Title: Limitations on the psi-epistemic view of quantum states

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Abstract: The "psi-epistemic" view is that the quantum state does not represent a state of the world, but a state of knowledge about the world. It is motivated, in part, by the observation of qualitative similarities between characteristic properties of non-orthogonal quantum wavefunctions and between overlapping classical probability distributions. It might be suggested that this gives a natural explanation for these properties, which seem puzzling for the alternative "psi-ontic" view. I will examine two such similarities, quantum state overlap and quantum state discrimination, and ask how far can we reproduce the quantitative values given by quantum theory. It will be shown that the psi-epistemic view cannot properly account for the quantitative values, and so must still rely on the same kind of explanations as the "psi-ontic" view.<strongr>

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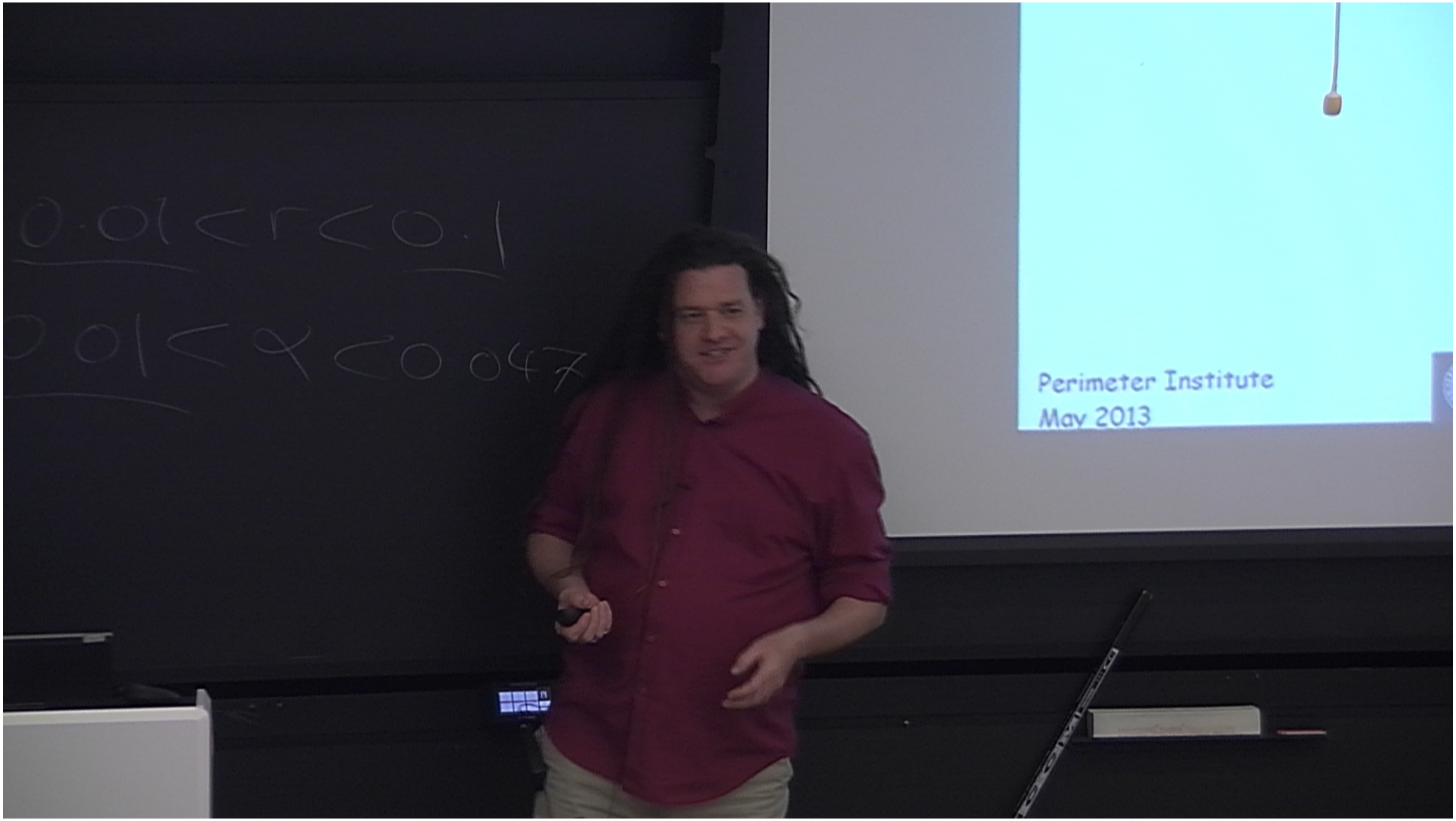
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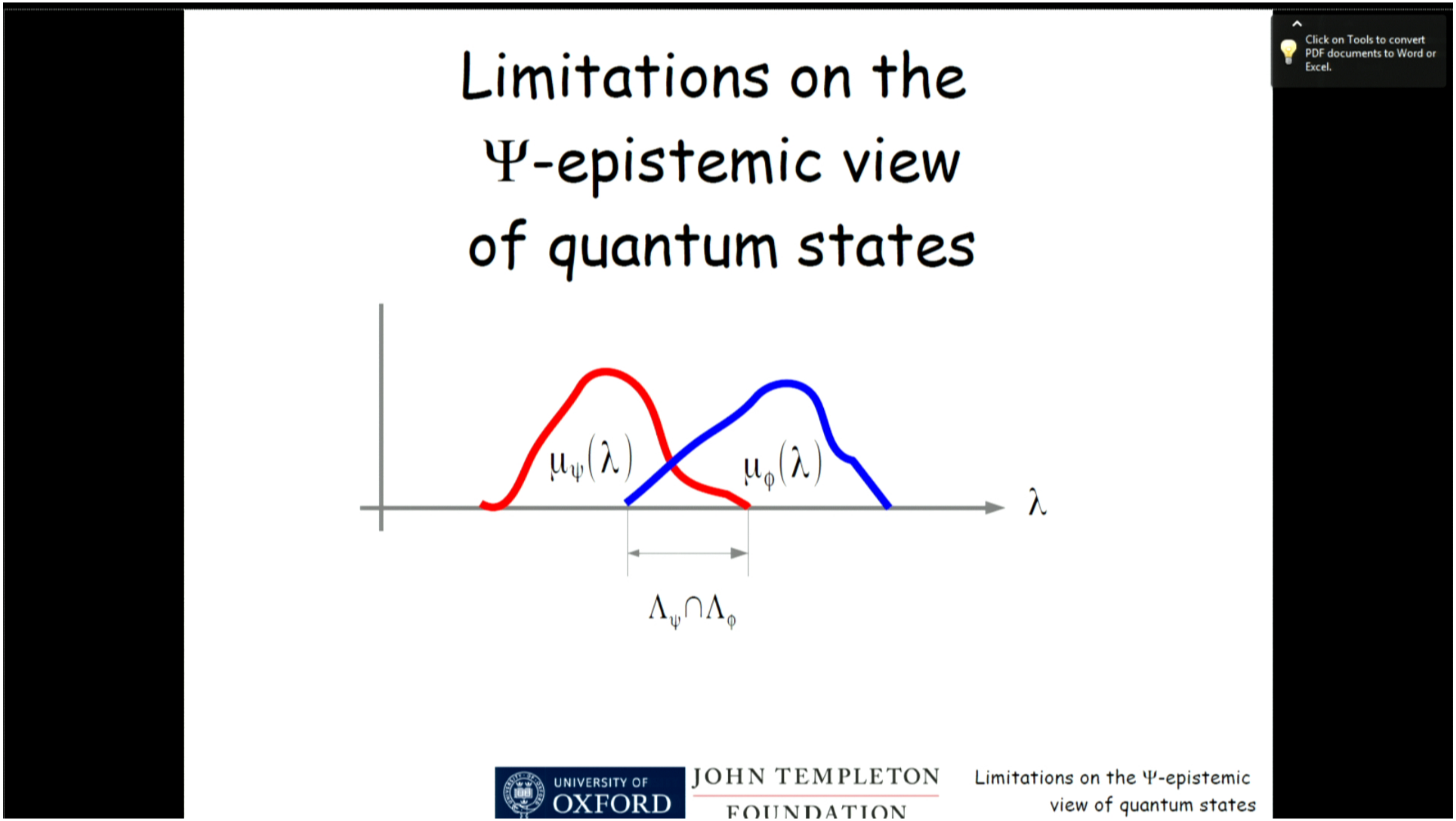

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Limitations on the Ψ-epistemic view of quantum states

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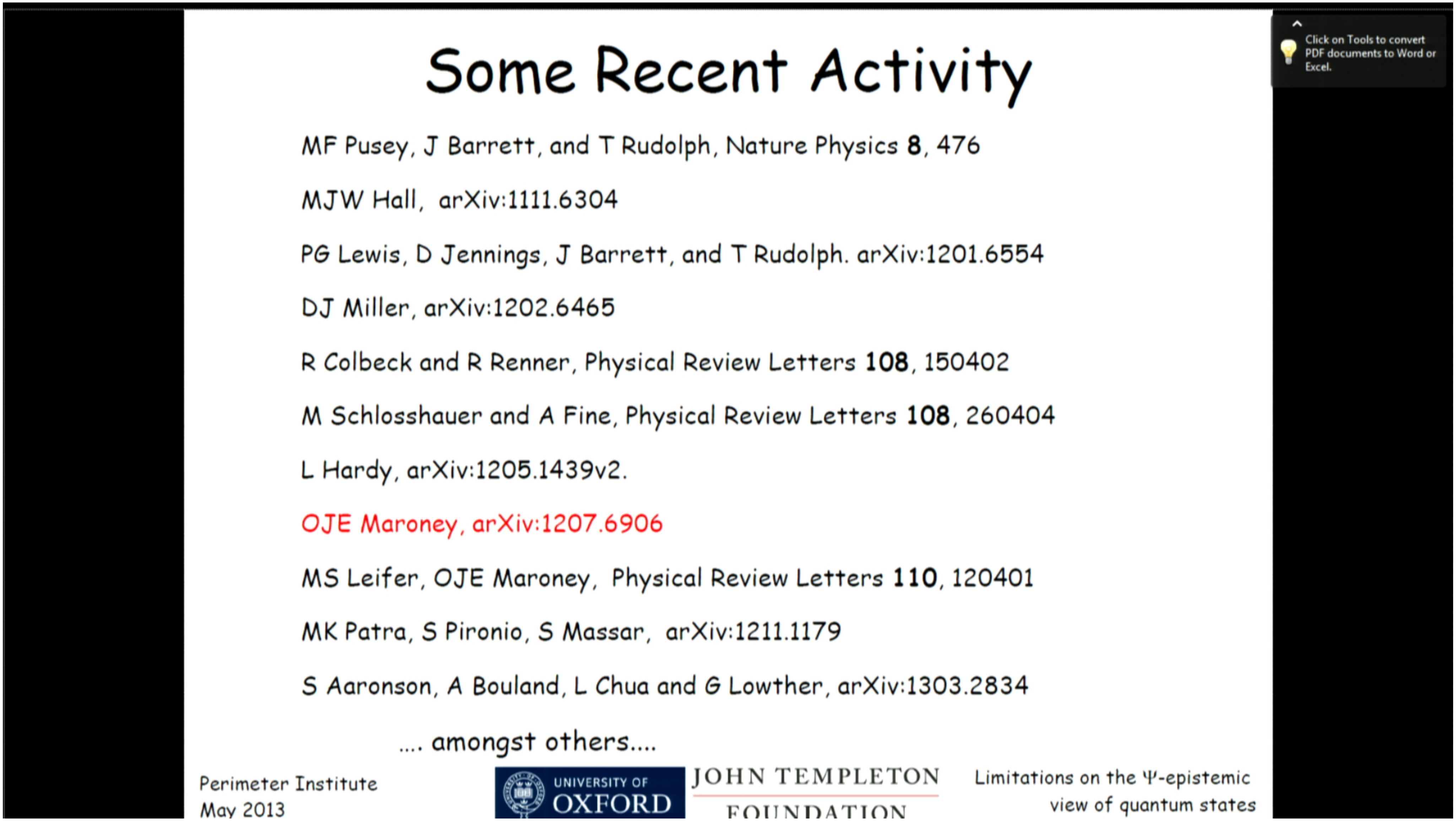

Some Recent Activity

MF Pusey, J Barrett, and T Rudolph, Nature Physics **8**, 476

MJW Hall, arXiv:1111.6304

PG Lewis, D Jennings, J Barrett, and T Rudolph. arXiv:1201.6554

DJ Miller, arXiv:1202.6465

R Colbeck and R Renner, Physical Review Letters **108**, 150402

M Schlosshauer and A Fine, Physical Review Letters **108**, 260404

L Hardy, arXiv:1205.1439v2.

OJE Maroney, arXiv:1207.6906

MS Leifer, OJE Maroney, Physical Review Letters **110**, 120401

MK Patra, S Pironio, S Massar, arXiv:1211.1179

S Aaronson, A Bouland, L Chua and G Lowther, arXiv:1303.2834

.... amongst others....

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- Ψ-epistemic theories
  - What are they? Why are they?
  - PBR and Hardy theorems
- Quantum State Overlap
  - Epistemic vs. Ontic explanation
  - Limitations of the epistemic explanation
- Quantum State Discrimination
  - Epistemic vs. Ontic explanation
  - Limitations of the epistemic explanation
- Where does that leave us?

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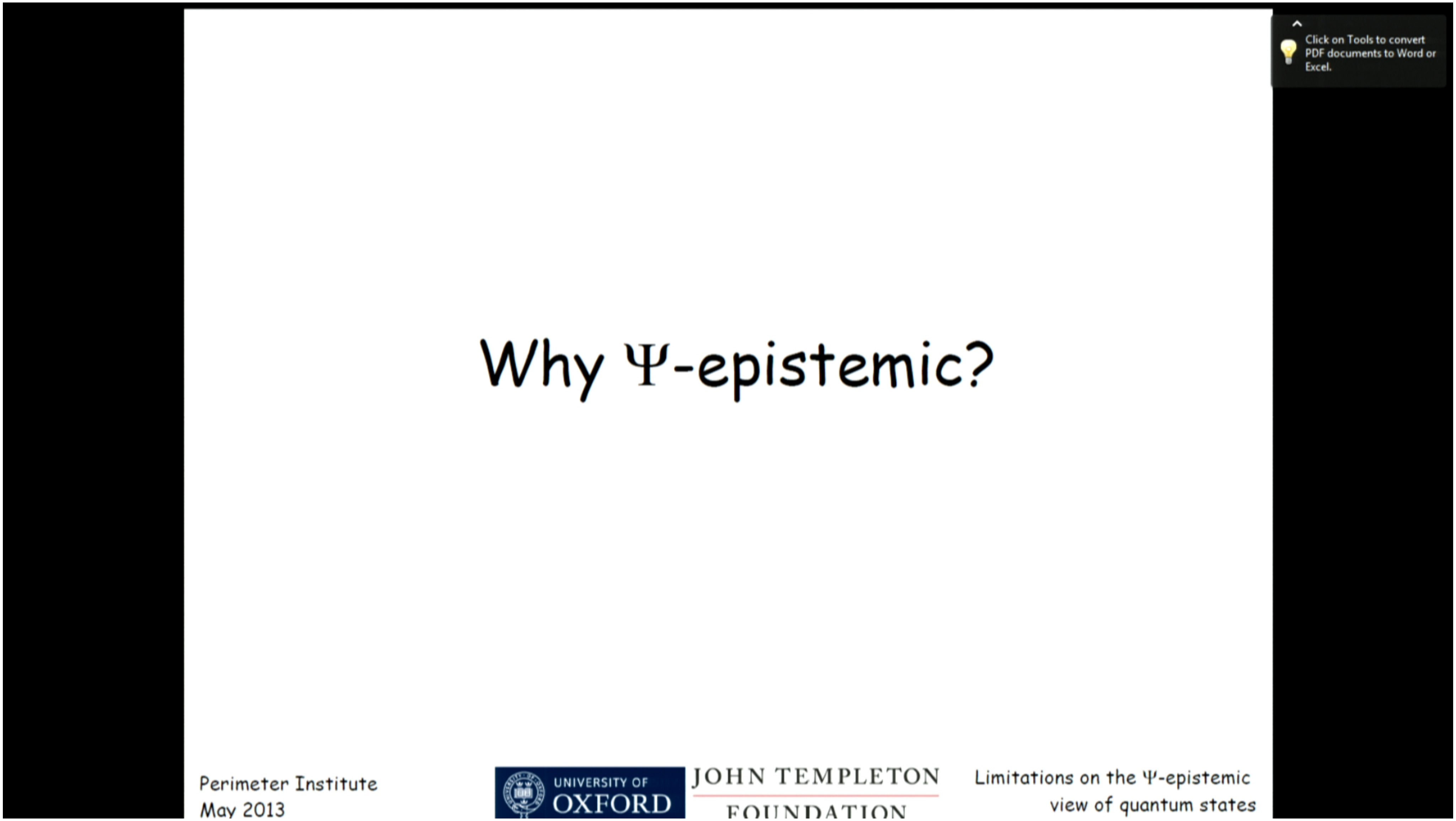

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Why Ψ-epistemic?

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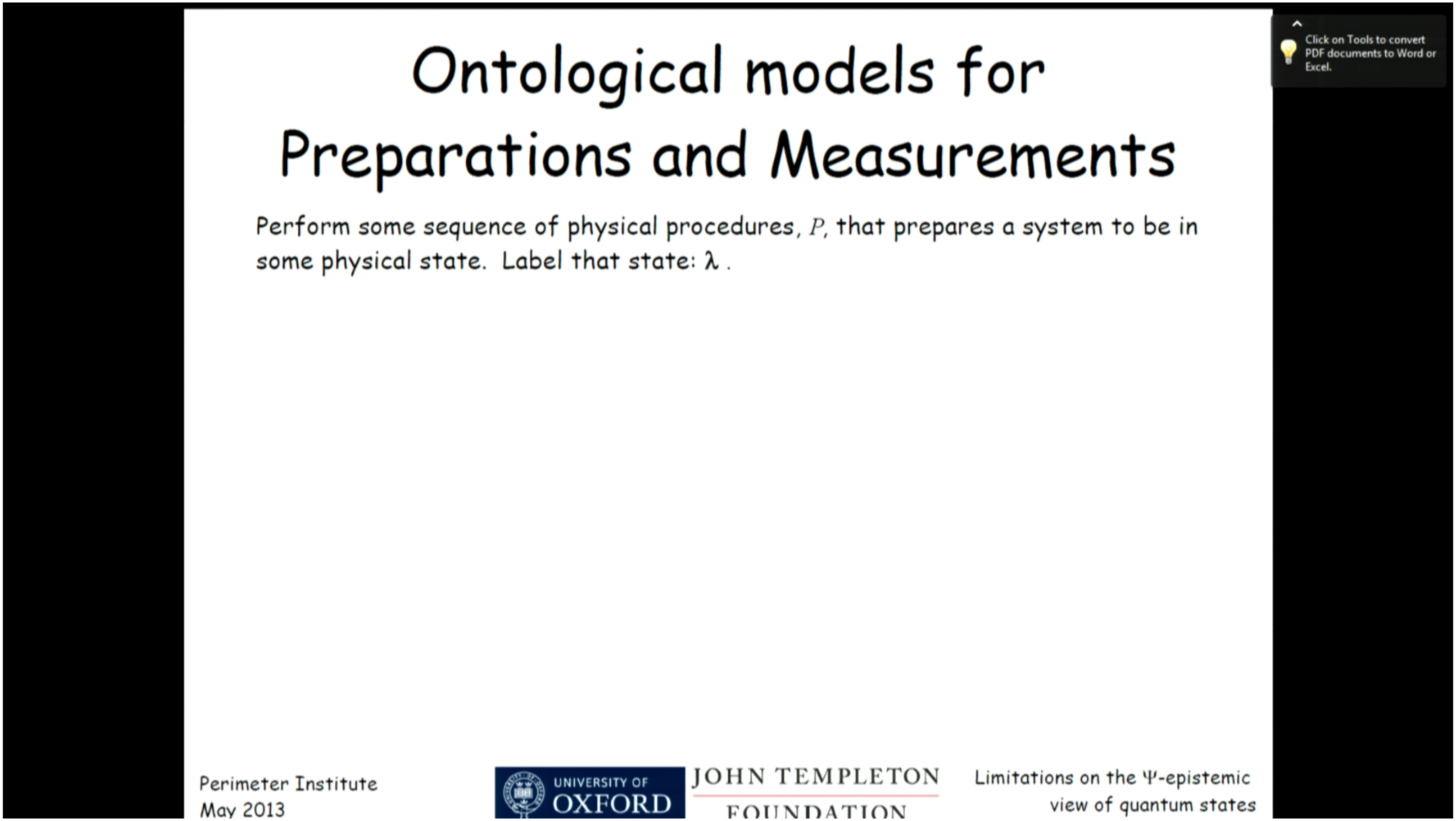

Ontological models for Preparations and Measurements

Perform some sequence of physical procedures, P, that prepares a system to be in some physical state. Label that state: λ .

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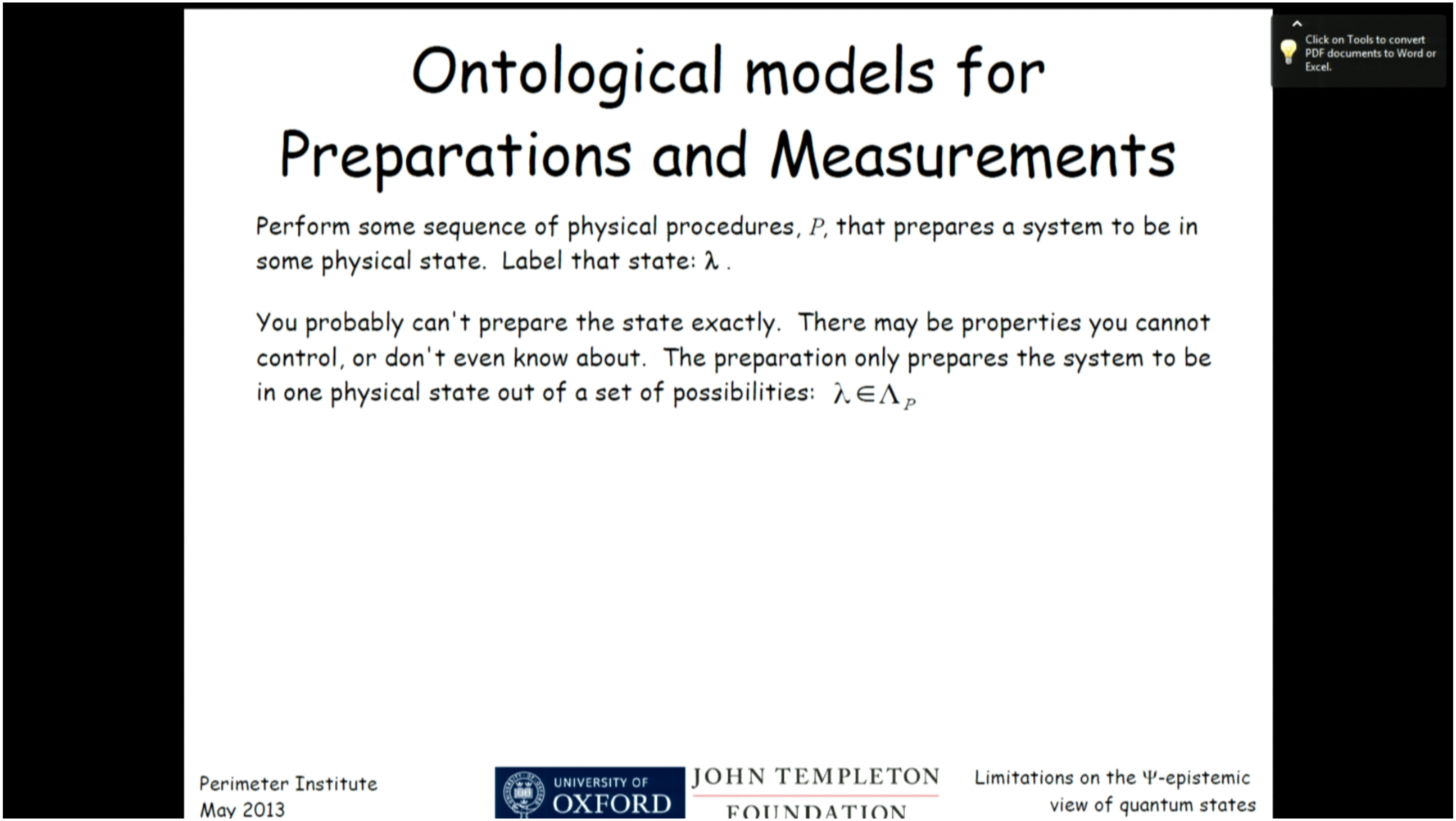

Ontological models for Preparations and Measurements

Perform some sequence of physical procedures, P, that prepares a system to be in some physical state. Label that state: λ .

You probably can't prepare the state exactly. There may be properties you cannot control, or don't even know about. The preparation only prepares the system to be in one physical state out of a set of possibilities: $\lambda \in \Lambda_P$

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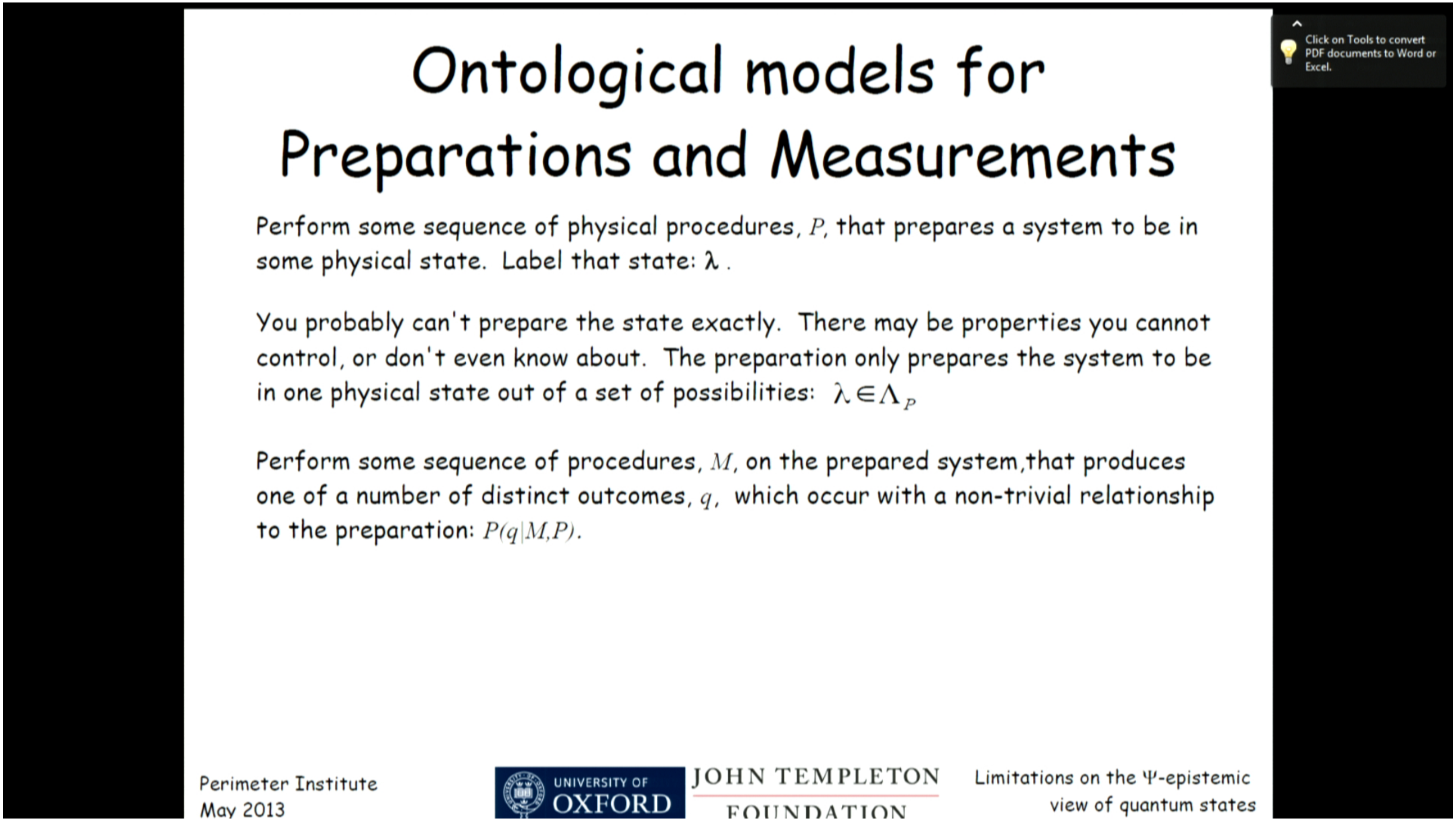

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Perform some sequence of procedures, M, on the prepared system,that produces one of a number of distinct outcomes, q, which occur with a non-trivial relationship to the preparation: $P(q|M,P)$.

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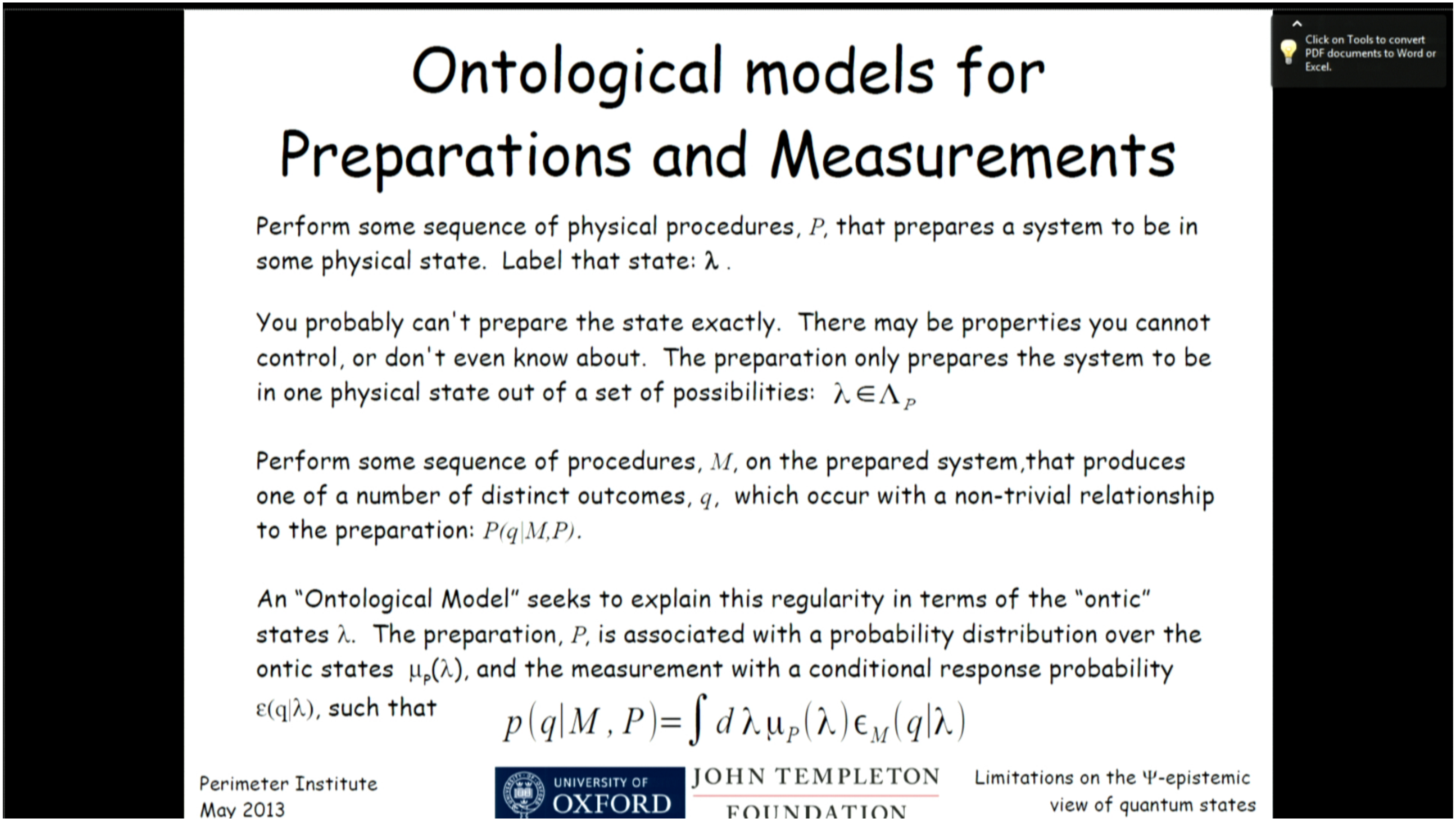

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An "Ontological Model" seeks to explain this regularity in terms of the "ontic" states λ. The preparation, P, is associated with a probability distribution over the ontic states $\mu_P(\lambda)$, and the measurement with a conditional response probability $\varepsilon(q|\lambda)$, such that

$$p(q|M,P) = \int d\lambda \, \mu_P(\lambda) \epsilon_M(q|\lambda)$$

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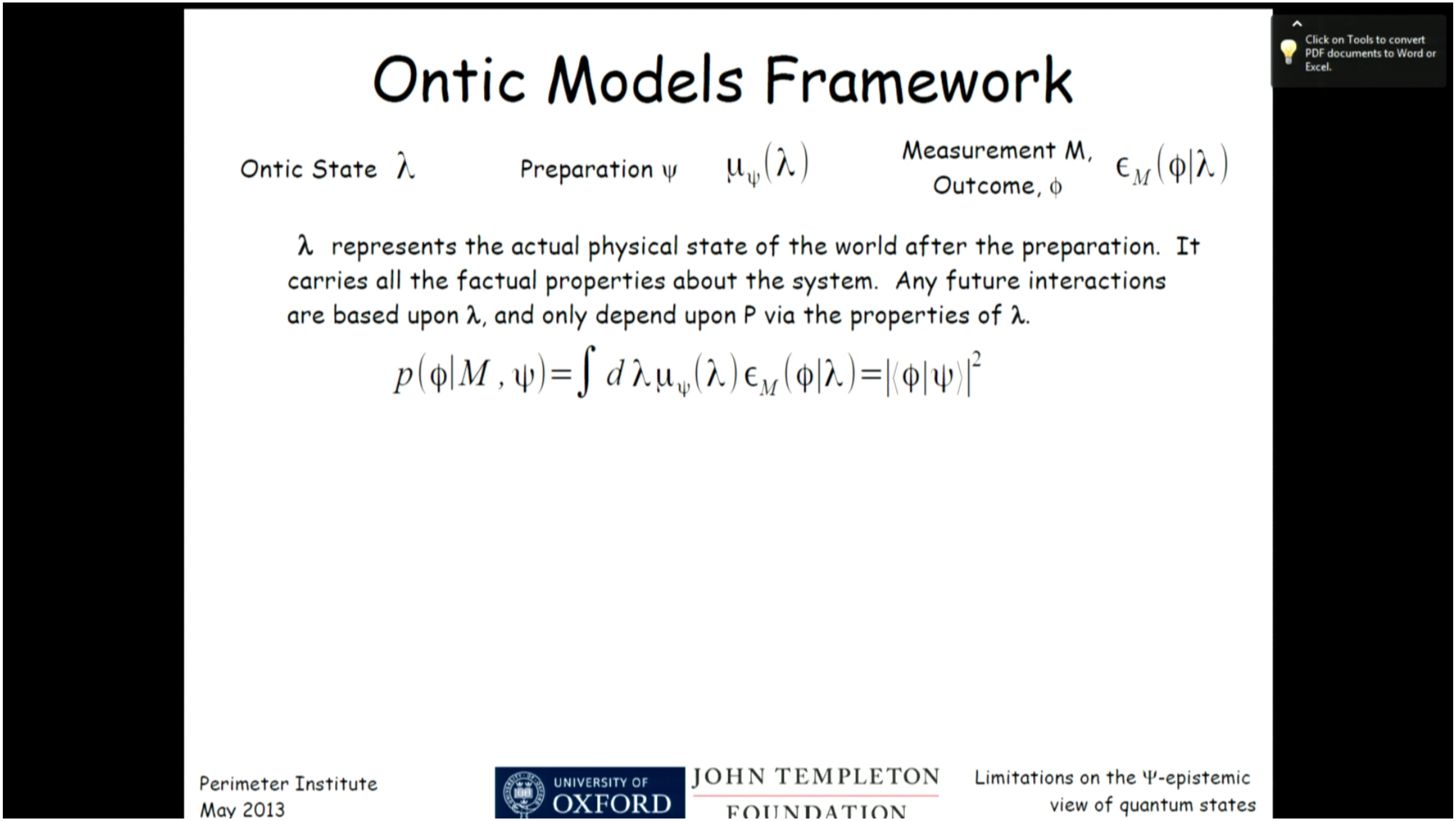

Ontic Models Framework

Ontic State λ

Preparation ψ $\mu_{\psi}(\lambda)$

Measurement M,
Outcome, ϕ $\epsilon_{M}(\phi|\lambda)$

λ represents the actual physical state of the world after the preparation. It carries all the factual properties about the system. Any future interactions are based upon λ, and only depend upon P via the properties of λ.

$$p(\phi|M,\psi)=\int d\lambda\,\mu_{\psi}(\lambda)\,\epsilon_{M}(\phi|\lambda)=|\langle\phi|\psi\rangle|^{2}$$

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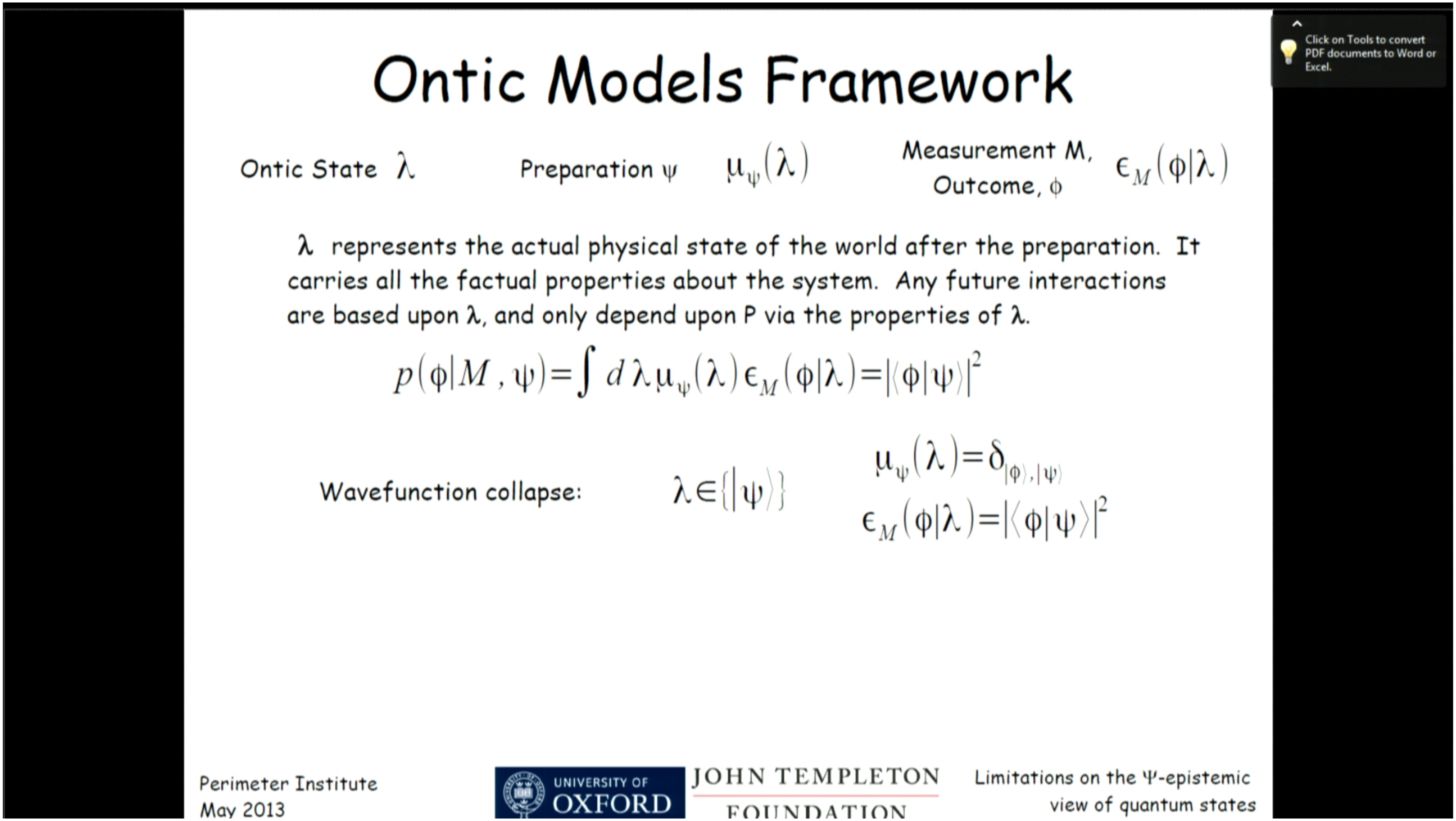

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Ontic Models Framework

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$$p(\phi|M,\psi)=\int d\lambda\,\mu_\psi(\lambda)\,\epsilon_M(\phi|\lambda)=|\langle\phi|\psi\rangle|^2$$

Wavefunction collapse: $\lambda\in\{|\psi\rangle\}$

$$\mu_\psi(\lambda)=\delta_{|\phi\rangle,|\psi\rangle}$$

$$\epsilon_M(\phi|\lambda)=|\langle\phi|\psi\rangle|^2$$

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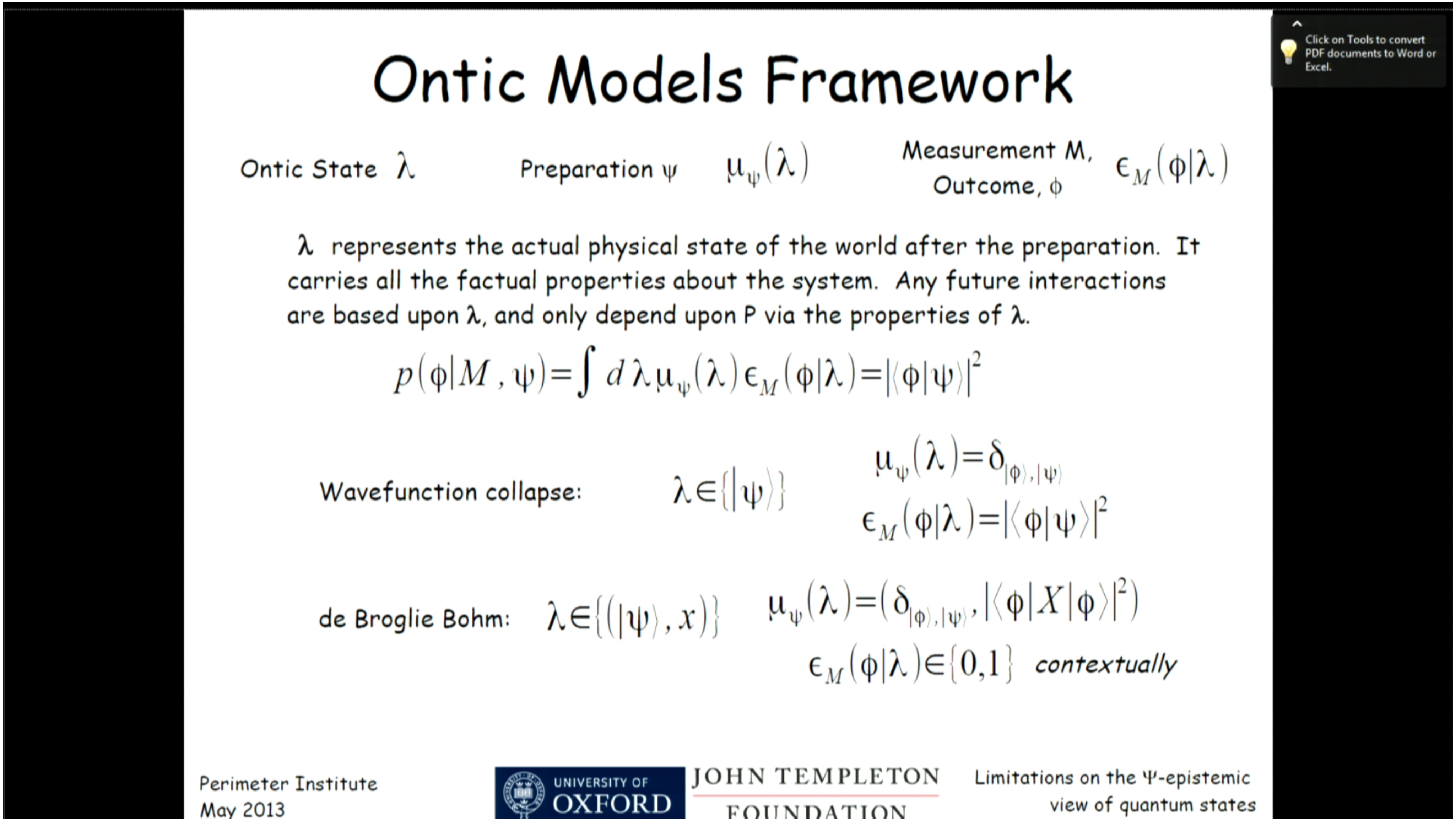

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$$\epsilon_M(\phi|\lambda)=|\langle\phi|\psi\rangle|^2$$

de Broglie Bohm: $\lambda\in\{(|\psi\rangle,x)\}$

$$\mu_\psi(\lambda)=(\delta_{|\phi\rangle,|\psi\rangle},|\langle\phi|X|\phi\rangle|^2)$$

$\epsilon_M(\phi|\lambda)\in\{0,1\}$ *contextually*

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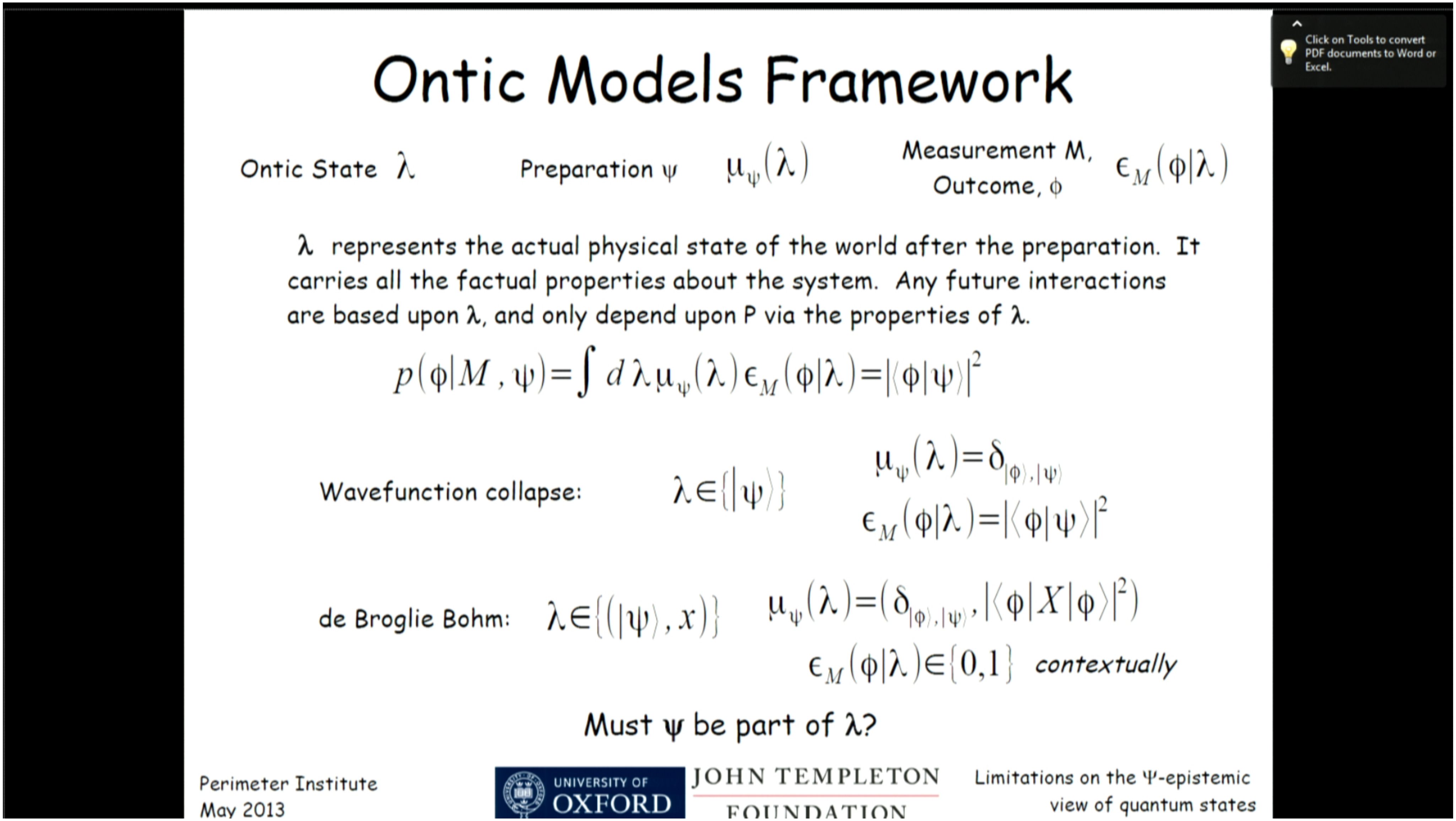

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Must ψ be part of λ?

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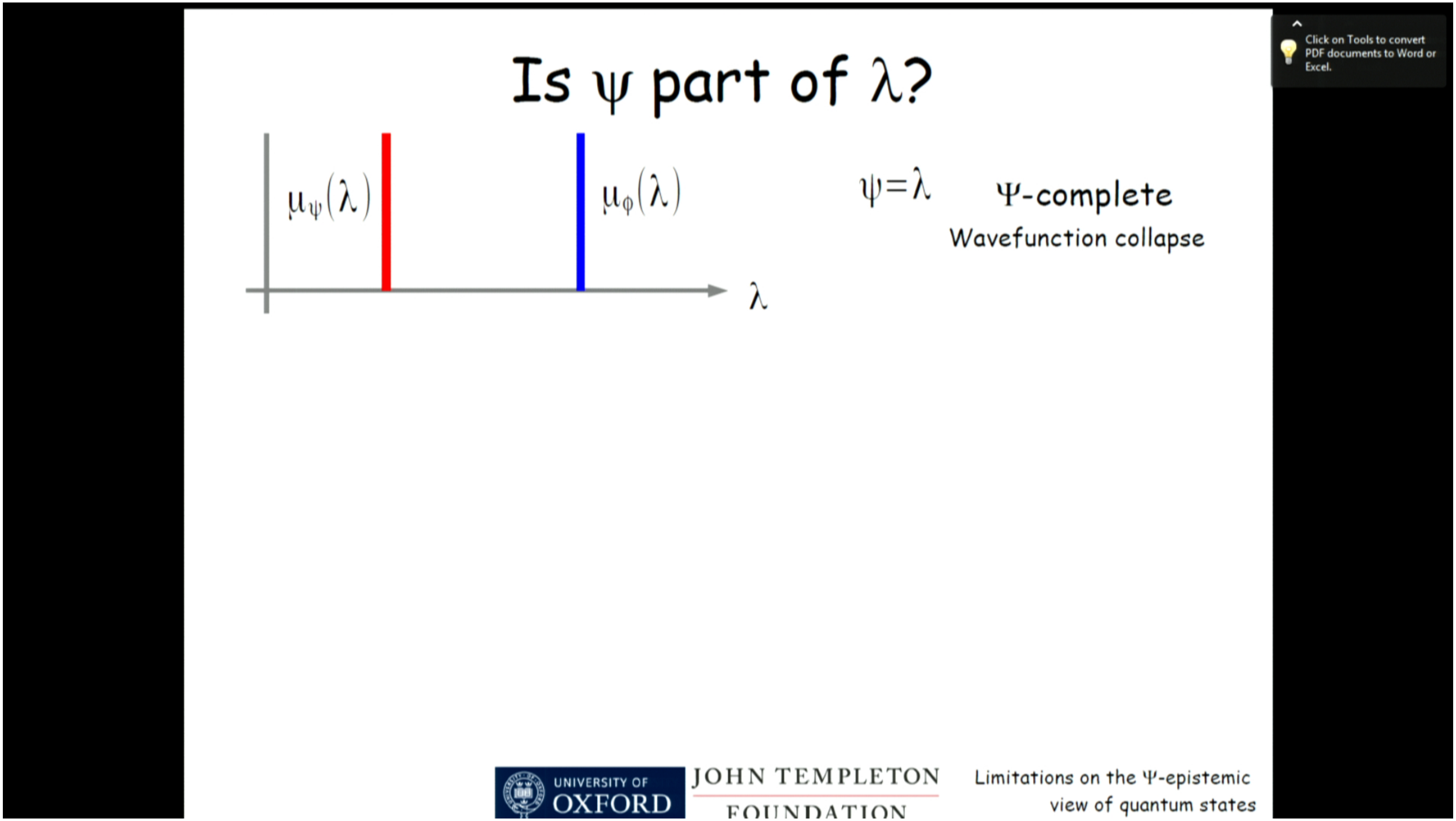

Is ψ part of λ?

