Title: Processing quantum information with relativistic motion of atoms

Date: May 13, 2013 04:00 PM

URL: http://pirsa.org/13050017

Abstract: We
show that particle detectors, such as 2-level atoms, in non-inertial motion (or in gravitational fields) could be used to build quantum gates for the processing of quantum information. Concretely, we show that through suitably chosen non-inertial trajectories of the detectors the interaction Hamiltonian's time dependence can be modulated to yield arbitrary rotations in the Bloch sphere due to relativistic quantum effects.

Ref. Phys.
Rev. Lett. 110, 160501 (2013)

Relativistic Quantum Information processing

E. M-M, D. Aasen, A. Kempf.
Phys. Rev. Lett. 110, 160501 (2013)

PiQuDos, May. 28th 2012

Bourses postdoctorales
Banting
Postdoctoral Fellowships

Eduardo Martín-Martínez
Inst. for Quantum Computing, UW & Perimeter Institute
Waterloo (Canada)

Happy Birthday Thomas!!!

The Unruh-DeWitt detector model

$$H_I = \lambda \left(\sigma^+ e^{i\Omega\tau} + \sigma^- e^{-i\Omega\tau}\right) \sum_{j=1}^{\infty} \left[a_j^\dagger e^{i\omega_j t(\tau)} + a_j e^{-i\omega_j t(\tau)}\right] \sin k_j x(\tau),$$

Models the interaction of a two-level system with a scalar field

$\sigma^+ a_j,\ \sigma^- a_j^\dagger$	Rotating-wave terms	$e^{i[\Omega\tau - \omega_j t(\tau)]}$
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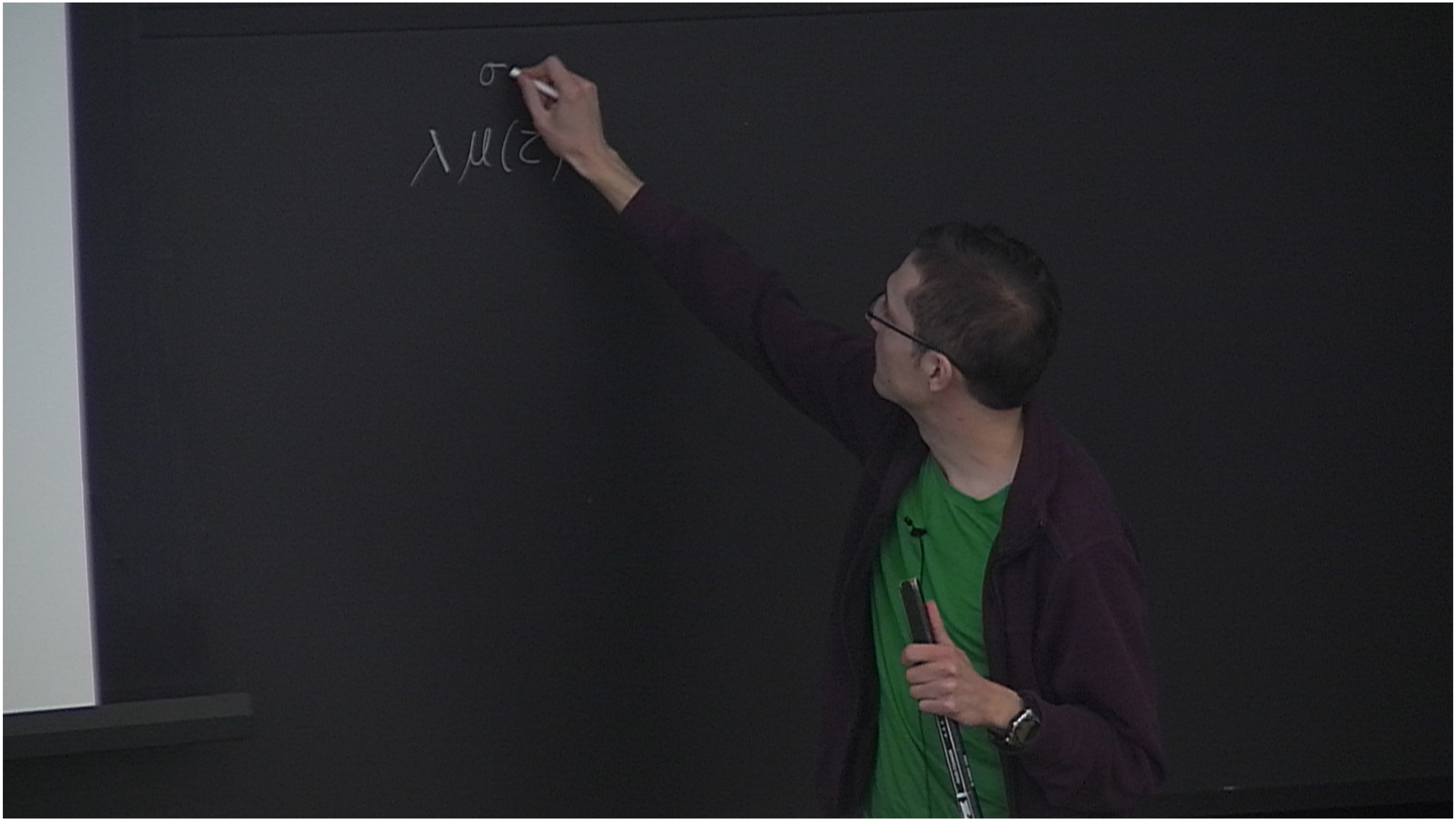
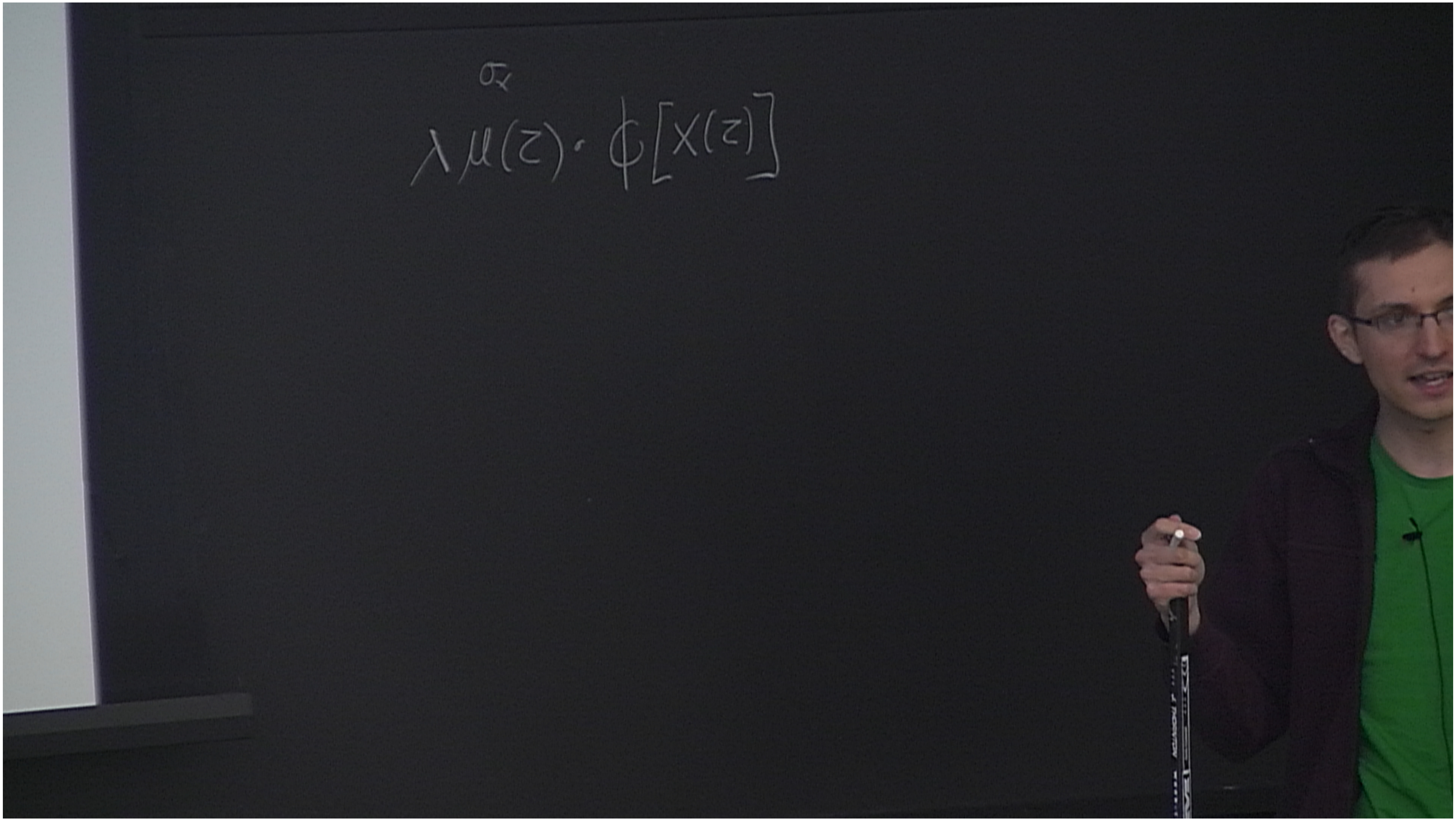
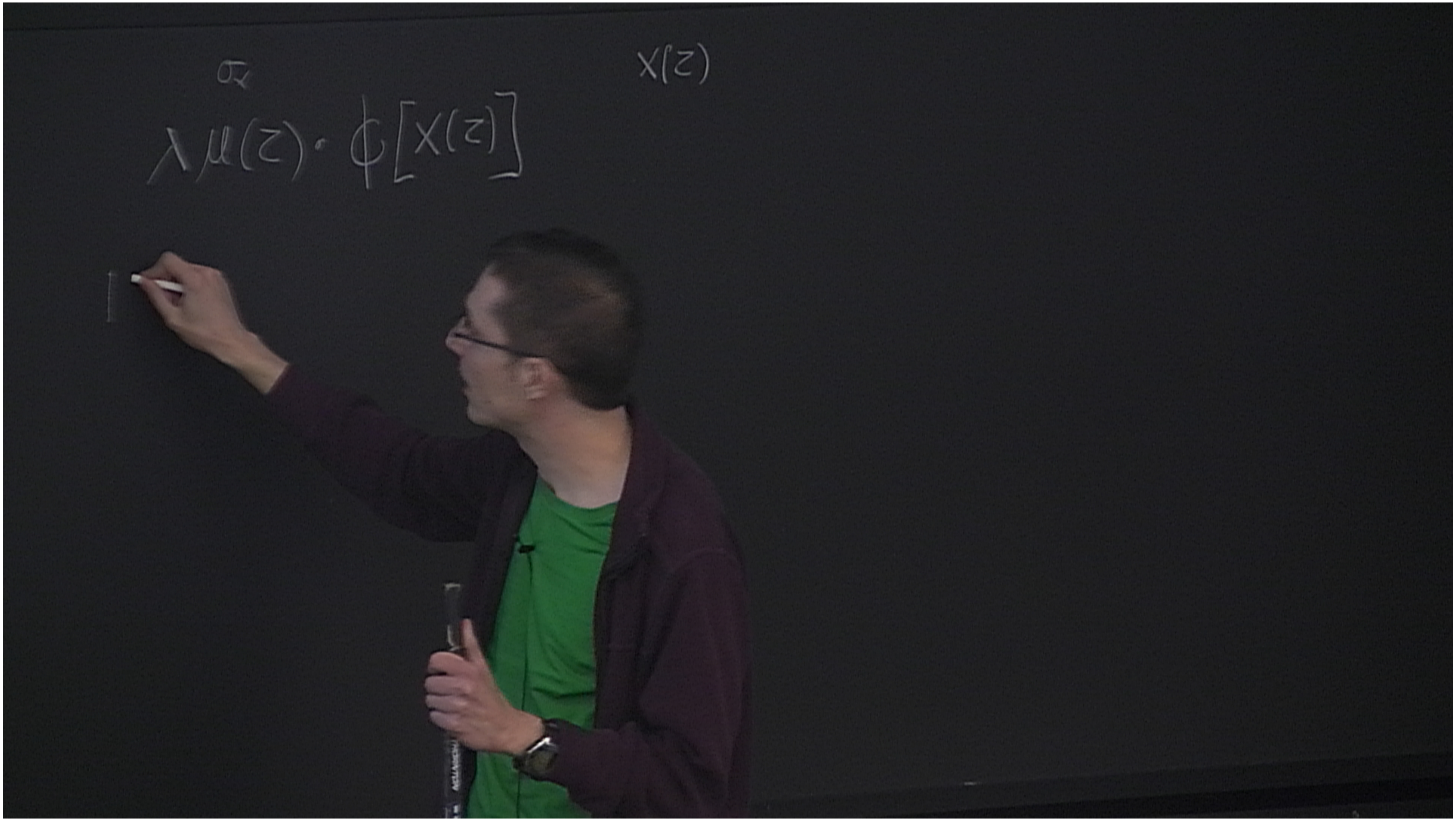
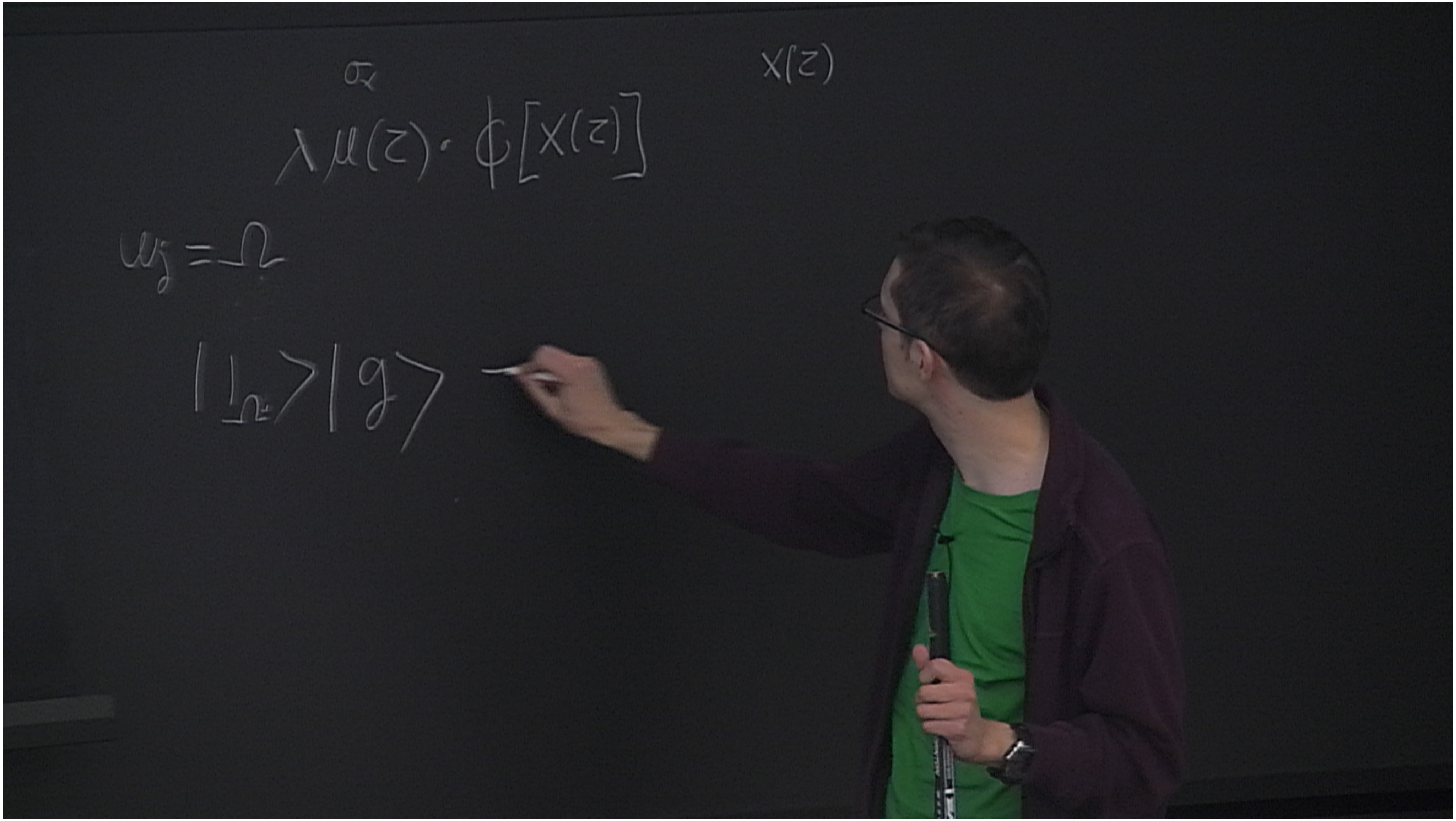
Detector at rest (or inertial): $x(\tau) = x_0, \quad t(\tau) = \tau$

