Title: Protected edge modes without symmetry

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Abstract: Some 2D quantum many-body systems with a bulk energy gap support gapless edge modes which are extremely robust. These modes cannot be gapped out or localized by general classes of interactions or disorder at the edge: they are "protected" by the structure of the bulk phase. Examples of this phenomena include quantum Hall states and 2D topological insulators, among others. Recently, much progress has been made in understanding protected edge modes in non-interacting fermion systems. However, less is known about the interacting case. A basic problem is to predict, for general interacting systems, when such edge modes are present or absent, and to identify the different physical mechanisms that underlie their stability. In this talk, I will discuss this problem in the simplest case: interacting fermion systems without any symmetry.

arXiv: 1301.7355
Protected edge modes without symmetry
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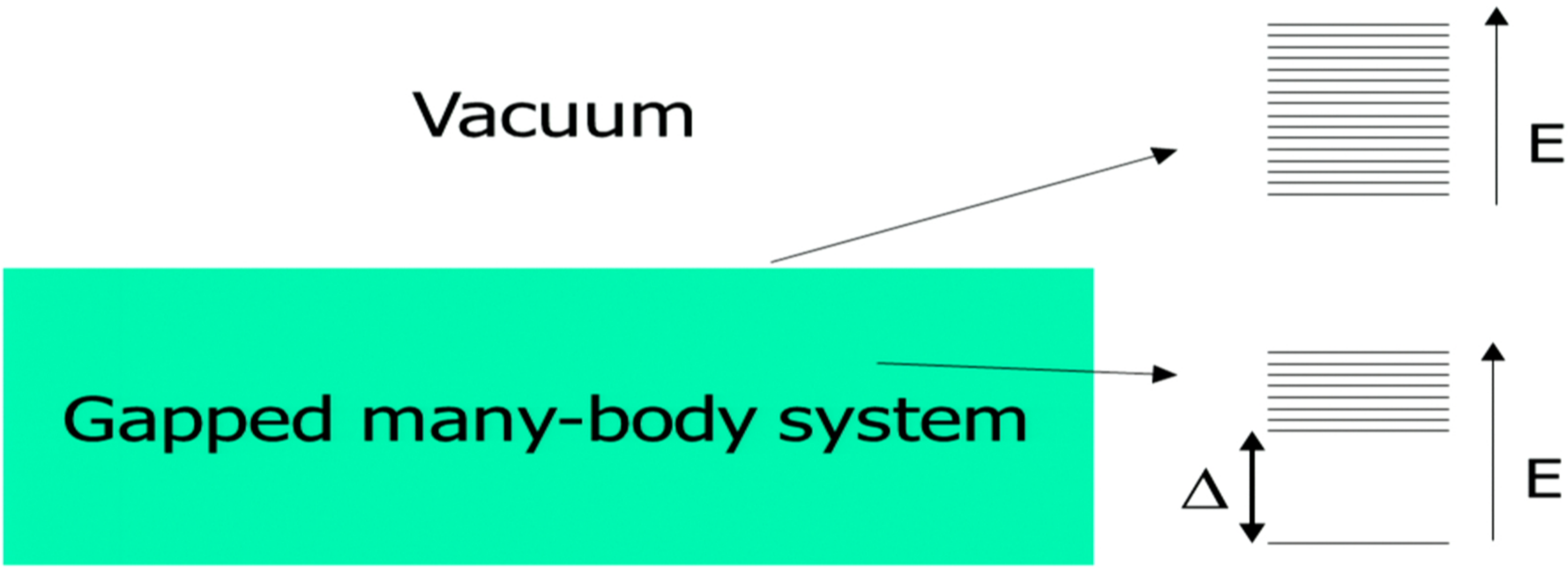
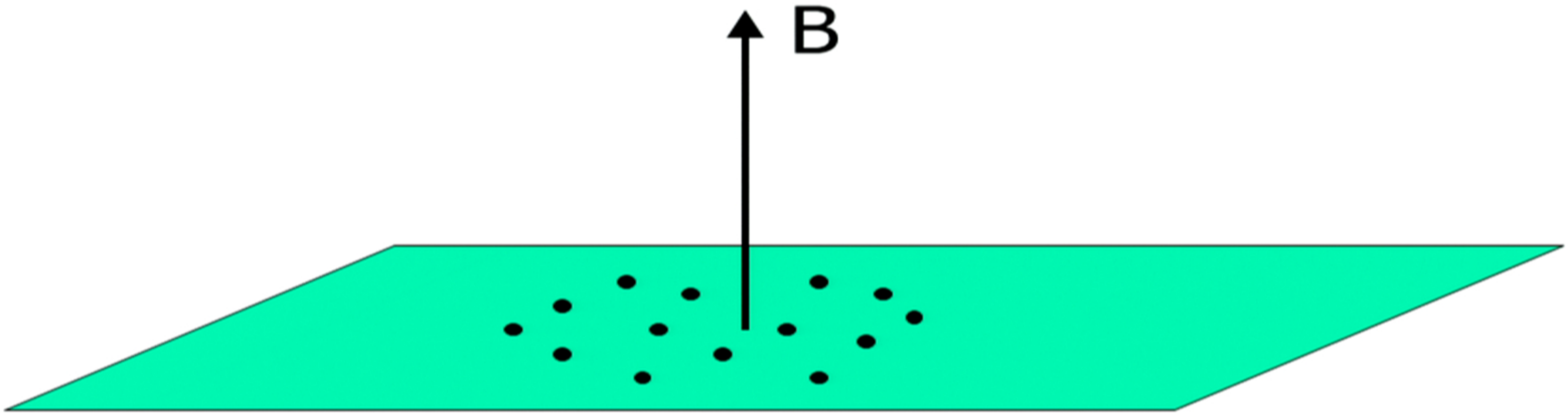
Protected edge modes



Integer quantum Hall states

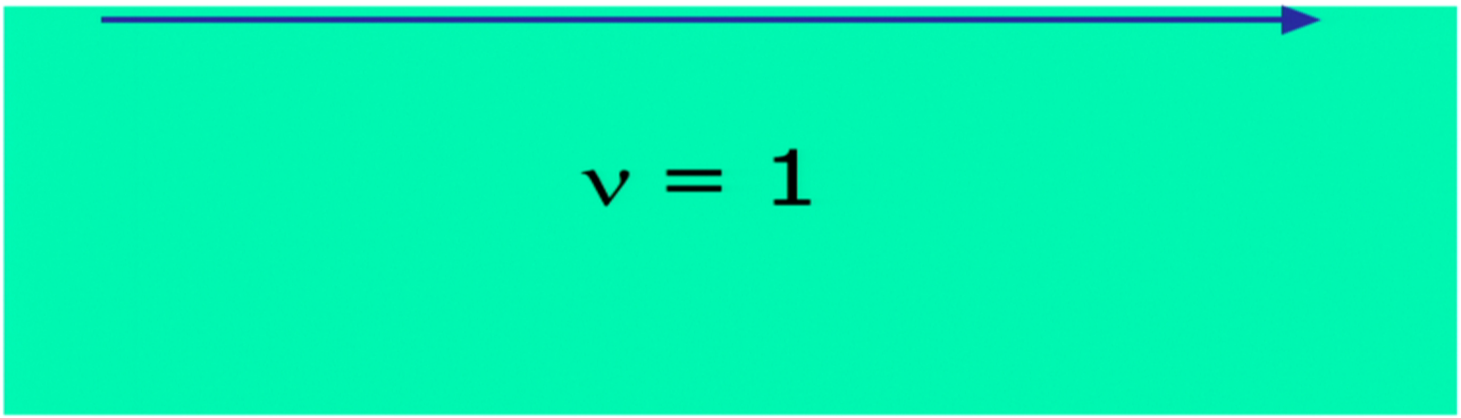
2D electron gas in a uniform magnetic field



$$H = (p - e A)^2/2m$$

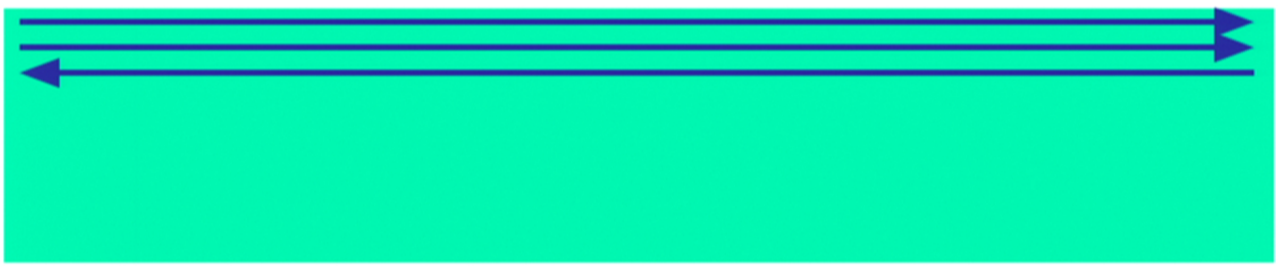
(Halperin, PRB, 1982)