$\psi = \lambda$ Ψ-complete

Wavefunction collapse

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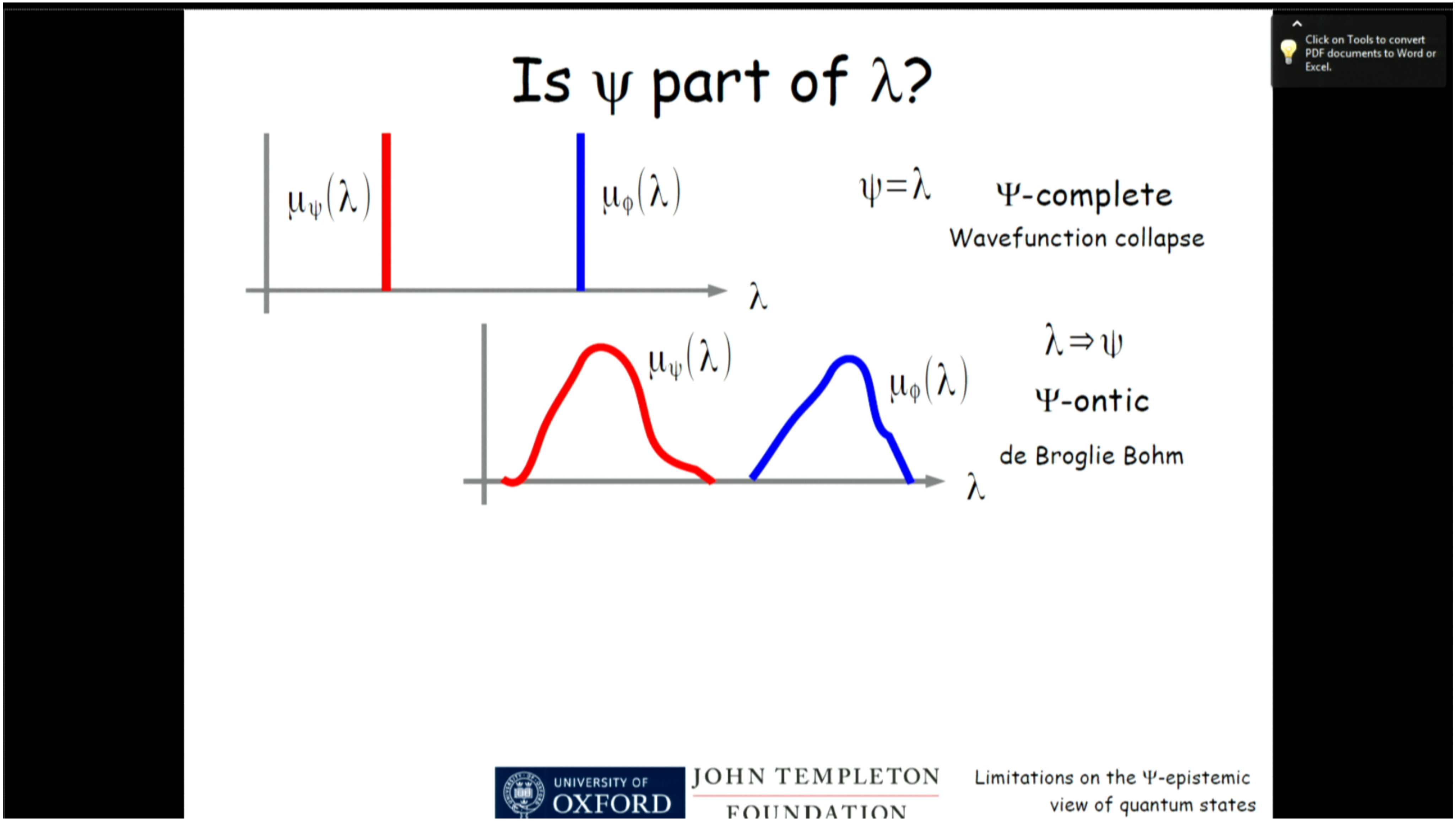

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Is ψ part of λ?

$\psi = \lambda$ **Ψ-complete**

Wavefunction collapse

$\lambda \Rightarrow \psi$

Ψ-ontic

de Broglie Bohm

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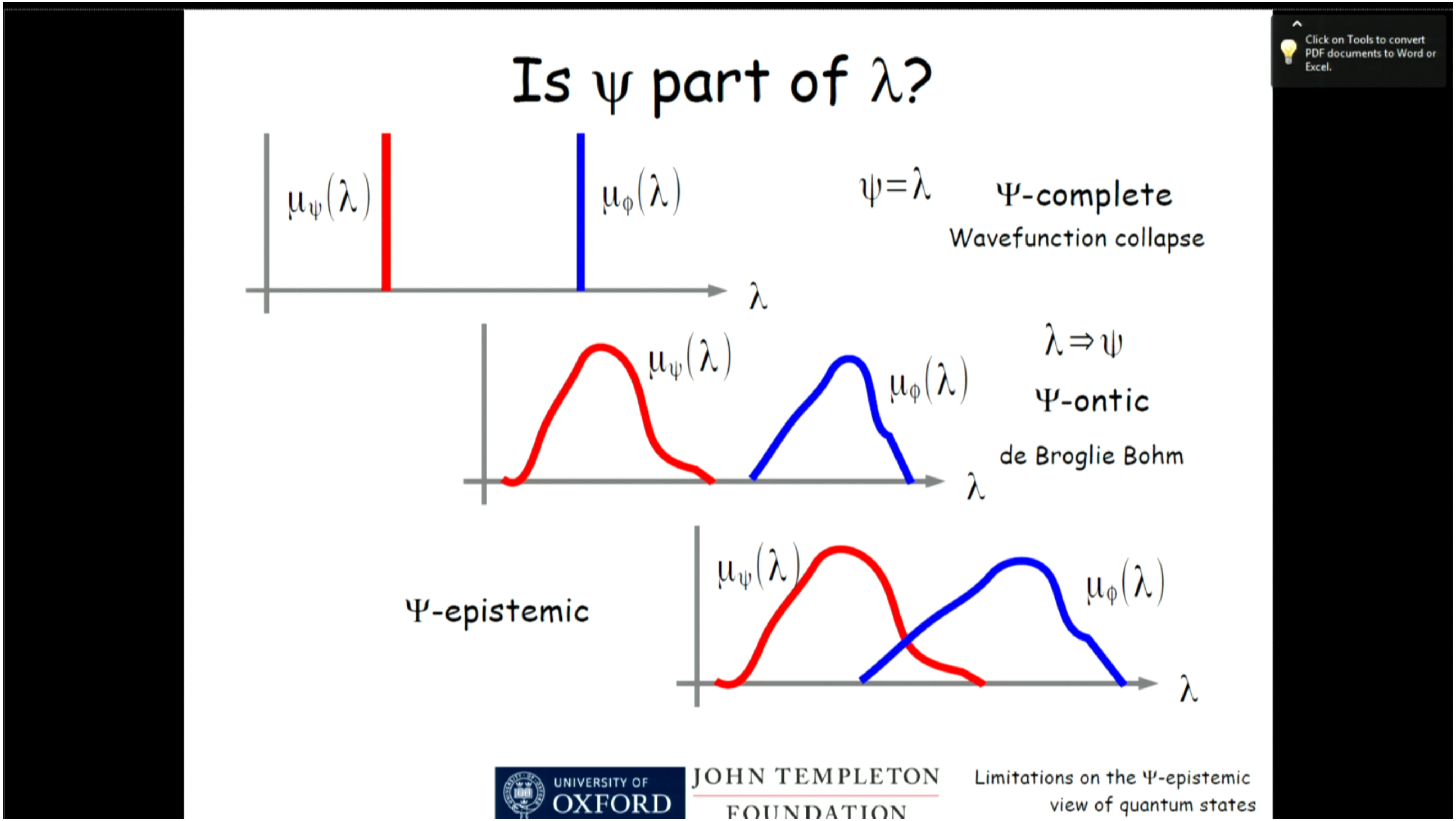

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Ψ-epistemic

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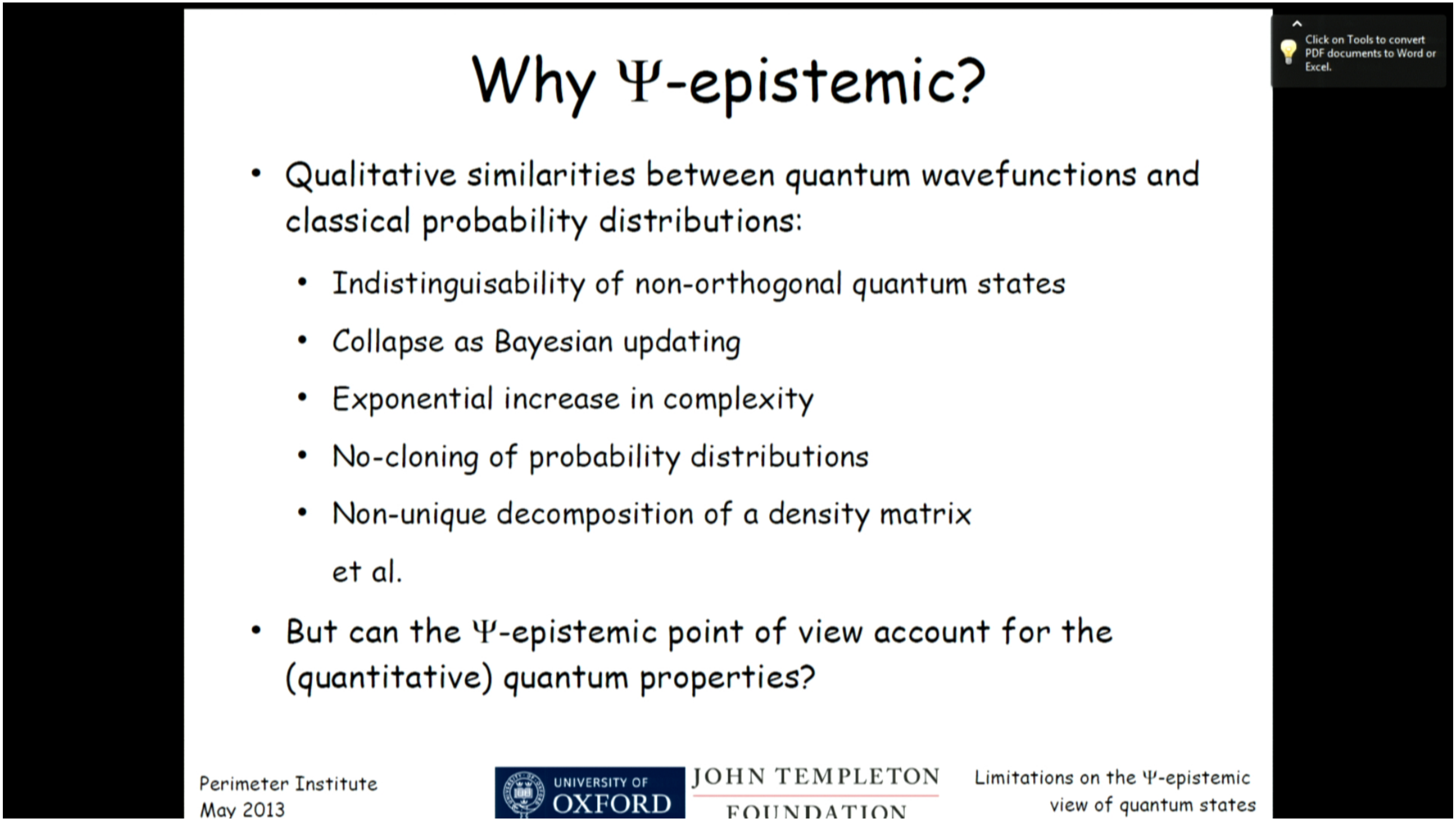

Why Ψ-epistemic?

- Qualitative similarities between quantum wavefunctions and classical probability distributions:
  - Indistinguisability of non-orthogonal quantum states
  - Collapse as Bayesian updating
  - Exponential increase in complexity
  - No-cloning of probability distributions
  - Non-unique decomposition of a density matrix

  et al.
- But can the Ψ-epistemic point of view account for the (quantitative) quantum properties?

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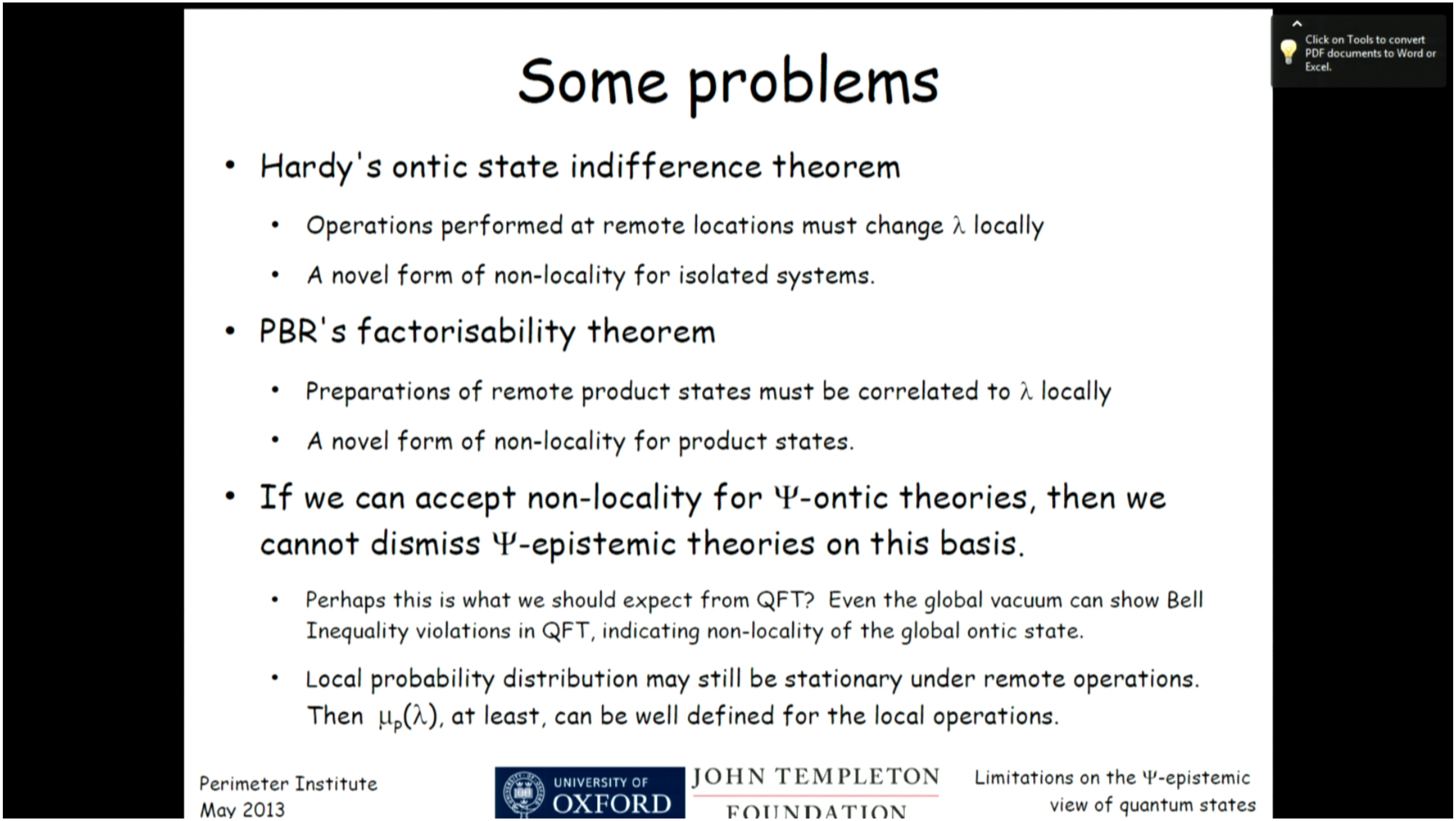

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Some problems

- Hardy's ontic state indifference theorem
  - Operations performed at remote locations must change λ locally
  - A novel form of non-locality for isolated systems.
- PBR's factorisability theorem
  - Preparations of remote product states must be correlated to λ locally
  - A novel form of non-locality for product states.
- If we can accept non-locality for Ψ-ontic theories, then we cannot dismiss Ψ-epistemic theories on this basis.
  - Perhaps this is what we should expect from QFT? Even the global vacuum can show Bell Inequality violations in QFT, indicating non-locality of the global ontic state.
  - Local probability distribution may still be stationary under remote operations. Then $\mu_p(\lambda)$, at least, can be well defined for the local operations.

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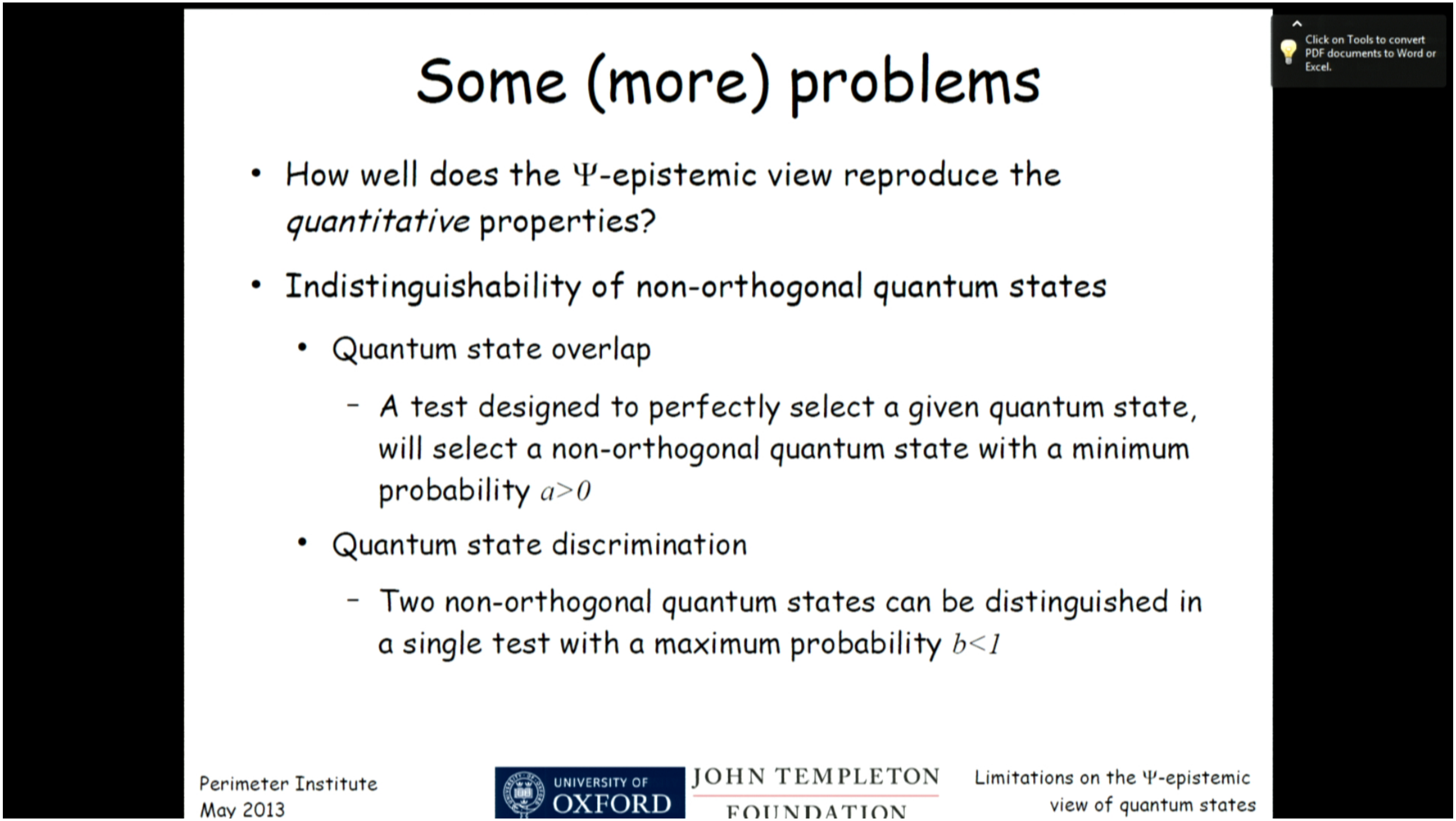

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Some (more) problems

- How well does the Ψ-epistemic view reproduce the *quantitative* properties?
- Indistinguishability of non-orthogonal quantum states
  - Quantum state overlap
    - A test designed to perfectly select a given quantum state, will select a non-orthogonal quantum state with a minimum probability $a>0$
  - Quantum state discrimination
    - Two non-orthogonal quantum states can be distinguished in a single test with a maximum probability $b<1$

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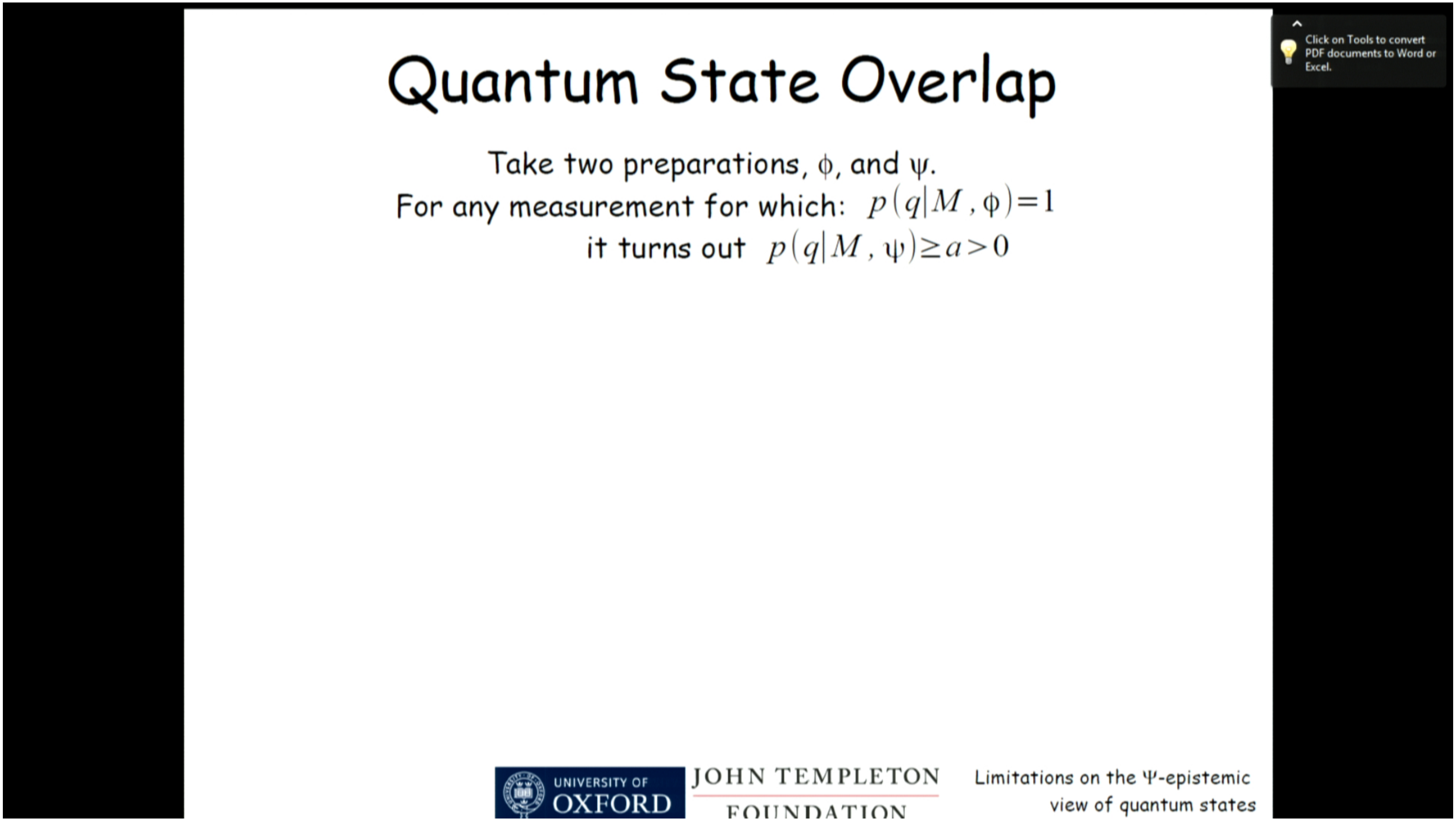

Quantum State Overlap

Take two preparations, ϕ, and ψ.

For any measurement for which: $p(q|M,\phi)=1$

it turns out $p(q|M,\psi)\geq a>0$

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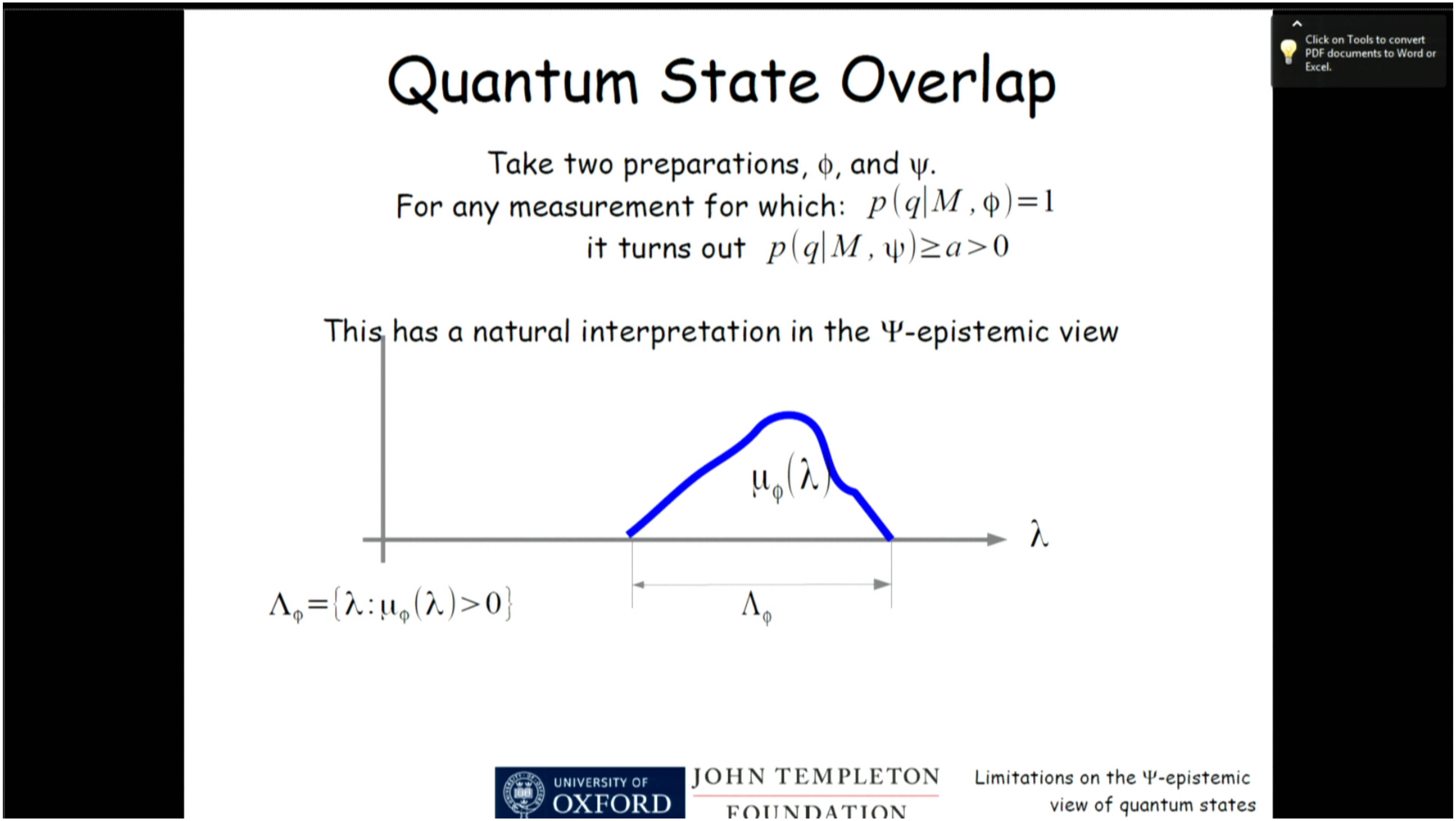

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This has a natural interpretation in the Ψ-epistemic view

$\Lambda_\phi=\{\lambda:\mu_\phi(\lambda)>0\}$

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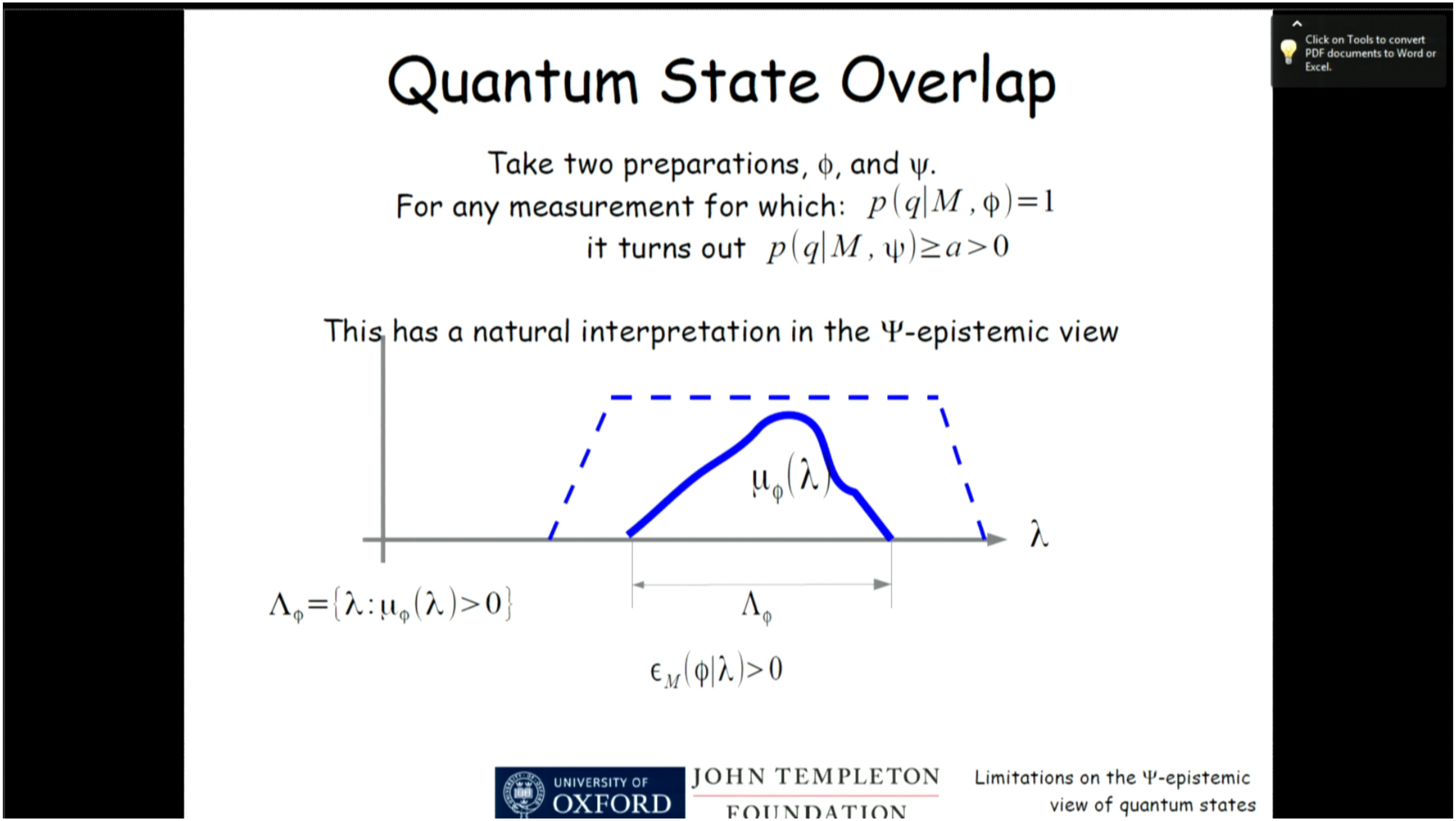

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$\epsilon_M(\phi|\lambda)>0$

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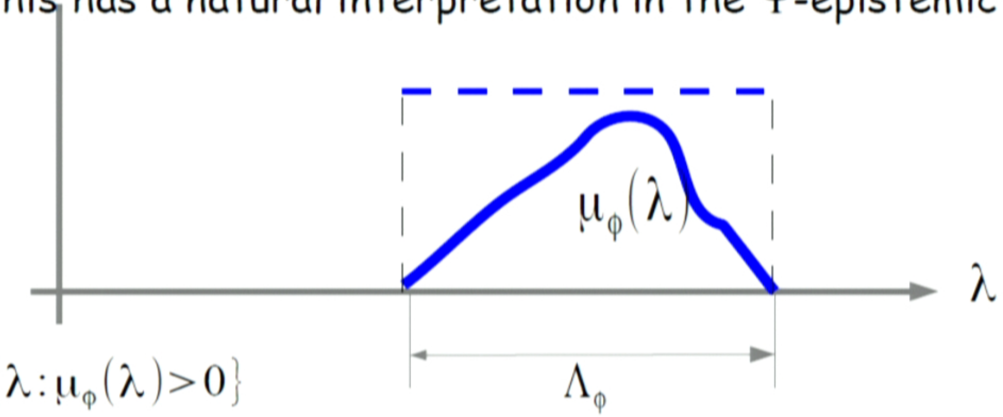

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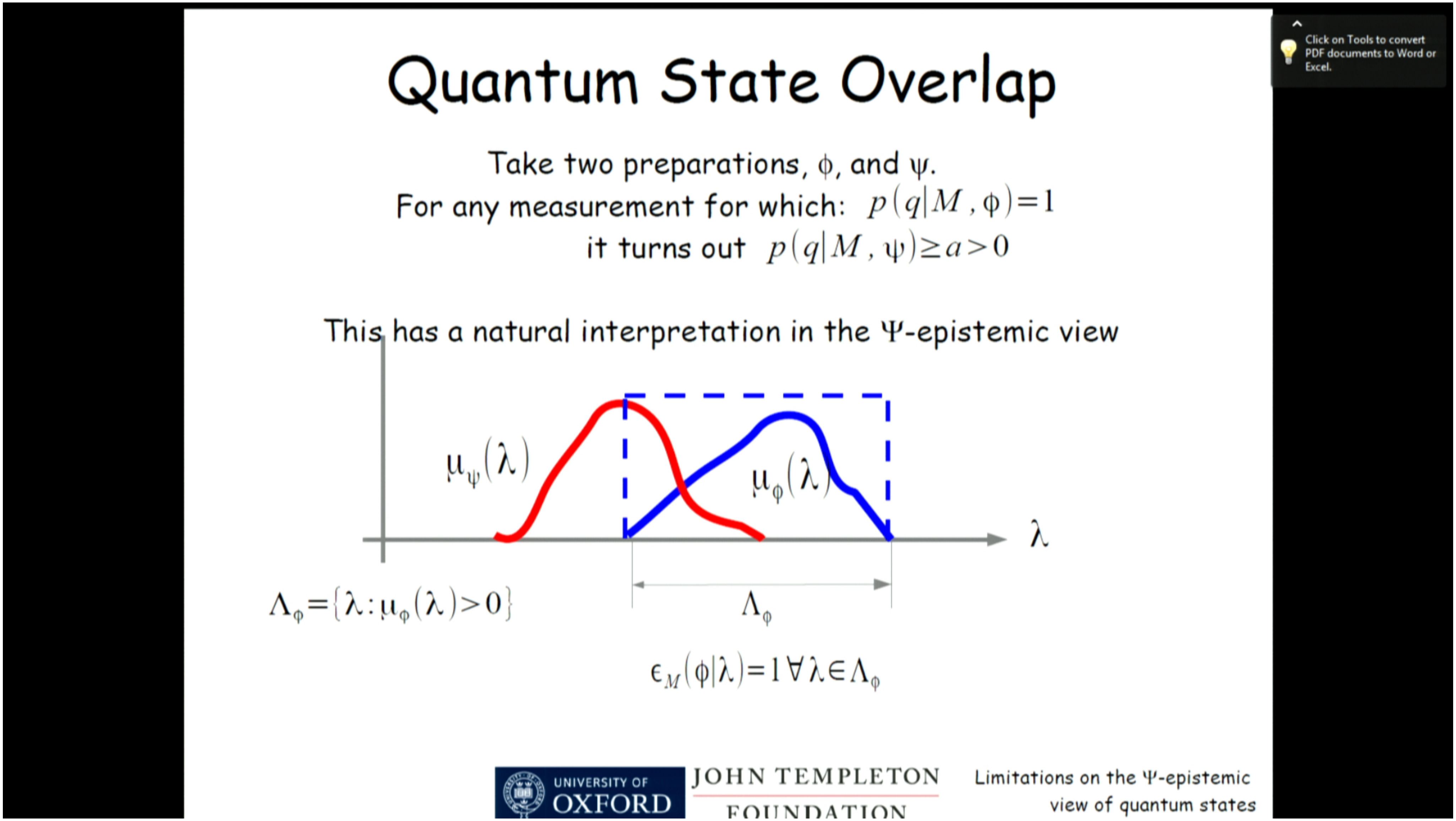

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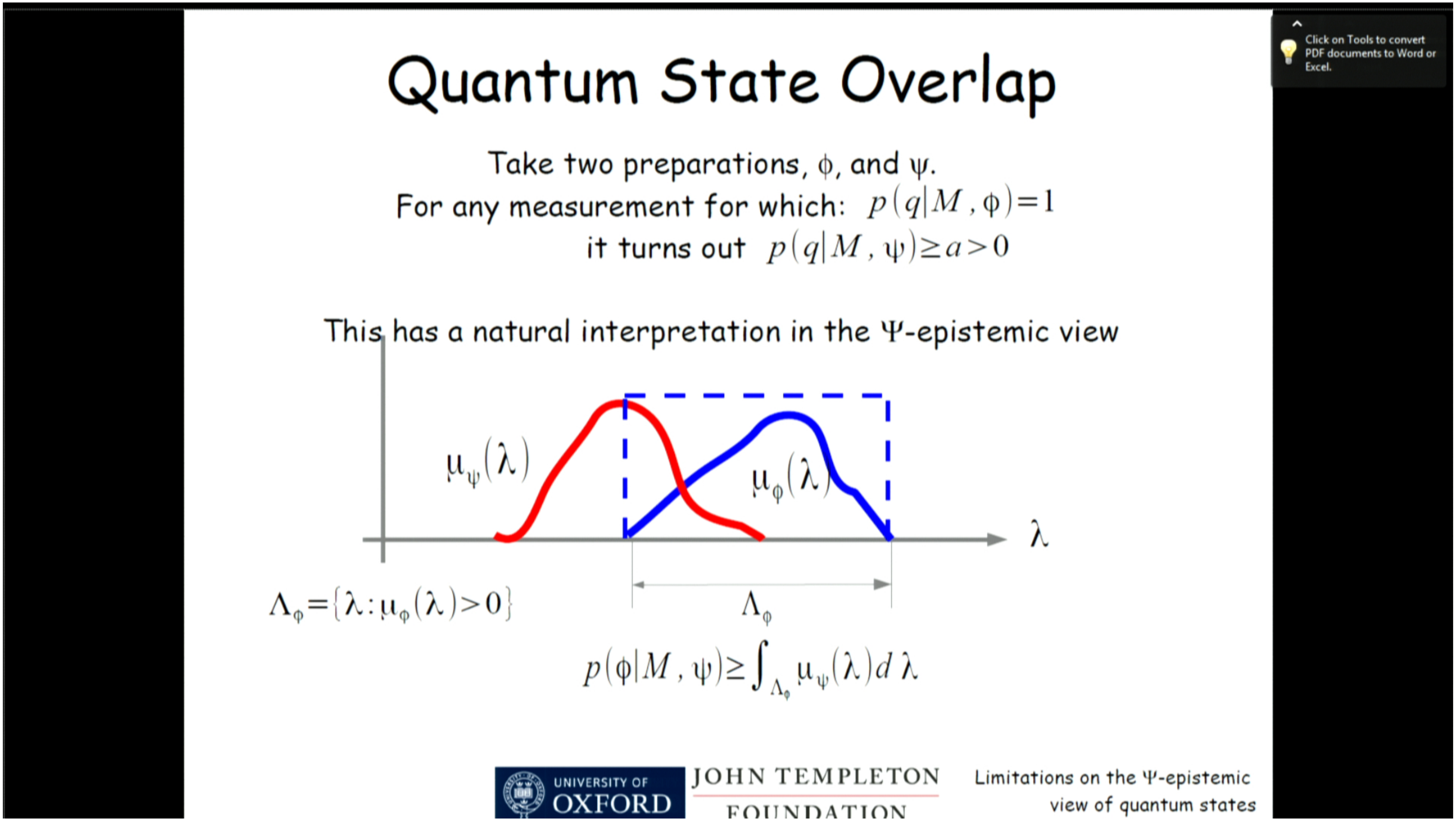

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For any measurement for which: $p(q|M,\phi)=1$

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This has a natural interpretation in the Ψ-epistemic view

$\Lambda_\phi=\{\lambda:\mu_\phi(\lambda)>0\}$

$$p(\phi|M,\psi)\geq\int_{\Lambda_\phi}\mu_\psi(\lambda)d\lambda$$

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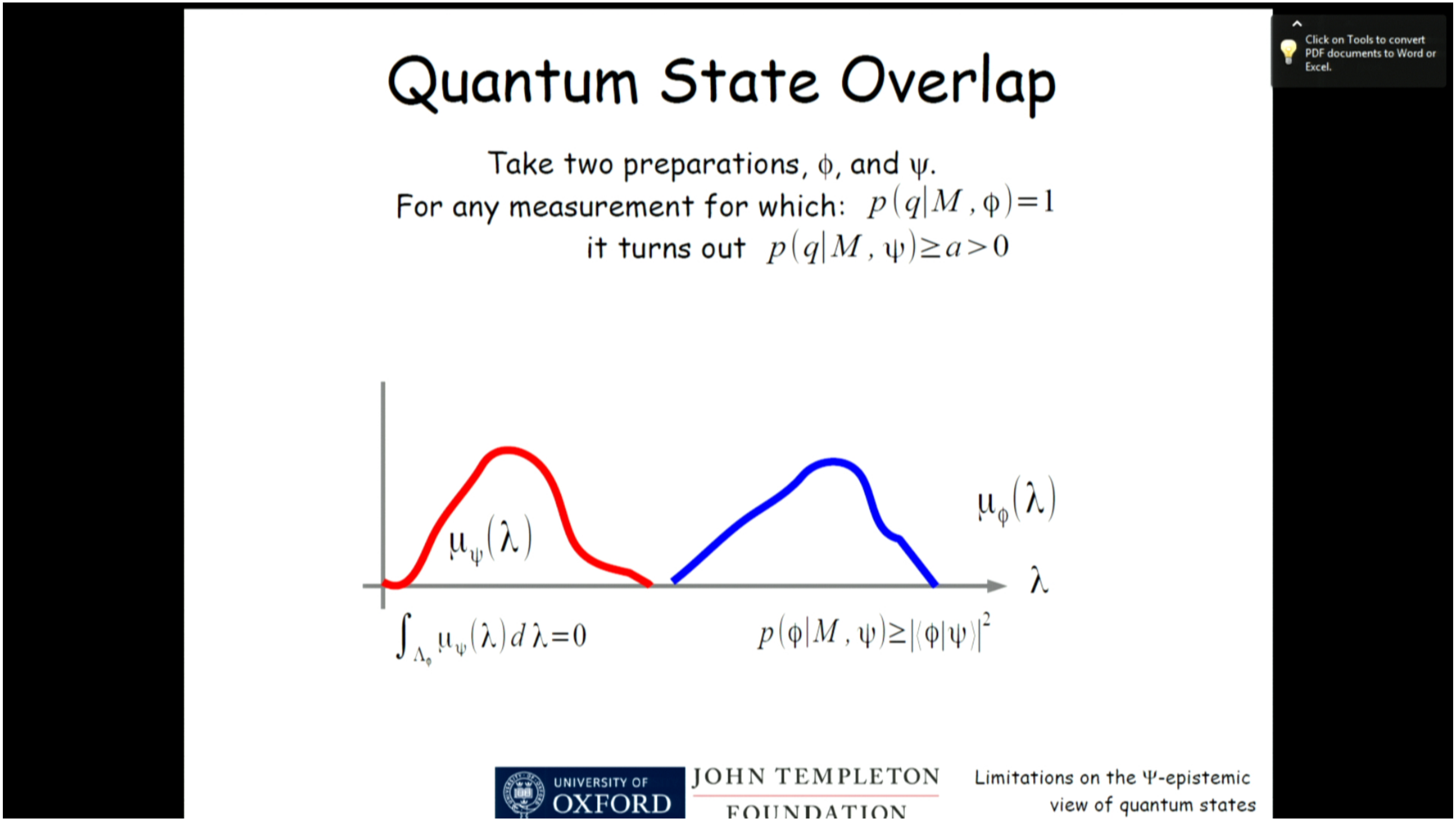

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For any measurement for which: $p(q|M,\phi)=1$

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$\int_{\Lambda_\phi}\mu_\psi(\lambda)d\lambda=0$

$p(\phi|M,\psi)\geq|\langle\phi|\psi\rangle|^2$

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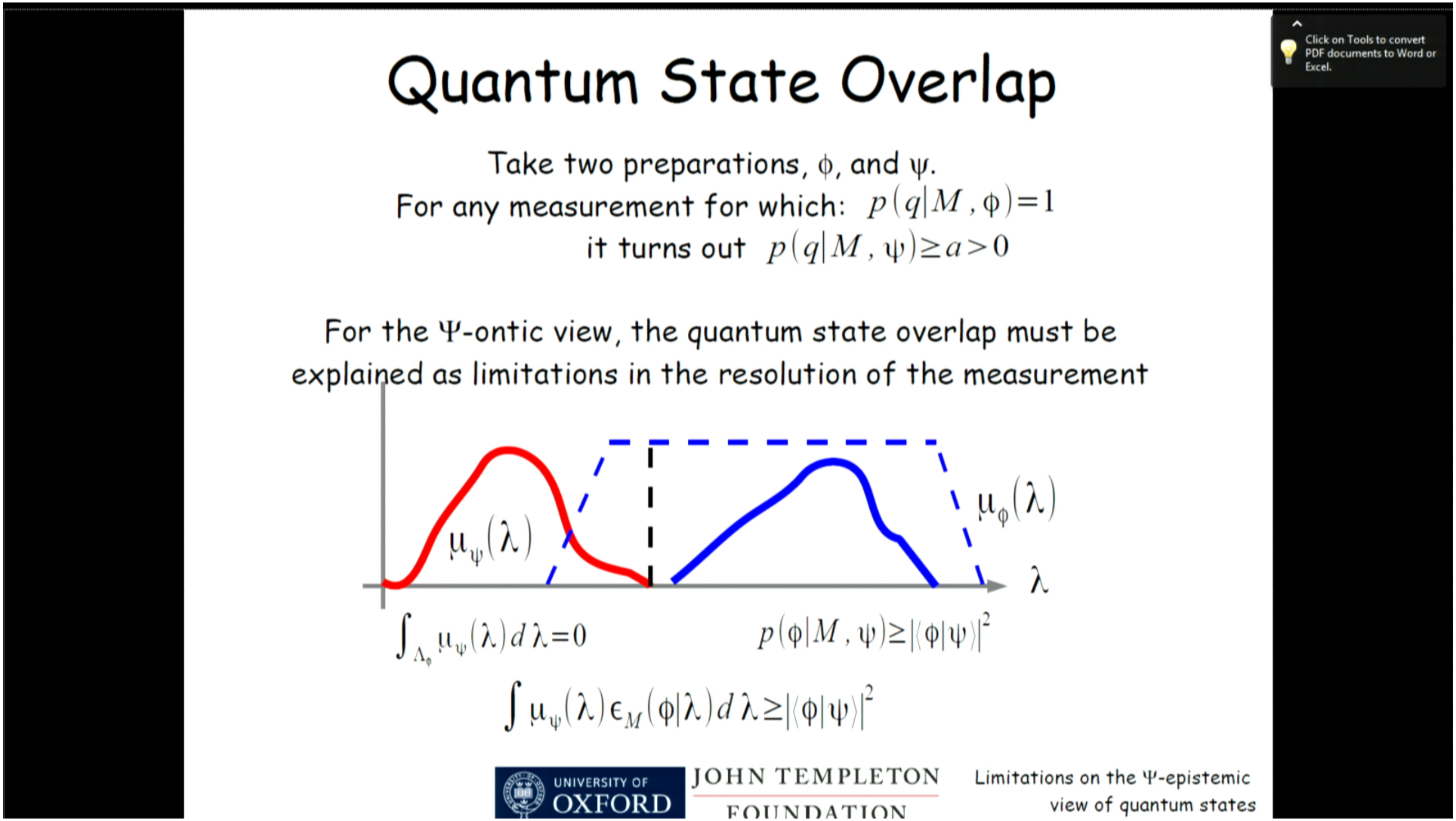

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Quantum State Overlap

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For the Ψ-ontic view, the quantum state overlap must be explained as limitations in the resolution of the measurement

$$\int_{\Lambda_\phi}\mu_\psi(\lambda)d\lambda=0$$

$$p(\phi|M,\psi)\geq|\langle\phi|\psi\rangle|^2$$

$$\int\mu_\psi(\lambda)\epsilon_M(\phi|\lambda)d\lambda\geq|\langle\phi|\psi\rangle|^2$$

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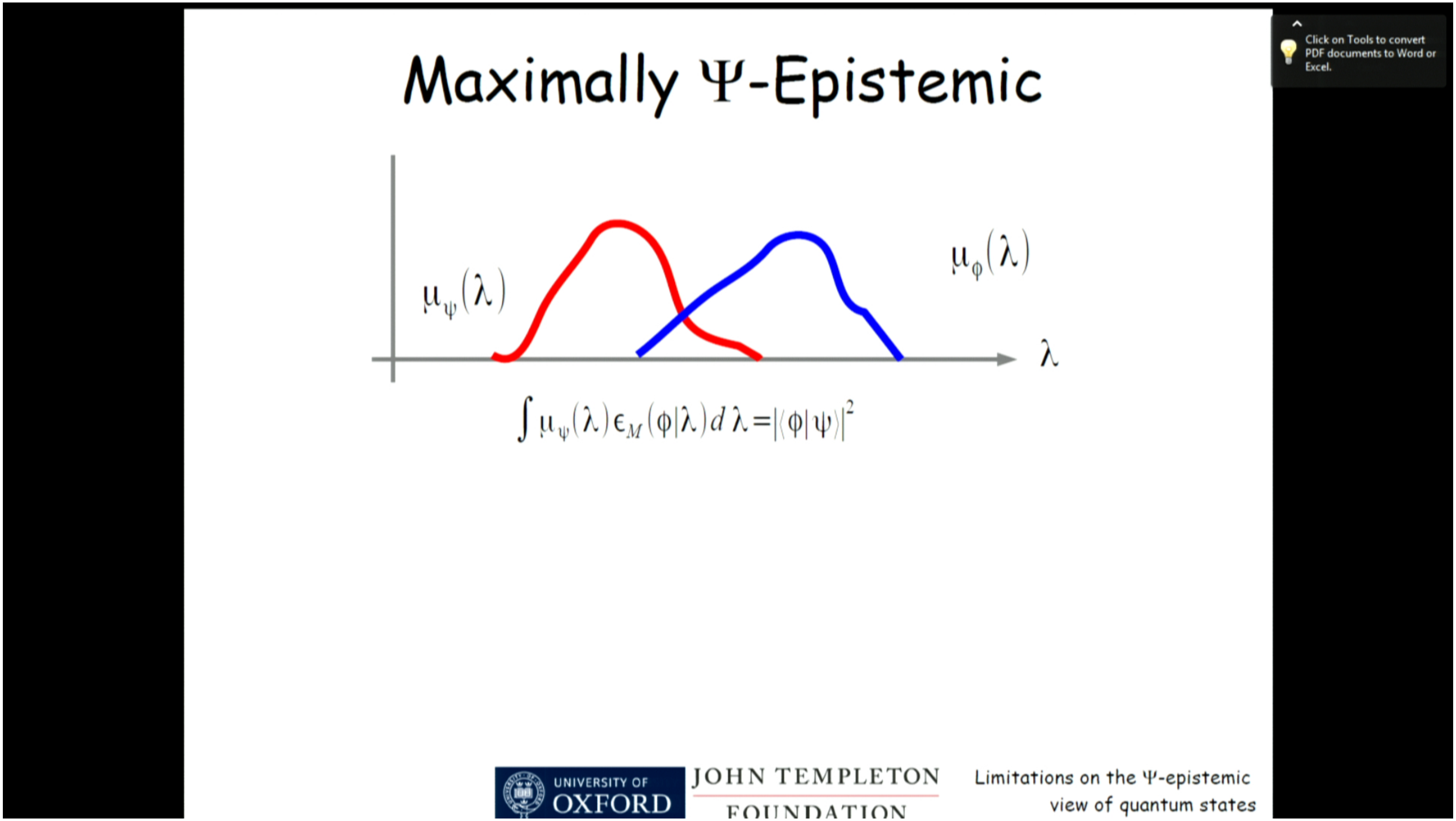

