Title: Gromov-Witten Invariants from 2D Gauge Theories

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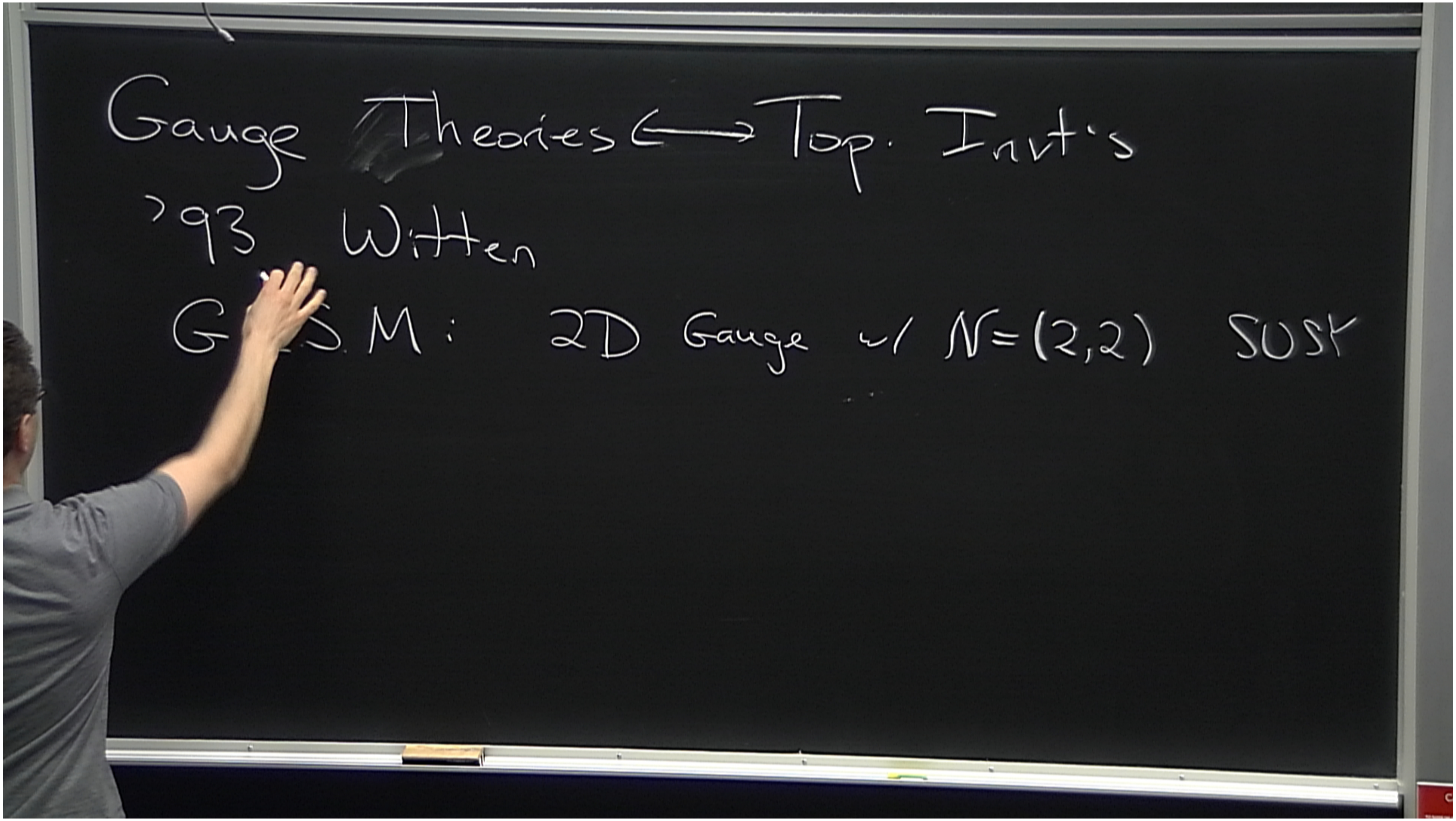
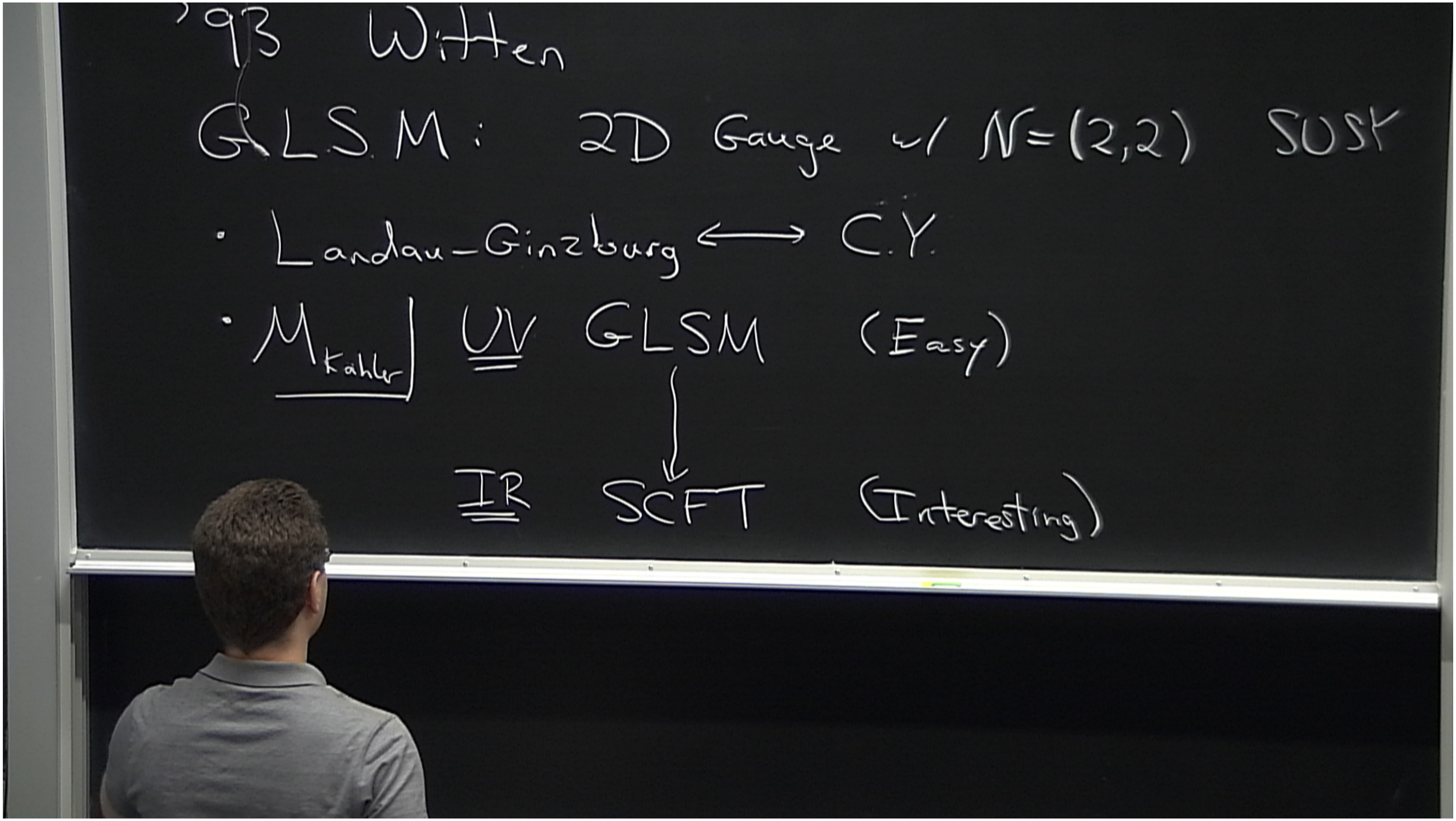
Abstract: It has been known for twenty years that a class of two-dimensional gauge theories are intimately connected to toric geometry, as well as to hypersurfaces or complete intersections in a toric varieties, and to generalizations thereof. Under renormalization group flow, the two-dimensional gauge theory flows to a conformal field theory that describes string propagation on the associated geometry. This provides a connection between certain quantities in the gauge theory and topological invariants of the associated geometry. In this talk, I will explain how recent results show that, for Calabi-Yau geometries, the partition function for each gauge theory computes the Kahler potential on the Kahler moduli of the associated geometry. The result is expressed in terms of a Barnes' integral and is readily evaluated in a series expansion around special points in the moduli space (e.g., large volume), providing a fairly efficient way to compute Gromov-Witten invariants of the associated geometry.