Integer quantum Hall edge



Origin of edge protection

$n_R - n_L \neq 0 \Rightarrow$ protected edge mode



$n_R = 2$
$n_L = 1$

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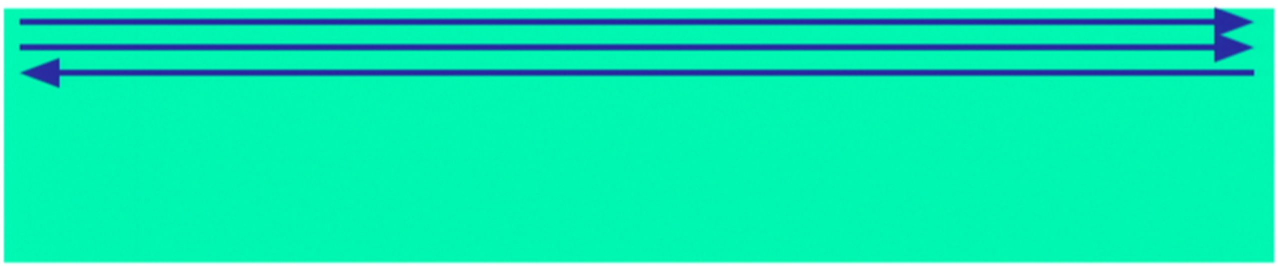


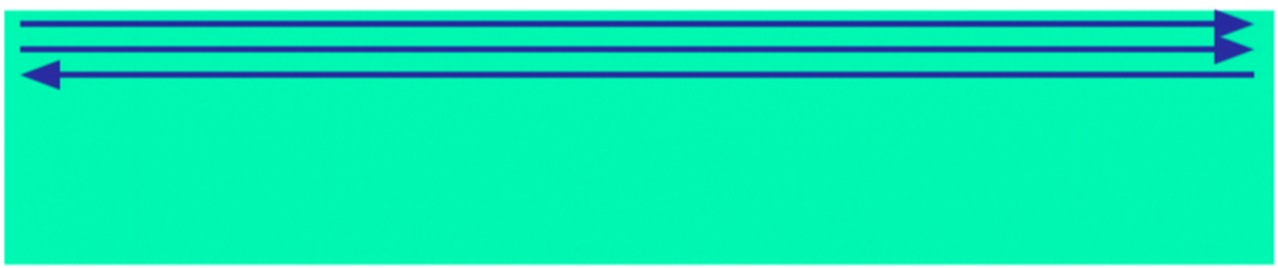
$n_R = 2$
$n_L = 1$

(Kane, Fisher, PRB, 1997)

Origin of edge protection

$n_R - n_L \neq 0 \Rightarrow$ protected edge mode

$\sim K_H$, "Thermal Hall conductance"



$n_R = 2$
$n_L = 1$

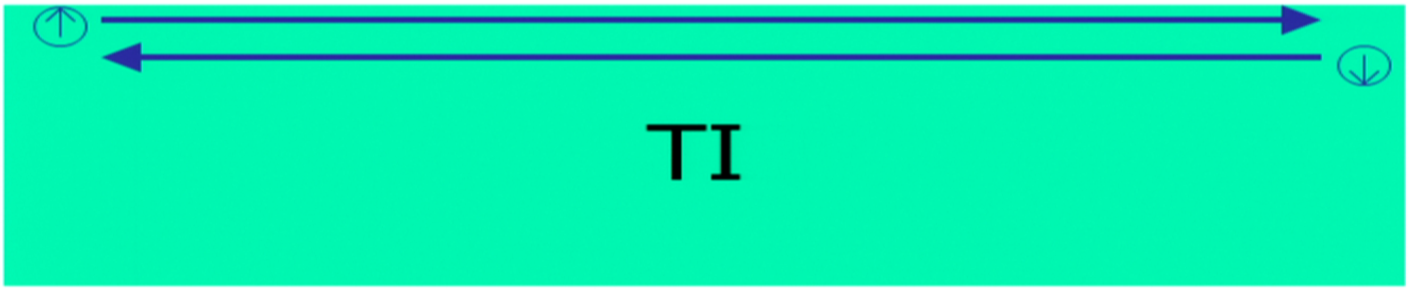
(Kane, Mele, PRL, 2005)

2D Topological insulators

- Band insulators
- Time reversal invariant
- Qualitatively different from conventional ins.
- Have protected edge modes

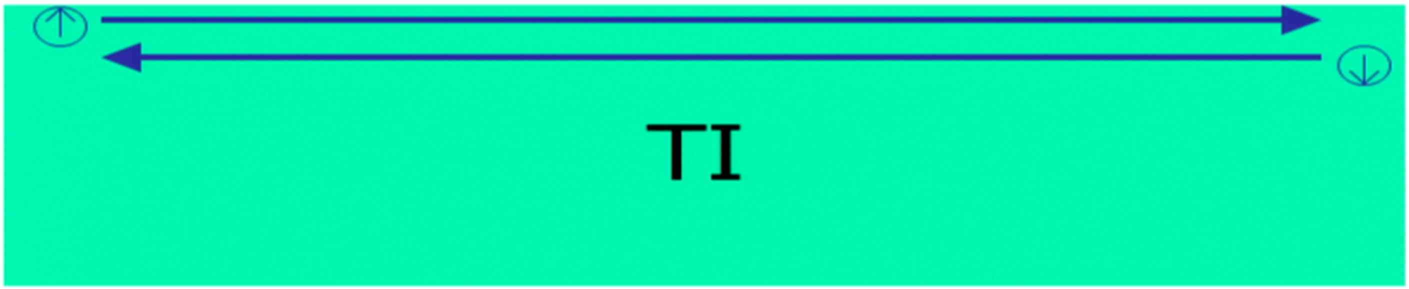
(Kane, Mele, PRL, 2005)

Topological insulator edge



(Kane, Mele, PRL, 2005)

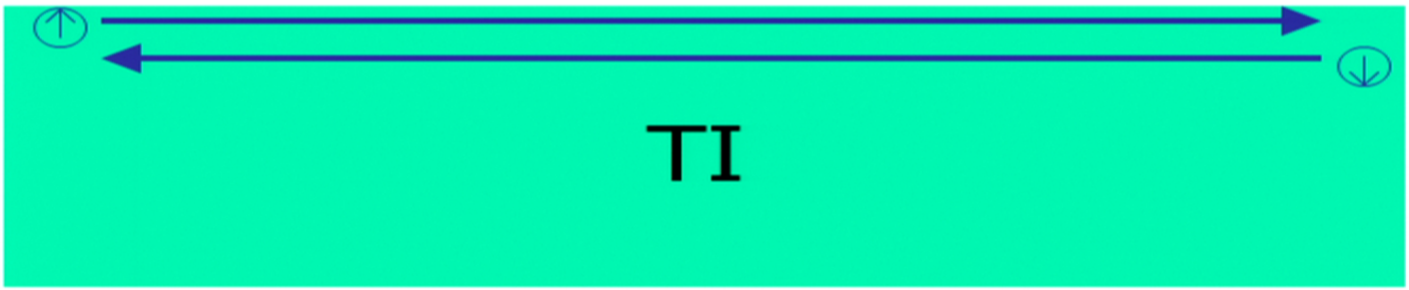
Topological insulator edge



Why is edge protected?

(Kane, Mele, PRL, 2005)

Topological insulator edge

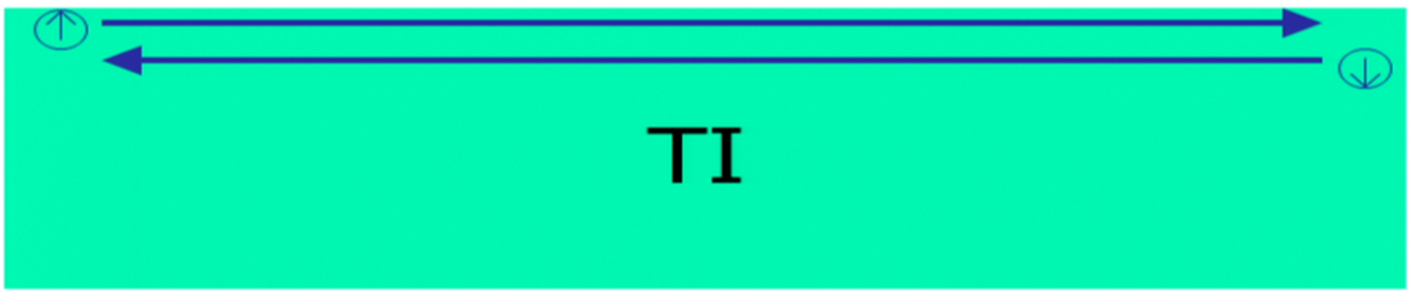


Why is edge protected?

