Title: 3d boson topological insulators and quantum spin liquids

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Abstract: I will discuss recent work on 3d Symmetry Protected Topological (SPT) phases of bosonic systems, and their implications for understanding the more exotic quantum spin liquid phases. First I will describe various characterizations of these 3d SPT phases, in particular their surface effective theories and (when applicable) bulk electromagnetic response. Next I will show how this understanding leads to several new insights into the theory of both 2d and 3d quantum spin liquids. Finally I will provide an explicit construction of several 3d SPT phases in a system of `coupled layers'. This includes a 3d SPT state that is beyond the existing cohomology classification of such states.

PERIMETER PI INSTITUTE FOR THEORETICAL PHYSICS

EMERGENCE AND ENTANGLEMENT II

Waterloo, 6th-10th May 2013

Organizers: Xiao-Gang Wen
Guifre Vidal

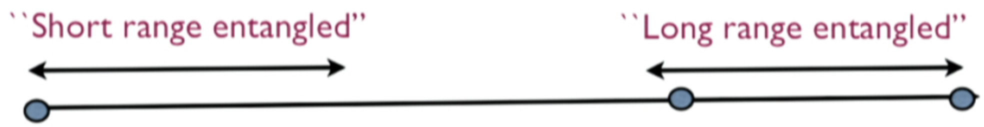
3d boson topological insulators and quantum spin liquids

T. Senthil (MIT)

Main collaborators: Ashvin Vishwanath, Chong Wang (MIT grad student), C. Xu

Other related work with M. Levin, N. Regnault, B. Swingle, J. Sau, A. Potter

Phases of quantum matter



Conventional
(trivial?)
phases
Eg: Band insulators,
superfluids,
antiferromagnets,
....

Gapped
`topologically ordered'
phases
Eg: FQHE

Gapless* phases/critical
points
Eg: Fermi/ non-fermi liquids

*Not just Goldstone

Phases of quantum matter

``Short range entangled''

``Long range entangled''

Conventional phases
Eg: Band insulators, superfluids, antiferromagnets, ...

Almost conventional phases: Topological insulators

Gapped `topologically ordered' phases
Eg: FQHE

Gapless phases/critical points
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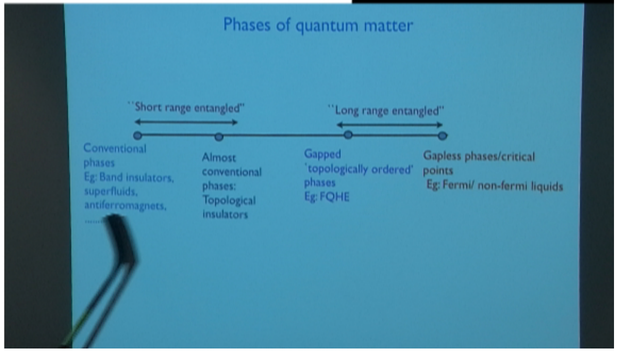

Phases of quantum matter

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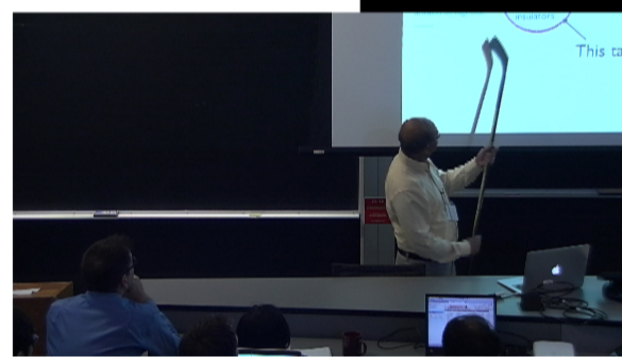
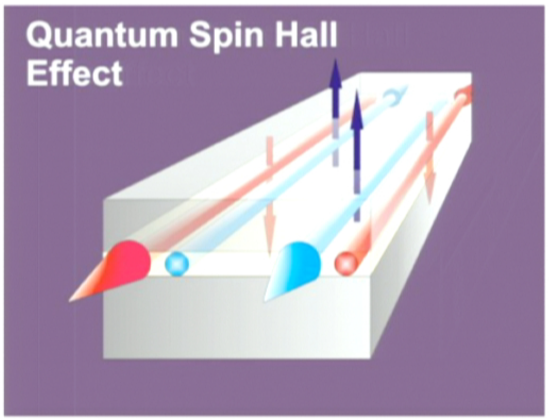
This talk



Modern topological insulators

Usual characterization: Non-trivial surface states with gapless excitations protected by some symmetry



More general point of view: Surface (d-1) field theory of TI realizes symmetry in a manner forbidden in `acceptable' lattice theory in strict (d-1) spacetime dimension.