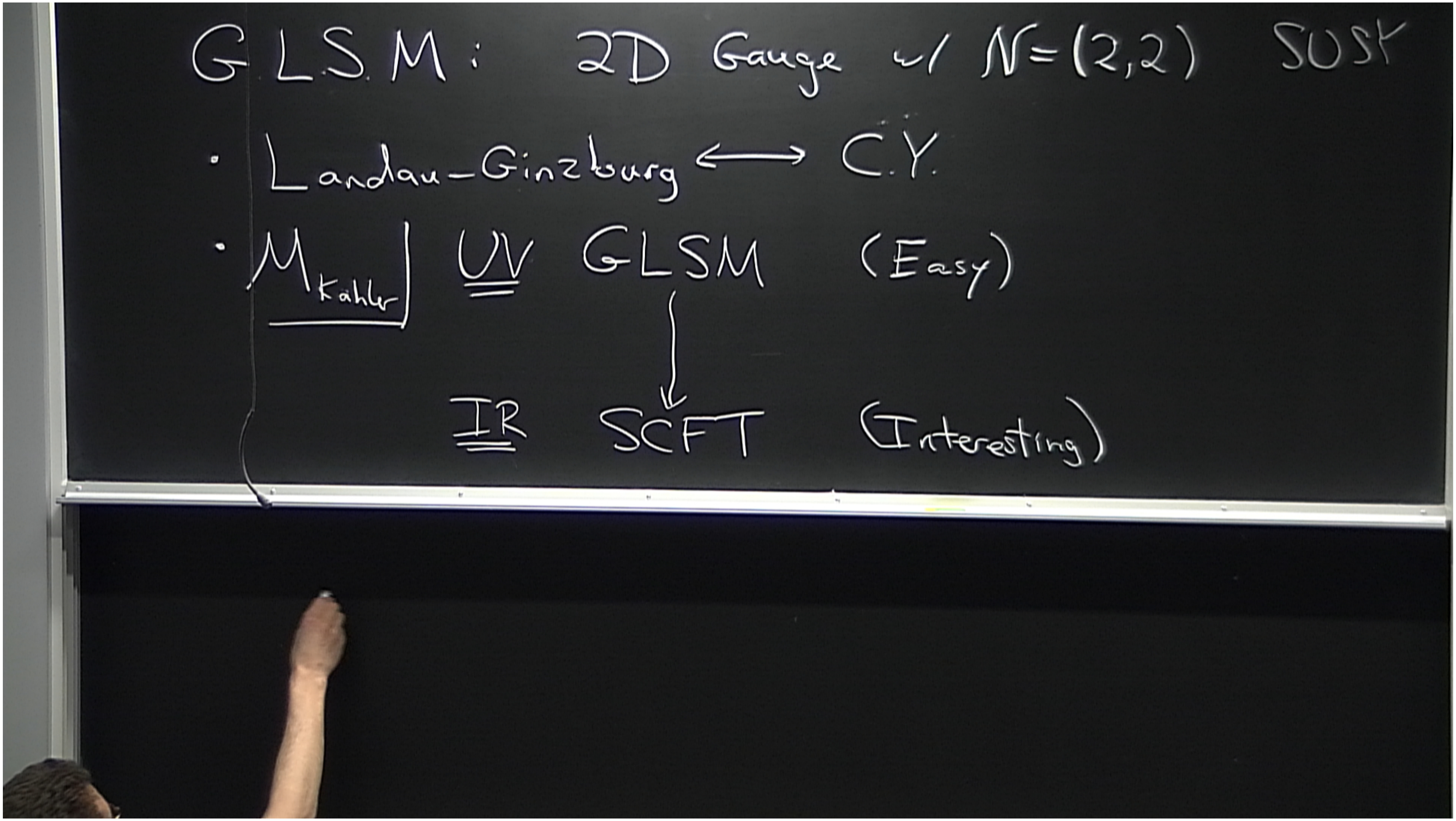

Gauge Theories ⟷ Top. Invt's
'93 Witten
G SM: 2D Gauge w/ N=(2,2) SUSY

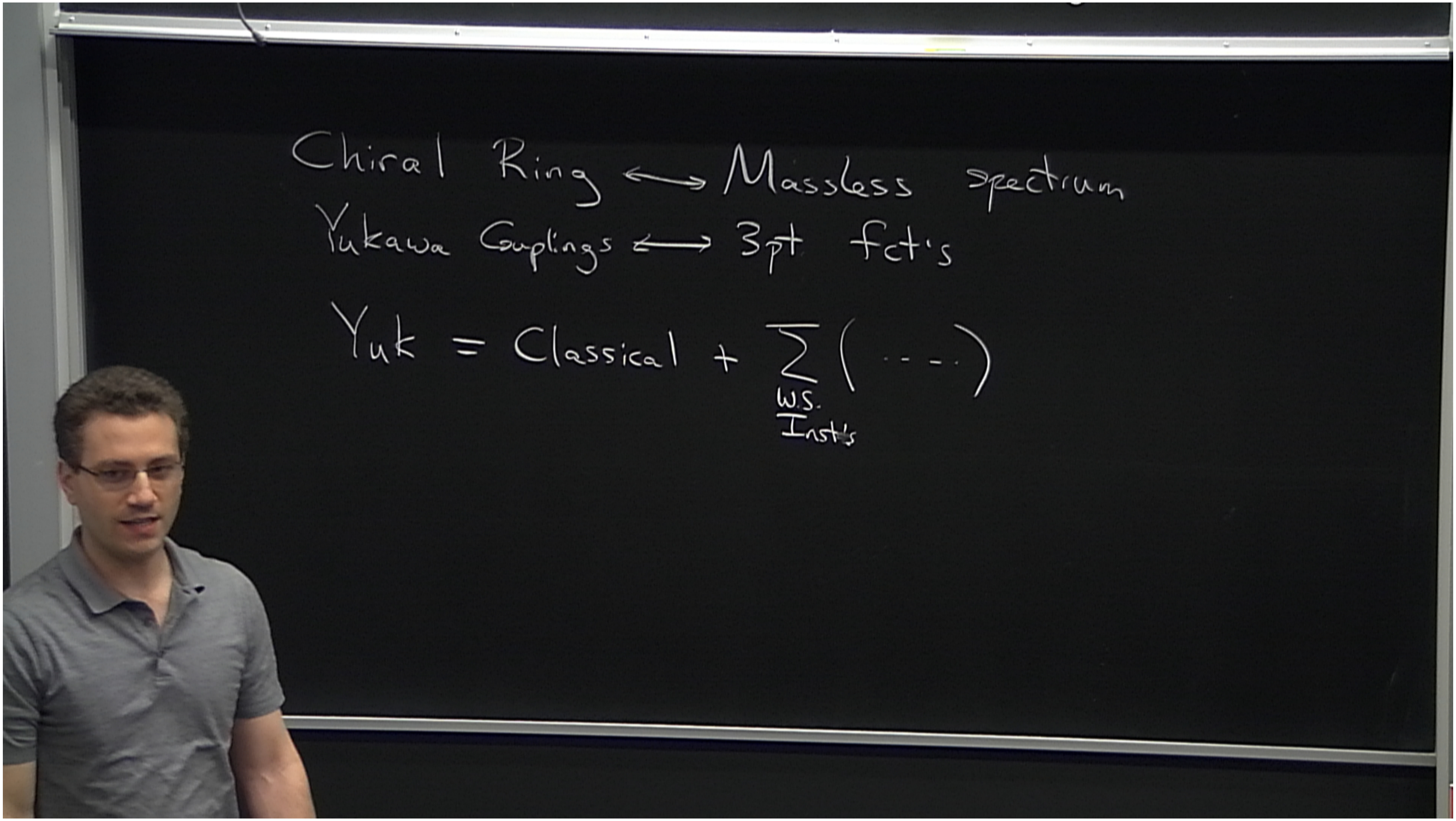
'93 Witten

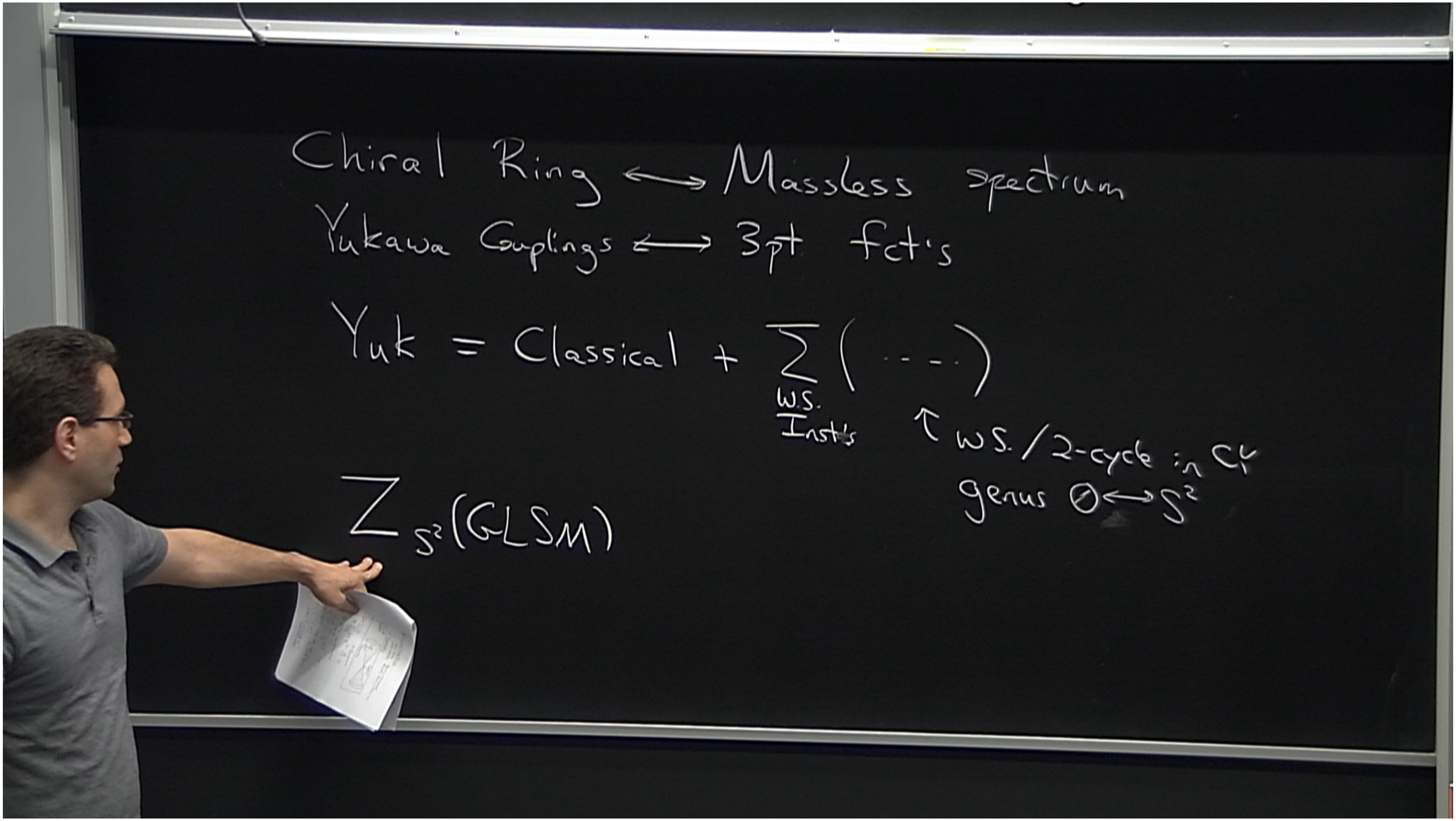
G.L.S.M: 2D Gauge w/ $\mathcal{N}=(2,2)$ SUSY

- Landau-Ginzburg $\longleftrightarrow$ C.Y.
- $\mathcal{M}_{\text{Kähler}}$


'93 Witten
G.L.S.M.: 2D Gauge w/ $\mathcal{N}=(2,2)$ SUSY
• Landau-Ginzburg ⟷ C.Y.
• $\underline{M_{\text{Kähler}}}$ | UV GLSM (Easy)
IR SCFT (Interesting)


GLSM: 2D Gauge w/ N=(2,2) SUSY
• Landau-Ginzburg ⟷ C.Y.
• M Kahler
UV GLSM (Easy)
IR SCFT (Interesting)


Chiral Ring ⟷ Massless spectrum
