Title: The effective field theory of general relativity, running couplings and Asymptotic Safety

Date: May 09, 2013 02:30 PM

URL: http://pirsa.org/13050003

Abstract: Effective field
theory techniques allow reliable quantum calculations in general relativity at low energy. After a review of these techniques, I will discuss the attempts to define the gravitational corrections to running gauge couplings and to the couplings of gravity itself. I will also describe an attempt to understand the relation between the effective field theory and Asymptotic Safety in the region where they overlap.

The effective field theory of general relativity, running couplings, and Asymptotic Safety

1) Some motivations

2) GR as an EFT

3) Gravitational corrections to gauge coupling running?

4) Can we define a good running G in the perturbative region?

5) AS to one loop matching to EFT

Overall goal: Understanding how gravity works in the perturbative regime
Does gravity lead to well-defined corrections to running couplings?

"Running" work with M. Anber (Toronto)

John Donoghue
Perimeter, May 9, 2013

A motivation for the gravity and running couplings:

PRL 96, 231601 (2006) PHYSICAL REVIEW LETTERS week ending 16 JUNE 2006

Gravitational Correction to Running of Gauge Couplings

Sean P. Robinson* and Frank Wilczek†
Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA
(Received 30 March 2006; published 15 June 2006)

We calculate the contribution of graviton exchange to the running of gauge couplings at lowest nontrivial order in perturbation theory. Including this contribution in a theory that features coupling constant unification does not upset this unification, but rather shifts the unification scale. When extrapolated formally, the gravitational correction renders all gauge couplings asymptotically free.

$$\beta(g, E) \equiv \frac{dg}{d\ln E} = -\frac{b_0}{(4\pi)^2} g^3 + a_0 \frac{E^2}{M_P^2} g,$$

$$G \sim \frac{1}{M_P^2} \sim \kappa^2$$

FIG. 1. A typical Feynman diagram for a gravitational process contributing to the renormalization of a gauge coupling at one loop. Curly lines represent gluons. Double lines represent gravitons. The three-gluon vertex ■ is proportional to g, while the gluon-graviton vertex ● is proportional to E/M_P.

A hint of asymptotic freedom for all couplings

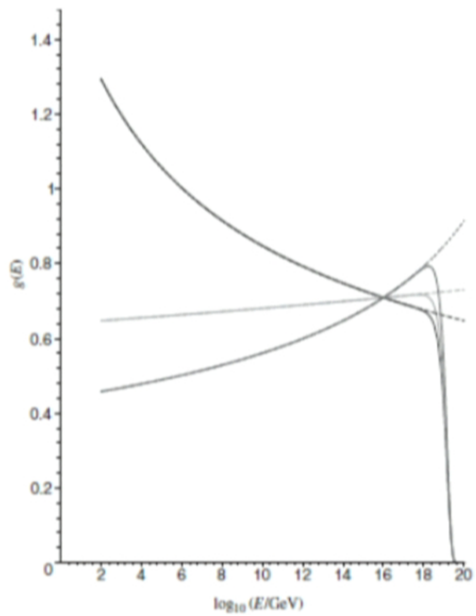


FIG. 2. When gravity is ignored, the three gauge couplings of a model theory evolve as the inverse logarithm of E at one-loop order (dashed curves). Initial values at 100 GeV were set so that the curves exactly intersect at approximately 10^{16} GeV. When gravity is included at one loop (solid curves), the couplings remain unified near 10^{16} GeV, but evolve rapidly towards weaker coupling at high E.

when the energy approaches the Planck scale, and soon after that one loses the right to neglect higher-order graviton exchanges. Though neglect of additional corrections is not justified beyond $E \ll M_P$, it is entertaining to consider some consequences of extrapolating Eq. (2) as it stands to these energies, taking into account $a_0 < 0$. The integral on the right-hand side converges as $E \to \infty$, and so Eq. (20) arises as an asymptotic relation. Thus, the effective coupling vanishes rapidly beyond the Planck scale, rendering the gauge sector approximately free at these energies. In

A hint of asymptotic freedom for all couplings

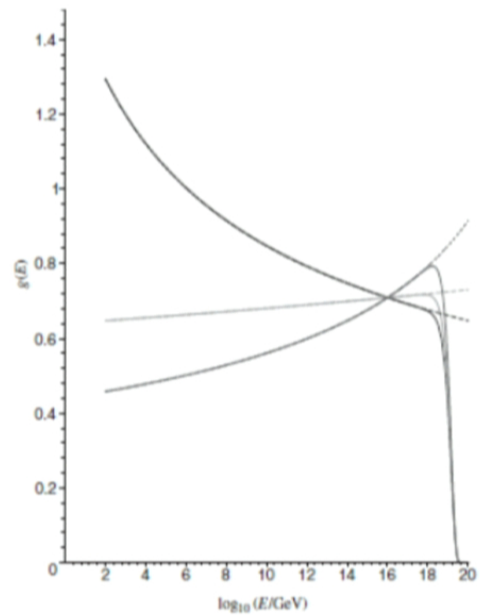


FIG. 2. When gravity is ignored, the three gauge couplings of a model theory evolve as the inverse logarithm of E at one-loop order (dashed curves). Initial values at 100 GeV were set so that the curves exactly intersect at approximately 10^{16} GeV. When gravity is included at one loop (solid curves), the couplings remain unified near 10^{16} GeV, but evolve rapidly towards weaker coupling at high E.

when the energy approaches the Planck scale, and soon after that one loses the right to neglect higher-order graviton exchanges. Though neglect of additional corrections is not justified beyond $E \ll M_P$, it is entertaining to consider some consequences of extrapolating Eq. (2) as it stands to these energies, taking into account $a_0 < 0$. The integral on the right-hand side converges as $E \to \infty$, and so Eq. (20) arises as an asymptotic relation. Thus, the effective coupling vanishes rapidly beyond the Planck scale, rendering the gauge sector approximately free at these energies. In

A Rough History:

Prehistory: Fradkin, Vilkovisky, Tseytlin, Diennes, Kiritsis, Kounnas...

Start of "modern era":

S. P. Robinson and F. Wilczek, "Gravitational correction to running of gauge couplings," Phys. Rev. Lett. **96**, 231601 (2006) [arXiv:hep-th/0509050].

Claims that couplings do not run
-analysis in dimensional regularization

A. R. Pietrykowski, "Gauge dependence of gravitational correction to running of gauge couplings," Phys. Rev. Lett. **98**, 061801 (2007) [arXiv:hep-th/0606208].

D. J. Toms, "Quantum gravity and charge renormalization," Phys. Rev. D **76**, 045015 (2007) [arXiv:0708.2990 [hep-th]].

D. Ebert, J. Plefka and A. Rodigast, "Absence of gravitational contributions to the running Yang-Mills coupling," Phys. Lett. B **660**, 579 (2008) [arXiv:0710.1002 [hep-th]].

Y. Tang and Y. L. Wu, "Gravitational Contributions to the Running of Gauge Couplings," arXiv:0807.0331 [hep-ph].

•

Claims that couplings do run:
- analysis using cutoff regularization

D. J. Toms, "Quantum gravitational contributions to quantum electrodynamics," Nature **468**, 56-59 (2010). [arXiv:1010.0793 [hep-th]].

H. -J. He, X. -F. Wang, Z. -Z. Xianyu, "Gauge-Invariant Quantum Gravity Corrections to Gauge Couplings via Vilkovisky-DeWitt Method and Gravity Assisted Gauge Unification," [arXiv:1008.1839 [hep-th]].

Y. Tang, Y. -L. Wu, "Quantum Gravitational Contributions to Gauge Field Theories,"
[arXiv:1012.0626 [hep-ph]].

S. Folkerts, D. F. Litim, J. M. Pawlowski, "Asymptotic freedom of Yang-Mills theory with gravity," [arXiv:1101.5552 [hep-th]].

Claims that running couplings do not work

M. M. Anber, J. F. Donoghue, M. El-Houssieny, "Running couplings and operator mixing in the gravitational corrections to coupling constants," Phys. Rev. D83, 124003 (2011) [arXiv:1011.3229 [hep-th]].

J. Ellis, N. E. Mavromatos, "On the Interpretation of Gravitational Corrections to Gauge Couplings," [arXiv:1012.4353 [hep-th]].

Agreement

D. J. Toms, "Quadratic divergences and quantum gravitational contributions to gauge coupling constants," Phys. Rev. D **84**, 084016 (2011).

What is going on?

1) Dim-reg vs cutoff regularization – why the difference?

2) Running with $(\text{Energy})^2$
   - dimensional coupling constant

3) Why don't other similar effective field theories use running couplings?

4) **Application in physical processes**
   - does the running coupling work?

Also motivation from gravity itself:

Many attempts to define a running $G_N(E)$:

- some are pretty clearly wrong
- some are more sophisticated

Asymptotic safety:

Weinberg

Hypothesis of UV fixed point

$$\sigma(\Lambda, G, c_i, p_a) = \frac{1}{\mu^2}\sigma(\frac{\Lambda}{\mu^4}, G\mu^2, \tilde{c}_i, \frac{p_a}{\mu})$$

Dimensionless running couplings – running to UV fixed point

$$\tilde{\lambda} \to \tilde{\lambda}^*, \qquad \tilde{G} = G\mu^2 \to \tilde{G}_*, \qquad \tilde{c}_i \to \tilde{c}_i^*$$

Goal is to make gravity UV finite – G is weaker at high energies

Also motivation from gravity itself:

Many attempts to define a running $G_N(E)$:

- some are pretty clearly wrong
- some are more sophisticated

Asymptotic safety:

Weinberg

Hypothesis of UV fixed point

$$\sigma(\Lambda, G, c_i, p_a) = \frac{1}{\mu^2}\sigma(\frac{\Lambda}{\mu^4}, G\mu^2, \tilde{c}_i, \frac{p_a}{\mu})$$

Dimensionless running couplings – running to UV fixed point

$$\tilde{\lambda} \to \tilde{\lambda}^*, \qquad \tilde{G} = G\mu^2 \to \tilde{G}_*, \qquad \tilde{c}_i \to \tilde{c}_i^*$$

Goal is to make gravity UV finite – G is weaker at high energies

AS is now a developing subfield:

- various truncations, methods
- Euclidean "Exact Functional Renormalization Group"

Reuter, Percacci,…..

$$\tilde{G} = G\mu^2 \qquad \tilde{G} \to \tilde{G}_* \qquad G(\mu) = \frac{G_N}{1 + aG_N\mu^2}$$

Running occurs even in the perturbative regime:

For example, Cosmological Constant

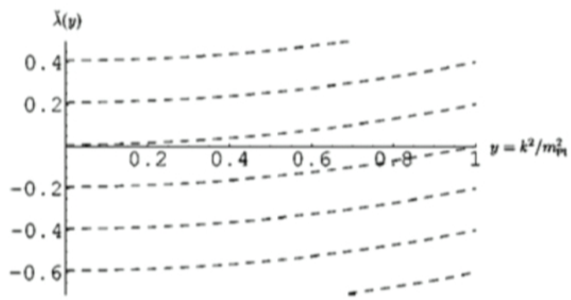


FIG. 3. Solution (4.5) to the naive flow equation for different initial values $\tilde{\lambda}(\hat{y})$ and $\tilde{G}(0)=1$.

Reuter - CC ~ density of water when at atomic scale !

Applications – cosmology
i.e. Entropy generation
from variable CC. (Reuter)

Can we see indications of AS in Lorentzian perturbation theory?

AS is now a developing subfield:

Reuter, Percacci,…..

- various truncations, methods
- Euclidean "Exact Functional Renormalization Group"

$$\tilde{G} = G\mu^2 \qquad \tilde{G} \to \tilde{G}_* \qquad G(\mu) = \frac{G_N}{1 + aG_N\mu^2}$$

Running occurs even in the perturbative regime:

For example, Cosmological Constant

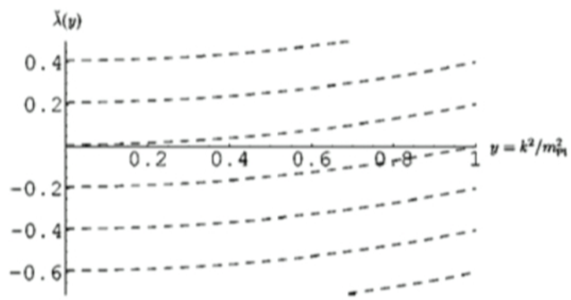


FIG. 3. Solution (4.5) to the naive flow equation for different initial values $\tilde{\lambda}(\hat{y})$ and $\tilde{G}(0) = 1$.

Reuter - CC ~ density of water when at atomic scale !

Applications – cosmology
i.e. Entropy generation
from variable CC. (Reuter)

Can we see indications of AS in Lorentzian perturbation theory?

General Relativity as a quantum Effective Field Theory

We do have a quantum theory of GR at ordinary energies

It has the form of an Effective Field Theory

Rigorous well-established procedure

EFT falls apart beyond Planck scale

Well-defined calculations at ordinary energy

A lot of portentous drivel has been written about the quantum theory of gravity, so I'd like to begin by making a fundamental observation about it that tends to be obfuscated. There is a perfectly well-defined quantum theory of gravity that agrees accurately with all available experimental data.

Frank Wilczek
Physics Today
August 2002

General Relativity as a quantum Effective Field Theory

We do have a quantum theory of GR at ordinary energies

It has the form of an Effective Field Theory

Rigorous well-established procedure

EFT falls apart beyond Planck scale

Well-defined calculations at ordinary energy

A lot of portentous drivel has been written about the quantum theory of gravity, so I'd like to begin by making a fundamental observation about it that tends to be obfuscated. There is a perfectly well-defined quantum theory of gravity that agrees accurately with all available experimental data.

Frank Wilczek
Physics Today
August 2002

Key Steps

1) **High energy effects are local** (when viewed at low E)

Example = W exchange

=> local 4 Fermi interaction

Even loops

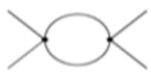
=> local mass counterterm

Low energy particles propagate long distances:

Photon:

← **Not** local

$$V \sim \frac{1}{q^2} \sim \frac{1}{r}$$



← Even in loops – cuts, imag. parts....

Result: High energy effects in **local** Lagrangian

$$L = g_1 L_1 + g_2 L_2 + g_3 L_3 + \ldots.$$

- Even if you don't know the true effect, you know that it is local
  - -use most general local Lagrangian

Key Steps

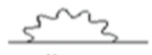
1) High energy effects are local (when viewed at low E)

Example = W exchange

=> local 4 Fermi interaction

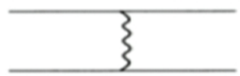
Even loops

=> local mass counterterm

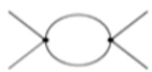


Low energy particles propagate long distances:

Photon:



← **Not** local $\qquad V \sim \frac{1}{q^2} \sim \frac{1}{r}$



Even in loops – cuts, imag. parts....