Other non-trivial examples?

Strongly correlated topological insulators

Interaction dominated phases as topological insulators?

Move away from the crutch of free fermion Hamiltonians and band topology.

This talk: Topological Insulators of bosons

Why study bosons?

1. Non-interacting bosons necessarily trivial - so must deal with an interacting theory right away

Necessitates thinking more generally about TI phases without the aid of a free fermion model.

2. Natural realizations in quantum spin systems

Is there a spin analog of a topological insulator, i.e a ``Topological Paramagnet'' (as distinct from a ``Quantum spin liquid'')?

(Other realizations: cold atoms)

3. Correlated bosons are stepping stone to correlated fermions (probably).

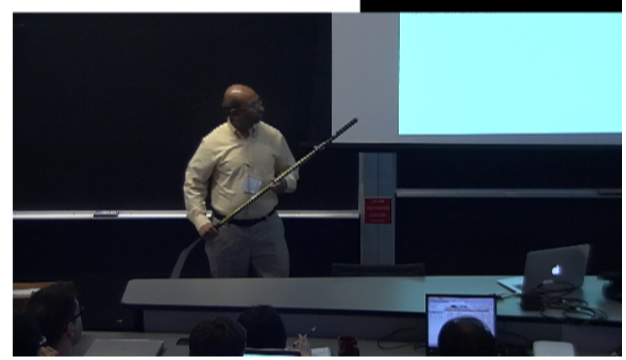
Topological Insulators of bosons?

Can bosons be in a ``topological insulator'' state with

(1) no fractionalized excitations or topological order

(2) a bulk gap

(3) non-trivial surface states protected by global (internal) symmetry?



An old example: Haldane spin chain

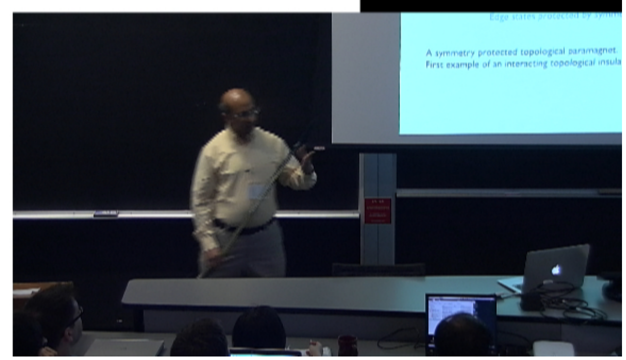
$$\mathcal{H} = J \sum_i \vec{S}_i . \vec{S}_{i+1}$$

$\vec{S}_i$: spin-1 operators

Unique ground state on ring, no bulk fractional excitations, but dangling spin-1/2 moments at edge.

Edge states protected by symmetry (SO(3) x time reversal)

A symmetry protected topological paramagnet.
First example of an interacting topological insulator.



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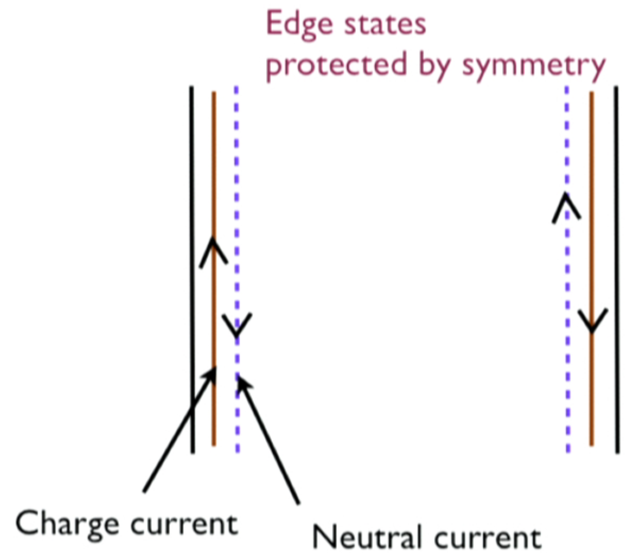
What about $d > 1$?

Yes! (according to recent progress in general abstract classification of short ranged entangled phases)

1. Cohomology classification (Chen, Liu, Gu, Wen, 2011)
2. ?? (Kitaev, unpublished)
3. Chern-Simons classification in $d = 2$ (Lu, Vishwanath, 2012)

A simple d = 2 example: Integer quantum Hall effect for bosons

Bosons with global U(1) symmetry admit integer quantum Hall state with no topological order.