Yukawa Couplings ⟷ 3pt fct's
Yuk = Classical + $\sum_{\text{W.S. Inst's}} (\ldots)$


Chiral Ring ⟷ Massless spectrum
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$\text{Yuk} = \text{Classical} + \sum_{\substack{\text{W.S.}\\ \text{Inst's}}} (\cdots)$
↑ W.S. / 2-cycle in CY
genus 0 ⟷ S^2
$Z_{S^2}(\text{GLSM})$

Chiral Ring $\longleftrightarrow$ Massless spectrum

Yukawa Couplings $\longleftrightarrow$ 3pt fct's

$$\text{Yuk} = \text{Classical} + \sum_{\substack{\text{W.S.}\\ \text{Inst's}}} \left(\cdots \right)$$

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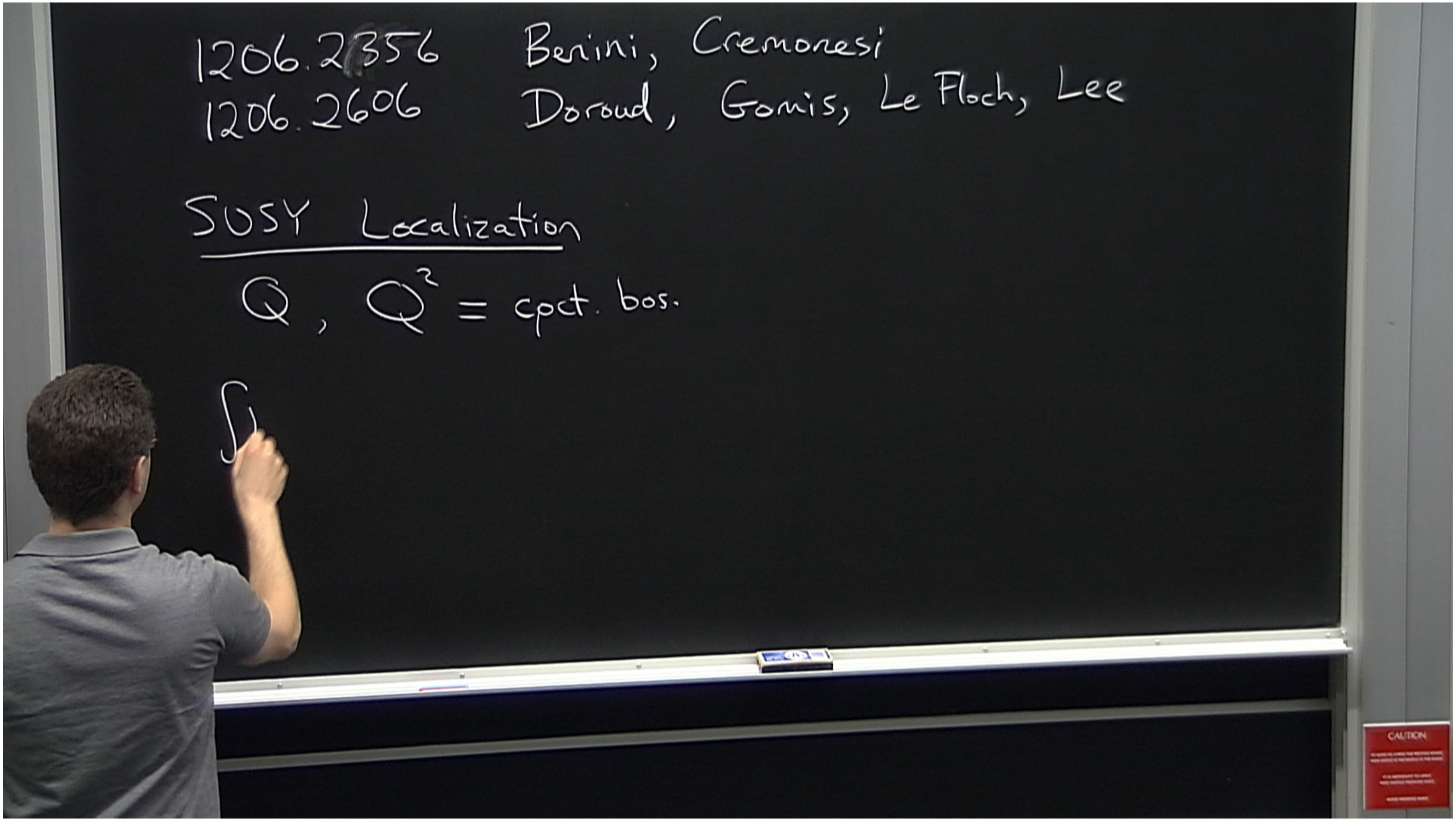
genus $0 \longleftrightarrow S^2$

