Title: Topological Order with a Twist: Ising Anyons from an Abelian Model

Date: May 06, 2013 11:45 AM

URL: http://pirsa.org/13050024

Abstract: Anyon models can be symmetric under some permutations of their topological charges. One can then conceive topological defects that, under monodromy, transform anyons according to a symmetry. We study the realization of such defects in the toric code model, showing that a process where defects are braided and fused has the same outcome as if they were Ising anyons.

PRL 105.030403 / arXiv:1004.1838
NJP 13.043005 / arXiv:1006.5260

Topological Order with a Twist

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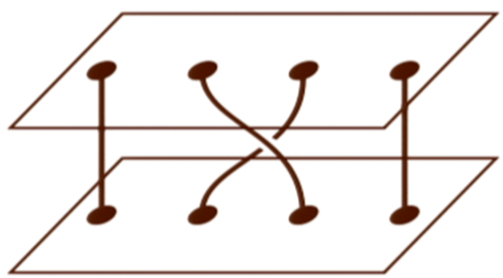
Monday, 6 May, 13

motivation

- TO: gapped, topological GS degeneracy
  - local indistinguishability
  - immune to local distortion
- good quantum memories: TQECC (no Hamiltonian)
- TQC: non-abelian anyon braiding



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motivation

- other option: gapped boundaries
- sometimes not enough...

- twists: defects from anyon symmetries

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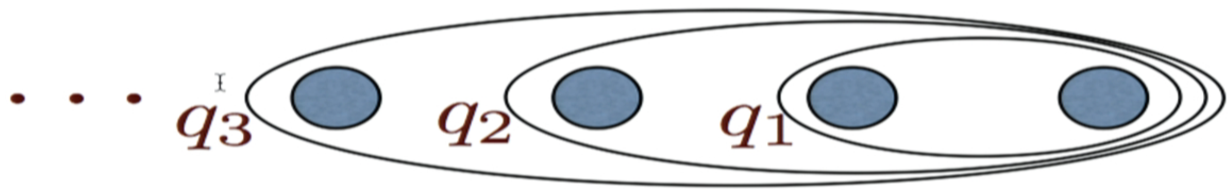
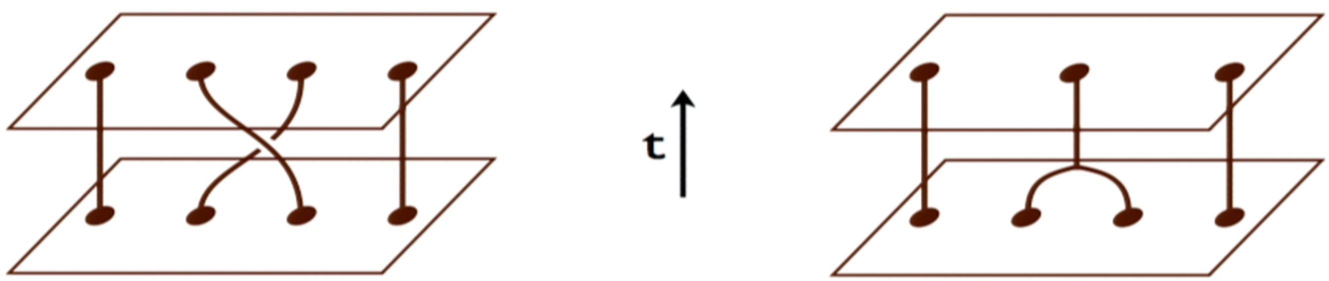
anyons

- TQC (Kitaev '03, Freedman et al '03)

encode in fusion channels



compute = braid measure = fuse

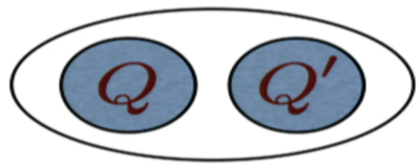


anyons

- ingredients of an anyon model:

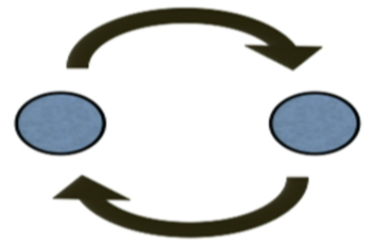
$Q \in \{a, b, \ldots\}$

topological charges



$Q \times Q' = q_1 + q_2 + \ldots$

fusion rules

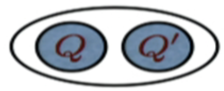


$|\psi\rangle \longrightarrow\ ?$

braiding rules

anyons

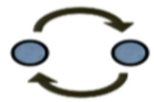
- ex.: **Ising anyons**
  - topological charges 

