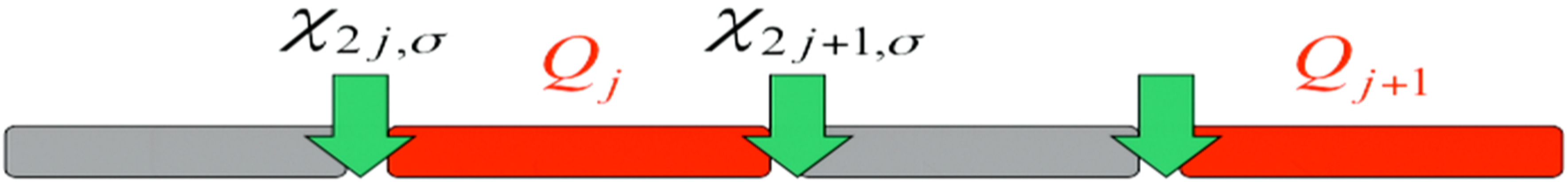

$$\chi_{r,\sigma}\left|q_1,\ldots,q_j,\ldots;s\right\rangle \propto \left|q_1,\ldots,q_j+1,\ldots;s+\sigma\right\rangle$$

# "Fractionalized Majorana operators"




$$\chi_{r,\sigma}\left|q_1,\ldots,q_j,\ldots;s\right\rangle \propto \left|q_1,\ldots,q_j+1,\ldots;s+\sigma\right\rangle$$

# "Fractionalized Majorana operators"




$$\chi_{r,\sigma}\left|q_1,\ldots,q_j,\ldots;s\right\rangle \propto \left|q_1,\ldots,q_j+1,\ldots;s+\sigma\right\rangle$$

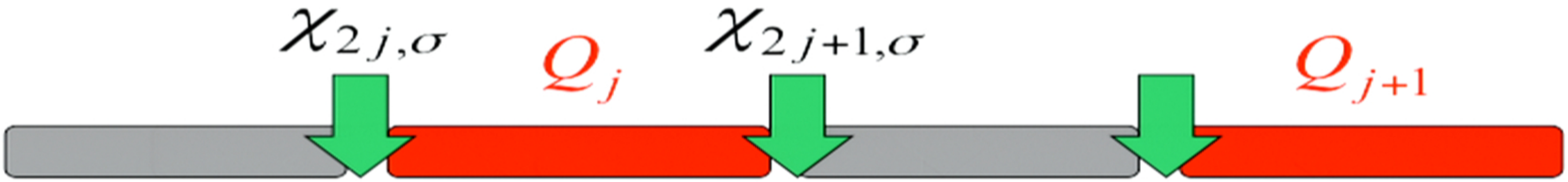
$$\left[H,\chi_{r\sigma}\right]=0$$

$$\left(\chi_{r\sigma}\right)^{2m}=1$$

$\chi_{r\sigma}$ have q.p commutation relations

$$\chi_{j,\sigma}\chi_{k,\uparrow}=e^{i\pi/m}\chi_{k,\uparrow}\chi_{j,\sigma}$$
$$\chi_{j,\sigma}\chi_{k,\downarrow}=e^{-i\pi/m}\chi_{k,\downarrow}\chi_{j,\sigma}$$

# "Fractionalized Majorana operators"




$$\chi_{r,\sigma}\left|q_1,\ldots,q_j,\ldots;s\right\rangle \propto \left|q_1,\ldots,q_j+1,\ldots;s+\sigma\right\rangle$$