$$Z_{S^2}(\text{GLSM})$$

1206.2356 Benini, Cremonesi

1206.2606 Doroud, Gomis, Le Floch, Lee

<u>SUSY Localization</u>


1206.2356 Benini, Cremonesi
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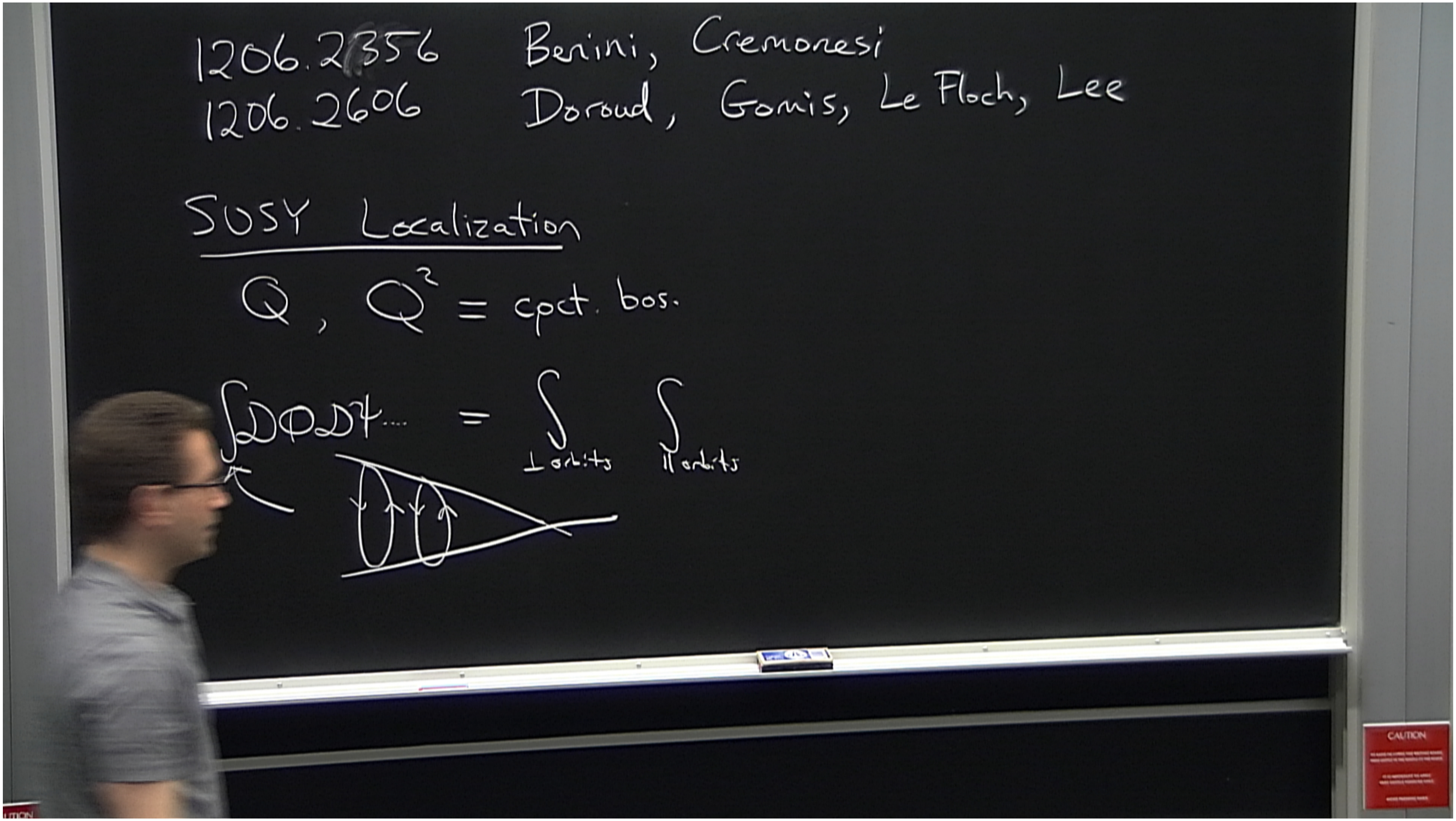
SUSY Localization
Q , Q² = cpct. bos.
∫


1206.2356 Benini, Cremonesi
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SUSY Localization
Q , Q² = cpct. bos.
∫DΦDΨ.... = ∫ ∫
⊥ orbits ∥ orbits

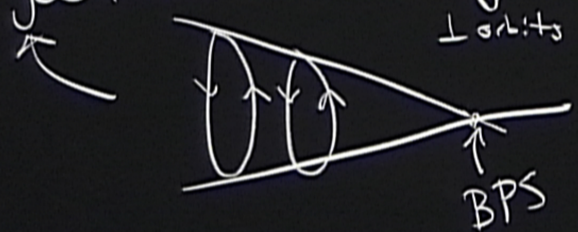
1206.2356 Benini, Cremonesi

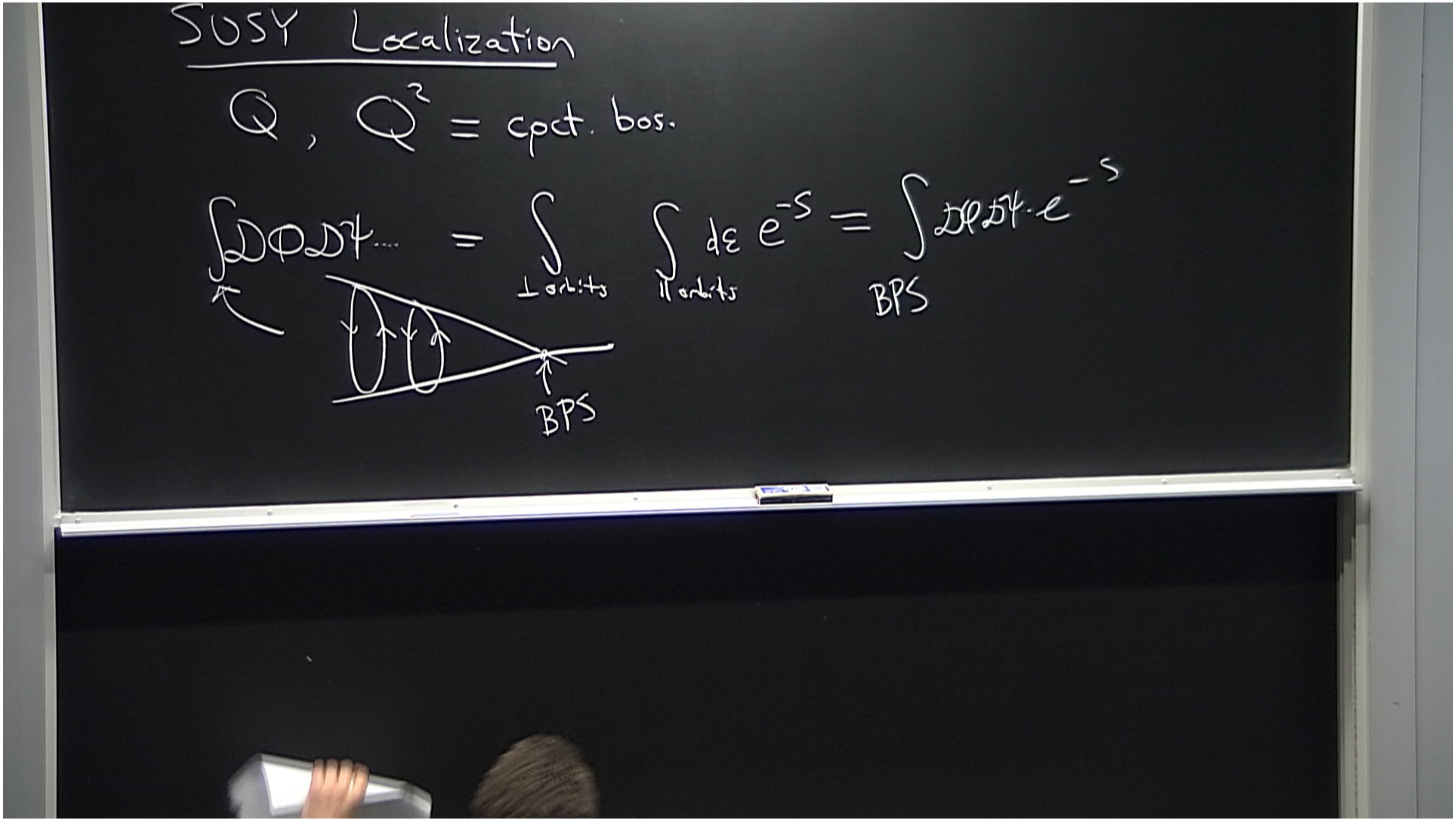
1206.2606 Doroud, Gomis, Le Floch, Lee

SUSY Localization

$Q \,,\; Q^2 = \text{cpct. bos.}$

$$\int \mathcal{D}\Phi \mathcal{D}\Psi \ldots = \int_{\perp \text{ orbits}} \int_{\parallel \text{ orbits}} d\varepsilon \, e^{-S} = \int_{BPS} \mathcal{D}\Phi \mathcal{D}\Psi \cdot e^{-S}$$




SUSY Localization
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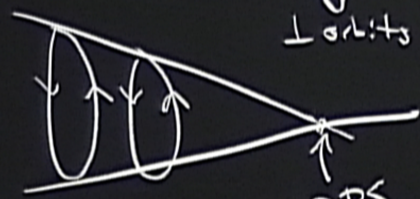
BPS

$$\int \mathcal{D}\phi\,\mathcal{D}\psi \ldots = \int_{\perp \text{ orbits}} \int_{\parallel \text{ orbits}} d\varepsilon\, e^{-S} = \int_{\text{BPS + Saddles of } Q\cdot V} \mathcal{D}\phi\,\mathcal{D}\psi \cdot e^{-S}$$



$$S \longrightarrow S + \lambda\, Q\cdot V$$

(i) $Q^2 \cdot V = 0$

(ii) $Q \cdot V$

$\underline{\lambda = 0}$

$\underline{\lambda = \infty}$

$Z_{S^2}(GLSM) \longrightarrow$ W.S. Inst.

