Title: Asymmetry protected emergent E8 symmetry

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Abstract: The E8 state of bosons is a 2+1d gapped phase of matter which has no topological entanglement entropy but has protected chiral edge states in the absence of any symmetry. This peculiar state is interesting in part because it sits at the boundary between short- and long-range entangled phases of matter. When the system is translation invariant and for special choices of parameters, the edge states form the chiral half of a 1+1d conformal field theory - an E8 level 1 Wess-Zumino-Witten model. However, in general the velocities of different edge channels are different and the system does not have conformal symmetry. We show that by considering the most general microscopic Hamiltonian, in particular by relaxing the constraint of translation invariance and adding disorder, conformal symmetry remerges in the low energy limit. The disordered fixed point has all velocities equal and is the E8 level 1 WZW model. Hence a highly entangled and highly symmetric system emerges, but only when the microscopic Hamiltonian is completely asymmetric.


$K =$

Asymmetry Protected Emergent E8 Symmetry

Brian Swingle

Entanglement and Emergence II

Perimeter, May 6, 2013

$$16 \times \frac{1}{16} = 1 \quad 120 + 128 = 248$$

$$3 - 4 < 0$$

SIMONS FOUNDATION

Acknowledgements

- Collaborators: Jay Sau, Senthil

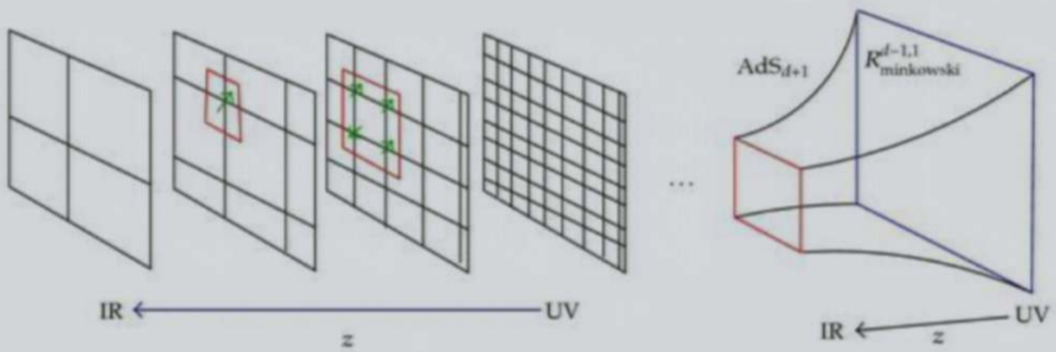
- References: Gross et al. (Het string), Di Francesco et al. (Big yellow book), Witten (Jones, NA Bos.), Kitaev (Exact anyons), Lu-Vishwanath (2012), Kane-Fisher (1995), informal sources

Plan

- ``Motivation
- ``Trivial" states of bosons
- Constructing the E8 state
- Disorder: asymmetry protected emergent E8 symmetry

(SSS '13 to appear)

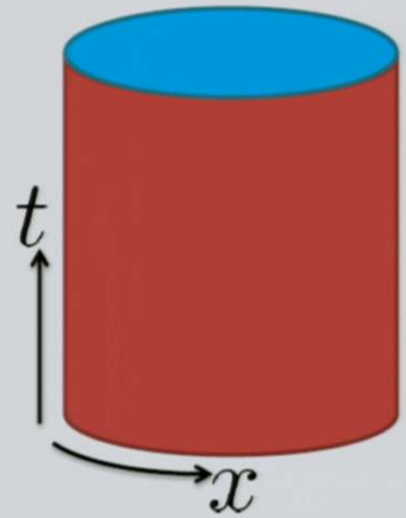
Big picture of quantum matter: entanglement and renormalization