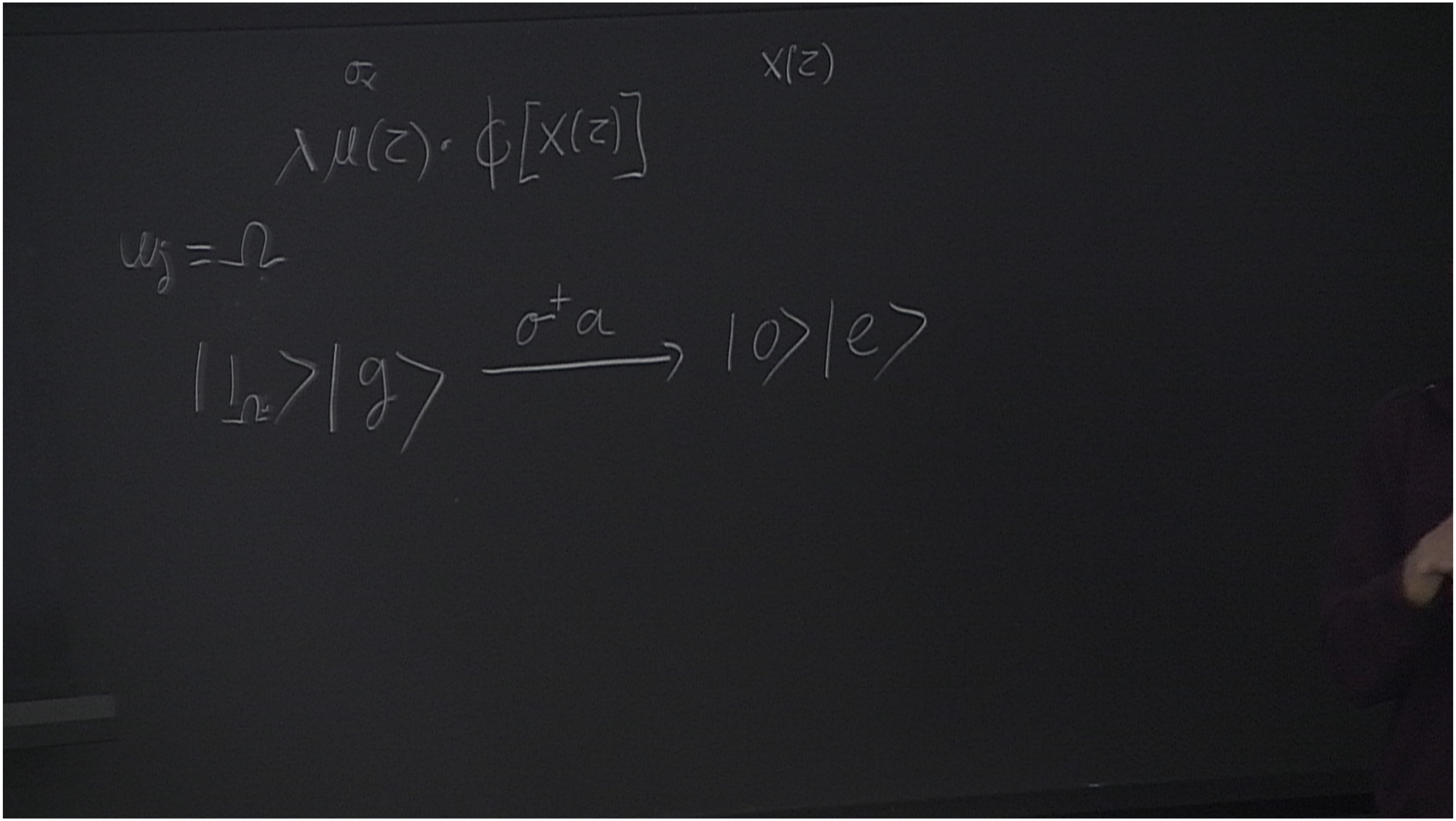
Counter-rotating transitions suppressed by highly oscillatory weight (for long times)