$\uparrow$

SUSY Local.

w/ H. Jockers, V. Kumar, J.L., D. Morrison, M. Romo
1208.6244

$$Z_{S^2}(GLSM) = |f_0|^2 e^{-K_{\text{Kähler}}}$$

$\uparrow$ intersection #'s
Gromov-Witten invt's

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intersection #'s

Gromov-Witten invt's

(2,2) GLSM

Chiral $\Phi \supset \varphi, \psi_\pm, F_\varphi$

$\underline{G}$: Vector $V \supset A_\mu, \sigma, \lambda_\pm, D$

Ex: Quintic $\mathbb{P}^4[5]$

	$\Phi^{i=1,\dots,5}$	P
$U(1)$	1	-5

$$W = P\, G_5(\Phi)$$

(2,2) GLSM

Chiral $\Phi \supset \varphi, \psi_\pm, F_\varphi$

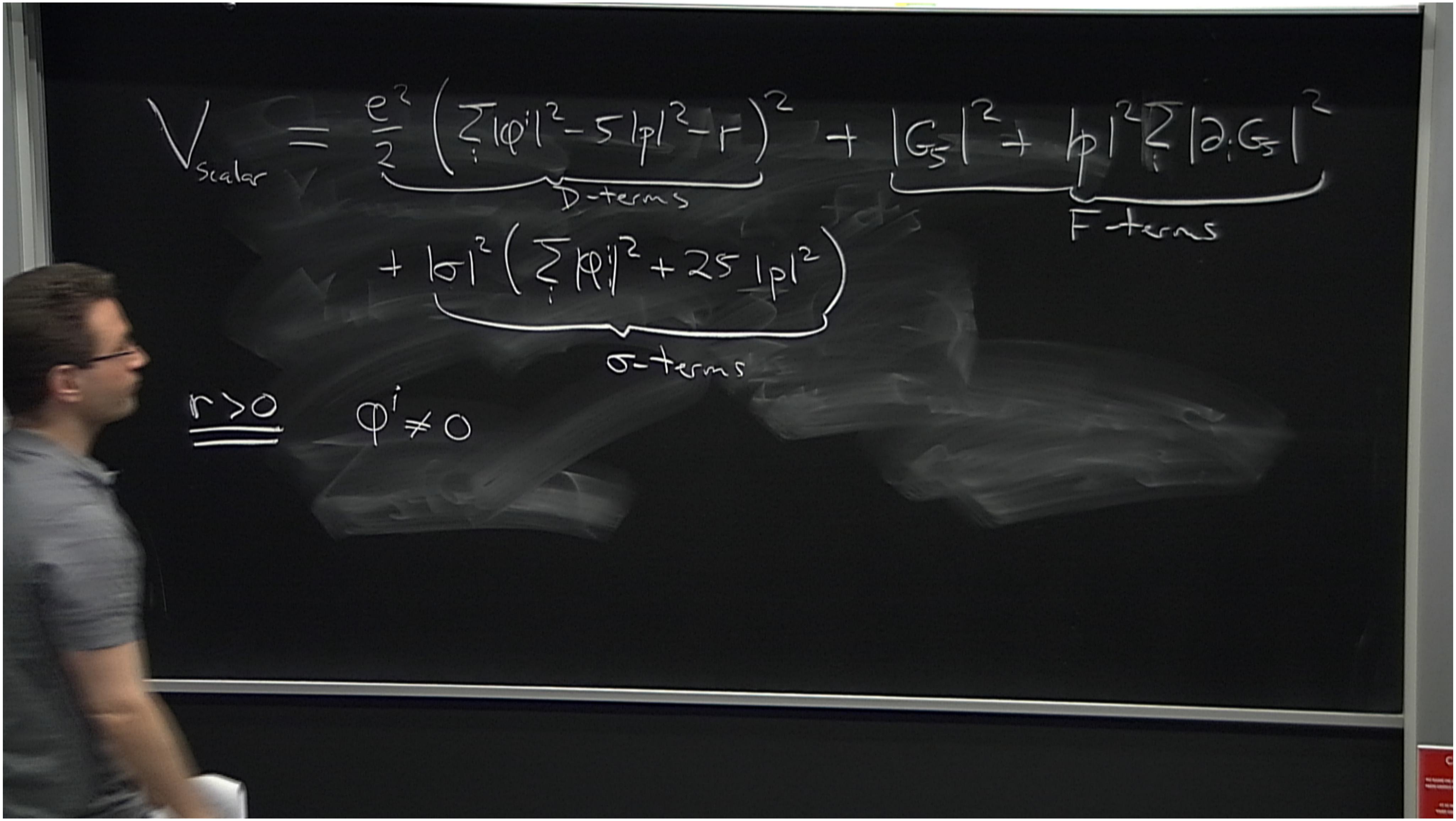
$\underline{G}$: Vector $V \supset A_\mu, \sigma, \lambda_\pm, D$

$\hookrightarrow \Sigma =$ twisted chiral; $\bar{D}_+ \Sigma = 0 = D_- \Sigma$

$\underline{\text{Ex}}$: Quintic $\mathbb{P}^4[5]$

$$\begin{array}{c|cc} & \Phi^{i=1,\dots,5} & P \\ \hline U(1) & 1 & -5 \end{array} \quad ; \quad \widetilde{W}(\Sigma) = t\,\Sigma, \quad t = ir + \frac{\theta}{2\pi} \ \text{(F.I.)}$$

$$W = P\, G_5(\Phi)$$



$V_{scalar} = \underbrace{\frac{e^2}{2}\left(\sum_i |\varphi^i|^2 - 5|p|^2 - r\right)^2}_{\text{D-terms}} + \underbrace{|G_5|^2 + |p|^2 \sum_i |\partial_i G_5|^2}_{\text{F-terms}}$
$+ \underbrace{|\sigma|^2\left(\sum_i |\varphi^i|^2 + 25|p|^2\right)}_{\sigma\text{-terms}}$
$r > 0 \quad \varphi^i \neq 0$

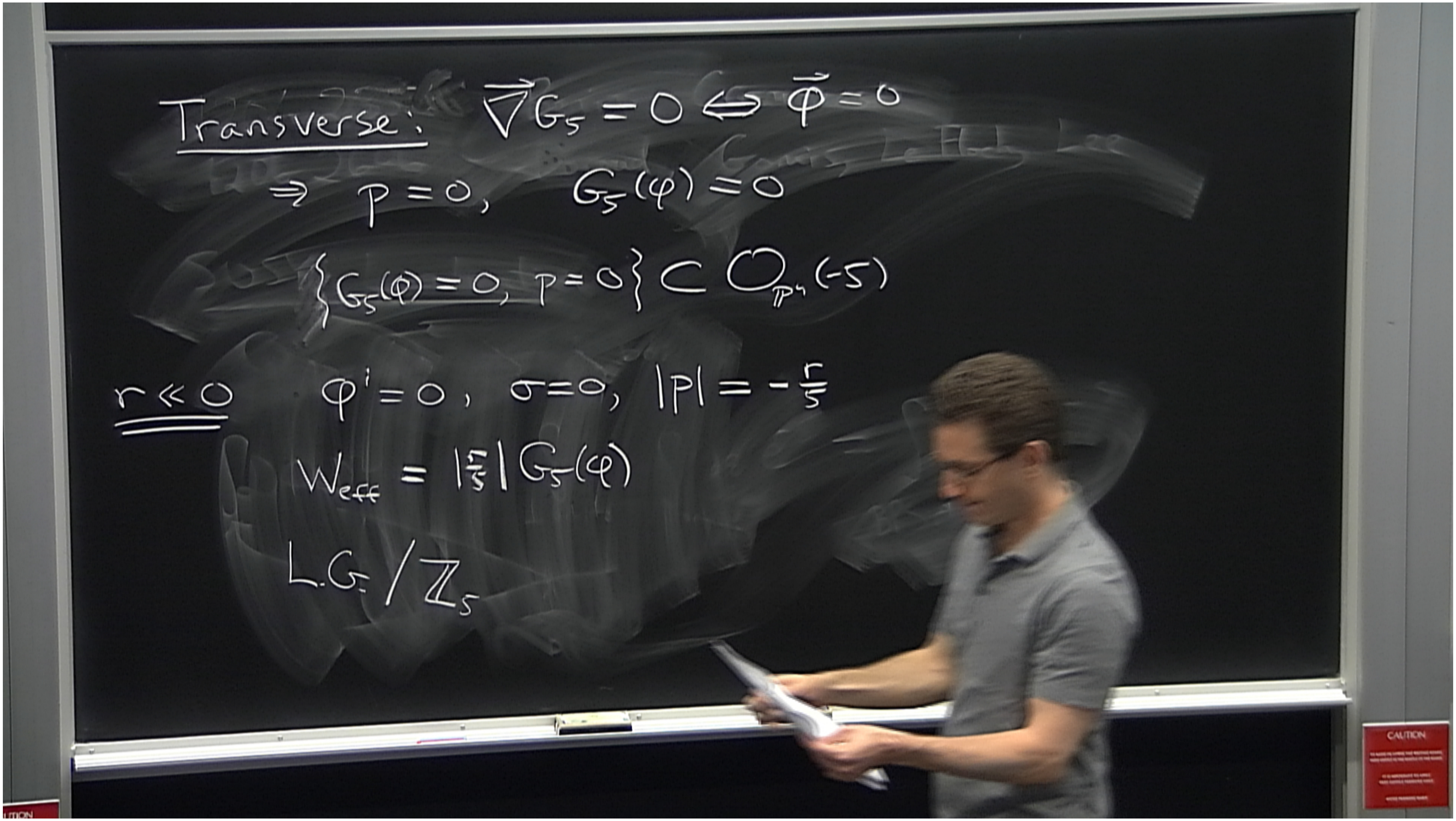
$$V_{\text{scalar}} = \underbrace{\frac{e^2}{2}\Big(\sum_i |\varphi^i|^2 - 5|p|^2 - r\Big)^2}_{\text{D-terms}} + \underbrace{|G_5|^2 + |p|^2 \sum_i |\partial_i G_5|^2}_{\text{F-terms}}$$