+

$$\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

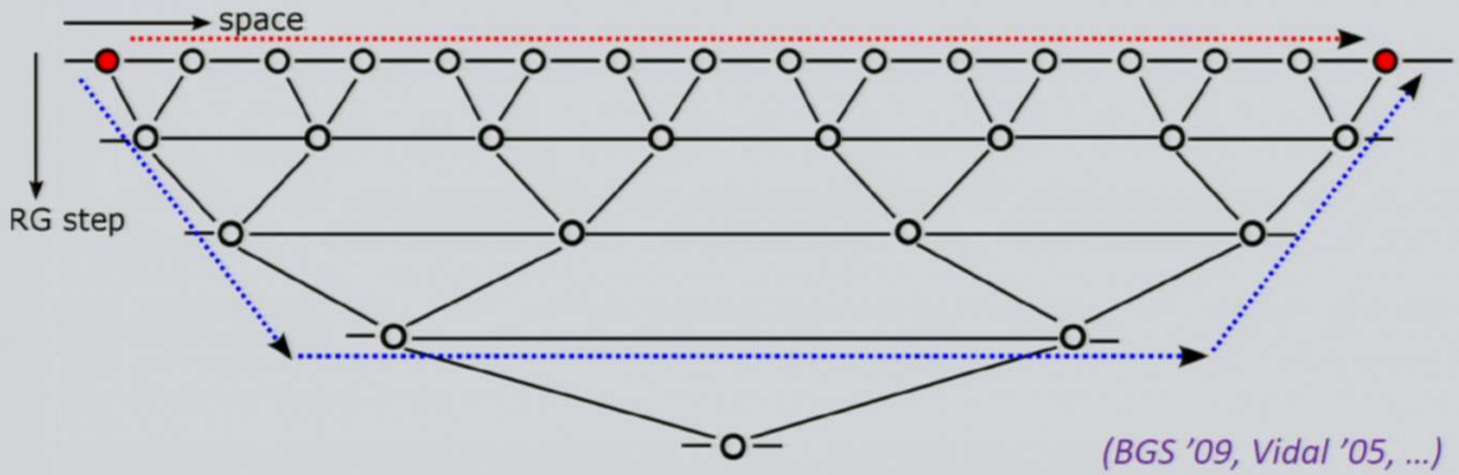
Experimental* fact: some systems of interacting qubits* are secretly theories of quantum gravity in an emergent space



(Maldacena-Witten-Polyakov -Klebanov-Gubser-...)

This fact is intimately tied to the structure of entanglement in quantum matter

Entanglement as the fabric of spacetime



(BGS '09, Vidal '05, ...)

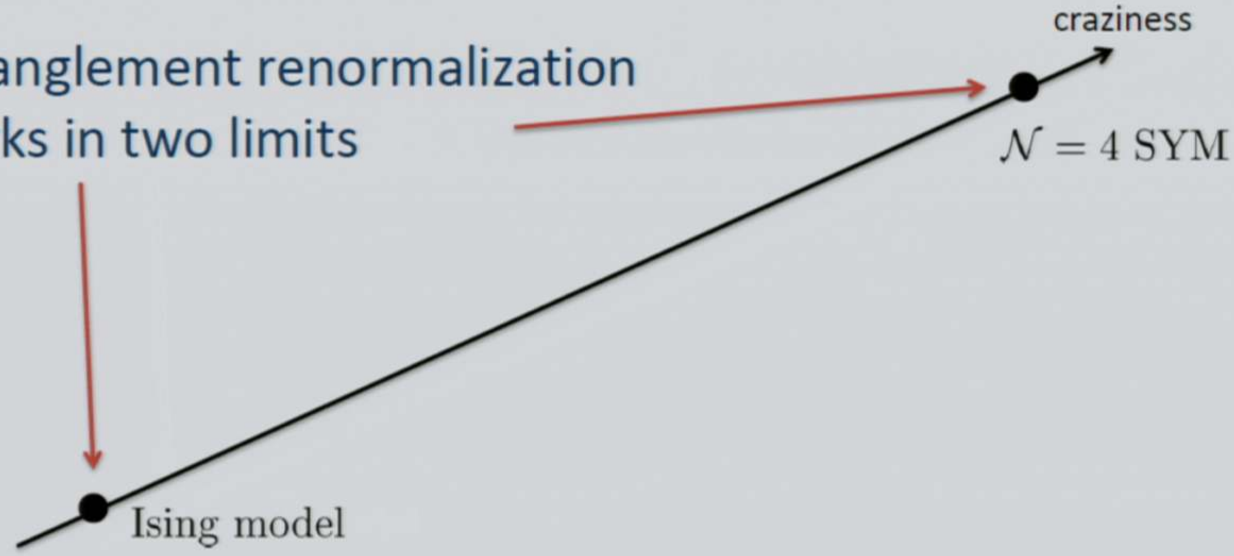
What's special about large N/strong coupling?