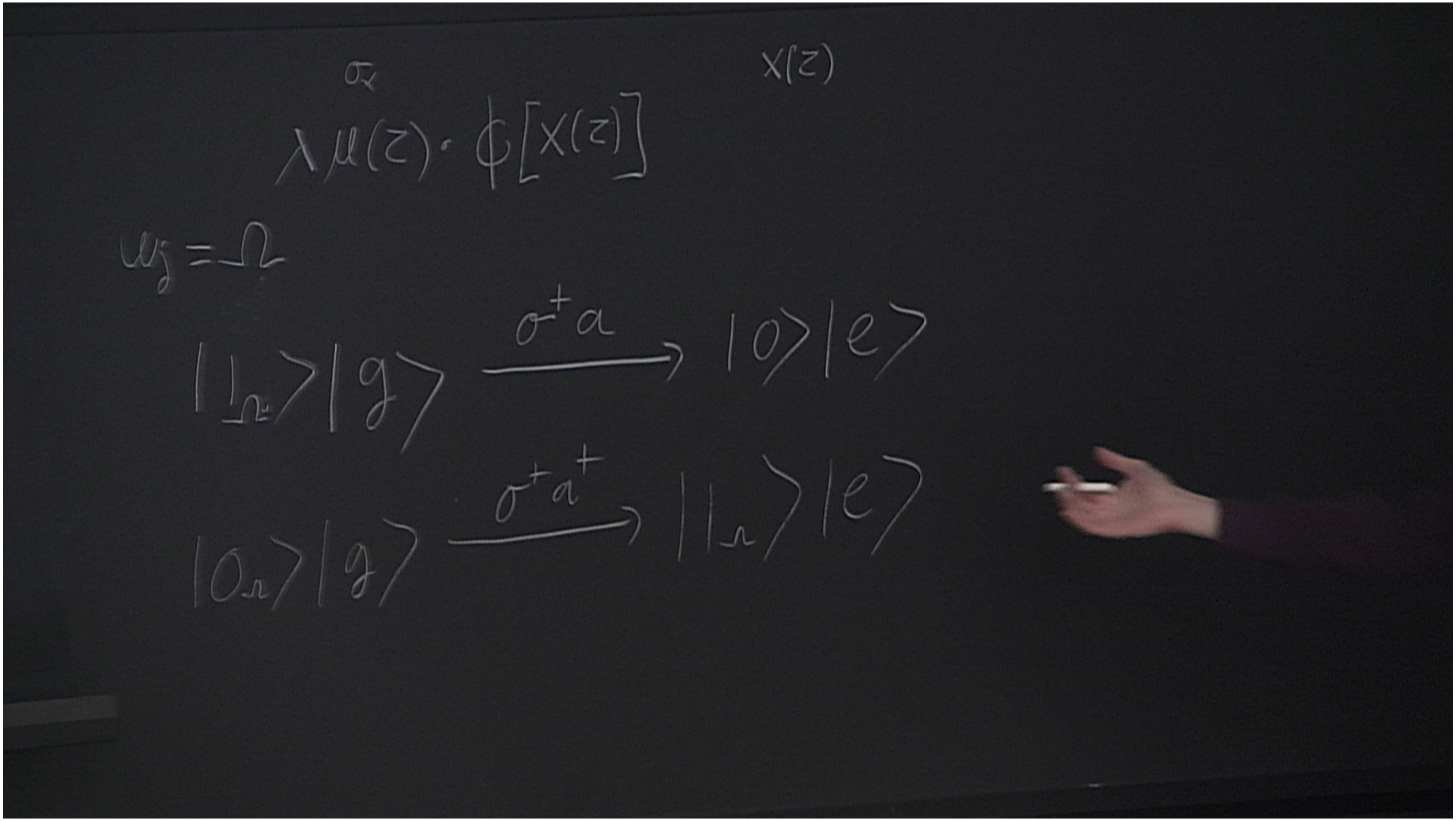

σ
λμ(z


σ_z
$\lambda \mu(z) \cdot \phi[X(z)]$


X(z)
λ μ(z) · φ[X(z)]


σz
x(z)
λμ(z)·φ[x(z)]
ωj = Ω
|1Ω⟩|g⟩


σ_z
$X(z)$
$\lambda \mu(z) \cdot \phi[X(z)]$
$\omega_j = \Omega$
$\sigma^+ a$
$|1_\Omega\rangle|g\rangle \rightarrow |0\rangle|e\rangle$


σz
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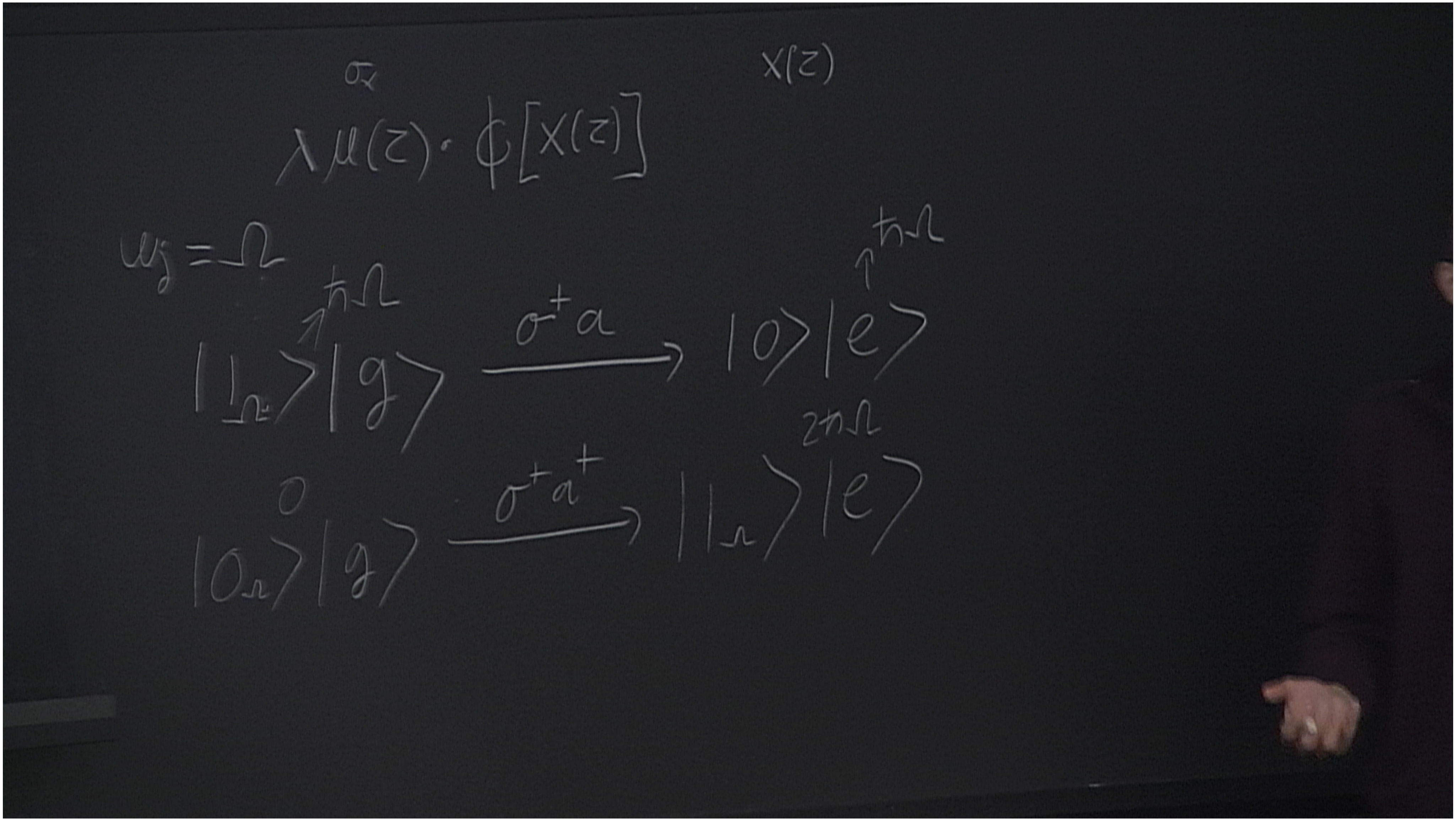
ωj = Ω
|1Ω⟩|g⟩ —σ⁺a→ |0⟩|e⟩
|0Ω⟩|g⟩ —σ⁺a⁺→ |1Ω⟩|e⟩


σz
X(z)
λ μ(z) · φ[X(z)]
ωj = Ω
ħΩ
σ+a
|1Ω>|g> → |0>|e>
ħΩ
2ħΩ
0
σ+a+
|0Ω>|g> → |1Ω>|e>

The Unruh-DeWitt detector model

$$H_I = \lambda \left(\sigma^+ e^{i\Omega\tau} + \sigma^- e^{-i\Omega\tau}\right) \sum_{j=1}^{\infty} \left[a_j^\dagger e^{i\omega_j t(\tau)} + a_j e^{-i\omega_j t(\tau)}\right] \sin k_j x(\tau).$$