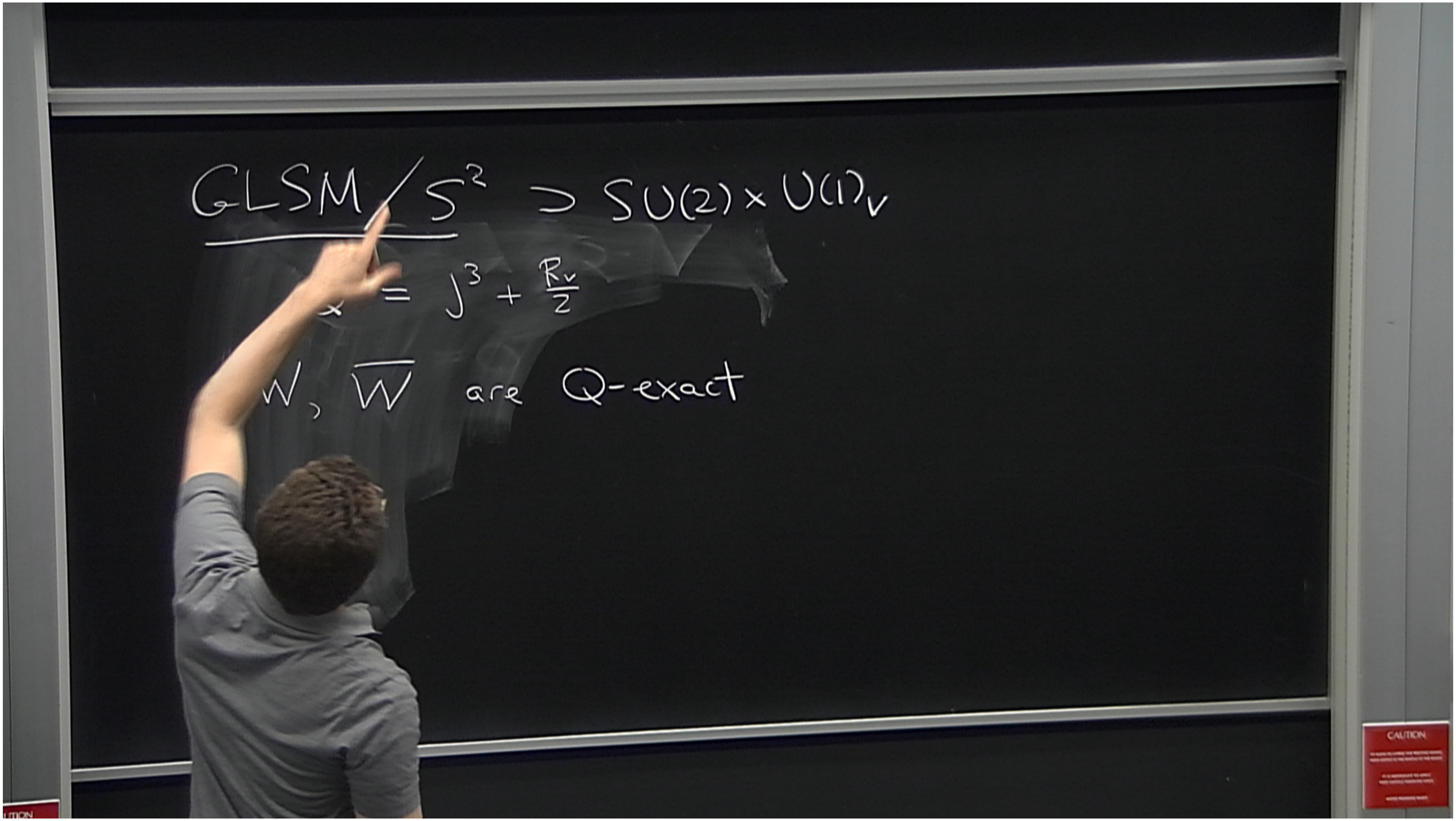
$$+ \underbrace{|\sigma|^2 \Big(\sum_i |\varphi^i|^2 + 25|p|^2\Big)}_{\sigma\text{-terms}}$$

$\underline{\underline{r>0}}$ $\quad \varphi^i \neq 0 \Rightarrow \sigma = 0;$

$$S^9/U(1) \cong \mathbb{P}^4 \ni [\varphi^i]$$

$\mathcal{O}_{\mathbb{P}^4}(-5)$


Transverse: $\vec{\nabla} G_5 = 0 \Leftrightarrow \vec{\varphi} = 0$
$\Rightarrow p = 0, \quad G_5(\varphi) = 0$
$\{G_5(\varphi) = 0, \ p = 0\} \subset \mathcal{O}_{\mathbb{P}^4}(-5)$
$r \ll 0$
$\varphi^i = 0, \ \sigma = 0, \ |p| = -\frac{r}{5}$
$W_{eff} = |\frac{r}{5}| G_5(\varphi)$
$LG / \mathbb{Z}_5$


GLSM / S^2 ⊃ SU(2) × U(1)_V
= J^3 + R_V/2
W, W̄ are Q-exact

$\underline{GLSM/S^2} \supset SU(2) \times U(1)_V$

$$Q^2 = J^3 + \frac{R_V}{2}$$

$W, \overline{W}$ are Q-exact

$\widetilde{W}, \overline{\widetilde{W}}$ are not Q-exact

$$\underline{GLSM/S^2} \supset SU(2) \times U(1)_V$$

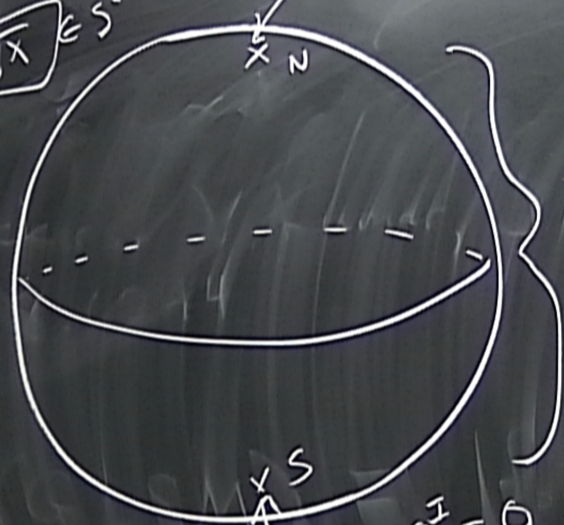
$$Q^2 = J^3 + \frac{R_V}{2}$$

$W, \overline{W}$ are Q-exact

$\widetilde{W}, \overline{\widetilde{W}}$ are not Q-exact

V_{Higgs} : $\sigma = 0$, $\boxed{x, \bar{x}} \in S^2$

$$\left.\begin{aligned} & D_x \phi^I = 0 \\ & \sum |\phi^I|^2 Q^I - r = -F \end{aligned}\right\} \text{Anti-}$$