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Maximally Ψ-Epistemic

$$\int \mu_\psi(\lambda)\epsilon_M(\phi|\lambda)d\lambda = |\langle\phi|\psi\rangle|^2$$

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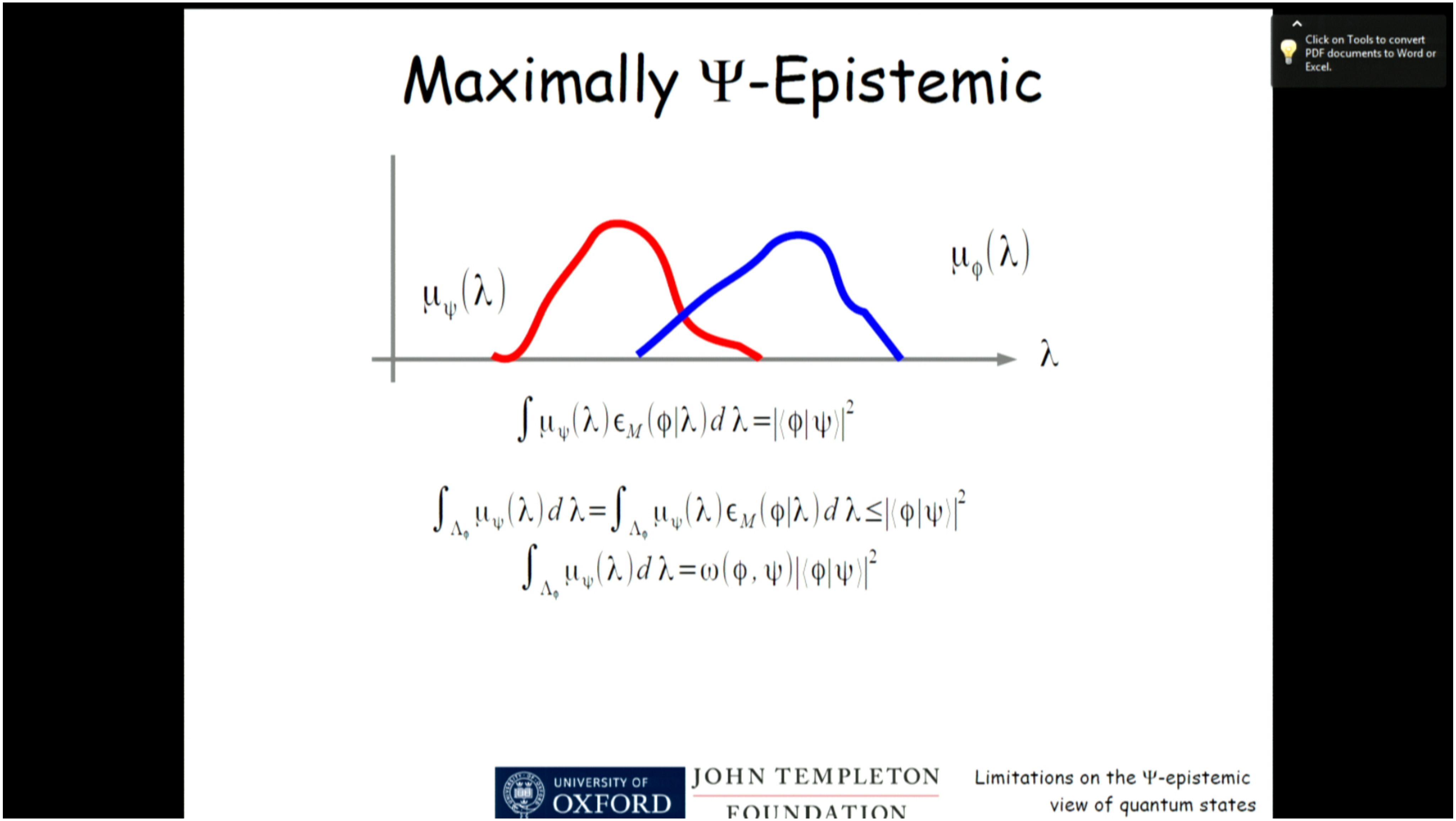

Maximally Ψ-Epistemic

$$\int \mu_\psi(\lambda)\epsilon_M(\phi|\lambda)\,d\lambda = |\langle\phi|\psi\rangle|^2$$

$$\int_{\Lambda_\phi} \mu_\psi(\lambda)\,d\lambda = \int_{\Lambda_\phi} \mu_\psi(\lambda)\epsilon_M(\phi|\lambda)\,d\lambda \leq |\langle\phi|\psi\rangle|^2$$

$$\int_{\Lambda_\phi} \mu_\psi(\lambda)\,d\lambda = \omega(\phi,\psi)|\langle\phi|\psi\rangle|^2$$

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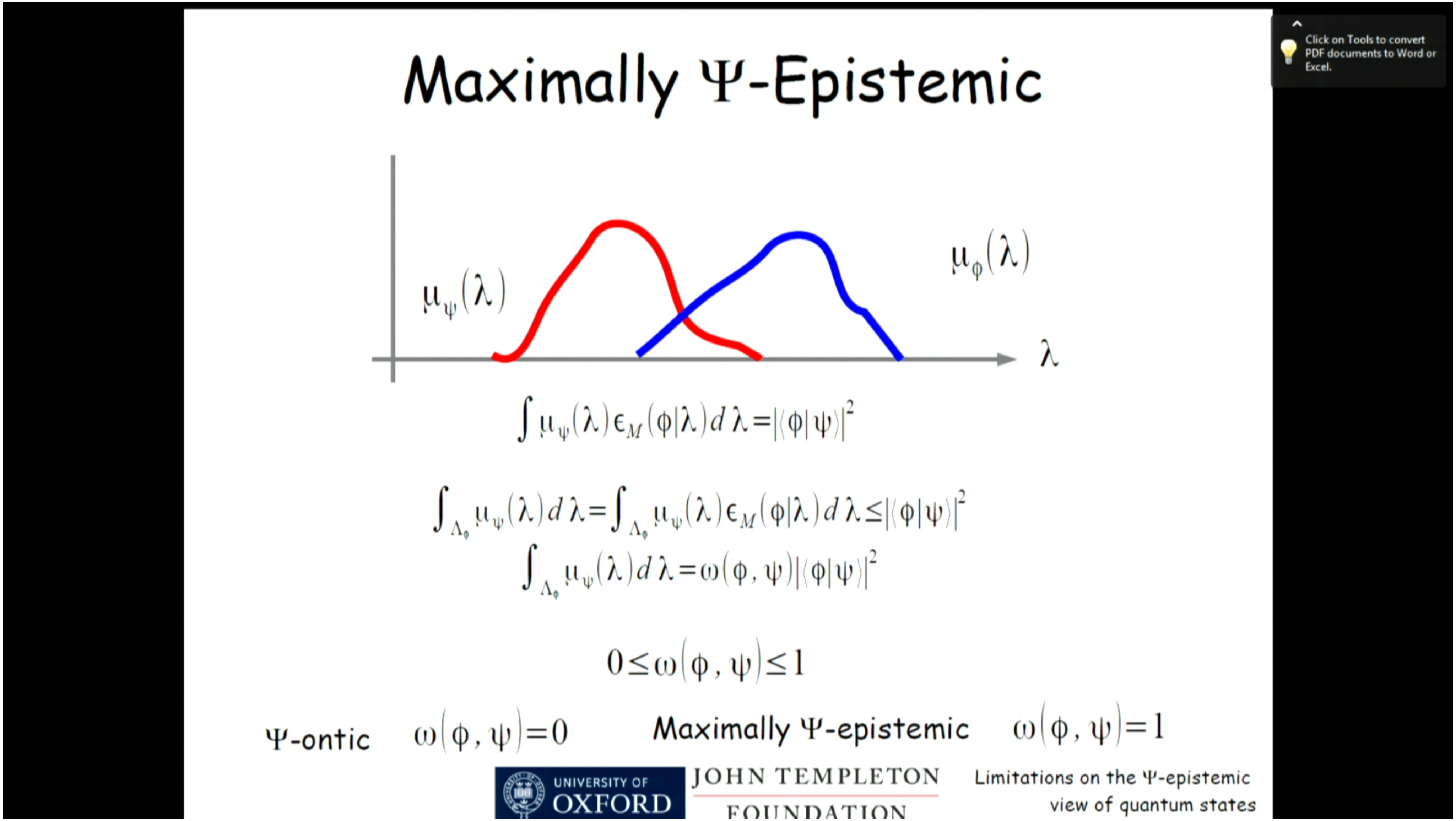

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$$\int \mu_\psi(\lambda)\epsilon_M(\phi|\lambda)d\lambda=|\langle\phi|\psi\rangle|^2$$

$$\int_{\Lambda_\phi}\mu_\psi(\lambda)d\lambda=\int_{\Lambda_\phi}\mu_\psi(\lambda)\epsilon_M(\phi|\lambda)d\lambda\le|\langle\phi|\psi\rangle|^2$$
$$\int_{\Lambda_\phi}\mu_\psi(\lambda)d\lambda=\omega(\phi,\psi)|\langle\phi|\psi\rangle|^2$$

$$0\le\omega(\phi,\psi)\le1$$

Ψ-ontic $\omega(\phi,\psi)=0$ Maximally Ψ-epistemic $\omega(\phi,\psi)=1$

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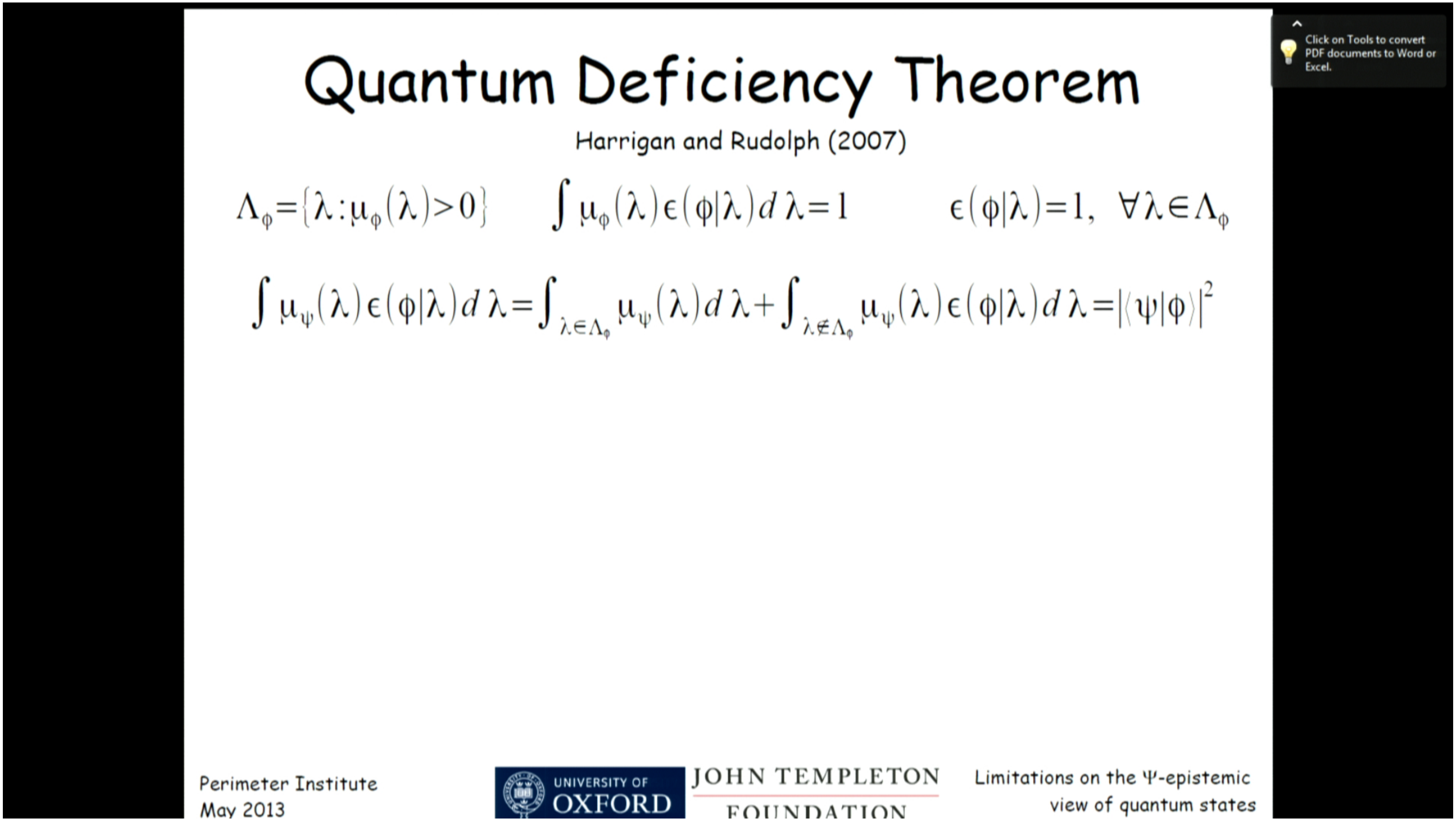

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Quantum Deficiency Theorem

Harrigan and Rudolph (2007)

$$\Lambda_\phi = \{\lambda : \mu_\phi(\lambda) > 0\} \qquad \int \mu_\phi(\lambda)\epsilon(\phi|\lambda)\,d\lambda = 1 \qquad \epsilon(\phi|\lambda) = 1, \ \forall \lambda \in \Lambda_\phi$$

$$\int \mu_\psi(\lambda)\epsilon(\phi|\lambda)\,d\lambda = \int_{\lambda\in\Lambda_\phi} \mu_\psi(\lambda)\,d\lambda + \int_{\lambda\notin\Lambda_\phi} \mu_\psi(\lambda)\epsilon(\phi|\lambda)\,d\lambda = |\langle\psi|\phi\rangle|^2$$

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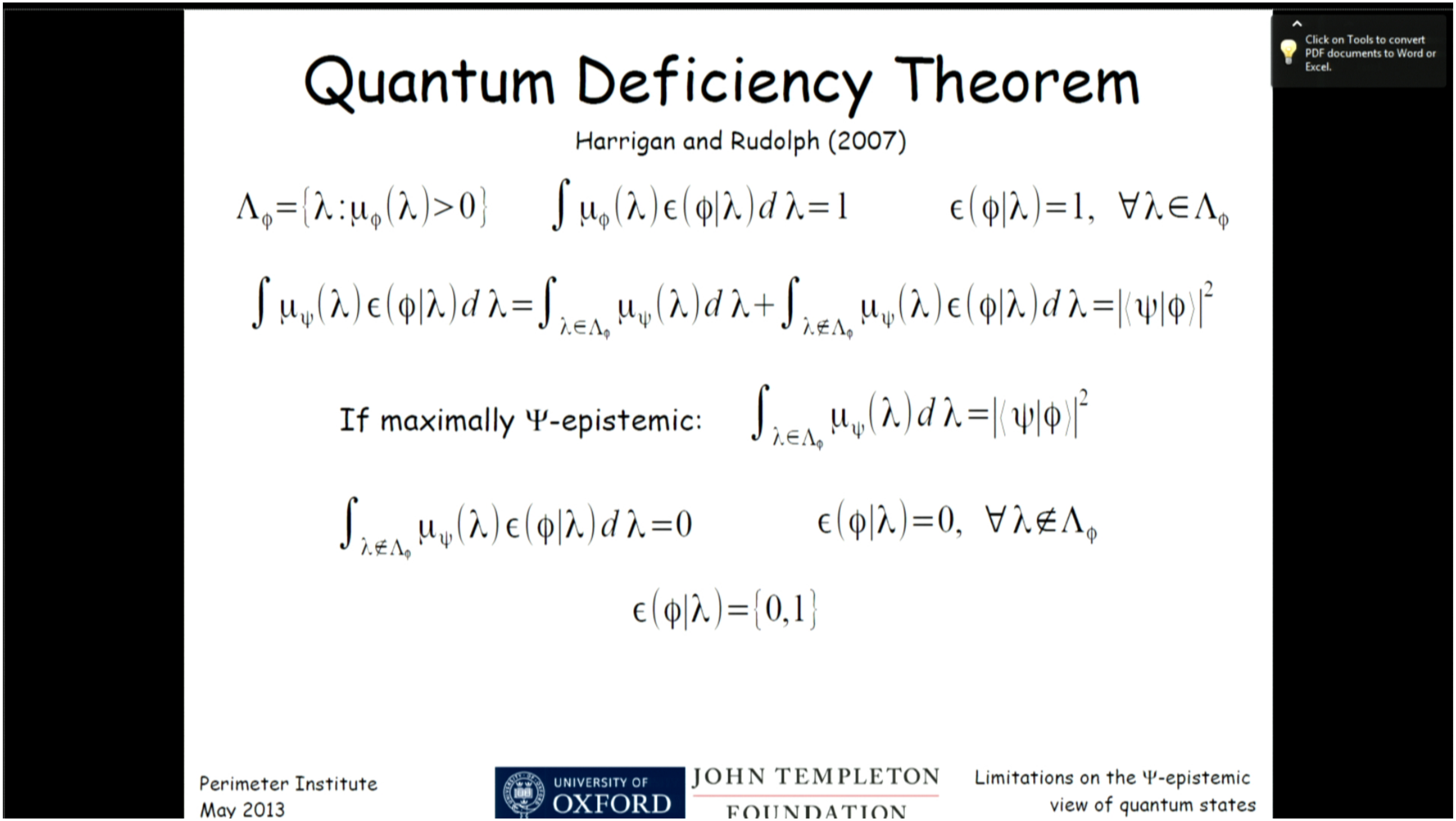

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If maximally Ψ-epistemic: $\int_{\lambda\in\Lambda_\phi} \mu_\psi(\lambda)\,d\lambda = |\langle\psi|\phi\rangle|^2$

$$\int_{\lambda\notin\Lambda_\phi} \mu_\psi(\lambda)\,\epsilon(\phi|\lambda)\,d\lambda = 0 \qquad \epsilon(\phi|\lambda) = 0, \ \forall \lambda \notin \Lambda_\phi$$

$$\epsilon(\phi|\lambda) = \{0,1\}$$

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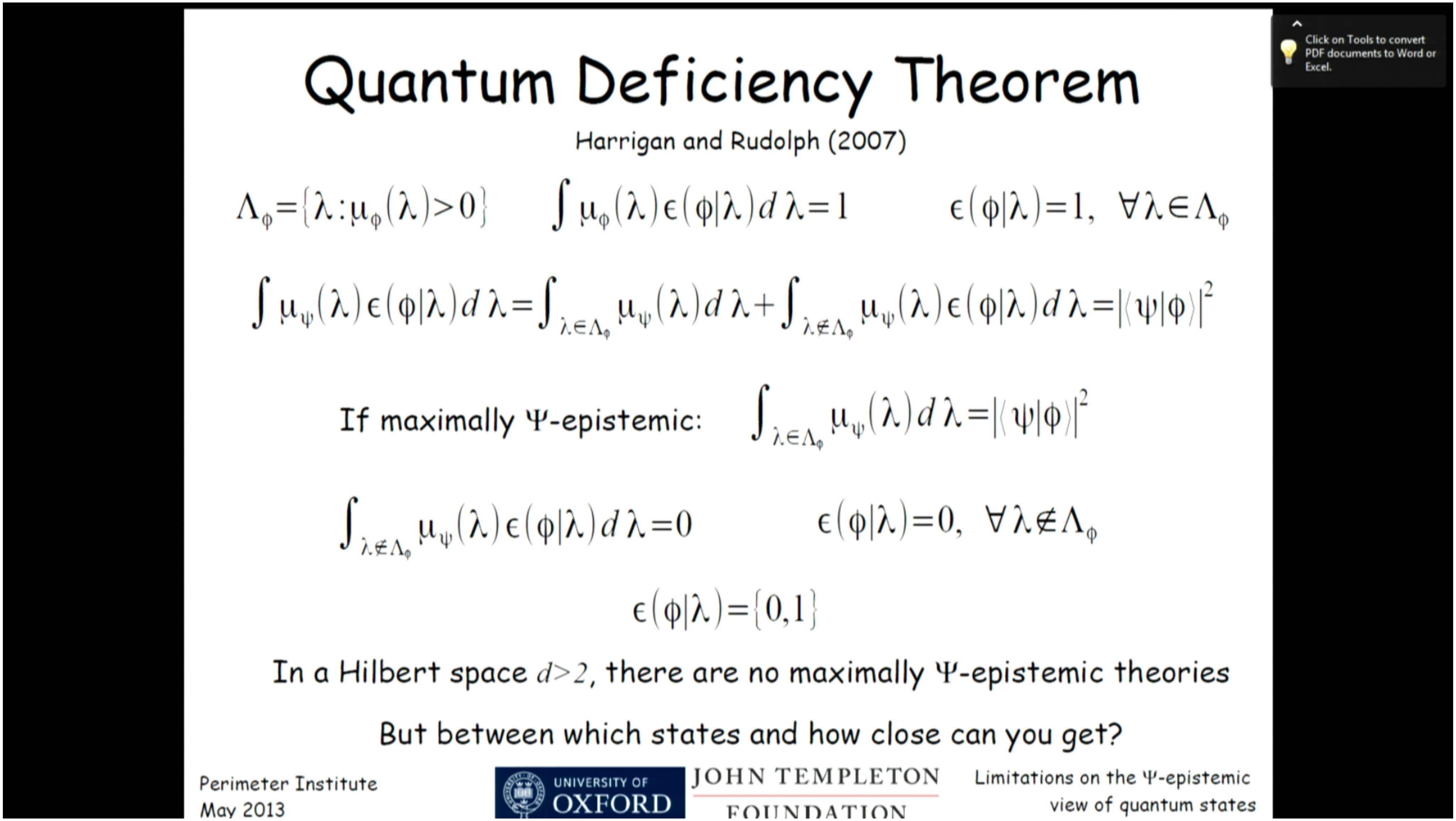

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In a Hilbert space $d>2$, there are no maximally Ψ-epistemic theories

But between which states and how close can you get?

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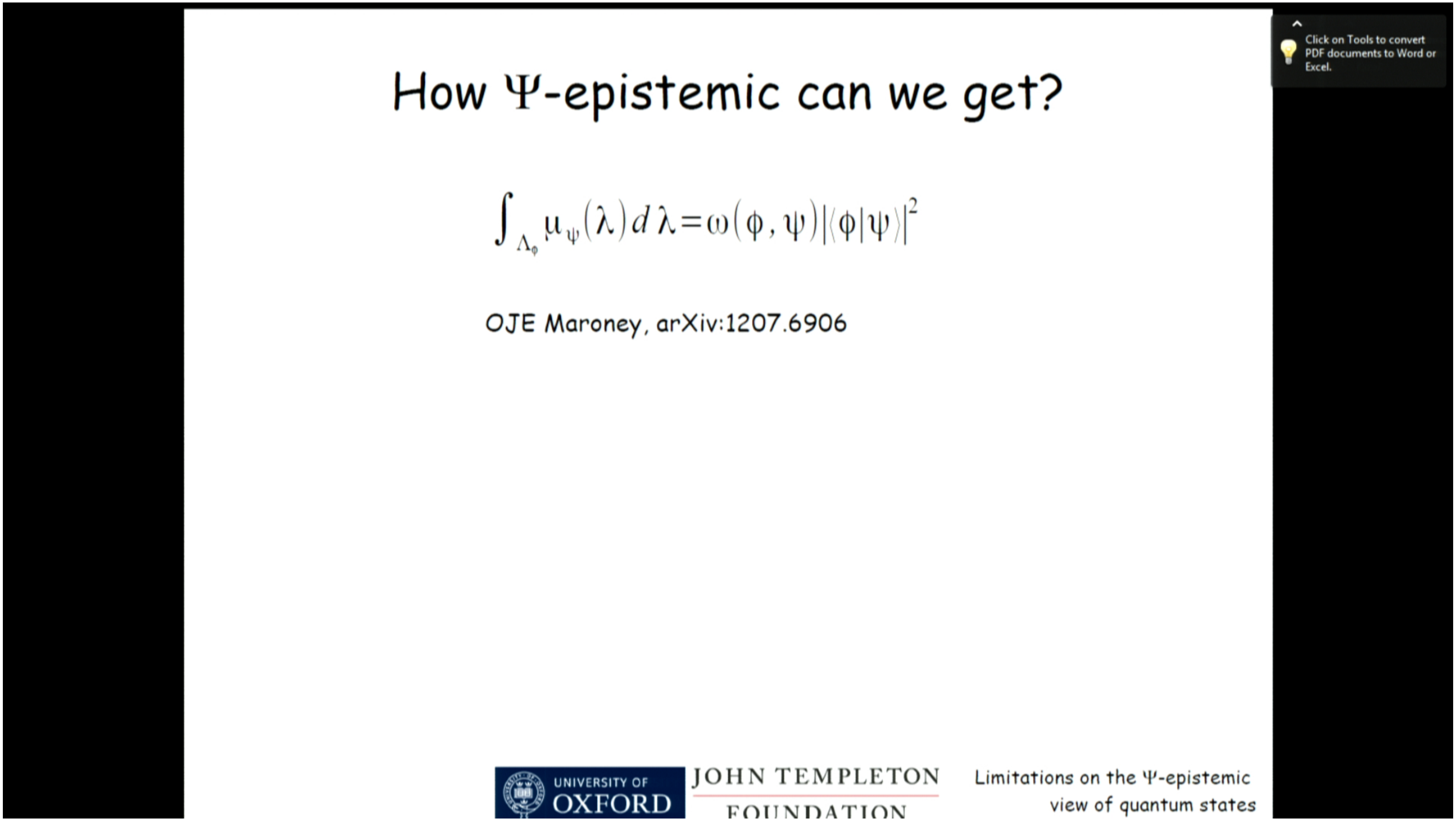

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How Ψ-epistemic can we get?

$$\int_{\Lambda_\phi} \mu_\psi(\lambda)\, d\lambda = \omega(\phi, \psi)|\langle\phi|\psi\rangle|^2$$

OJE Maroney, arXiv:1207.6906

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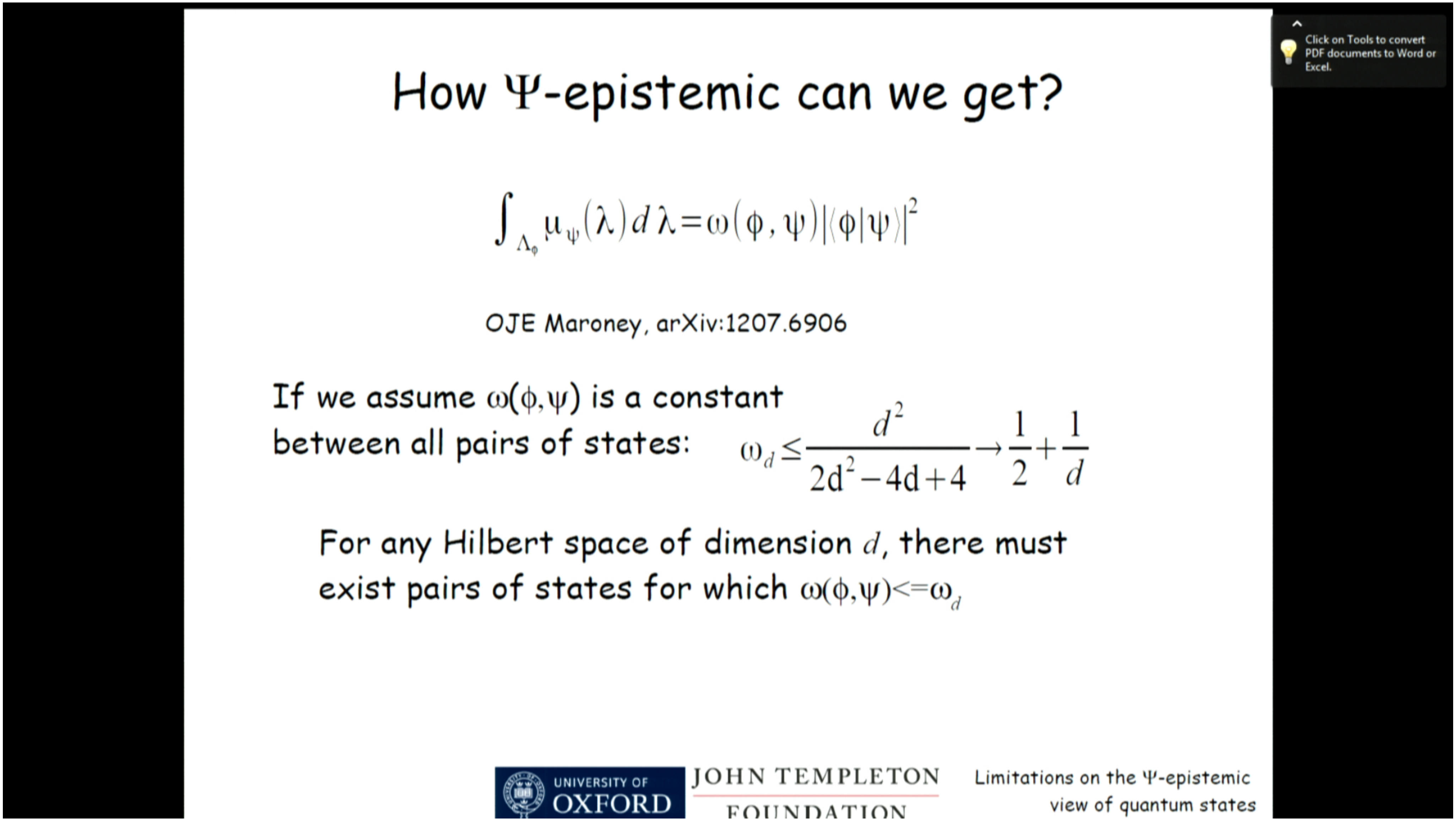

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$$\int_{\Lambda_\phi} \mu_\psi(\lambda)\, d\lambda = \omega(\phi,\psi)|\langle\phi|\psi\rangle|^2$$

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If we assume $\omega(\phi,\psi)$ is a constant between all pairs of states:

$$\omega_d \leq \frac{d^2}{2d^2-4d+4} \to \frac{1}{2}+\frac{1}{d}$$

For any Hilbert space of dimension d, there must exist pairs of states for which $\omega(\phi,\psi) <= \omega_d$

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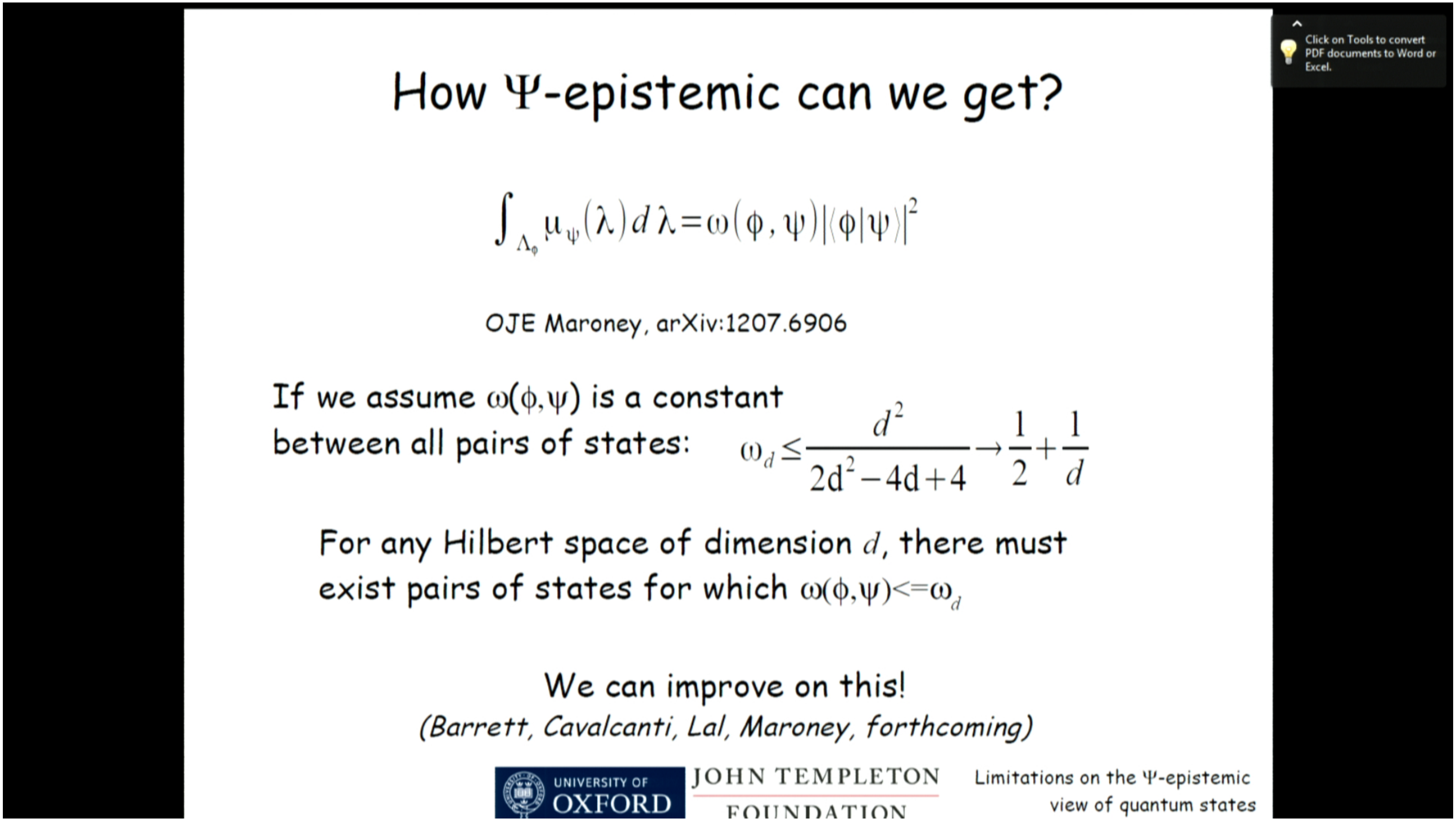

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We can improve on this!

(Barrett, Cavalcanti, Lal, Maroney, forthcoming)

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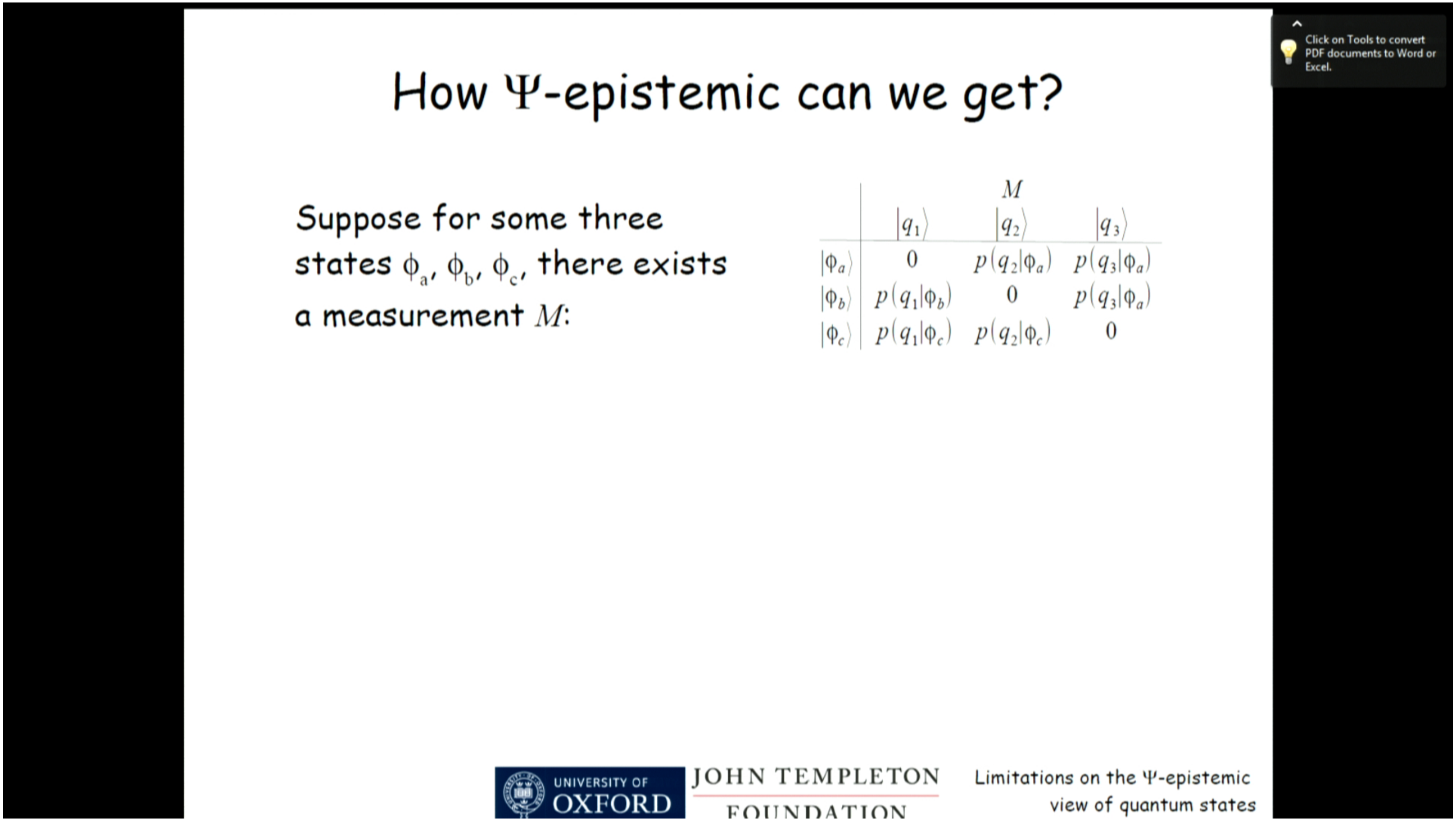

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Suppose for some three states ϕ_a, ϕ_b, ϕ_c, there exists a measurement M:

		M	
	$\lvert q_1\rangle$	$\lvert q_2\rangle$	$\lvert q_3\rangle$
$\lvert\phi_a\rangle$	0	$p(q_2\lvert\phi_a)$	$p(q_3\lvert\phi_a)$
$\lvert\phi_b\rangle$	$p(q_1\lvert\phi_b)$	0	$p(q_3\lvert\phi_a)$
$\lvert\phi_c\rangle$	$p(q_1\lvert\phi_c)$	$p(q_2\lvert\phi_c)$	0

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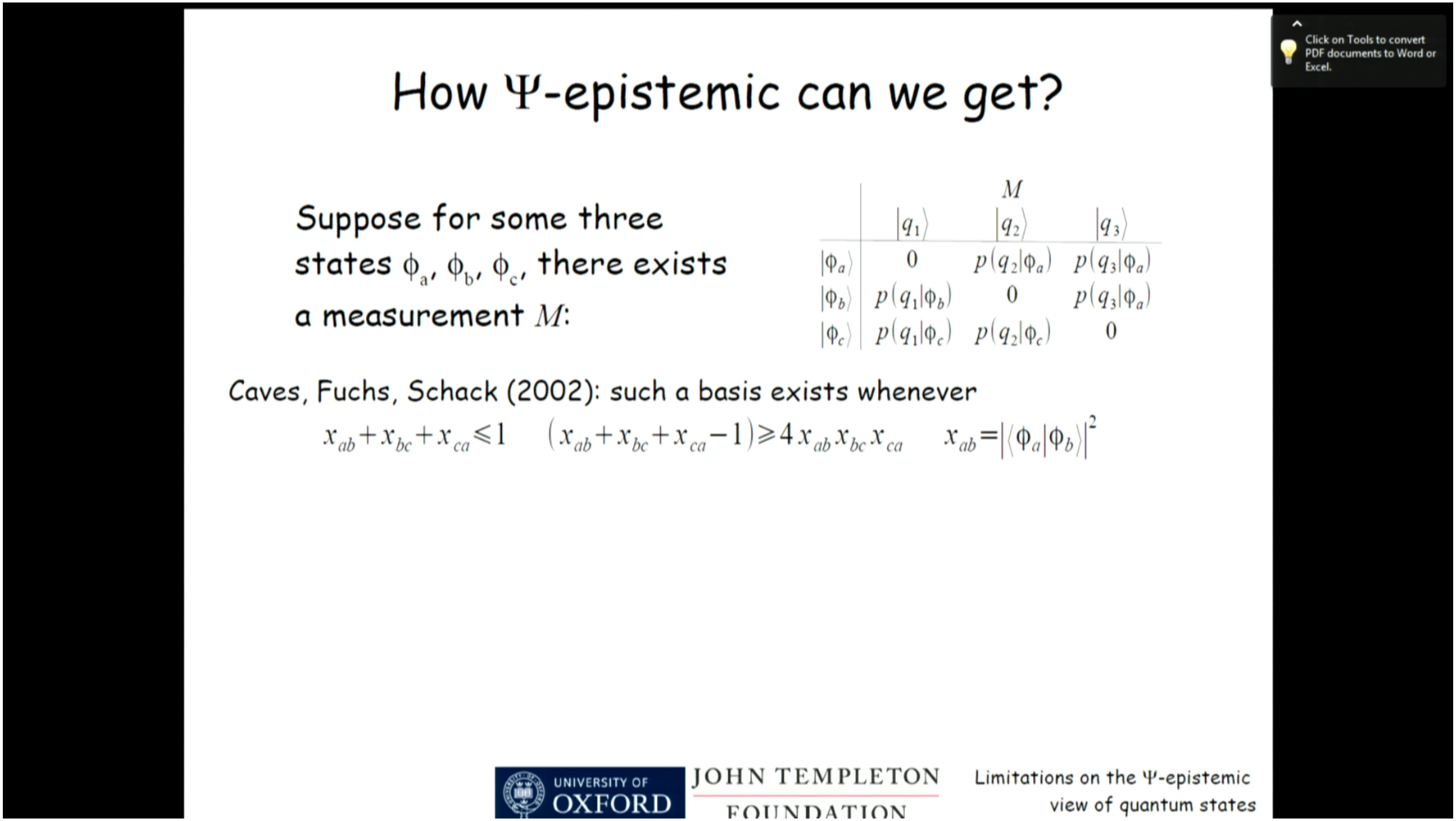

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$\|\phi_c\rangle$	$p(q_1\|\phi_c)$	$p(q_2\|\phi_c)$	0

Caves, Fuchs, Schack (2002): such a basis exists whenever

$$x_{ab}+x_{bc}+x_{ca}\leqslant 1 \qquad (x_{ab}+x_{bc}+x_{ca}-1)\geqslant 4x_{ab}x_{bc}x_{ca} \qquad x_{ab}=|\langle\phi_a|\phi_b\rangle|^2$$

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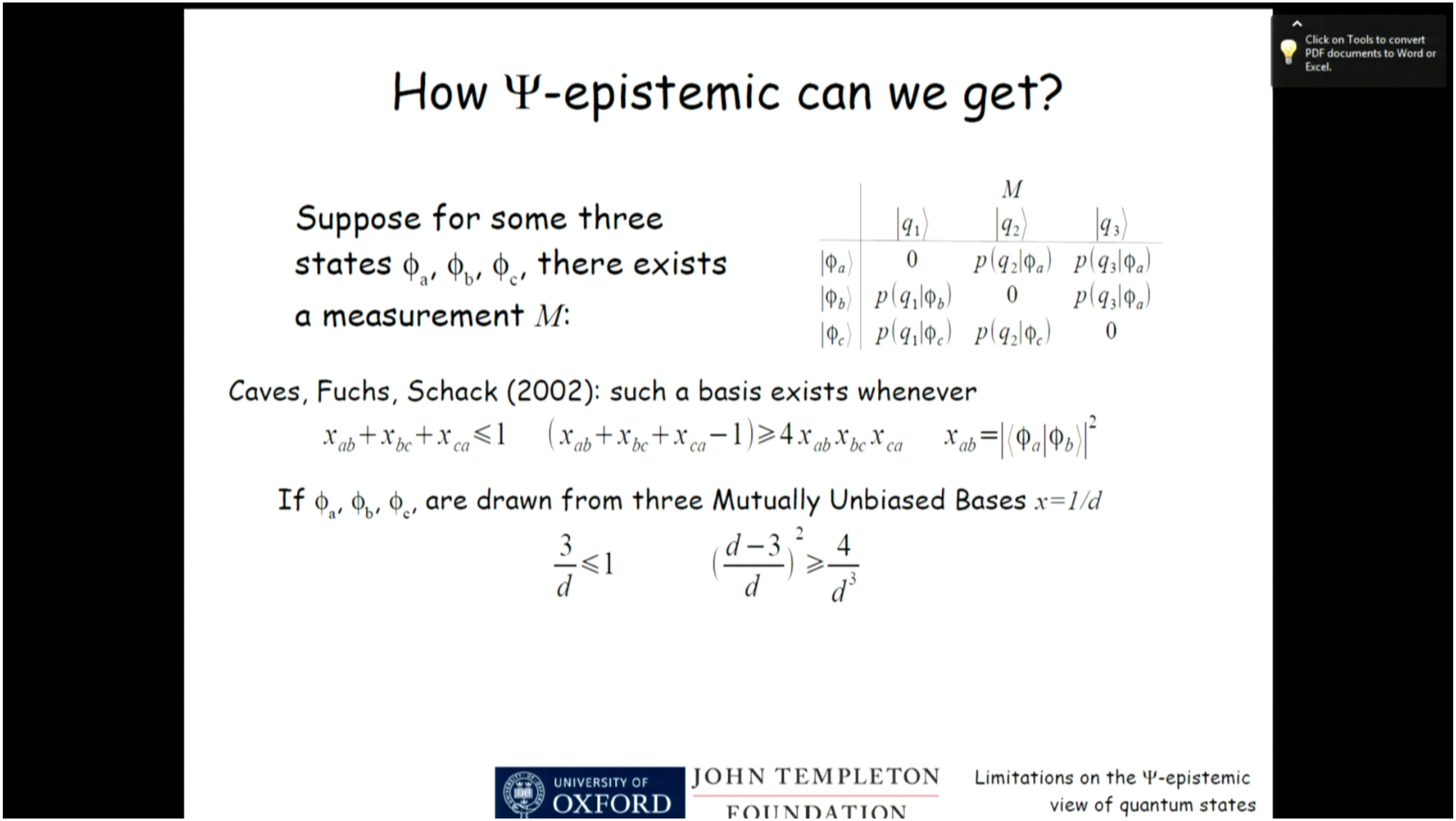

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Suppose for some three states ϕ_a, ϕ_b, ϕ_c, there exists a measurement M:

$$\begin{array}{c|ccc} & & M & \\ & |q_1\rangle & |q_2\rangle & |q_3\rangle \\ \hline |\phi_a\rangle & 0 & p(q_2|\phi_a) & p(q_3|\phi_a) \\ |\phi_b\rangle & p(q_1|\phi_b) & 0 & p(q_3|\phi_a) \\ |\phi_c\rangle & p(q_1|\phi_c) & p(q_2|\phi_c) & 0 \end{array}$$

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If ϕ_a, ϕ_b, ϕ_c, are drawn from three Mutually Unbiased Bases $x=1/d$

$$\frac{3}{d}\leqslant 1 \qquad \left(\frac{d-3}{d}\right)^2\geqslant\frac{4}{d^3}$$

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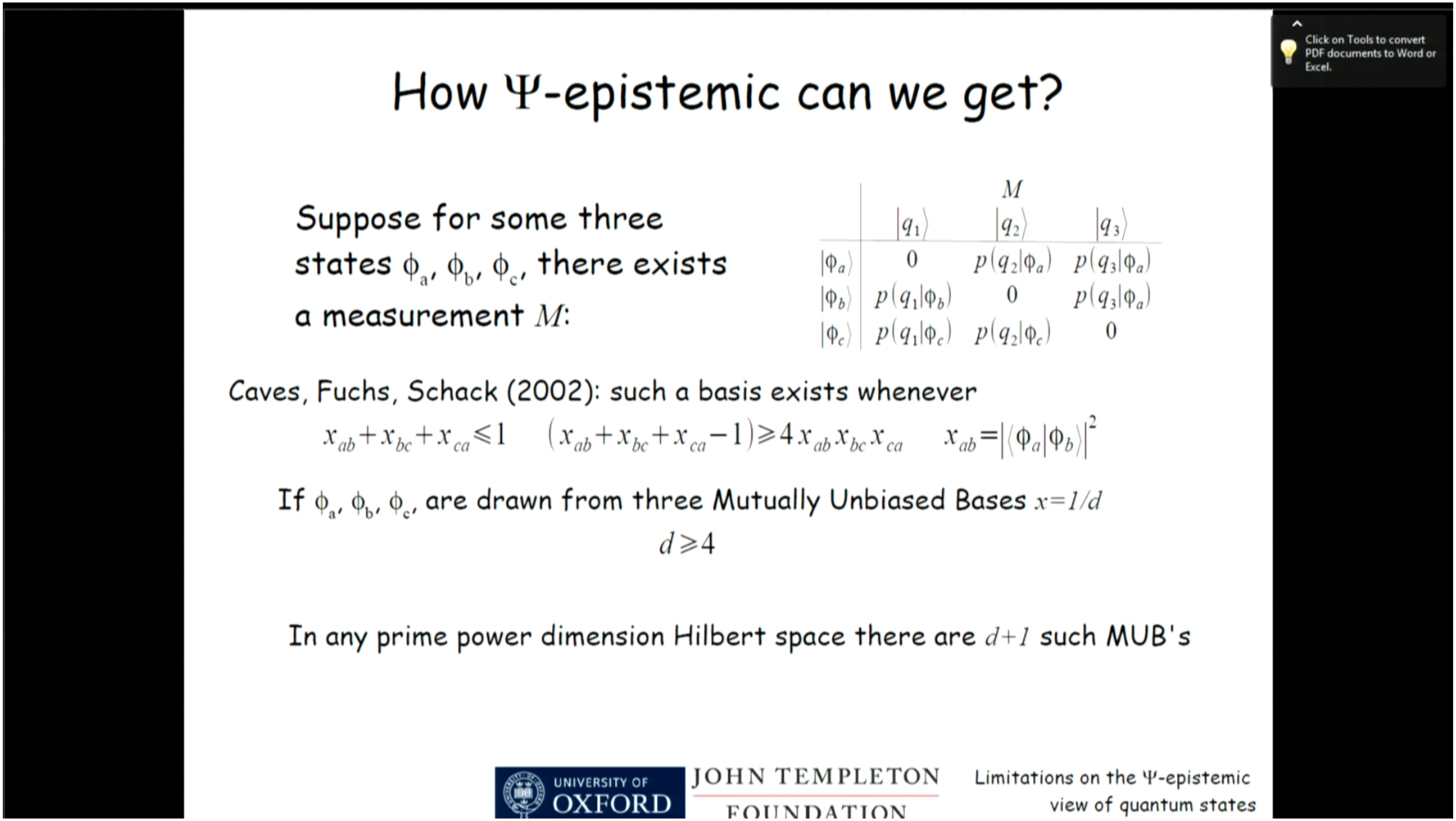

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If ϕ_a, ϕ_b, ϕ_c, are drawn from three Mutually Unbiased Bases $x=1/d$

$$d\geqslant 4$$

In any prime power dimension Hilbert space there are $d+1$ such MUB's

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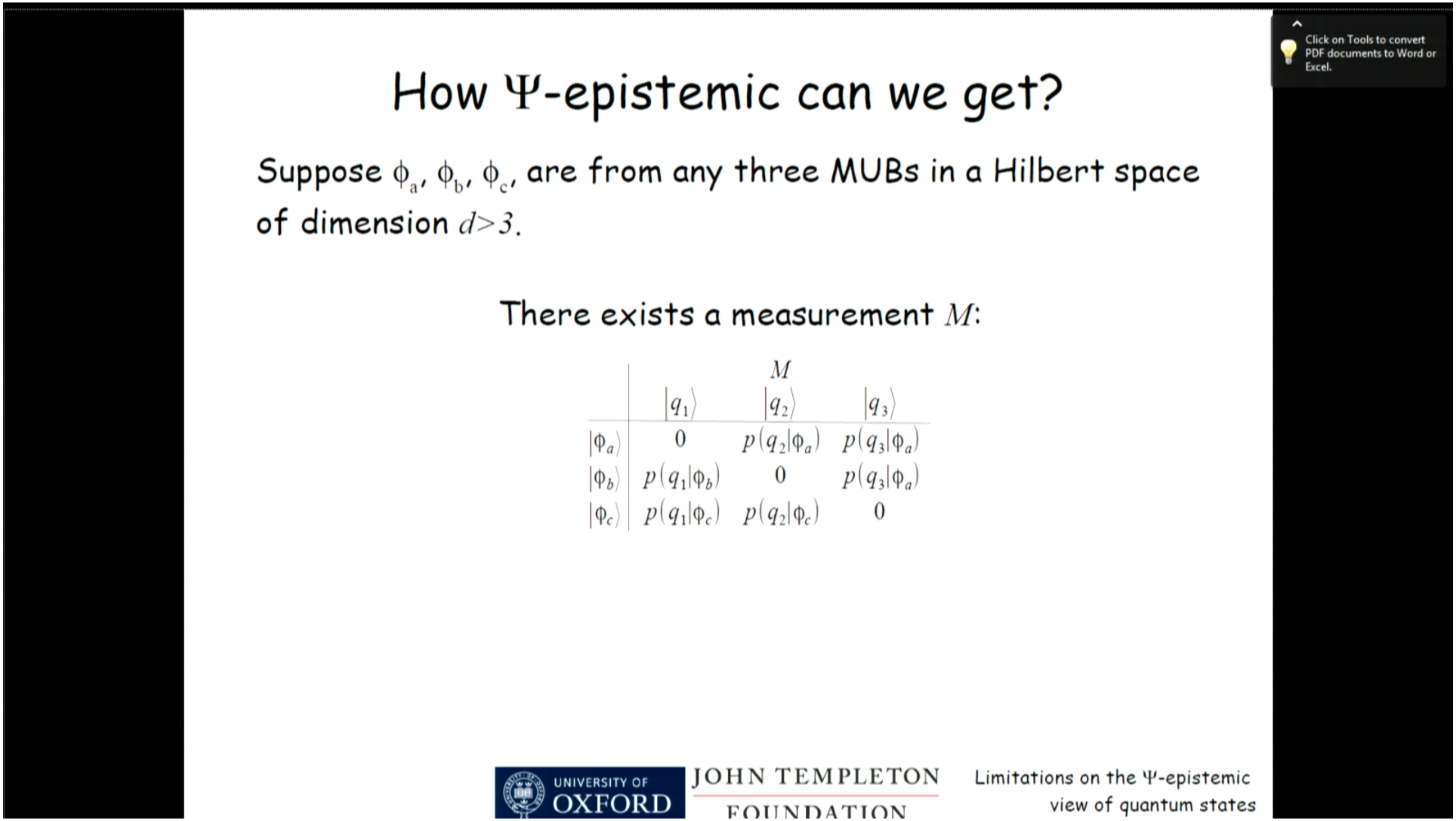

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How Ψ-epistemic can we get?