- Local RG circuit = general feature of quantum matter?
- Extra features: *(obtained from tensor network picture: BGS '12)*
  - Sparse spectrum of operator dimensions
  - Non-fluctuating geometry
- Dynamics: vN entropy -> thermal entropy, long timescales emerge from Hagedorn dos

(Behan-Larjo-Lashkari-BGS-vanRaamsdonk '13)

Theorem (physics)

Entanglement renormalization works in two limits



Intermediate Value Theorem implies entanglement renormalization works everywhere

This talk

- Goal: understand better entanglement and renormalization
- Many kinds of entanglement, e.g. SRE vs LRE, with edge states, gapped vs gapless, with symmetry, …
- This talk: what is the simplest LRE state? What does its entanglement look like? What is the low energy physics?

the E8 state

RULES OF THE GAME

Rules of the game (v1)

- Rule 1: microscopic Hilbert space is a tensor product of local bosonic spaces
- Rule 2: local Hamiltonian in d=2 dimensions
- Rule 3: no symmetry besides translation invariance
- Rule 4: bulk gap
- Rule 5: no anyon excitations, etc.

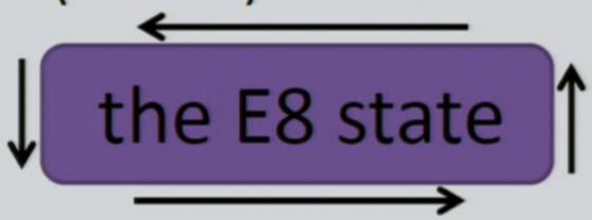
What is the simplest state of bosons?

$$|GS\rangle = \bigotimes_r |\beta_r\rangle$$

Mean-field superfluid
or Mott insulator (no symmetry)

What is the simplest ``long-range entangled" state of bosons?

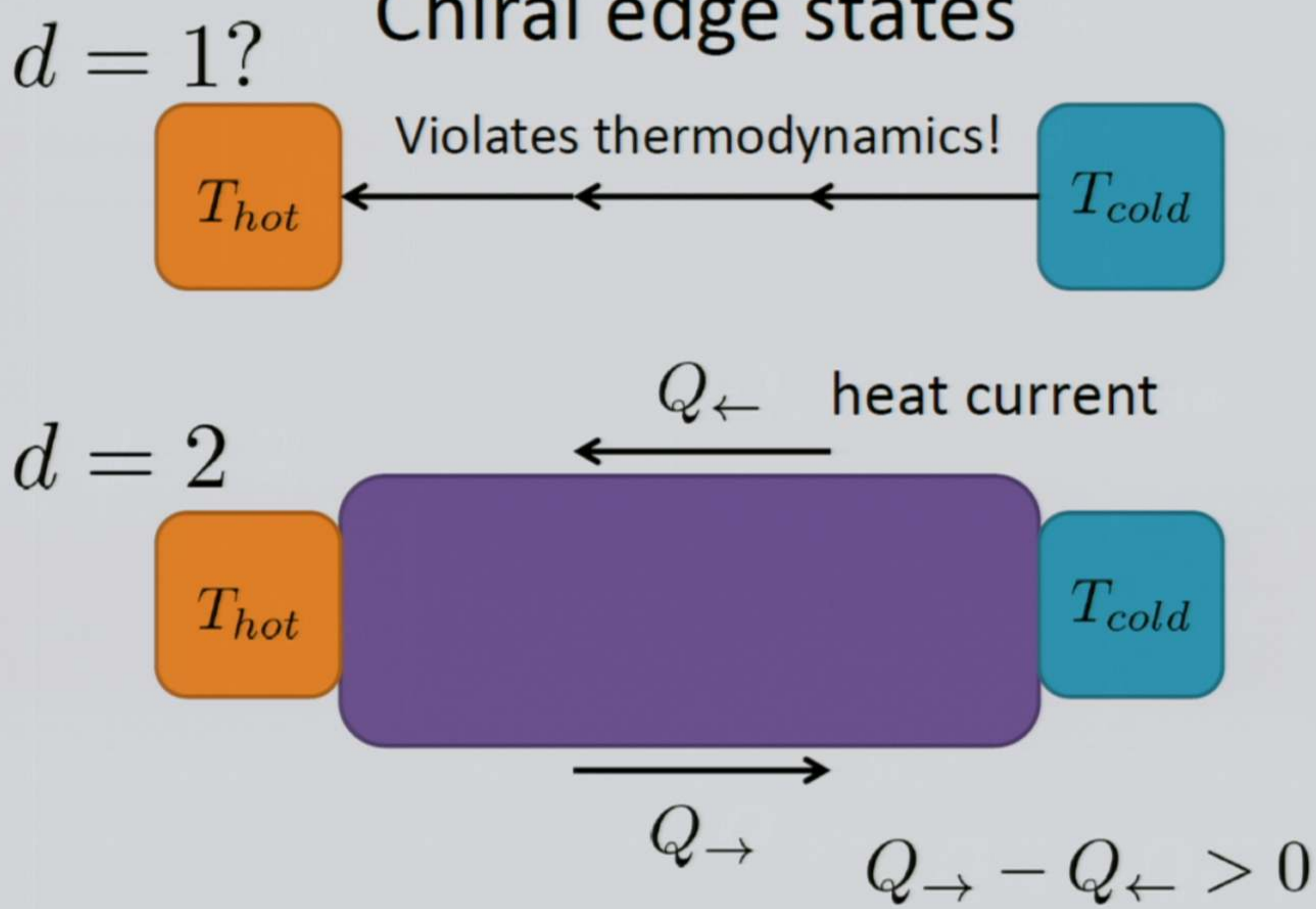
- With no bulk anyons, no symmetry, and a gap, is there anything?
- It turns out (in d=2) there is …