$$\left[H,\chi_{r\sigma}\right]=0$$

$$\left(\chi_{r\sigma}\right)^{2m}=1$$

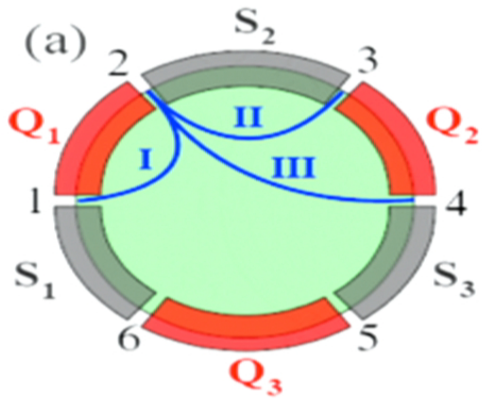
$\chi_{r\sigma}$ have q.p commutation relations

$$\chi_{j,\sigma}\chi_{k,\uparrow}=e^{i\pi/m}\chi_{k,\uparrow}\chi_{j,\sigma}$$
$$\chi_{j,\sigma}\chi_{k,\downarrow}=e^{-i\pi/m}\chi_{k,\downarrow}\chi_{j,\sigma}$$

**1D model of "Parafermions": P. Fendley, arXiv:1209.0472**

# Braiding

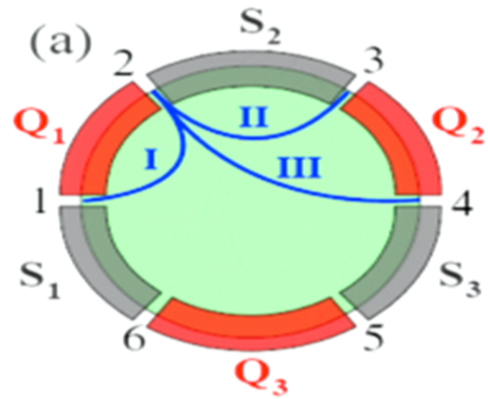
## Braiding domain walls 3 and 4:




For an arbitrary coupling of any three domain walls, the ground state degeneracy remains $(2m)^2$ as long as only one spin species is allowed to tunnel.

# Braiding

Braiding domain walls 3 and 4:



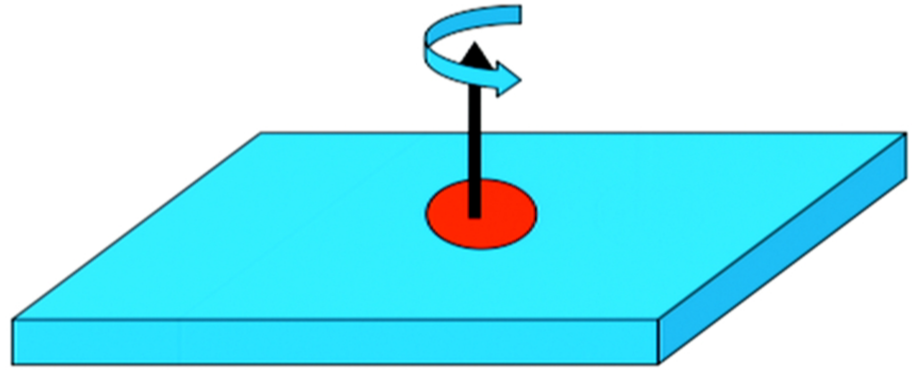

For an arbitrary coupling of any three domain walls, the ground state degeneracy remains $(2m)^2$ as long as only one spin species is allowed to tunnel.