$$H' = U\, c^{*}_{\uparrow}\, c_{\downarrow} + h.c$$

(Kane, Mele, PRL, 2005)

Topological insulator edge



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Prohibited by time reversal symmetry

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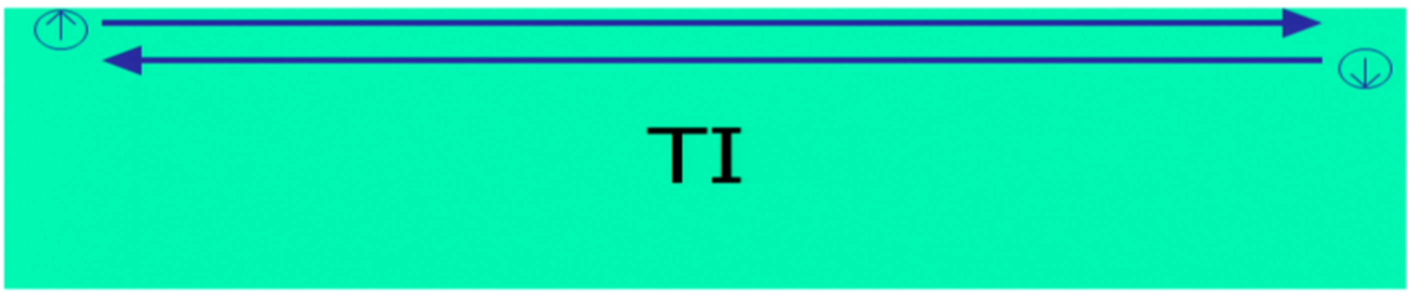

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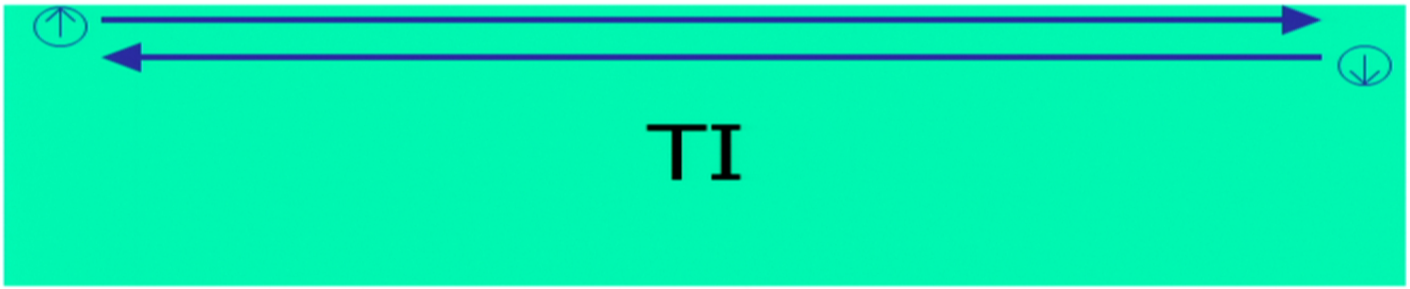
Prohibited by time reversal symmetry

$H' = \Delta\, c_{\uparrow}\, c_{\downarrow} + h.c$

(Kane, Mele, PRL, 2005)

Topological insulator edge



Why is edge protected?

$$H' = U\, c^*_{\uparrow}\, c_{\downarrow} + h.c$$

Prohibited by time reversal symmetry

$$H' = \Delta\, c_{\uparrow}\, c_{\downarrow} + h.c$$

Prohibited by charge conserv. symmetry

Edge protection mechanisms

1. $n_R - n_L \neq 0$ (e.g. IQH states)

2. Symmetry (e.g. topological insulators)

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2. Symmetry (e.g. topological insulators)

Are there other mechanisms?

Edge protection mechanisms

Can a system with no symmetry and with $n_R = n_L$ have a protected edge?

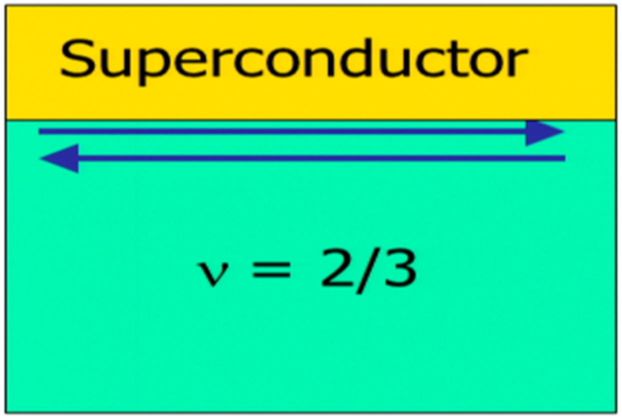
Edge protection mechanisms

Can a system with no symmetry and with $n_R = n_L$ have a protected edge?

Yes! This happens in (some) "fractionalized" systems

(ML, arXiv:1301.7355)

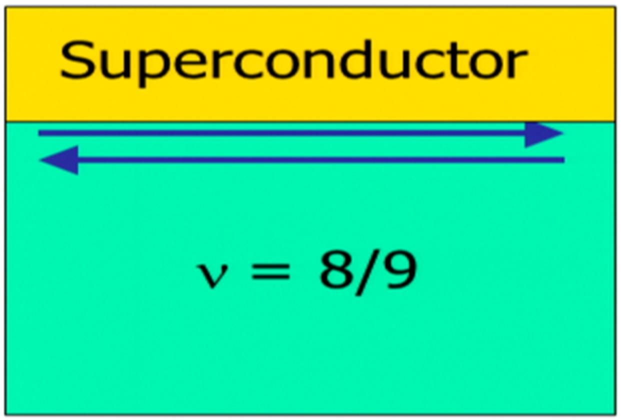
Examples



$$n_R - n_L = 0$$

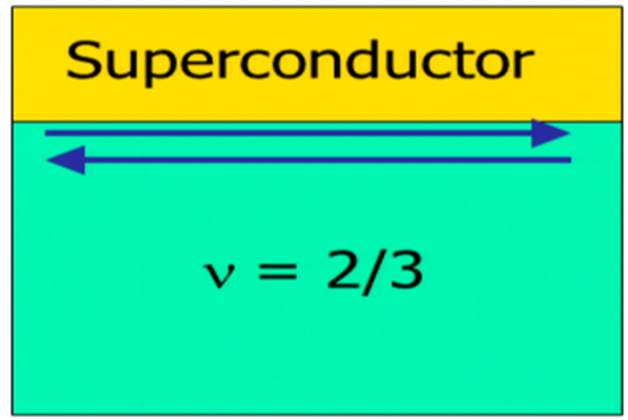
Protected edge

Examples



$n_R - n_L = 0$

No protected edge

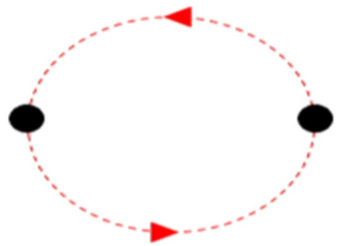


$n_R - n_L = 0$

Protected edge

Fractional statistics

$e^{i\theta} \neq \pm 1$



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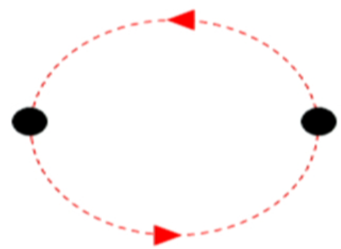
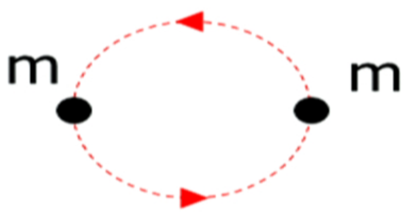
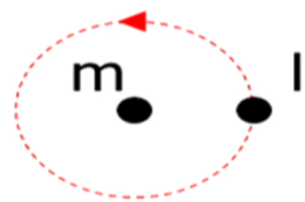


To fully characterize (abelian) fractional statistics, specify:



$e^{i\theta_m}$



$e^{i\theta_{lm}}$

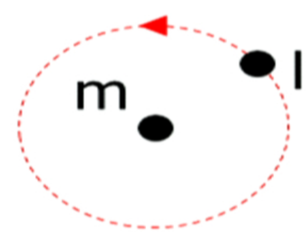
General criterion

A system with $n_R = n_L$ can have a gapped edge iff there exists a subset of quasiparticles $\mathbf{M} = \{\mathbf{m_1}, \mathbf{m_2}, \ldots\}$ satisfying:

(1) $e^{i\theta_{\mathbf{mm'}}} = 1$ for any $\mathbf{m}, \mathbf{m'} \in \mathbf{M}$

(2) If $\mathbf{l} \notin \mathbf{M}$, then there exists $\mathbf{m} \in \mathbf{M}$ with $e^{i\theta_{\mathbf{lm}}} \neq 1$