This state has

1. Bulk gap, no anyons, etc.
2. Chiral edge states $\kappa_{xy} = 8$

Chiral edge states



K-matrix framework

- Introduce gauge fields A, normally $j^\mu = \epsilon^{\mu\nu\lambda}\partial_\nu A_\lambda$ is a conserved current, but we can break all symmetries *(many people, e.g. Lu-Vishwanath '12)*
- Real virtue: edge description
- Requirements on K: symmetric, even, det(K)=1

$$S_{bulk} = \int d^2x dt\, \epsilon^{\mu\nu\lambda} A^a_\mu K_{ab} \partial_\nu A^b_\lambda$$

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E8 K-matrix theory

$$\mathcal{S}_{E_8} = \frac{1}{4\pi}\int dx dt \left[K_{ab}\partial_t\phi^a\partial_x\phi^b - V_{ab}\partial_x\phi^a\partial_x\phi^b\right]$$

Requirements on K: symmetric, even, det(K)=1

$$K = \begin{pmatrix} 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 2 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$

$$\kappa_{xy} = 8$$

E8 Lie algebra

- 248 generators
- 8 "Cartan" generators (like J_z)
- 240 raising/lowering generators (like $J_\pm$)

$$[J_z, J_\pm] = \pm J_\pm$$

$$[H^a, H^b] = 0,\ a, b = 1, ..., 8$$

$$[H^a, E^\alpha] = \alpha^a E^\alpha$$

$$K = \text{inner products}$$

"roots"

E8 K-matrix theory

- EOM: $K_{ab}\partial_t\partial_x\phi = V_{ab}\partial_x^2\phi$
- Uniform velocities $V_{ab} = vK_{ab}$

- Current algebra $\Delta = 1$

$$H^a \to \partial_x\phi^a,\ E^\alpha \to e^{in_a\phi^a}$$

- Partition function

$$Z = \mathrm{tr}\left(e^{-\beta H_{S^1}}\right) = q^{-1/3}\left(1 + 248q + ...\right)$$

$$q = e^{-\beta v/(2\pi L)}$$

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Translation invariant and generic

$$V_{ab} \neq vK_{ab}$$

1. No conformal or E8 symmetry
2. Many velocities
3. Still have thermal Hall effect

Can we do better? Yes!
Intuition: randomly mix different edge modes, common velocity emerges?