Result: High energy effects in **local** Lagrangian

$$L = g_1 L_1 + g_2 L_2 + g_3 L_3 + \ldots.$$

Even if you don’t know the true effect, you know that it is local
-use most general local Lagrangian

Key Steps

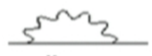
1) **High energy effects are local** (when viewed at low E)

Example = W exchange

=> local 4 Fermi interaction

Even loops

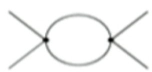
=> local mass counterterm



Low energy particles propagate long distances:

Photon:

← **Not** local $V \sim \frac{1}{q^2} \sim \frac{1}{r}$



Even in loops – cuts, imag. parts....

Result: High energy effects in **local** Lagrangian

$$L = g_1 L_1 + g_2 L_2 + g_3 L_3 + \dots$$

- Even if you don't know the true effect, you know that it is local
  -use most general local Lagrangian

Key Steps

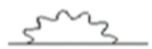
1) **High energy effects are local** (when viewed at low E)

Example = W exchange

=> local 4 Fermi interaction

Even loops

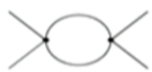
=> local mass counterterm



Low energy particles propagate long distances:

Photon:

← **Not** local $V \sim \frac{1}{q^2} \sim \frac{1}{r}$



← Even in loops – cuts, imag. parts....

Result: High energy effects in **local** Lagrangian

$$L = g_1 L_1 + g_2 L_2 + g_3 L_3 + \ldots.$$

- Even if you don't know the true effect, you know that it is local
  -use most general local Lagrangian

General Procedure

1) Identify Lagrangian
- -- most general (given symmetries)
- -- order by energy expansion

2) **Calculate and renormalize**
- -- start with lowest order
- -- renormalize parameters

3) **Phenomenology**
- -- measure parameters
- -- residual relations are predictions

Note: Two differences from textbook renormalizable field theory:
1) no restriction to renormalizable terms only
2) energy expansion

Gravity as an effective theory

Weinberg
JFD

Both General Relativity and Quantum Mechanics known and tested over common range of scales

Is there an incompatibility **at those scales** ?

Or are problems only at uncharted high energies?

Need to study GR with a careful consideration of scales

Parameters

$$S_{grav} = \int d^4x \sqrt{-g} \left\{ \Lambda + \frac{2}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \dots \right\}$$

1) Λ = cosmological constant

$$\Lambda = (1.2 \pm 0.4) \times 10^{-123} M_P^4$$

$$M_P = 1.22 \times 10^{19}\ \text{GeV}$$

-this is observable only on cosmological scales
-neglect for rest of talk
-interesting aspects

2) Newton's constant

$$\kappa^2 = 32\pi G$$

3) Curvature –squared terms c_1, c_2

- studied by Stelle
- modify gravity at very small scales
-essentially unconstrained by experiment

$$c_1, c_2 \leq 10^{74}$$

Quantization

“**Easy” to quantize gravity**:

-Covariant quantization   Feynman deWitt

-gauge fixing

-ghosts fields

-Background field method   ‘t Hooft Veltman

-retains symmetries of GR

-path integral

Background field:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \kappa h_{\mu\nu}$$

$$g^{\mu\nu} = \bar{g}^{\mu\nu} - \kappa h^{\mu\nu} + \kappa^2 h^{\mu}_{\lambda} h^{\lambda\nu} + \ldots$$

Expand around this background:

$$S_{grav} = \int d^4x \sqrt{-\bar{g}} \left[\frac{2\bar{R}}{\kappa^2} + \mathcal{L}_g^{(1)} + \mathcal{L}_g^{(2)} + \ldots \right]$$

$$\begin{aligned}
\mathcal{L}_g^{(1)} &= \frac{h_{\mu\nu}}{\kappa} \left[\bar{g}^{\mu\nu} \bar{R} - 2\bar{R}^{\mu\nu} \right] \\
\mathcal{L}_g^{(2)} &= \frac{1}{2} h_{\mu\nu;\alpha} h^{\mu\nu;\alpha} - \frac{1}{2} h_{;\alpha} h^{;\alpha} + h_{;\alpha} h^{\alpha\beta}{}_{;\beta} - h_{\mu\beta;\alpha} h^{\mu\alpha;\beta} \\
&\quad + \bar{R} \left(\frac{1}{4} h^2 - \frac{1}{2} h_{\mu\nu} h^{\mu\nu} \right) + \left(2h^{\lambda}_{\mu} h_{\nu\lambda} - h h_{\mu\nu} \right) \bar{R}^{\mu\nu}
\end{aligned}$$

Linear term vanishes by Einstein Eq. $\bar{R}^{\mu\nu} - \frac{1}{2}\bar{g}^{\mu\nu}\bar{R} = -\frac{\kappa^2}{4} T^{\mu\nu}$

Gauge fixing:

-harmonic gauge

$$\mathcal{L}_{gf} = \sqrt{-\bar{g}} \left\{ \left(h_{\mu\nu}{}^{;\nu} - \frac{1}{2} h_{;\mu} \right) \left(h^{\mu\lambda}{}_{;\lambda} - h^{;\mu} \right) \right\}$$

$$h \equiv h^{\lambda}_{\lambda}$$

Ghost fields:

$$\mathcal{L}_{ghost} = \sqrt{-\bar{g}} \eta^{*\mu} \left\{ \eta_{\mu;\lambda}{}^{;\lambda} - \bar{R}_{\mu\nu} \eta^{\nu} \right\}$$

vector fields
anticommuting,
in loops only

Interesting note:
Feynman introduced
ghost fields in GR
before F-P in YM

Renormalization

One loop calculation: 'tHooft and Veltman

$$Z[\phi, J] = Tr ln D$$

Divergences are local:

$$\Delta\mathcal{L}_0^{(1)} = \frac{1}{8\pi^2}\frac{1}{\epsilon}\left\{\frac{1}{120}R^2 + \frac{7}{20}R_{\mu\nu}R^{\mu\nu}\right\} \quad \epsilon = 4-d$$

dim. reg. preserves symmetry

Renormalize parameters in general action:

$$c_1^{(r)} = c_1 + \frac{1}{960\pi^2\epsilon}$$
$$c_2^{(r)} = c_2 + \frac{7}{160\pi^2\epsilon}$$

Pure gravity "one loop finite" since $R_{\mu\nu}$=0

Note: Two loop calculation known in pure gravity Goroff and Sagnotti

$$\Delta\mathcal{L}^{(2)} = \frac{209\,\kappa}{2880(16\pi^2)^2}\frac{1}{\epsilon}\sqrt{-g}R^{\alpha\beta}{}_{\gamma\delta}R^{\gamma\delta}{}_{\rho\sigma}R^{\rho\sigma}{}_{\alpha\beta}$$

Order of six derivatves

Renormalization

One loop calculation: 'tHooft and Veltman

$$Z[\phi, J] = Tr ln D$$

Divergences are local:

$$\Delta\mathcal{L}_0^{(1)} = \frac{1}{8\pi^2}\frac{1}{\epsilon}\left\{\frac{1}{120}R^2 + \frac{7}{20}R_{\mu\nu}R^{\mu\nu}\right\} \qquad \epsilon = 4 - d$$

dim. reg. preserves symmetry

Renormalize parameters in general action:

$$c_1^{(r)} = c_1 + \frac{1}{960\pi^2\epsilon}$$
$$c_2^{(r)} = c_2 + \frac{7}{160\pi^2\epsilon}$$

Pure gravity "one loop finite" since $R_{\mu\nu}$=0

Note: Two loop calculation known in pure gravity — Goroff and Sagnotti

$$\Delta\mathcal{L}^{(2)} = \frac{209\,\kappa}{2880(16\pi^2)^2}\frac{1}{\epsilon}\sqrt{-g}R^{\alpha\beta}{}_{\gamma\delta}R^{\gamma\delta}{}_{\rho\sigma}R^{\rho\sigma}{}_{\alpha\beta}$$

Order of six derivatves

What are the quantum predictions?

Not the divergences

- they come from the Planck scale
- unreliable part of theory

Not the parameters

- local terms in L
- we would have to measure them

$$Amp \sim q^2 \ln(-q^2) \quad , \quad \sqrt{-q^2}$$

Low energy propagation

- not the same as terms in the Lagrangian
- most always non-analytic dependence in momentum space
- can't be Taylor expanded – can't be part of a local Lagrangian
- long distance in coordinate space

What are the quantum predictions?

Not the divergences

- they come from the Planck scale
- unreliable part of theory

Not the parameters

- local terms in L
- we would have to measure them

$$Amp \sim q^2 \ln(-q^2) \quad , \quad \sqrt{-q^2}$$

Low energy propagation

- not the same as terms in the Lagrangian
- most always non-analytic dependence in momentum space
- can't be Taylor expanded – can't be part of a local Lagrangian
- long distance in coordinate space

Corrections to Newtonian Potential

JFD 1994
JFD, Holstein,
Bjerrum-Bohr 2002
Khriplovich and Kirilin
Other references later

Here discuss scattering potential of two heavy masses.

$$\begin{aligned} \langle f|T|i\rangle &\equiv (2\pi)^4\delta^{(4)}(p-p')(\mathcal{M}(q)) \\ &= -(2\pi)\delta(E-E')\langle f|\tilde{V}(\mathbf{q})|i\rangle \end{aligned}$$

Potential found using from

$$V(\mathbf{x}) = \frac{1}{2m_1}\frac{1}{2m_2}\int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{x}}\mathcal{M}(\vec{q})$$

Classical potential has been well studied

Iwasaki
Gupta-Radford
Hiida-Okamura

Parameter free and divergence free

Recall: divergences like local Lagrangian ~R^2

Also unknown parameters in local Lagrangian ~c_1,c_2

But this generates only "short distance term"

Note: R^2 has 4 derivatives $R^2 \sim q^4$

Then:

Treating R^2 as perturbation

$$V_{R^2} \sim G^2 Mm \ \frac{1}{q^2}\, q^4\, \frac{1}{q^2} \sim \text{const.} \rightarrow G^2 Mm\delta^3(x)$$

Local lagrangian gives only short range terms

Parameter free and divergence free

Recall: divergences like local Lagrangian $\sim R^2$

Also unknown parameters in local Lagrangian $\sim c_1, c_2$

But this generates only "short distance term"

Note: R^2 has 4 derivatives $\qquad R^2 \sim q^4$

<u>Then:</u>

Treating R^2 as perturbation

$$V_{R^2} \sim G^2 Mm \ \frac{1}{q^2}\ q^4\ \frac{1}{q^2} \sim \text{const.} \rightarrow G^2 Mm \delta^3(x)$$

Local lagrangian gives only short range terms

What to expect:

General expansion:

$$V(r) = -\frac{GMm}{r}\left[1 + a\frac{G(M+m)}{rc^2} + b\frac{G\hbar}{r^2c^3}\right] + cG^2Mm\delta^3(r)$$

Classical expansion parameter

Quantum expansion parameter

Short range

Relation to momentum space:

$$\int \frac{d^3q}{(2\pi)^3}\, e^{i\mathbf{q}\cdot\mathbf{r}} \frac{1}{|\mathbf{q}|^2} = \frac{1}{4\pi r}$$

$$\int \frac{d^3q}{(2\pi)^3}\, e^{i\mathbf{q}\cdot\mathbf{r}} \frac{1}{|\mathbf{q}|} = \frac{1}{2\pi^2 r^2}$$

$$\int \frac{d^3q}{(2\pi)^3}\, e^{i\mathbf{q}\cdot\mathbf{r}} \ln(\mathbf{q}^2) = \frac{-1}{2\pi r^3}$$

Momentum space amplitudes:

$$V(q^2) = \frac{GMm}{q^2}\left[1 + a'G(M+m)\sqrt{-q^2} + b'G\hbar q^2\ln(-q^2) + c'Gq^2\right]$$

Classical

quantum

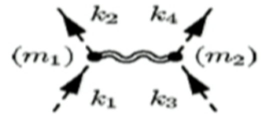
short range

Non-analytic

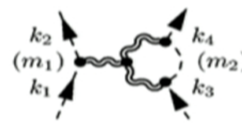
analytic

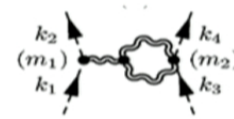
The calculation:

Lowest order:

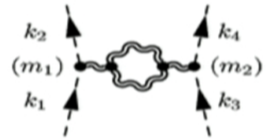


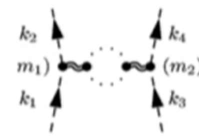
Vertex corrections:



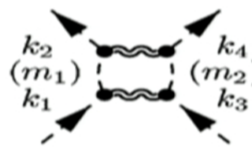


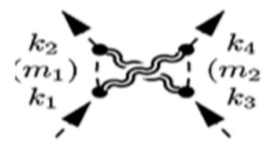
Vacuum polarization:
(Duff 1974)



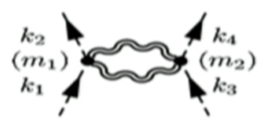


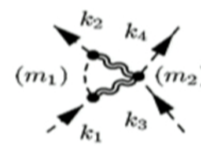
Box and crossed box





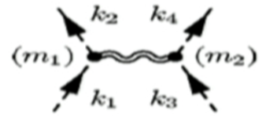
Others:



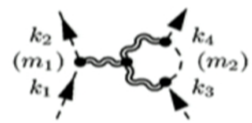


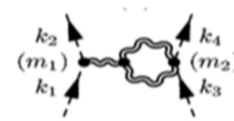
The calculation:

Lowest order:

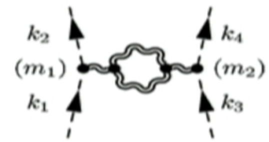


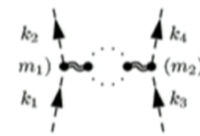
Vertex corrections:



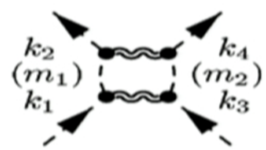


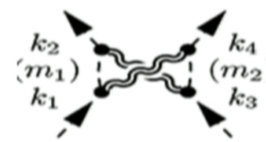
Vacuum polarization:
(Duff 1974)



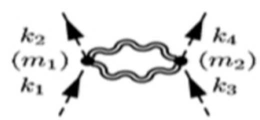


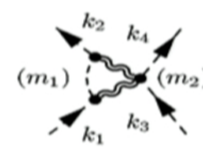
Box and crossed box





Others:





Results:

Pull out non-analytic terms:
-for example the vertex corrections:

$$M_{5(a)+5(b)}(\vec{q}) = 2G^2 m_1 m_2 \left(\frac{\pi^2(m_1+m_2)}{|\vec{q}|} + \frac{5}{3}\log \vec{q}^2 \right)$$
$$M_{5(c)+5(d)}(\vec{q}) = -\frac{52}{3} G^2 m_1 m_2 \log \vec{q}^2$$

Sum diagrams:

$$V(r) = -\frac{Gm_1m_2}{r}\left[1 + 3\frac{G(m_1+m_2)}{r} + \frac{41}{10\pi}\frac{G\hbar}{r^2}\right]$$

Gives precession
of Mercury, etc
(Iwasaki ;
Gupta + Radford)

Quantum
correction

Results:

Pull out non-analytic terms:
-for example the vertex corrections:

$$M_{5(a)+5(b)}(\vec{q}) = 2G^2 m_1 m_2 \left(\frac{\pi^2(m_1+m_2)}{|\vec{q}|} + \frac{5}{3}\log \vec{q}^2 \right)$$
$$M_{5(c)+5(d)}(\vec{q}) = -\frac{52}{3} G^2 m_1 m_2 \log \vec{q}^2$$

Sum diagrams:

$$V(r) = -\frac{G m_1 m_2}{r}\left[1 + 3\frac{G(m_1+m_2)}{r} + \frac{41}{10\pi}\frac{G\hbar}{r^2}\right]$$

Gives precession of Mercury, etc (Iwasaki ; Gupta + Radford)

Quantum correction

Comments

1) Both classical and quantum emerge from a one loop calculation!
   - classical first done by Gupta and Radford (1980)

1) Unmeasurably small correction:
   - best perturbation theory known(!)