 $\{\mathbf{1}, \sigma, \psi\}$
  - fusion rules 

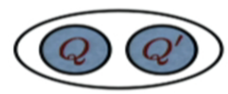
$$\sigma \times \sigma = \mathbf{1} + \psi, \qquad \sigma \times \psi = \sigma, \qquad \psi \times \psi = \mathbf{1}.$$

- the total charge of two distant σ-s is $\mathbf{1}$ or ψ:
  - if far appart, global qubit
- fusion space: $2n$ σ-s $\rightarrow$ n qubits

anyons

- braiding rules: 
  - we can describe braiding up to a phase with a **Majorana operator** per σ
    - Majorana operators are self-adjoint c_i with

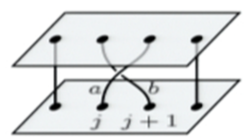
$$c_j c_k + c_k c_j = 2\delta_{jk}$$

    - total charge of j-th and j+1-th σ-s: 

$$-ic_j c_{j+1}$$

    - braiding: $c_j \longrightarrow c_{j+1}$

$$c_{j+1} \longrightarrow -c_j$$



  - not universal, but we can use distillation (Bravyi '06)

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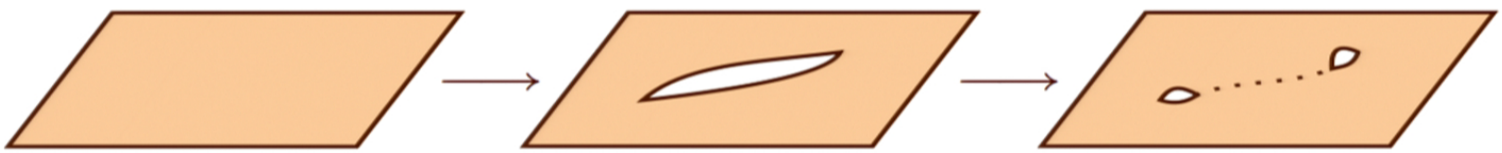
twists

- **anyon symmetry**: charge permutation leaving the anyon model invariant

$$q \longrightarrow \pi(q)$$

- imagine ‘cutting’ the anyons’ 2D world and gluing it again up to a symmetry



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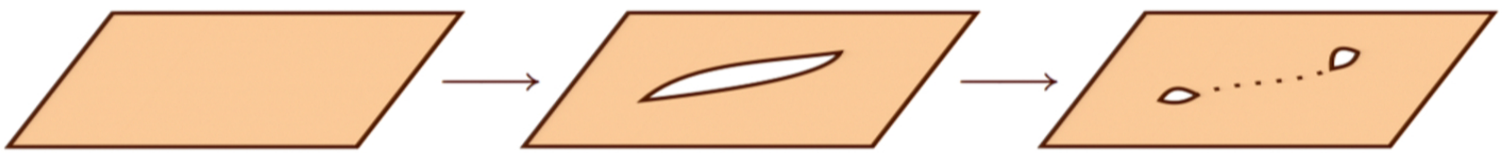

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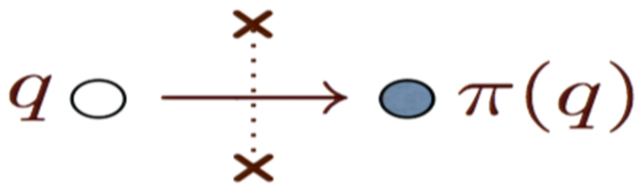
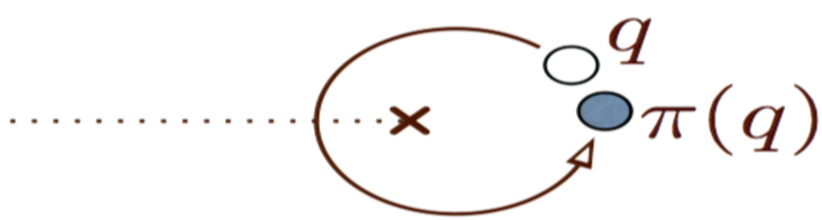
twists