Electrical Hall conductivity $\sigma_{xy} = 2$

1. Counterpropagating edge states but only one branch transports charge.

2. Thermal Hall conductivity = 0

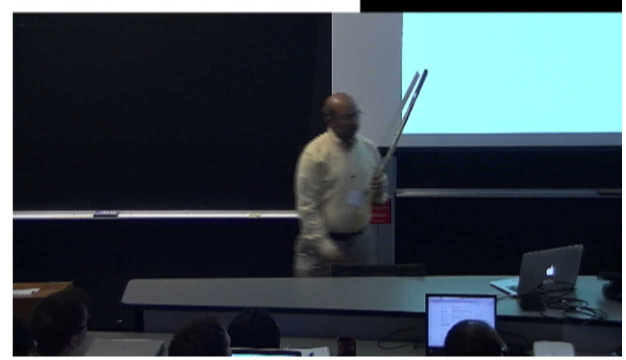
3. Realize: 2-component bosons at total filling factor 2

Lu, Vishwanath; TS, M. Levin; Liu, Wen; Geraedts, Motrunich, many others

Exact diagonalization: strong evidence for delta function repulsion (Furukawa, Ueda; Wu, Jain; Regnault, TS, 2013).

Comments

- State described has $\sigma_{xy} = 2, \kappa_{xy} = 0$.
  Can obtain states with $\sigma_{xy} = 2n, \kappa_{xy} = 0$ by taking copies.
- For bosons, IQHE necessarily has σ_{xy} even.



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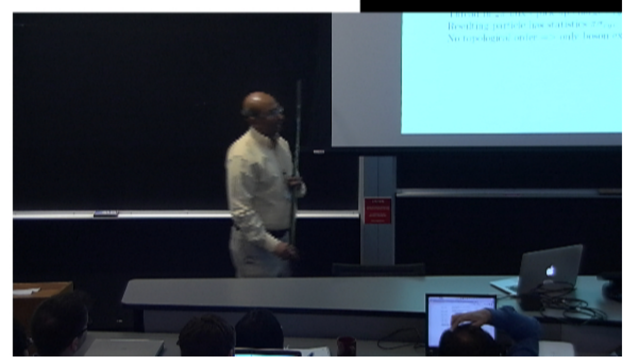
Simple argument (TS, Levin 12).
Thread in 2π flux - pick up charge σ_{xy}.
Resulting particle has statistics $\pi\sigma_{xy}$.
No topological order => only boson excitations, so σ_{xy} even.



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Note: Other boson states with non-zero κ_{xy} exist even in the absence of *any* symmetry for $\kappa_{xy} = 8n$. ``E8 state'': edge is E8 CFT

Apparently well known (........, Kitaev 06, Lu, Vishwanath 12; see Swingle talk)

Time reversal symmetric topological insulators of bosons in 3d

- Quantized magneto-electric effect (axion angle $\theta = 2\pi$ or even 0)
- emergent fermionic (or other exotic) vortices at surface, ...

Review: free fermion 3d topological insulators

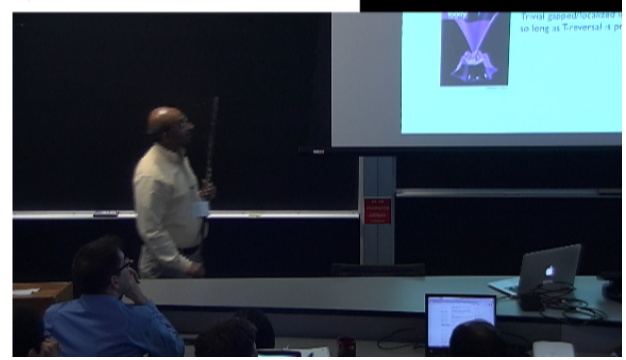
Characterize by

1. presence/absence of non-trivial surface states

2. EM response

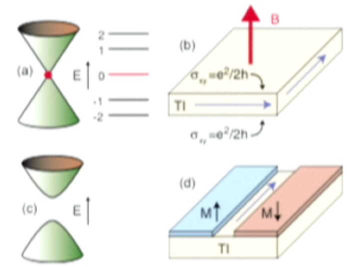
Surface states: Odd number of Dirac cones

Trivial gapped/localized insulator not possible at surface so long as T-reversal is preserved (even with disorder)



Review: free fermion topological insulators

EM response: Surface quantum Hall Effect



If surface gapped by B-field/proximity to magnetic insulator, surface Hall conductance

$$\sigma_{xy} = \left(n + \frac{1}{2}\right) \frac{e^2}{h}$$

Domain wall between opposite T-breaking regions: chiral edge mode of 2d fermion IQHE

Review: Free fermion topological insulators
Axion Electrodynamics

Qi, Hughes, Zhang, 09
Essin, Moore, Vanderbilt, 09

EM response of *any* 3d insulator

$$\mathcal{L}_{eff} = \mathcal{L}_{Max} + \mathcal{L}_{\theta} \quad (1)$$
$$\mathcal{L}_{\theta} = \frac{\theta}{4\pi^2}\vec{E}.\vec{B}$$

Under $\mathcal{T}$-reversal, $\theta \to -\theta$.
Periodicity $\theta \to \theta + 2\pi$: only $\theta = n\pi$ consistent with $\mathcal{T}$-reversal.
Domain wall with $\theta = 0$ insulator: Surface quantum Hall effect

$$\sigma_{xy} = \frac{\theta}{2\pi}$$

Free fermion TI: $\theta = \pi$.
Interpretation of periodicity:
$\theta \to \theta + 2\pi$: deposit a 2d fermion IQHE at surface.
Not a distinct state.