3) Quantum loop well behaved - no conflict of GR and QM

4) Other calculations
   (Radikowski, Duff, JFD; Muzinich and Vokos; Hamber and Liu; Akhundov, Bellucci, and Sheikh ; Khriplovich and Kirilin)
   -other potentials or mistakes

5) Why not understood 30 years ago?
   - power of effective field theory reasoning

Comments

1) Both classical and quantum emerge from a one loop calculation!
   - classical first done by Gupta and Radford (1980)

1) Unmeasurably small correction:
   - best perturbation theory known(!)

3) Quantum loop well behaved - no conflict of GR and QM

4) Other calculations
   (Radikowski, Duff, JFD; Muzinich and Vokos; Hamber and Liu; Akhundov, Bellucci, and Sheikh ; Khriplovich and Kirilin)
   -other potentials or mistakes

5) Why not understood 30 years ago?
   - power of effective field theory reasoning

Results:

Pull out non-analytic terms:
-for example the vertex corrections:

$$M_{5(a)+5(b)}(\vec{q}) = 2G^2 m_1 m_2 \left(\frac{\pi^2(m_1+m_2)}{|\vec{q}|} + \frac{5}{3}\log \vec{q}^2 \right)$$

$$M_{5(c)+5(d)}(\vec{q}) = -\frac{52}{3} G^2 m_1 m_2 \log \vec{q}^2$$

Sum diagrams:

$$V(r) = -\frac{Gm_1m_2}{r}\left[1 + 3\frac{G(m_1+m_2)}{r} + \frac{41}{10\pi}\frac{G\hbar}{r^2}\right]$$

Gives precession of Mercury, etc (Iwasaki ; Gupta + Radford)

Quantum correction

Comments

1) Both classical and quantum emerge from a one loop calculation!
   - classical first done by Gupta and Radford (1980)

1) Unmeasurably small correction:
   - best perturbation theory known(!)

3) Quantum loop well behaved - no conflict of GR and QM

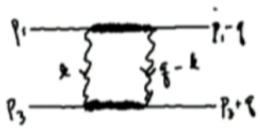
4) Other calculations
   (Radikowski, Duff, JFD; Muzinich and Vokos; Hamber and Liu; Akhundov, Bellucci, and Sheikh ; Khriplovich and Kirilin)
   -other potentials or mistakes

5) Why not understood 30 years ago?
   - power of effective field theory reasoning

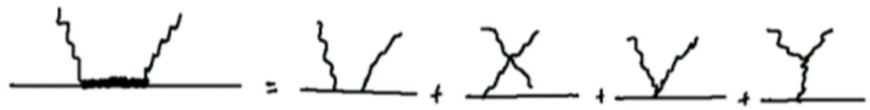
“Breaking news” – finished last night

Loops without loops - dispersive techniques:

-on-shell cut involves gravitational Compton amplitude



with



$$\rho_g(s,t) = \frac{-1}{32\pi}\int \frac{d\Omega_k}{4\pi}\,\mathcal{M}_A^{\mu\nu,\lambda\sigma}(p_1,-k,p_1-q,q-k)\mathcal{M}_B^{\alpha\beta,\gamma\delta}(p_3,-k,p_3+q,k-q)P_{\mu\nu,\alpha\beta}P_{\lambda\sigma,\gamma\delta}$$

On-shell gravity amplitudes are squares of gauge theory amplitudes – consistency check

Reproduce usual result

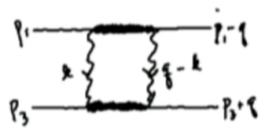
$$V(r) = -\frac{GMm}{r}\left[1 + 3\frac{G(M+m)}{rc^2} + \frac{41}{10\pi}\frac{G\hbar}{r^2c^3}\right]$$

VERY strong check on loop calculation

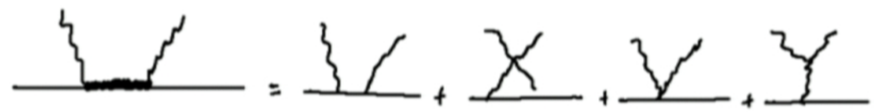
Working on full modern helicity formalism calculation (with E. Bjerrum-Bohr and P. Vanhove)

"Breaking news" – finished last night
Loops without loops - dispersive techniques:

-on-shell cut involves gravitational Compton amplitude



with



$$\rho_g(s,t) = \frac{-1}{32\pi}\int \frac{d\Omega_k}{4\pi}\, \mathcal{M}_A^{\mu\nu,\lambda\sigma}(p_1,-k,p_1-q,q-k)\mathcal{M}_B^{\alpha\beta,\gamma\delta}(p_3,-k,p_3+q,k-q)\,P_{\mu\nu,\alpha\beta}P_{\lambda\sigma,\gamma\delta}$$

On-shell gravity amplitudes are squares of gauge theory amplitudes – consistency check

Reproduce usual result

$$V(r) = -\frac{GMm}{r}\left[1 + 3\frac{G(M+m)}{rc^2} + \frac{41}{10\pi}\frac{G\hbar}{r^2c^3}\right]$$

VERY strong check on loop calculation
Working on full modern helicity formalism calculation (with E. Bjerrum-Bohr and P. Vanhove)

Graviton –graviton scattering

Fundamental quantum gravity process

Lowest order amplitude:

$$\mathcal{A}^{tree}(++;++) = \frac{i}{4}\frac{\kappa^2 s^3}{tu}$$

Cooke;
Behrends Gastmans
Grisaru et al

One loop:

Incredibly difficult using field theory

Dunbar and Norridge –string based methods! (just tool, not full string theory)

$$\begin{aligned}
\mathcal{A}^{1-loop}(++;--) &= -i\,\frac{\kappa^4}{30720\pi^2}\left(s^2+t^2+u^2\right)\\
\mathcal{A}^{1-loop}(++;+-) &= -\frac{1}{3}\mathcal{A}^{1-loop}(++;--)\\
\mathcal{A}^{1-loop}(++;++) &= \frac{\kappa^2}{4(4\pi)^{2-\epsilon}}\frac{\Gamma^2(1-\epsilon)\Gamma(1+\epsilon)}{\Gamma(1-2\epsilon)}\mathcal{A}^{tree}(++;++)\times(s\,t\,u) \qquad (3)\\
&\times\left[\frac{2}{\epsilon}\left(\frac{\ln(-u)}{st}+\frac{\ln(-t)}{su}+\frac{\ln(-s)}{tu}\right)+\frac{1}{s^2}f\left(\frac{-t}{s},\frac{-u}{s}\right)\right.\\
&\left.+2\left(\frac{\ln(-u)\ln(-s)}{su}+\frac{\ln(-t)\ln(-s)}{tu}+\frac{\ln(-t)\ln(-s)}{ts}\right)\right]
\end{aligned}$$

where

$$\begin{aligned}
f\left(\frac{-t}{s},\frac{-u}{s}\right) &= \frac{(t+2u)(2t+u)\left(2t^4+2t^3u-t^2u^2+2tu^3+2u^4\right)}{s^6}\left(\ln^2\frac{t}{u}+\pi^2\right)\\
&+\frac{(t-u)\left(341t^4+1609t^3u+2566t^2u^2+1609tu^3+341u^4\right)}{30s^5}\ln\frac{t}{u}\\
&+\frac{1922t^4+9143t^3u+14622t^2u^2+9143tu^3+1922u^4}{180s^4}, \qquad (4)
\end{aligned}$$

Graviton –graviton scattering

Fundamental quantum gravity process

Lowest order amplitude:

$$\mathcal{A}^{tree}(++;++) = \frac{i}{4}\frac{\kappa^2 s^3}{tu}$$

Cooke;
Behrends Gastmans
Grisaru et al

One loop:

Incredibly difficult using field theory

Dunbar and Norridge –string based methods! (just tool, not full string theory)

$$\begin{aligned}
\mathcal{A}^{1-loop}(++;--) &= -i\,\frac{\kappa^4}{30720\pi^2}\left(s^2+t^2+u^2\right) \\
\mathcal{A}^{1-loop}(++;+-) &= -\frac{1}{3}\mathcal{A}^{1-loop}(++;--) \\
\mathcal{A}^{1-loop}(++;++) &= \frac{\kappa^2}{4(4\pi)^{2-\epsilon}}\frac{\Gamma^2(1-\epsilon)\Gamma(1+\epsilon)}{\Gamma(1-2\epsilon)}\mathcal{A}^{tree}(++;++)\times(s\,t\,u) \\
&\quad\times\left[\frac{2}{\epsilon}\left(\frac{\ln(-u)}{st}+\frac{\ln(-t)}{su}+\frac{\ln(-s)}{tu}\right)+\frac{1}{s^2}f\left(\frac{-t}{s},\frac{-u}{s}\right)\right. \\
&\quad\left.+2\left(\frac{\ln(-u)\ln(-s)}{su}+\frac{\ln(-t)\ln(-s)}{tu}+\frac{\ln(-t)\ln(-s)}{ts}\right)\right]
\end{aligned} \tag{3}$$

where

$$\begin{aligned}
f\left(\frac{-t}{s},\frac{-u}{s}\right) &= \frac{(t+2u)(2t+u)\left(2t^4+2t^3u-t^2u^2+2tu^3+2u^4\right)}{s^6}\left(\ln^2\frac{t}{u}+\pi^2\right) \\
&\quad+\frac{(t-u)\left(341t^4+1609t^3u+2566t^2u^2+1609tu^3+341u^4\right)}{30s^5}\ln\frac{t}{u} \\
&\quad+\frac{1922t^4+9143t^3u+14622t^2u^2+9143tu^3+1922u^4}{180s^4},
\end{aligned} \tag{4}$$

Graviton –graviton scattering

Fundamental quantum gravity process

Lowest order amplitude:

$$\mathcal{A}^{tree}(++;++) = \frac{i}{4}\frac{\kappa^2 s^3}{tu}$$

Cooke;
Behrends Gastmans
Grisaru et al

One loop:

Incredibly difficult using field theory

Dunbar and Norridge –string based methods! (just tool, not full string theory)

$$\begin{aligned}
\mathcal{A}^{1-loop}(++;--) &= -i\,\frac{\kappa^4}{30720\pi^2}\left(s^2+t^2+u^2\right)\\
\mathcal{A}^{1-loop}(++;+-) &= -\frac{1}{3}\mathcal{A}^{1-loop}(++;--)\\
\mathcal{A}^{1-loop}(++;++) &= \frac{\kappa^2}{4(4\pi)^{2-\epsilon}}\frac{\Gamma^2(1-\epsilon)\Gamma(1+\epsilon)}{\Gamma(1-2\epsilon)}\mathcal{A}^{tree}(++;++)\times(s\,t\,u) \qquad (3)\\
&\times\left[\frac{2}{\epsilon}\left(\frac{\ln(-u)}{st}+\frac{\ln(-t)}{su}+\frac{\ln(-s)}{tu}\right)+\frac{1}{s^2}f\left(\frac{-t}{s},\frac{-u}{s}\right)\right.\\
&\left.+2\left(\frac{\ln(-u)\ln(-s)}{su}+\frac{\ln(-t)\ln(-s)}{tu}+\frac{\ln(-t)\ln(-s)}{ts}\right)\right]
\end{aligned}$$

where

$$\begin{aligned}
f\left(\frac{-t}{s},\frac{-u}{s}\right) &= \frac{(t+2u)(2t+u)\left(2t^4+2t^3u-t^2u^2+2tu^3+2u^4\right)}{s^6}\left(\ln^2\frac{t}{u}+\pi^2\right)\\
&+\frac{(t-u)\left(341t^4+1609t^3u+2566t^2u^2+1609tu^3+341u^4\right)}{30s^5}\ln\frac{t}{u}\\
&+\frac{1922t^4+9143t^3u+14622t^2u^2+9143tu^3+1922u^4}{180s^4}, \qquad (4)
\end{aligned}$$

Infrared safe:

JFD + Torma

The $1/\epsilon$ is from infrared
 -soft graviton radiation
 -made finite in usual way
 $1/\epsilon$ -> ln(1/resolution) (gives scale to loops)
 -cross section finite

$$\left(\frac{d\sigma}{d\Omega}\right)_{tree} + \left(\frac{d\sigma}{d\Omega}\right)_{rad.} + \left(\frac{d\sigma}{d\Omega}\right)_{nonrad.} = \tag{29}$$
$$= \frac{\kappa^4 s^5}{2048\pi^2 t^2 u^2}\left\{1 + \frac{\kappa^2 s}{16\pi^2}\left[\ln\frac{-t}{s}\ln\frac{-u}{s} + \frac{tu}{2s^2} f\left(\frac{-t}{s}, \frac{-u}{s}\right)\right.\right.$$
$$\left.\left. - \left(\frac{t}{s}\ln\frac{-t}{s} + \frac{u}{s}\ln\frac{-u}{s}\right)\left(3\ln(2\pi^2) + \gamma + \ln\frac{s}{\Lambda^2} + \frac{\sum_{ij}\eta_i\eta_j\mathcal{F}^{(1)}(\gamma_{ij})}{\sum_{ij}\eta_i\eta_j\mathcal{F}^{(0)}(\gamma_{ij})}\right)\right]\right\}.$$

* finite

Beautiful result:
 -low energy theorem of quantum gravity

Integrated curvature qualitatively explains IR issues:

- curvature builds up between horizon and spatial infinity
- singularities due to evolution of any curvature to long enough distance

<u>But how do we treat this in EFT?</u>

Maybe singularities can be treated as gravitational sources

- excise a region around the singularity
- include a coupling to the boundary
- analogy Skyrmions in ChPTh

But distant horizons?