- across the cut, charges change:



- topologically, the cut location is unphysical.
- endpoints are meaningful: under monodromy they permute charges → **twists**



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twists

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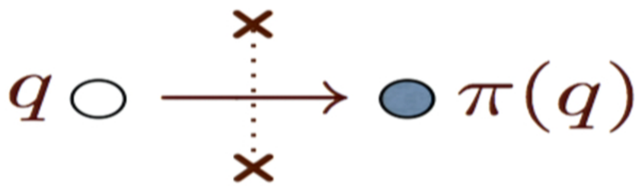

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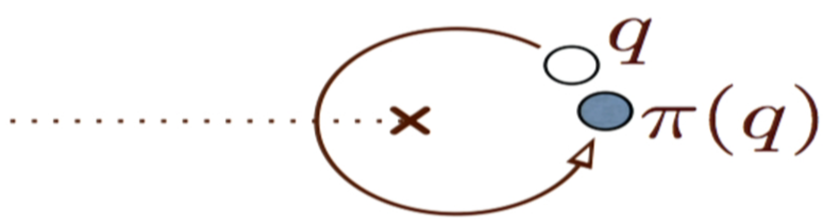

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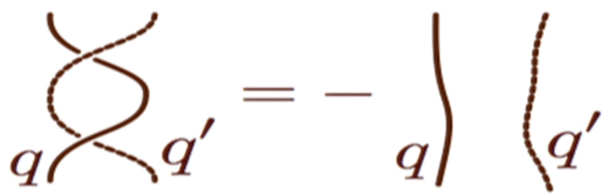
twists

- ex.: quantum double of Z_2 (**toric code**)
- charges: $\{\mathbf{1}, e, m, \epsilon\}$
- fusion: $e \times m = \epsilon \qquad e \times \epsilon = m \qquad m \times \epsilon = e$

$$e \times e = m \times m = \epsilon \times \epsilon = \mathbf{1}$$

- braiding: $e, m \longrightarrow \text{bosons} \qquad \epsilon \longrightarrow \text{fermion}$

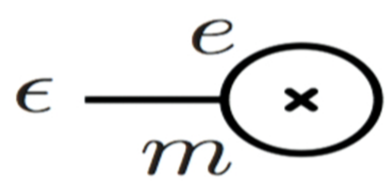


$$\mathbf{1} \neq q \neq q' \neq \mathbf{1}$$

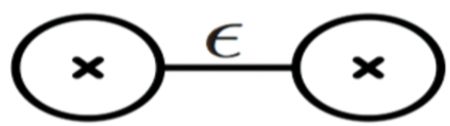
- nontrivial **symmetry**: $e \leftrightarrow m$

twists

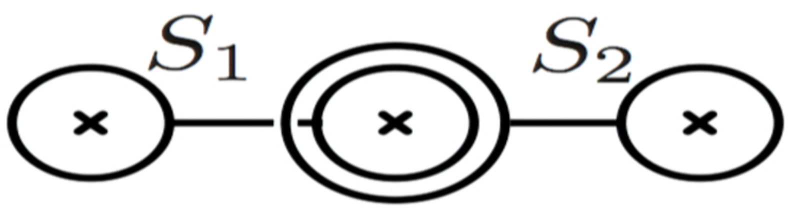
- twists are sinks/sources for fermions:



- vacuum to vacuum processes...



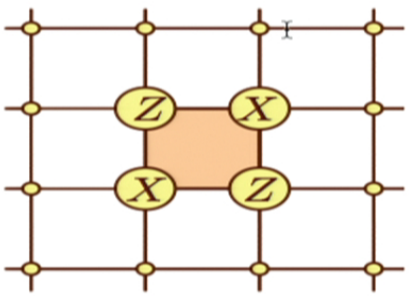
- ...lead to **topological degeneracy**:



$$S_1 S_2 = -S_2 S_1$$

toric code

- toric code (Kitaev '97, Wen '03):
  - qubits form a square lattice
  - 4-local check operators at plaquettes