Rules of the game (v2)

- Rule 1: microscopic Hilbert space is a tensor product of local bosonic spaces
- Rule 2: local Hamiltonian in d=2 dimensions
- Rule 3: no symmetry, no translation invariance
- Rule 4: bulk gap
- Rule 5: no anyon excitations, etc.

System now has no symmetry …

How to treat disorder?

Breaks E8 ...

$$\mathcal{S} = \mathcal{S}_{E_8} + \int dxdt \frac{1}{4\pi} \partial_x \phi^a \delta V_{ab} \partial_x \phi^b$$
$$+ \int dxdt\, g_a(x) \partial_x \phi^a + \int dxdt\, g_n(x) \cos(n_a \phi^a) + \ldots$$

Relatively easy to deal with

Harder ...

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Breaks E8 ...

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Harder ...

FERMION CONSTRUCTION

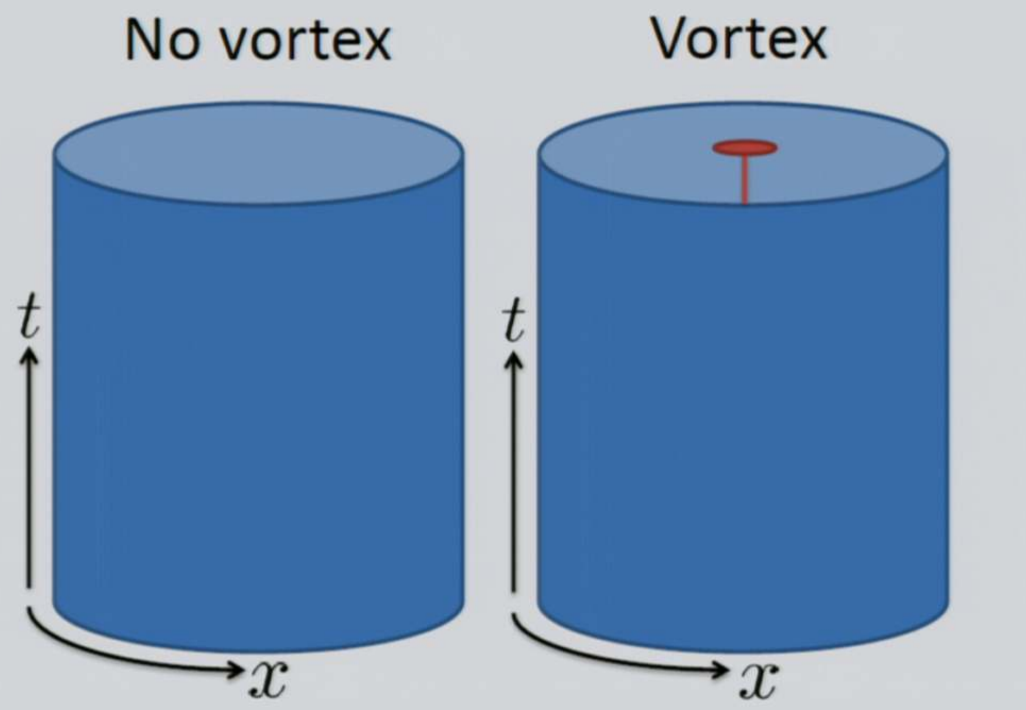
Starting point: SO(16) fermion theory

Bulk = 16 copies of p+ip state $\gamma^i, i = 1, ..., 16$

1. All fermions coupled to a Z2 gauge field
2. All fermions see the same Z2 vortex σ

$$\mathcal{S}_{edge} = \int dxdt\gamma^T(i\partial_t + iv\partial_x)\gamma$$

$$\kappa_{xy} = 16 \times \frac{1}{2}$$



$$\gamma(x + 2\pi L) = -\gamma(x) \quad \gamma(x + 2\pi L) = \gamma(x)$$

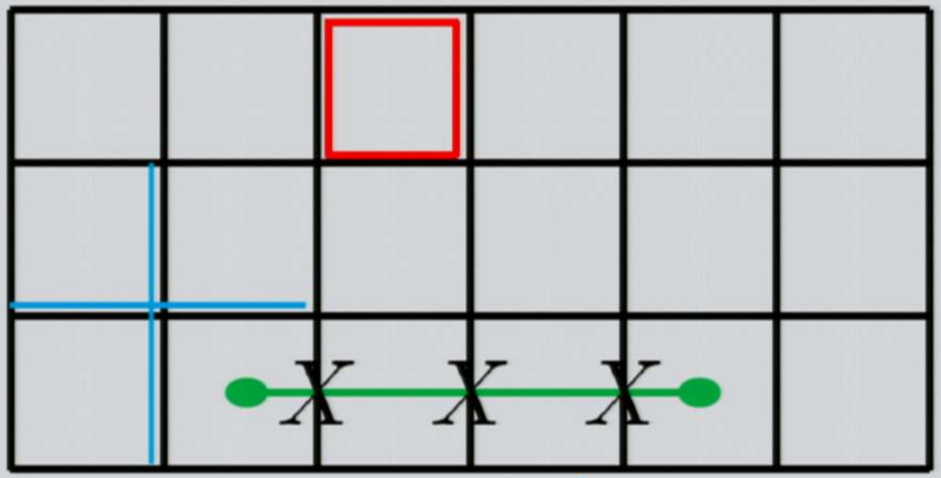
Vortex condensation

- To reach a state with no topological order, we must confine the fermions
- To so do, we ``condense" bosonic vortices

$$s(\sigma) = 16 \times \frac{1}{16}$$

(high energy literature, Kitaev '05)

- Roughly speaking, we make the gap to single vortex excitations zero, and let them move around freely *(SSS '13)*



 $= ZZZZ$

 $= XXXX$

$$n_p = \frac{1 - ZZZZ_p}{2}$$