# Braiding: general arguments

**Why** $\exp\left(i\frac{\pi}{2m}q_2^2\right)$ **?**

**Topological spin**

$$\theta_a = e^{i2\pi s_a}$$



# Braiding: general arguments

**Two types of particles:**

$$X \times X = 0 + 1 + \ldots 2m - 1$$

$$q_1 \times q_2 = q_1 + q_2 \bmod 2m$$

# Braiding: general arguments

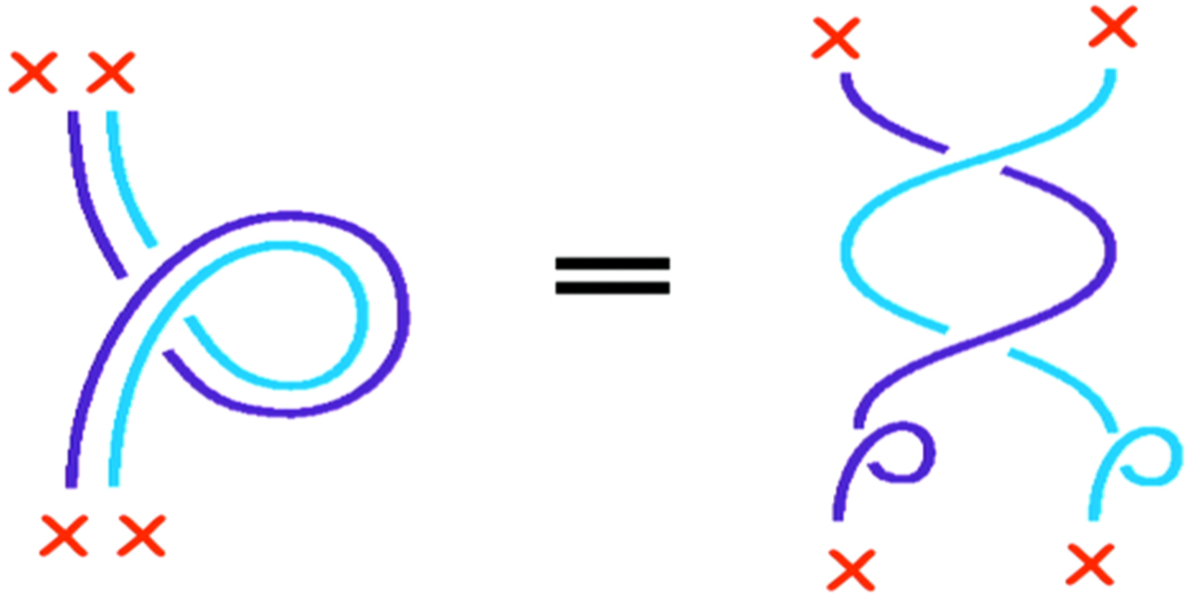
**Two types of particles:**

$$X \times X = 0 + 1 + \ldots 2m - 1$$

$$q_1 \times q_2 = q_1 + q_2 \bmod 2m$$

$\theta_X =$  $\theta_q =$ 

# Braiding: general arguments



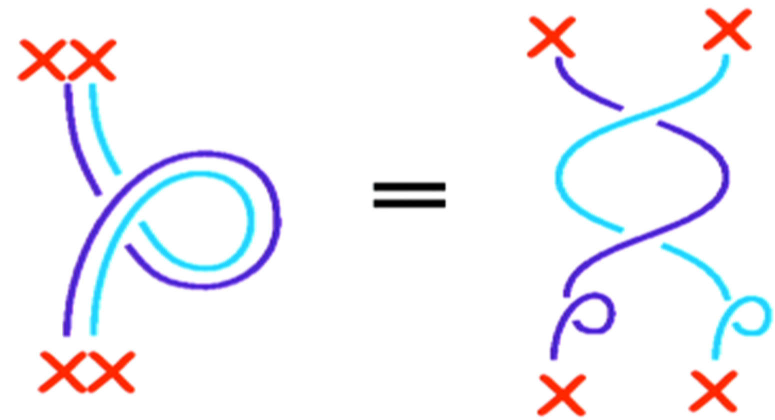
$$\theta_q = \theta_X^2 U^2(q)$$

# Conclusion

**A paradigm for new, "non-Majorana" phases on gapped edges of 2D topological states.**

$$U_{34} = \exp\left(i\frac{\pi m}{2}\hat{Q}_2^2\right) = \exp\left(i\frac{\pi}{2m}q_2^2\right)$$

$$Q_2 = \frac{1}{m}q_2, \quad q_2 = 0, \ldots, 2m-1$$



$$\theta_q = \theta_X^2 U^2(q)$$