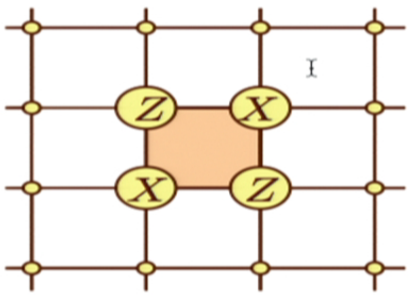
$$C_k := X_k Z_{k+\mathbf{i}} X_{k+\mathbf{i}+\mathbf{j}} Z_{k+\mathbf{j}}$$

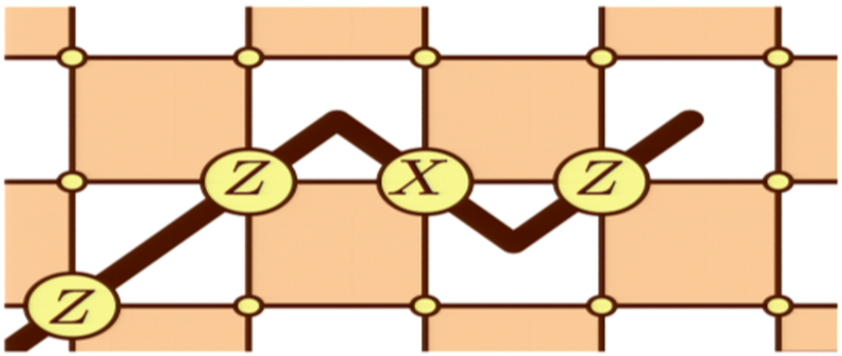
$$C_k |\psi\rangle = |\psi\rangle$$

- Hamiltonian version: $H := -\sum_k C_k$
  - excitations live at plaquettes

toric code

- **string operators** create/destroy excitations at their endpoints





- two types of strings/excitations: e (light) and m (dark)

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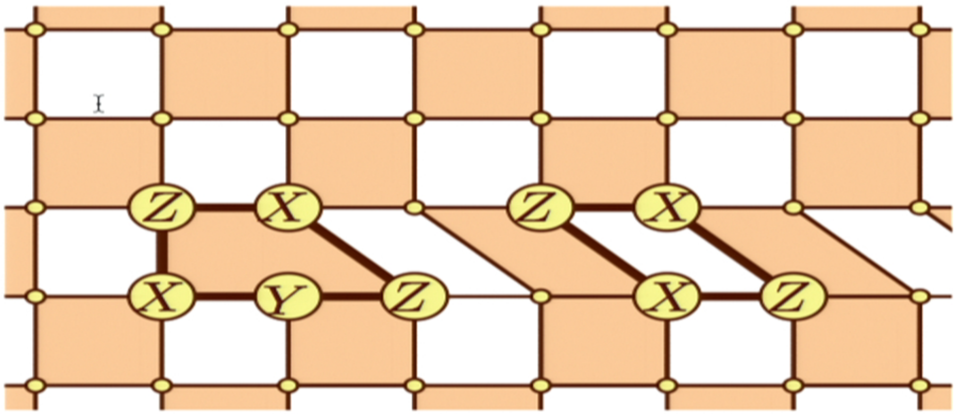
toric code

- twists amount to **dislocations**



- twists can be locally created in PAIRS only

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toric code

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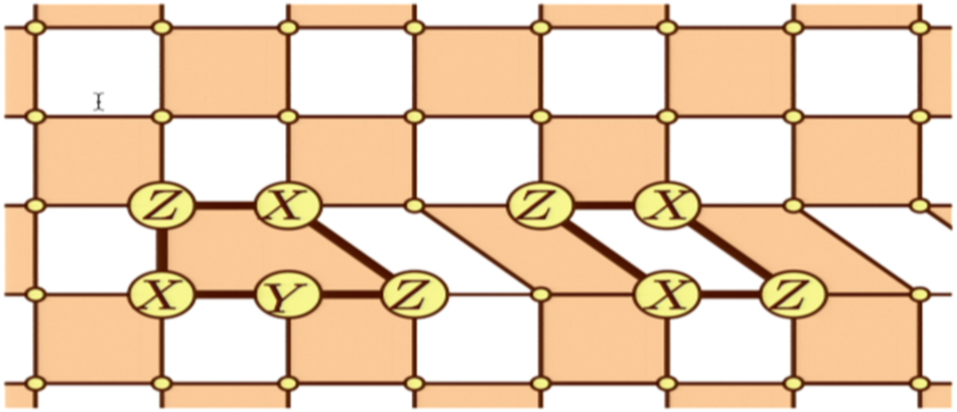