$$H = H_{Z_2} - \mu \sum_p n_p + V \sum_{<pp'>} n_p n_{p'} - w \sum_{<rr'>} X_{<rr'>}$$

Partition function

- Compute the partition of the full system but ignoring gapped bulk states … *(SSS '13)*

$$Z_{condensed} = \underbrace{\mathrm{tr}\left(\frac{1+(-1)^F}{2}e^{-\beta H_A}\right)}_{\text{Even vortex sector}} + \underbrace{\mathrm{tr}\left(\frac{1+(-1)^F}{2}e^{-\beta H_P}\right)}_{\substack{\text{Odd vortex sector} \\ \text{(assumed gapless)}}}$$

$$Z_{condensed} = q^{-1/3}(1 + 120q + \ldots) + q^{2/3}(128 + \ldots)$$

$$Z_{condensed} = q^{-1/3}(1 + 248q + \ldots)$$

Partition function

- Compute the partition of the full system but ignoring gapped bulk states ... *(SSS '13)*

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Even vortex sector — Odd vortex sector (assumed gapless)

$$Z_{condensed} = q^{-1/3}(1 + 120q + ...) + q^{2/3}(128 + ...)$$

$$Z_{condensed} = q^{-1/3}(1 + 248q + ...)$$

Current algebra

$$SO(16): \boxed{\gamma^i \gamma^j} \qquad \frac{16\times 15}{2} = 120$$

$$\gamma^{2j-1} + i\gamma^{2j} \equiv \psi^j, \ \psi^j = e^{i\theta^j}$$

$$\sigma_q = \boxed{\exp\left(\frac{i}{2}\sum_j (-1)^{q_j}\theta^j\right)}$$

$$\Delta_{\sigma_q} = \frac{1}{2}\sum_j \left(\frac{(-1)^{q_j}}{2}\right)^2 = 1$$

$$2^7 = 128, \text{ fixed } (-1)^F$$

Disorder

$$\mathcal{S}'_{edge} = \mathcal{S}_0 + \int dxdt \left[\gamma^T \delta v(x) i\partial_x \gamma + i\gamma^T M(x)\gamma\right] + ...$$

1. Breaks SO(16) invariance
2. Assume disorder is random and short-range correlated
3. Edge still cannot be gapped (chiral)

What is the IR fate of this theory?