Boson topological insulators: EM response

Vishwanath, TS, 2012

For bosons, $\theta = 2\pi$ is distinct from $\theta = 0$.

Surface Hall conductivity $\sigma_{xy} = 1$, i.e, half of 2d boson IQHE state.
-cannot be obtained by depositing 2d boson IQHE state (which has σ_{xy} even).
Surface of $\theta = 2\pi$ requires the 3d bulk.

$\theta \to \theta + 4\pi$ is trivial.

Later: Depending on how T-reversal is realized distinct bosonic TIs exist with $\theta = 2\pi$ or even with $\theta = 0$.

Plan

1. Effective field theory of surface?

Key idea: dual vortex theory with unusual vortex

2. Simple explicit construction of 3d SPT phases

Coupled layers of known 2d phases.

3. Boson topological insulators as a window into more exotic phases.

New insights into gapless quantum spin liquids.

Effective Field Theory of surface states of boson TI?

Eg: Fermion TI - surface effective field theory: free fermions with odd number of Dirac cones + interactions

What is analog for bosons?

(Mostly) focus on symmetry $U(1) \times Z_2^T$ (appropriate for spin systems with XY symmetry) to show widest range of phenomena.

Symmetry $U(1) \rtimes Z_2^T$ (appropriate for bosons such as He4) is also easily understood within same framework but realizes only subset of phenomena.

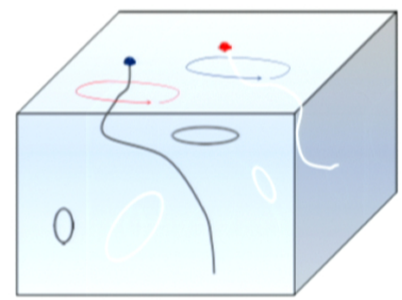
Other instructive example: just Z_2^T (boson "topological superconductor").

Key requirement of surface theory

Trivial symmetry preserving insulator not possible at surface.

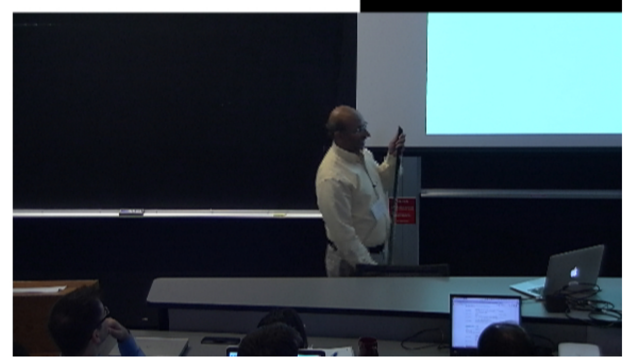
Convenient implementation through dual vortex point of view.

Description of surface in terms of point vortices = points where vortex lines of bulk penetrate surface.



Conventional 2d bosons: charge-vortex duality

$$\mathcal{L}_d = \mathcal{L}[\Phi, a_\mu] + \frac{1}{2\pi}\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda A_\mu$$



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Bosonic vortex

Dasgupta, Halperin, '80
Peskin, Stone, '80
Fisher, Lee, '89

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Physical boson current

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Physical boson current

external probe gauge field

Boson superfluid = vortex insulator

Mott insulator = vortex condensate

Dasgupta, Halperin, '80
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Dual description of boson TI surface

Mott insulator = vortex condensate

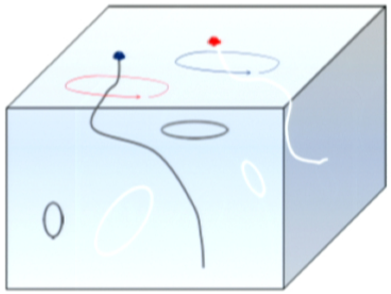
Boson TI surface: Demand that there is no trivial vortex that can condense to give a trivial insulator.