$e^{i\theta_{\mathbf{lm}}}$



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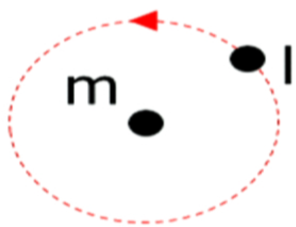
"Lagrangian subgroup"

$e^{i\theta_{\mathbf{lm}}}$



Examples

$\underline{\nu = 8/9:}$ Quasiparticle types = {0, e/9, 2e/9,..., 8e/9}

M = {0, 3e/9, 6e/9} is a Lagrangian subgroup

$\Rightarrow$ edge can be gapped

$\underline{\nu = 2/3:}$ Quasiparticle types = {0, e/3, 2e/3}

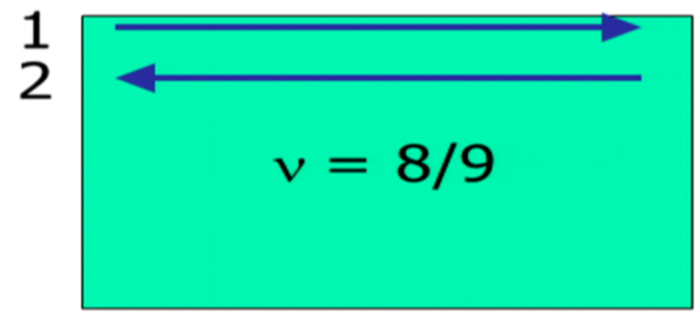
No Lagrangian subgroup **M**

$\Rightarrow$ edge is protected

Microscopic analysis: $\nu = 8/9$

$$L = 1/4\pi\ \partial_x\phi_1(\partial_t\phi_1 - v_1\partial_x\phi_1)$$

$$+1/4\pi\ \partial_x\phi_2(-9\partial_t\phi_2 - v_2\partial_x\phi_2)$$



Electron operators: $\psi_1 = e^{i\phi_1}$, $\psi_2 = e^{-9i\phi_2}$

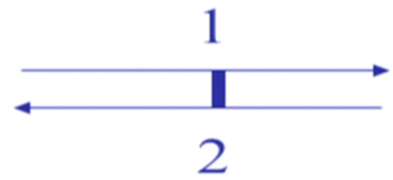
Microscopic analysis: $\nu = 8/9$

$$L = 1/4\pi \, \partial_x \Phi^T \left(\mathbf{K} \, \partial_t \Phi - \mathbf{V} \, \partial_x \Phi \right)$$

$$\Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \quad \mathbf{K} = \begin{pmatrix} 1 & 0 \\ 0 & -9 \end{pmatrix} \quad \mathbf{V} = \begin{pmatrix} v_1 & 0 \\ 0 & v_2 \end{pmatrix}$$

Microscopic analysis: $\nu = 8/9$

Simplest scattering terms:



$$\text{“}U\ \psi_1^p\ \psi_2^q + \text{h.c.”} = U\ \text{Cos}(p\phi_1 - 9q\phi_2)$$

Will this term gap the edge?

(Haldane, PRL, 1995)

Null vector criterion

Can gap the edge if

$$(p\ q) \begin{pmatrix} 1 & 0 \\ 0 & -9 \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} = 0$$

Guarantees that we can change variables to $\theta = p\phi_1 - 9q\phi_2$, $\phi = q\phi_1 + p\phi_2$ with:

$$L \sim \partial_x\phi\ \partial_t\theta - v^2 (\partial_x\phi)^2 - v^2 (\partial_x\theta)^2 + U \cos(\theta)$$

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Null vector criterion

$$(p\ q) \begin{pmatrix} 1 & 0 \\ 0 & -9 \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} = 0$$

$$p^2 - 9\,q^2 = 0$$

Solution: $(p\ q) = (3\ {-1})$

$\Rightarrow$ $U \cos(3\phi_1 + 9\phi_2)$ can gap edge.

Null vector criterion: $\nu = 2/3$

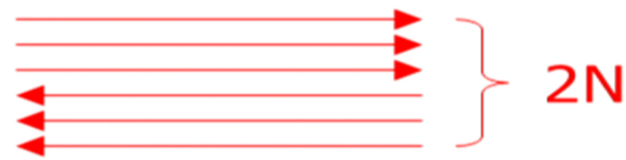
$$(p\ q) \begin{pmatrix} 1 & 0 \\ 0 & -3 \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} = 0$$

$$\leftrightarrow p^2 - 3\,q^2 = 0$$

No integer solutions.

$\Rightarrow$ (simple) scattering terms cannot gap edge!

General case

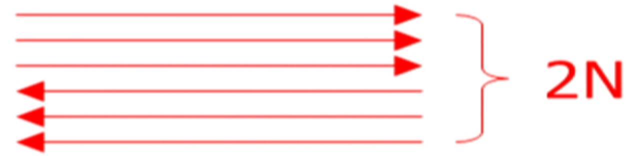


Edge can be gapped iff there exist $\{\Lambda_1,\ldots,\Lambda_N\}$ satisfying:

$$\Lambda_i^T \mathbf{K} \Lambda_j = 0 \qquad \text{for all } i,j \qquad \mathbf{(*)}$$

Can show **(*)** has a solution iff there is a Lagrangian subgroup **M**

General case



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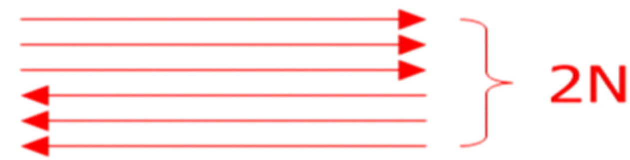
Problems with derivation

1. Only considered simplest kind of backscattering terms

$\Rightarrow$ proof that $\nu = 2/3$ edge is protected is not complete

2. Physical interpretation is unclear

General case



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Annihilating particles at a gapped edge

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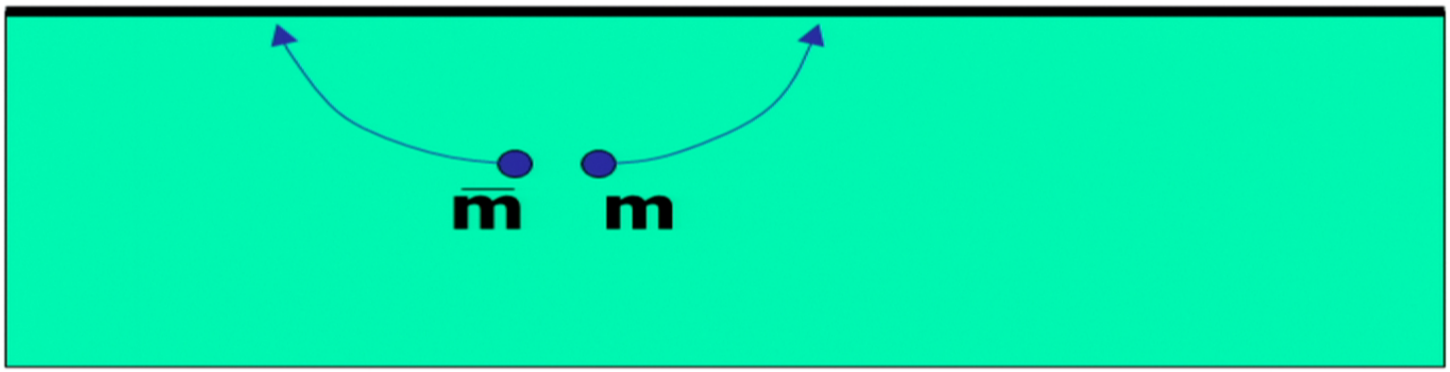
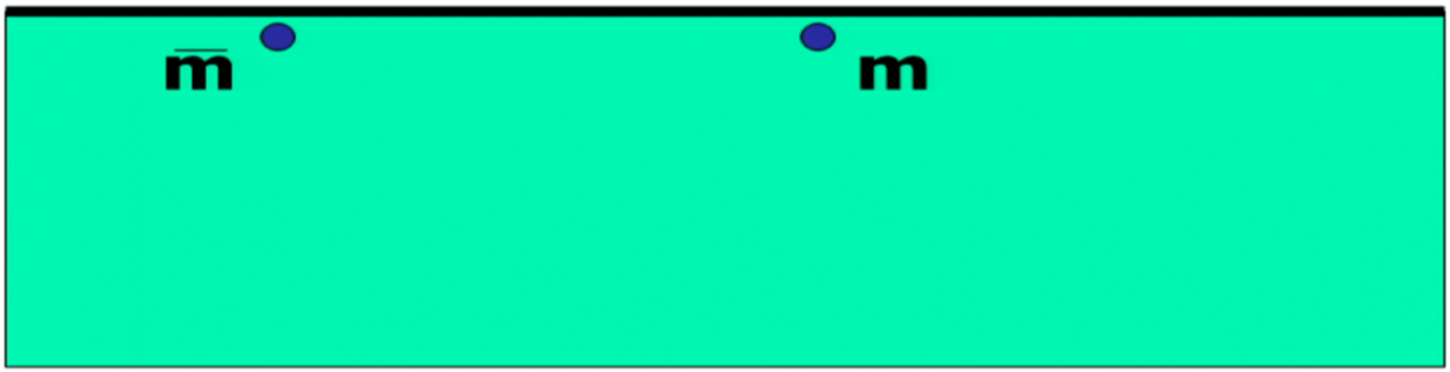