Interpolation

$$\left.\begin{aligned} & D_{\bar{x}} \phi^I = 0 \\ & \sum |\phi^I|^2 Q^I - r = +F \end{aligned}\right\} \text{Vortex eqn's (A)}$$

$\underline{\underline{V_{\text{Coulomb}}}}$: $\varphi^I = D = F_\varphi = D_\mu \sigma = 0$

$$\mathcal{F} = \frac{B}{2\pi}\,\Omega_2\,, \quad B \in \text{Cartan}$$

$$\mathrm{Re}(\sigma) = -\frac{B}{R}$$

$$[\mathrm{Im}(\sigma), B] = 0$$

$$\hookrightarrow \mathrm{Im}(\sigma) \in \text{Cartan}$$

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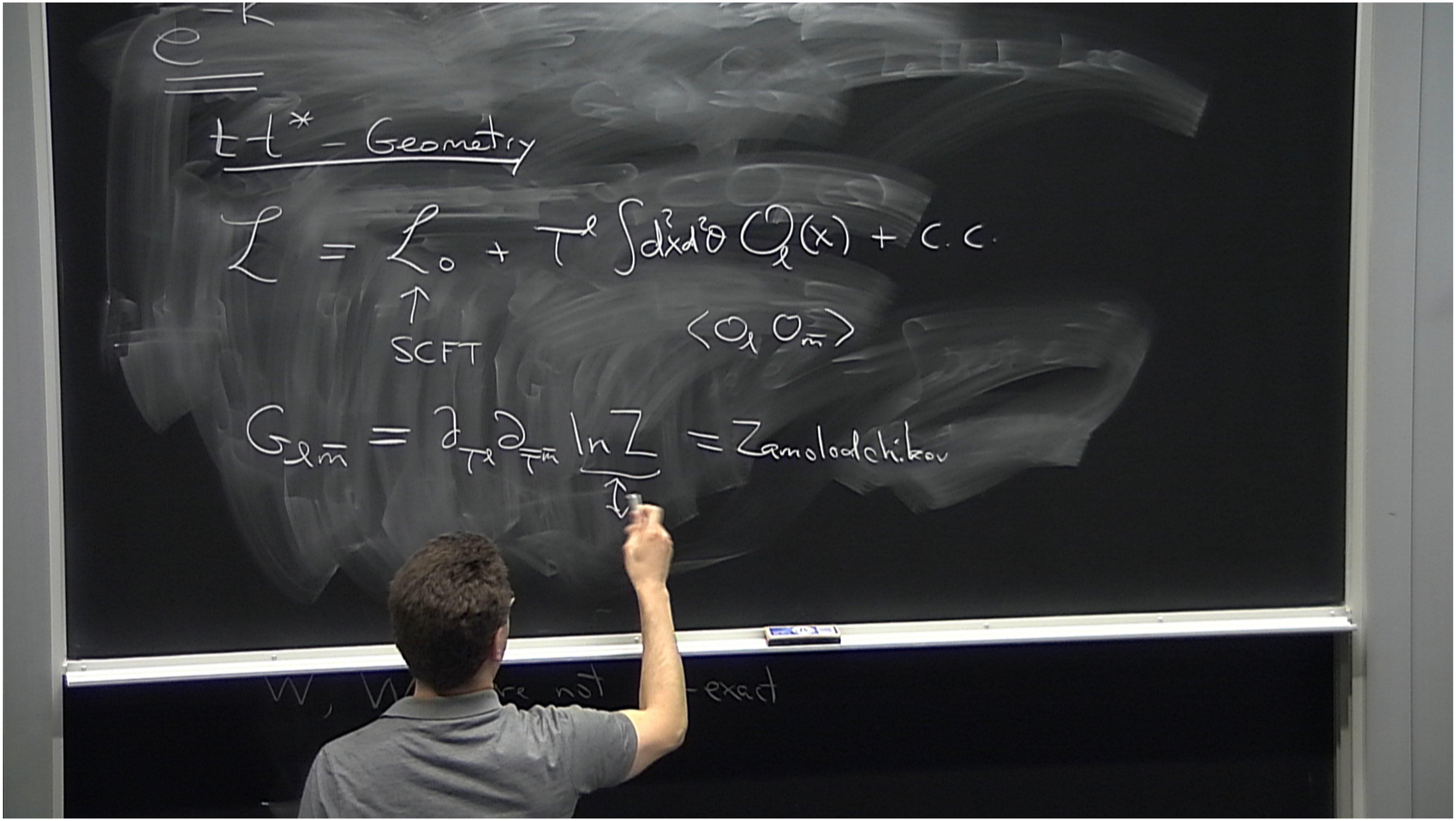
$$Z_{S^2} = \frac{1}{|W_G|} \sum_{B} \int \frac{d^{rk(G)}\sigma_a}{(2\pi)^{rk(G)}} Z_{Cl} Z_{Vec} \prod_{\Phi} Z_{\Phi}$$

$$Z_{Vec} = \prod_{\alpha>0} \left(\frac{\alpha(B)^2}{4} + \alpha(\sigma)^2 \right)$$

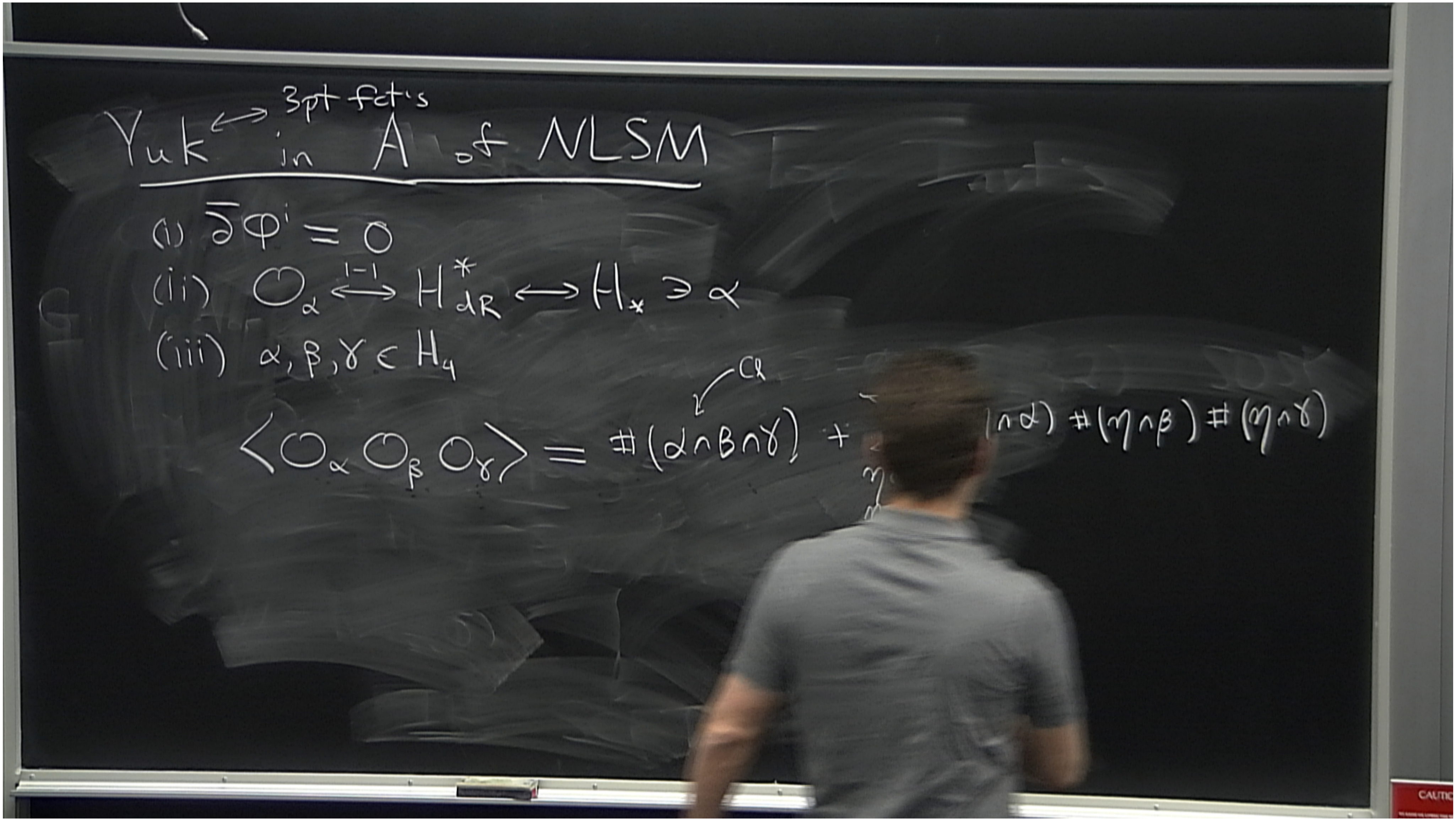
$$Z_{\Phi} = \prod_{w \in \rho_{\Phi}} \frac{\Gamma\left(\frac{q_{\Phi}}{2} - i w(\sigma) - \frac{w(B)}{2}\right)}{\Gamma\left(1 - \frac{q_{\Phi}}{2} + i w(\sigma) - \frac{w(B)}{2}\right)}$$

$$Z_{Cl} = e^{-4\pi i r \mathrm{Tr}(\sigma) - i\theta \mathrm{Tr}(B)}$$



tt* - Geometry
$\mathcal{L} = \mathcal{L}_0 + \tau^\ell \int d^2x\, d^2\theta\, \mathcal{O}_\ell(x) + c.c.$
SCFT
$\langle \mathcal{O}_\ell \mathcal{O}_{\bar m} \rangle$
$G_{\ell \bar m} = \partial_{\tau^\ell} \partial_{\bar\tau^{\bar m}} \ln Z$ = Zamolodchikov