Implement: vortices are

(i) Kramers doublet bosons

(ii) fermions but Kramers singlet

(iii) fermion and Kramers doublet (corresponding to different TIs)



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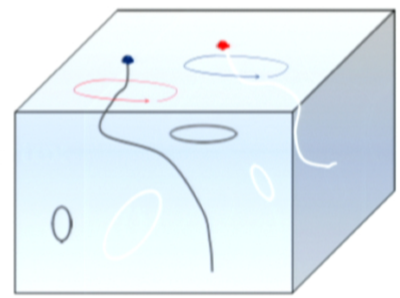

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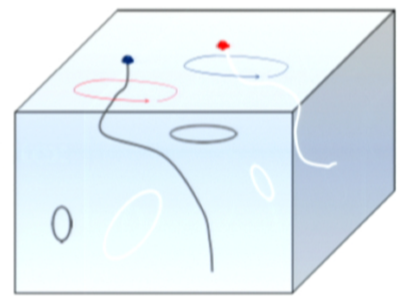

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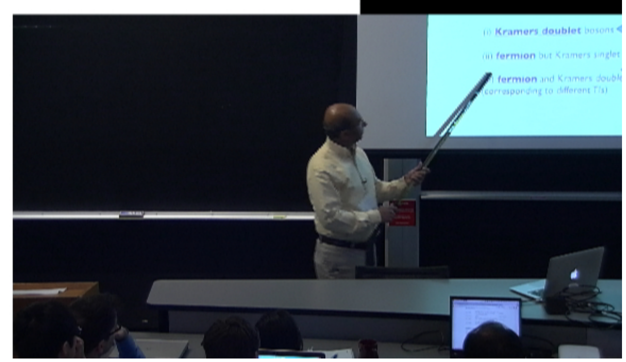


Surface Landau-Ginzburg effective field theory

Vishwanath, TS, 12
Wang, TS, 13

Example 1

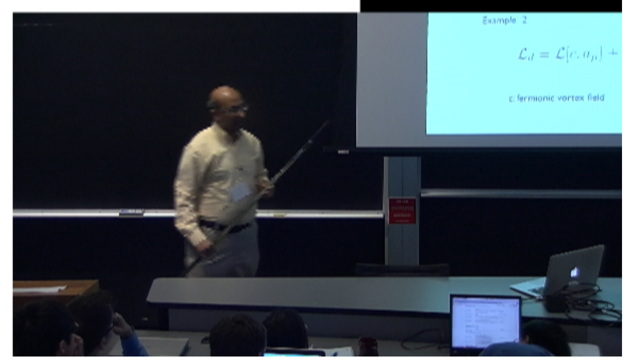
$$\mathcal{L}_d = \mathcal{L}[\Phi_\alpha, a_\mu] + \frac{1}{2\pi}\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda A_\mu$$

Φ_α, $(\alpha = 1, 2)$: Kramers doublet bosonic vortex

Example 2

$$\mathcal{L}_d = \mathcal{L}[c, a_\mu] + \frac{1}{2\pi}\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda A_\mu$$

c: fermionic vortex field



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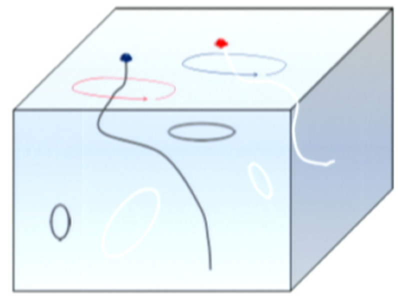

Implement: vortices are

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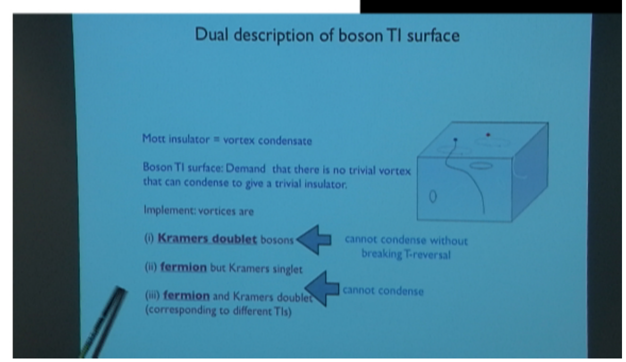

Surface Landau-Ginzburg effective field theory

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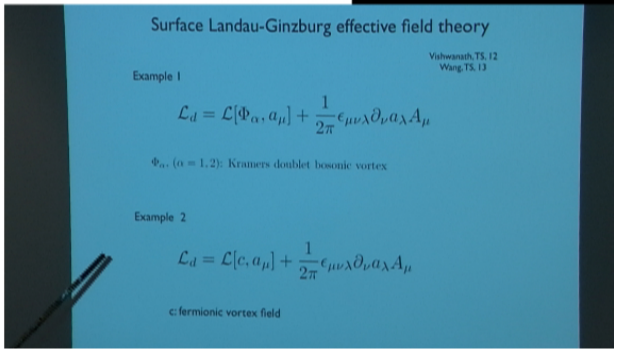

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c: fermionic vortex field

Discuss bit more

More insight: bulk ground state wavefunction*

C. Xu, TS, 13

\`Dual' view of bosons in terms of vortex loops.