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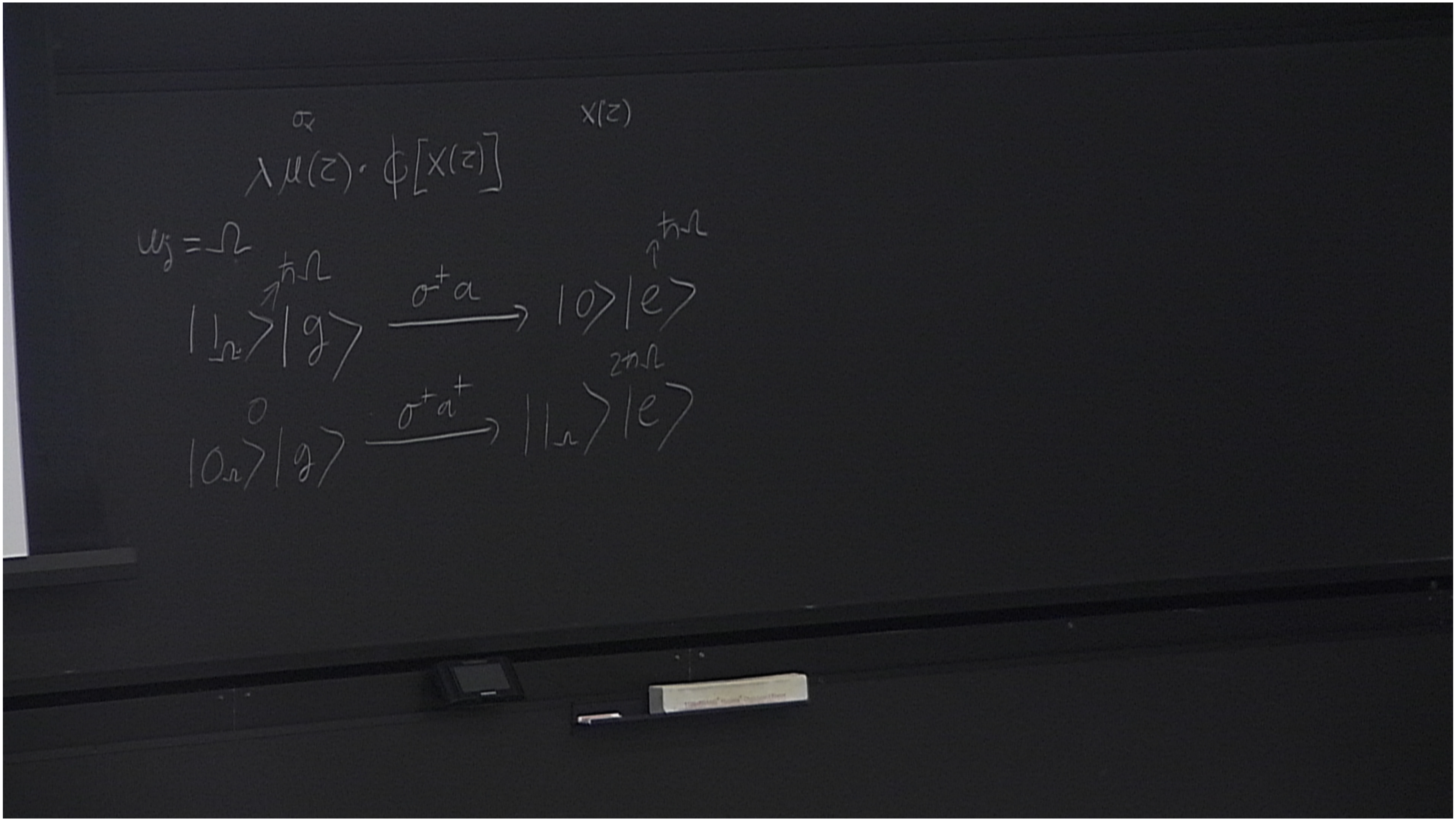
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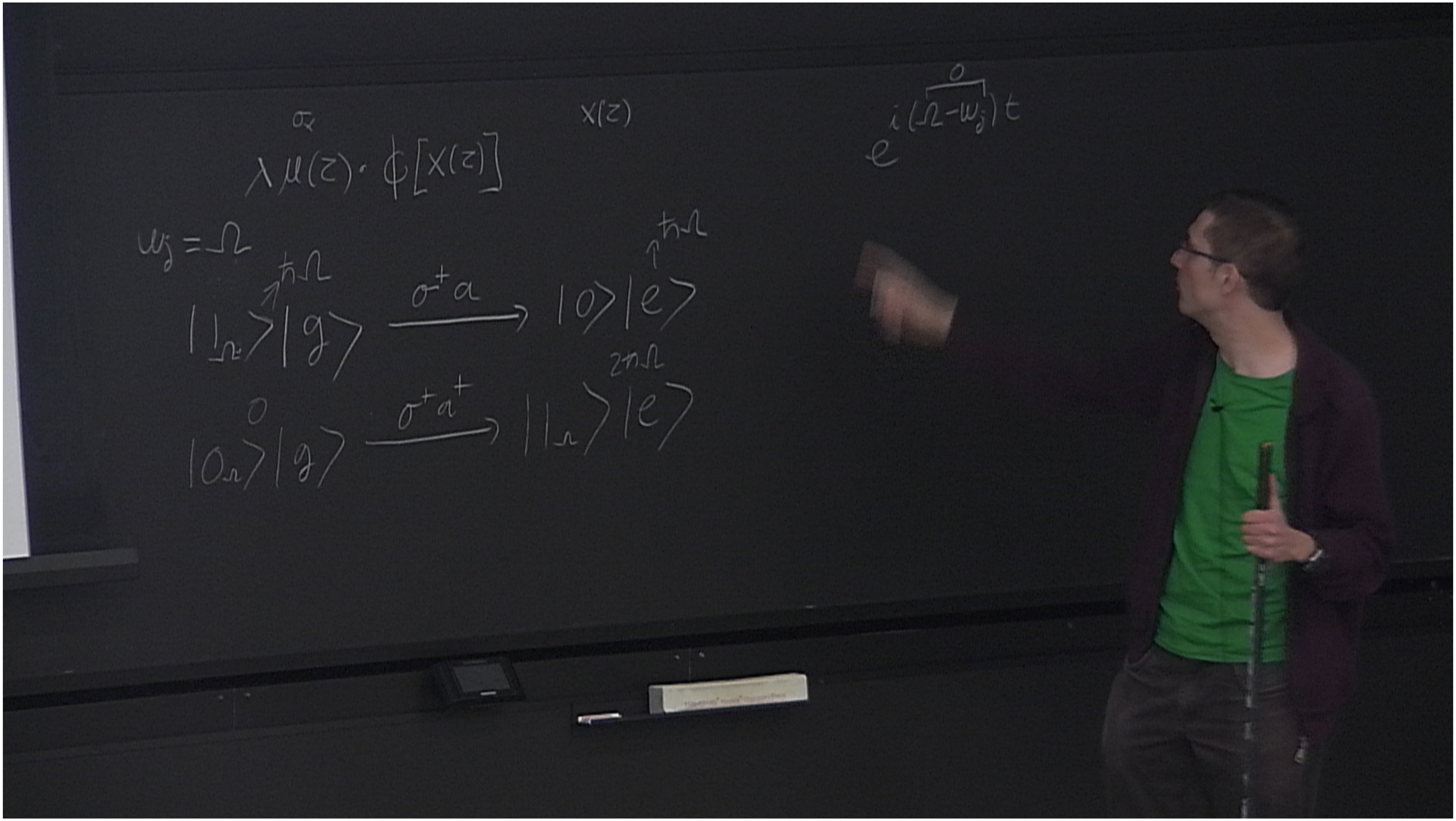
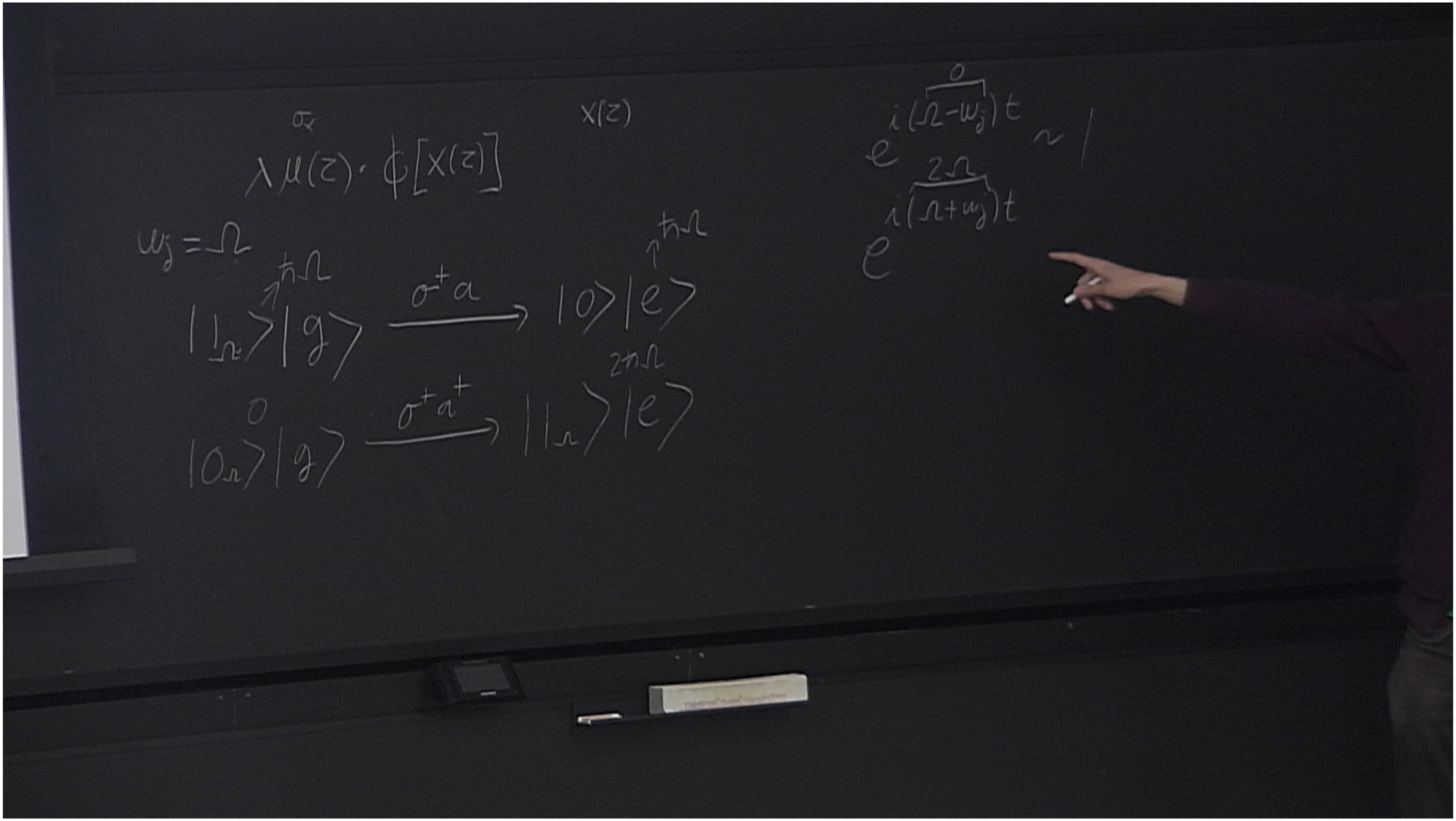
$\hbar\Omega$
$\hbar\Omega$
$|1_\Omega\rangle|g\rangle \xrightarrow{\sigma^+ a} |0\rangle|e\rangle$
0
$2\hbar\Omega$
$|0_\Omega\rangle|g\rangle \xrightarrow{\sigma^+ a^+} |1_\Omega\rangle|e\rangle$