Suppose ϕ_a, ϕ_b, ϕ_c, are from any three MUBs in a Hilbert space of dimension $d>3$.

There exists a measurement M:

		M	
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$\lvert\phi_a\rangle$	0	$p(q_2\lvert\phi_a)$	$p(q_3\lvert\phi_a)$
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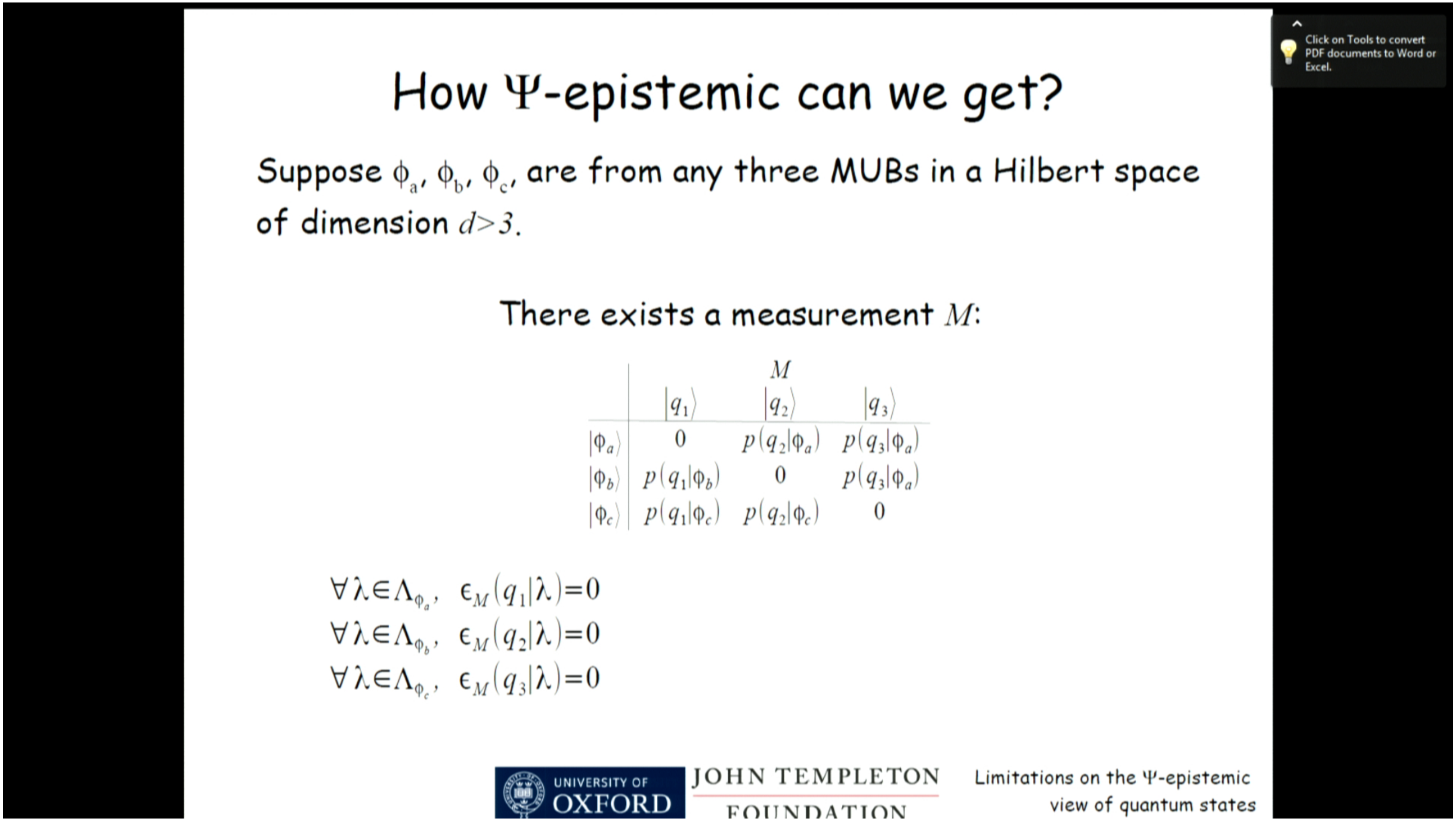

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$\lvert\phi_c\rangle$	$p(q_1\lvert\phi_c)$	$p(q_2\lvert\phi_c)$	0

$$\forall\lambda\in\Lambda_{\phi_a},\quad \epsilon_M(q_1|\lambda)=0$$
$$\forall\lambda\in\Lambda_{\phi_b},\quad \epsilon_M(q_2|\lambda)=0$$
$$\forall\lambda\in\Lambda_{\phi_c},\quad \epsilon_M(q_3|\lambda)=0$$

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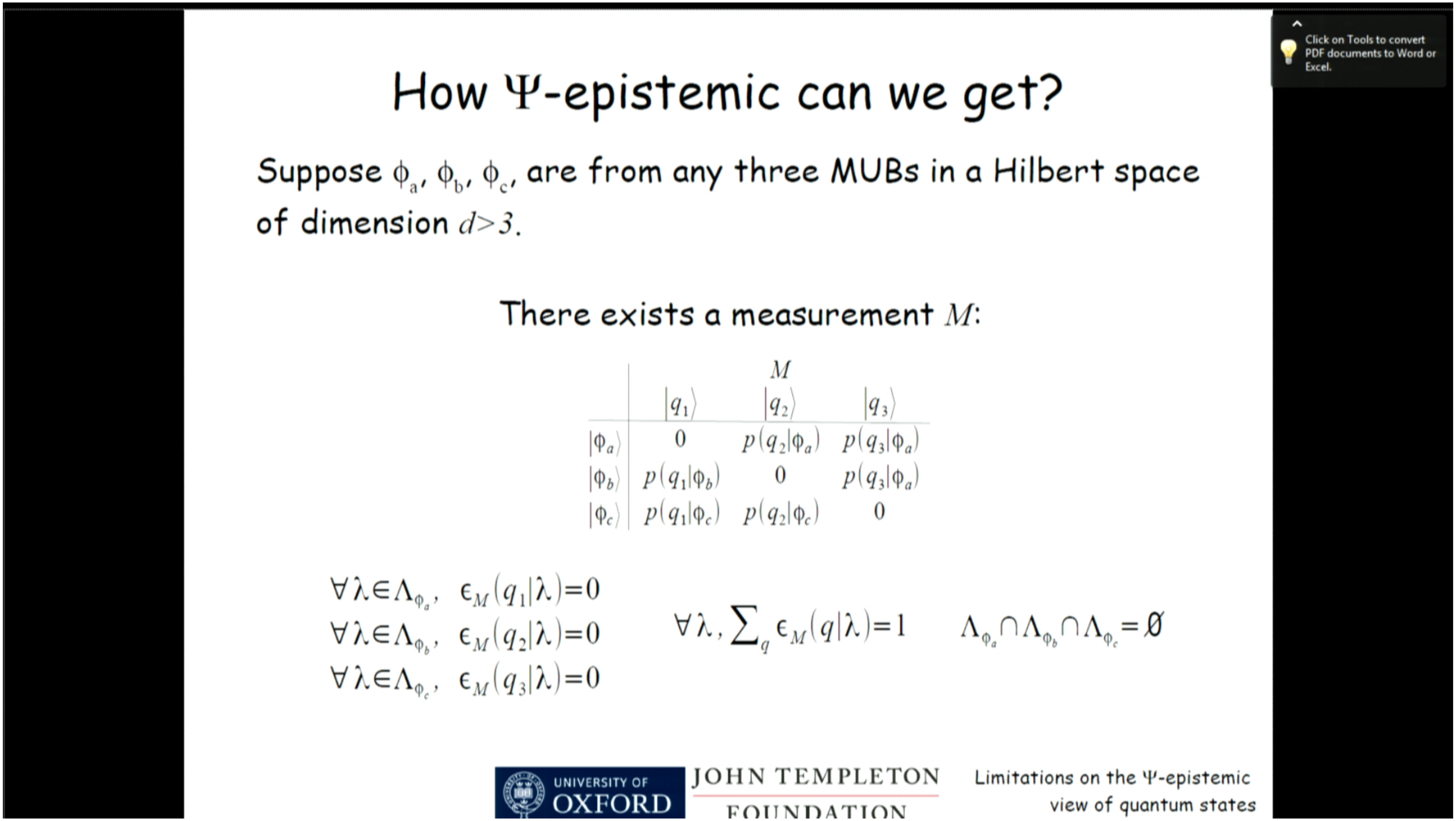

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$$\forall\lambda\in\Lambda_{\phi_c},\ \ \epsilon_M(q_3|\lambda)=0$$

$$\forall\lambda,\sum_q\epsilon_M(q|\lambda)=1$$

$$\Lambda_{\phi_a}\cap\Lambda_{\phi_b}\cap\Lambda_{\phi_c}=\emptyset$$

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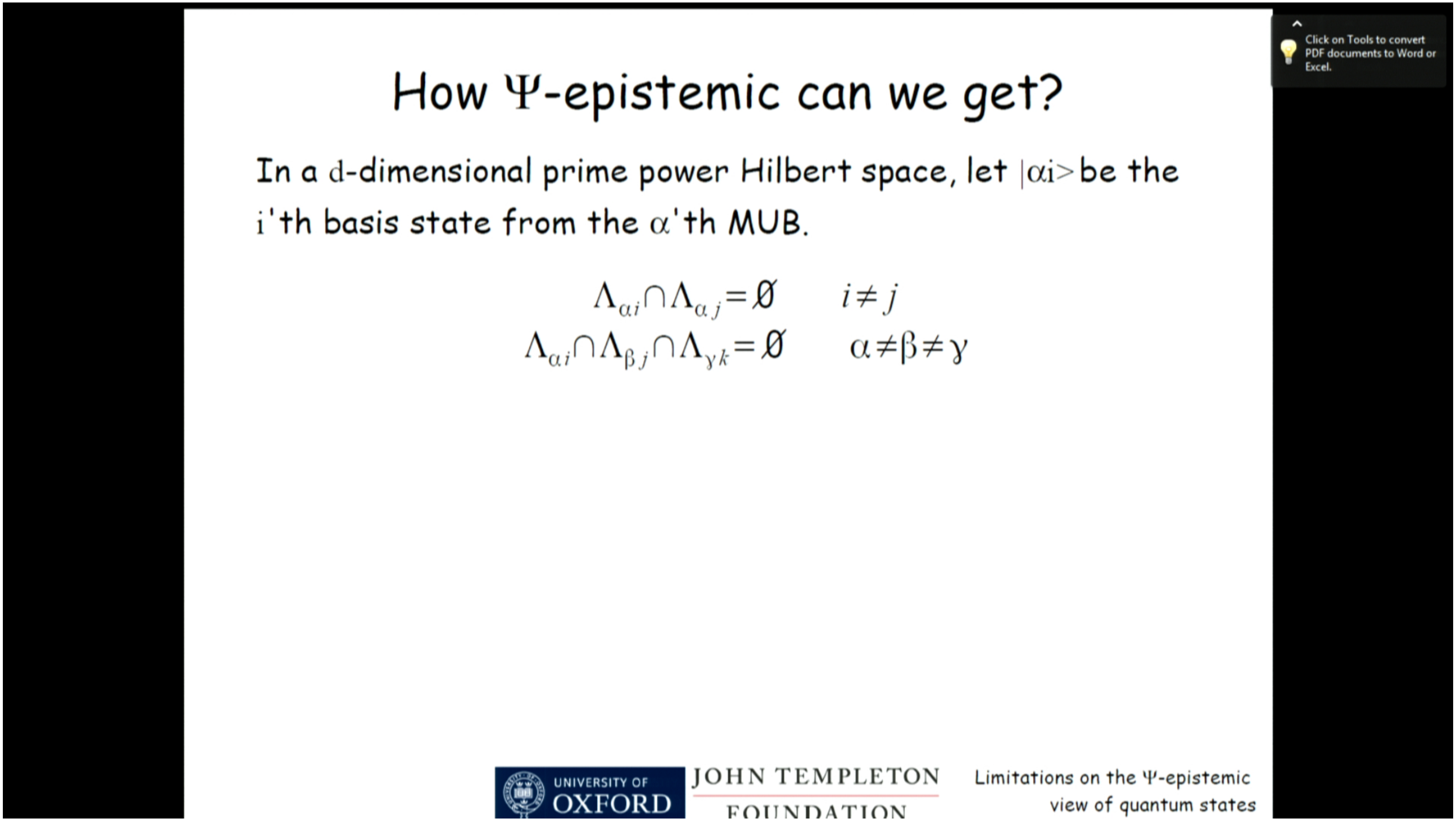

How Ψ-epistemic can we get?

In a d-dimensional prime power Hilbert space, let $|\alpha i>$ be the i'th basis state from the α'th MUB.

$$\Lambda_{\alpha i} \cap \Lambda_{\alpha j} = \emptyset \qquad i \neq j$$

$$\Lambda_{\alpha i} \cap \Lambda_{\beta j} \cap \Lambda_{\gamma k} = \emptyset \qquad \alpha \neq \beta \neq \gamma$$

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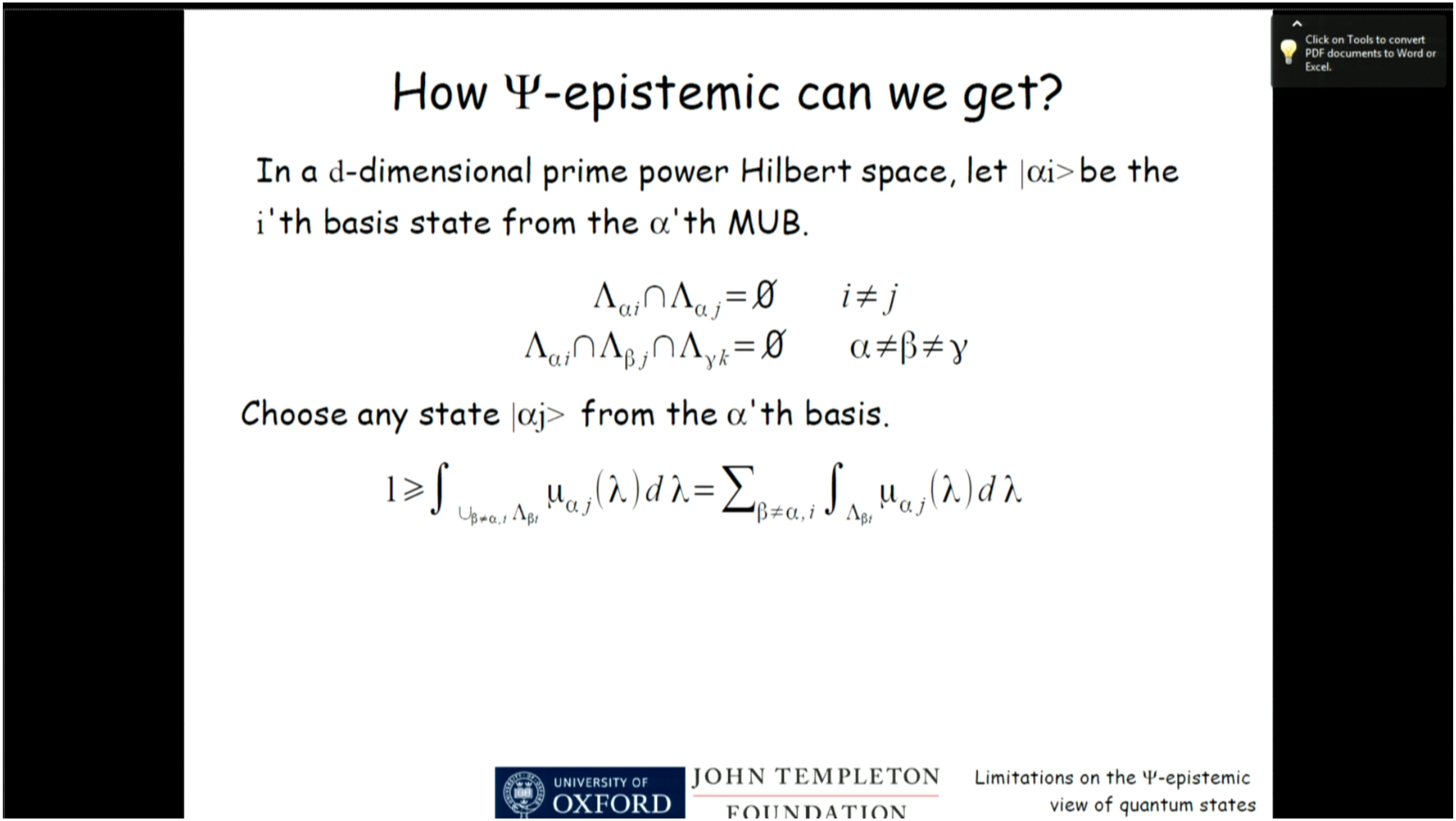

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Choose any state $|\alpha j\rangle$ from the α'th basis.

$$1 \geqslant \int_{\cup_{\beta \neq \alpha, i} \Lambda_{\beta i}} \mu_{\alpha j}(\lambda) d\lambda = \sum_{\beta \neq \alpha, i} \int_{\Lambda_{\beta i}} \mu_{\alpha j}(\lambda) d\lambda$$

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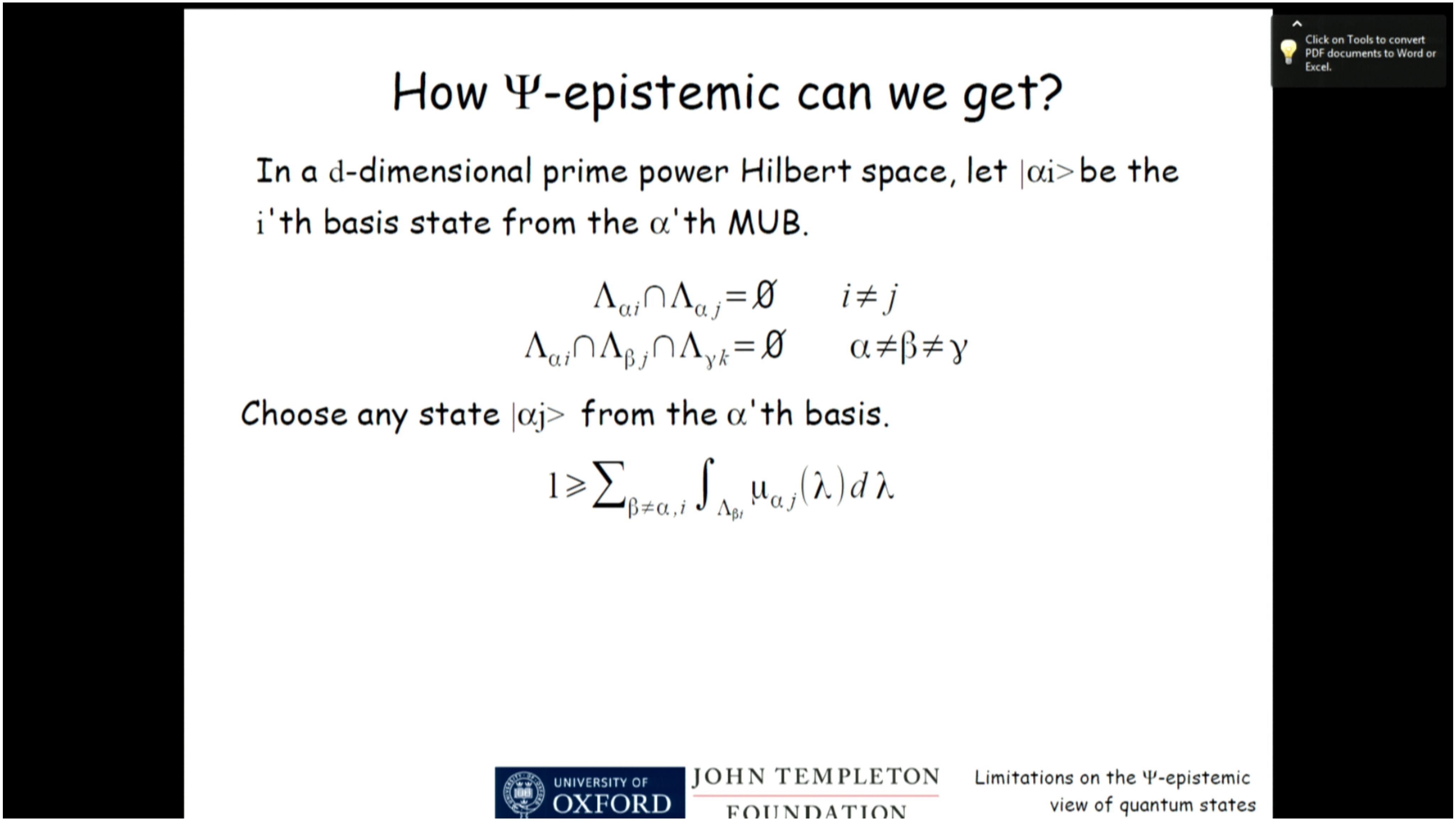

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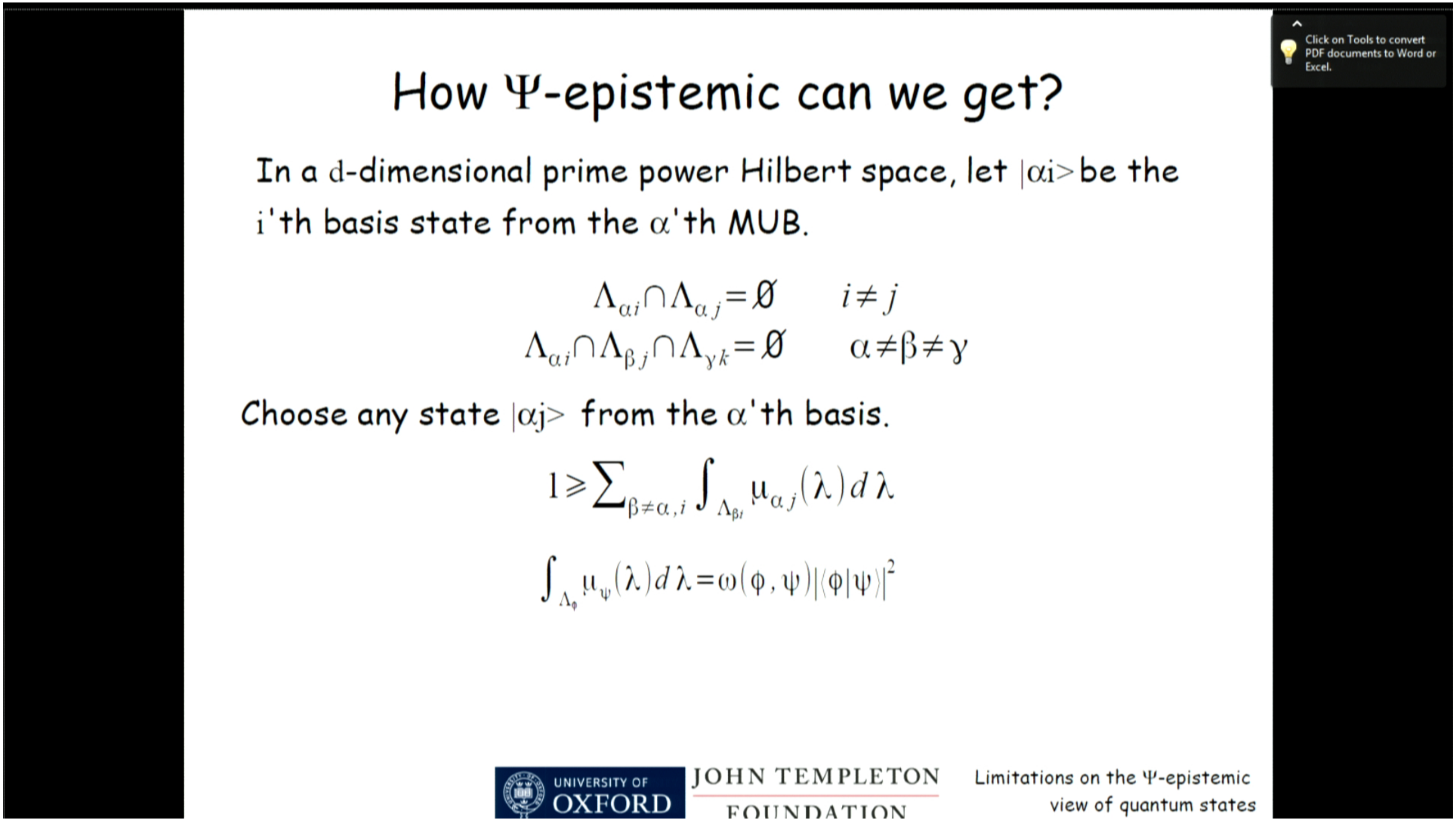

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$$\int_{\Lambda_{\phi}}\mu_{\psi}(\lambda)\,d\lambda=\omega(\phi,\psi)|\langle\phi|\psi\rangle|^{2}$$

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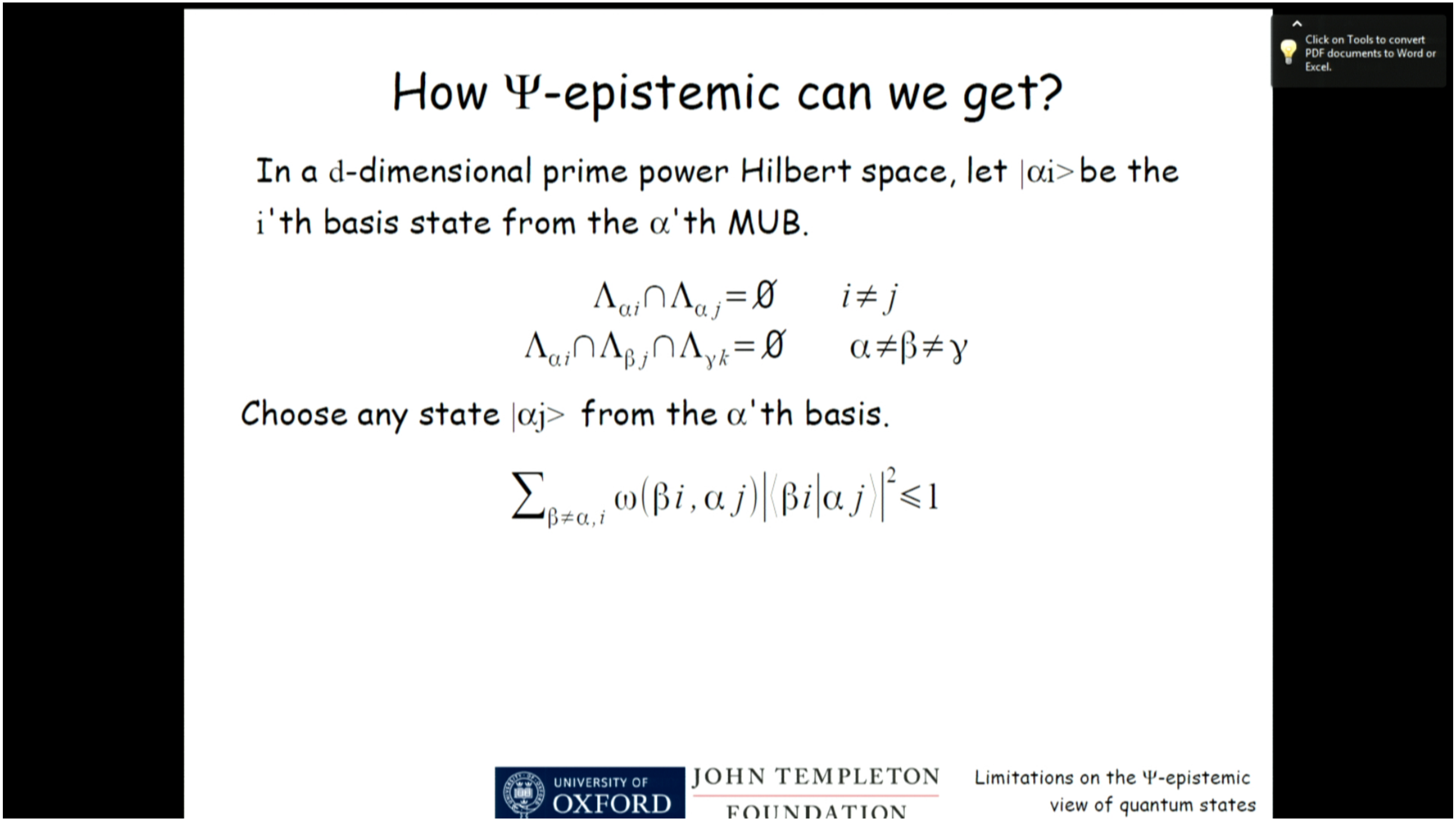

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$$\sum_{\beta\neq\alpha,i}\omega(\beta i,\alpha j)\left|\langle\beta i|\alpha j\rangle\right|^2\leqslant 1$$

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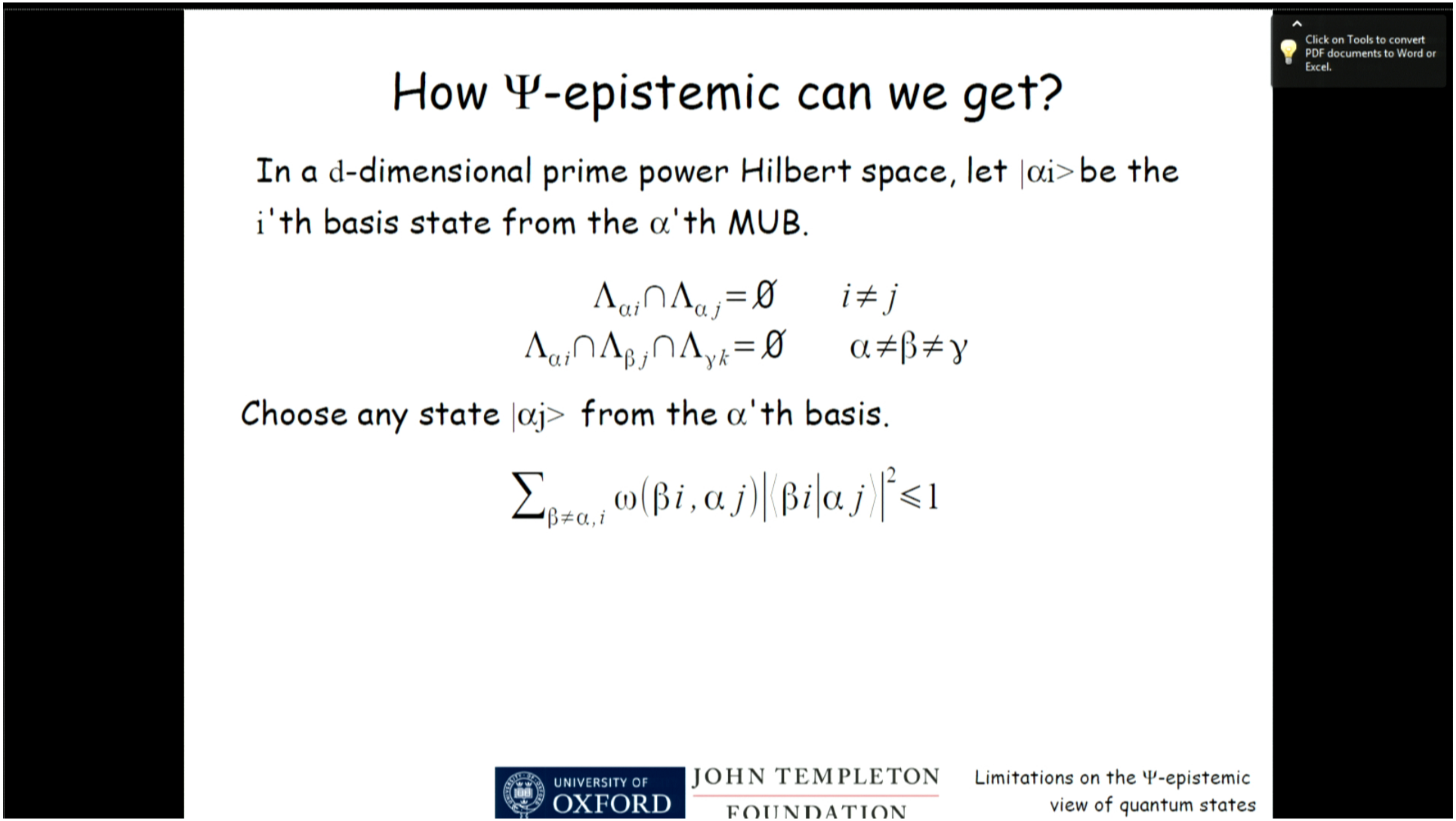

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$$\sum_{\beta \neq \alpha, i} \omega(\beta i, \alpha j) \left| \langle \beta i | \alpha j \rangle \right|^2 \leqslant 1$$

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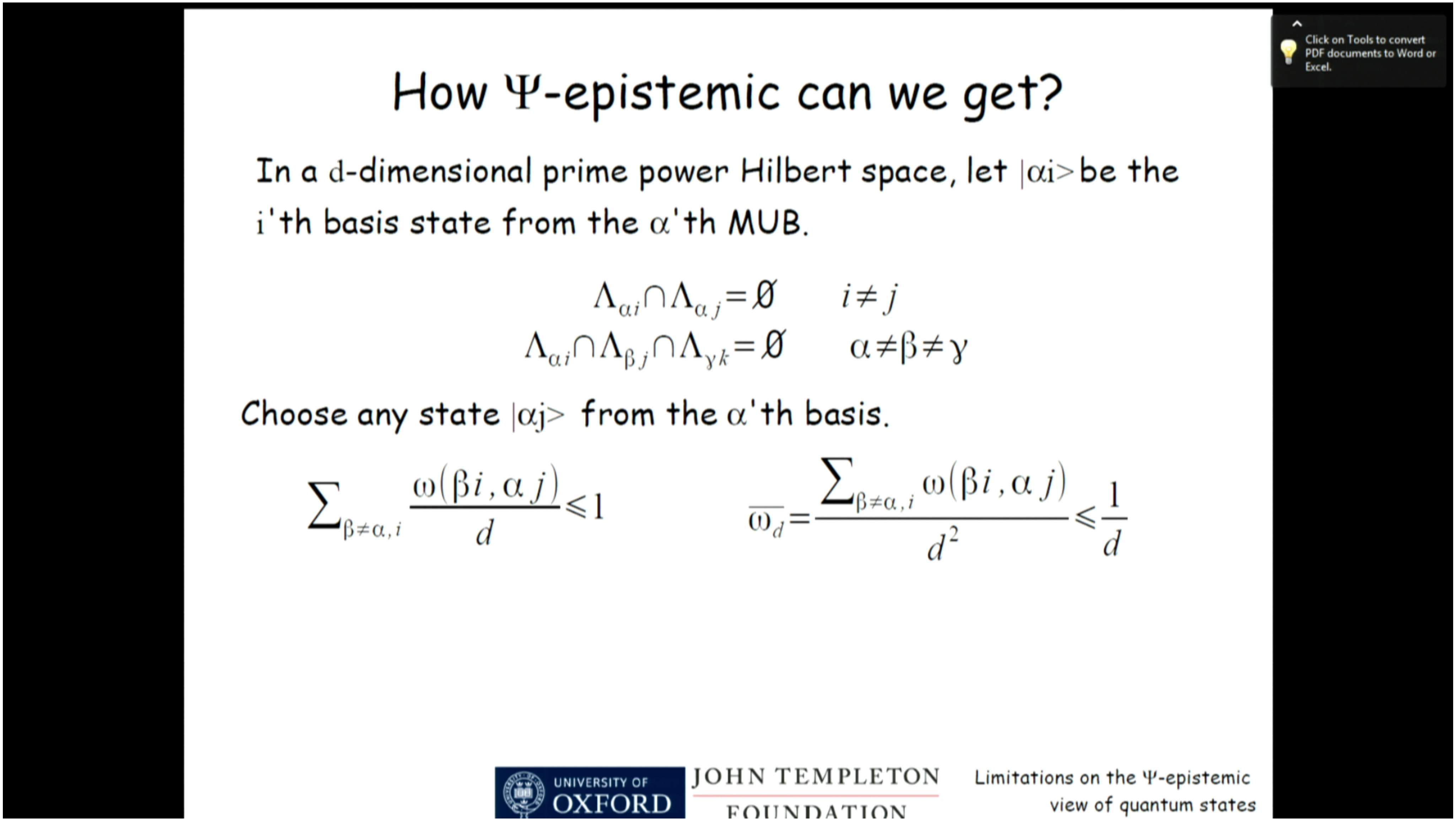

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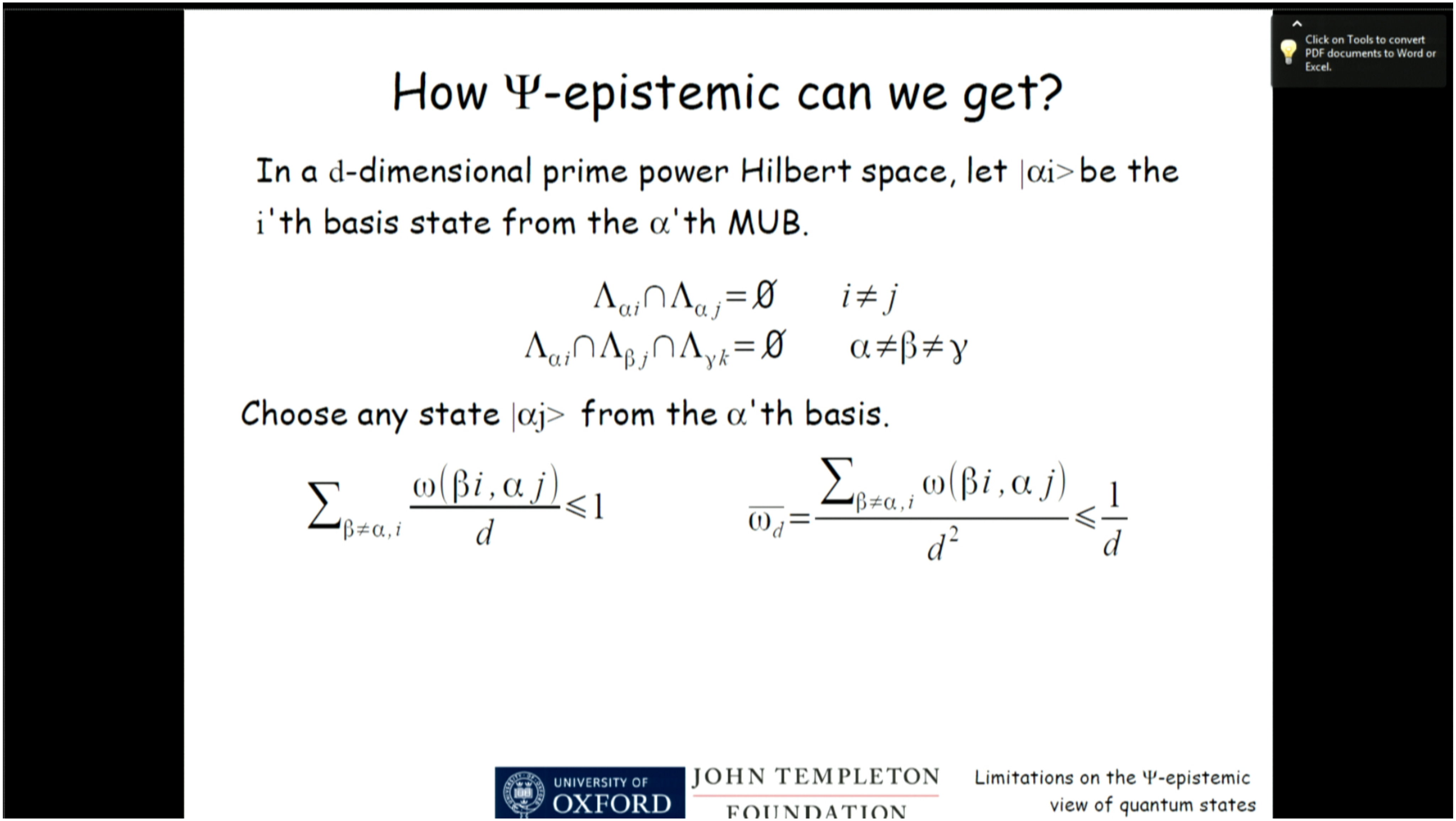

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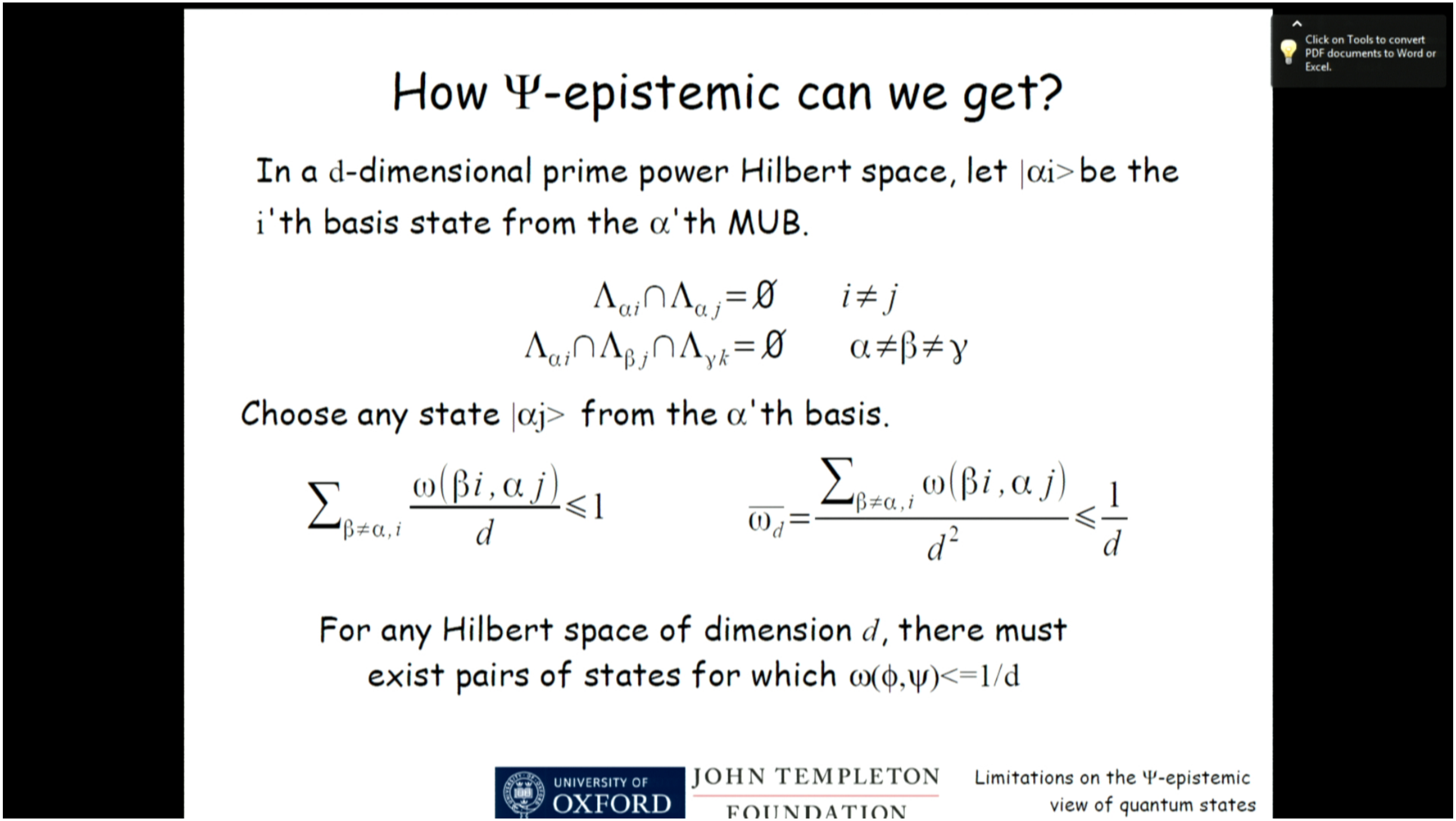

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Choose any state $|\alpha j\rangle$ from the α'th basis.

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For any Hilbert space of dimension d, there must exist pairs of states for which $\omega(\phi,\psi)<=1/d$

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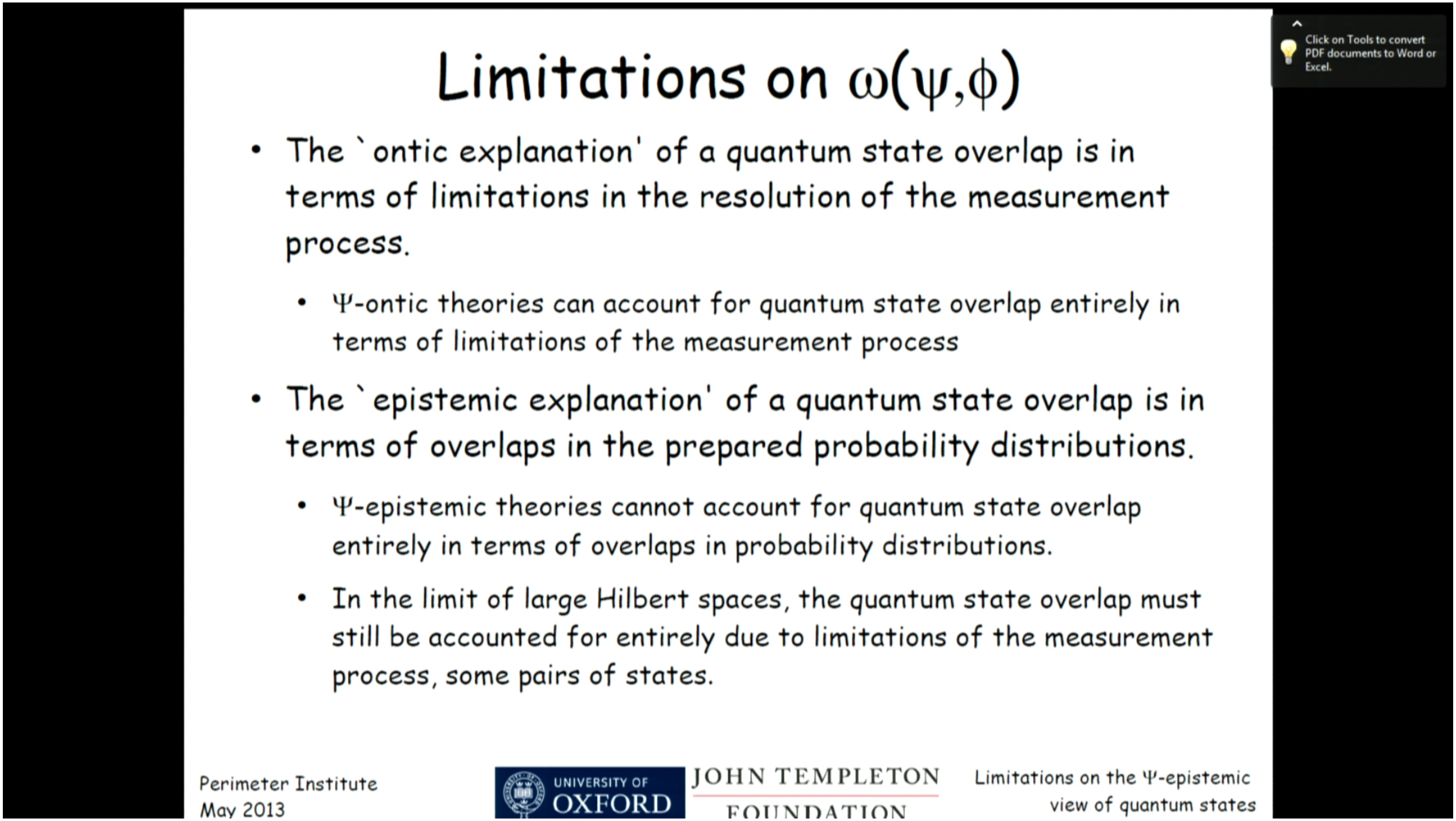
Limitations on the Ψ-epistemic view of quantum states



Limitations on $\omega(\psi,\phi)$

- The `ontic explanation' of a quantum state overlap is in terms of limitations in the resolution of the measurement process.
  - Ψ-ontic theories can account for quantum state overlap entirely in terms of limitations of the measurement process
- The `epistemic explanation' of a quantum state overlap is in terms of overlaps in the prepared probability distributions.
  - Ψ-epistemic theories cannot account for quantum state overlap entirely in terms of overlaps in probability distributions.
  - In the limit of large Hilbert spaces, the quantum state overlap must still be accounted for entirely due to limitations of the measurement process, some pairs of states.