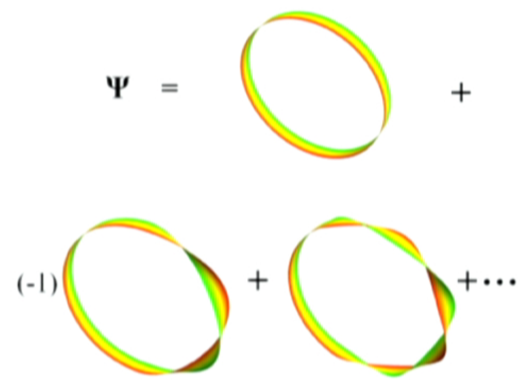
Superfluid: Vortex loops have tension => costly

Insulator: Vortex loops proliferate.

Conventional insulator: Ground state wavefunction - vortex loop gas with positive weights.

Topological insulator: Vortex is a \`\`ribbon''

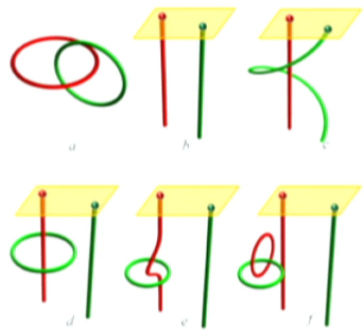
Vortex ribbon loop gas with phase (-1) for each self-linking of ribbon.



*Derive from bulk effective field theories proposed by Vishwanath, TS (see C. Xu talk)

Comments

1. Naturally point of penetration of vortex at surface is fermion.



2. Other topological insulating phase: vortex line core is Haldane spin chain.

End point of vortex: Kramers doublet.

Surface phase structure

$$\mathcal{L}_d = \mathcal{L}[c, a_\mu] + \frac{1}{2\pi}\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda A_\mu$$

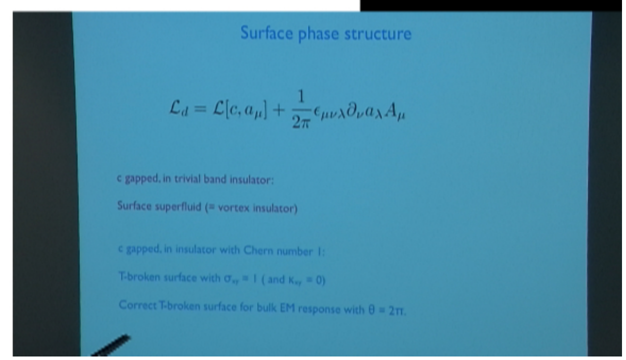
c gapped, in trivial band insulator:

Surface superfluid (= vortex insulator)

c gapped, in insulator with Chern number 1:

T-broken surface with $\sigma_{xy} = 1$ (and $\kappa_{xy} = 0$)

Correct T-broken surface for bulk EM response with $\theta = 2\pi$.

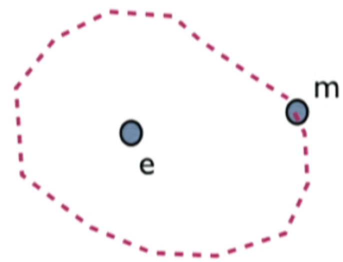


Symmetry preserving surface phases
Eg: surface topological order

Simplest: Paired vortex condensation
=> Z2 surface topological order (``toric code")

Usual topological quasiparticles e, m, ε(= e+m)

charge 1/2 bosons — charge-0 fermion



Phase of π

Surface is in a `quantum spin liquid' phase...........

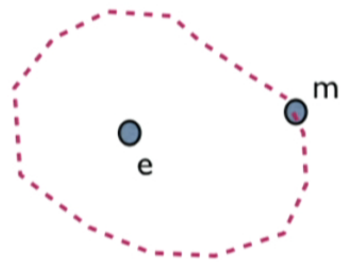
However symmetry is realized in a manner prohibited in strict 2d.

Strict 2d Z2 states: describe by mutual Chern-Simons theory.
Charge assignment as above breaks T-reversal.

Surface topological order of 3d SPTs

3d SPT surface can have topological order though bulk does not.

Resulting symmetry preserving gapped surface state realizes symmetry `anamolously'.