- perhaps non-perturbatively small??

Apparent conflict between EFT and gravity contribution to running couplings:

Gravitational corrections modify **different** operator
- at higher order in energy expansion
- R^2 rather than R

But certainly **logically possible**
– renormalize at higher energy scale E

$$\begin{aligned} \text{Amp}_i &= a_i g^2 + b_i g^2 \kappa^2 q^2 \\ &= a_i g^2\left(1 + \frac{b_i}{a_i}\kappa^2 E^2\right) + b_i g^2 \kappa^2 (q^2 - E^2) \\ &= a_i g^2(E) + b_i g^2(E)\kappa^2(q^2 - E^2), \end{aligned}$$

R+W

Result.—At this point the Gaussian integrals over the quantum fields in Eq. (10) are formally defined, but the resulting functional determinants contain ultraviolet divergences. We subtract them at a reference energy E_0. We find the one-loop effective action at energy scale E is

$$S_{\text{eff}}[g, a] \approx -\frac{1}{4}\int d^4x\left[\frac{1}{g^2} + \frac{\kappa^2}{g^2}\frac{3}{(4\pi)^2}(E^2 - E_0^2) + \frac{b_0}{(4\pi)^2}\ln\frac{E^2}{E_0^2}\right]F^{\mathbf{a}}_{ab}F^{\mathbf{a}ab}, \quad (18)$$

where b_0 depends on the gauge and matter content independently of whether gravitation is included in the calculation. Taking E differentially close to E_0, we read off the one-loop β function

$$\beta(g, E) = -\frac{b_0}{(4\pi)^2}g^3 - 3\frac{\kappa^2}{(4\pi)^2}gE^2. \quad (19)$$

Conditions for success:

1) Definition is **useful**
   - like $\alpha_s(q^2)$ sums up a set of radiative corrections
2) Definition is **universal**
   - like $\alpha_s(q^2)$ comes from the universal renormalization of α_s

We will find two big obstructions

1) The crossing problem – kinematic

q^2 does not have a definite sign - renormalize at $q^2 = +E^2$ or $q^2 = -E^2$.

- occurs differently in different processes – with different signs
- since really higher order operator

2) The universality problem

- b_i/a_i highly process dependent
- no definition works for other process
- since really not renormalization of original operator

$$\begin{aligned}\mathrm{Amp}_i &= a_i g^2 + b_i g^2 \kappa^2 q^2 \\ &= a_i g^2\left(1 + \frac{b_i}{a_i}\kappa^2 E^2\right) + b_i g^2 \kappa^2 (q^2 - E^2) \\ &= a_i g^2(E) + b_i g^2(E)\kappa^2(q^2 - E^2),\end{aligned}$$

My claim: No definition is useful and universal (in perturbative regime)

Conditions for success:

1) Definition is **useful**

- like $\alpha_s(q^2)$ sums up a set of radiative corrections

2) Definition is **universal**

- like $\alpha_s(q^2)$ comes from the universal renormalization of α_s

We will find two big obstructions

1) The crossing problem – kinematic

q^2 does not have a definite sign - renormalize at $q^2 = +E^2$ or $q^2 = -E^2$.

- occurs differently in different processes – with different signs
- since really higher order operator

$$\begin{aligned}\mathrm{Amp}_i &= a_i g^2 + b_i g^2 \kappa^2 q^2 \\ &= a_i g^2\left(1 + \frac{b_i}{a_i}\kappa^2 E^2\right) + b_i g^2 \kappa^2 (q^2 - E^2) \\ &= a_i g^2(E) + b_i g^2(E)\kappa^2 (q^2 - E^2),\end{aligned}$$

2) **The universality problem**

- b_i/a_i highly process dependent
- no definition works for other process
- since really not renormalization of original operator

My claim: No definition is useful and universal (in perturbative regime)

Standard EFT practice and Renormalization Group

$$U = exp[i\frac{\tau \cdot \phi}{F_\pi}]$$

Closest analogy is chiral perturbation theory:
- also carries dimensionful coupling and similar energy expansion

$$\mathcal{L} = F^2 Tr(\partial_\mu U \partial^\mu U^\dagger) + \ell_1 [Tr(\partial_\mu U \partial^\mu U^\dagger)]^2 + \ell_2 Tr(\partial_\mu U \partial_\nu U^\dagger)\, Tr(\partial^\mu U \partial^\nu U^\dagger) + ...$$

- renormalization and general behavior is analogous to GR

$$\Delta\mathcal{L} = \frac{1}{192\pi^2(d-4)} \left[[Tr(D_\mu U D^\mu U^\dagger)]^2 + 2Tr(D_\mu U D_\nu U^\dagger)\, Tr(D^\mu U D^\nu U^\dagger) \right]$$

RGE: (Weinberg 1979, Colangelo, Buchler, Bijnens et al, M. Polyakov et al)

- Physics is independent of scale μ in dim. reg
- One loop – $1/\epsilon$ goes into renormalizaton of ℓ_i
  - comes along with specified ln μ and ln q^2 dependence
- Even better at two loops
  - two loops (hard) gives q^4/ϵ^2 terms – correlated with $q^4 \ln^2 q^2/\mu^2$
  - cancelled by one loop (easy) calculation using ℓ_i
  - RGE fixes leading $(q^2 \ln q^2)^n$ behavior

Standard EFT practice and Renormalization Group

$$U = exp[i\frac{\tau \cdot \phi}{F_\pi}]$$

Closest analogy is chiral perturbation theory:

- also carries dimensionful coupling and similar energy expansion

$$\mathcal{L} = F^2 Tr(\partial_\mu U \partial^\mu U^\dagger) + \ell_1 [Tr(\partial_\mu U \partial^\mu U^\dagger)]^2 + \ell_2 Tr(\partial_\mu U \partial_\nu U^\dagger)\, Tr(\partial^\mu U \partial^\nu U^\dagger) + ...$$

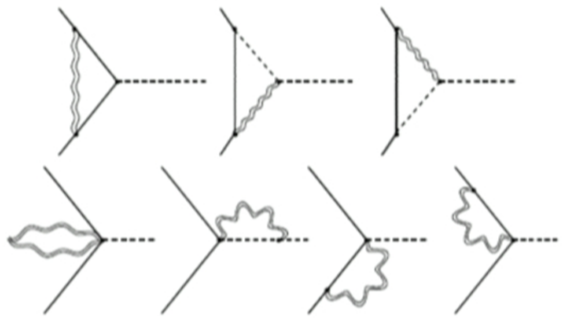
- renormalization and general behavior is analogous to GR

$$\Delta\mathcal{L} = \frac{1}{192\pi^2(d-4)}\left[[Tr(D_\mu U D^\mu U^\dagger)]^2 + 2Tr(D_\mu U D_\nu U^\dagger)\; Tr(D^\mu U D^\nu U^\dagger)\right]$$

RGE: (Weinberg 1979, Colangelo, Buchler, Bijnens et al, M. Polyakov et al)

- Physics is independent of scale μ in dim. reg
- One loop – $1/\epsilon$ goes into renormalizaton of ℓ_i
  - comes along with specified $\ln \mu$ and $\ln q^2$ dependence
- Even better at two loops
  - two loops (hard) gives q^4/ϵ^2 terms – correlated with $q^4 \ln^2 q^2/\mu^2$
  - cancelled by one loop (easy) calculation using ℓ_i
  - RGE fixes leading $(q^2 \ln q^2)^n$ behavior

Direct calculation



Vertex (fermions on shell) found to be:

$$\mathcal{V} = \bar{u}\left[e\gamma^\mu + a(q^2)e\kappa^2 q^2 \gamma^\mu\right]u$$

with

$$a(q^2) = a_0\left[\frac{1}{\epsilon} + \frac{1}{2}\ln 4\pi - \frac{\gamma}{2} - \frac{1}{2}\ln(-q^2/\mu^2)\right]$$

Physical process:

FIG. 4: Tree diagram for the on-shell scattering processes involving fermion.The filled circle denotes the set of vertex renormalization diagrams.

Overall matrix element:

$$\mathcal{M} = \bar{u}\left[e^2\gamma^\mu + e^2 a(q^2)\kappa^2 q^2\gamma^\mu\right] u \, \frac{1}{q^2} \, \bar{u}\gamma_\mu u + h.c. + c_2\bar{u}\gamma^\mu u\bar{u}\gamma_\mu u$$
$$= \bar{u}\gamma^\mu u \, \bar{u}\gamma_\mu u \left[\frac{e^2}{q^2} + \left(c_2^r(\mu) - e^2 a_0\kappa^2 \ln(-q^2/\mu^2)\right)\right]$$

Think of the Lamb shift

Describes the two reactions:

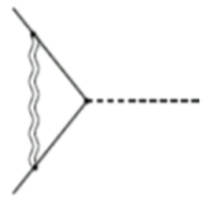
$$q^2 > 0 \quad \text{for} f + \bar{f} \to f + \bar{f}$$
$$< 0 \quad \text{for} f + f \to f + f$$

Renormalization of higher order operator:

$$c_2^r = c_2 - a_0\left[\frac{1}{\epsilon} + \frac{1}{2}\ln 4\pi - \frac{\gamma}{2}\right]$$

Lamb shift analogy:

- Corrections to vertex diagram gives q^2 dependent terms



$$\mathcal{V}^\mu = \bar{u}\left[e\gamma^\mu\left(1 + a\frac{q^2}{m_e^2}\right)\right]u$$

$$\mathcal{M} = \bar{u}\left[e^2\gamma^\mu\left(1 + a\frac{q^2}{m_e^2}\right)\right]u\ \frac{1}{q^2}\ \bar{u}\gamma_\mu u$$

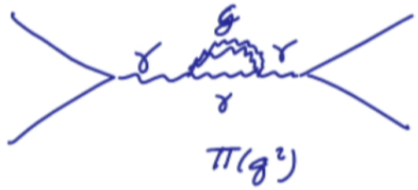
Leads to a contact interaction:

$$\bar{\psi}\gamma_\mu\psi\bar{\psi}\gamma^\mu\psi$$

Influences S states only

- Not counted as a running coupling

Similar in the modification of photon properties



Photon propagator correction:

$$\Pi = c\kappa^2 q^4$$

Like

$$\mathcal{L}_{h.o.} = -k\partial_\mu F^{\mu\nu}\partial^\lambda F_{\lambda\nu}$$

Again looks like contact interaction:

$$\mathcal{M} = \bar{u}\gamma^\mu u \, \frac{1}{q^2} \, \left[e^2 + e^2 c\kappa^2 q^2\right] \, \frac{1}{q^2} \, \bar{u}\gamma_\mu u + c_2 \bar{u}\gamma^\mu u \bar{u}\gamma_\mu u$$

Can this be packaged as a running coupling?

Propose:

$$e^2(M^2) = e^2\left[1 + b_0\kappa^2 M^2\right]$$

Is the amplitude equal to?

$$\begin{aligned}\mathcal{M} \quad ?=? \quad & \bar{u}\gamma^\mu u\ \bar{u}\gamma_\mu u\left[\frac{e^2(M^2)}{q^2} + (c_2')\right)\right] \\ ?=? \quad & \bar{u}\gamma^\mu u\ \bar{u}\gamma_\mu u\left[\frac{e^2}{q^2} + \frac{e^2 2b_0\kappa^2 M^2}{q^2} + (c_2')\right)\right]\end{aligned}$$

$$\begin{aligned} q^2 &> 0 \ \text{for} f + \bar{f} \to f + \bar{f} \\ &< 0 \ \text{for} f + f \to f + f \end{aligned}$$

Recall

$$\begin{aligned}\mathcal{M} &= \bar{u}\left[e^2\gamma^\mu + e^2 a(q^2)\kappa^2 q^2\gamma^\mu\right] u\ \frac{1}{q^2}\ \bar{u}\gamma_\mu u + h.c. + c_2\bar{u}\gamma^\mu u\bar{u}\gamma_\mu u \\ &= \bar{u}\gamma^\mu u\ \bar{u}\gamma_\mu u\left[\frac{e^2}{q^2} + \left(c_2^r(\mu) - e^2 a_0\kappa^2\ln(-q^2/\mu^2)\right)\right]\end{aligned}$$

You can make the definition work for either process but not for both

- No universal definition

Can this be packaged as a running coupling?

Propose:

$$e^2(M^2) = e^2\left[1 + b_0\kappa^2 M^2\right]$$

Is the amplitude equal to?

$$\mathcal{M} \;\; ?=? \;\; \bar{u}\gamma^\mu u\, \bar{u}\gamma_\mu u\left[\frac{e^2(M^2)}{q^2} + (c_2')\right]$$

$$?=? \;\; \bar{u}\gamma^\mu u\, \bar{u}\gamma_\mu u\left[\frac{e^2}{q^2} + \frac{e^2 2b_0\kappa^2 M^2}{q^2} + (c_2')\right]$$

$$\begin{aligned} q^2 &> 0 \;\; \text{for} f + \bar{f} \to f + \bar{f} \\ &< 0 \;\; \text{for} f + f \to f + f \end{aligned}$$

Recall

$$\begin{aligned} \mathcal{M} &= \bar{u}\left[e^2\gamma^\mu + e^2 a(q^2)\kappa^2 q^2\gamma^\mu\right] u\,\frac{1}{q^2}\,\bar{u}\gamma_\mu u + h.c. + c_2\bar{u}\gamma^\mu u\bar{u}\gamma_\mu u \\ &= \bar{u}\gamma^\mu u\,\bar{u}\gamma_\mu u\left[\frac{e^2}{q^2} + \left(c_2^r(\mu) - e^2 a_0\kappa^2\ln(-q^2/\mu^2)\right)\right] \end{aligned}$$

You can make the definition work for either process but not for both

- No universal definition

Other forms of non-universality:

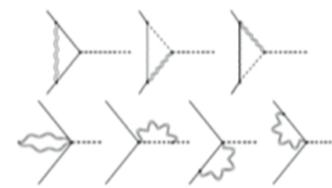
Other processes have other divergences and other operators:

Lowest order:

$$\mathcal{L}_{l.o} = g\bar{\psi}\gamma_\mu\psi A^\mu$$

Different higher order operator is relevant

$$\mathcal{L}_{h.o.} = c_3 A^\mu \bar{\psi}\gamma_\mu \partial^2 \psi$$



Calculation of the vertex corrections:

$$\mathcal{V} = \bar{u}\left[e\gamma^\mu + b(p^2)e\kappa^2 p^2 \gamma^\mu\right] u$$