- twists can be locally created in PAIRS only

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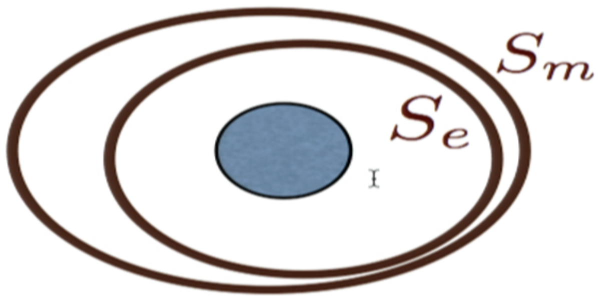
rules

- no twists (or even number) → 4 possible charges



	$\mathbf{1}$	e	m	ϵ
S_e	+1	+1	-1	-1
S_m	+1	-1	+1	-1

- a twist (or an odd number) → 2 possible charges

	σ_+	σ_-
S	+i	-i

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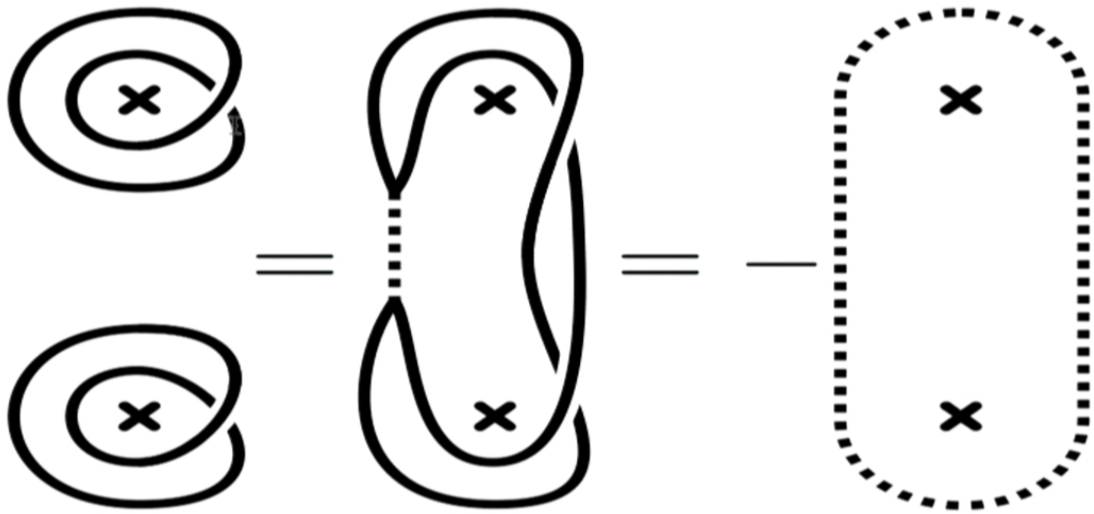
rules



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rules

- non-abelian fusion rules! (but not anyons!!)

$$\sigma_\pm \times \sigma_\pm = 1 + \epsilon \qquad \sigma_\pm \times \sigma_\mp = e + m$$

$$\sigma_\pm \times \epsilon = \sigma_\pm \qquad \sigma_\pm \times e = \sigma_\pm \times m = \sigma_\mp$$

- we recover **Ising** rules:

$$\sigma_+ \times \sigma_+ = \mathbf{1} + \epsilon \qquad \sigma_+ \times \epsilon = \sigma \qquad \epsilon \times \epsilon = \mathbf{1}$$

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rules

- all closed string ops can be expressed in terms of a set of open string ops → Majorana operators

$$c_j c_k + c_k c_j = 2\delta_{jk}$$

- braiding is also Ising-like!

$$c_j \longrightarrow c_{j+1}$$
$$c_{j+1} \longrightarrow -c_j$$

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conclusions & other results

- anyon symmetries allow to introduce twists
- increased computational power
- general defects in 2D (Kitaev, Kong '12)
- genons: universal TQC from non-universal layers (Barkeshli, Jian, Qi '13)
- anyon condensation on a line (You, Jian, Wen '13)
- twists in 3D TO (Mesaros, Kim, Ran '13)
- ...

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