1210.6022 ~ Gomis, Lee


Yuk $\longleftrightarrow$ 3pt fct's in A of NLSM
(i) $\bar{\partial}\varphi^i = 0$
(ii) $\mathcal{O}_\alpha \overset{1-1}{\longleftrightarrow} H^*_{dR} \longleftrightarrow H_* \ni \alpha$
(iii) $\alpha, \beta, \gamma \in H_4$
Cl
$\langle \mathcal{O}_\alpha \mathcal{O}_\beta \mathcal{O}_\gamma \rangle = \#(\alpha \cap \beta \cap \gamma) +$
$\cap\alpha)\ \#(\eta\cap\beta)\ \#(\eta\cap\gamma)$

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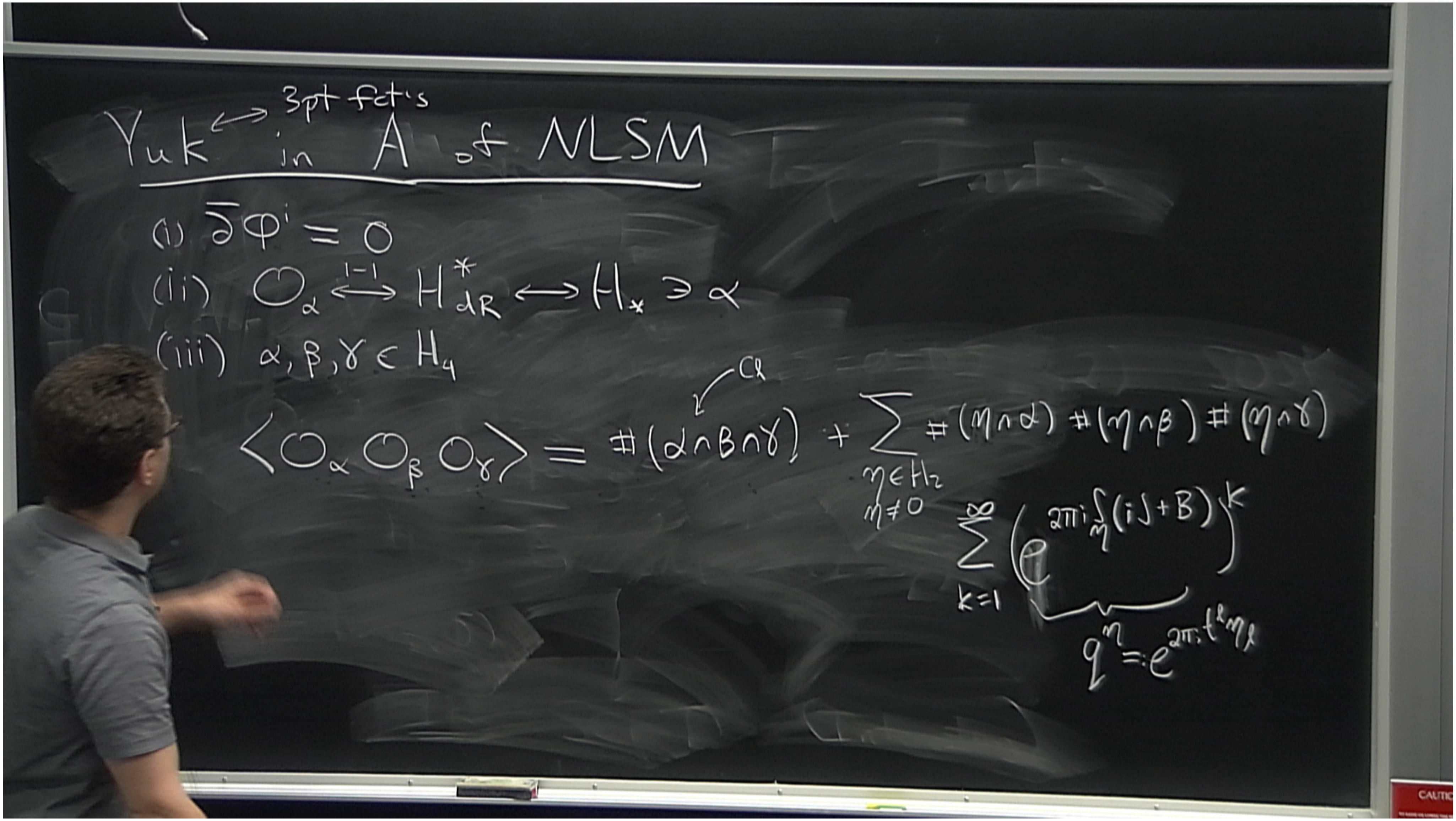
$$\langle \mathcal{O}_\alpha \mathcal{O}_\beta \mathcal{O}_\gamma \rangle = \#(\alpha \overset{cl}{\cap} \beta \cap \gamma) + \sum_{\substack{\eta \in H_2 \\ \eta \neq 0}} \#(\eta \cap \alpha)\,\#(\eta \cap \beta)\,\#(\eta \cap \gamma)\; \underbrace{e^{2\pi i \int_\eta (iJ + B)}}_{q^\eta =: e^{2\pi i t^\ell \eta_\ell}}$$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$\langle \mathcal{O}_\alpha \mathcal{O}_\beta \mathcal{O}_\gamma \rangle = \#(\alpha\cap\beta\cap\gamma) + \sum_{\substack{\eta\in H_2 \\ \eta\neq 0}} \#(\eta\cap\alpha)\,\#(\eta\cap\beta)\,\#(\eta\cap\gamma)$
$\sum_{k=1}^{\infty} \Big(\underbrace{e^{2\pi i \int_\eta (iJ+B)}}_{q^\eta =: e^{2\pi i t^\ell \eta_\ell}}\Big)^k$

Yuk $\overset{\text{3pt fct's}}{\longleftrightarrow}$ in $\mathcal{A}$ of NLSM

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G.W. Invt

$$\mathrm{Li}_s(q) \equiv \sum_{k=1}^{\infty} \frac{q^k}{k^s}$$

$$e^{-K_{\text{Kähler}}} = -\frac{i}{6} K_{\ell m n}(t^\ell - \bar{t}^\ell)(t^m - \bar{t}^m)(t^n - \bar{t}^n) + \frac{\zeta(3)}{4\pi^3}\chi + \frac{2i}{(2\pi i)^3} \sum_{\substack{\eta \in H_2 \\ \neq 0}} N_\eta \left(\mathrm{Li}_3(q^\eta) + c.c.\right) + \cdots$$

$$Z_{\mathbb{P}^4[5]} = e^{\frac{4\pi r}{5}} \sum_{m \in \mathbb{Z}} e^{-i\theta m} \int_{\frac{1}{5}-i\infty}^{\frac{1}{5}+i\infty} \frac{d\tau}{2\pi i} e^{4\pi r \tau} \frac{\Gamma(\tau - \frac{m}{2})^5}{\Gamma(1-\tau-\frac{m}{2})^5} \frac{\Gamma(1-5\tau+\frac{5m}{2})}{\Gamma(5\tau+\frac{5m}{2})}$$

$$Z_{\mathbb{P}^4[5]} \overset{\text{L.V.}}{=\!=} \oint \frac{d\varepsilon}{2\pi i} (z\bar{z})^{\frac{1}{5}-\varepsilon} \frac{\pi^4 \sin(5\pi\varepsilon)}{\sin^5(\pi\varepsilon)} \left| \sum_{k=0}^{\infty} (-1)^{5k} z^k \frac{\Gamma(1+5k-5\varepsilon)}{\Gamma(1+k-\varepsilon)^5} \right|^2$$

$$|f_0|^2 e^{-K(t,\bar{t})} = Z_{S^2}(z,\bar{z})$$

$$z \to e^{-2\pi r + i\theta}$$

L.V. $z=0$,

$$t^\ell = \frac{\ln z^\ell}{2\pi i} + f^\ell(z)$$

holo in z