Different value for the correction (verified in Yukawa case)

$$b(q^2) \neq a(q^2)$$

- Different correction to matrix element

$$\mathcal{M} \;=\; e^2\epsilon_\mu\epsilon_\nu\left(\bar{u}\gamma^\mu\left[1 + b((q+p_1)^2)\kappa^2(q+p_1)^2\right]\frac{1}{\not{q}+\not{p}_1}\gamma^\nu u + h.c. + c_3\bar{u}\gamma^\mu(\not{q}+\not{p}_1)\gamma^\nu\gamma_\mu u\right)$$

Other forms of non-universality:

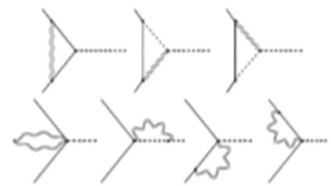
Other processes have other divergences and other operators:

Lowest order:

$$\mathcal{L}_{l.o} = g\bar{\psi}\gamma_\mu \psi A^\mu$$

Different higher order operator is relevant

$$\mathcal{L}_{h.o.} = c_3 A^\mu \bar{\psi}\gamma_\mu \partial^2 \psi$$



Calculation of the vertex corrections:

$$\mathcal{V} = \bar{u}\left[e\gamma^\mu + b(p^2)e\kappa^2 p^2 \gamma^\mu\right] u$$

Different value for the correction (verified in Yukawa case)

$$b(q^2) \neq a(q^2)$$

- Different correction to matrix element

$$\mathcal{M} = e^2\epsilon_\mu\epsilon_\nu \left(\bar{u}\gamma^\mu \left[1 + b((q+p_1)^2)\kappa^2(q+p_1)^2\right] \frac{1}{\not{q}+\not{p}_1}\gamma^\nu u + h.c. + c_3\bar{u}\gamma^\mu(\not{q}+\not{p}_1)\gamma^\nu\gamma_\mu u\right)$$

Standard EFT practice and Renormalization Group

Closest analogy is chiral perturbation theory: $U = exp[i\frac{\tau \cdot \phi}{F_\pi}]$

- also carries dimensionful coupling and similar energy expansion

$$\mathcal{L} = F^2 Tr(\partial_\mu U \partial^\mu U^\dagger) + \ell_1 [Tr(\partial_\mu U \partial^\mu U^\dagger)]^2 + \ell_2 Tr(\partial_\mu U \partial_\nu U^\dagger)\, Tr(\partial^\mu U \partial^\nu U^\dagger) + \ldots\ldots$$

- renormalization and general behavior is analogous to GR

$$\Delta\mathcal{L} = \frac{1}{192\pi^2(d-4)} \left[[Tr(D_\mu U D^\mu U^\dagger)]^2 + 2Tr(D_\mu U D_\nu U^\dagger)\, Tr(D^\mu U D^\nu U^\dagger) \right]$$

RGE: (Weinberg 1979, Colangelo, Buchler, Bijnens et al, M. Polyakov et al)

- Physics is independent of scale μ in dim. reg
- One loop – $1/\epsilon$ goes into renormalizaton of ℓ_i
  - comes along with specified $\ln \mu$ and $\ln q^2$ dependence
- Even better at two loops
  - two loops (hard) gives q^4/ϵ^2 terms – correlated with $q^4 \ln^2 q^2/\mu^2$
  - cancelled by one loop (easy) calculation using ℓ_i
  - RGE fixes leading $(q^2 \ln q^2)^n$ behavior

This has been explored in depth:

TABLE I: Table of $I = 0$ LL coefficients for the $4D$ σ-model, $\omega_{nl}^{I=0} \cdot (N-1)^{-1}$

$n \setminus l$	0	2	4
1	1		
2	$\frac{N}{2} - \frac{1}{9}$	$\frac{5}{18}$	
3	$\frac{N^2}{4} - \frac{61N}{144} + \frac{59}{144}$	$-\frac{13N}{144} + \frac{13}{48}$	
4	$\frac{N^3}{8} - \frac{631N^2}{2700} + \frac{46279N}{194400} - \frac{13309}{194400}$	$\frac{173N^2}{2160} - \frac{4313N}{38880} + \frac{5333}{38880}$	$\frac{N^2}{200} - \frac{49N}{5400} + \frac{8}{675}$
5	$\frac{N^4}{16} - \frac{136N^3}{675} + \frac{2498743N^2}{7776000} - \frac{3083771N}{11664000} + \frac{619889}{4665600}$	$-\frac{1417N^3}{40320} + \frac{481367N^2}{3628800} - \frac{727373N}{4082400} + \frac{1071107}{6531840}$	$-\frac{N^3}{280} + \frac{9787N^2}{756000} - \frac{449681N}{27216000} + \frac{81007}{5443200}$

For our purposes:

- Lowest order operator does not run
- Higher order operator gets renormalized
- With renormalization comes $\ln \mu$ dependence
- Can exploit for leading high power x leading log
- Tracks higher order log dependence ($q^2 \ln q^2$)
- Multiple higher order operators – different processes have different effects

Consider gravity corrections to gauge interactions:

(Anber, El Houssienny, JFD)

-we have done this in great detail for Yukawa
- I will be schematic for gauge interactions in order to highlight key points

Lowest order operator:

$$\mathcal{L}_{l.o} = g\bar{\psi}\gamma_\mu\psi A^\mu \qquad \mathcal{L}_{l.o.} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

Higher order operator

$$\mathcal{L}_{h.o} = c_2\bar{\psi}\gamma_\mu\psi\partial^2 A^\mu \qquad \mathcal{L}_{h.o.} = -k\partial_\mu F^{\mu\nu}\partial^\lambda F_{\lambda\nu}$$

Equations of motion

$$\partial^2 A^\mu = J^\mu$$

Equivalent contact operator:

$$\mathcal{L}_{h.o} = c_2\bar{\psi}\gamma_\mu\psi\bar{\psi}\gamma^\mu\psi = c_2 J_\mu J^\mu$$

Test cases for running of G:

Anber, JFD

1) Graviton propagator – vacuum polarization

2) Graviton-graviton scattering

3) Massless scalars scattering gravitationally
   a) identical scalars – permutation symmetry
   b) non-identical scalars – channel dependence

4) Massive gravitational potential

1) Vacuum polarization in the graviton propagator:

Including vacuum polarization and renormalizing

$$\Delta \mathcal{L} = \frac{\sqrt{g}}{16\pi^2 \epsilon}\left[\frac{1}{120}R^2 + \frac{7}{20}R_{\alpha\beta}R^{\alpha\beta}\right]$$

we find the one-loop corrected propagator to be

$$L^{\mu\nu}(q) = \eta^{\mu\nu} - q^\mu q^\nu / q^2.$$

$$i\mathcal{D}'^{\alpha\beta,\mu\nu} = i\mathcal{D}^{\alpha\beta,\mu\nu} + i\mathcal{D}^{\alpha\beta,\gamma\delta} i\Pi_{\gamma\delta,\rho\tau} i\mathcal{D}^{\rho\tau,\mu\nu}$$
$$= \frac{i}{2q^2}(1 + 2B(q^2))[L^{\alpha\mu}L^{\beta\nu} + L^{\alpha\nu}L^{\beta\mu} - L^{\alpha\beta}L^{\mu\nu}] - i\frac{A(q^2)}{4}L^{\alpha\beta}L^{\mu\nu}.$$

$$A(q^2) = -\frac{1}{30\pi}G\ln\left(\frac{-q^2}{\mu_1^2}\right) - \frac{7}{10\pi}G\ln\left(\frac{-q^2}{\mu_2^2}\right),$$
$$B(q^2) = \frac{7}{40\pi}Gq^2\ln\left(\frac{-q^2}{\mu_2^2}\right).$$

Not a unique definition:

a) 00,00 component (non-rel. masses)

$$G(q^2) = G\left[1 + \frac{1}{60\pi}Gq^2\ln\left(\frac{-q^2}{\mu_2^2}\right) + \frac{7}{10\pi}Gq^2\ln\left(\frac{-q^2}{\mu_1^2}\right)\right]$$

b) Proportional to original propagator

$$G(q^2) = G(1 + 2B(q^2)) = G\left(1 + \frac{7}{20\pi}Gq^2\ln\left(\frac{-q^2}{\mu_2^2}\right)\right).$$

Has **crossing problem**: which sign of q^2 should be used?

If Euclidean-ized, $q^2 \rightarrow -q_E^2$ the effective strength **increases**

Also know that μ_i drop out in pure gravity

1) Vacuum polarization in the graviton propagator:

Including vacuum polarization and renormalizing

$$\Delta\mathcal{L} = \frac{\sqrt{g}}{16\pi^2\epsilon}\left[\frac{1}{120}R^2 + \frac{7}{20}R_{\alpha\beta}R^{\alpha\beta}\right]$$

we find the one-loop corrected propagator to be

$$\begin{aligned} i\mathcal{D}'^{\alpha\beta,\mu\nu} &= i\mathcal{D}^{\alpha\beta,\mu\nu} + i\mathcal{D}^{\alpha\beta,\gamma\delta} i\Pi_{\gamma\delta,\rho\tau} i\mathcal{D}^{\rho\tau,\mu\nu} \\ &= \frac{i}{2q^2}(1 + 2B(q^2))[L^{\alpha\mu}L^{\beta\nu} + L^{\alpha\nu}L^{\beta\mu} \\ &\quad - L^{\alpha\beta}L^{\mu\nu}] - i\frac{A(q^2)}{4}L^{\alpha\beta}L^{\mu\nu}. \end{aligned}$$

$$L^{\mu\nu}(q) = \eta^{\mu\nu} - q^\mu q^\nu/q^2.$$

$$A(q^2) = -\frac{1}{30\pi}G\ln\left(\frac{-q^2}{\mu_1^2}\right) - \frac{7}{10\pi}G\ln\left(\frac{-q^2}{\mu_2^2}\right).$$

$$B(q^2) = \frac{7}{40\pi}Gq^2\ln\left(\frac{-q^2}{\mu_2^2}\right).$$

Not a unique definition:

a) 00,00 component (non-rel. masses)

$$G(q^2) = G\left[1 + \frac{1}{60\pi}Gq^2\ln\left(\frac{-q^2}{\mu_2^2}\right) + \frac{7}{10\pi}Gq^2\ln\left(\frac{-q^2}{\mu_1^2}\right)\right].$$

b) Proportional to original propagator

$$G(q^2) = G(1 + 2B(q^2)) = G\left(1 + \frac{7}{20\pi}Gq^2\ln\left(\frac{-q^2}{\mu_2^2}\right)\right).$$

Has **crossing problem**: which sign of q^2 should be used?

If Euclidean-ized, $q^2 \to -q_E^2$ the effective strength **increases**

Also know that μ_i drop out in pure gravity

2) Lets look at graviton –graviton scattering

Lowest order amplitude:

$$\mathcal{A}^{tree}(++;++) = \frac{i}{4}\frac{\kappa^2 s^3}{tu}$$

One loop: Dunbar and Norridge

$$\begin{aligned}
\mathcal{A}^{1-loop}(++;--) &= -i\,\frac{\kappa^4}{30720\pi^2}\left(s^2+t^2+u^2\right) \\
\mathcal{A}^{1-loop}(++;+-) &= -\frac{1}{3}\mathcal{A}^{1-loop}(++;--) \\
\mathcal{A}^{1-loop}(++;++) &= \frac{\kappa^2}{4(4\pi)^{2-\epsilon}}\,\frac{\Gamma^2(1-\epsilon)\Gamma(1+\epsilon)}{\Gamma(1-2\epsilon)}\,\mathcal{A}^{tree}(++;++)\times(s\,t\,u) \qquad (3) \\
&\times\left[\frac{2}{\epsilon}\left(\frac{\ln(-u)}{st}+\frac{\ln(-t)}{su}+\frac{\ln(-s)}{tu}\right)+\frac{1}{s^2}f\left(\frac{-t}{s},\frac{-u}{s}\right)\right. \\
&\left.+2\left(\frac{\ln(-u)\ln(-s)}{su}+\frac{\ln(-t)\ln(-s)}{tu}+\frac{\ln(-t)\ln(-s)}{ts}\right)\right]
\end{aligned}$$

where

$$\begin{aligned}
f\left(\frac{-t}{s},\frac{-u}{s}\right) &= \frac{(t+2u)(2t+u)\left(2t^4+2t^3u-t^2u^2+2tu^3+2u^4\right)}{s^6}\left(\ln^2\frac{t}{u}+\pi^2\right) \\
&+\frac{(t-u)\left(341t^4+1609t^3u+2566t^2u^2+1609tu^3+341u^4\right)}{30s^5}\ln\frac{t}{u} \\
&+\frac{1922t^4+9143t^3u+14622t^2u^2+9143tu^3+1922u^4}{180s^4}, \qquad (4)
\end{aligned}$$

4) Two different types of massless particles

$\underline{A + B \to A + B}$

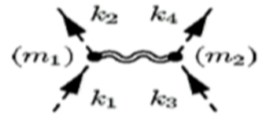
$$\mathcal{M}_{\text{tree}} = \frac{i\kappa^2 su}{4t},$$

$$\mathcal{M}_h = i\frac{\kappa^4}{(4\pi)^2}\Big[\frac{1}{8}\Big(\frac{s^3}{t}\ln(-s)\ln(-t) + \frac{u^3}{t}\ln(-u)\ln(-t)\Big) - \frac{1}{16t}(s^3 + u^3 + tsu)\ln(-t) + \frac{1}{16}(s^2\ln^2(-s) + u^2\ln^2(-u))$$
$$+ \frac{us}{16t}(s\ln^2(-s) + t\ln^2(-t) + u\ln^2(-u)) + \frac{1}{240}(71us - 11t^2)\ln(-t) - \frac{1}{16}(s^2\ln(-s) + u^2\ln(-u))\Big], \qquad (4$$

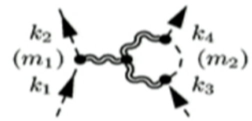
This reaction is not crossing symmetric

Components of log in matter coupling

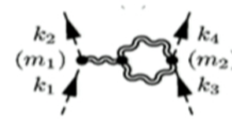
Lowest order:



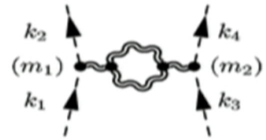
Vertex corrections:



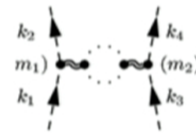
-42/3



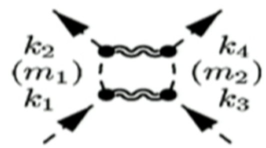
Vacuum polarization:
(Duff 1974)



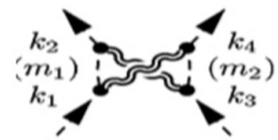
+43/15



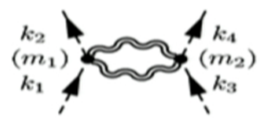
Box and crossed box



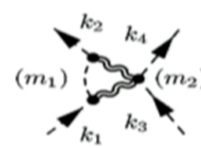
+94/3



Others:



+44-56



Summary – gravitational running in EFT

1) No useful, universal running coupling seen in perturbative calculations

2) Kinematic ambiguity resurfaces

3) Tasks for asymptotic safety program
   - continuing back to Lorentzian spacetimes
   - addition of matter couplings
   - universality of effects
   - matching to perturbative results

Summary – gravitational running in EFT

1) No useful, universal running coupling seen in perturbative calculations

2) Kinematic ambiguity resurfaces

3) Tasks for asymptotic safety program
   - continuing back to Lorentzian spacetimes
   - addition of matter couplings
   - universality of effects
   - matching to perturbative results

Summary – gravitational running in EFT

1) No useful, universal running coupling seen in perturbative calculations

2) Kinematic ambiguity resurfaces

3) Tasks for asymptotic safety program
   - continuing back to Lorentzian spacetimes
   - addition of matter couplings
   - universality of effects
   - matching to perturbative results

Summary – gravitational running in EFT

1) No useful, universal running coupling seen in perturbative calculations

2) Kinematic ambiguity resurfaces

3) Tasks for asymptotic safety program
   - continuing back to Lorentzian spacetimes
   - addition of matter couplings
   - universality of effects
   - matching to perturbative results

5) Gravity matter coupling – non-relativistic masses:

Recall: $$V(r) = -\frac{Gm_1 m_2}{r}\left[1 + 3\frac{G(m_1+m_2)}{r} + \frac{41}{10\pi}\frac{G\hbar}{r^2}\right]$$

Including all diagrams:

$$\text{“}G(r)\text{”} = G\left[1 + \frac{41}{10\pi}\frac{G}{r}\right]$$

Excluding box plus crossed box:

$$\text{“}G(r)\text{”} = G\left[1 - \frac{347}{60\pi}\frac{G}{r}\right]$$

Vac. pol. + vertex corrections:

$$\text{“}G(r)\text{”} = G\left[1 - \frac{167}{15\pi}\frac{G}{r}\right]$$

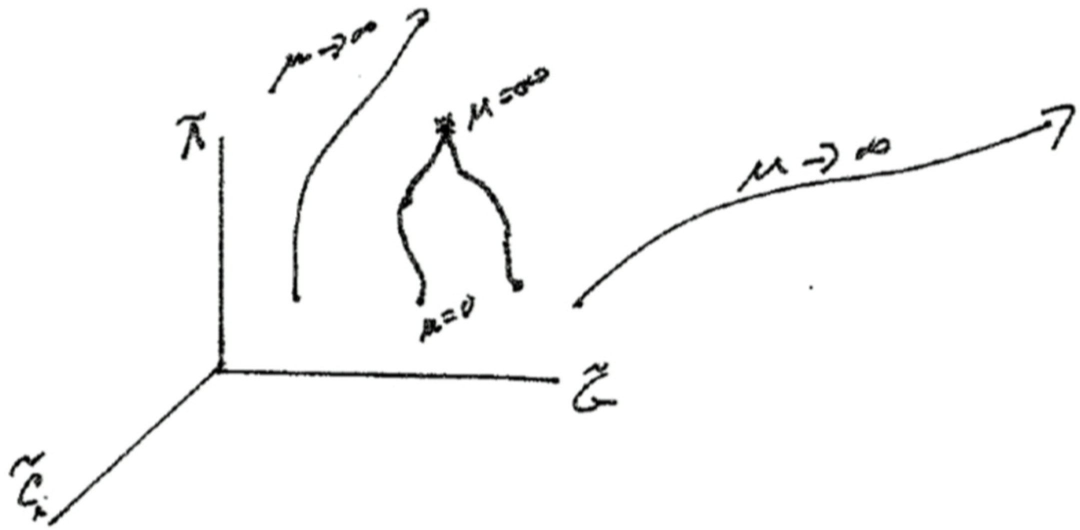
- **Only vacuum polarization”**

$$\text{“}G(r)\text{”} = G\left[1 + \frac{43}{15\pi}\frac{G}{r}\right]$$

Asymptotic Safety in practice/ to one loop

Thanks for discussions with Percacci, Cordello, Reuter but they are not to be held responsible

Defining a Euclidean theory – scaling to $\mu = 0, \mu = \infty$:
Integrate out modes above μ



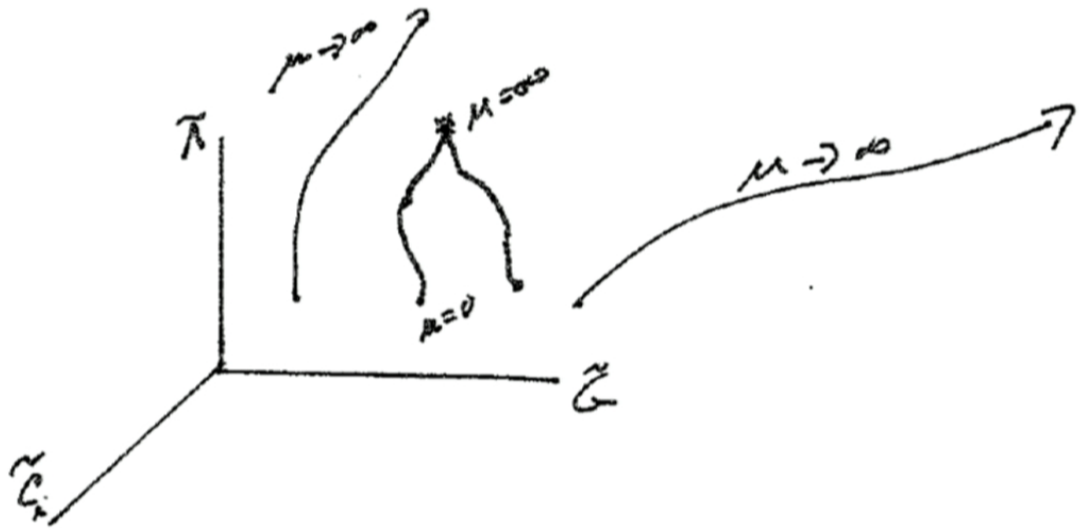
$$\tilde{G} = G\mu^2$$

UV fixed point = **finite** bare theory

Asymptotic Safety in practice/ to one loop

Thanks for discussions with Percacci, Cordello, Reuter but they are not to be held responsible

Defining a Euclidean theory – scaling to $\mu = 0, \mu = \infty$:
Integrate out modes above μ



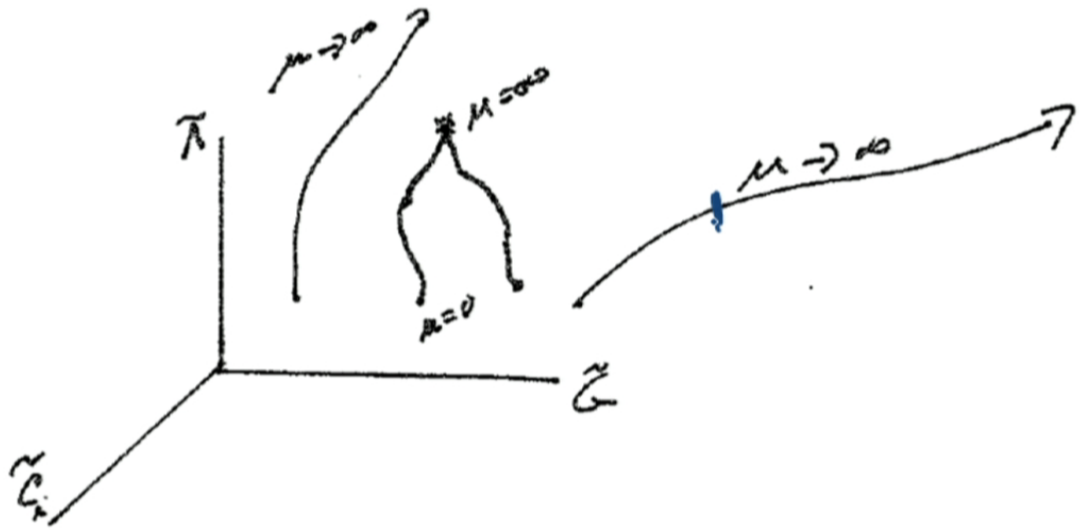
$\tilde{G} = G\mu^2$

UV fixed point = **finite** bare theory

Asymptotic Safety in practice/ to one loop

Thanks for discussions with Percacci, Cordello, Reuter but they are not to be held responsible

Defining a Euclidean theory – scaling to $\mu = 0, \mu = \infty$:
Integrate out modes above μ



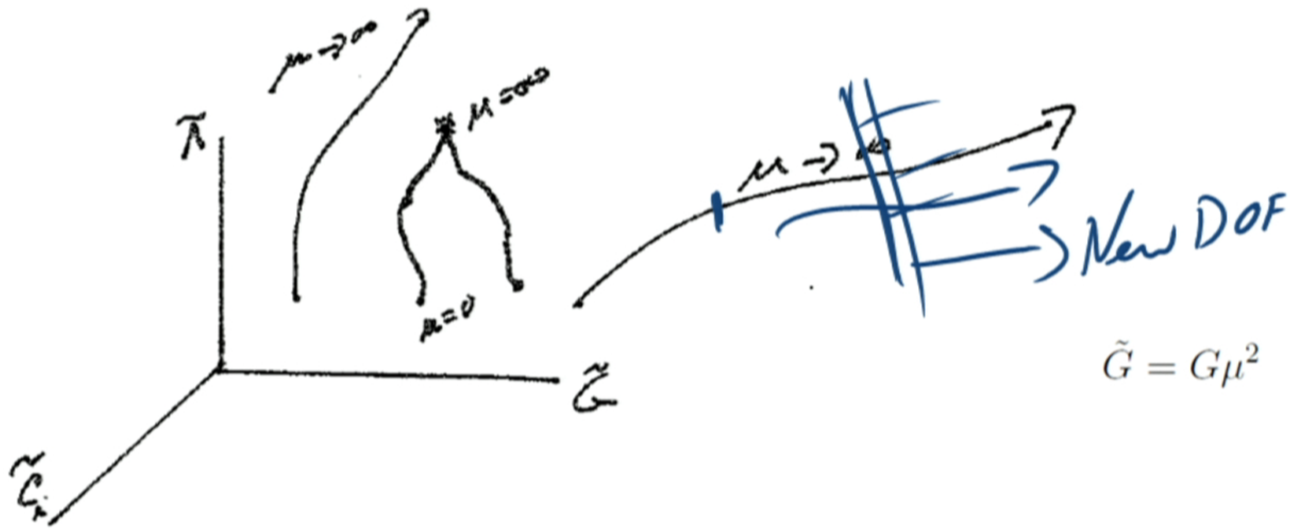
$\tilde{G} = G\mu^2$

UV fixed point = **finite** bare theory

Asymptotic Safety in practice/ to one loop

Thanks for discussions with Percacci, Cordello, Reuter but they are not to be held responsible

Defining a Euclidean theory – scaling to $\mu = 0, \mu = \infty$:
Integrate out modes above μ



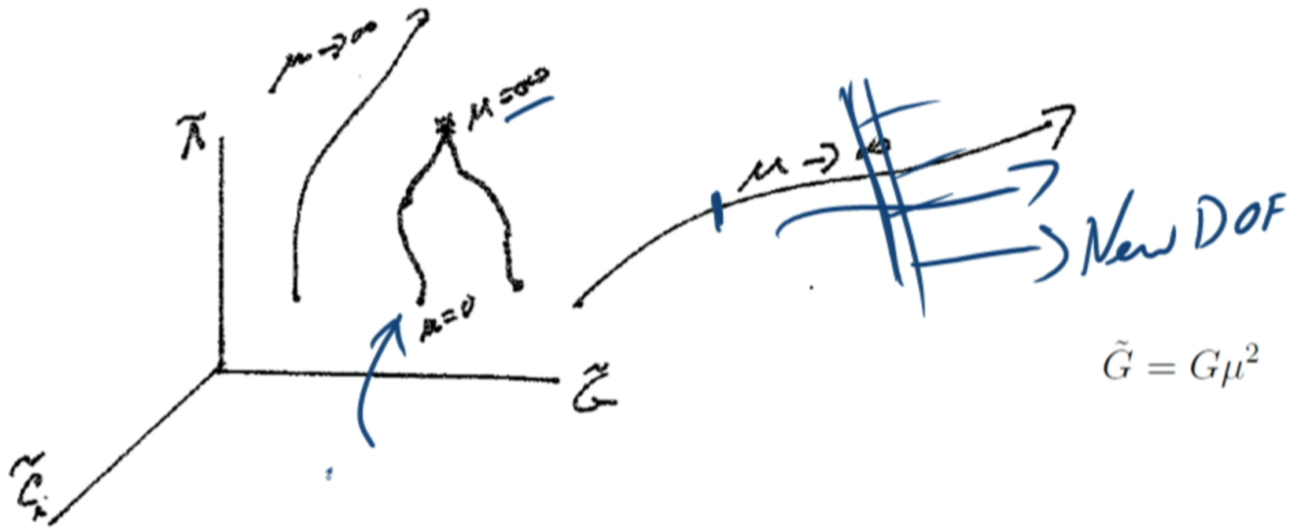
$\tilde{G} = G\mu^2$

UV fixed point = **finite** bare theory

Asymptotic Safety in practice/ to one loop

Thanks for discussions with Percacci, Cordello, Reuter but they are not to be held responsible

Defining a Euclidean theory – scaling to $\mu = 0, \mu = \infty$:
Integrate out modes above μ