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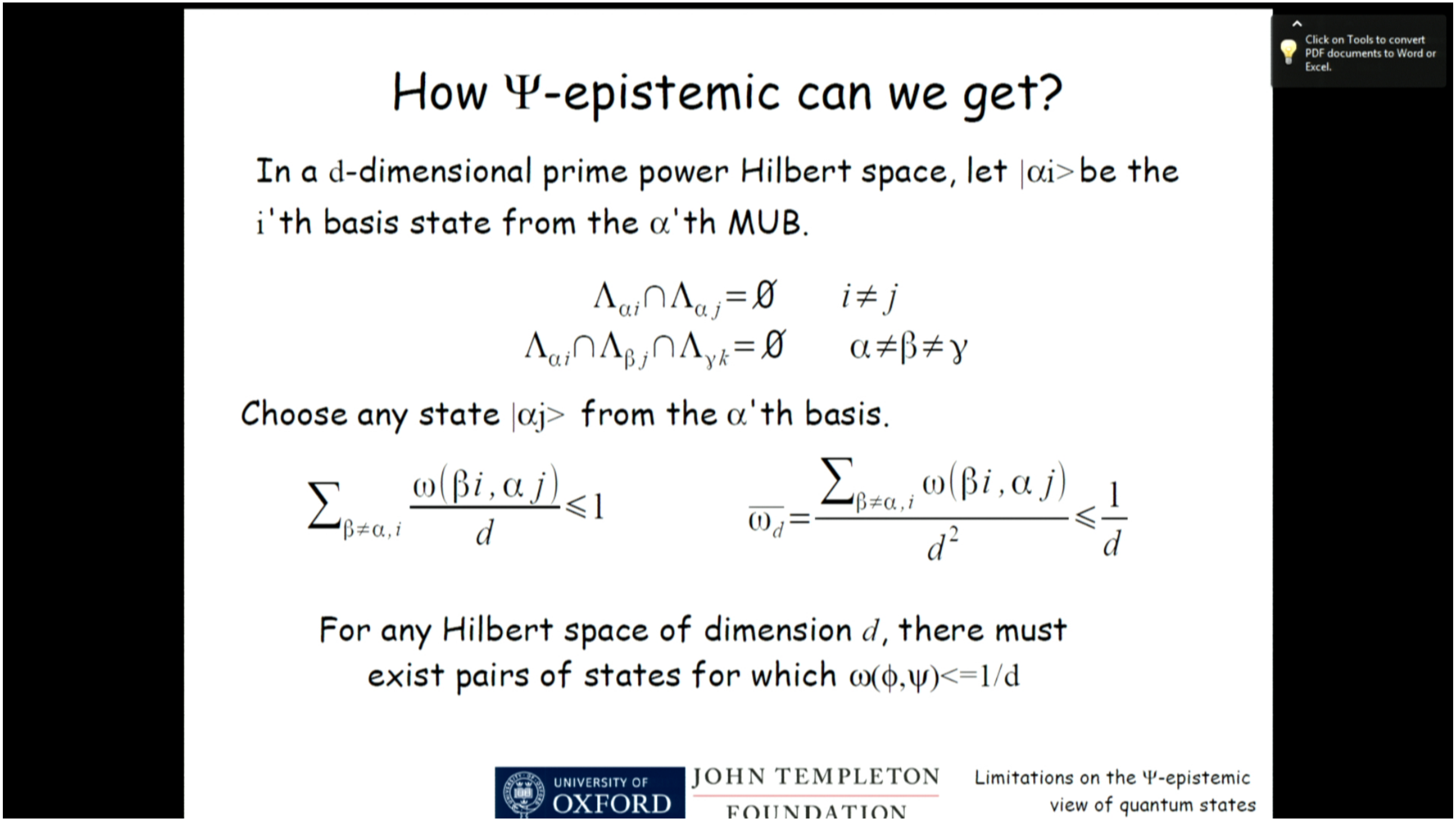

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How Ψ-epistemic can we get?

In a d-dimensional prime power Hilbert space, let $|\alpha i\rangle$ be the i'th basis state from the α'th MUB.

$$\Lambda_{\alpha i}\cap\Lambda_{\alpha j}=\emptyset \qquad i\neq j$$

$$\Lambda_{\alpha i}\cap\Lambda_{\beta j}\cap\Lambda_{\gamma k}=\emptyset \qquad \alpha\neq\beta\neq\gamma$$

Choose any state $|\alpha j\rangle$ from the α'th basis.

$$\sum_{\beta\neq\alpha,i}\frac{\omega(\beta i,\alpha j)}{d}\leqslant 1 \qquad\qquad \overline{\omega_d}=\frac{\sum_{\beta\neq\alpha,i}\omega(\beta i,\alpha j)}{d^2}\leqslant\frac{1}{d}$$

For any Hilbert space of dimension d, there must exist pairs of states for which $\omega(\phi,\psi)<=1/d$

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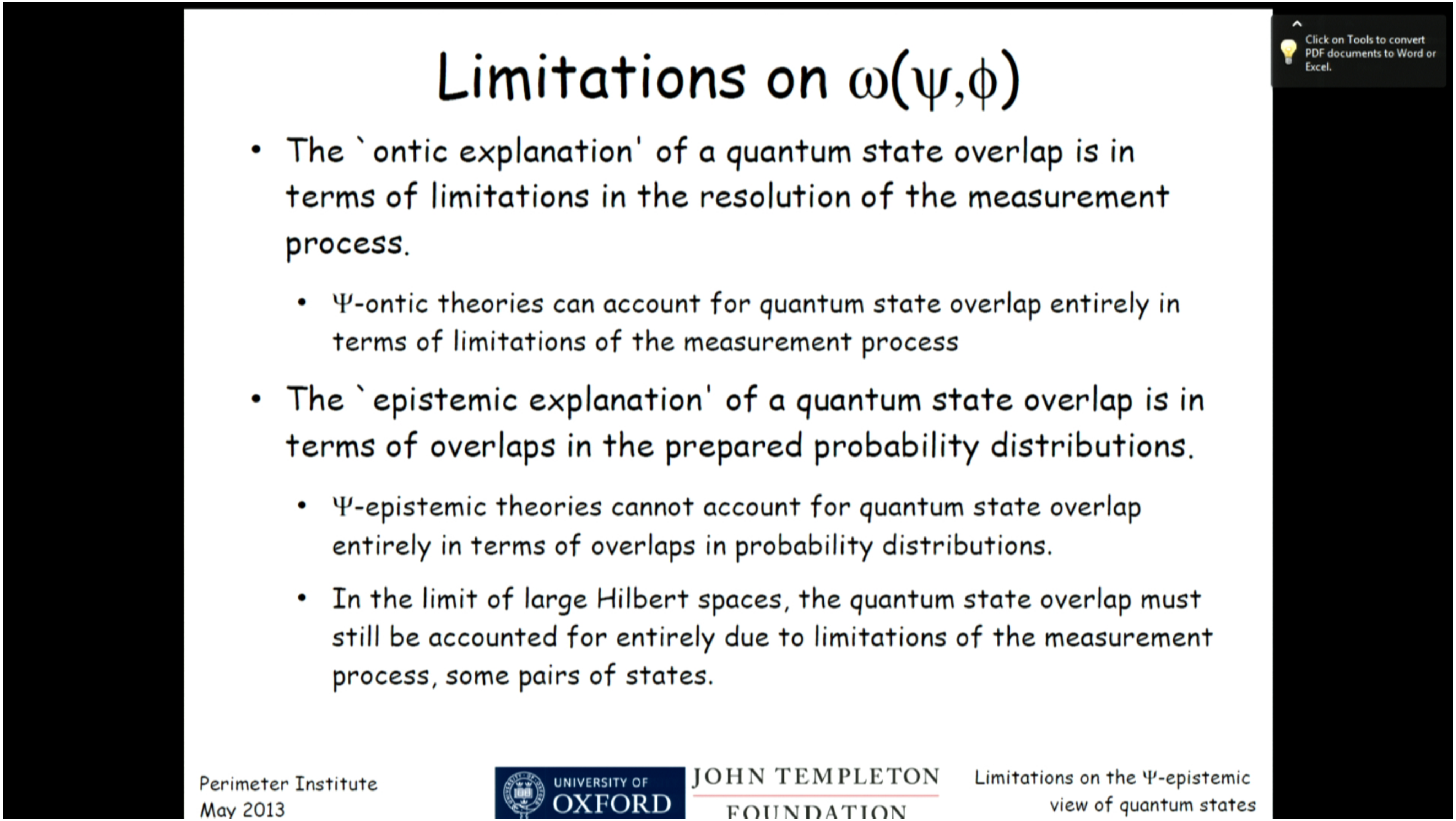

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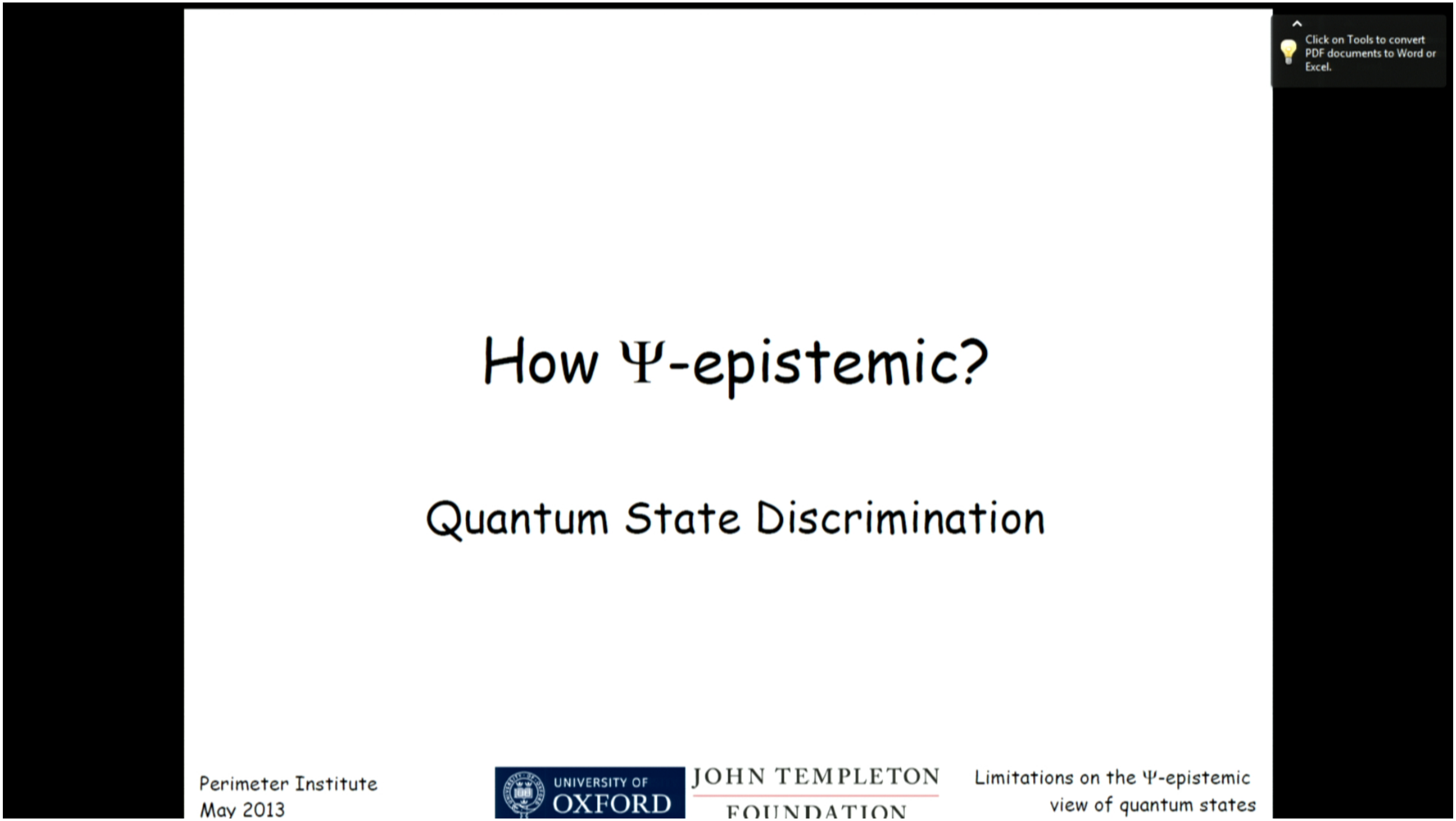

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How Ψ-epistemic?

Quantum State Discrimination

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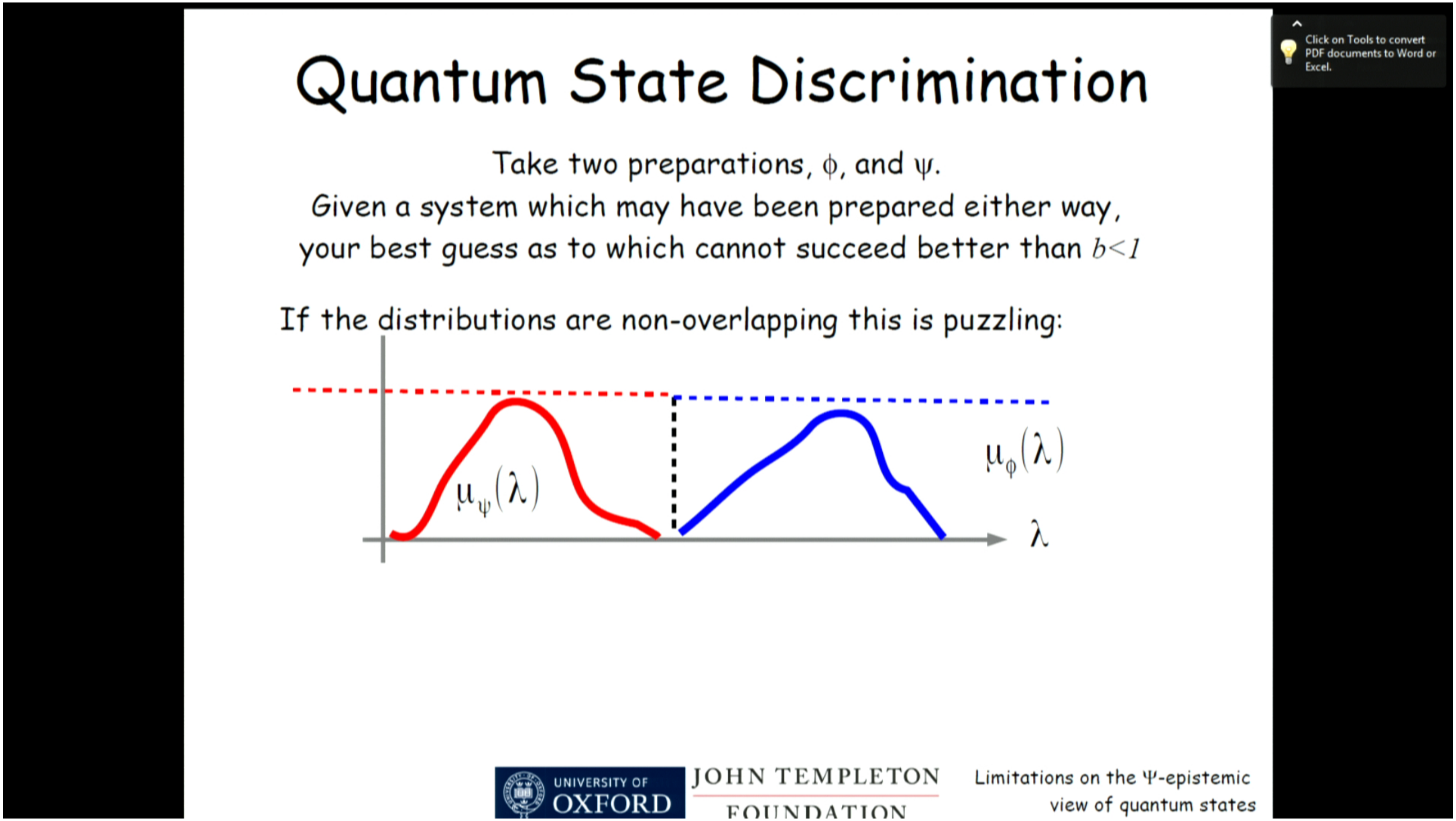
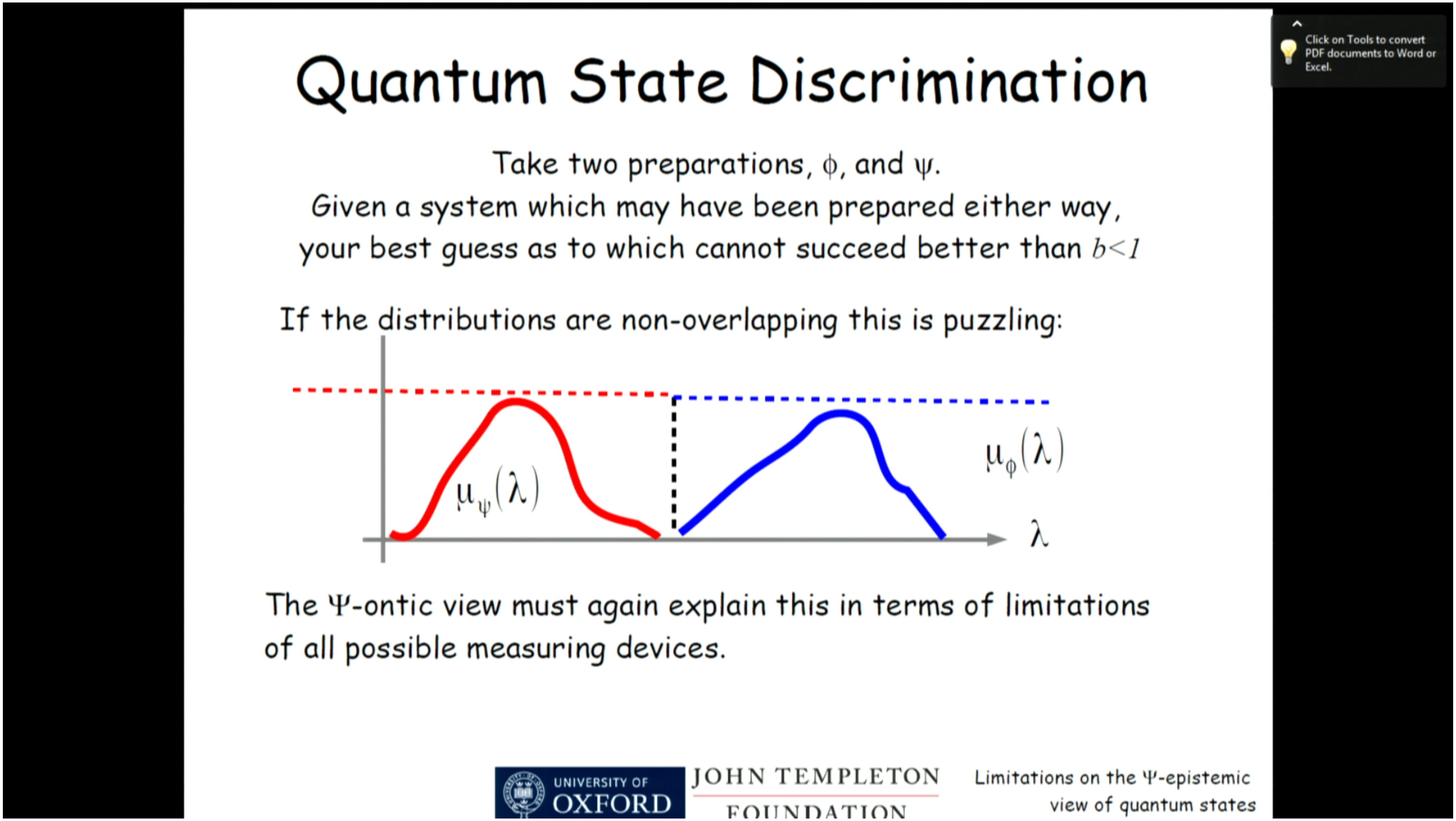


Quantum State Discrimination

Take two preparations, ϕ, and ψ.
Given a system which may have been prepared either way,
your best guess as to which cannot succeed better than $b<1$

If the distributions are non-overlapping this is puzzling:

Limitations on the Ψ-epistemic
view of quantum states



Quantum State Discrimination

Take two preparations, ϕ, and ψ.
Given a system which may have been prepared either way,
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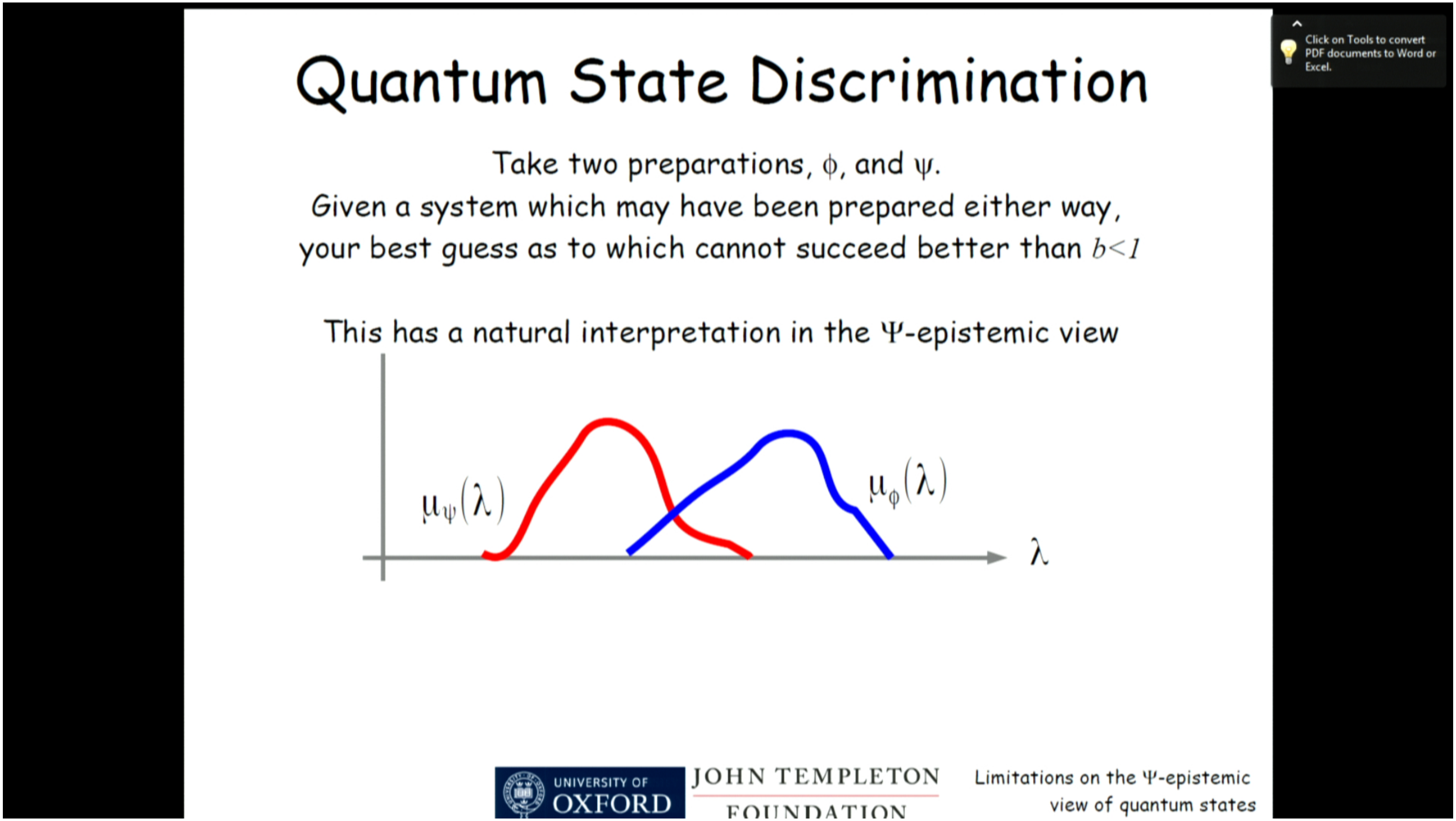
If the distributions are non-overlapping this is puzzling:

The Ψ-ontic view must again explain this in terms of limitations of all possible measuring devices.

Limitations on the Ψ-epistemic view of quantum states



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Take two preparations, ϕ, and ψ.
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This has a natural interpretation in the Ψ-epistemic view

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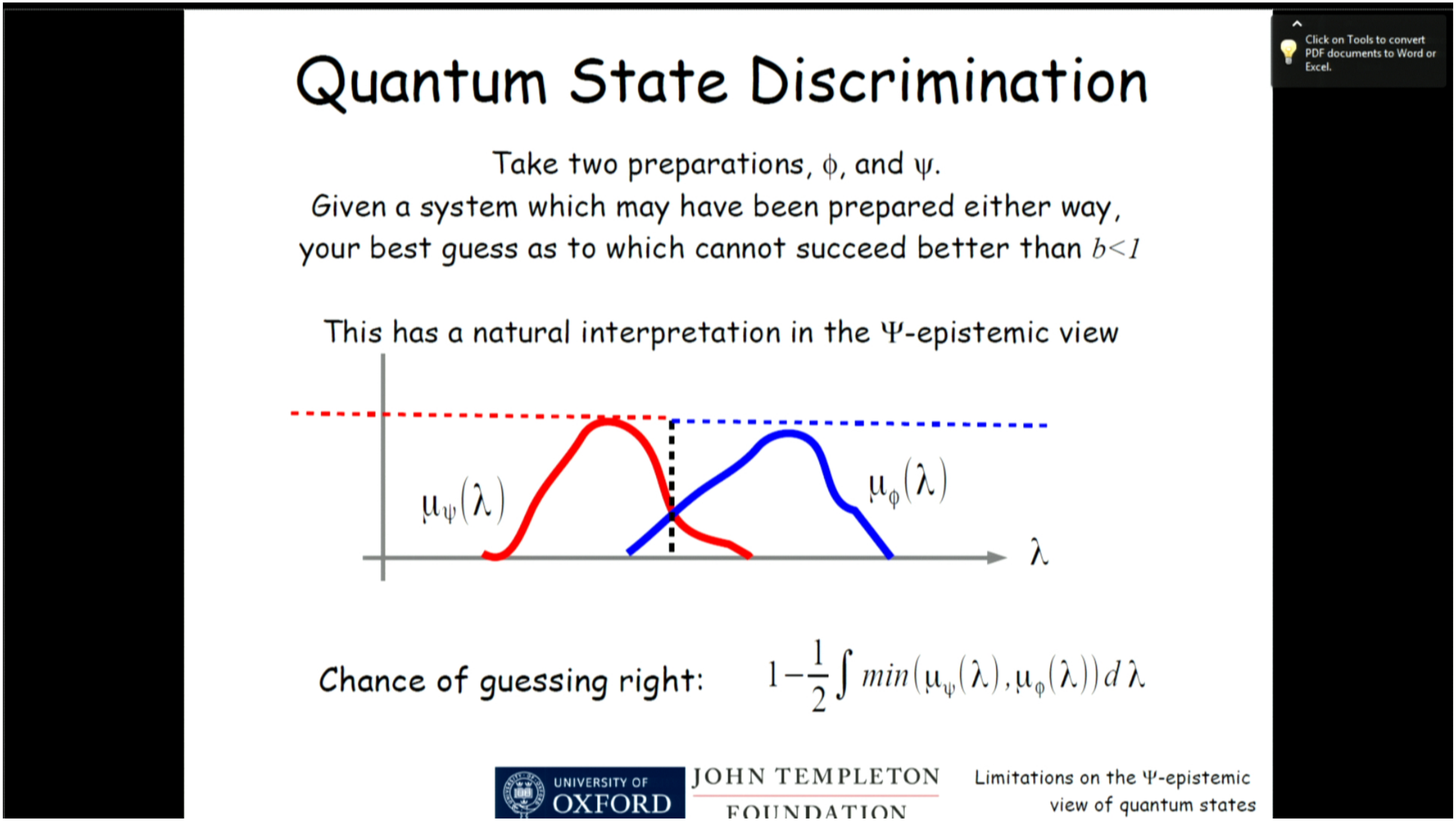


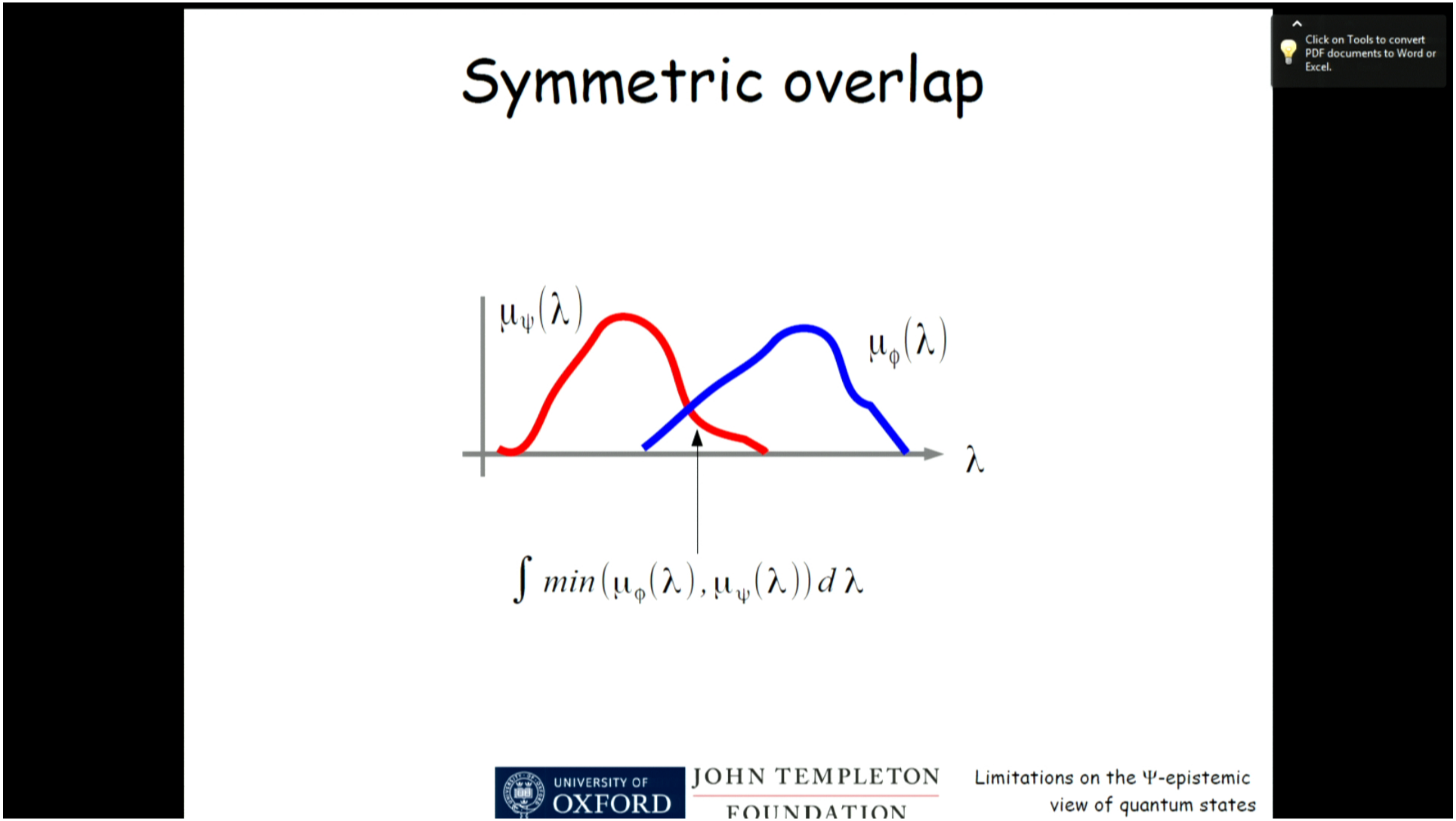
Quantum State Discrimination

Take two preparations, ϕ, and ψ.
Given a system which may have been prepared either way,
your best guess as to which cannot succeed better than $b<1$

This has a natural interpretation in the Ψ-epistemic view

Chance of guessing right: $1-\frac{1}{2}\int min(\mu_\psi(\lambda),\mu_\phi(\lambda))\,d\lambda$

Limitations on the Ψ-epistemic view of quantum states



Symmetric overlap

$$\int min(\mu_\phi(\lambda), \mu_\psi(\lambda))\, d\lambda$$

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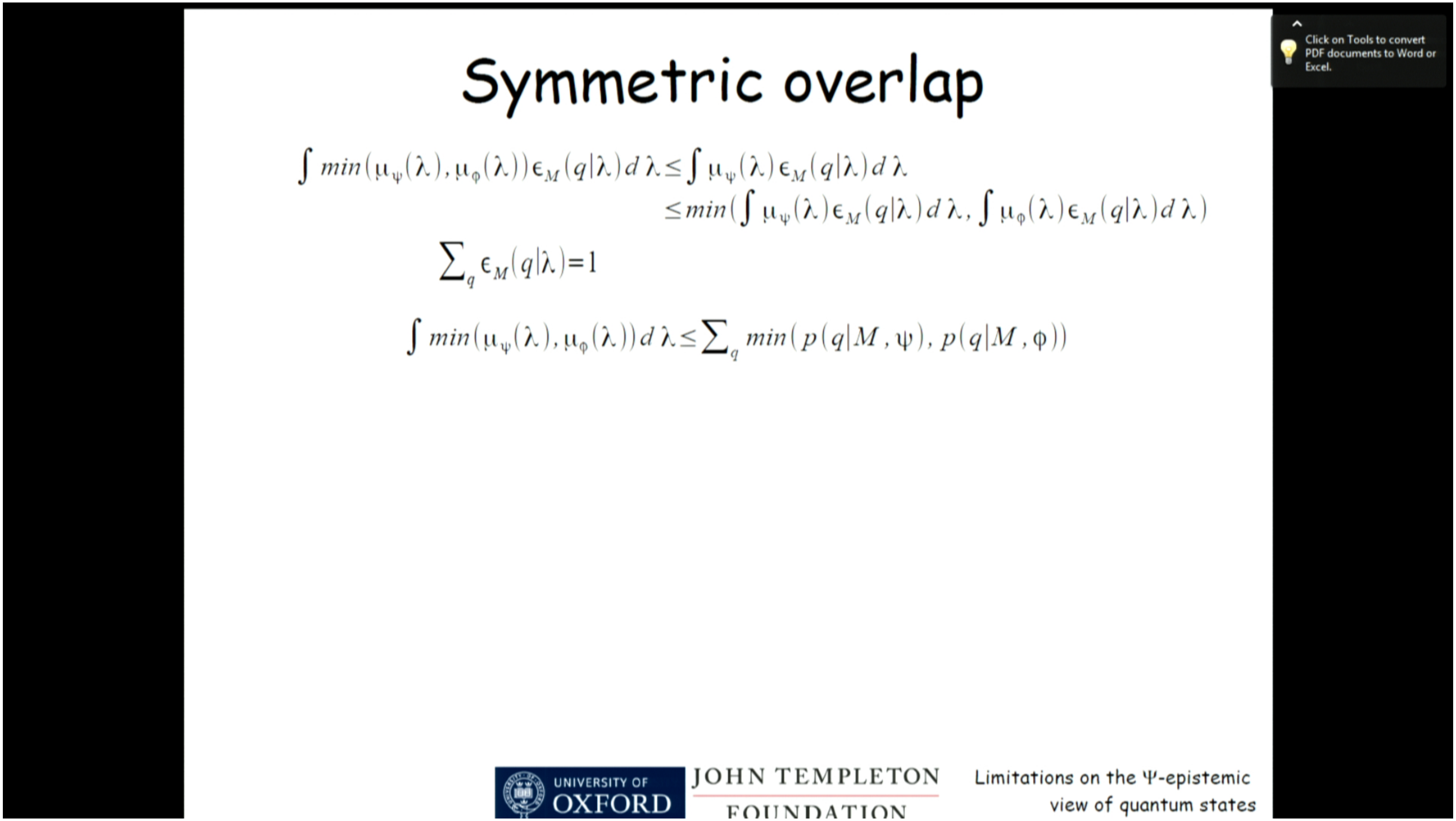
view of quantum states



Symmetric overlap

$$\int min(\mu_\psi(\lambda),\mu_\phi(\lambda))\epsilon_M(q|\lambda)d\lambda \le \int \mu_\psi(\lambda)\epsilon_M(q|\lambda)d\lambda$$

$$\le min(\int \mu_\psi(\lambda)\epsilon_M(q|\lambda)d\lambda, \int \mu_\phi(\lambda)\epsilon_M(q|\lambda)d\lambda)$$

$$\sum_q \epsilon_M(q|\lambda)=1$$

$$\int min(\mu_\psi(\lambda),\mu_\phi(\lambda))d\lambda \le \sum_q min(p(q|M,\psi),p(q|M,\phi))$$

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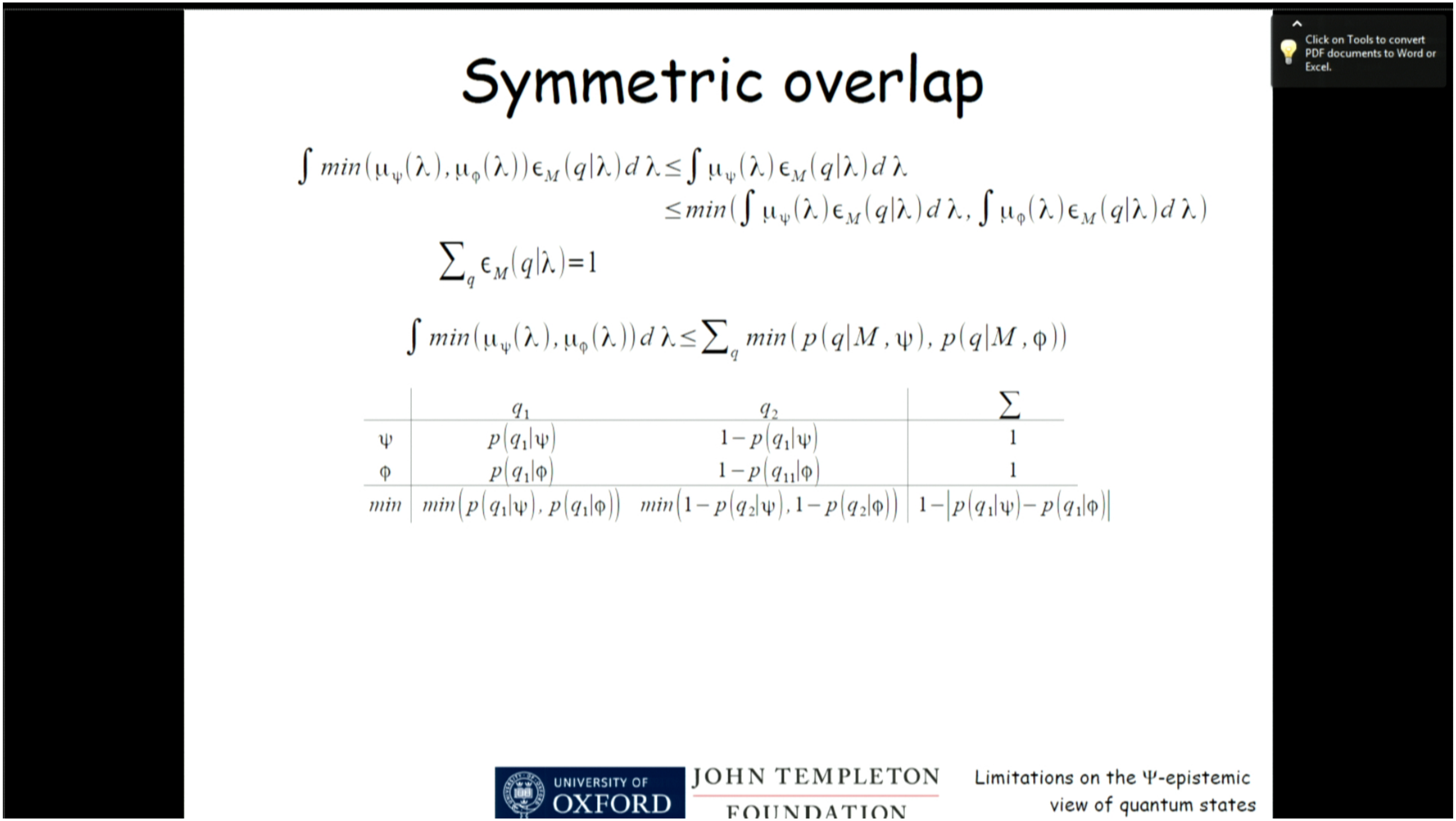


Symmetric overlap

$$\int min(\mu_\psi(\lambda),\mu_\phi(\lambda))\epsilon_M(q|\lambda)d\lambda\leq\int\mu_\psi(\lambda)\epsilon_M(q|\lambda)d\lambda$$

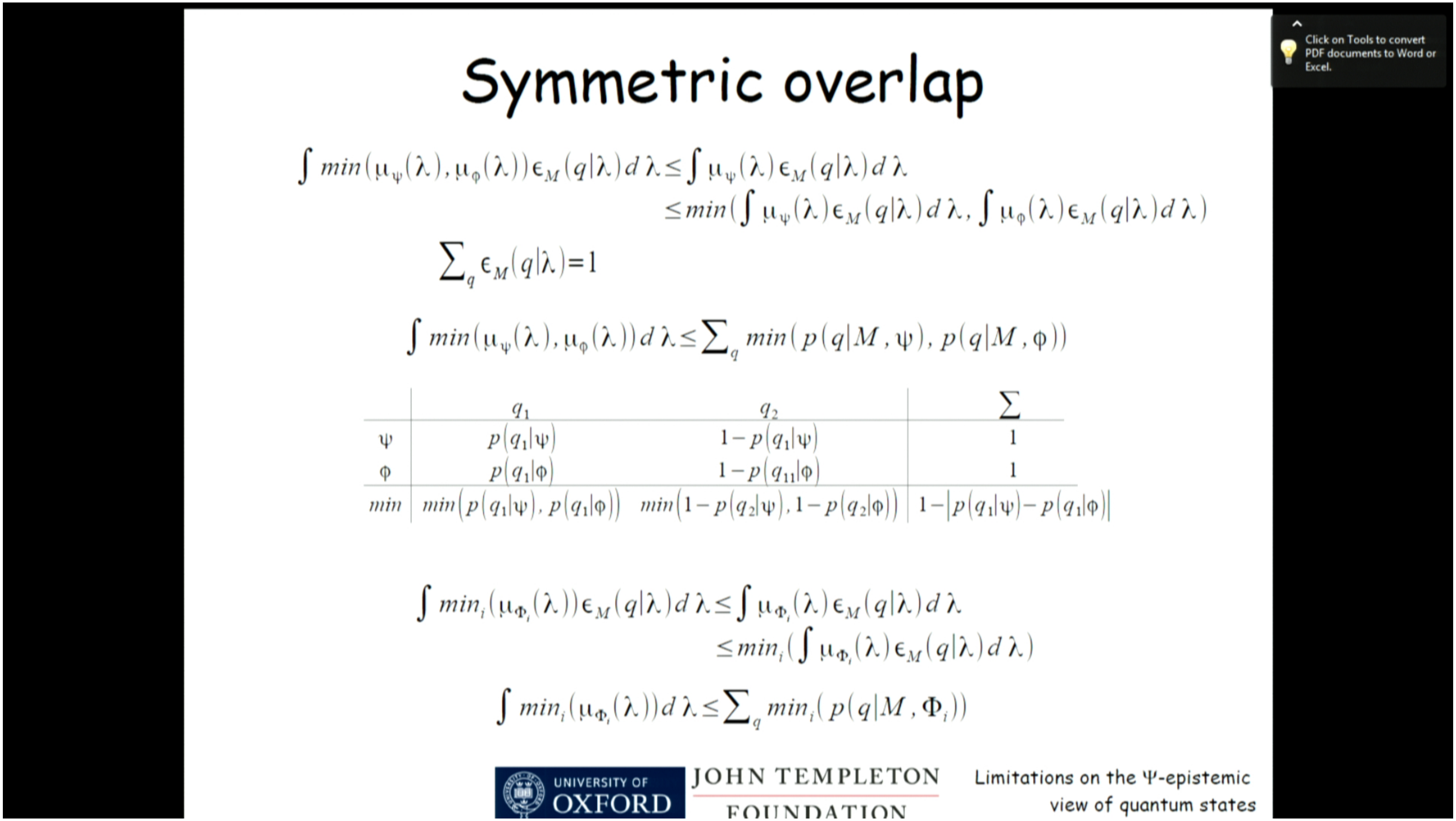
$$\leq min(\int\mu_\psi(\lambda)\epsilon_M(q|\lambda)d\lambda,\int\mu_\phi(\lambda)\epsilon_M(q|\lambda)d\lambda)$$

$$\sum_q\epsilon_M(q|\lambda)=1$$

$$\int min(\mu_\psi(\lambda),\mu_\phi(\lambda))d\lambda\leq\sum_q min(p(q|M,\psi),p(q|M,\phi))$$

	q_1	q_2	$\sum$
ψ	$p(q_1\|\psi)$	$1-p(q_1\|\psi)$	1
ϕ	$p(q_1\|\phi)$	$1-p(q_{11}\|\phi)$	1
min	$min(p(q_1\|\psi),p(q_1\|\phi))$	$min(1-p(q_2\|\psi),1-p(q_2\|\phi))$	$1-\|p(q_1\|\psi)-p(q_1\|\phi)\|$

Limitations on the Ψ-epistemic view of quantum states



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Symmetric overlap

$$\int min(\mu_\psi(\lambda),\mu_\phi(\lambda))\epsilon_M(q|\lambda)d\lambda \leq \int \mu_\psi(\lambda)\epsilon_M(q|\lambda)d\lambda$$
$$\leq min(\int \mu_\psi(\lambda)\epsilon_M(q|\lambda)d\lambda, \int \mu_\phi(\lambda)\epsilon_M(q|\lambda)d\lambda)$$

$$\sum_q \epsilon_M(q|\lambda)=1$$

$$\int min(\mu_\psi(\lambda),\mu_\phi(\lambda))d\lambda \leq \sum_q min(p(q|M,\psi), p(q|M,\phi))$$

	q_1	q_2	$\sum$
ψ	$p(q_1\|\psi)$	$1-p(q_1\|\psi)$	1
ϕ	$p(q_1\|\phi)$	$1-p(q_{11}\|\phi)$	1
min	$min(p(q_1\|\psi), p(q_1\|\phi))$	$min(1-p(q_2\|\psi), 1-p(q_2\|\phi))$	$1-\|p(q_1\|\psi)-p(q_1\|\phi)\|$

$$\int min_i(\mu_{\Phi_i}(\lambda))\epsilon_M(q|\lambda)d\lambda \leq \int \mu_{\Phi_i}(\lambda)\epsilon_M(q|\lambda)d\lambda$$
$$\leq min_i(\int \mu_{\Phi_i}(\lambda)\epsilon_M(q|\lambda)d\lambda)$$

$$\int min_i(\mu_{\Phi_i}(\lambda))d\lambda \leq \sum_q min_i(p(q|M,\Phi_i))$$

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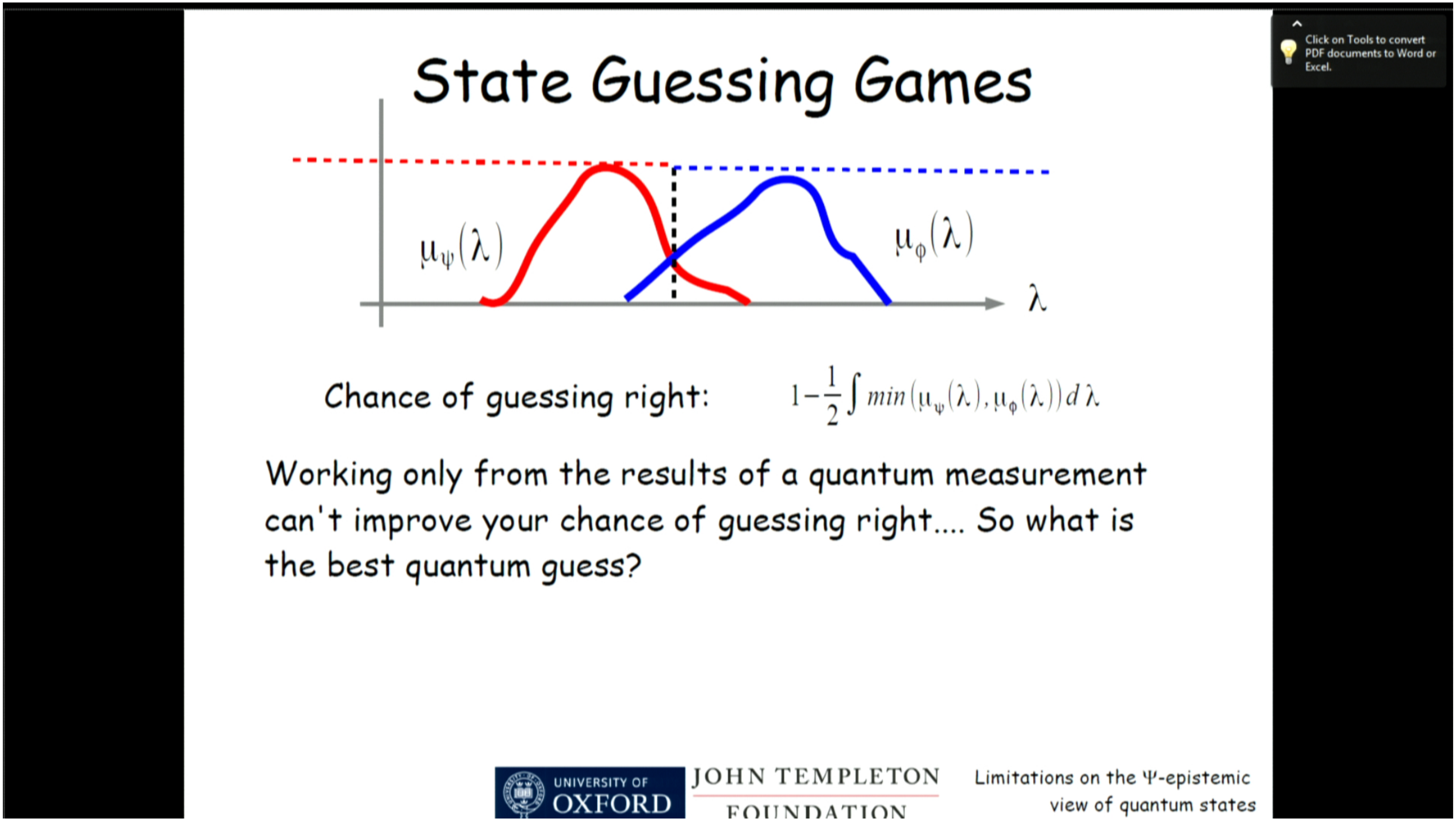

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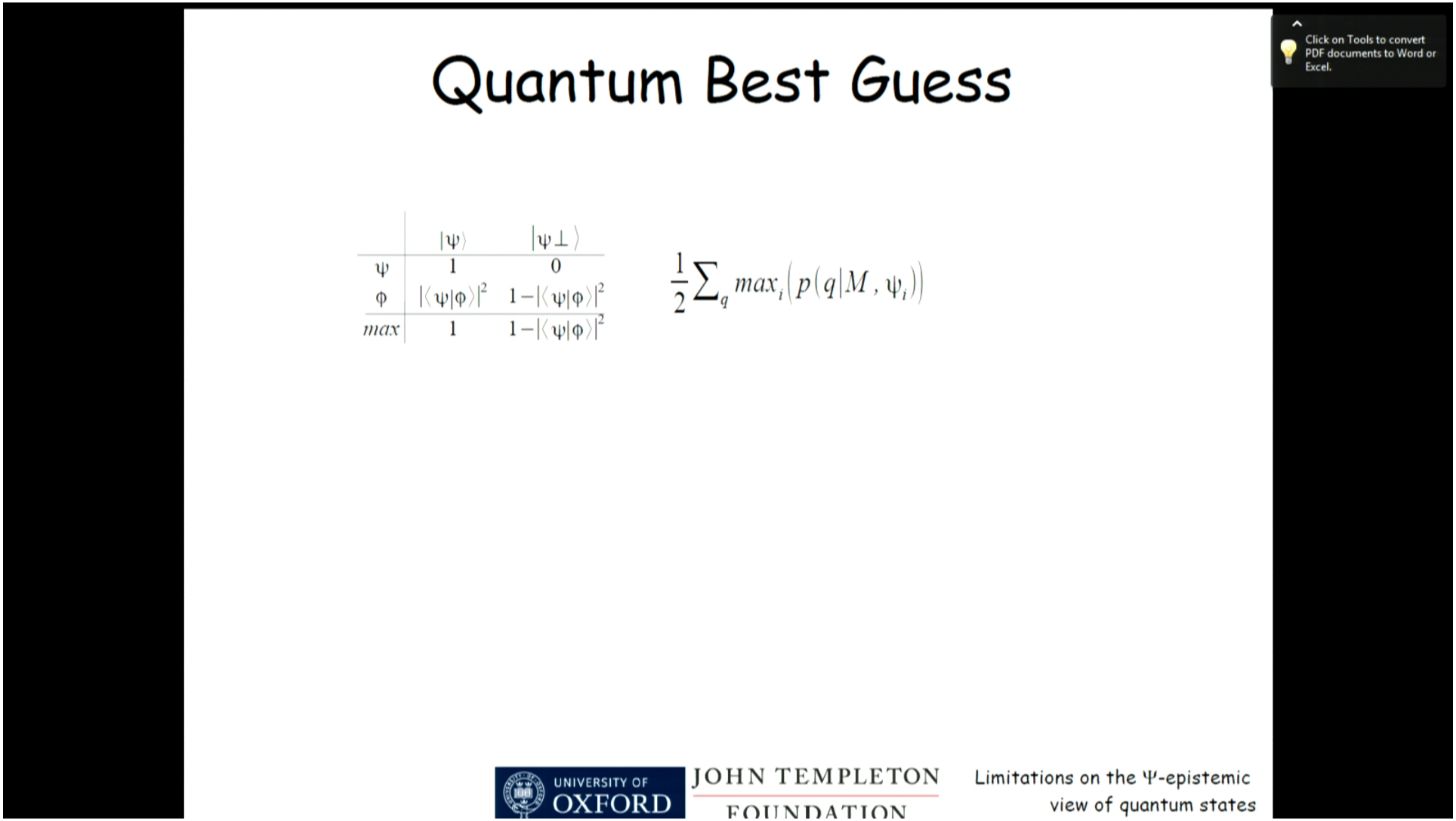
State Guessing Games

Chance of guessing right: $1-\frac{1}{2}\int min(\mu_\psi(\lambda),\mu_\phi(\lambda))d\lambda$

Working only from the results of a quantum measurement can't improve your chance of guessing right.... So what is the best quantum guess?