σz
λ μ(τ) · φ[x(τ)]
x(τ)
e^{i(Ω-ωj)t}
0
ωj = Ω
ħΩ
|1Ω>|g> σ+a → |0>|e>
ħΩ
0
|0Ω>|g> σ+a+ → |1Ω>|e>
2ħΩ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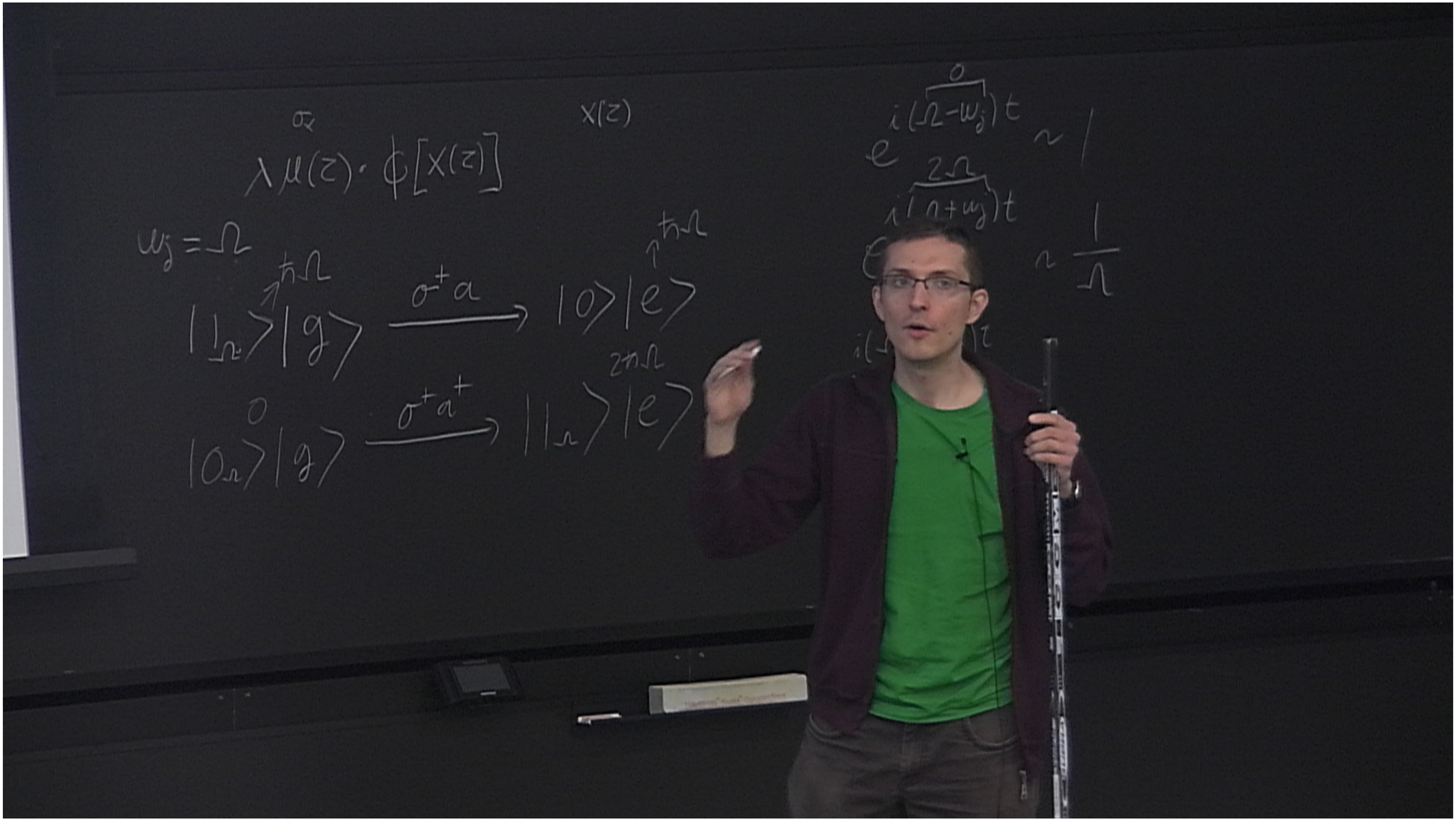
|1Ω⟩|g⟩ σ⁺a → |0⟩|e⟩
|0Ω⟩|g⟩ σ⁺a⁺ → |1Ω⟩|e⟩
e^{i(Ω−ωj)t} ~ 1
e^{i(Ω+ωj)t}



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ω_j = Ω
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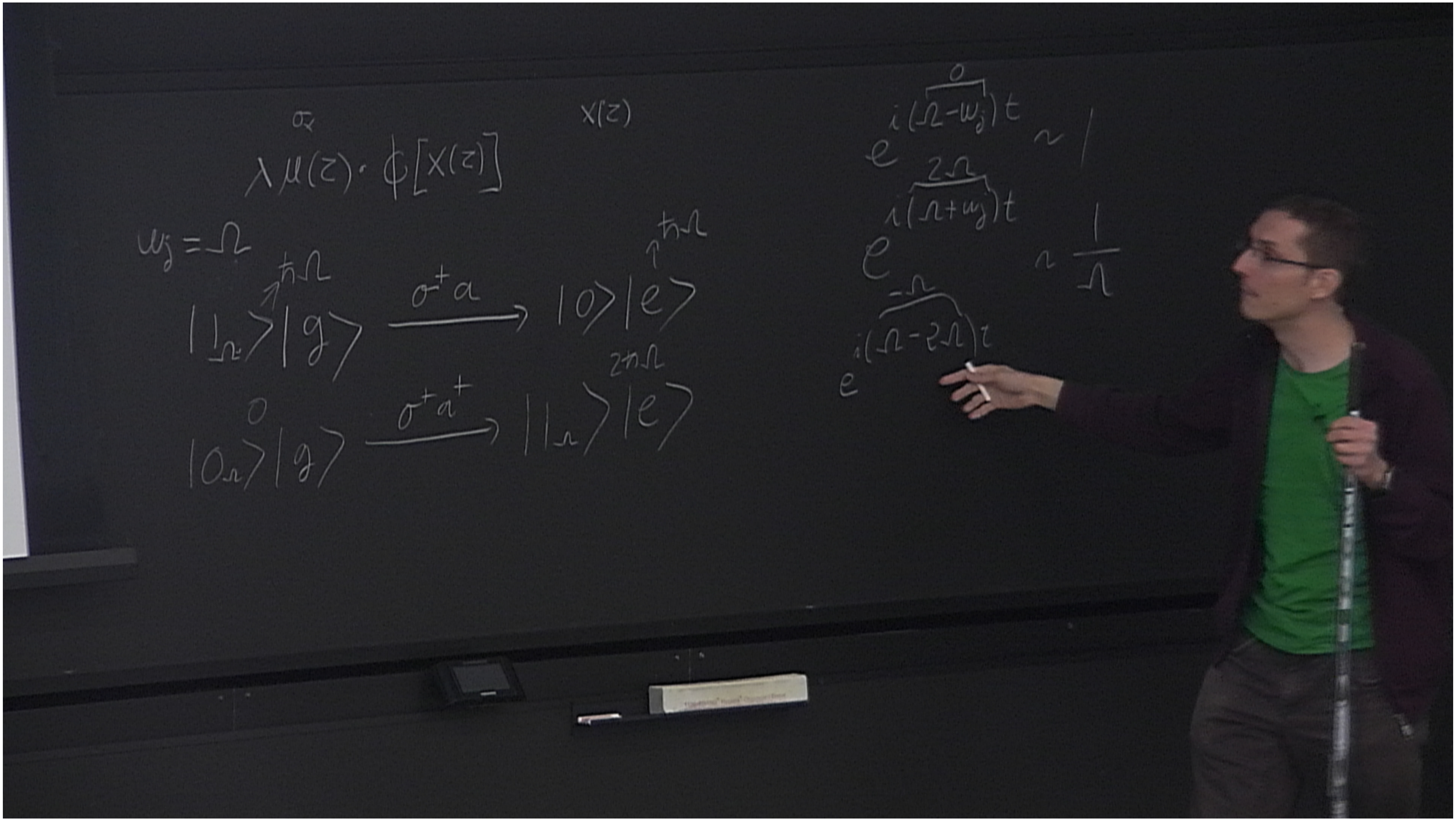

$\lambda \mu(\tau) \cdot \phi[x(\tau)]$
$\omega_j = \Omega$
$|1_\Omega\rangle|g\rangle \xrightarrow{\sigma^+ a} |0\rangle|e\rangle$
$|0_\Omega\rangle|g\rangle \xrightarrow{\sigma^+ a^+} |1_\Omega\rangle|e\rangle$
$e^{i(\Omega-\omega_j)t} \sim 1$
$e^{i(\Omega+\omega_j)t} \sim \frac{1}{\Omega}$
$e^{i(\Omega-2\Omega)t}$