Annihilating particles at a gapped edge



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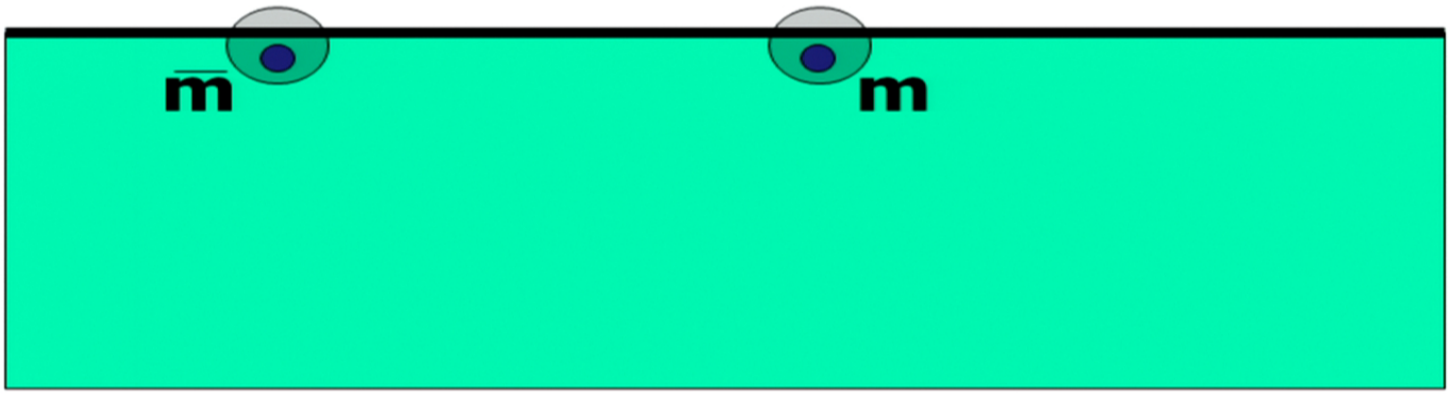

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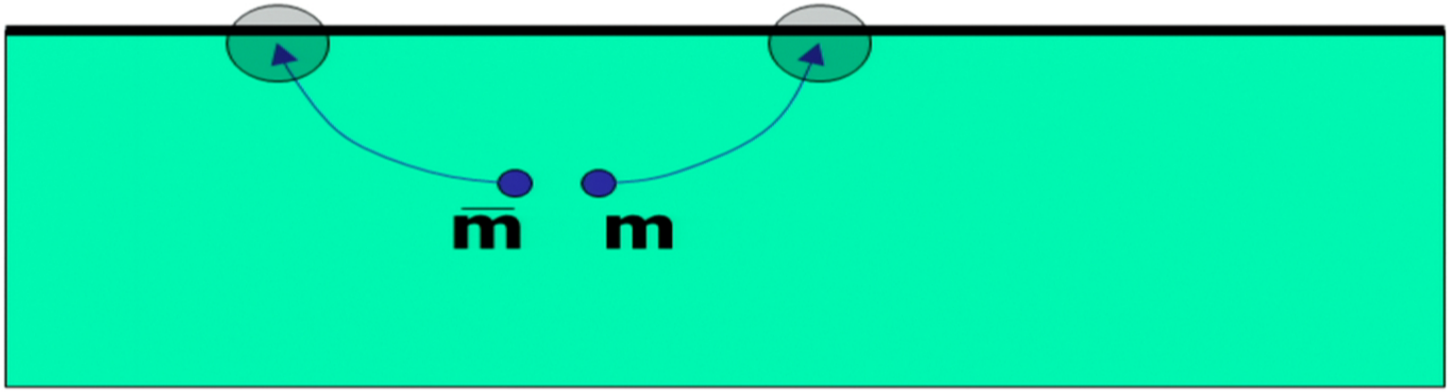
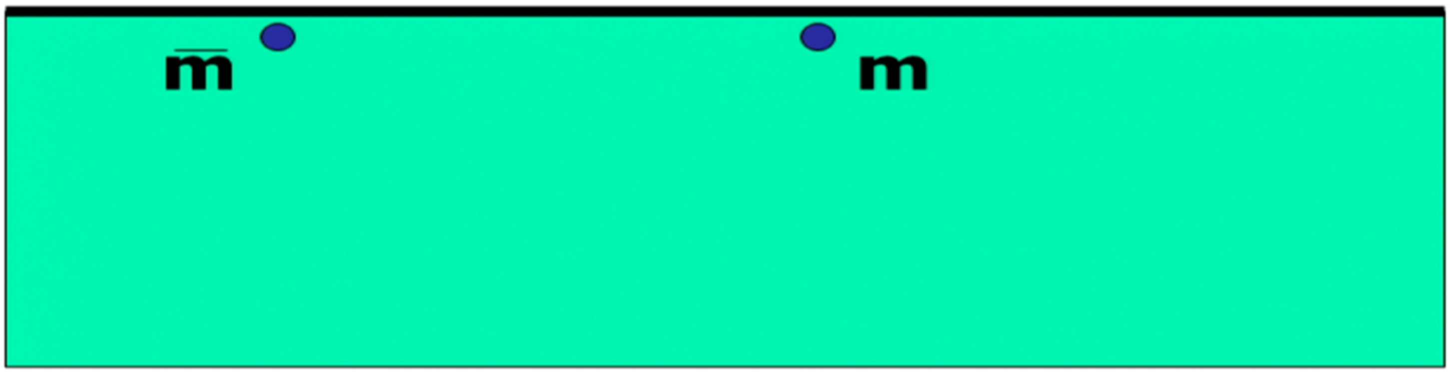


"**m**, $\bar{\mathbf{m}}$ can be annihilated at the edge"

Annihilating particles at a gapped edge



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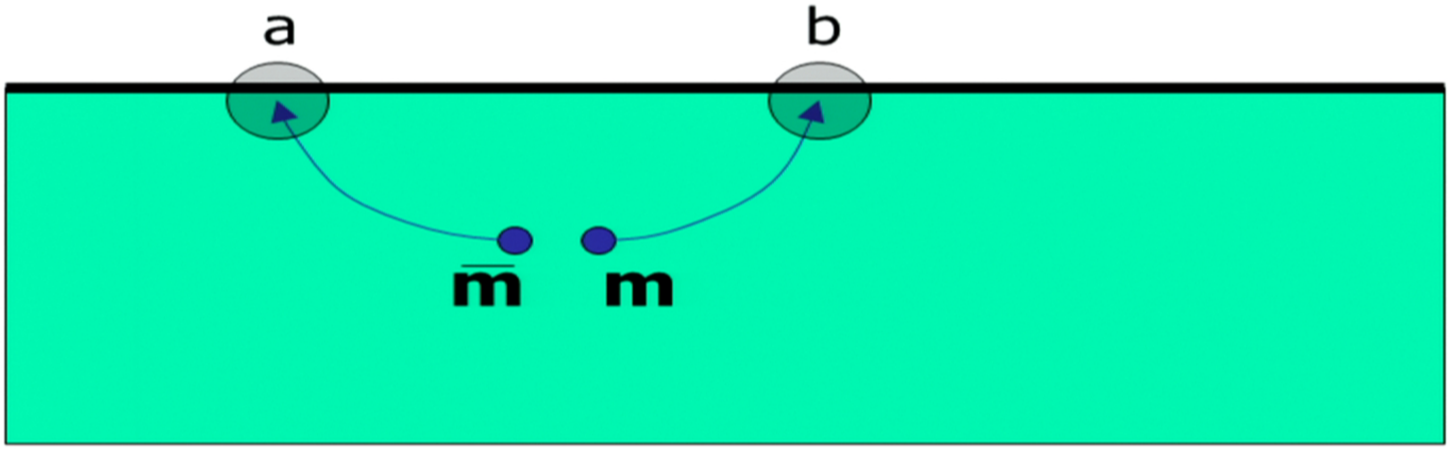


"**m**, $\bar{\mathbf{m}}$ can be annihilated at the edge"