Limitations on the Ψ-epistemic view of quantum states



Quantum Best Guess

	$\|\psi\rangle$	$\|\psi\perp\rangle$
ψ	1	0
φ	$\|\langle\psi\|\varphi\rangle\|^2$	$1-\|\langle\psi\|\varphi\rangle\|^2$
max	1	$1-\|\langle\psi\|\varphi\rangle\|^2$

$$\frac{1}{2}\sum_q max_i\left(p\left(q|M,\psi_i\right)\right)$$

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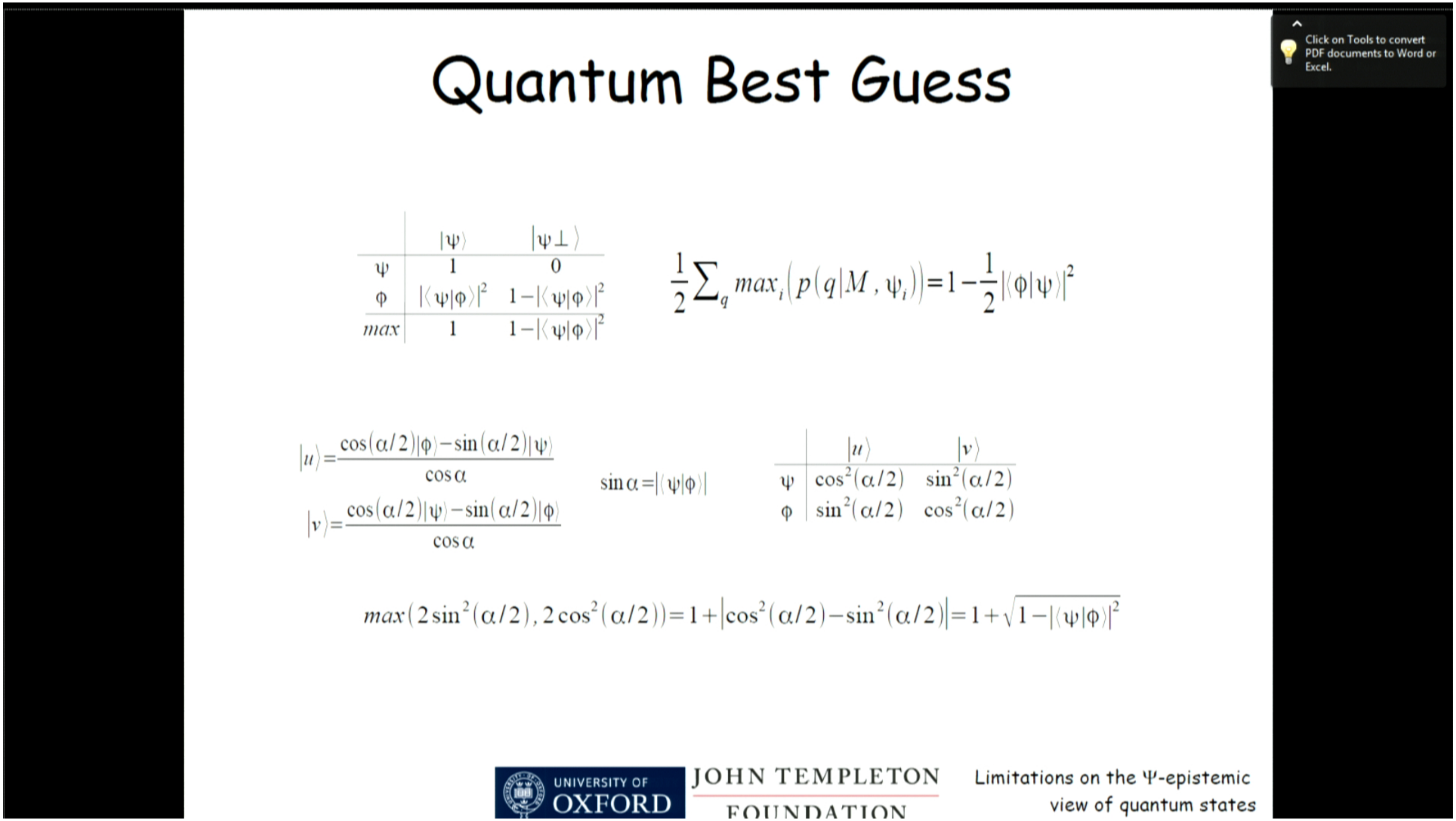

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Quantum Best Guess

	$\vert\psi\rangle$	$\vert\psi\perp\rangle$
ψ	1	0
ϕ	$\vert\langle\psi\vert\phi\rangle\vert^2$	$1-\vert\langle\psi\vert\phi\rangle\vert^2$
max	1	$1-\vert\langle\psi\vert\phi\rangle\vert^2$

$$\frac{1}{2}\sum_q max_i\left(p(q|M,\psi_i)\right)=1-\frac{1}{2}|\langle\phi|\psi\rangle|^2$$

$$|u\rangle=\frac{\cos(\alpha/2)|\phi\rangle-\sin(\alpha/2)|\psi\rangle}{\cos\alpha}$$

$$|v\rangle=\frac{\cos(\alpha/2)|\psi\rangle-\sin(\alpha/2)|\phi\rangle}{\cos\alpha}$$

$$\sin\alpha=|\langle\psi|\phi\rangle|$$

	$\vert u\rangle$	$\vert v\rangle$
ψ	$\cos^2(\alpha/2)$	$\sin^2(\alpha/2)$
ϕ	$\sin^2(\alpha/2)$	$\cos^2(\alpha/2)$

$$max(2\sin^2(\alpha/2),2\cos^2(\alpha/2))=1+\left|\cos^2(\alpha/2)-\sin^2(\alpha/2)\right|=1+\sqrt{1-|\langle\psi|\phi\rangle|^2}$$

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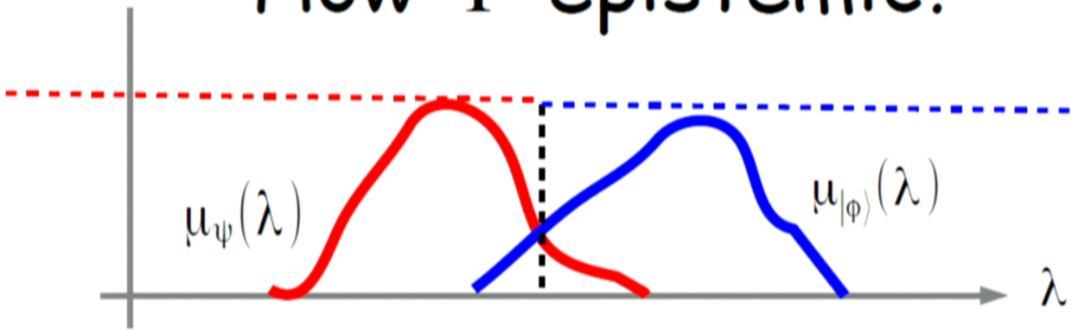
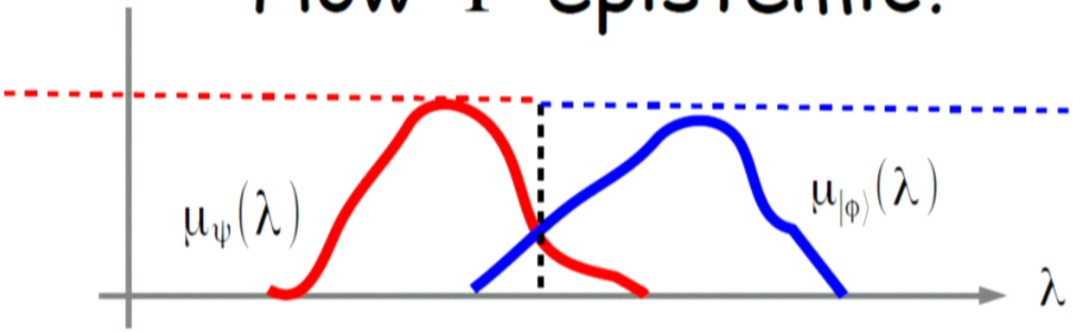
Limitations on the Ψ-epistemic view of quantum states

How Ψ-epistemic?



$$1-\frac{1}{2}\int min(\mu_\psi(\lambda),\mu_\phi(\lambda))d\lambda \geq \frac{1}{2}(1+\sqrt{1-|\langle\psi|\phi\rangle|^2})$$

How Ψ-epistemic?



$$1-\frac{1}{2}\int min(\mu_\psi(\lambda),\mu_\phi(\lambda))d\lambda \geq \frac{1}{2}(1+\sqrt{1-|\langle\psi|\phi\rangle|^2})$$

$$\int min(\mu_\psi(\lambda),\mu_\phi(\lambda))d\lambda \leq (1-\sqrt{1-|\langle\psi|\phi\rangle|^2}) < |\langle\psi|\phi\rangle|^2$$

$$\int min(\mu_\psi(\lambda),\mu_\phi(\lambda))d\lambda = \varpi(\psi,\phi)(1-\sqrt{1-|\langle\psi|\phi\rangle|^2})$$

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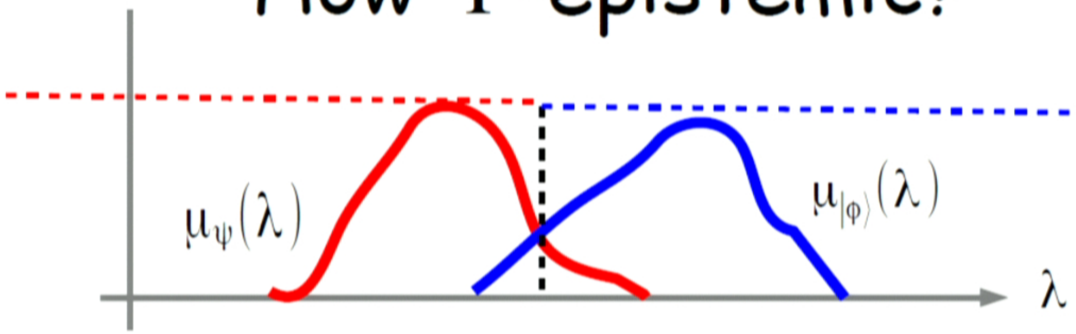

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$$\int min(\mu_\psi(\lambda),\mu_\phi(\lambda))d\lambda\leq(1-\sqrt{1-|\langle\psi|\phi\rangle|^2})<|\langle\psi|\phi\rangle|^2$$

$$\int min(\mu_\psi(\lambda),\mu_\phi(\lambda))d\lambda=\varpi(\psi,\phi)(1-\sqrt{1-|\langle\psi|\phi\rangle|^2})$$

$$0\leq\varpi(\phi,\psi)\leq 1$$

Ψ-ontic $\varpi(\phi,\psi)=0$ Maximally Ψ-epistemic $\varpi(\phi,\psi)=1$

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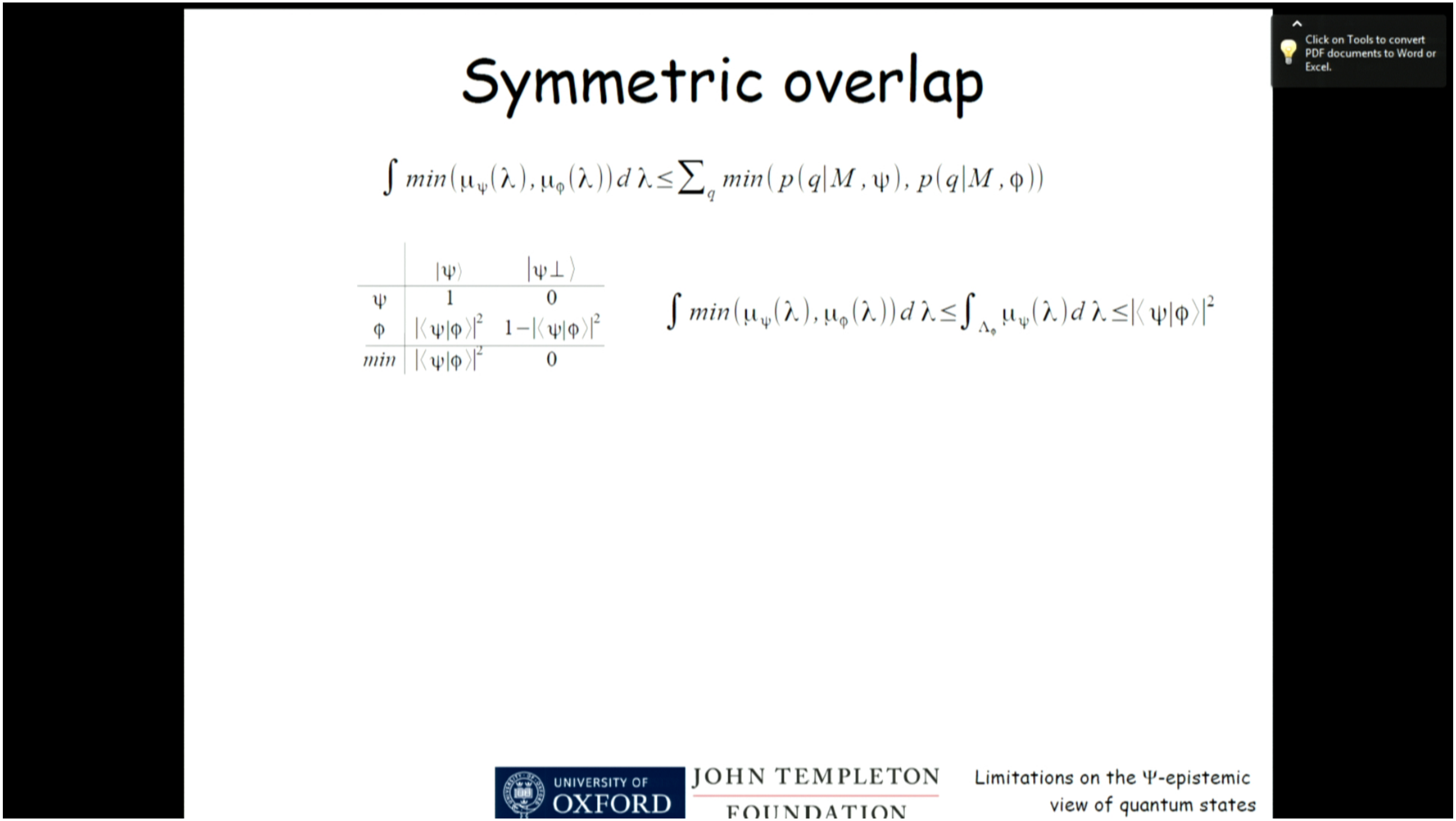

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Symmetric overlap

$$\int min(\mu_\psi(\lambda),\mu_\phi(\lambda))d\lambda \leq \sum_q min(p(q|M,\psi),p(q|M,\phi))$$

	$|\psi\rangle$	$|\psi\perp\rangle$
ψ	1	0
ϕ	$|\langle\psi|\phi\rangle|^2$	$1-|\langle\psi|\phi\rangle|^2$
min	$|\langle\psi|\phi\rangle|^2$	0

$$\int min(\mu_\psi(\lambda),\mu_\phi(\lambda))d\lambda \leq \int_{\Lambda_\phi} \mu_\psi(\lambda)d\lambda \leq |\langle\psi|\phi\rangle|^2$$

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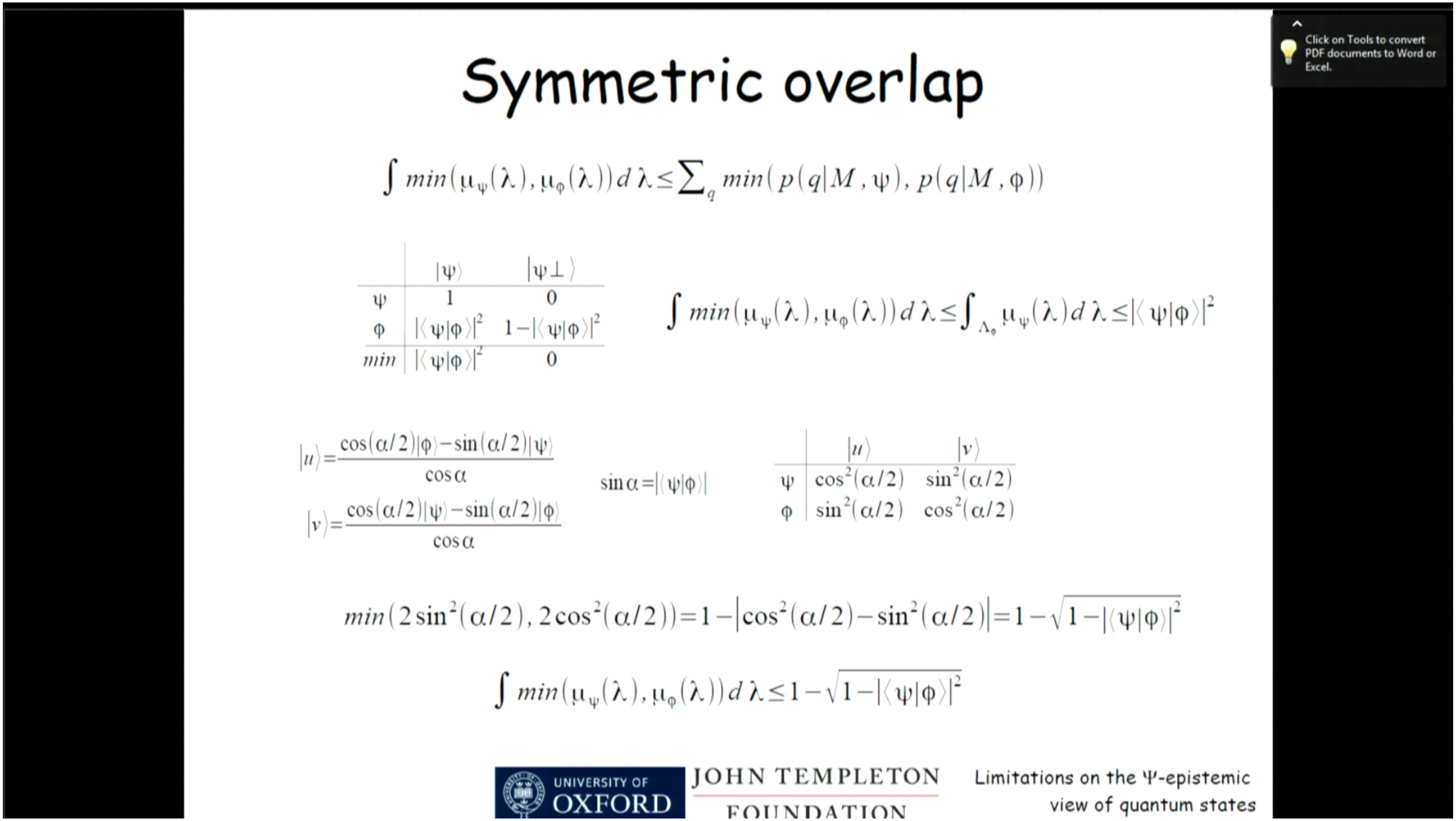

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min	$\vert\langle\psi\vert\phi\rangle\vert^2$	0

$$\int min(\mu_\psi(\lambda),\mu_\phi(\lambda))\,d\lambda \le \int_{\Lambda_\phi} \mu_\psi(\lambda)\,d\lambda \le |\langle\psi|\phi\rangle|^2$$

$$|u\rangle = \frac{\cos(\alpha/2)|\phi\rangle - \sin(\alpha/2)|\psi\rangle}{\cos\alpha}$$

$$|v\rangle = \frac{\cos(\alpha/2)|\psi\rangle - \sin(\alpha/2)|\phi\rangle}{\cos\alpha}$$

$$\sin\alpha = |\langle\psi|\phi\rangle|$$

	$\vert u\rangle$	$\vert v\rangle$
ψ	$\cos^2(\alpha/2)$	$\sin^2(\alpha/2)$
ϕ	$\sin^2(\alpha/2)$	$\cos^2(\alpha/2)$

$$min(2\sin^2(\alpha/2),\,2\cos^2(\alpha/2)) = 1 - \left|\cos^2(\alpha/2) - \sin^2(\alpha/2)\right| = 1 - \sqrt{1 - |\langle\psi|\phi\rangle|^2}$$

$$\int min(\mu_\psi(\lambda),\mu_\phi(\lambda))\,d\lambda \le 1 - \sqrt{1 - |\langle\psi|\phi\rangle|^2}$$

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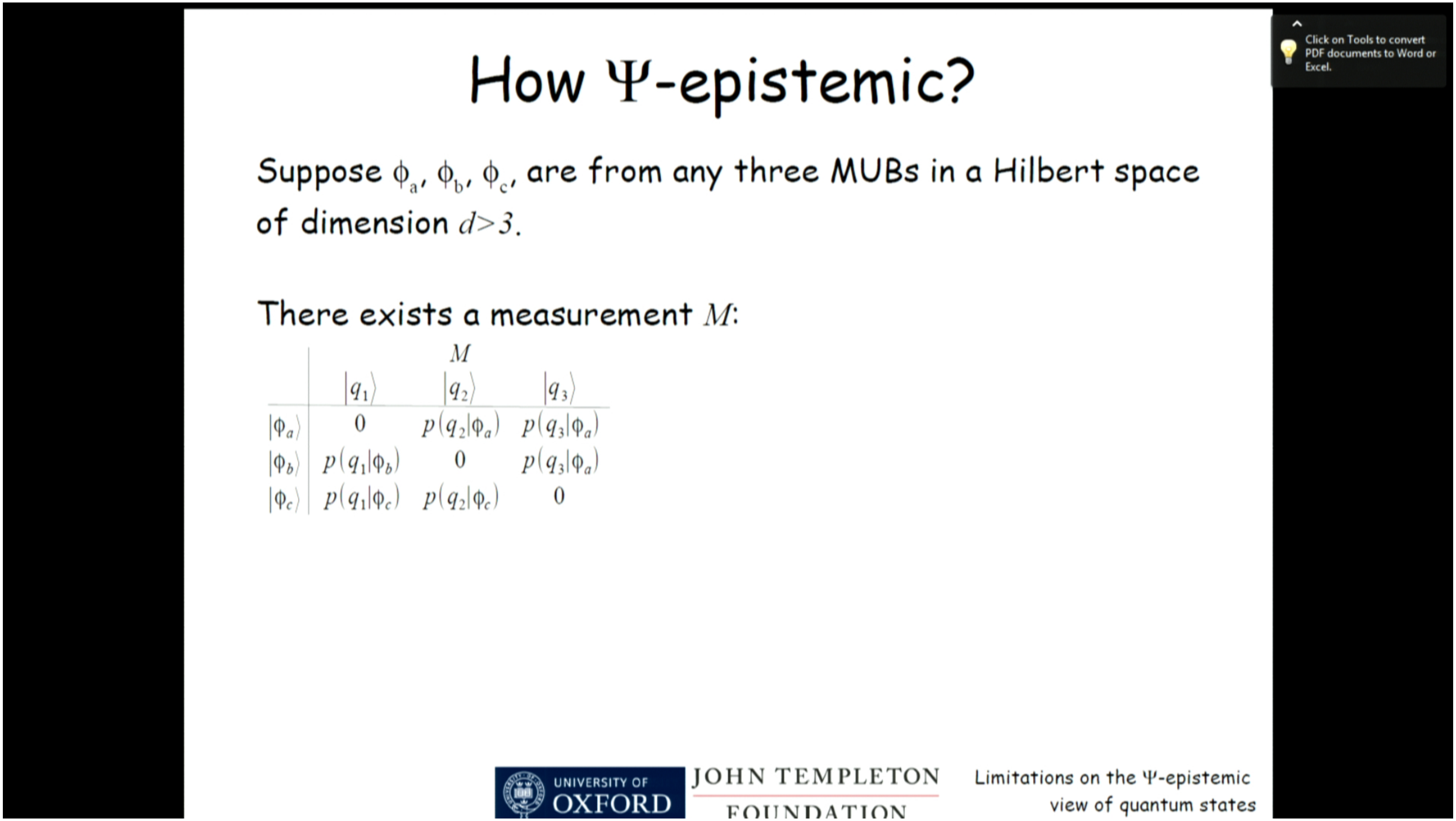

How Ψ-epistemic?

Suppose ϕ_a, ϕ_b, ϕ_c, are from any three MUBs in a Hilbert space of dimension $d>3$.

There exists a measurement M:

		M	
	$\lvert q_1\rangle$	$\lvert q_2\rangle$	$\lvert q_3\rangle$
$\lvert\phi_a\rangle$	0	$p(q_2\vert\phi_a)$	$p(q_3\vert\phi_a)$
$\lvert\phi_b\rangle$	$p(q_1\vert\phi_b)$	0	$p(q_3\vert\phi_a)$
$\lvert\phi_c\rangle$	$p(q_1\vert\phi_c)$	$p(q_2\vert\phi_c)$	0

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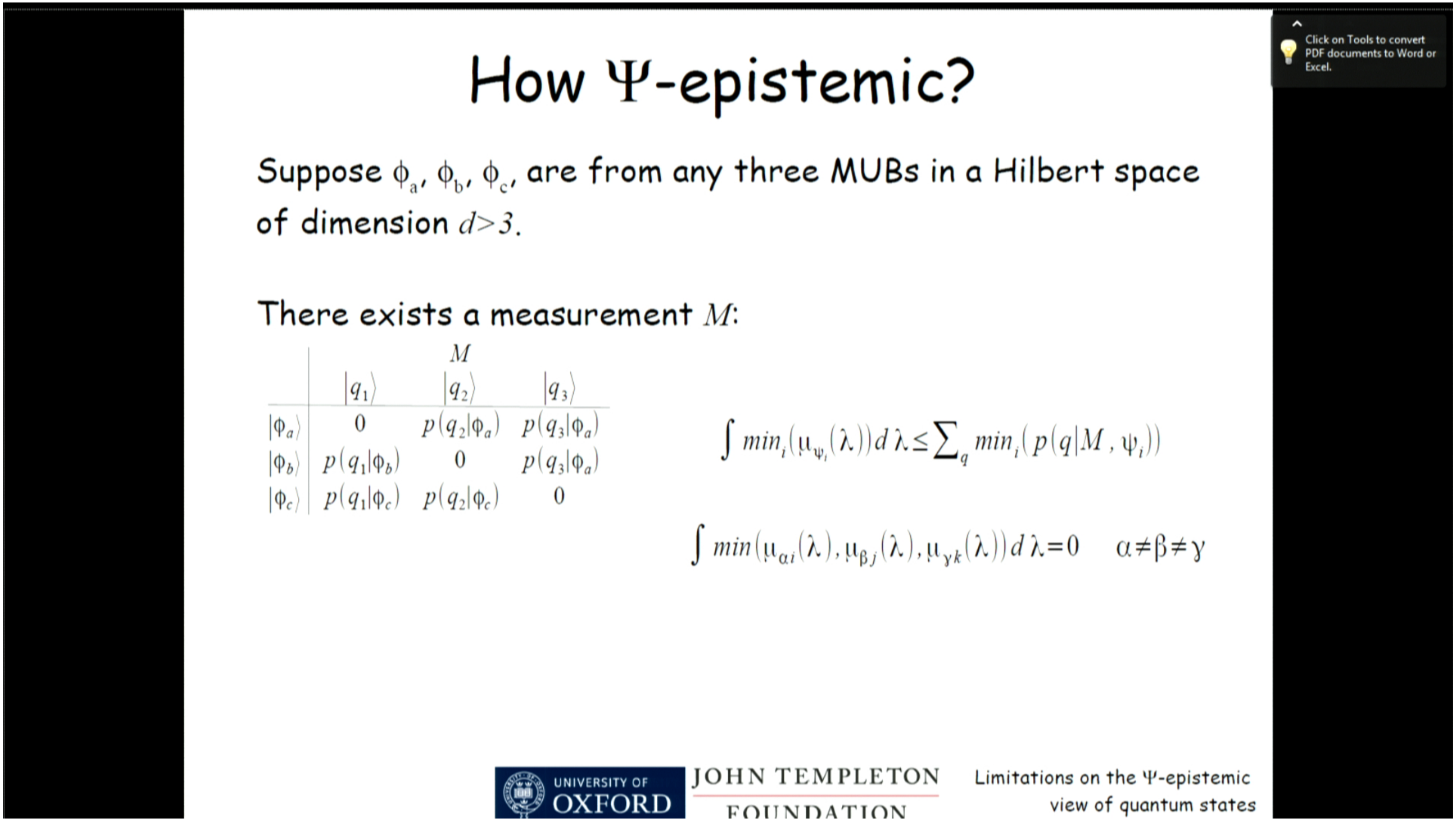

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$\|\phi_a\rangle$	0	$p(q_2\|\phi_a)$	$p(q_3\|\phi_a)$
$\|\phi_b\rangle$	$p(q_1\|\phi_b)$	0	$p(q_3\|\phi_a)$
$\|\phi_c\rangle$	$p(q_1\|\phi_c)$	$p(q_2\|\phi_c)$	0

$$\int min_i(\mu_{\psi_i}(\lambda))d\lambda \leq \sum_q min_i(p(q|M,\psi_i))$$

$$\int min(\mu_{\alpha i}(\lambda),\mu_{\beta j}(\lambda),\mu_{\gamma k}(\lambda))d\lambda=0 \quad \alpha\neq\beta\neq\gamma$$

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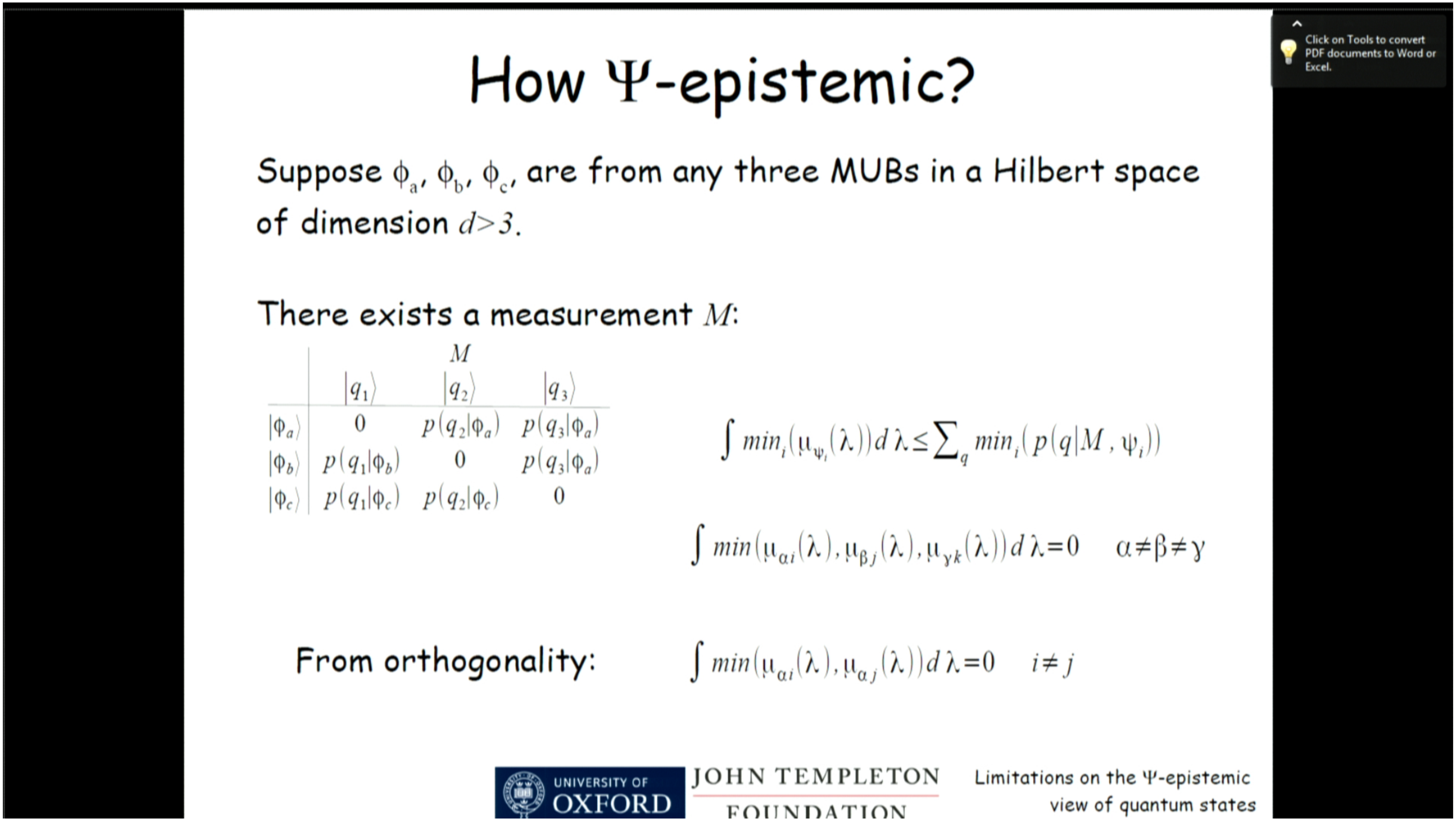

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$\lvert\phi_b\rangle$	$p(q_1\lvert\phi_b)$	0	$p(q_3\lvert\phi_a)$
$\lvert\phi_c\rangle$	$p(q_1\lvert\phi_c)$	$p(q_2\lvert\phi_c)$	0

$$\int min_i(\mu_{\psi_i}(\lambda))d\lambda \leq \sum_q min_i(p(q|M,\psi_i))$$

$$\int min(\mu_{\alpha i}(\lambda),\mu_{\beta j}(\lambda),\mu_{\gamma k}(\lambda))d\lambda=0 \quad \alpha\neq\beta\neq\gamma$$

From orthogonality:

$$\int min(\mu_{\alpha i}(\lambda),\mu_{\alpha j}(\lambda))d\lambda=0 \quad i\neq j$$

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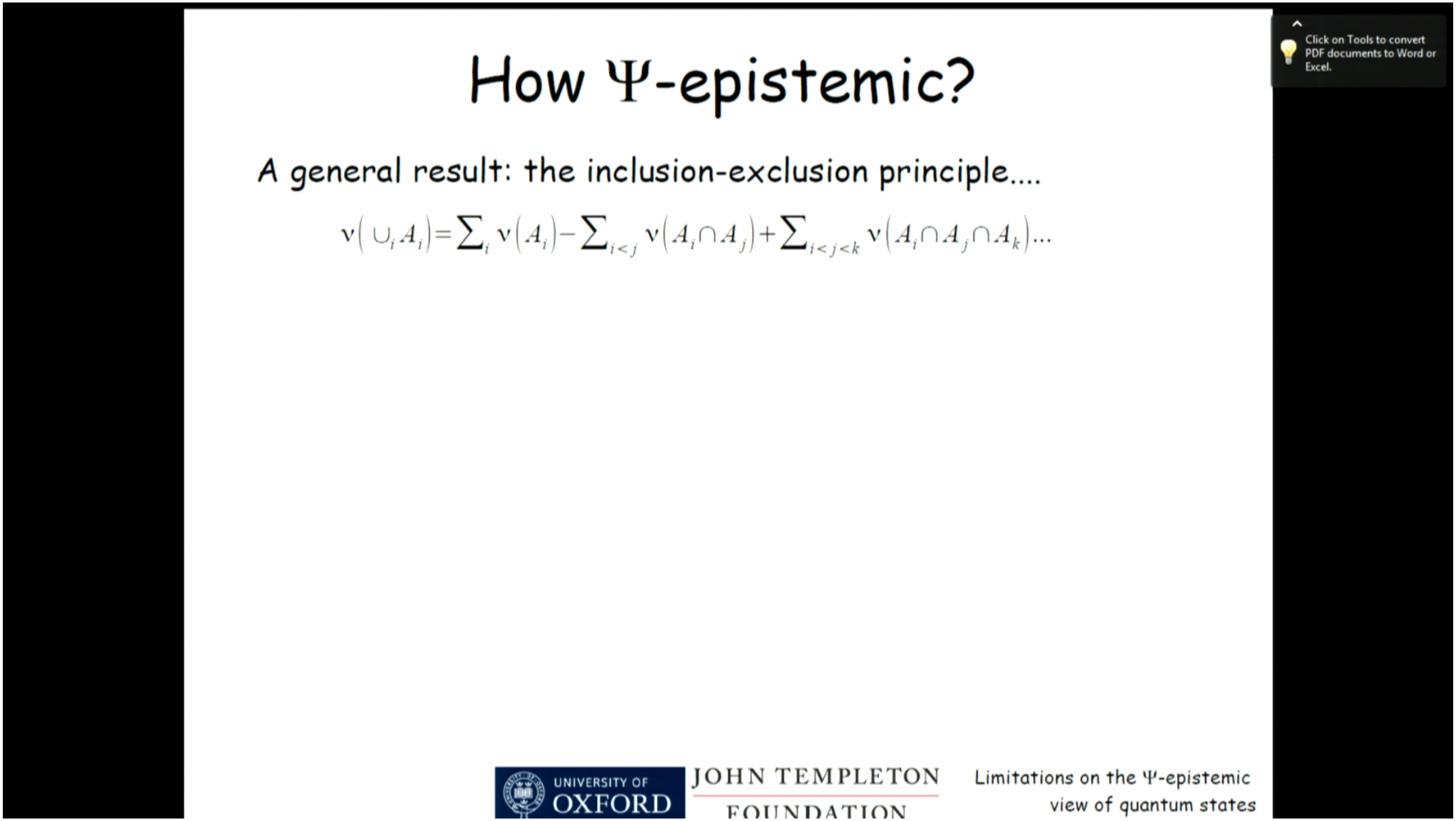

How Ψ-epistemic?

A general result: the inclusion-exclusion principle....

$$\nu\left(\cup_i A_i\right)=\sum_i \nu\left(A_i\right)-\sum_{i<j}\nu\left(A_i\cap A_j\right)+\sum_{i<j<k}\nu\left(A_i\cap A_j\cap A_k\right)\ldots$$

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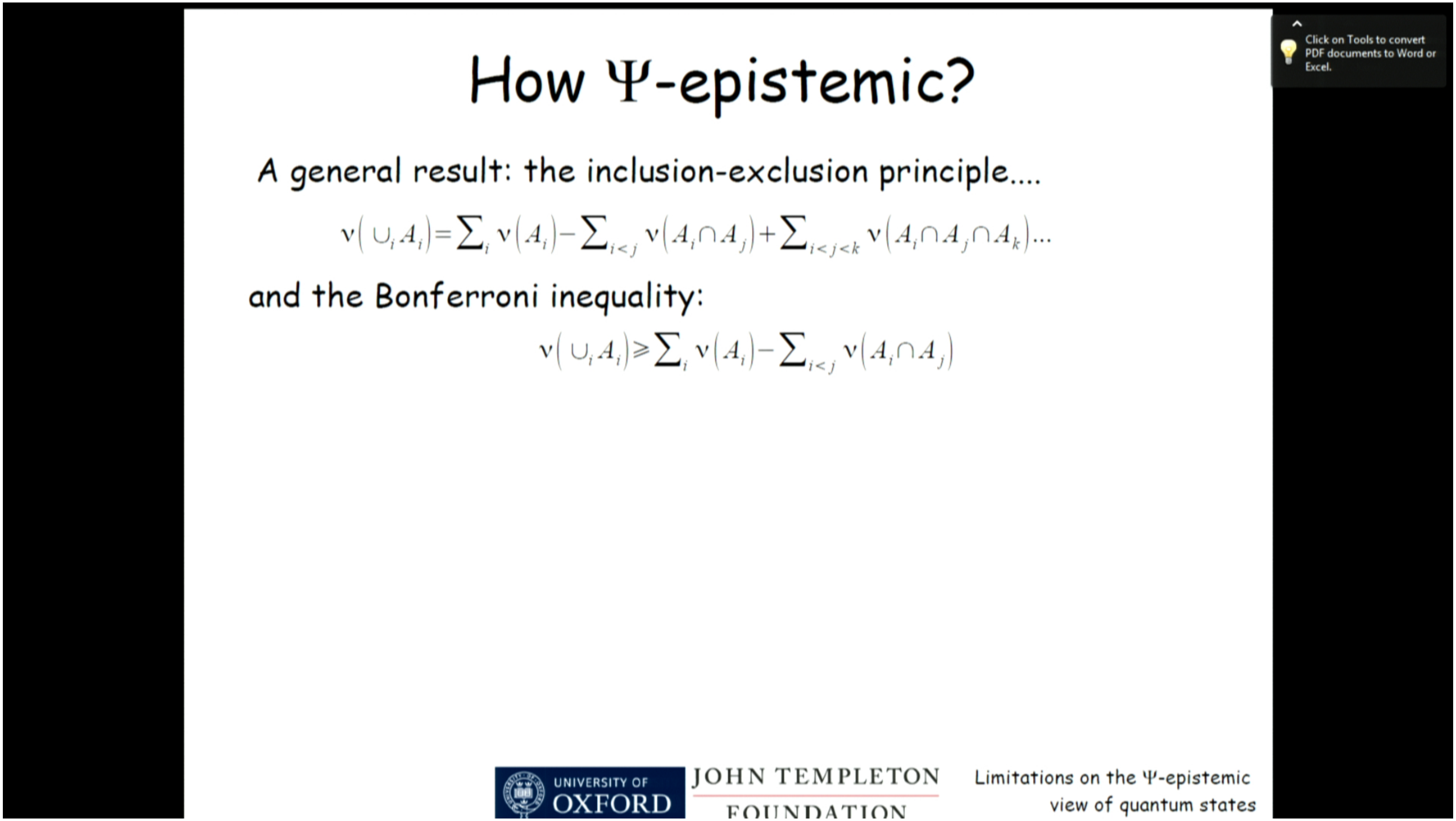
Limitations on the Ψ-epistemic view of quantum states



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and the Bonferroni inequality:

$$\nu\left(\cup_i A_i\right)\geqslant\sum_i \nu\left(A_i\right)-\sum_{i<j} \nu\left(A_i\cap A_j\right)$$

Limitations on the Ψ-epistemic
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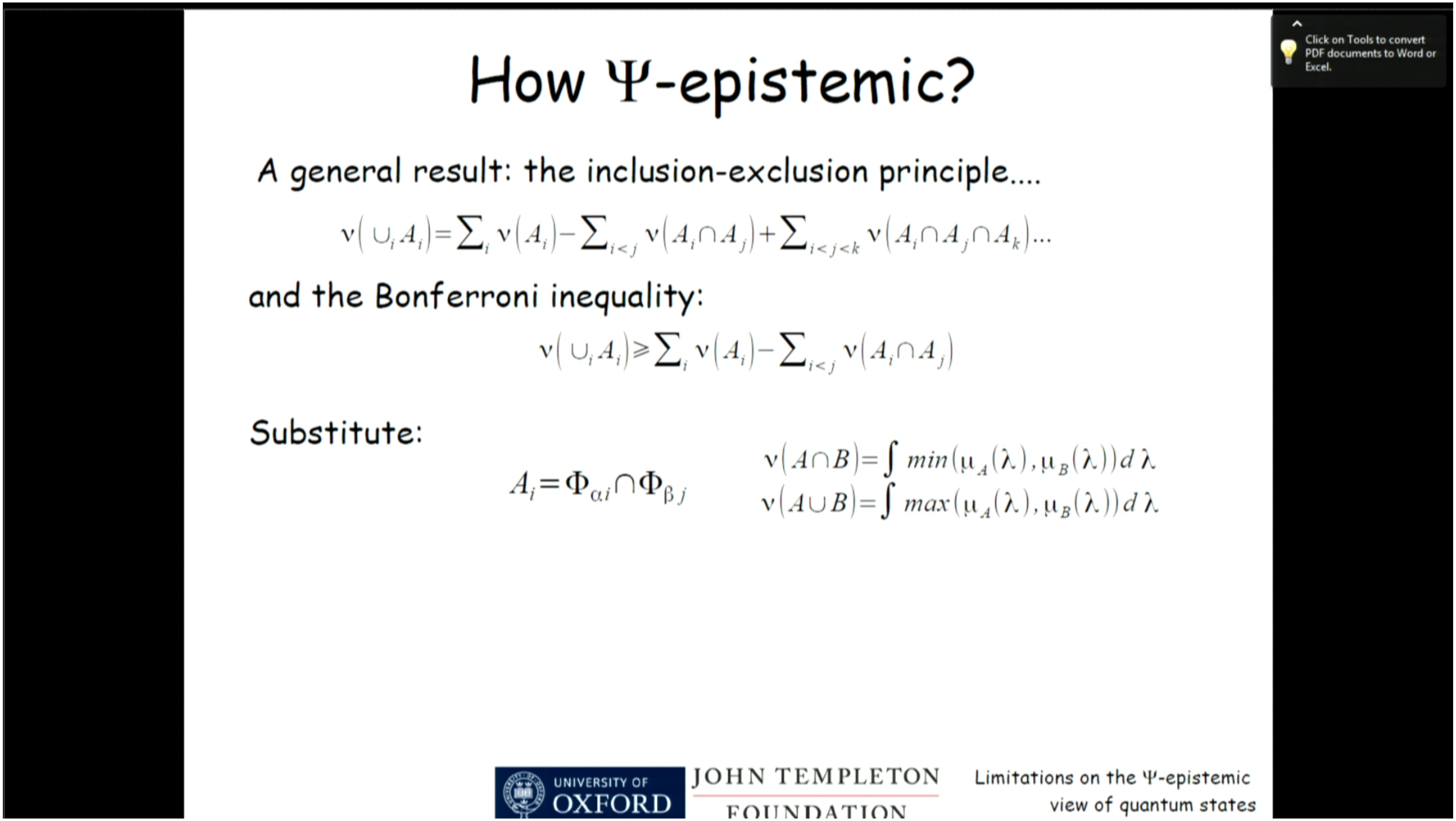

How Ψ-epistemic?