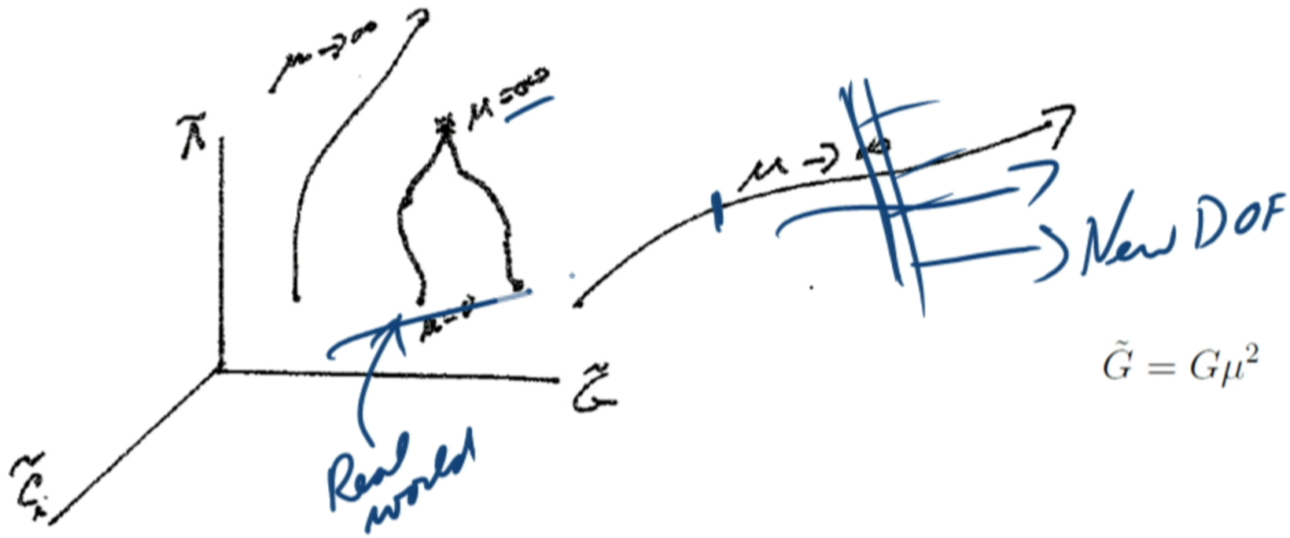
$\tilde{G} = G\mu^2$

UV fixed point = **finite** bare theory

Asymptotic Safety in practice/ to one loop

Thanks for discussions with Percacci, Cordello, Reuter but they are not to be held responsible

Defining a Euclidean theory – scaling to $\mu = 0, \mu = \infty$:
Integrate out modes above μ



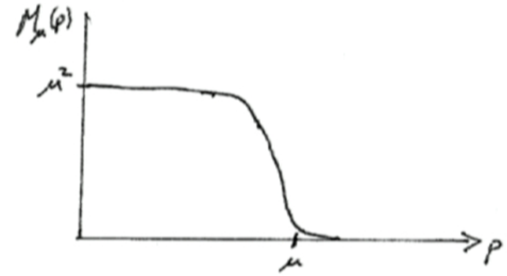
$\tilde{G} = G\mu^2$

UV fixed point = **finite** bare theory

In practice:

Expand around background field $$g_{\alpha\beta} = \bar{g}_{\alpha\beta} + h_{\alpha\beta}$$

Define function that suppresses all low momentum modes, integrate out high modes
- bi-linear – mass-like



$$\Delta\mathcal{L} = h(p) M_{\{\mu\}}(p) h(p)$$

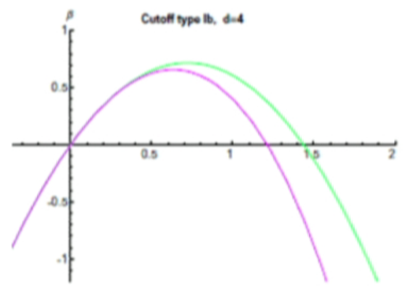
Calculate effective action variation with scale

$$\mu \frac{\partial}{\partial \mu} \Gamma_{\{\mu\}} = \frac{1}{2} Tr \left[\frac{1}{D + M_{\{\mu\}}} \right] \mu \frac{\partial}{\partial \mu} M_{\{\mu\}}$$

Truncate action to manageable set of terms
- simplest is just the Einstein action
- locate UV fixed pt.

$$\mu \frac{\partial \tilde{G}}{\partial \mu} = 2\tilde{G} - \frac{167}{15\pi} \tilde{G}^2$$

$$\tilde{G} = G\mu^2$$



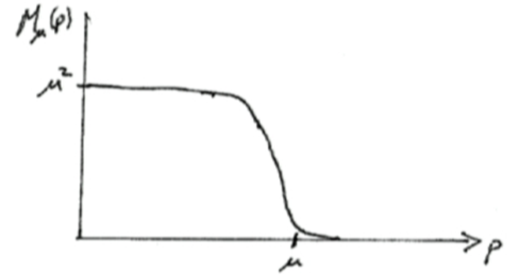
Running G $$G(\mu) = \frac{G_N}{1 + \frac{167}{30\pi} G_N \mu^2}$$

In practice:

Expand around background field $g_{\alpha\beta} = \bar{g}_{\alpha\beta} + h_{\alpha\beta}$

Define function that suppresses all low momentum modes, integrate out high modes
- bi-linear – mass-like

$$\Delta\mathcal{L} = h(p) M_{\{\mu\}}(p) h(p)$$

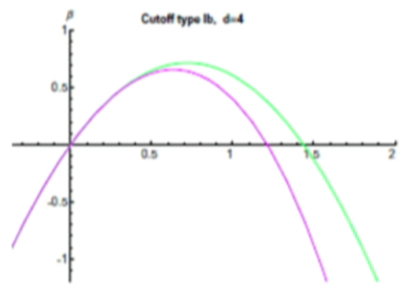


Calculate effective action variation with scale

$$\mu \frac{\partial}{\partial \mu} \Gamma_{\{\mu\}} = \frac{1}{2} Tr \left[\frac{1}{D + M_{\{\mu\}}} \right] \mu \frac{\partial}{\partial \mu} M_{\{\mu\}}$$

Truncate action to manageable set of terms
- simplest is just the Einstein action
- locate UV fixed pt.

$$\mu \frac{\partial \tilde{G}}{\partial \mu} = 2\tilde{G} - \frac{167}{15\pi} \tilde{G}^2 \qquad \tilde{G} = G\mu^2$$

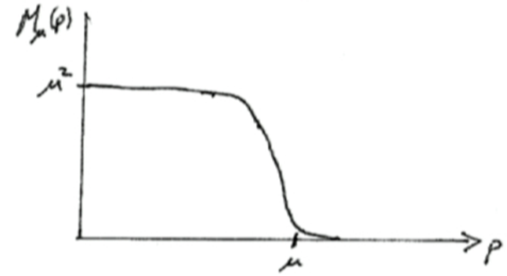


Running G $\quad G(\mu) = \dfrac{G_N}{1 + \frac{167}{30\pi} G_N \mu^2}$

In practice:

Expand around background field $g_{\alpha\beta} = \bar{g}_{\alpha\beta} + h_{\alpha\beta}$

Define function that suppresses all low momentum modes, integrate out high modes
- bi-linear – mass-like



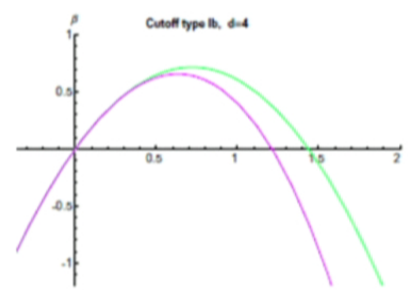
$$\Delta\mathcal{L} = h(p) M_{\{\mu\}}(p) h(p)$$

Calculate effective action variation with scale

$$\mu \frac{\partial}{\partial\mu} \Gamma_{\{\mu\}} = \frac{1}{2} Tr \left[\frac{1}{D + M_{\{\mu\}}} \right] \mu \frac{\partial}{\partial\mu} M_{\{\mu\}}$$

Truncate action to manageable set of terms
- simplest is just the Einstein action
- locate UV fixed pt.

$$\mu \frac{\partial \tilde{G}}{\partial \mu} = 2\tilde{G} - \frac{167}{15\pi}\tilde{G}^2 \qquad \tilde{G} = G\mu^2$$



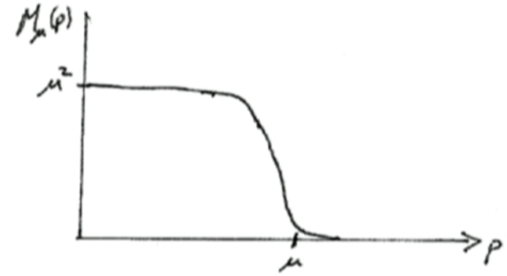
Running G $\quad G(\mu) = \dfrac{G_N}{1 + \frac{167}{30\pi} G_N \mu^2}$

In practice:

Expand around background field $$g_{\alpha\beta} = \bar{g}_{\alpha\beta} + h_{\alpha\beta}$$

Define function that suppresses all low momentum modes, integrate out high modes
- bi-linear – mass-like

$$\Delta\mathcal{L} = h(p) M_{\{\mu\}}(p) h(p)$$



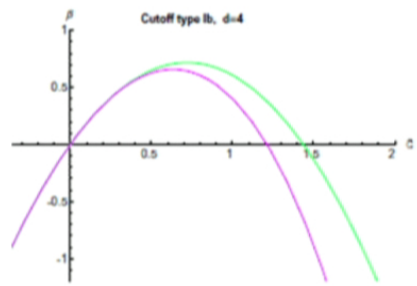
Calculate effective action variation with scale

$$\mu \frac{\partial}{\partial\mu} \Gamma_{\{\mu\}} = \frac{1}{2}\, Tr \left[\frac{1}{D + M_{\{\mu\}}} \right] \mu \frac{\partial}{\partial\mu} M_{\{\mu\}}$$

Truncate action to manageable set of terms
- simplest is just the Einstein action
- locate UV fixed pt.

$$\mu \frac{\partial \tilde{G}}{\partial \mu} = 2\tilde{G} - \frac{167}{15\pi}\tilde{G}^2$$

$$\tilde{G} = G\mu^2$$

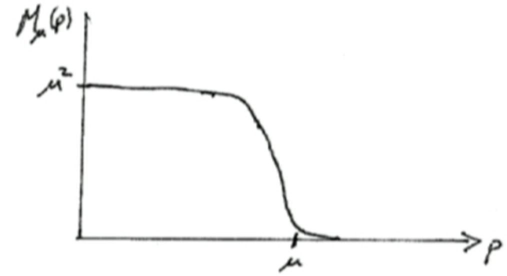


Running G $$G(\mu) = \frac{G_N}{1 + \frac{167}{30\pi} G_N \mu^2}$$

In practice:

Expand around background field $$g_{\alpha\beta} = \bar{g}_{\alpha\beta} + h_{\alpha\beta}$$

Define function that suppresses all low momentum modes, integrate out high modes
- bi-linear – mass-like



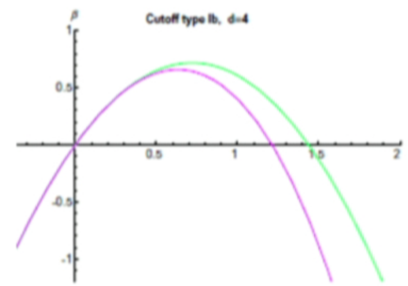
$$\Delta\mathcal{L} = h(p)M_{\{\mu\}}(p)h(p)$$

Calculate effective action variation with scale

$$\mu\frac{\partial}{\partial\mu}\,\Gamma_{\{\mu\}} = \frac{1}{2}\,Tr\left[\frac{1}{D+M_{\{\mu\}}}\right]\,\mu\frac{\partial}{\partial\mu}M_{\{\mu\}}$$

Truncate action to manageable set of terms
- simplest is just the Einstein action
- locate UV fixed pt.

$$\mu\frac{\partial\tilde{G}}{\partial\mu} = 2\tilde{G} - \frac{167}{15\pi}\tilde{G}^2 \qquad \tilde{G} = G\mu^2$$



Running G $$G(\mu) = \frac{G_N}{1+\frac{167}{30\pi}G_N\mu^2}$$

Comments:

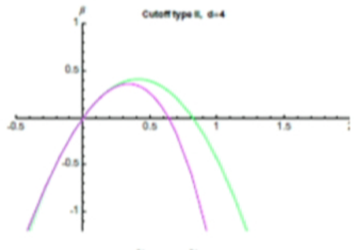
Many acknowledged challenges:

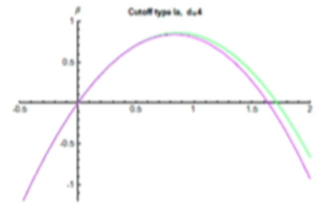
Infinite number of couplings

Different truncations

Dependence on cutoff function

Different Newton couplings





Much work to address challenges

Comments:

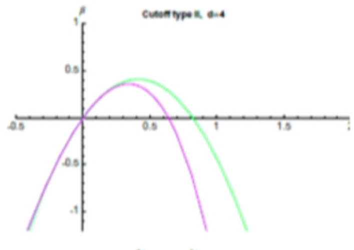
Many acknowledged challenges:

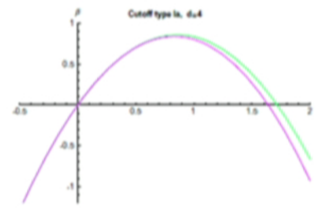
Infinite number of couplings

Different truncations

Dependence on cutoff function

Different Newton couplings





Much work to address challenges

Comments:

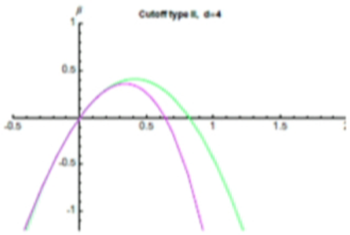
Many acknowledged challenges:

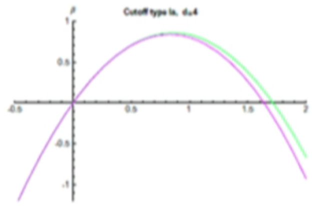
Infinite number of couplings

Different truncations

Dependence on cutoff function

Different Newton couplings



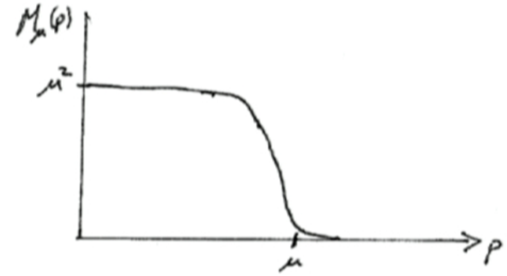


Much work to address challenges

In practice:

Expand around background field $g_{\alpha\beta} = \bar{g}_{\alpha\beta} + h_{\alpha\beta}$

Define function that suppresses all low momentum modes, integrate out high modes
- bi-linear – mass-like



$$\Delta \mathcal{L} = h(p) M_{\{\mu\}}(p) h(p)$$

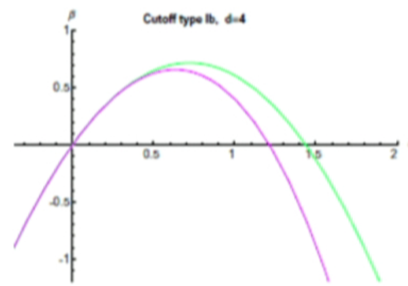
Calculate effective action variation with scale

$$\mu \frac{\partial}{\partial \mu} \Gamma_{\{\mu\}} = \frac{1}{2} Tr \left[\frac{1}{D + M_{\{\mu\}}} \right] \mu \frac{\partial}{\partial \mu} M_{\{\mu\}}$$