Removing disorder (Kane-Fisher trick)

$$\mathcal{S}'_{edge} = \mathcal{S}_0 + \int dxdt \left[\gamma^T \delta v(x) i\partial_x \gamma + i\gamma^T M(x)\gamma\right] + ...$$

M(x) looks like a static SO(16) gauge potential; can we "gauge" it away?

$$\gamma(x,t) = O(x,t)\tilde{\gamma}(x,t)$$

$$\{O\partial_x O^T, v_0 + \delta v\} = 2M$$

Disorder RG

$$\mathcal{S}'_{edge} = \mathcal{S}_0[\tilde{\gamma}] + \int dxdt\tilde{\gamma}^T \left[O^T\delta v(x)O\right] i\partial_x\tilde{\gamma} + ...$$

Random term with zero average

Look at its disorder averaged square!

$$\int dxdtD(x)\hat{\mathcal{O}}(x,t),\ \overline{D} = 0$$

Disorder RG

$$\int dxdt D(x)\hat{\mathcal{O}}(x,t),\ \overline{D}=0$$

$$\overline{\int dxdt \int dx'dt' D(x)D(x')\hat{\mathcal{O}}(x,t)\hat{\mathcal{O}}(x',t')}$$
$$= \int dxdtdx'dt' \overline{D(x)D(x')}\hat{\mathcal{O}}(x,t)\hat{\mathcal{O}}(x',t')$$
$$= \int dxdtdx'dt' W\delta(x-x')\hat{\mathcal{O}}(x,t)\hat{\mathcal{O}}(x',t')$$
$$= W\int dxdtdt'\hat{\mathcal{O}}(x,t)\hat{\mathcal{O}}(x,t')$$

$$\boxed{\Lambda\frac{dW}{d\Lambda} = (3-2\Delta_{\mathcal{O}})W}$$

(Kane-Fisher '95)

$$\Lambda \frac{dW}{d\Lambda} = (3 - 2\Delta_{\mathcal{O}})W$$

$$\mathcal{S}'_{edge} = \mathcal{S}_0[\tilde{\gamma}] + \int dx dt \tilde{\gamma}^T \left[O^T \delta v(x) O\right] i\partial_x \tilde{\gamma} + ...$$

1. Velocity perturbation irrelevant
2. Interactions are also irrelevant

… thus the SO(16) disordered fixed is stable!

(SSS '13)

Then condense vortices …

- On a finite size circle we want

$$O(x + 2\pi L) = O(x)$$

- We slightly adjust the problem …

$$\{O\partial_x O^T, v_0 + \delta v\} = 2(M - \delta M)$$

- Can still condense vortices, residual small SO(16) gauge field *(SSS '13)*

$$\lim_{L\to\infty} \frac{Z_{dis+cond}}{Z_{E_8}} = 1$$

ONE FINAL PICTURE ...

Disordered E8 WZNW model

$$\mathcal{S} = \int dxdt\, \mathrm{tr}\left(g^{-1}\partial_t g\, g^{-1}\partial_x g\right) + i\mathcal{S}_{WZW} + \ldots$$

Chiral non-linear sigma model
with target space E8 group manifold

$$\ldots = \int dxdt\, \mathrm{tr}(g^{-1}\partial_x g\, M(x)), \text{ etc.}$$

Can we deal with this theory directly?
It would be the most elegant and
symmetric route ...

Removing disorder

$$J(x) = g^{-1}\partial_x g$$

$$[J^a(x), J^b(y)] = ig^{ab}\delta'(x-y) + if^{ab}_c J^c(y)\delta(x-y)$$

$$H = \frac{1}{2}\int dx \mathrm{tr}(J(x)J(x)) + \int dx \mathrm{tr}(J(x)M(x)) + ...$$

1. Redefine currents preserving algebra:

$$J = R\tilde{J} + Q$$

2. Choose Q to remove disorder term
3. Remaining terms have zero average, squares are irrelevant *(SSS '13)*

Recap

I've shown you … *(SSS '13)*

- three ways to construct the E8 state
- and two arguments that, with disorder, the low energy theory has enhanced symmetry

Asymmetry protected emergent E8 symmetry

Some thoughts …

- Bulk-edge correspondence implies that the same symmetry emerges in the "entanglement spectrum"
- Can we have (emergent symmetry) protected phases, e.g. in gapless systems?
- Definite prediction for huge symmetry should this system be realized in the lab

THANKS!