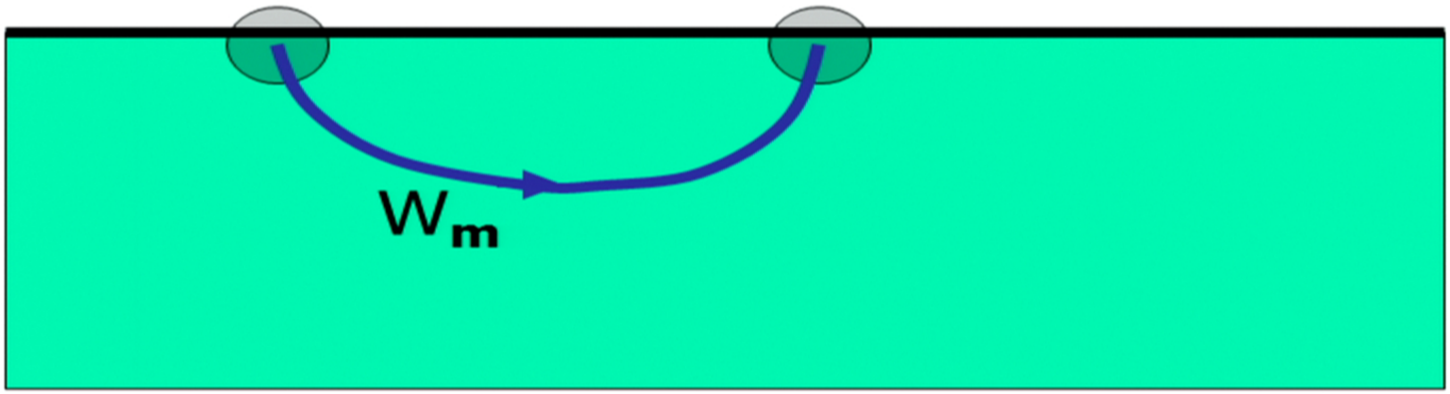
Annihilating particles at a gapped edge

Define:

$$\mathbf{M} = \{\mathbf{m} : \mathbf{m} \text{ can be annihilated at edge}\}$$

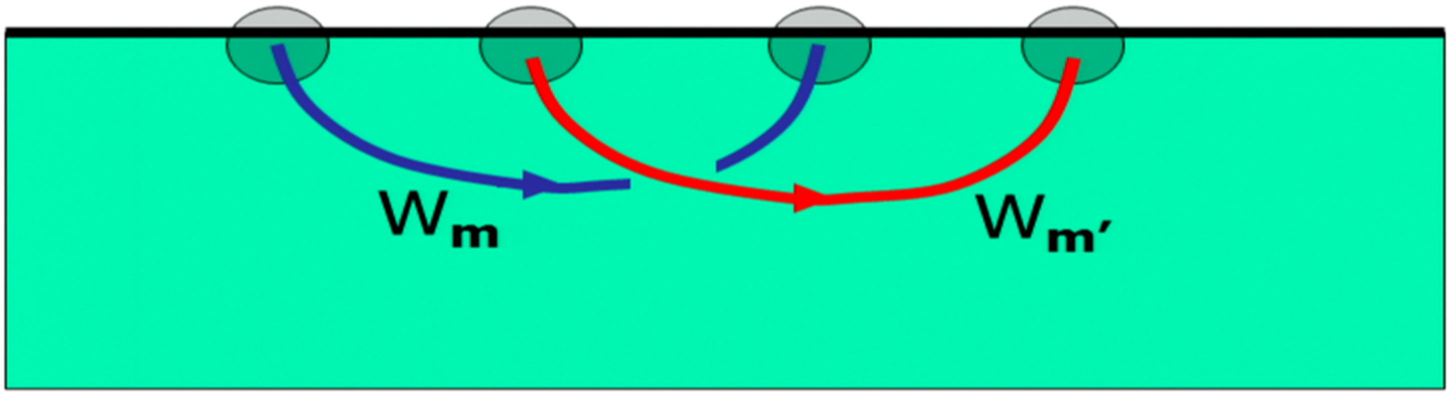
Claim: **M** is a Lagrangian subgroup

Constraints from fractional statistics



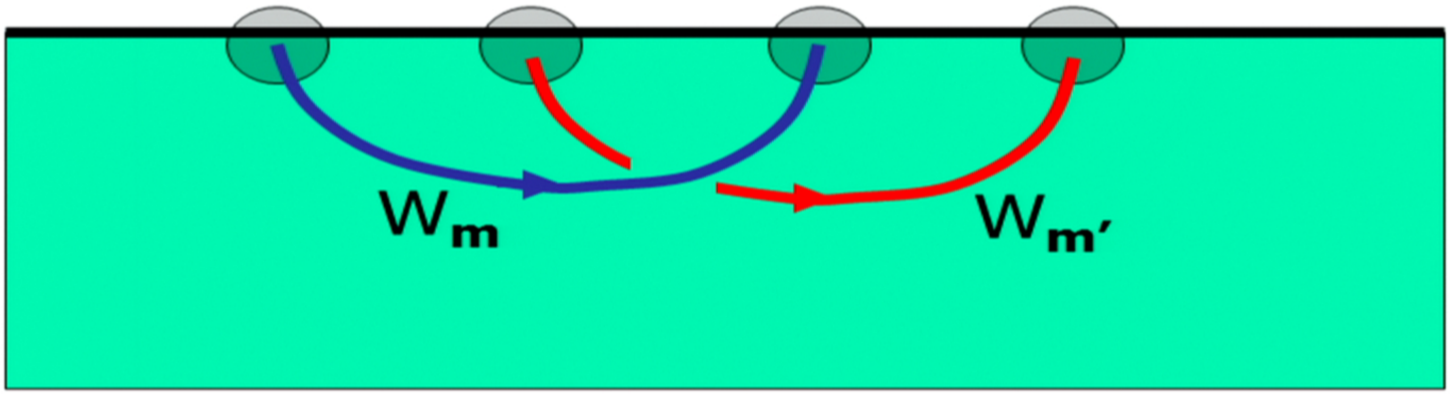
$$W_{\mathbf{m}} |0\rangle = |0\rangle$$