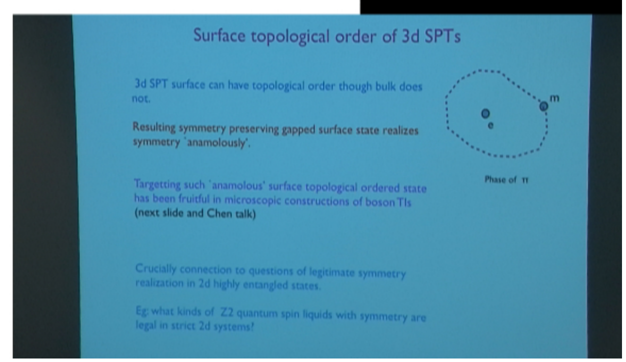


Phase of π

Targetting such `anamolous' surface topological ordered state has been fruitful in microscopic constructions of boson TIs (next slide and Chen talk)

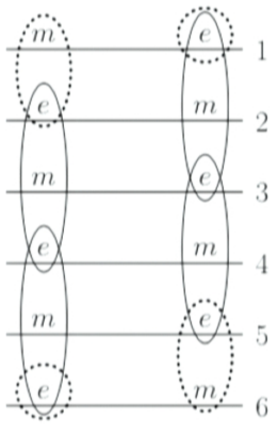
Crucially connection to questions of legitimate symmetry realization in 2d highly entangled states.

Eg: what kinds of Z2 quantum spin liquids with symmetry are legal in strict 2d systems?



Explicit construction of a 3d boson SPT

C. Wang, TS, 2013



1. Each layer: e is charge 1/2, m is charge 0.

2. Condense $e_i \, m_{i+1} \, e_{i+2}$ (all self and mutual bosons)

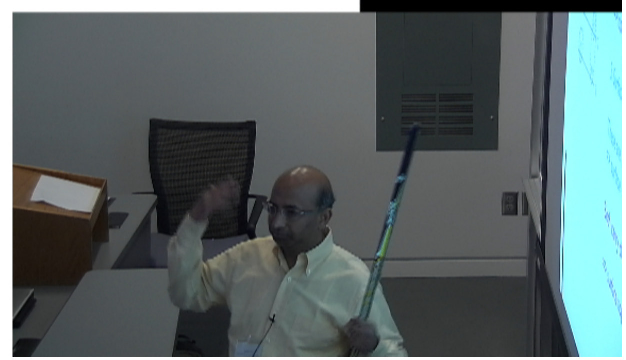
=> confine all bulk quasiparticles

3. Surface: e_1, $m_1 e_2$ survive (at top surface)

These are mutual semions, and self-bosons
=> surface Z_2 topological order

Both carry charge-1/2

=> surface topological order of 3d SPT.



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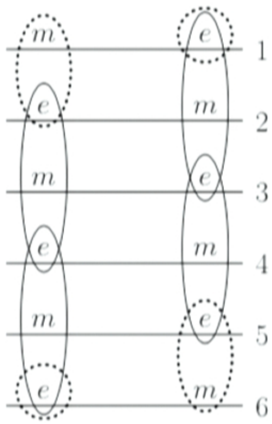

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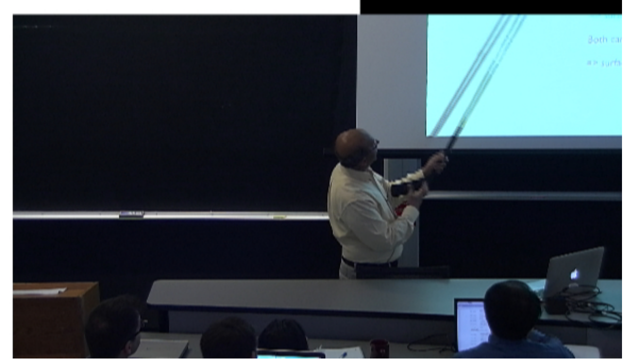
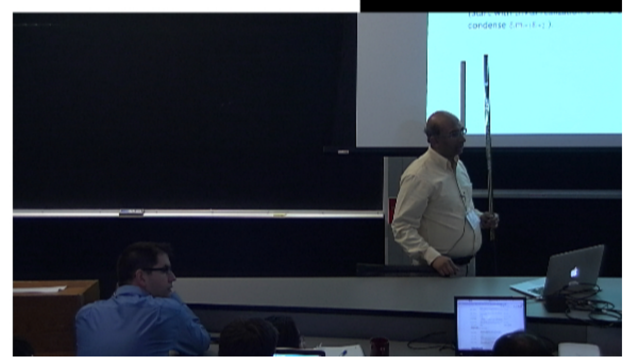


Comments

1. This coupled layer construction can be generalized to all the SPTs with various symmetries studied in Vishwanath, TS, PR X, 2013.

2. Nice example: very simple construction of SPT state with only T-reversal that is `beyond cohomology' classification of Chen, Gu, Liu, and Wen (2011).

(Start with trivial realization of T-reversal in each 2d layer, condense $\varepsilon_i m_{i+1} \varepsilon_{i+2}$).