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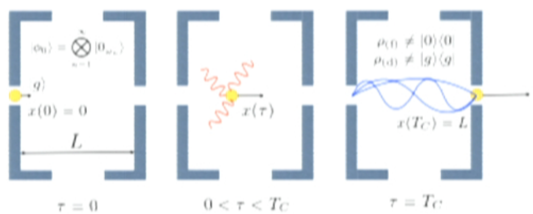
What if the trajectory is constantly accelerated?



$$x(\tau) = a^{-1}(\cosh a\tau - 1)$$

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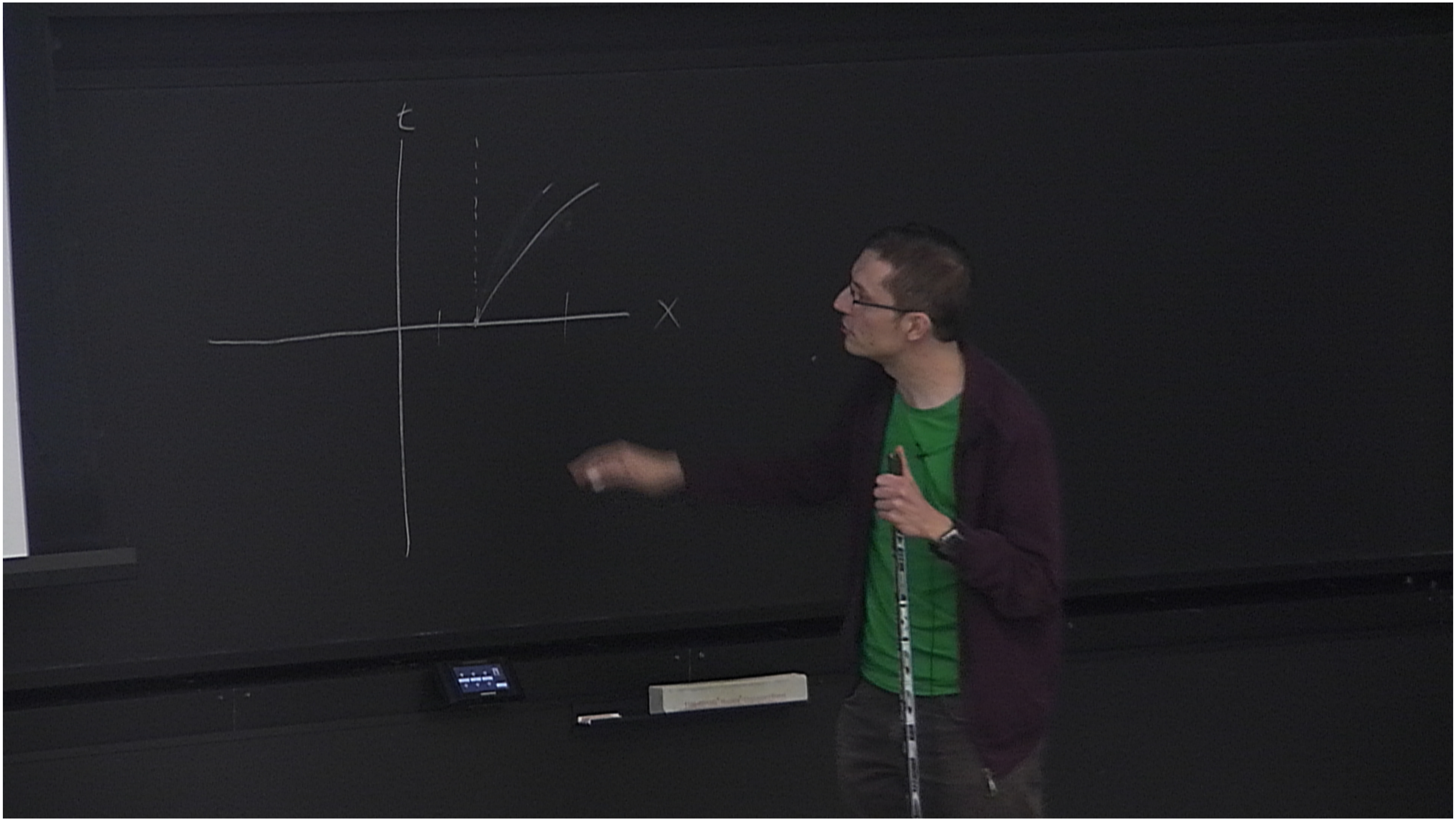
Both highly oscillatory and become not-negligible very soon.

Counter-rotating terms allow for vacuum excitations


t
x

Accelerated atom in the vacuum

$$H_I = \lambda \left(\sigma^+ e^{i\Omega\tau} + \sigma^- e^{-i\Omega\tau}\right) \sum_{j=1}^{\infty} \left[a_j^\dagger e^{i\omega_j t(\tau)} + a_j e^{-i\omega_j t(\tau)}\right] \sin k_j x(\tau),$$

$$I_{\pm,j} \equiv I_{\pm,j}(T) = \int_0^T d\tau e^{i[\pm\Omega\tau + \omega_j t(\tau)]} \sin\left[k_j x(\tau)\right].$$

For an accelerated detector

$$x(\tau) = a^{-1}(\cosh a\tau - 1)$$

$$t(\tau) = a^{-1}\sinh a\tau$$

- The phases in $I_{\pm,j}$ depend very nontrivially on time:
- The resonance condition is time dependent.
- The counter-rotating and rotating terms quickly become comparable.

An accelerated detector probing the vacuum detects field quanta due to the contribution of the counter-rotating terms. This is the 'Unruh effect'

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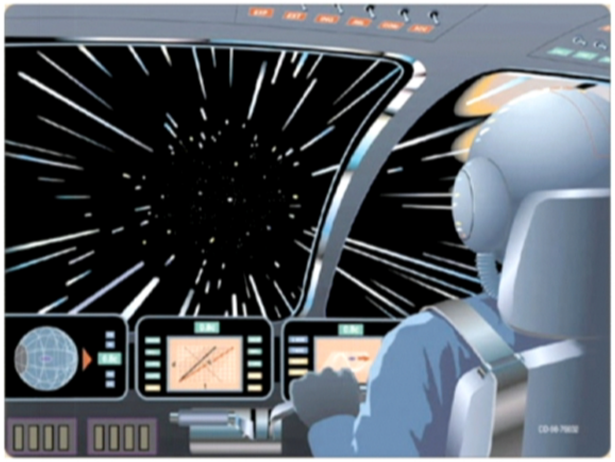
THE UNRUH EFFECT

Too small to be detected!!

Inertial frame | Accelerated frame



- Alice Observes the field vacuum.
- Rob observes a thermal bath of temperature $T_{\mathrm{U}} \propto a$

E. Martín-Martínez, I. Fuentes and R.B. Mann. Physical Review Letters 107, 131301 (2011)

Effects on quantum information

Particle detectors and QI

Standard view: Acceleration/gravity introduces noise

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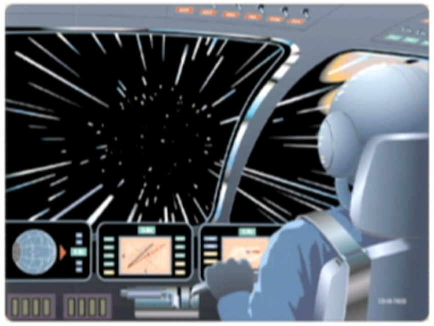
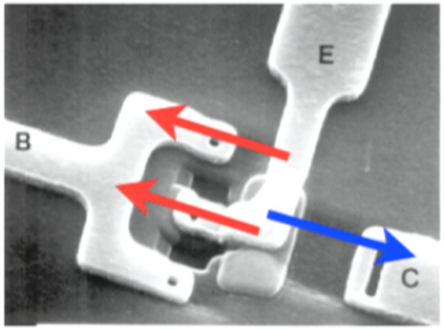
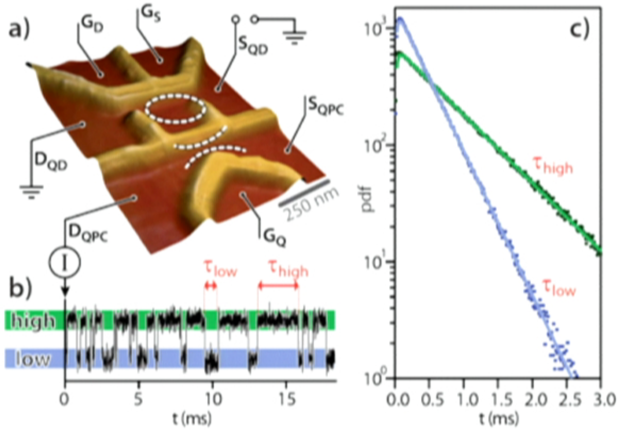


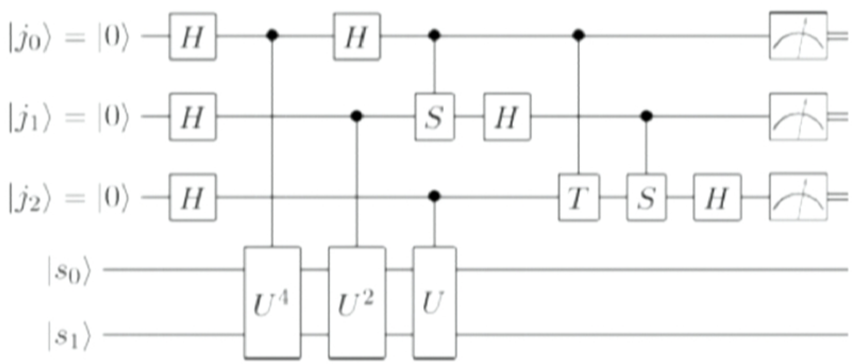
In the past the same arguments were used against quantum effects in classical computing





Effects on quantum information

But now we take advantage of quantum effects to go beyond what classical computers can do