Constraints from fractional statistics



$$W_{\mathbf{m'}} W_{\mathbf{m}} |0\rangle = |0\rangle$$

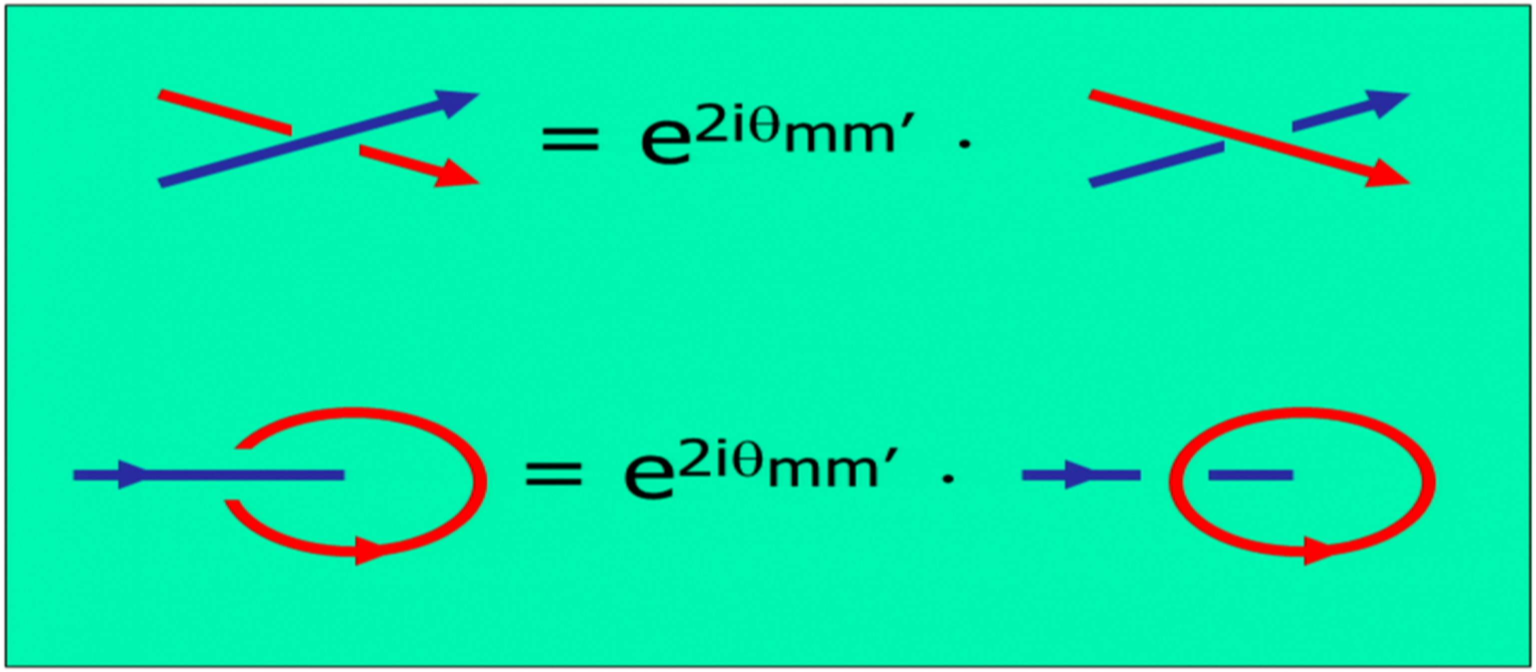
Constraints from fractional statistics



$$W_m W_{m'} |0\rangle = |0\rangle$$

$$\Rightarrow W_m W_{m'} |0\rangle = W_{m'} W_m |0\rangle$$

String operator algebra



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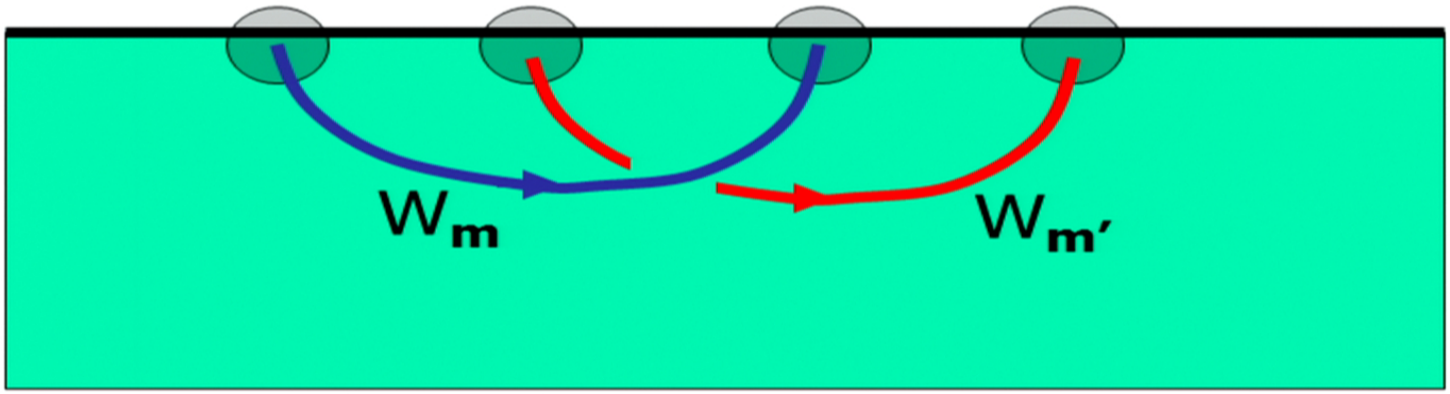
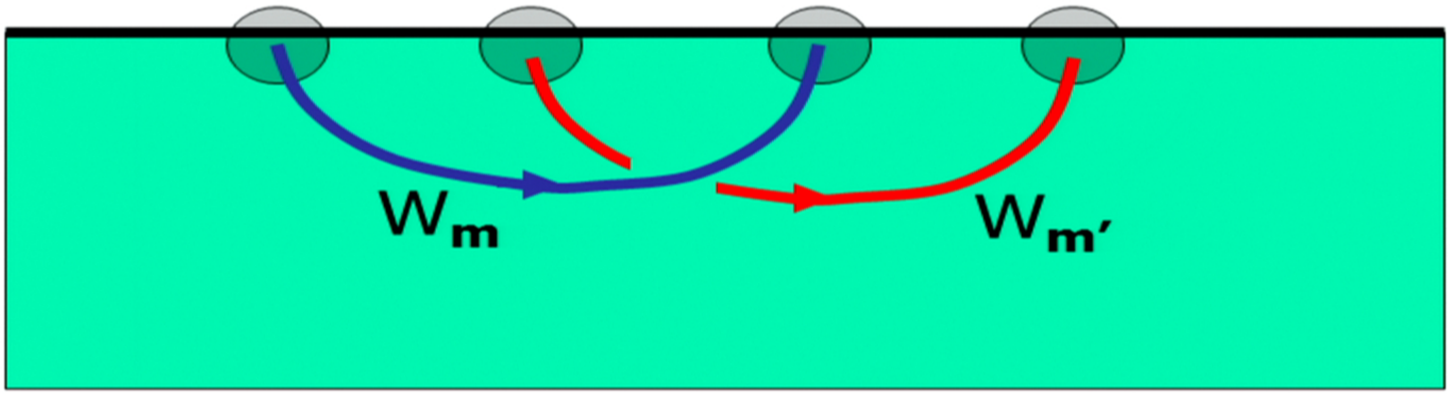


$$\Rightarrow W_{\mathbf{m}} W_{\mathbf{m'}} |0\rangle = e^{i\theta_{\mathbf{mm'}}} W_{\mathbf{m'}} W_{\mathbf{m}} |0\rangle$$

$$\therefore \; e^{i\theta_{mm'}} = 1 \text{ for all } \mathbf{m}, \mathbf{m'} \in \mathbf{M}$$

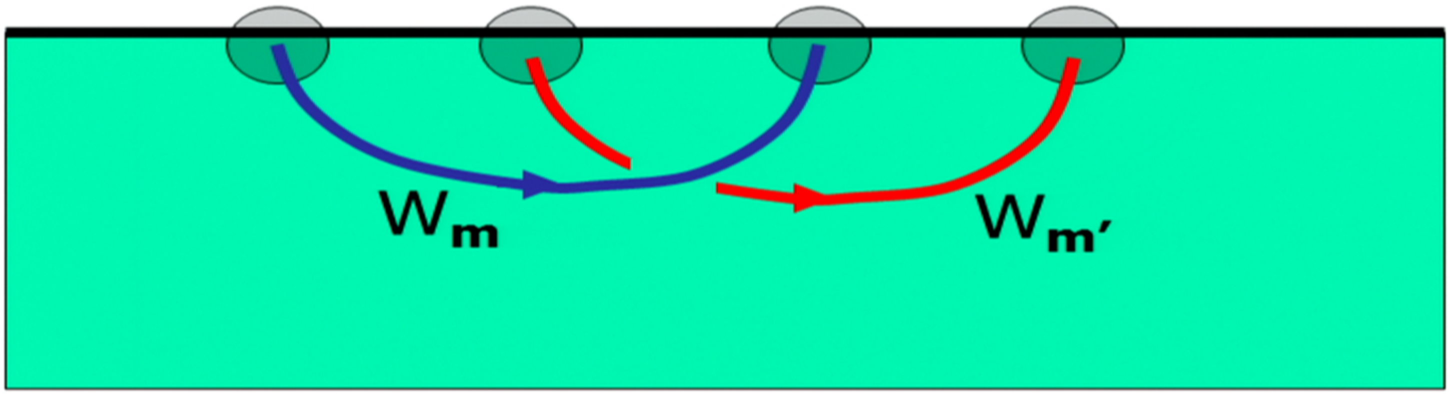
Constraints from fractional statistics



$$W_{\mathbf{m}} W_{\mathbf{m}'} |0\rangle = |0\rangle$$

$$\Rightarrow \quad W_{\mathbf{m}} W_{\mathbf{m}'} |0\rangle = W_{\mathbf{m}'} W_{\mathbf{m}} |0\rangle$$