A general result: the inclusion-exclusion principle....

$$\nu\left(\cup_i A_i\right)=\sum_i \nu\left(A_i\right)-\sum_{i<j}\nu\left(A_i\cap A_j\right)+\sum_{i<j<k}\nu\left(A_i\cap A_j\cap A_k\right)\ldots$$

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Substitute:

$$A_i=\Phi_{\alpha i}\cap\Phi_{\beta j}$$

$$\nu\left(A\cap B\right)=\int min\left(\mu_A(\lambda),\mu_B(\lambda)\right)d\lambda$$
$$\nu\left(A\cup B\right)=\int max\left(\mu_A(\lambda),\mu_B(\lambda)\right)d\lambda$$

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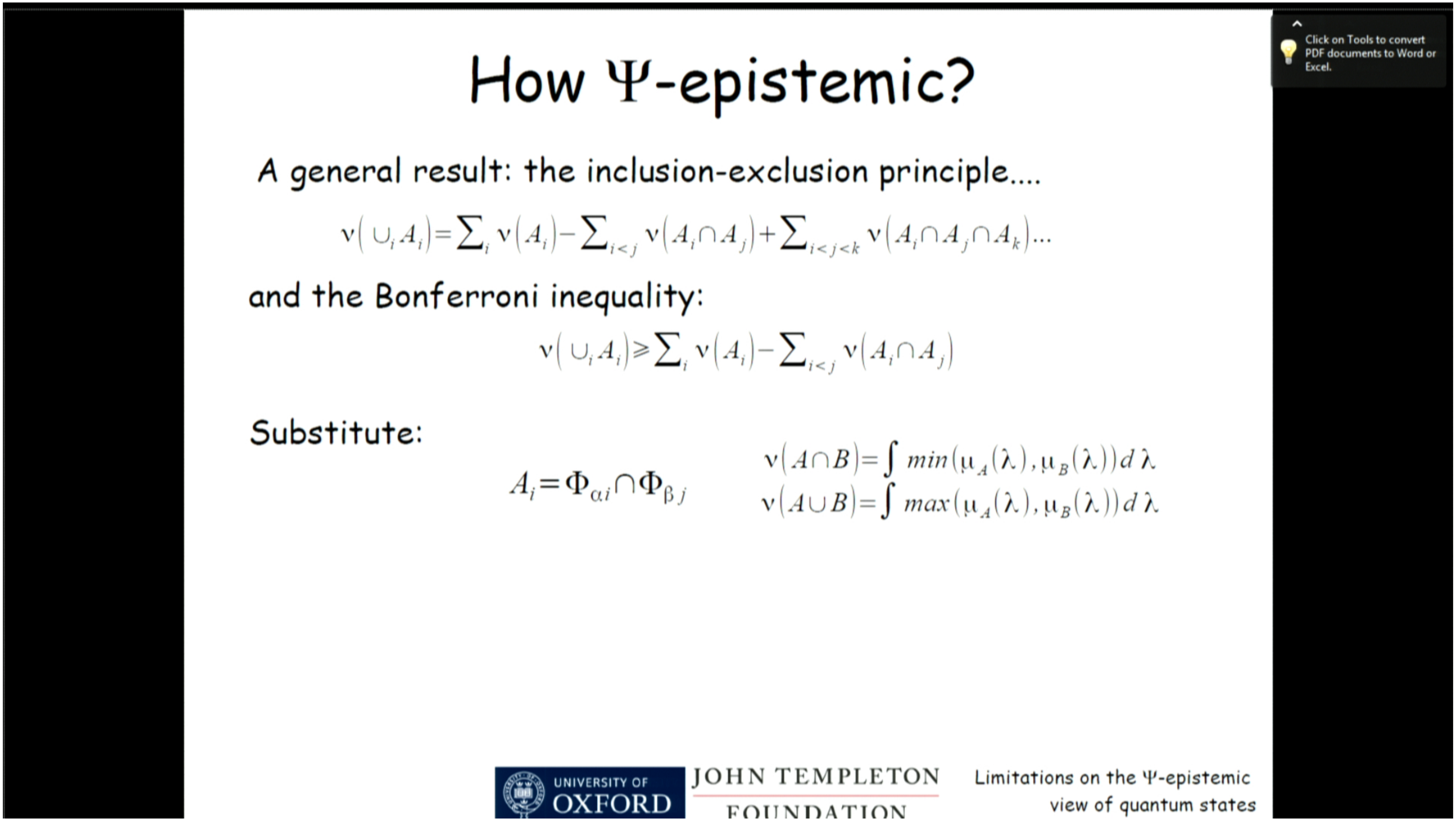

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Limitations on the Ψ-epistemic view of quantum states

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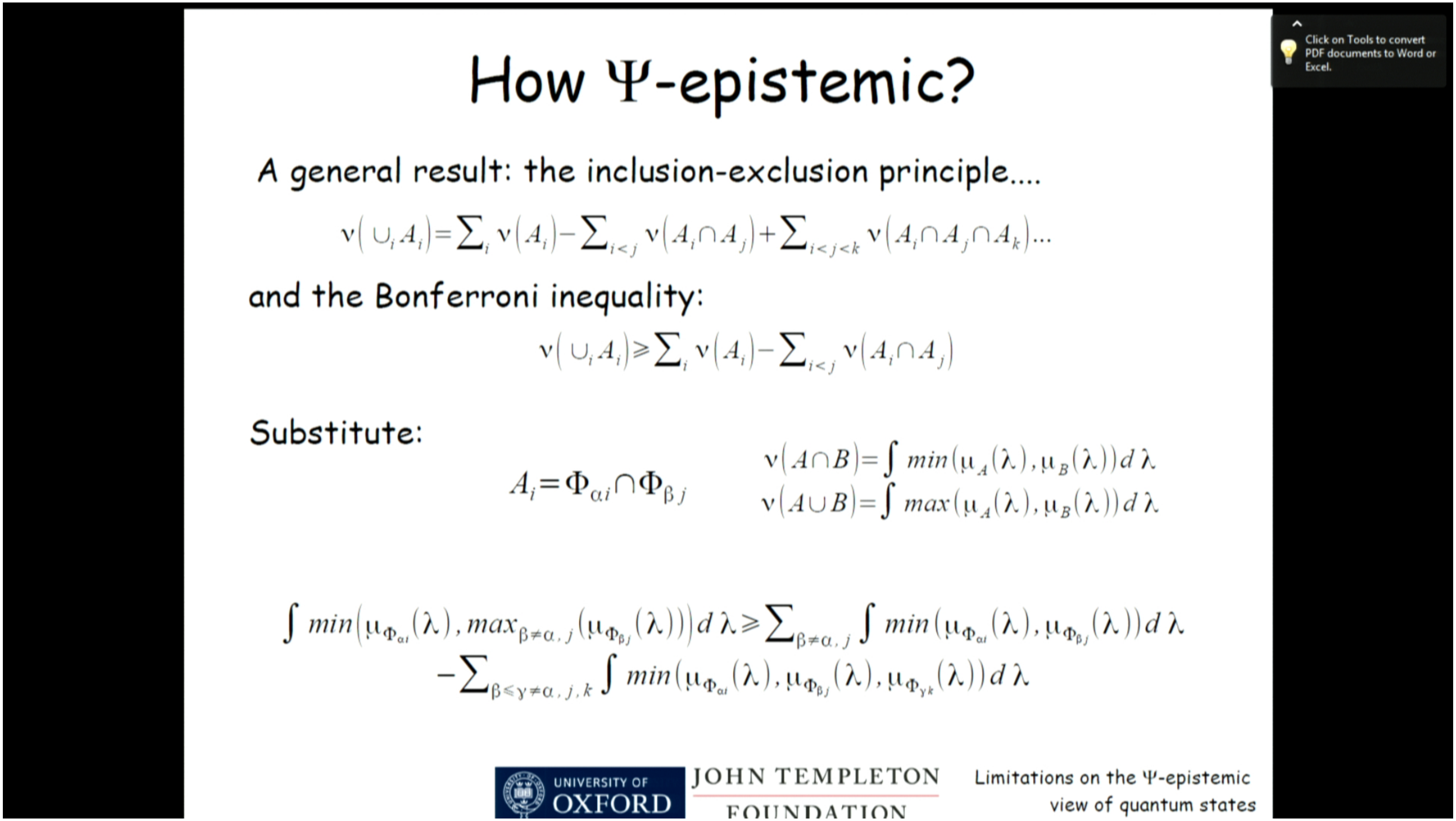

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$$\int min\left(\mu_{\Phi_{\alpha i}}(\lambda), max_{\beta \neq \alpha, j}\left(\mu_{\Phi_{\beta j}}(\lambda)\right)\right) d\lambda \geqslant \sum_{\beta \neq \alpha, j} \int min\left(\mu_{\Phi_{\alpha i}}(\lambda), \mu_{\Phi_{\beta j}}(\lambda)\right) d\lambda$$

$$-\sum_{\beta \leqslant \gamma \neq \alpha, j, k} \int min\left(\mu_{\Phi_{\alpha i}}(\lambda), \mu_{\Phi_{\beta j}}(\lambda), \mu_{\Phi_{\gamma k}}(\lambda)\right) d\lambda$$

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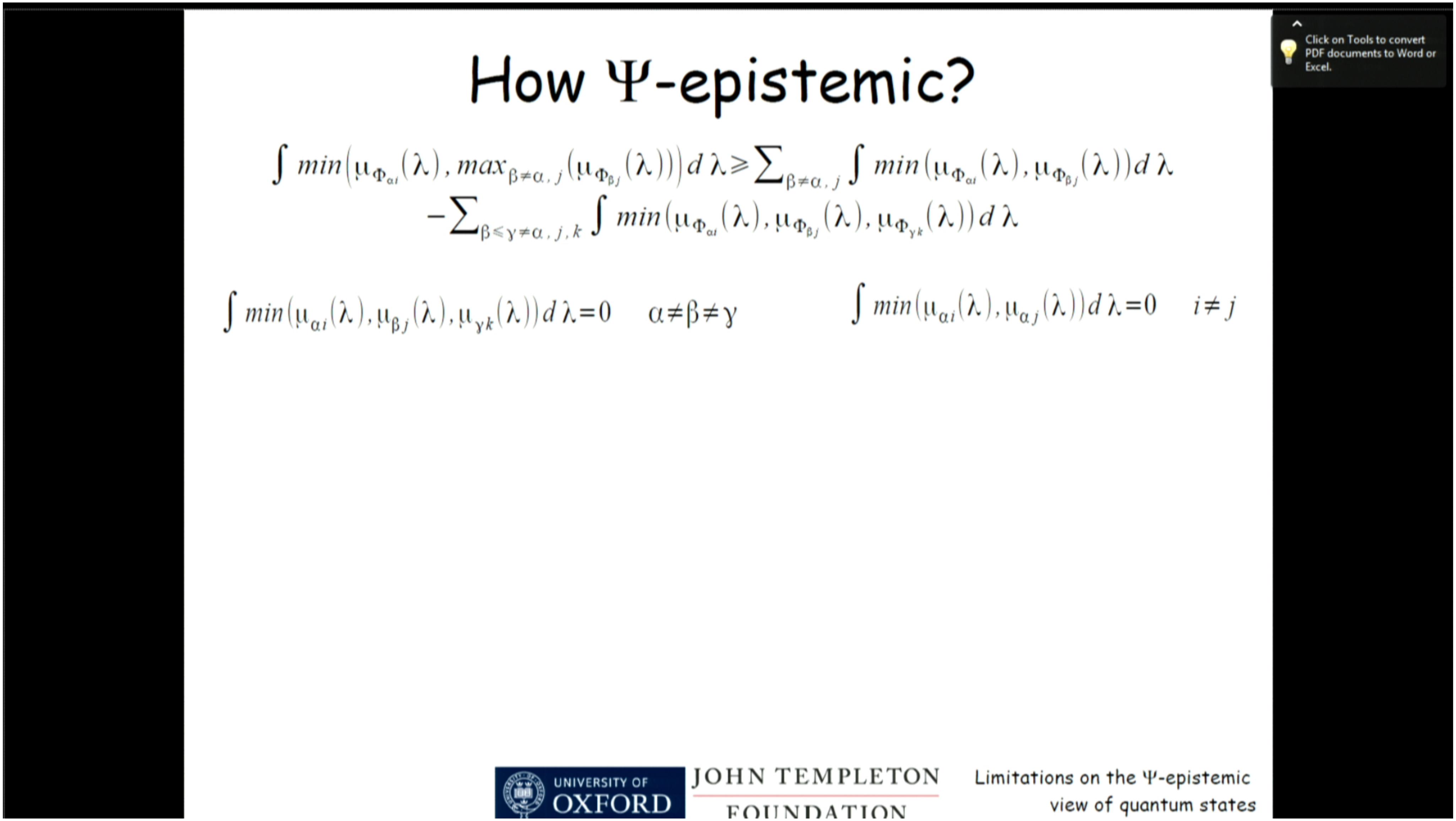

How Ψ-epistemic?

$$\int min\left(\mu_{\Phi_{\alpha i}}(\lambda), max_{\beta\neq\alpha,j}\left(\mu_{\Phi_{\beta j}}(\lambda)\right)\right) d\lambda \geqslant \sum_{\beta\neq\alpha,j}\int min\left(\mu_{\Phi_{\alpha i}}(\lambda), \mu_{\Phi_{\beta j}}(\lambda)\right) d\lambda$$

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$$\int min\left(\mu_{\alpha i}(\lambda), \mu_{\beta j}(\lambda), \mu_{\gamma k}(\lambda)\right) d\lambda = 0 \quad \alpha\neq\beta\neq\gamma$$

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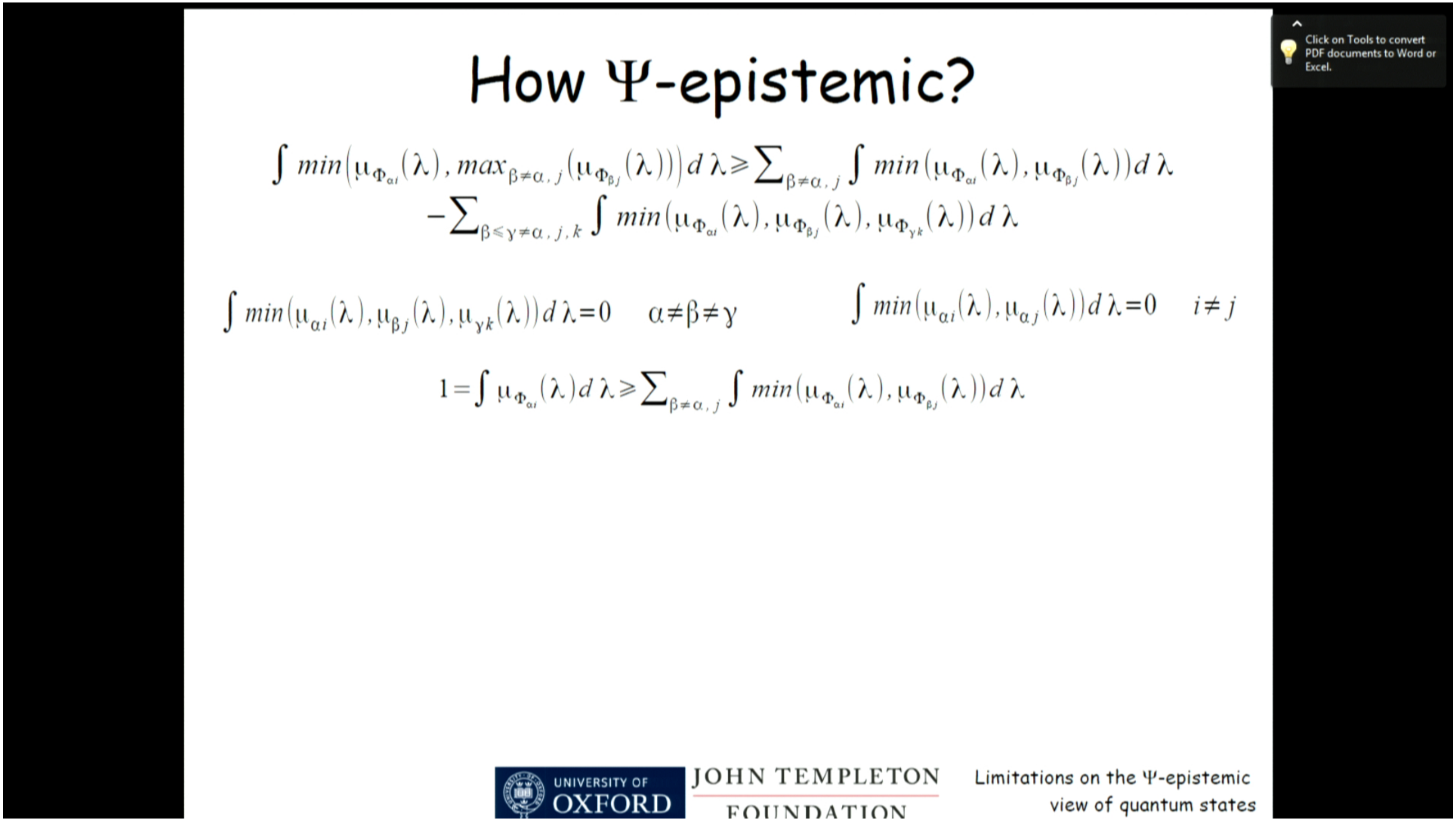

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$$1=\int \mu_{\Phi_{\alpha i}}(\lambda)d\lambda \geqslant \sum_{\beta\neq\alpha,j}\int min\left(\mu_{\Phi_{\alpha i}}(\lambda),\mu_{\Phi_{\beta j}}(\lambda)\right)d\lambda$$

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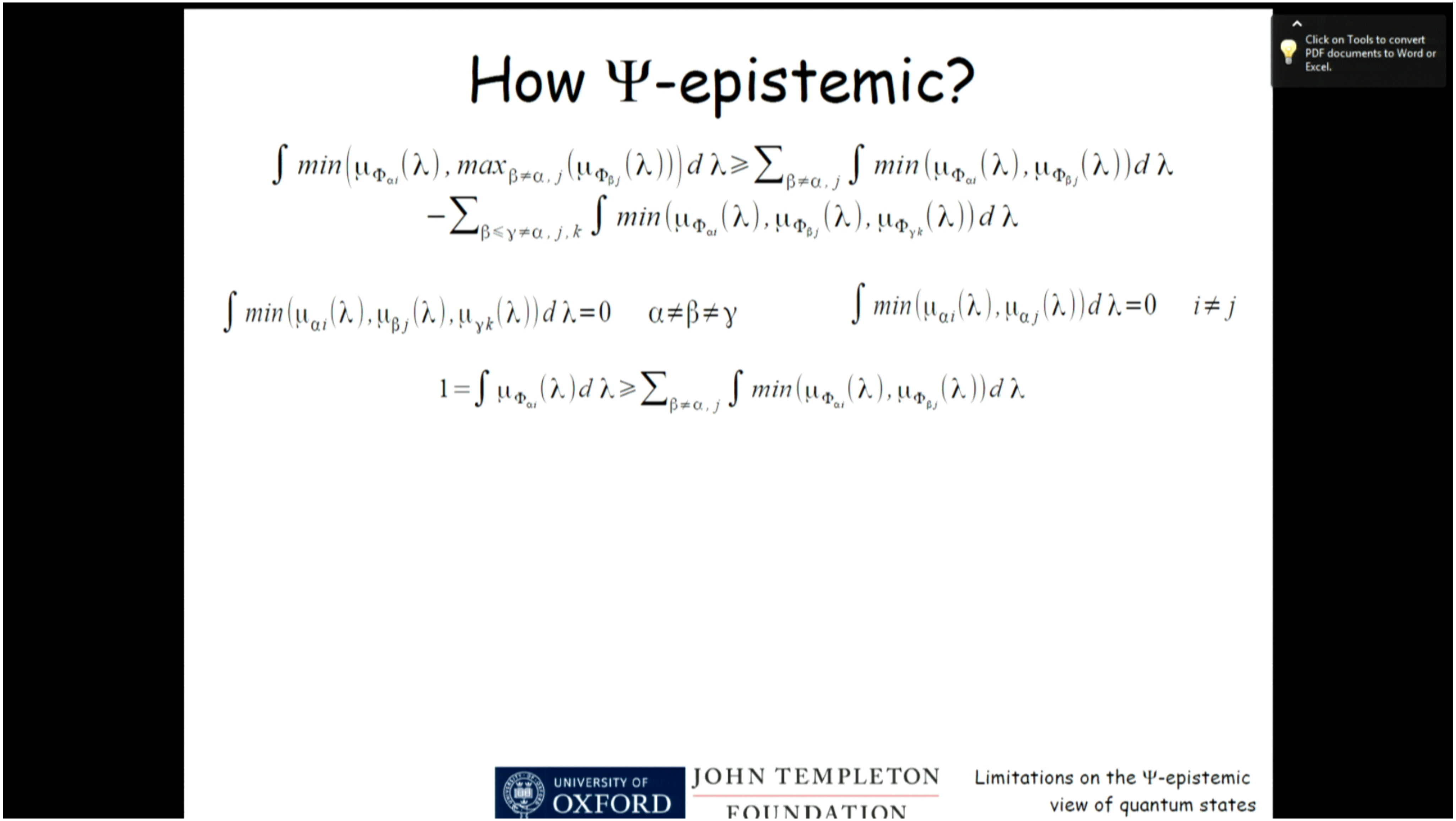

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How Ψ-epistemic?

$$\int min\left(\mu_{\Phi_{\alpha i}}(\lambda), max_{\beta\neq\alpha,j}(\mu_{\Phi_{\beta j}}(\lambda))\right) d\lambda \geqslant \sum_{\beta\neq\alpha,j}\int min(\mu_{\Phi_{\alpha i}}(\lambda),\mu_{\Phi_{\beta j}}(\lambda))d\lambda$$
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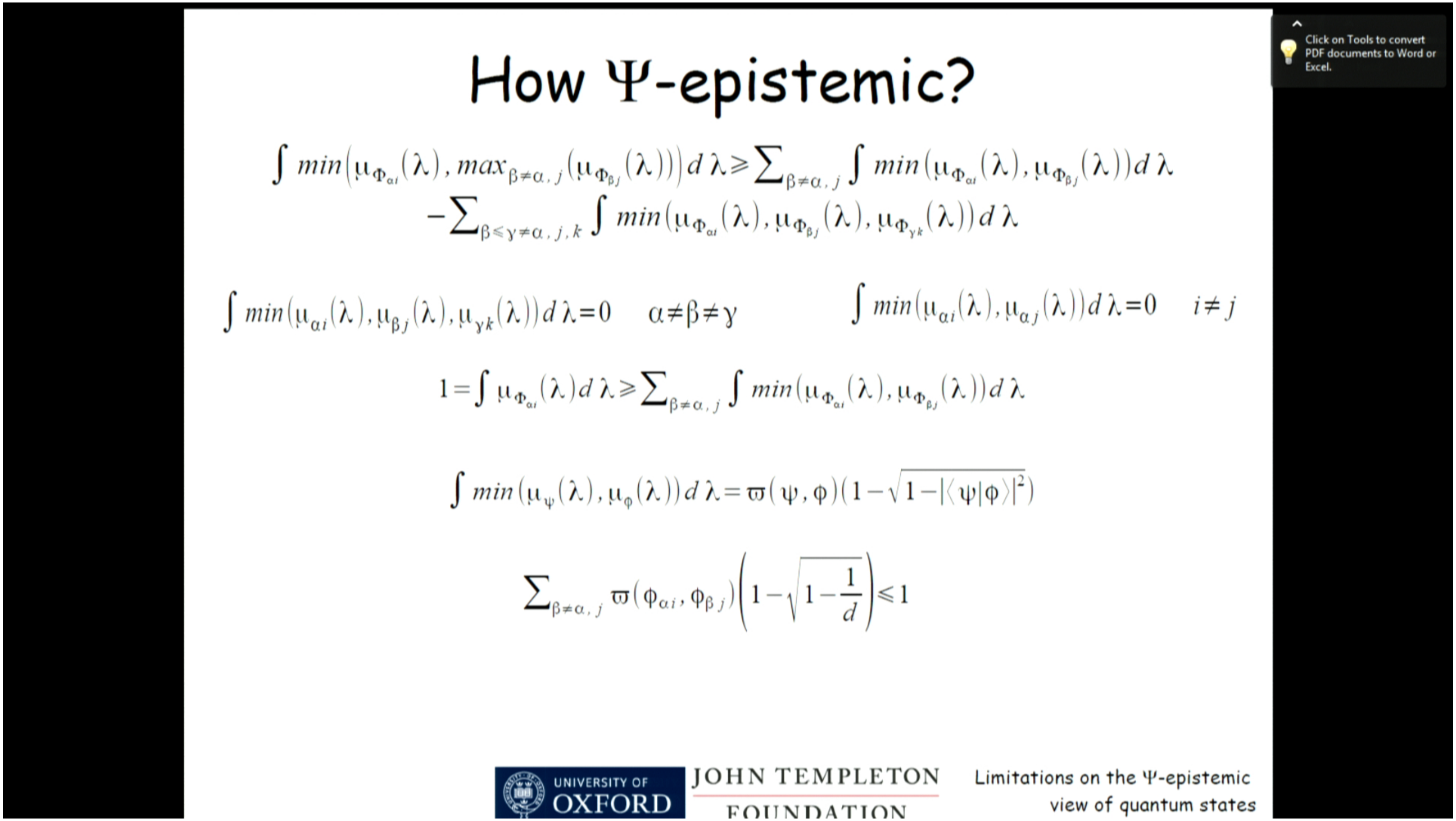

How Ψ-epistemic?

$$\int min\left(\mu_{\Phi_{\alpha i}}(\lambda), max_{\beta\neq\alpha,\,j}(\mu_{\Phi_{\beta j}}(\lambda))\right) d\lambda \geqslant \sum_{\beta\neq\alpha,\,j} \int min(\mu_{\Phi_{\alpha i}}(\lambda), \mu_{\Phi_{\beta j}}(\lambda)) d\lambda$$

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$$\int min(\mu_{\alpha i}(\lambda), \mu_{\beta j}(\lambda), \mu_{\gamma k}(\lambda)) d\lambda = 0 \quad \alpha\neq\beta\neq\gamma \qquad \int min(\mu_{\alpha i}(\lambda), \mu_{\alpha j}(\lambda)) d\lambda = 0 \quad i\neq j$$

$$1 = \int \mu_{\Phi_{\alpha i}}(\lambda) d\lambda \geqslant \sum_{\beta\neq\alpha,\,j} \int min(\mu_{\Phi_{\alpha i}}(\lambda), \mu_{\Phi_{\beta j}}(\lambda)) d\lambda$$

$$\int min(\mu_{\psi}(\lambda), \mu_{\varphi}(\lambda)) d\lambda = \varpi(\psi, \varphi)(1 - \sqrt{1 - |\langle\psi|\varphi\rangle|^2})$$

$$\sum_{\beta\neq\alpha,\,j} \varpi(\varphi_{\alpha i}, \varphi_{\beta j})\left(1 - \sqrt{1 - \frac{1}{d}}\right) \leqslant 1$$

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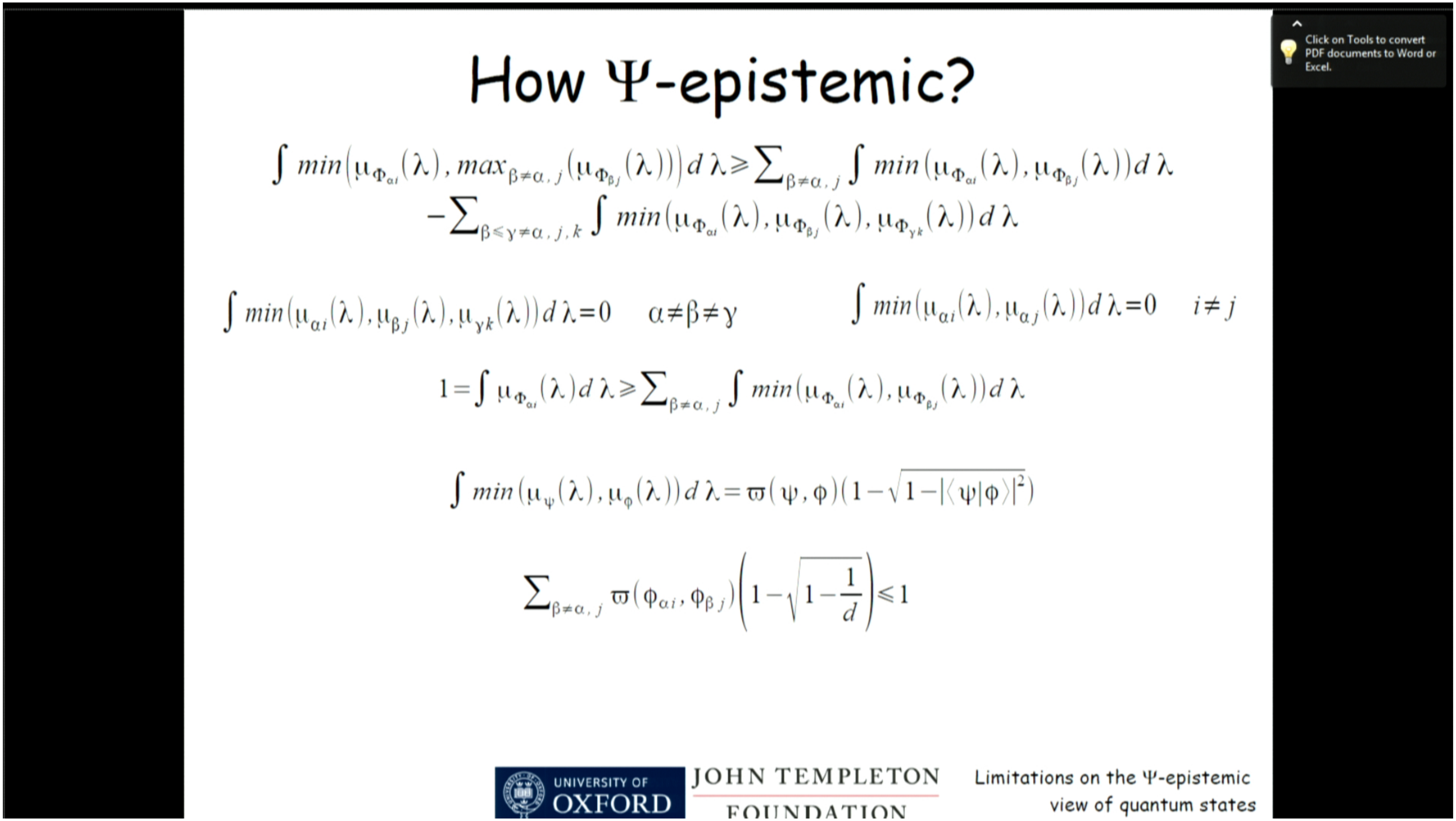

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How Ψ-epistemic?

$$\int min\left(\mu_{\Phi_{\alpha i}}(\lambda), max_{\beta\neq\alpha,j}(\mu_{\Phi_{\beta j}}(\lambda))\right)d\lambda \geqslant \sum\nolimits_{\beta\neq\alpha,j}\int min(\mu_{\Phi_{\alpha i}}(\lambda),\mu_{\Phi_{\beta j}}(\lambda))d\lambda$$
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$$\int min(\mu_{\alpha i}(\lambda),\mu_{\beta j}(\lambda),\mu_{\gamma k}(\lambda))d\lambda=0 \quad \alpha\neq\beta\neq\gamma \qquad \int min(\mu_{\alpha i}(\lambda),\mu_{\alpha j}(\lambda))d\lambda=0 \quad i\neq j$$

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$$\int min(\mu_\psi(\lambda),\mu_\phi(\lambda))d\lambda=\varpi(\psi,\phi)(1-\sqrt{1-|\langle\psi|\phi\rangle|^2})$$

$$\sum\nolimits_{\beta\neq\alpha,j}\varpi(\phi_{\alpha i},\phi_{\beta j})\left(1-\sqrt{1-\frac{1}{d}}\right)\leqslant 1$$

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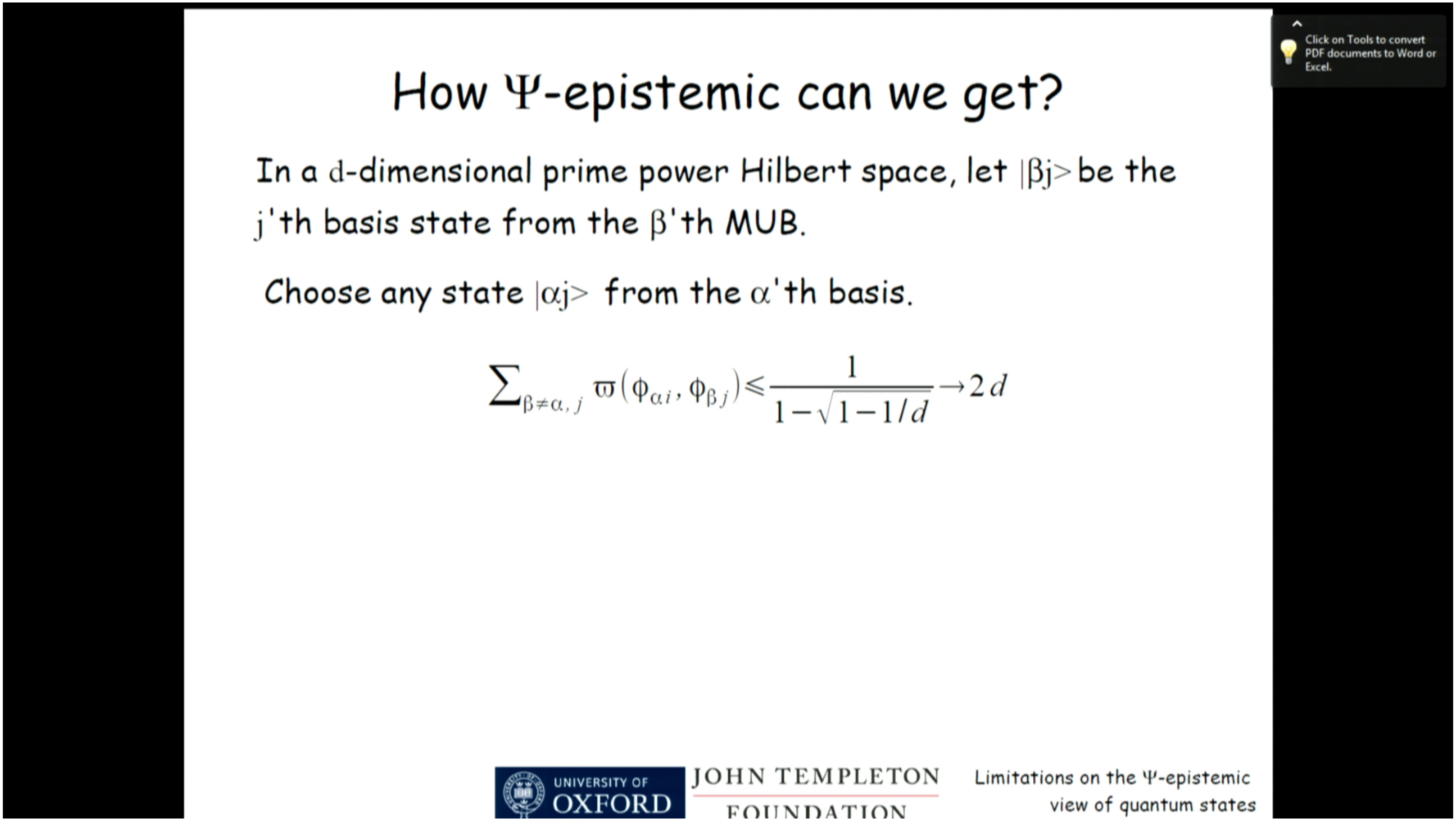

How Ψ-epistemic can we get?

In a d-dimensional prime power Hilbert space, let $|\beta j\rangle$ be the j'th basis state from the β'th MUB.

Choose any state $|\alpha j\rangle$ from the α'th basis.

$$\sum_{\beta\neq\alpha,\, j} \varpi(\phi_{\alpha i}, \phi_{\beta j}) \leqslant \frac{1}{1-\sqrt{1-1/d}} \to 2d$$

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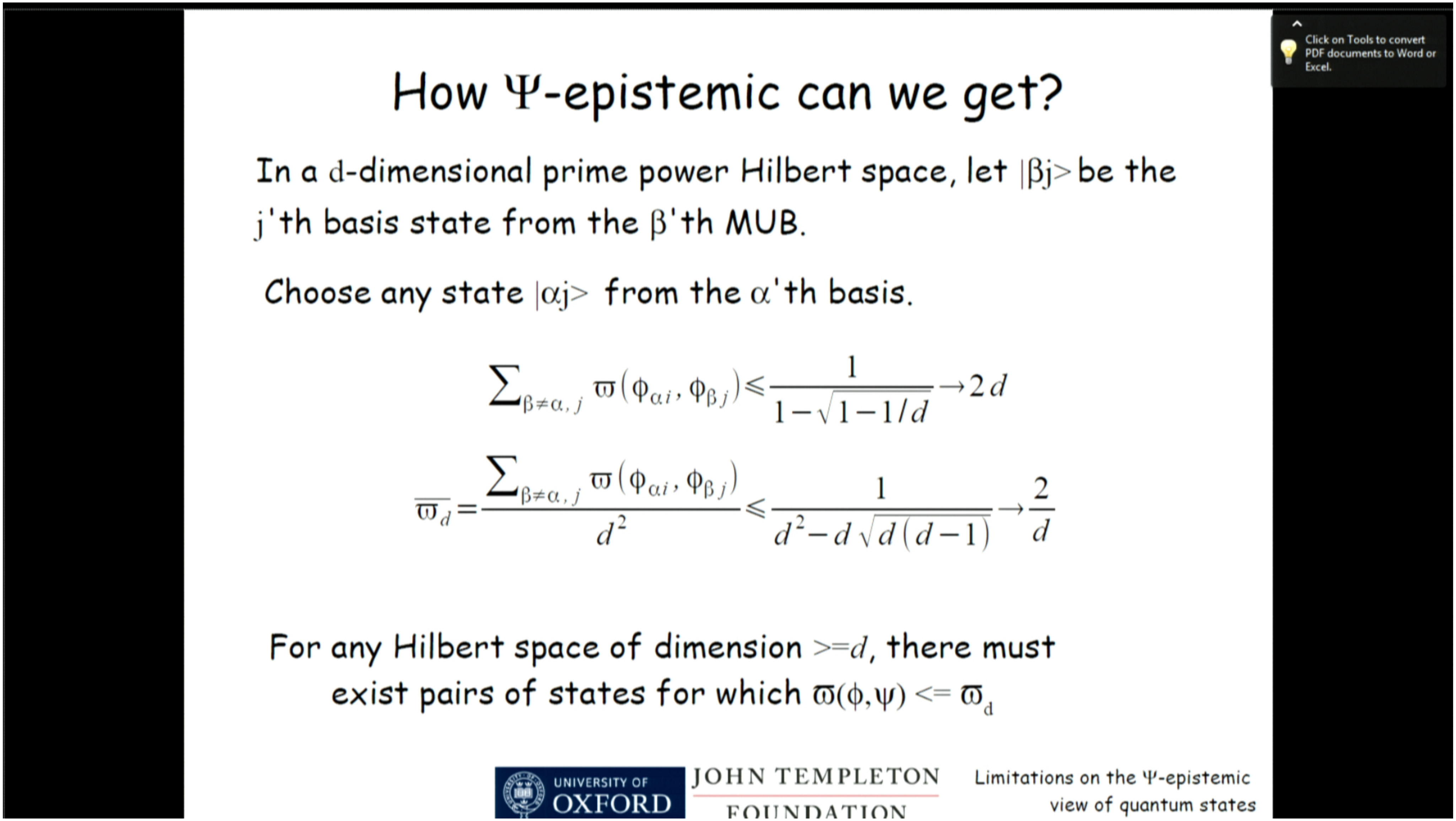

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How Ψ-epistemic can we get?

In a d-dimensional prime power Hilbert space, let |βj> be the j'th basis state from the β'th MUB.

Choose any state |αj> from the α'th basis.

$$\sum_{\beta\neq\alpha,\,j} \varpi\left(\phi_{\alpha i},\phi_{\beta j}\right) \leqslant \frac{1}{1-\sqrt{1-1/d}} \rightarrow 2d$$

$$\overline{\varpi}_d = \frac{\sum_{\beta\neq\alpha,\,j} \varpi\left(\phi_{\alpha i},\phi_{\beta j}\right)}{d^2} \leqslant \frac{1}{d^2-d\sqrt{d\left(d-1\right)}} \rightarrow \frac{2}{d}$$

For any Hilbert space of dimension >=d, there must exist pairs of states for which $\varpi(\phi,\psi)$ <= ϖ_d

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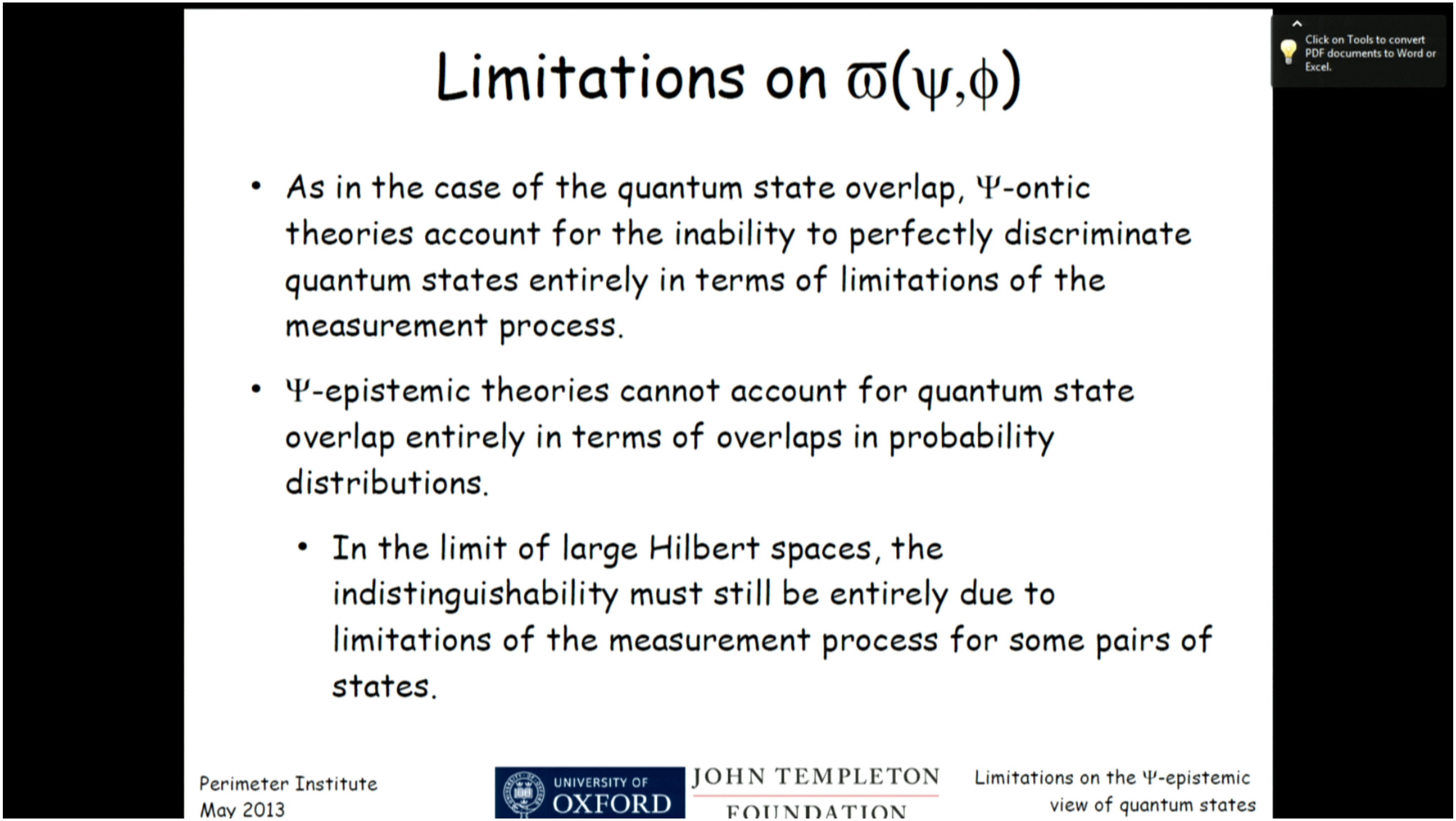
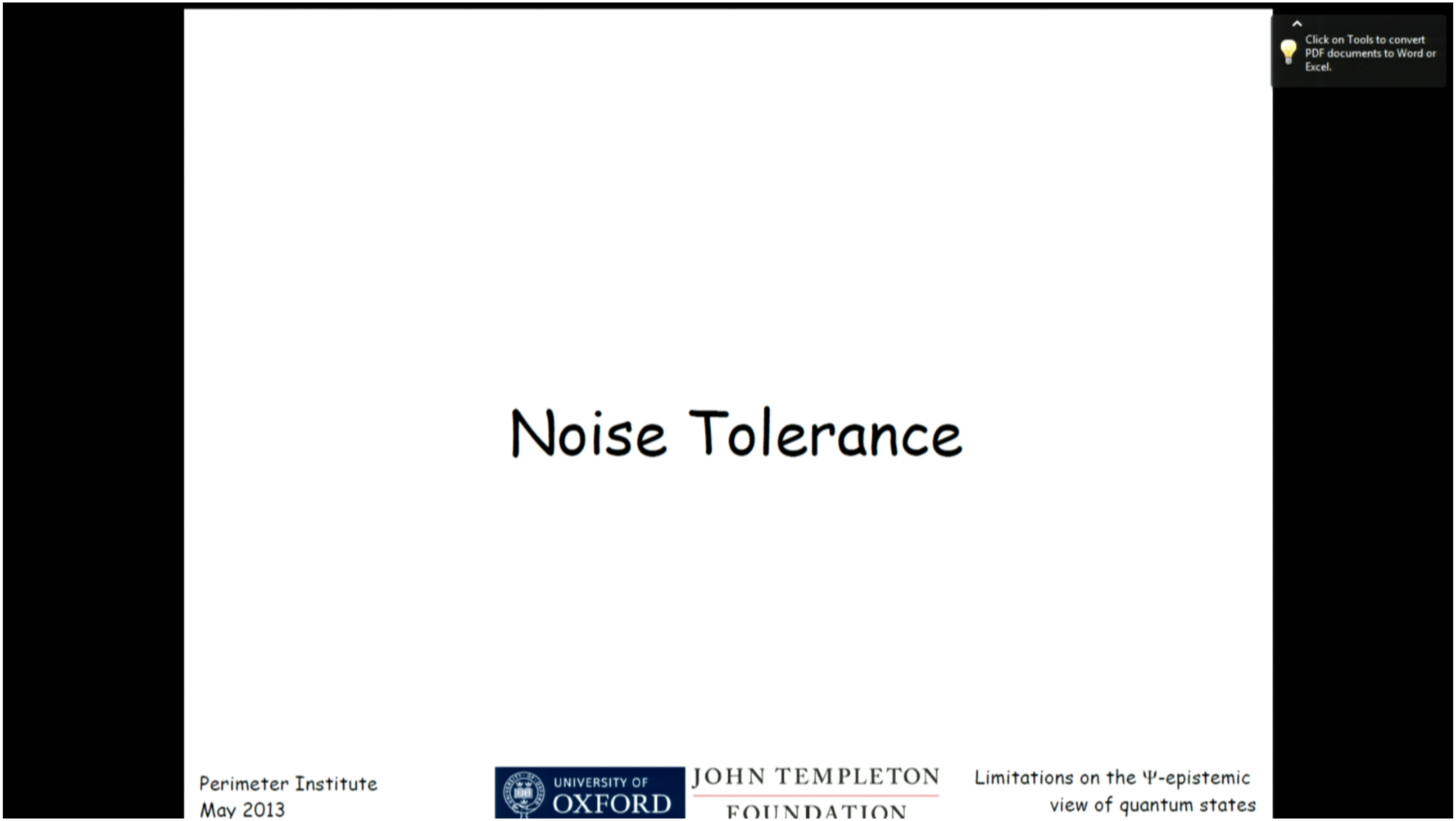


Limitations on $\varpi(\psi,\phi)$

- As in the case of the quantum state overlap, Ψ-ontic theories account for the inability to perfectly discriminate quantum states entirely in terms of limitations of the measurement process.
- Ψ-epistemic theories cannot account for quantum state overlap entirely in terms of overlaps in probability distributions.
  - In the limit of large Hilbert spaces, the indistinguishability must still be entirely due to limitations of the measurement process for some pairs of states.

Perimeter Institute
May 2013

Limitations on the Ψ-epistemic view of quantum states



Noise Tolerance

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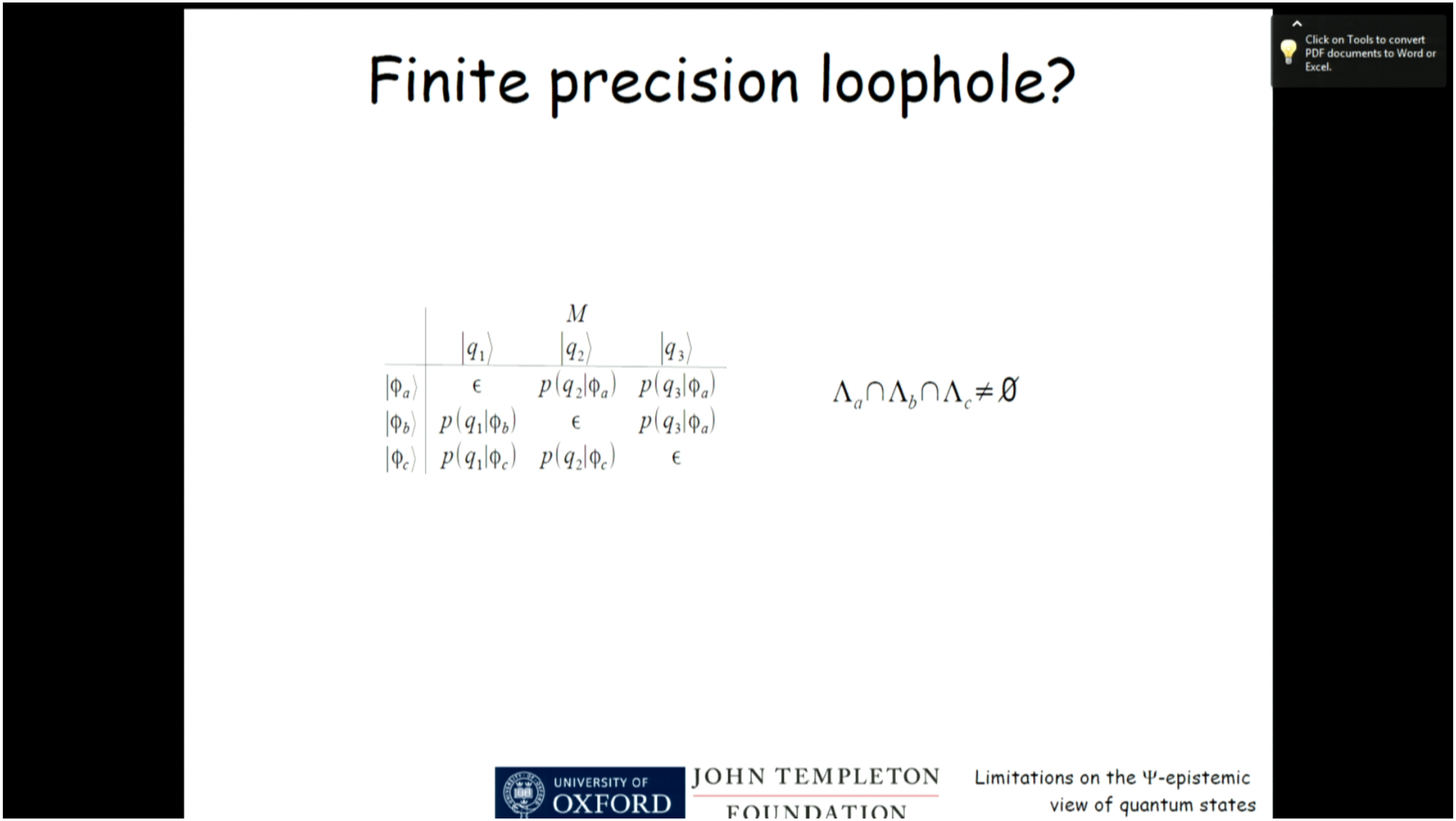

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Finite precision loophole?