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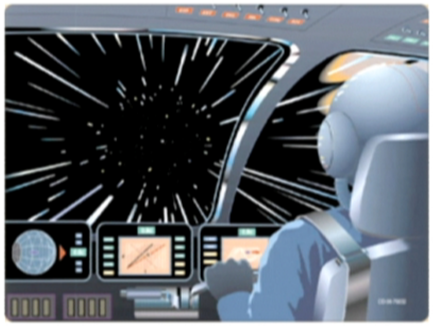

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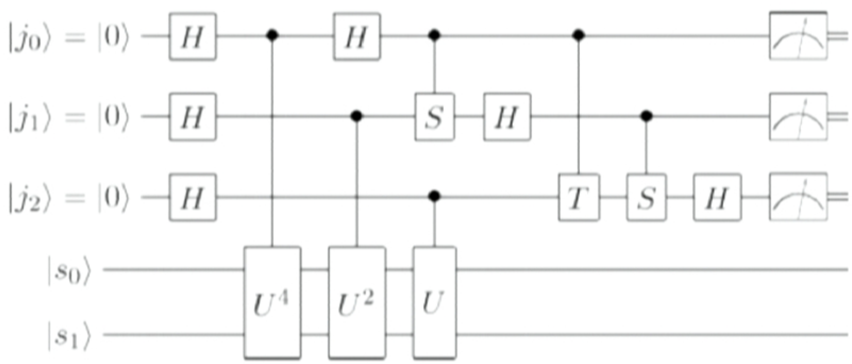
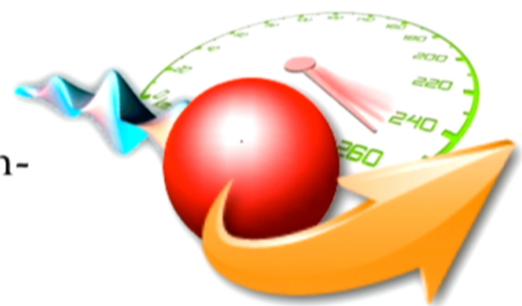


Can general relativistic effects be used to get computational advantage over non-relativistic settings?



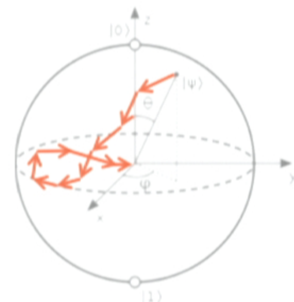
Roaming the Bloch Sphere (Vacuum)

Good news:

- It is possible to move in the Bloch sphere controlling acceleration

Bad news

- Affine transformation: $\Delta\vec{b}_T = M\vec{b}_0 + \vec{z}$
- Mixedness at the same order in λ
- Bad idea to process information



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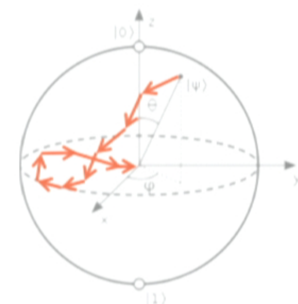

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Universal gates out of relativistic motion

One can achieve arbitrary rotations by preparing coherent states in one of the modes of the cavity. $\rho_0 = \rho_{0,(\mathrm{d})} \otimes |\alpha_{\omega_i}\rangle\langle\alpha_{\omega_i}| \bigotimes_{j\neq i} |0_{\omega_j}\rangle\langle 0_{\omega_j}|$.

In this case the leading order is given by $\rho_T^{(1)}$

$$\rho_{T,(\mathrm{d})} = \rho_{0,(\mathrm{d})} + \frac{\lambda}{i}\left[(A+A^*)\sigma_x\rho_{0,(\mathrm{d})} + i(A-A^*)\sigma_y\rho_{0,(\mathrm{d})} - \mathrm{H.c.}\right]$$

- $I_\pm$ corresponds to the mode where the coherent state is prepared
- $A = \alpha^* I_+ + \alpha I_-^*$.

This transformation is an infinitesimal rotation on the Bloch sphere around the axis defined by the direction of the (unnormalized) vector

$$\vec{n} = (A + A^*, i(A - A^*), 0),$$

and the magnitude of the rotation is

$$\delta = 2\lambda|\vec{n}| = 4\lambda|\alpha||e^{-i\,\mathrm{Arg}\,\alpha} I_+ + e^{i\,\mathrm{Arg}\,\alpha} I_-^*|.$$

- We can perform unitary rotations thus introducing no mixedness at leading order in the coupling strength.

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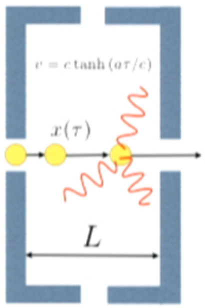
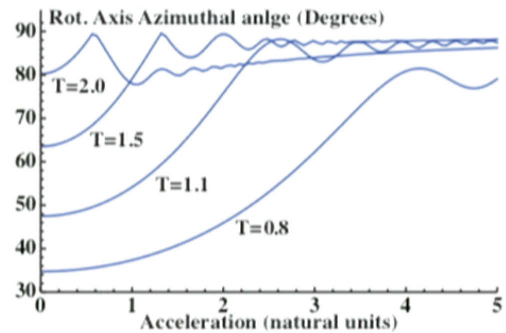
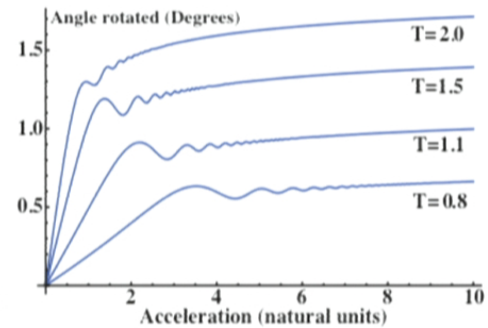


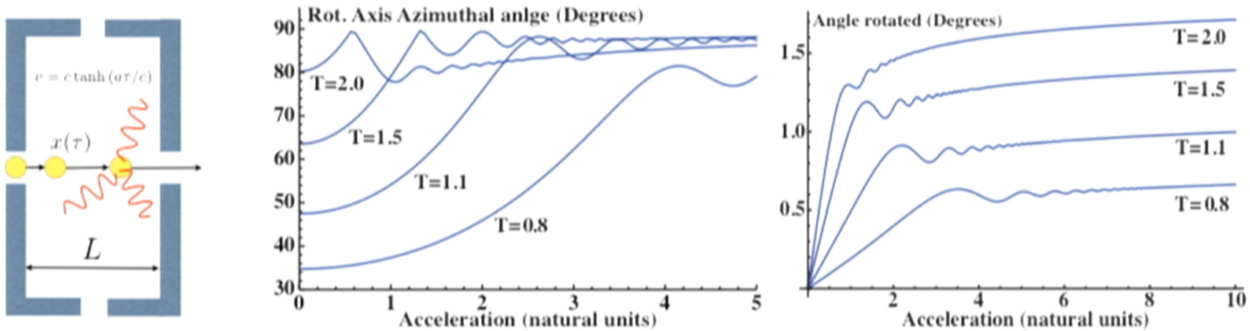




Rotation angle and magnitude can be controlled varying interaction time and acceleration

Independent rotations can be made

Universal gates out of relativistic motion



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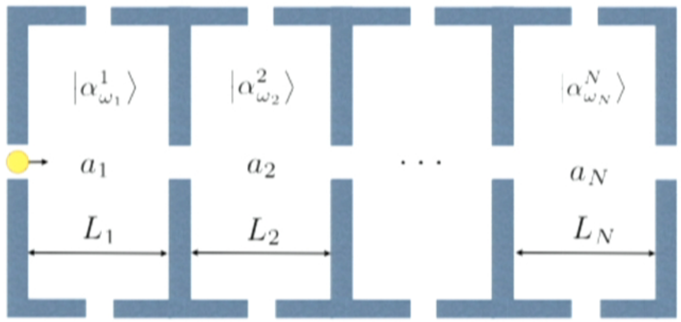
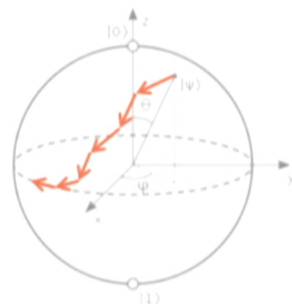
Independent rotations can be made

Roaming the Bloch sphere

Out of the composition of such rotations, an arbitrary trajectory in the Bloch sphere can be tailored by letting the atom describe accelerated trajectories through an array of cavities



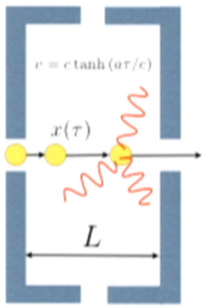


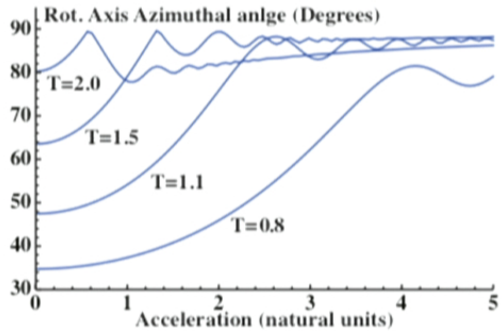
Cavity crossing time controlled by the length of every cavity

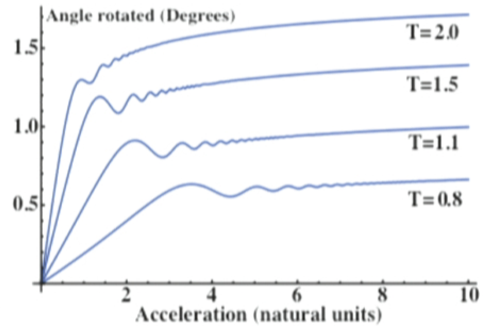
$$T_C = c\tilde{a}^{-1} \operatorname{arccosh}\left[\left(Lac^{-2}+1\right)\right]$$