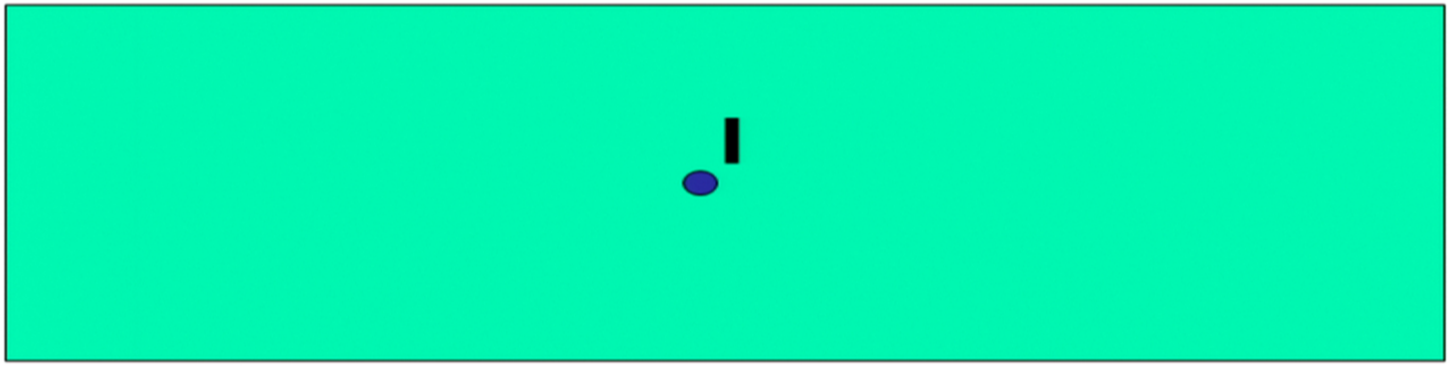
Constraints from fractional statistics



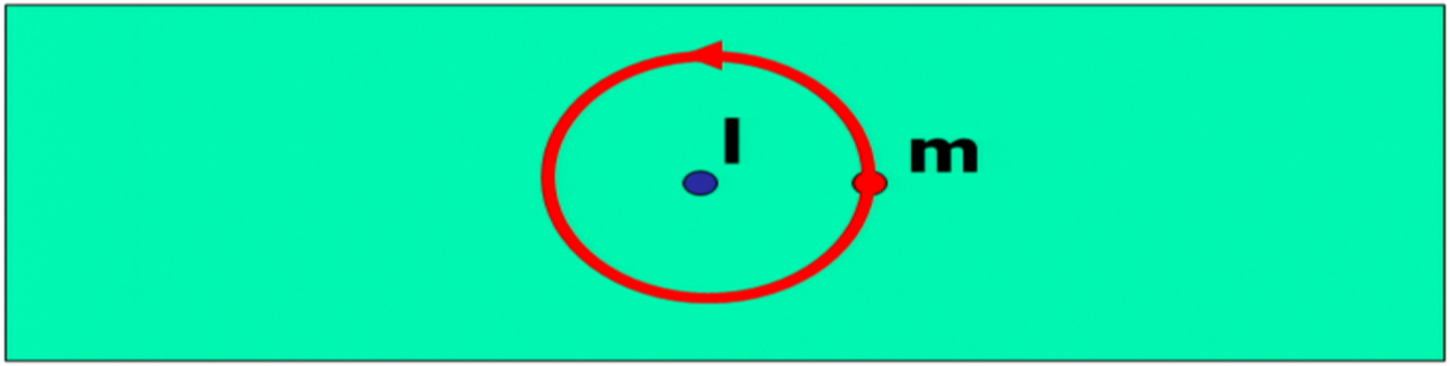
$$\Rightarrow W_{\mathbf{m}} W_{\mathbf{m'}} |0\rangle = e^{i\theta_{\mathbf{mm'}}} W_{\mathbf{m'}} W_{\mathbf{m}} |0\rangle$$

$\therefore \; e^{i\theta_{mm'}} = 1$ for all $\mathbf{m}, \mathbf{m'} \in \mathbf{M}$

Braiding non-degeneracy in bulk

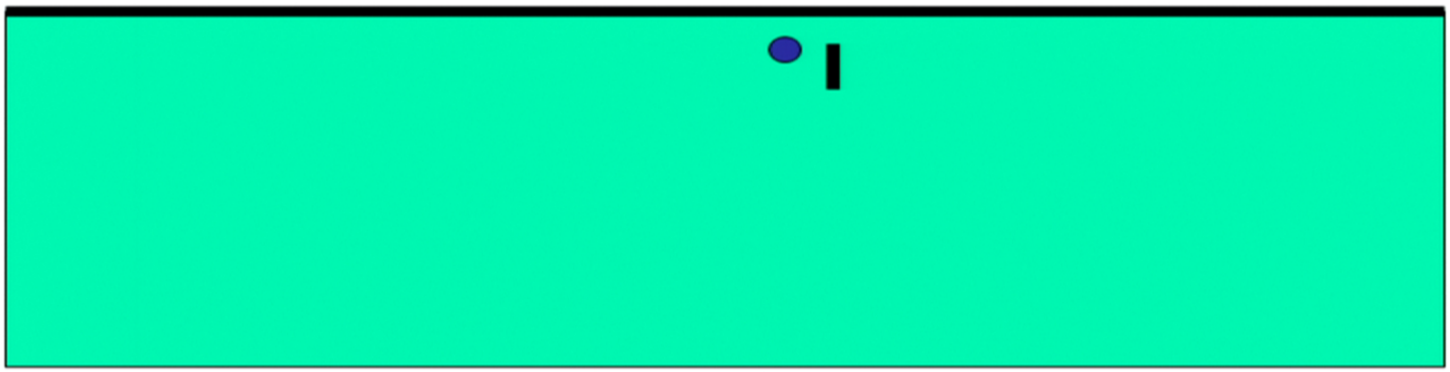


Braiding non-degeneracy in bulk



If **l** can't be annihilated (in bulk) then there exists **m** with $e^{i\theta_{\mathbf{lm}}} \neq 1$

Braiding non-degeneracy at a gapped edge



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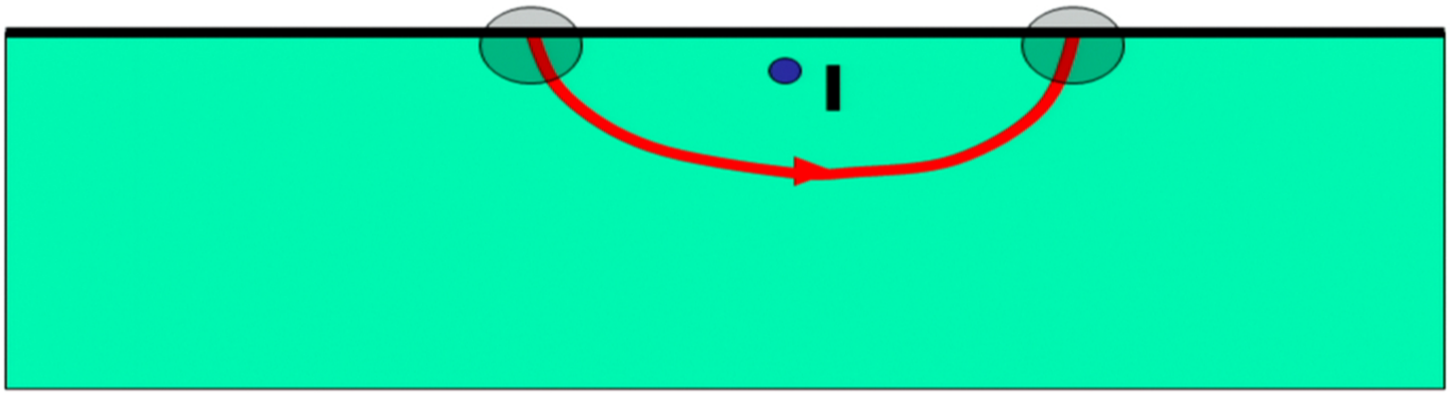
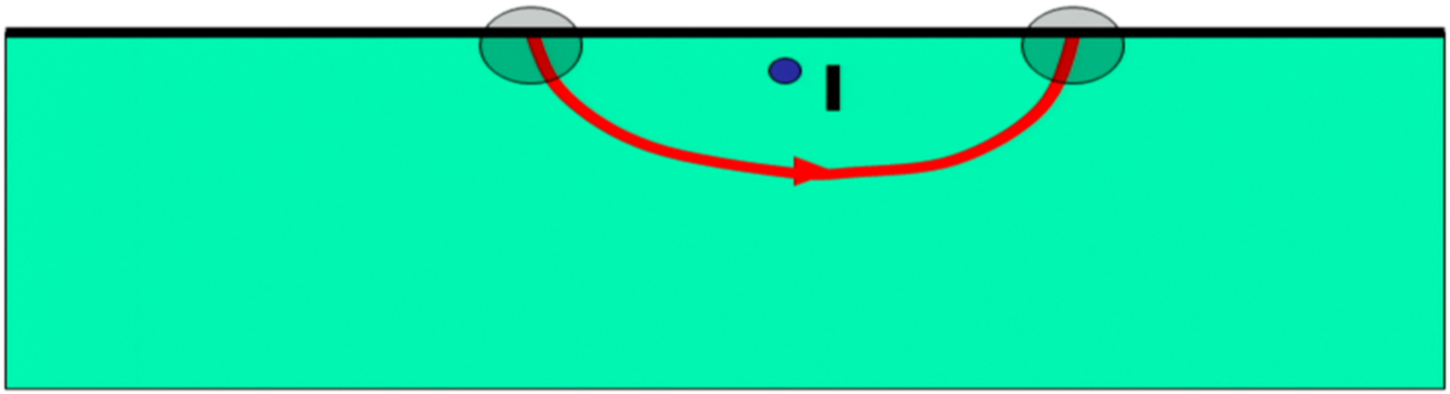


If **l** can't be annihilated at edge then there exists **m** which CAN be annihilated at edge with $e^{i\theta_{lm}} \neq 1$

Braiding non-degeneracy at a gapped edge



If **l** can't be annihilated at edge then there exists **m** which CAN be annihilated at edge with $e^{i\theta_{lm}} \neq 1$

$\Rightarrow$ **M** satisfies condition (2)

Summary

- Systems with $n_L = n_R$ and no symmetry can have a protected edge
- Edge protection originates from structure of fractional statistics in bulk
- Derived general criterion for when a system with abelian statistics has a protected edge