Comments

1. This coupled layer construction can be generalized to all the SPTs with various symmetries studied in Vishwanath, TS, PR X, 2013.

2. Nice example: very simple construction of SPT state with only T-reversal that is `beyond cohomology' classification of Chen, Gu, Liu, and Wen (2011).

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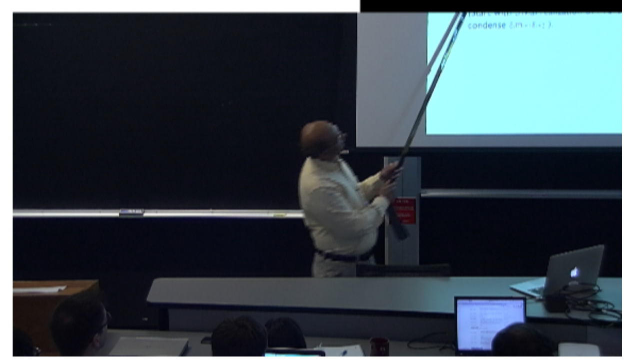


Beyond cohomology state

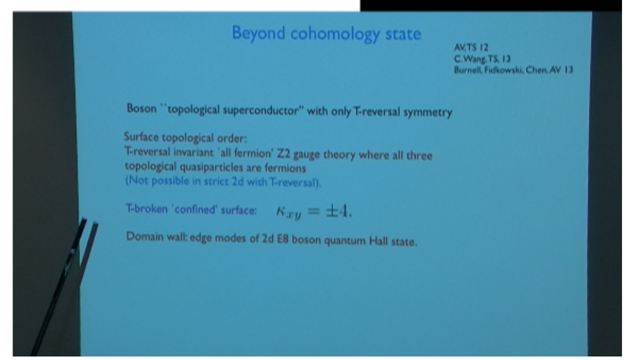
AV, TS 12
C. Wang, TS, 13
Burnell, Fidkowski, Chen, AV 13

Boson ``topological superconductor'' with only T-reversal symmetry

Surface topological order:
T-reversal invariant `all fermion' Z2 gauge theory where all three topological quasiparticles are fermions
(Not possible in strict 2d with T-reversal).

T-broken `confined' surface: $\kappa_{xy} = \pm 4.$

Domain wall: edge modes of 2d E8 boson quantum Hall state.



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A different characterization of 3d boson TIs with global U(1) symmetry

Bulk external sources of vortex lines behave similar to boundary vortex
=> fermions or Kramers doublets or both.

Alternate: Couple external gauge field to global U(1) charge*.

Bulk magnetic monopole is fermion/Kramers doublet or both. (Wang, TS 13)

More formal - `bulk-edge correspondence':
bulk monopole created at surface by surface vortex of dual LGW theory:

*Generalization to 3d of a similar result about 2d SPTs by Levin-Gu.

See also: different argument for 3d for $\theta = 2\pi$ TI (Metlitski et al, 13, M.P.A. Fisher talk).

Boson topological insulators as a new window into gapless quantum spin liquids

Some implications-I
Absence of gapless quantum vortex fluids in 2d (with T-reversal symmetry)

In a strictly 2d boson/spin system, suppose the vortex is a fermion. (Larkin, 1990s, M.P.A. Fisher, Motrunich, Alicea, 2006-08, ...)

=> can get exotic `vortex metal' as a highly entangled gapless `beyond quasiparticle' state of matter.

Interesting applications to quantum spin liquid theory, eg, on Kagome lattice (Fisher et al).

With T-reversal such a state can only occur at surface of 3d topological insulator => cannot occur in strict 2d.

Without T-reversal can actually occur in strict 2d (Galitski et al, 2006).

Some implications -II
New view point on gapless U(1) quantum spin liquids in 3+1-d with T-reversal

Standard view:
Excitations: Propagating photon
Electric particle: boson/fermion, Kramers singlet/doublet
Magnetic particle: boson, trivial internal quantum numbers

Electric-magnetic duality

e-particle: boson trivial quantum numbers
m-particle: spin-1/2 boson/fermion

=> dual e particle is in a boson topological insulator phase.

Alternate view of original gauge theory: monopole topological insulator.
Different monopole topological insulator phases: bose/fermi statistics, Kramers doublet/singlet for original e-particle

Summary

1. 3d boson TI phases with T-reversal invariance: Axion angle $\theta = 2\pi$ or 0.

Fermion/Kramers surface vortices and associated exotic phases.
Surface field theory: symmetry implementation not allowed in strict 2d.
``No-go" constraint on some theories of highly entangled matter.

2. New insights into quantum spin liquids, both gapped and gapless

3. Most important open question:

what kinds of real quantum magnets/other systems are suitable to host these bosonic SPT phases in 3d?