Accelerations can alternate sign

Natural Units





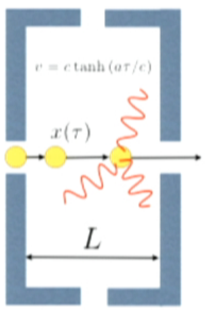


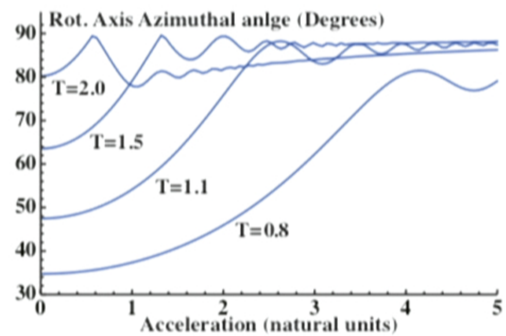
The natural unit of acceleration is $\dfrac{\Omega c}{\pi}$

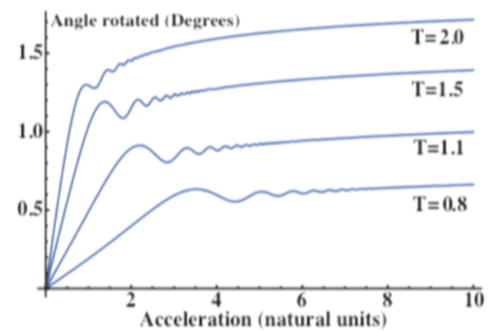
GHz: $10^{16} g$

Hz: $10^{7} g$

Natural Units







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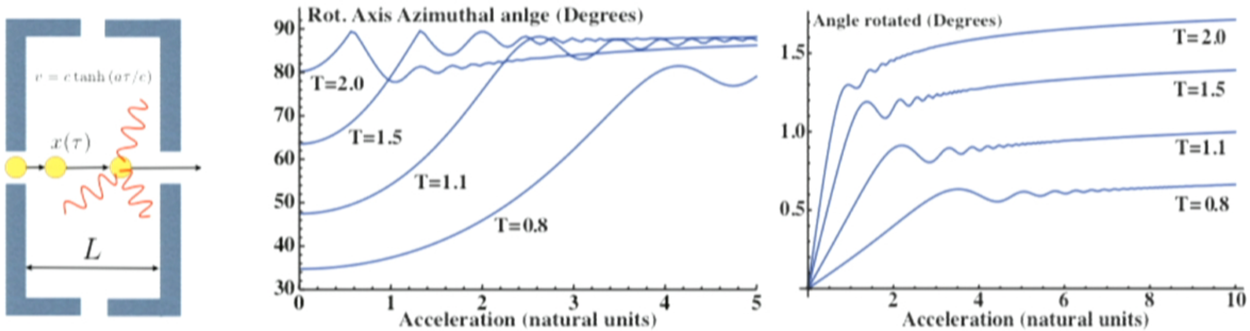
Natural Units

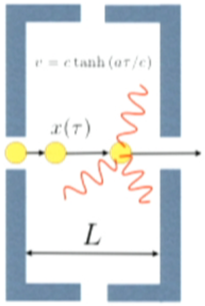


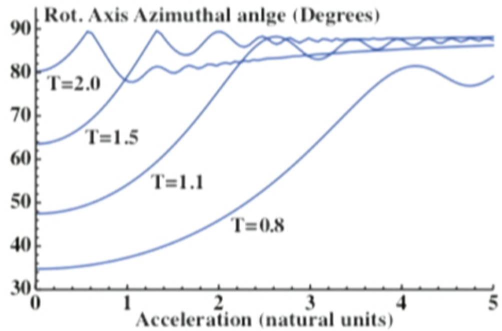
The natural unit of acceleration is $\dfrac{\Omega c}{\pi}$

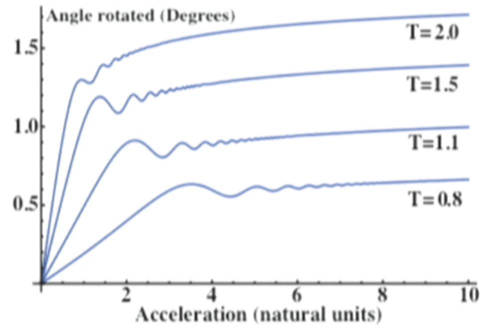
GHz: $10^{16} g$

Hz: $10^{7} g$

Natural Units







The natural unit of acceleration is $\dfrac{\Omega c}{\pi}$

GHz: $10^{16}g$

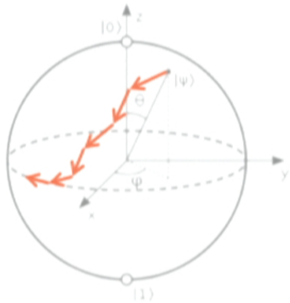
Hz: $10^{7}g$

Two-Qubit Gates

We can build non trivial 2 qubit operations by letting two atoms inside a cavity at the same time

Non-trivial two-qubit operations + arbitrary rotations yield, in principle, universal QC

Conclusions and Outlook



-General relativistic time dilation and space contraction introduces a time dependence in the interaction Hamiltonian

-This can be used to drive the interaction and perform controlled rotations in the Bloch sphere

-We can implement universal 1-qubit gates

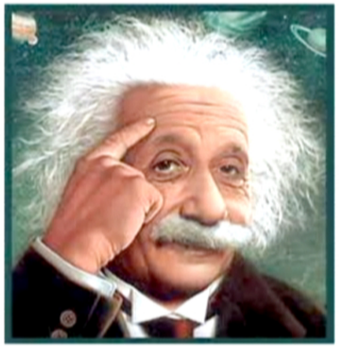
Conclusions and Outlook

-It can work the opposite way: Detect the variations to detect the Unruh effect

Movement around the Bloch sphere easier to detect that excitation probability

Tell me the resolution of your experiment, I will give you the acceleration you would need

Conclusions and Outlook



We have proved that general relativistic effects can be beneficial to process quantum information

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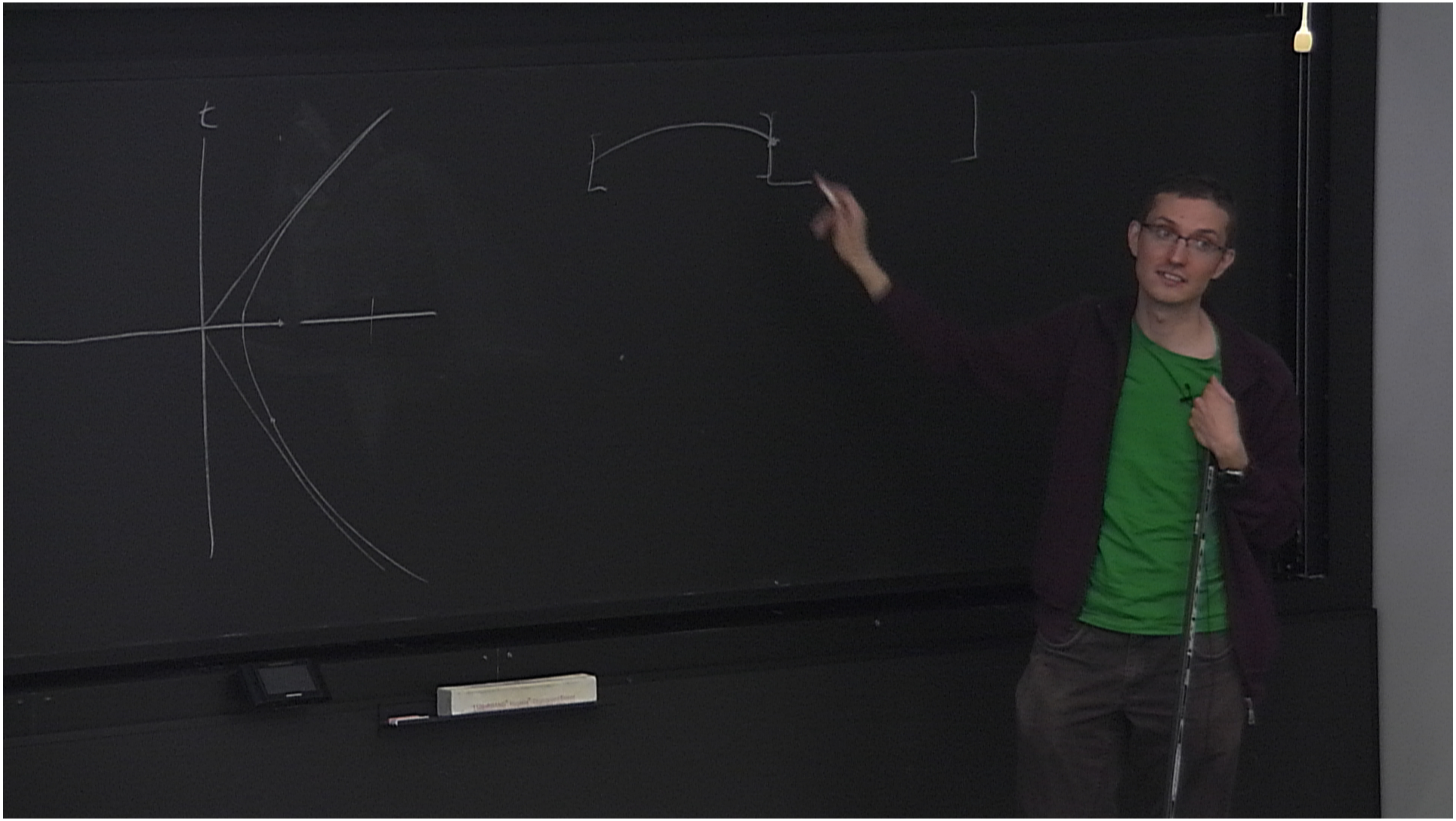
IQC's Selected Advanced Topics in Quantum Information

- **MODULE A :** "Introduction to Relativistic Quantum Information" by Eduardo Martin-Martinez

Abstract: This short course will consist of an introduction to Relativistic Quantum Information (RQI). By combining quantum information theoretic methods with special and general relativistic quantum theory, RQI produces results that range from theoretical insights into the quantum structure of space and time to concrete implementations of quantum computing algorithms. We will introduce the basic concepts, review current techniques and most important results in the field and begin to explore the state of the art and current open problems in Relativistic Quantum Information.

May 14, 16, 21, 23   9 to 10:20

University of Waterloo: QNC 1201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