		M	
	$\|q_1\rangle$	$\|q_2\rangle$	$\|q_3\rangle$
$\|\phi_a\rangle$	ϵ	$p(q_2\|\phi_a)$	$p(q_3\|\phi_a)$
$\|\phi_b\rangle$	$p(q_1\|\phi_b)$	ϵ	$p(q_3\|\phi_a)$
$\|\phi_c\rangle$	$p(q_1\|\phi_c)$	$p(q_2\|\phi_c)$	ϵ

$$\Lambda_a \cap \Lambda_b \cap \Lambda_c \neq \emptyset$$

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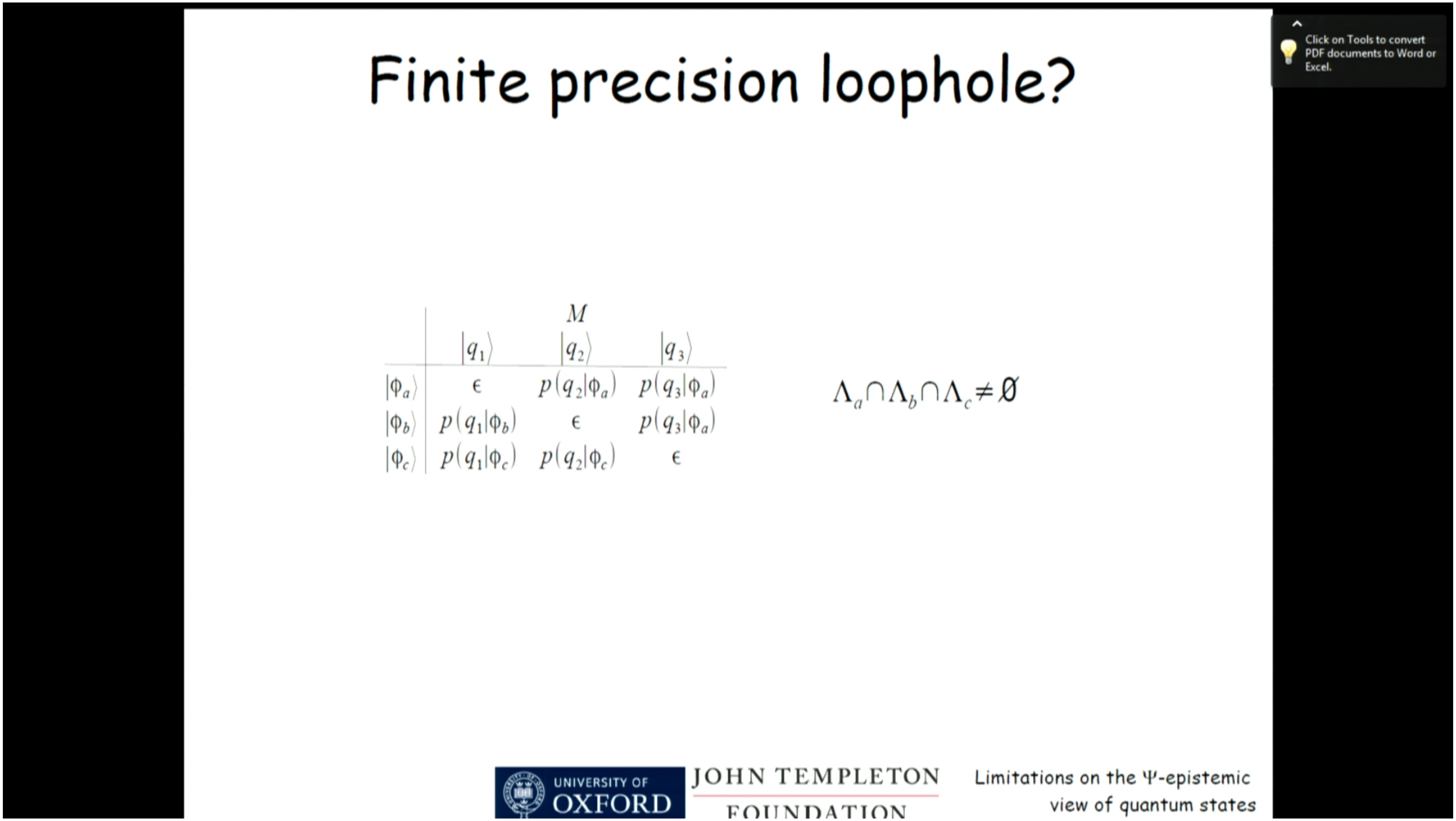

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$\lvert\phi_b\rangle$	$p(q_1\vert\phi_b)$	ϵ	$p(q_3\vert\phi_a)$
$\lvert\phi_c\rangle$	$p(q_1\vert\phi_c)$	$p(q_2\vert\phi_c)$	ϵ

$$\Lambda_a \cap \Lambda_b \cap \Lambda_c \neq \emptyset$$

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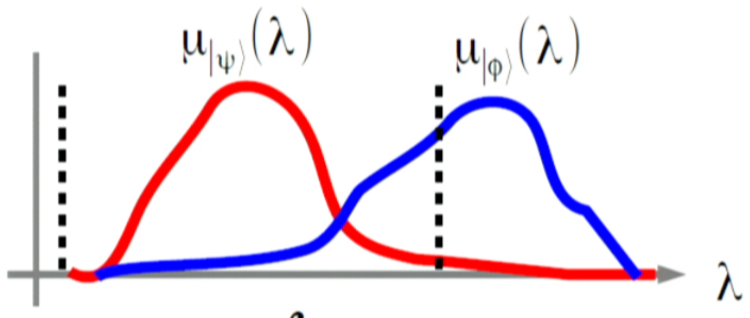
Quantum State Overlap



$$\int_{\Lambda_\psi} \mu_\phi(\lambda)\, d\lambda$$

	$\|\psi\rangle$	$\|\psi\perp\rangle$
$\|\psi\rangle$	$1-\epsilon$	ϵ
$\|\phi\rangle$	$\|\langle\psi\|\phi\rangle\|^2$	$1-\|\langle\psi\|\phi\rangle\|^2$

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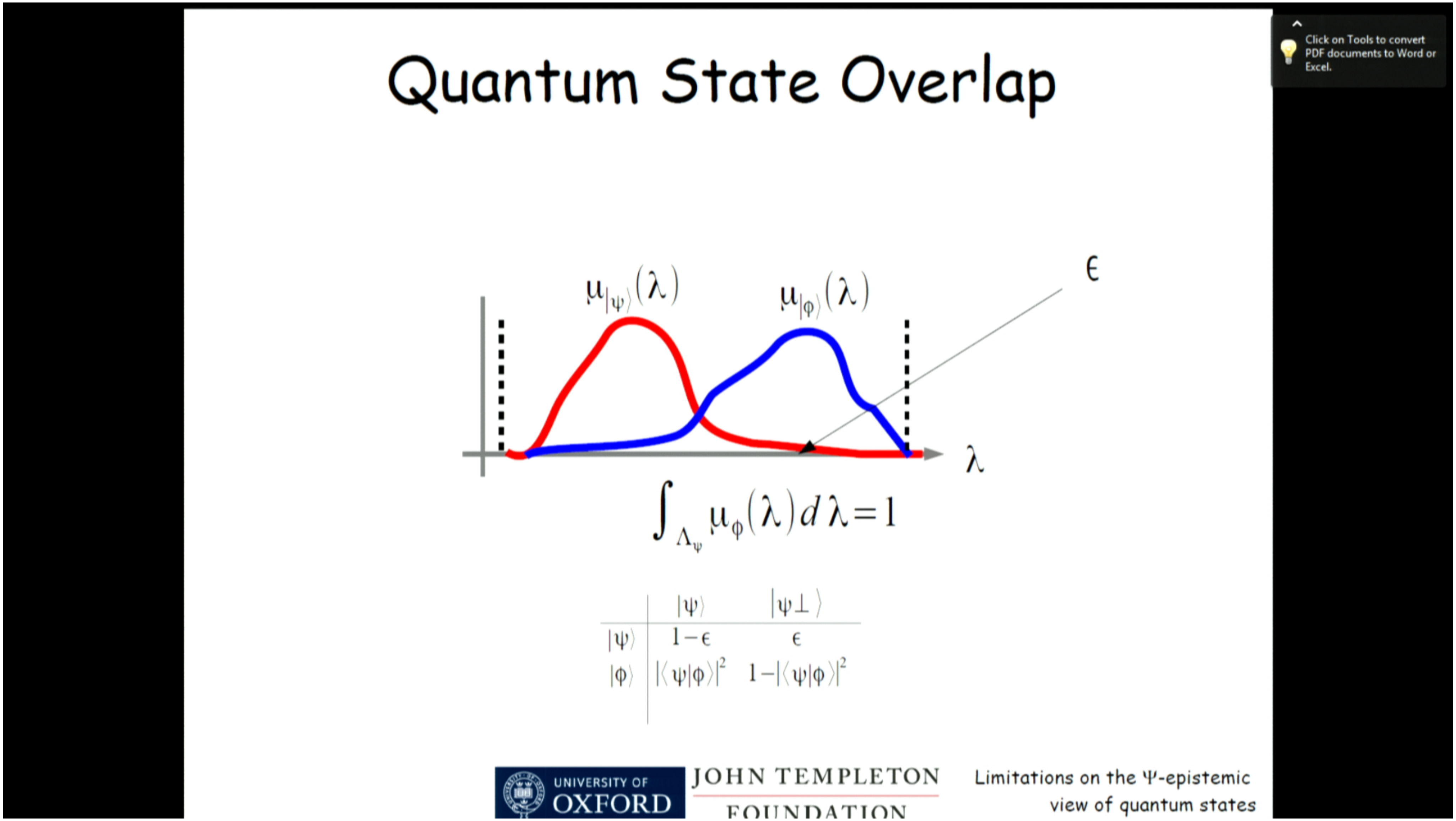

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Quantum State Overlap

$$\int_{\Lambda_\psi} \mu_\phi(\lambda)\, d\lambda = 1$$

	$\vert\psi\rangle$	$\vert\psi\perp\rangle$
$\vert\psi\rangle$	$1-\epsilon$	ϵ
$\vert\phi\rangle$	$\vert\langle\psi\vert\phi\rangle\vert^2$	$1-\vert\langle\psi\vert\phi\rangle\vert^2$

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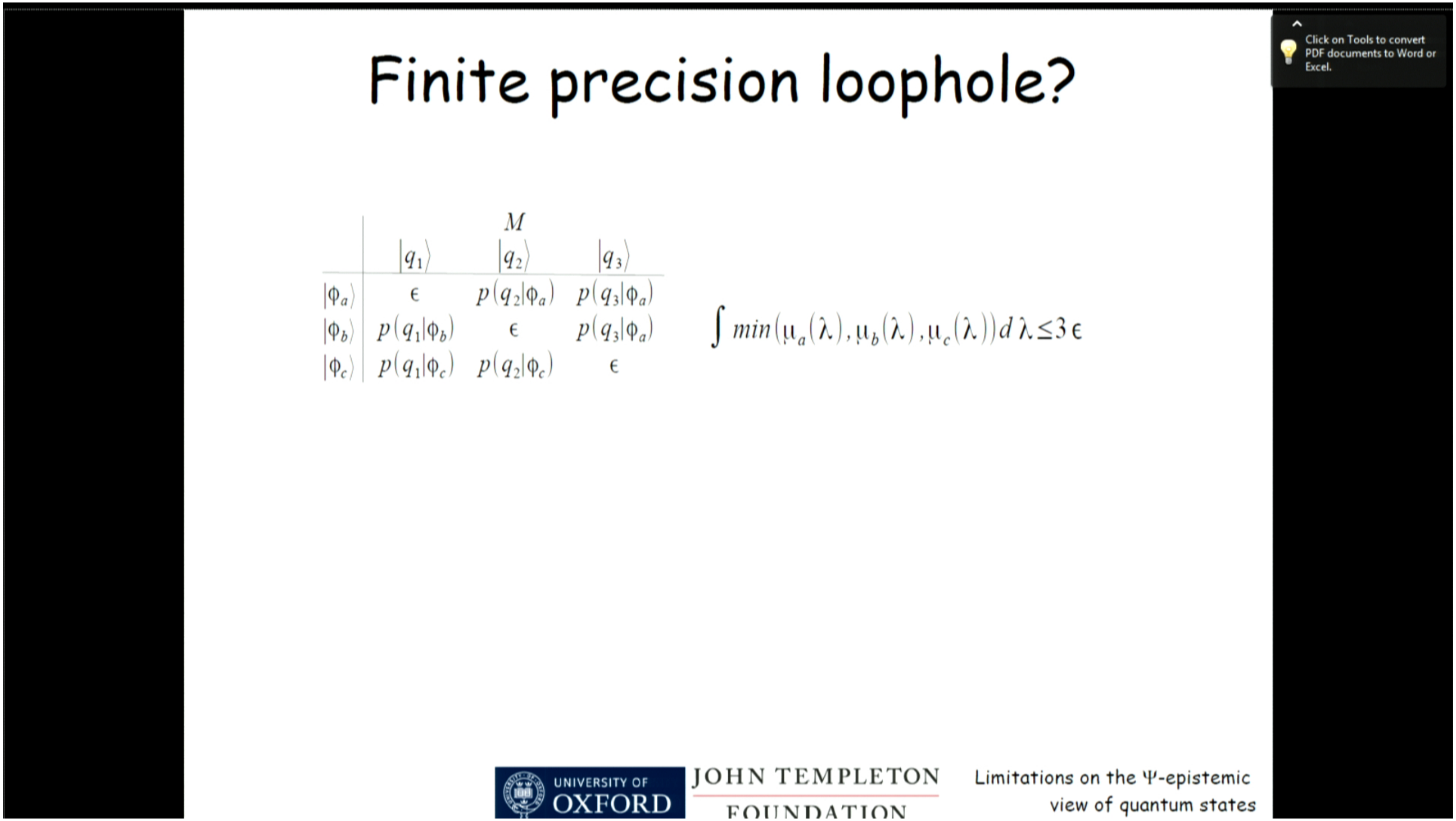

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$\|\phi_c\rangle$	$p(q_1\|\phi_c)$	$p(q_2\|\phi_c)$	ϵ

$$\int min(\mu_a(\lambda),\mu_b(\lambda),\mu_c(\lambda))d\lambda\leq 3\epsilon$$

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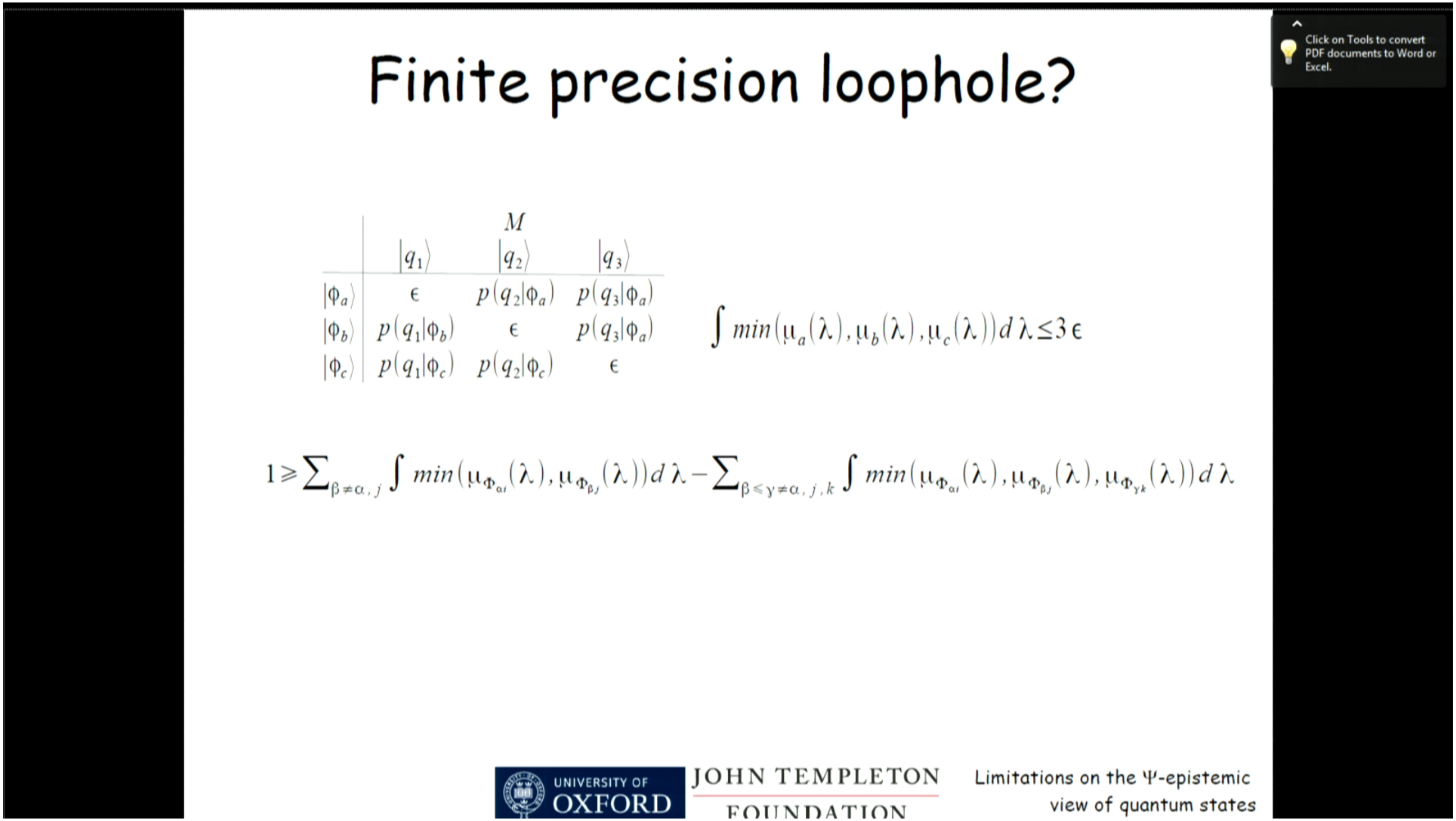

Finite precision loophole?

$$
\begin{array}{c|ccc}
 & & M & \\
 & |q_1\rangle & |q_2\rangle & |q_3\rangle \\
\hline
|\phi_a\rangle & \epsilon & p(q_2|\phi_a) & p(q_3|\phi_a) \\
|\phi_b\rangle & p(q_1|\phi_b) & \epsilon & p(q_3|\phi_a) \\
|\phi_c\rangle & p(q_1|\phi_c) & p(q_2|\phi_c) & \epsilon
\end{array}
$$

$$\int min(\mu_a(\lambda),\mu_b(\lambda),\mu_c(\lambda))d\lambda \leq 3\epsilon$$

$$1 \geqslant \sum_{\beta\neq\alpha,j} \int min(\mu_{\Phi_{\alpha i}}(\lambda),\mu_{\Phi_{\beta j}}(\lambda))d\lambda - \sum_{\beta\leqslant\gamma\neq\alpha,j,k} \int min(\mu_{\Phi_{\alpha i}}(\lambda),\mu_{\Phi_{\beta j}}(\lambda),\mu_{\Phi_{\gamma k}}(\lambda))d\lambda$$

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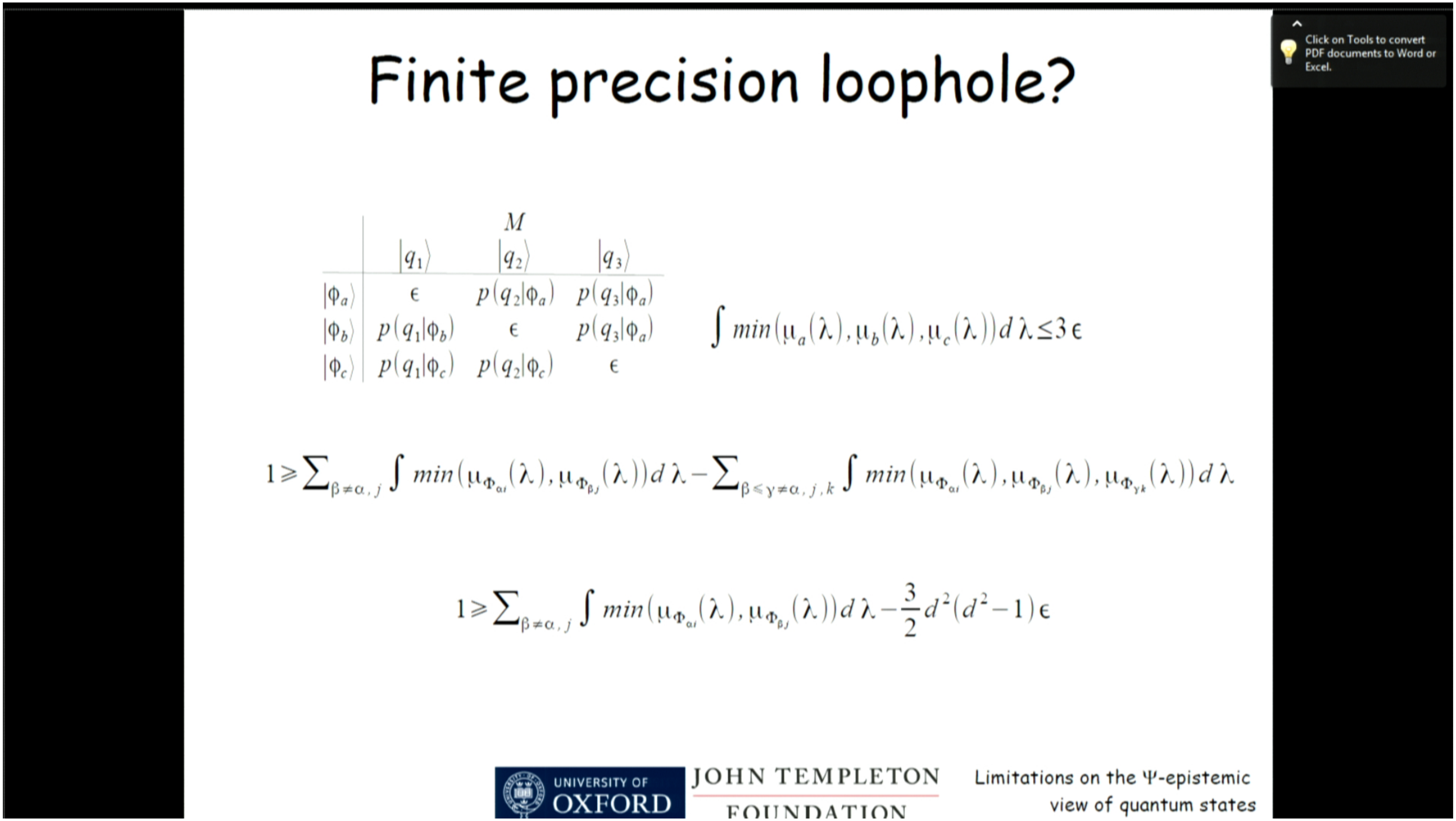

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$\lvert\phi_c\rangle$	$p(q_1\mid\phi_c)$	$p(q_2\mid\phi_c)$	ϵ

$$\int min(\mu_a(\lambda),\mu_b(\lambda),\mu_c(\lambda))d\lambda\leq 3\epsilon$$

$$1\geqslant\sum_{\beta\neq\alpha,j}\int min(\mu_{\Phi_{\alpha i}}(\lambda),\mu_{\Phi_{\beta j}}(\lambda))d\lambda-\sum_{\beta\leqslant\gamma\neq\alpha,j,k}\int min(\mu_{\Phi_{\alpha i}}(\lambda),\mu_{\Phi_{\beta j}}(\lambda),\mu_{\Phi_{\gamma k}}(\lambda))d\lambda$$

$$1\geqslant\sum_{\beta\neq\alpha,j}\int min(\mu_{\Phi_{\alpha i}}(\lambda),\mu_{\Phi_{\beta j}}(\lambda))d\lambda-\frac{3}{2}d^2(d^2-1)\epsilon$$

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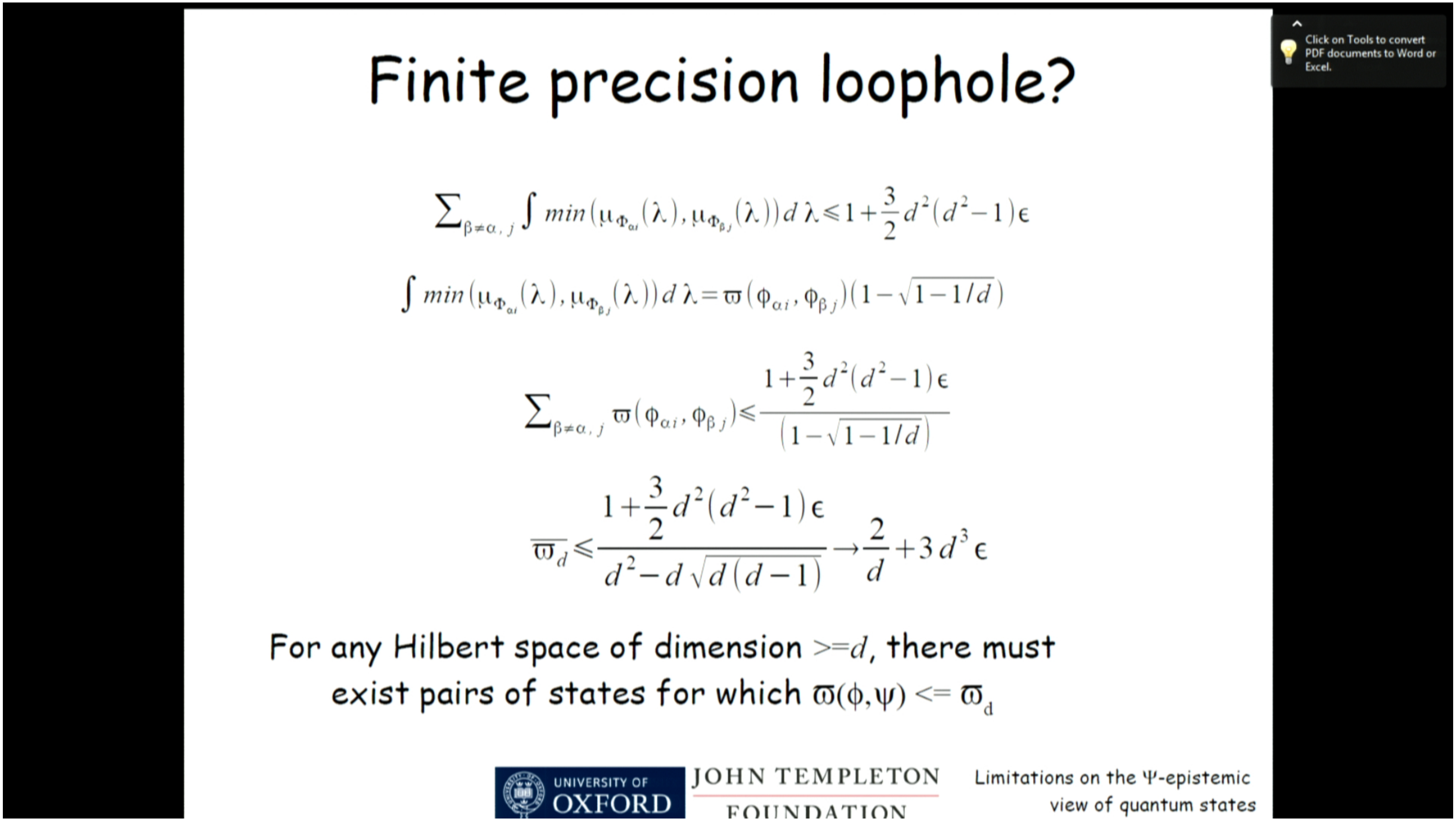

Finite precision loophole?

$$\sum_{\beta\neq\alpha,j}\int min(\mu_{\Phi_{\alpha i}}(\lambda),\mu_{\Phi_{\beta j}}(\lambda))d\lambda\leqslant 1+\frac{3}{2}d^2(d^2-1)\epsilon$$

$$\int min(\mu_{\Phi_{\alpha i}}(\lambda),\mu_{\Phi_{\beta j}}(\lambda))d\lambda=\varpi(\phi_{\alpha i},\phi_{\beta j})(1-\sqrt{1-1/d})$$

$$\sum_{\beta\neq\alpha,j}\varpi(\phi_{\alpha i},\phi_{\beta j})\leqslant\frac{1+\frac{3}{2}d^2(d^2-1)\epsilon}{(1-\sqrt{1-1/d})}$$

$$\overline{\varpi}_d\leqslant\frac{1+\frac{3}{2}d^2(d^2-1)\epsilon}{d^2-d\sqrt{d(d-1)}}\rightarrow\frac{2}{d}+3d^3\epsilon$$

For any Hilbert space of dimension >=d, there must exist pairs of states for which $\varpi(\phi,\psi)$ <= ϖ_d

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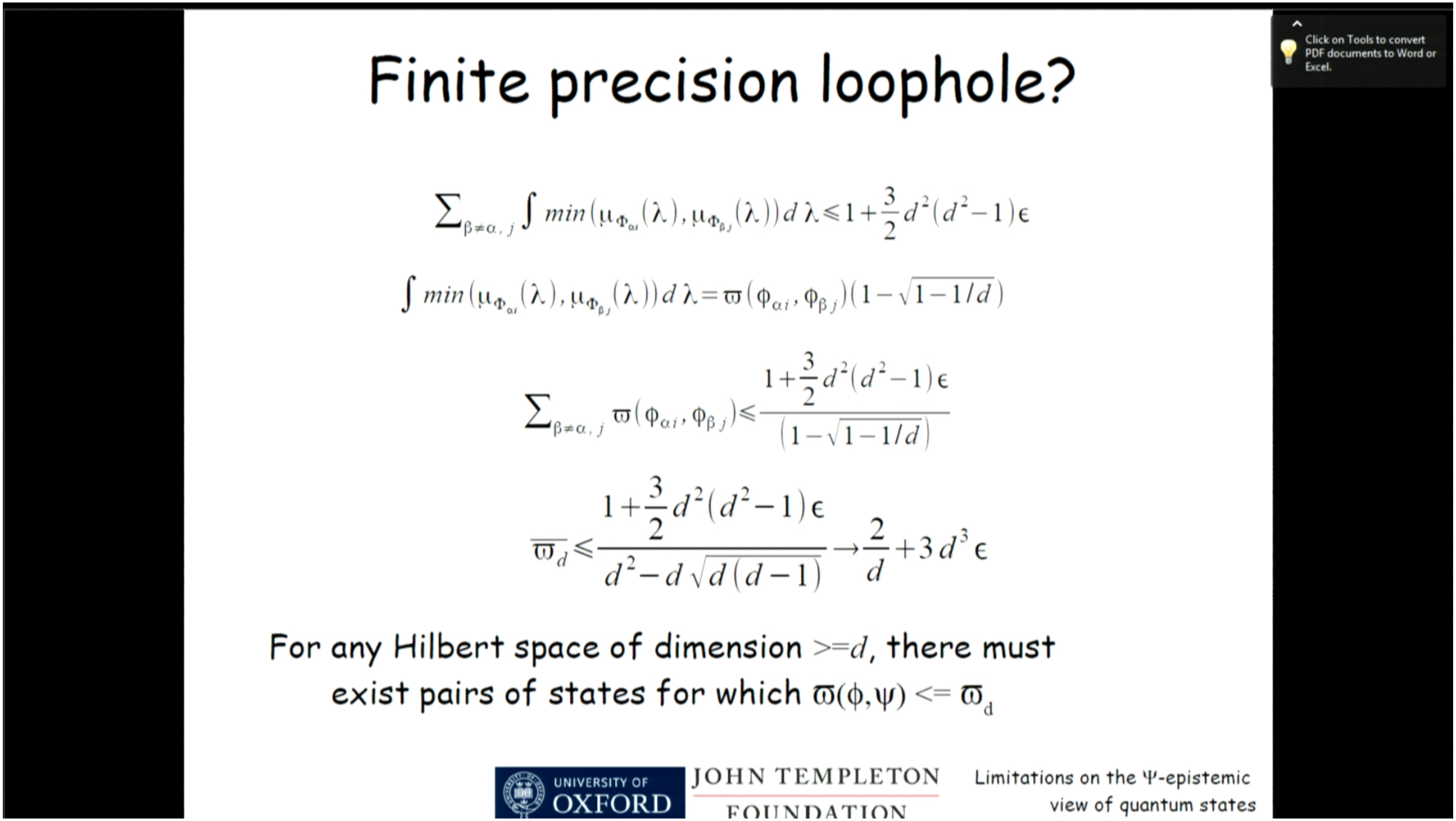

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For any Hilbert space of dimension $>=d$, there must exist pairs of states for which $\varpi(\phi,\psi) <= \varpi_d$

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Limitations on the Ψ-epistemic view of quantum states

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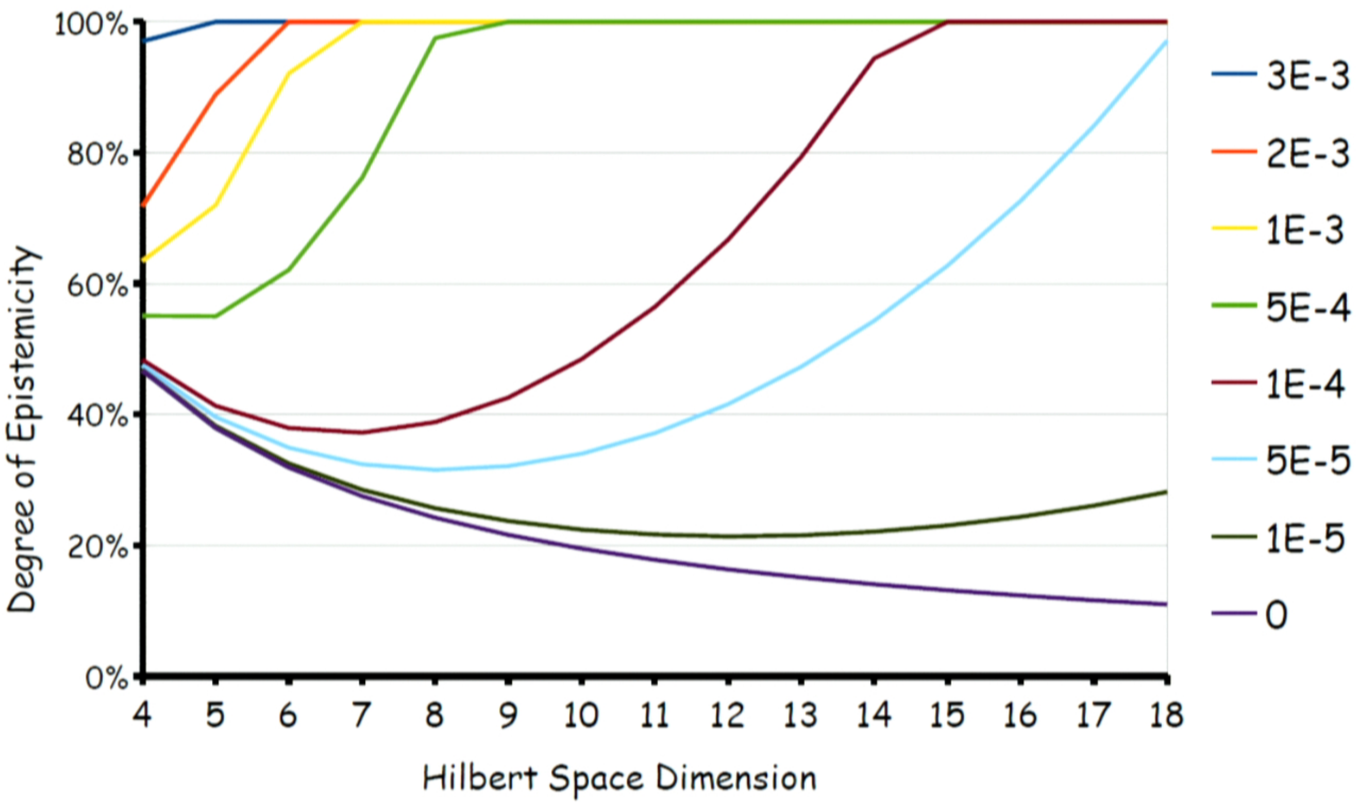
Error in measurement



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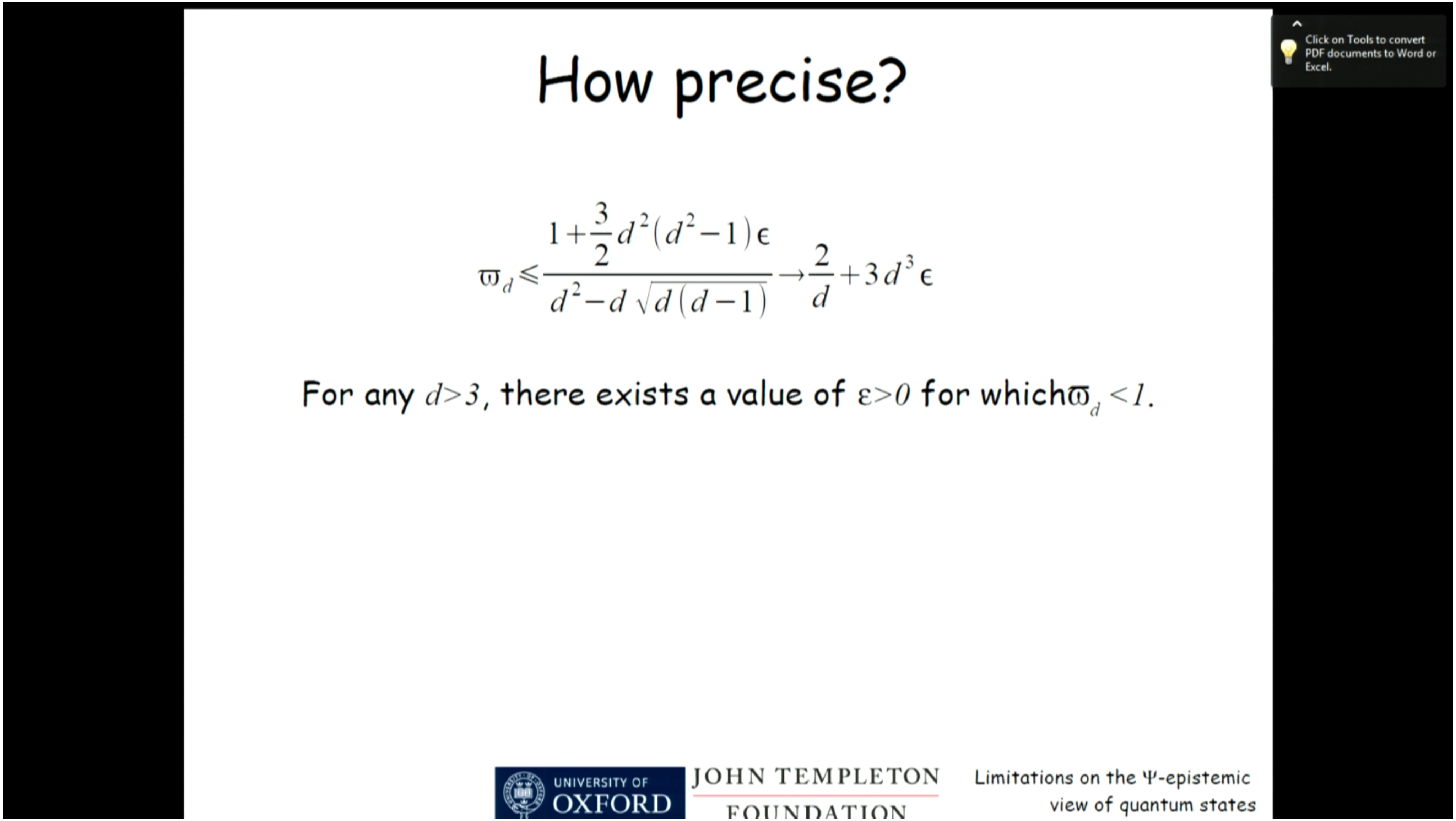

How precise?

$$\varpi_d \leqslant \frac{1+\frac{3}{2}d^2\left(d^2-1\right)\epsilon}{d^2-d\sqrt{d\left(d-1\right)}} \rightarrow \frac{2}{d}+3d^3\epsilon$$

For any $d>3$, there exists a value of $\varepsilon>0$ for which $\varpi_d<1$.

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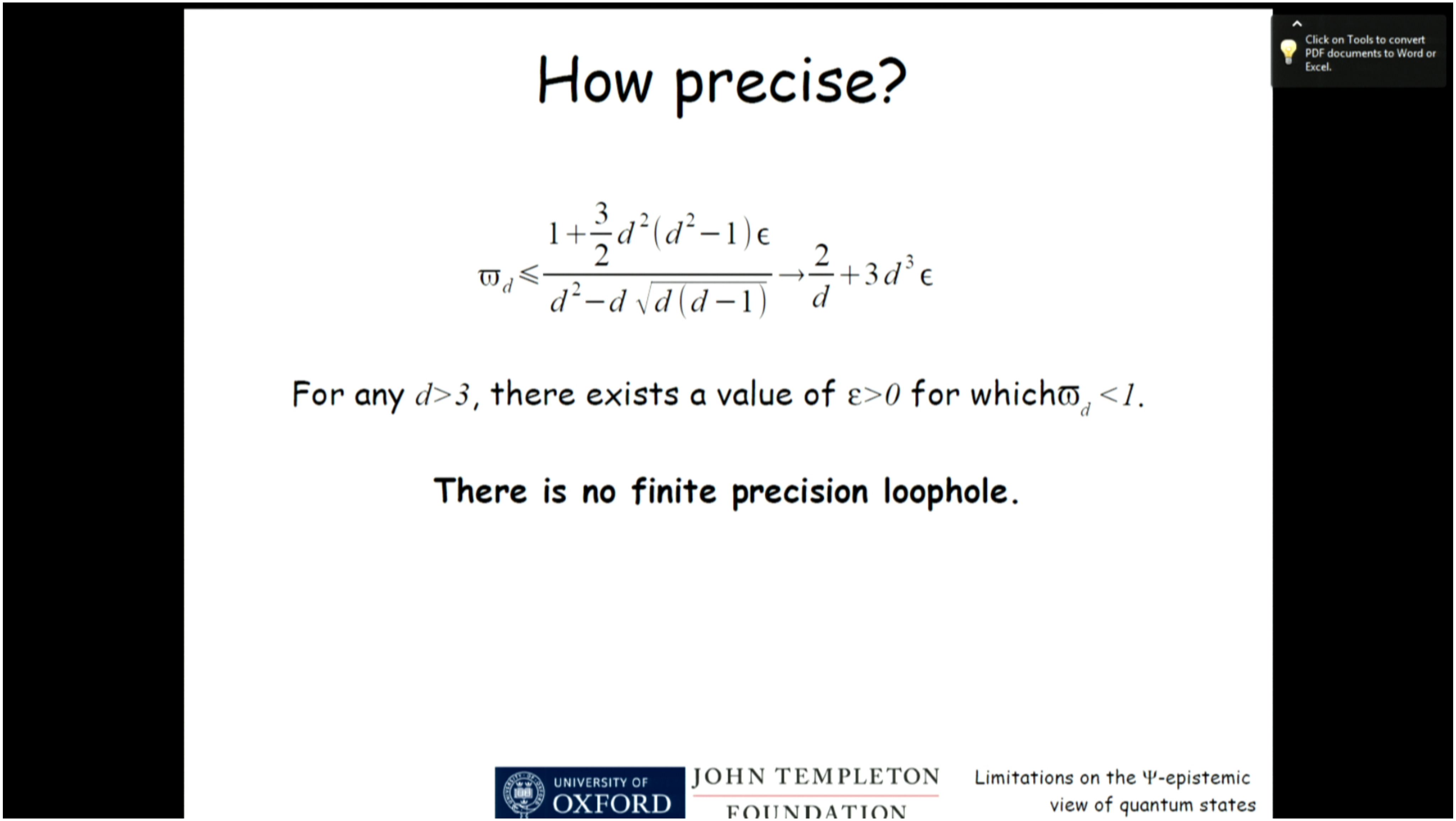

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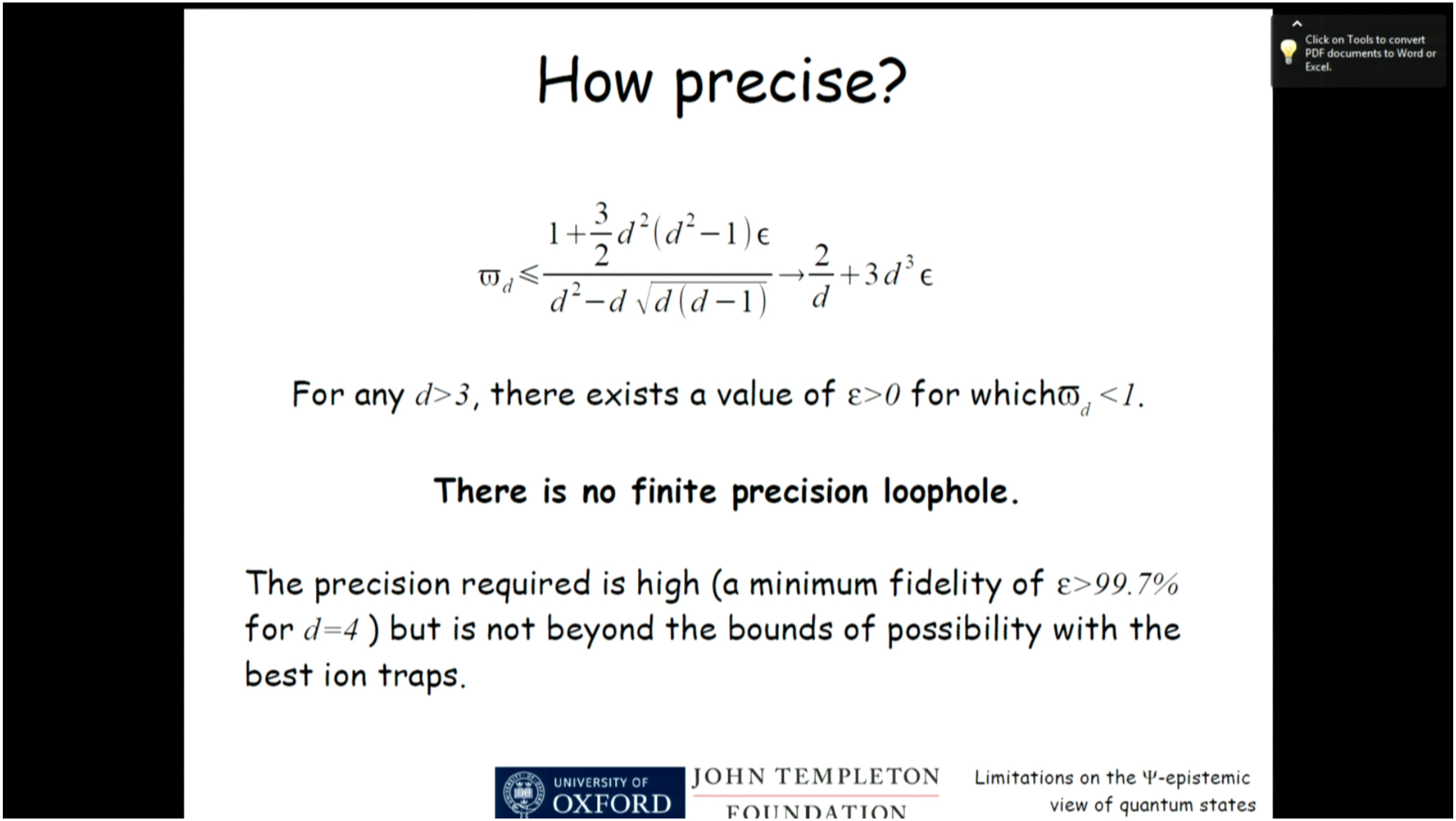

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The precision required is high (a minimum fidelity of $\varepsilon>99.7\%$ for $d=4$) but is not beyond the bounds of possibility with the best ion traps.

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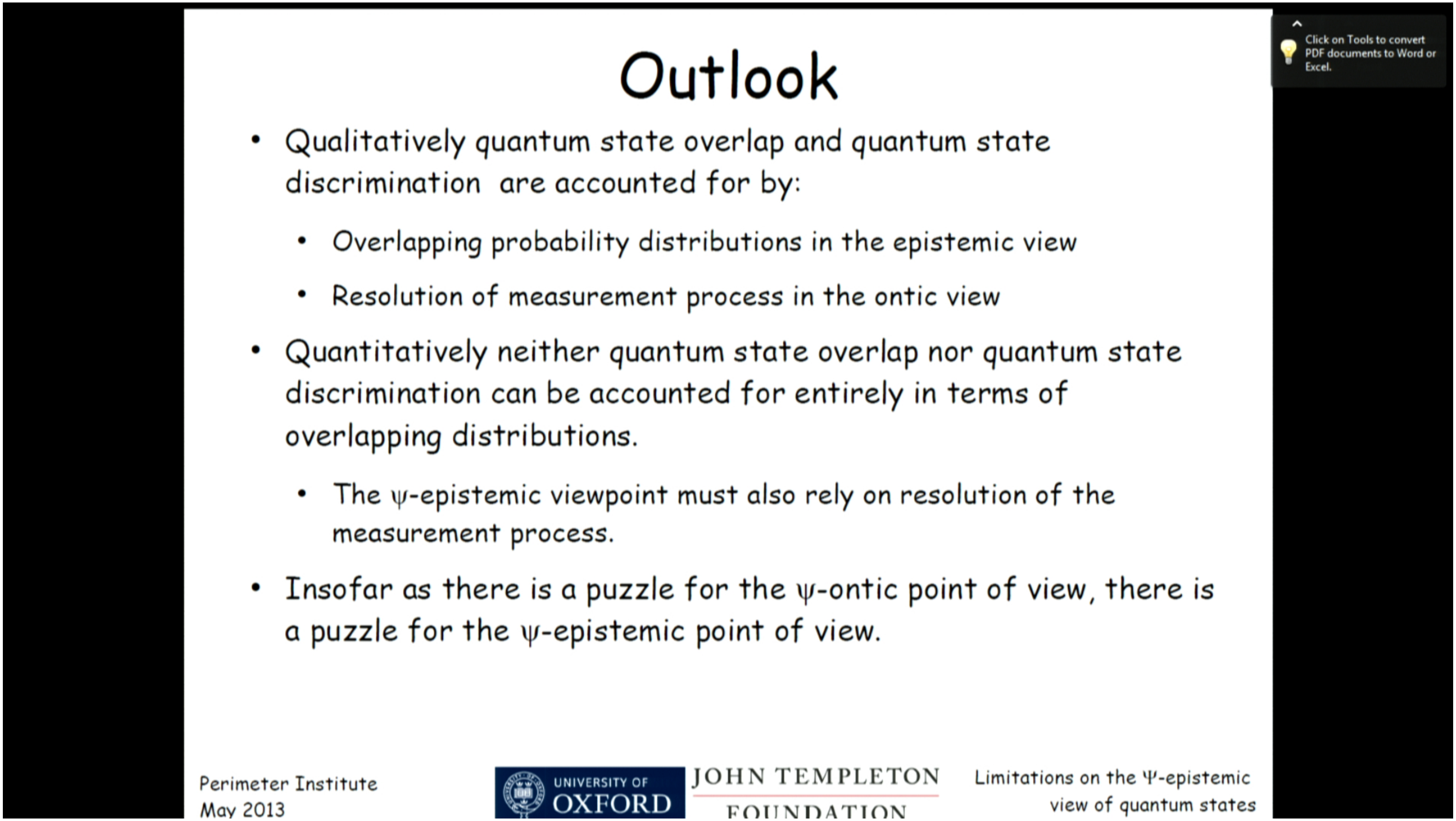

Outlook

- Qualitatively quantum state overlap and quantum state discrimination are accounted for by:
  - Overlapping probability distributions in the epistemic view
  - Resolution of measurement process in the ontic view
- Quantitatively neither quantum state overlap nor quantum state discrimination can be accounted for entirely in terms of overlapping distributions.
  - The ψ-epistemic viewpoint must also rely on resolution of the measurement process.
- Insofar as there is a puzzle for the ψ-ontic point of view, there is a puzzle for the ψ-epistemic point of view.

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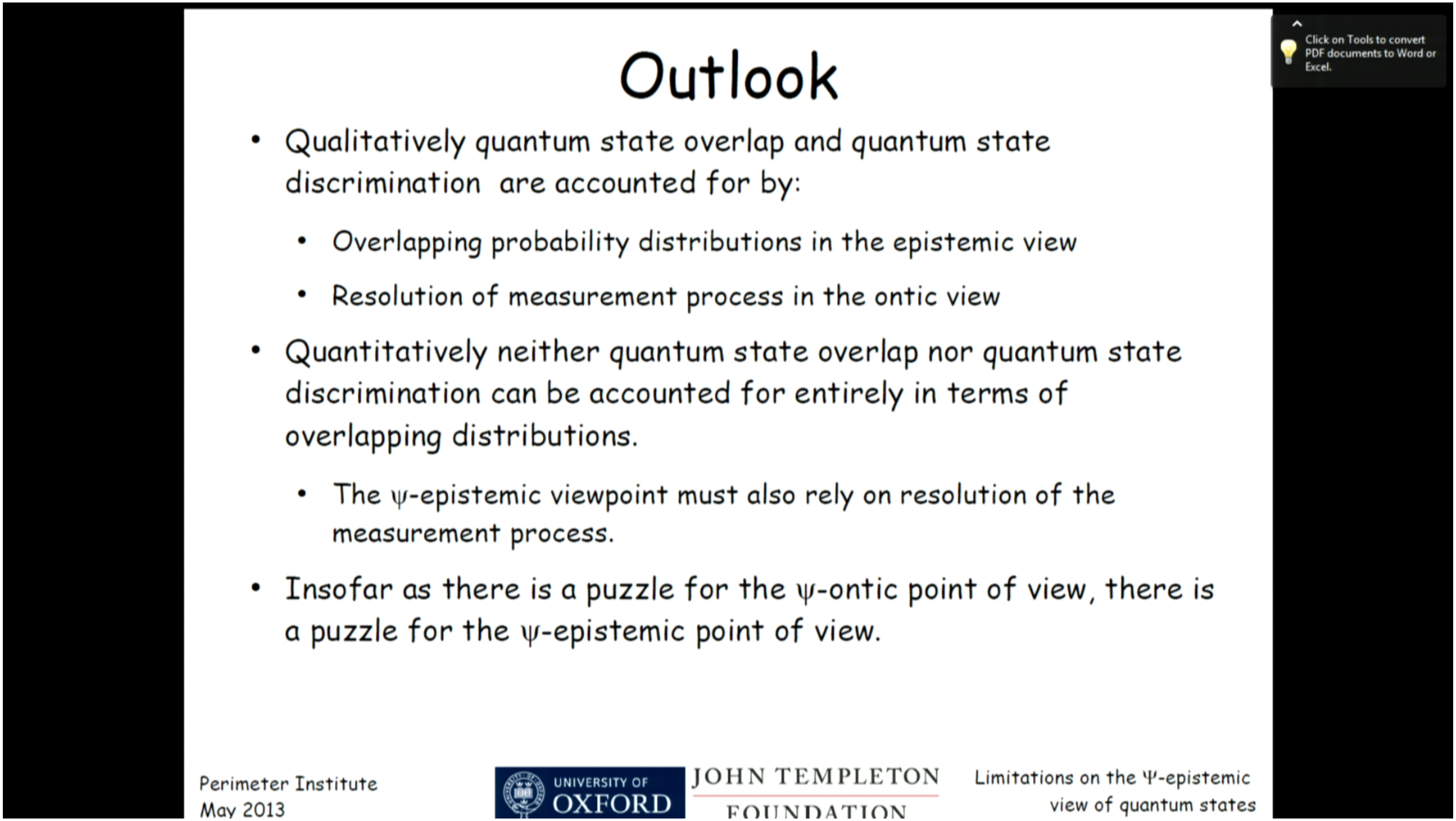

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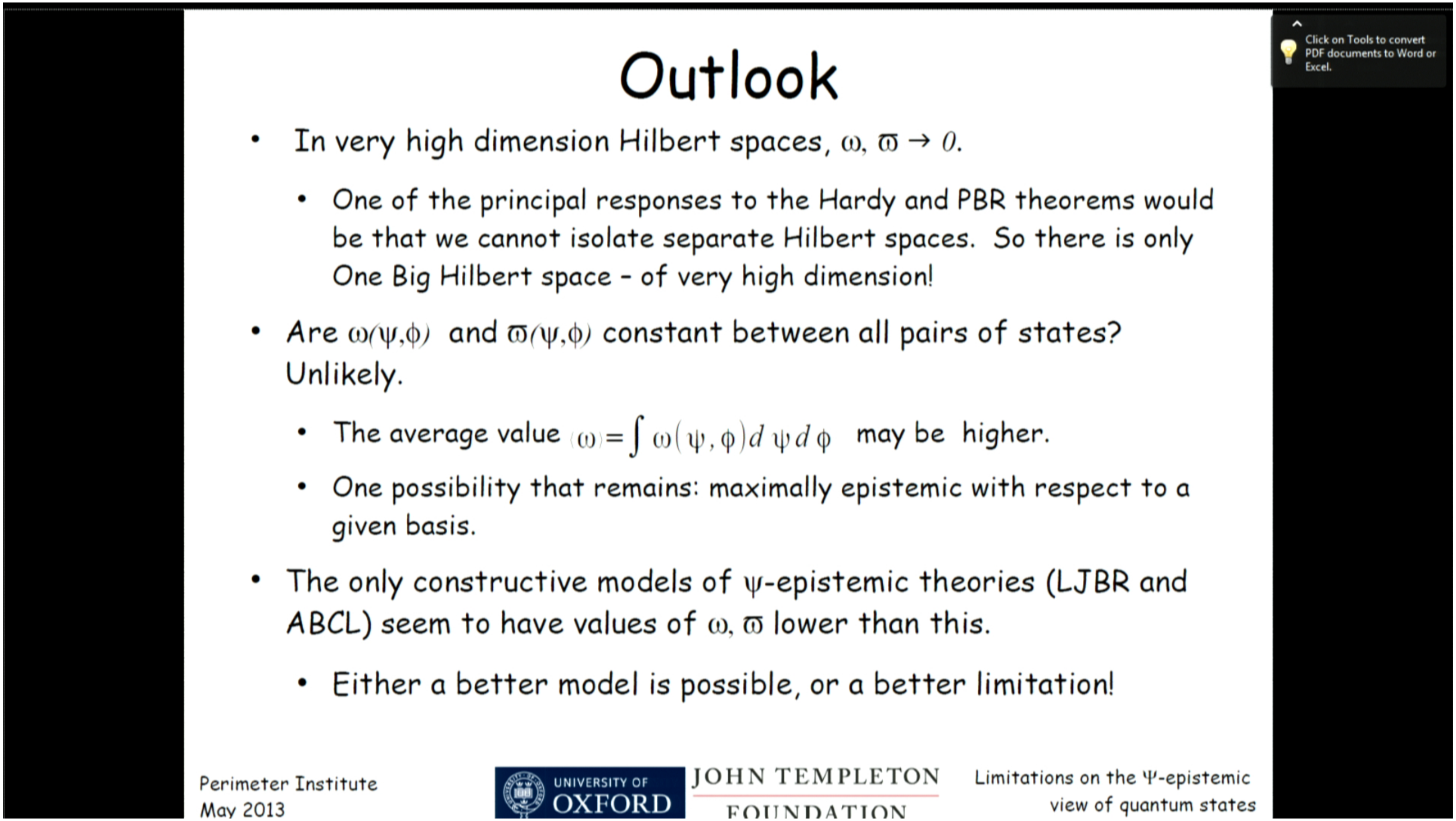

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Outlook

- In very high dimension Hilbert spaces, $\omega, \varpi \to 0$.
  - One of the principal responses to the Hardy and PBR theorems would be that we cannot isolate separate Hilbert spaces. So there is only One Big Hilbert space - of very high dimension!
- Are $\omega(\psi,\phi)$ and $\varpi(\psi,\phi)$ constant between all pairs of states? Unlikely.
  - The average value $\langle\omega\rangle = \int \omega(\psi,\phi) d\psi d\phi$ may be higher.
  - One possibility that remains: maximally epistemic with respect to a given basis.
- The only constructive models of ψ-epistemic theories (LJBR and ABCL) seem to have values of ω, ϖ lower than this.
  - Either a better model is possible, or a better limitation!

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