Truncate action to manageable set of terms
- simplest is just the Einstein action
- locate UV fixed pt.

$$\mu \frac{\partial \tilde{G}}{\partial \mu} = 2\tilde{G} - \frac{167}{15\pi} \tilde{G}^2$$

$$\tilde{G} = G\mu^2$$



Running G $G(\mu) = \dfrac{G_N}{1 + \frac{167}{30\pi} G_N \mu^2}$

Matching to Effective Field Theory

- one-loop running down to $\mu = 0$

Running of effective action:

$$\partial_t \, \Gamma_{\{\mu\}} = -\frac{1}{2} \, \partial_t \, Tr \ln \left[D(\bar{\phi}) + M_{\{\mu\}} \right]$$

Expand w.r.t background field to get individual matrix elements

- evaluated with running coupling

$$\partial_t \, \mathcal{M} = -\frac{1}{2} G^2(\mu^2) \partial_t \, Tr \left[V \frac{1}{D + M_{\{\mu\}}} V \frac{1}{D + M_{\{\mu\}}} \right]$$

Integrate from fixed point down to $\mu = 0$

$$\mathcal{M}_{\mu=0} = \mathcal{M}_{\infty} + \frac{1}{2} \int_0^{\infty} \frac{d\mu}{\mu} G^2(\mu) \; \mu \frac{\partial}{\partial \mu} \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2 + M_{\{\mu\}}(q^2)} \frac{1}{(q+p)^2 + M_{\{\mu\}}((q+p)^2)} f(q,p)$$

Matching to Effective Field Theory

- one-loop running down to $\mu = 0$

Running of effective action:

$$\partial_t \, \Gamma_{\{\mu\}} = -\frac{1}{2} \, \partial_t \, Tr \ln \left[D(\bar{\phi}) + M_{\{\mu\}} \right]$$

Expand w.r.t background field to get individual matrix elements

- evaluated with running coupling

$$\partial_t \, \mathcal{M} = -\frac{1}{2} G^2(\mu^2) \partial_t \, Tr \left[V \frac{1}{D + M_{\{\mu\}}} V \frac{1}{D + M_{\{\mu\}}} \right]$$

Integrate from fixed point down to $\mu = 0$

$$\mathcal{M}_{\mu=0} = \mathcal{M}_\infty + \frac{1}{2} \int_0^\infty \frac{d\mu}{\mu} G^2(\mu) \; \mu \frac{\partial}{\partial \mu} \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2 + M_{\{\mu\}}(q^2)} \frac{1}{(q+p)^2 + M_{\{\mu\}}((q+p)^2)} f(q,p)$$

Matching to Effective Field Theory

- one-loop running down to $\mu = 0$

Running of effective action:

$$\partial_t \, \Gamma_{\{\mu\}} = -\frac{1}{2} \, \partial_t \, Tr \ln \left[D(\bar{\phi}) + M_{\{\mu\}} \right]$$

Expand w.r.t background field to get individual matrix elements

- evaluated with running coupling

$$\partial_t \, \mathcal{M} = -\frac{1}{2} G^2(\mu^2) \partial_t \, Tr \left[V \frac{1}{D + M_{\{\mu\}}} V \frac{1}{D + M_{\{\mu\}}} \right]$$

Integrate from fixed point down to $\mu = 0$

$$\mathcal{M}_{\mu=0} = \mathcal{M}_{\infty} + \frac{1}{2} \int_0^{\infty} \frac{d\mu}{\mu} G^2(\mu) \; \mu \frac{\partial}{\partial \mu} \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2 + M_{\{\mu\}}(q^2)} \frac{1}{(q+p)^2 + M_{\{\mu\}}((q+p)^2)} f(q,p)$$

Matching to Effective Field Theory

- one-loop running down to $\mu = 0$

Running of effective action:

$$\partial_t \, \Gamma_{\{\mu\}} = -\frac{1}{2} \, \partial_t \, Tr \ln \left[D(\bar{\phi}) + M_{\{\mu\}} \right]$$

Expand w.r.t background field to get individual matrix elements

- evaluated with running coupling

$$\partial_t \, \mathcal{M} = -\frac{1}{2} G^2(\mu^2) \partial_t \, Tr \left[V \frac{1}{D + M_{\{\mu\}}} V \frac{1}{D + M_{\{\mu\}}} \right]$$

Integrate from fixed point down to $\mu = 0$

$$\mathcal{M}_{\mu=0} = \mathcal{M}_\infty + \frac{1}{2} \int_0^\infty \frac{d\mu}{\mu} G^2(\mu) \; \mu \frac{\partial}{\partial \mu} \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2 + M_{\{\mu\}}(q^2)} \frac{1}{(q+p)^2 + M_{\{\mu\}}((q+p)^2)} f(q,p)$$

Matching to Effective Field Theory

- one-loop running down to $\mu = 0$

Running of effective action:

$$\partial_t \, \Gamma_{\{\mu\}} = -\frac{1}{2} \, \partial_t \, Tr \ln \left[D(\bar{\phi}) + M_{\{\mu\}} \right]$$

Expand w.r.t background field to get individual matrix elements

- evaluated with running coupling

$$\partial_t \, \mathcal{M} = -\frac{1}{2} G^2(\mu^2) \partial_t \, Tr \left[V \frac{1}{D + M_{\{\mu\}}} V \frac{1}{D + M_{\{\mu\}}} \right]$$

Integrate from fixed point down to $\mu = 0$

$$\mathcal{M}_{\mu=0} = \mathcal{M}_{\infty} + \frac{1}{2} \int_0^{\infty} \frac{d\mu}{\mu} G^2(\mu) \; \mu \frac{\partial}{\partial \mu} \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2 + M_{\{\mu\}}(q^2)} \frac{1}{(q+p)^2 + M_{\{\mu\}}((q+p)^2)} f(q,p)$$

Matching to Effective Field Theory

- one-loop running down to $\mu = 0$

Running of effective action:

$$\partial_t \, \Gamma_{\{\mu\}} = -\frac{1}{2} \, \partial_t \, Tr \ln \left[D(\bar{\phi}) + M_{\{\mu\}} \right]$$

Expand w.r.t background field to get individual matrix elements

- evaluated with running coupling

$$\partial_t \, \mathcal{M} = -\frac{1}{2} G^2(\mu^2) \partial_t \, Tr \left[V \frac{1}{D + M_{\{\mu\}}} V \frac{1}{D + M_{\{\mu\}}} \right]$$

Integrate from fixed point down to $\mu = 0$

$$\mathcal{M}_{\mu=0} = \mathcal{M}_{\infty} + \frac{1}{2} \int_0^{\infty} \frac{d\mu}{\mu} G^2(\mu) \; \mu \frac{\partial}{\partial \mu} \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2 + M_{\{\mu\}}(q^2)} \frac{1}{(q+p)^2 + M_{\{\mu\}}((q+p)^2)} f(q,p)$$

Reproduces structure of effective field theory

$$\mathcal{M}_{\mu=0} = \mathcal{M}_{\infty} + \frac{1}{2}\int_0^{\infty} \frac{d\mu}{\mu} G^2(\mu)\ \mu\frac{\partial}{\partial\mu}\int \frac{d^4q}{(2\pi)^4}\frac{1}{q^2+M_{\{\mu\}}(q^2)}\frac{1}{(q+p)^2+M_{\{\mu\}}((q+p)^2)} f(q,p)$$

External momentum can be continued to Lorentzian space

Long distance parts of loops reproduced
-at low scales $G(\mu) \sim G_N = \text{const.}$

$$\int_0 \frac{d\mu}{\mu}\ \mu\frac{\partial}{\partial\mu}[...] \sim \frac{1}{2}G_N^2 \int \frac{d^4q}{(2\pi)^4}\frac{1}{q^2}\frac{1}{(q+p)^2} f(q,p)$$

High energy dependence on $G(\mu)$ appears local at low energy
- match to coefficients of EFT

More formally shown by 'non-local heat kernel expansion' — Satz, Cordello, Mazzitelli; Cordello, Zanusso

Regularized version of EFT

Reproduces structure of effective field theory

$$\mathcal{M}_{\mu=0} = \mathcal{M}_{\infty} + \frac{1}{2}\int_0^{\infty} \frac{d\mu}{\mu} G^2(\mu)\ \mu\frac{\partial}{\partial\mu}\int \frac{d^4q}{(2\pi)^4}\frac{1}{q^2 + M_{\{\mu\}}(q^2)}\frac{1}{(q+p)^2 + M_{\{\mu\}}((q+p)^2)} f(q,p)$$

External momentum can be continued to Lorentzian space

Long distance parts of loops reproduced
-at low scales $G(\mu) \sim G_N = \text{const.}$

$$\int_0 \frac{d\mu}{\mu}\ \mu\frac{\partial}{\partial\mu}[...] \sim \frac{1}{2}G_N^2 \int \frac{d^4q}{(2\pi)^4}\frac{1}{q^2}\frac{1}{(q+p)^2} f(q,p)$$

High energy dependence on $G(\mu)$ appears local at low energy
- match to coefficients of EFT

More formally shown by 'non-local heat kernel expansion'

Satz, Cordello, Mazzitelli
Cordello, Zanusso

Regularized version of EFT

Reproduces structure of effective field theory

$$\mathcal{M}_{\mu=0} = \mathcal{M}_\infty + \frac{1}{2}\int_0^\infty \frac{d\mu}{\mu} G^2(\mu)\ \mu\frac{\partial}{\partial\mu}\int \frac{d^4q}{(2\pi)^4}\frac{1}{q^2+M_{\{\mu\}}(q^2)}\frac{1}{(q+p)^2+M_{\{\mu\}}((q+p)^2)}f(q,p)$$

External momentum can be continued to Lorentzian space

Long distance parts of loops reproduced

-at low scales $G(\mu) \sim G_N = \text{const.}$

$$\int_0 \frac{d\mu}{\mu}\ \mu\frac{\partial}{\partial\mu}[...] \sim \frac{1}{2}G_N^2\int \frac{d^4q}{(2\pi)^4}\frac{1}{q^2}\frac{1}{(q+p)^2}f(q,p)$$

High energy dependence on $G(\mu)$ appears local at low energy

- match to coefficients of EFT

More formally shown by 'non-local heat kernel expansion'

Satz, Cordello, Mazzitelli
Cordello, Zanusso

Regularized version of EFT

Reproduces structure of effective field theory

$$\mathcal{M}_{\mu=0} = \mathcal{M}_\infty + \frac{1}{2}\int_0^\infty \frac{d\mu}{\mu} G^2(\mu)\ \mu\frac{\partial}{\partial\mu}\int \frac{d^4q}{(2\pi)^4}\frac{1}{q^2+M_{\{\mu\}}(q^2)}\frac{1}{(q+p)^2+M_{\{\mu\}}((q+p)^2)} f(q,p)$$

External momentum can be continued to Lorentzian space

Long distance parts of loops reproduced
-at low scales $G(\mu) \sim G_N = \text{const.}$

$$\int_0 \frac{d\mu}{\mu}\ \mu\frac{\partial}{\partial\mu}[...] \sim \frac{1}{2}G_N^2 \int \frac{d^4q}{(2\pi)^4}\frac{1}{q^2}\frac{1}{(q+p)^2} f(q,p)$$

High energy dependence on $G(\mu)$ appears local at low energy
- match to coefficients of EFT

More formally shown by 'non-local heat kernel expansion'

Satz, Cordello, Mazzitelli
Cordello, Zanusso

Regularized version of EFT

But not clear that Weinberg's original goal is satisfied

$$\sigma(\Lambda, G, c_i, p_a) = \frac{1}{\mu^2}\sigma(\frac{\Lambda}{\mu^4}, G\mu^2, \tilde{c}_i, \frac{p_a}{\mu})$$

All processes well behaved

This is **not** the form of the Lorentzian result in the perturbative region

In principle, end up with EFT with infinite numbers of local terms in Lagrangian, but with related coefficients and with regularized loops.

Will this make infinite number of reactions well-behaved in all kinematic regions?

Reproduces structure of effective field theory

$$\mathcal{M}_{\mu=0} = \mathcal{M}_{\infty} + \frac{1}{2}\int_0^{\infty} \frac{d\mu}{\mu} G^2(\mu)\ \mu\frac{\partial}{\partial\mu}\int \frac{d^4q}{(2\pi)^4}\frac{1}{q^2 + M_{\{\mu\}}(q^2)}\frac{1}{(q+p)^2 + M_{\{\mu\}}((q+p)^2)} f(q,p)$$

External momentum can be continued to Lorentzian space

Long distance parts of loops reproduced

-at low scales $G(\mu) \sim G_N = \text{const.}$

$$\int_0 \frac{d\mu}{\mu}\ \mu\frac{\partial}{\partial\mu}[...] \sim \frac{1}{2}G_N^2 \int \frac{d^4q}{(2\pi)^4}\frac{1}{q^2}\frac{1}{(q+p)^2} f(q,p)$$

High energy dependence on $G(\mu)$ appears local at low energy

- match to coefficients of EFT

More formally shown by ‘non-local heat kernel expansion’

Satz, Cordello, Mazzitelli
Cordello, Zanusso

Regularized version of EFT

But not clear that Weinberg's original goal is satisfied

$$\sigma(\Lambda, G, c_i, p_a) = \frac{1}{\mu^2}\sigma(\frac{\Lambda}{\mu^4}, G\mu^2, \tilde{c}_i, \frac{p_a}{\mu})$$

All processes well behaved

This is **not** the form of the Lorentzian result in the perturbative region

In principle, end up with EFT with infinite numbers of local terms in Lagrangian, but with related coefficients and with regularized loops.

Will this make infinite number of reactions well-behaved in all kinematic regions?

Summary of AS section

AS appears capable of reproducing EFT

In principle predictive (relationships among couplings). In practice, not yet.

But, Euclidean running with μ does not translate into running with energy in the physical perturbative region

Implication - not appropriate to assume running G, Λ in FLRW applications

Still unsure if Weinberg's asymptotic hypothesis emerges from present AS practice
- well defined Lorentzian QFT at high scales?

Summary of AS section

AS appears capable of reproducing EFT

In principle predictive (relationships among couplings). In practice, not yet.

But, Euclidean running with μ does not translate into running with energy in the physical perturbative region

Implication - not appropriate to assume running G, Λ in FLRW applications

Still unsure if Weinberg's asymptotic hypothesis emerges from present AS practice
- well defined Lorentzian QFT at high scales?

Overall:

EFT treatment useful in perturbative region

Running couplings with gravity fail two criteria:
Not **useful** – do not encapsulate a well defined set of quantum corrections
- e.g. the crossing problem even within related reactions

Not **universal** – not a renormalization of the basic coupling
- quantum effects very different in different reactions

Many cutoff calculations mis-applied

Asymptotic Safety
- compatible with EFT treatment
- but no special definition of running couplings in perturbative region.