

Title: Higgs Boson in Condensed Matter: From Polaron to Topological Insulator

Date: Apr 25, 2013 04:30 PM

URL: <http://pirsa.org/13040132>

Abstract: &nbsp;In this talk I will briefly review the polaron physics, which has helped theorists to conceive the BCS theory of conventional superconductors as well as experimentalists to discover high temperature superconductors in the cuprates. Specifically I will talk about how charge carriers obtain their  
masses from coupling to the phonon field in one, two, three or higher dimensions. More recently, there is increasing interest in topological insulators where a gap can be opened which may suggest new version of Higgs mechanism in condensed matter.

# Outline

- Superconductivity
- Polaron
- Wave function
- Effective mass
- Adiabatic limit
- MoS<sub>2</sub> and silicene
- Topological insulator

# Superfluids

## Flow without friction

- No energy loss
- persistent flow
- Doesn't want to rotate



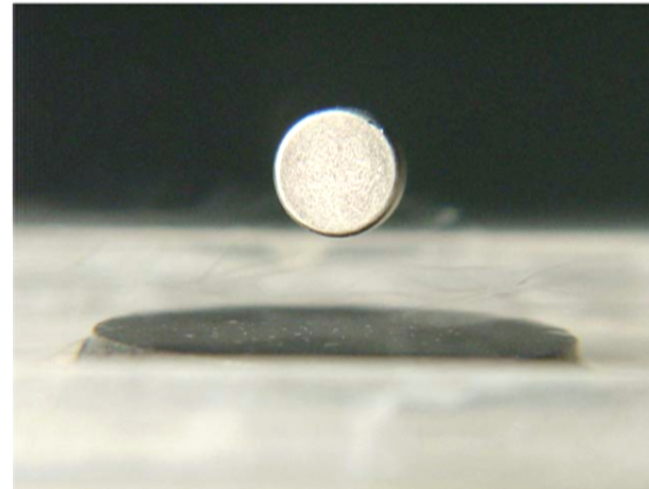
Onnes 1908,  
Kapitza, Allen & Misener 1938  
(1978)



# Superconductors

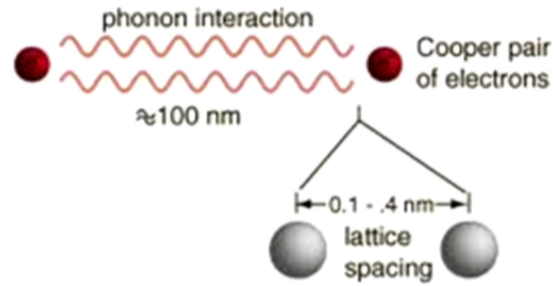
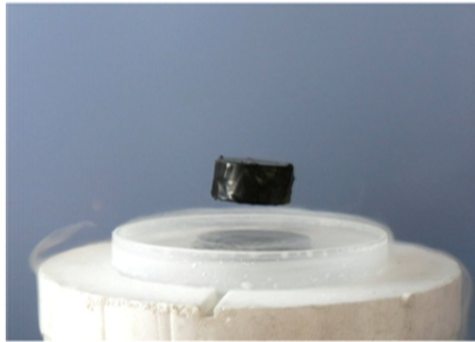
## Current without resistance

- No energy loss
- persistent currents
- expels magnetic fields



Onnes 1911 (1913)  
Müller & Bednorz 1987





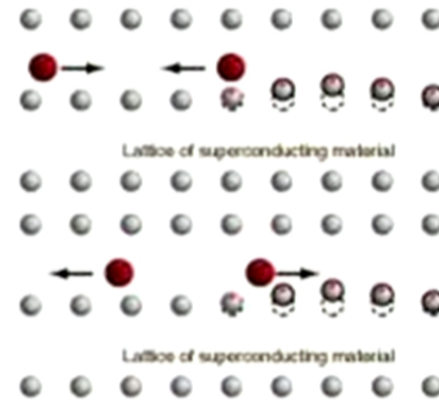
J. Bardeen



L.N. Cooper



J.R. Schrieffer

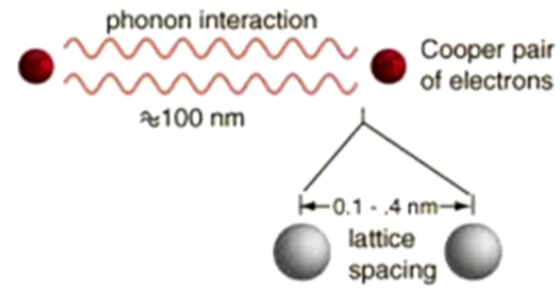
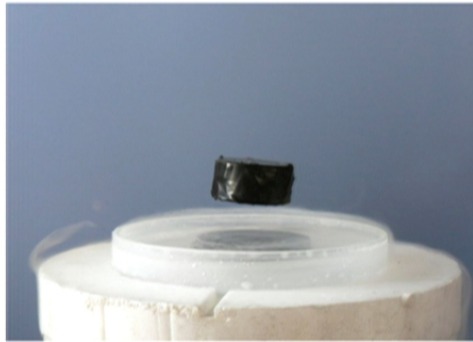


Jointly developed BCS theory for superconductivity in 1957



Nobel prize 1972





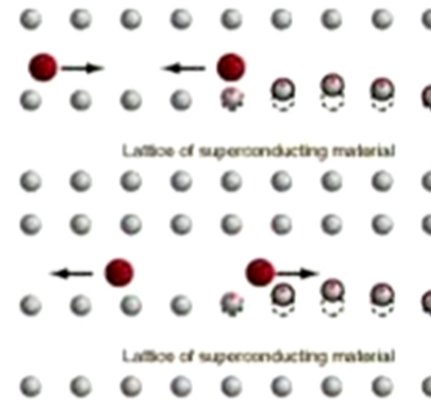
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Nobel Prize 1987



J. Georg Bednorz,  
West Germany, 1950-



K. Alexander Müller,  
Switzerland, 1927-

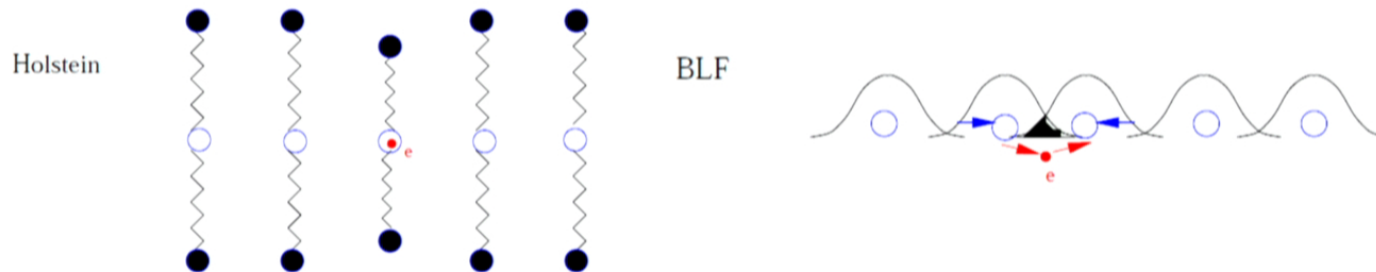
**High-temperature superconductivity discovered in 1986,  
theory in 2032 ?**

In their Nobel Prize lecture, they mentioned they were stimulated by the idea that if “an electron and a surrounding lattice distortion with a high effective mass can travel through the lattice as a whole, and a strong electron-lattice coupling exists an insulator could be turned into a high temperature superconductor”.

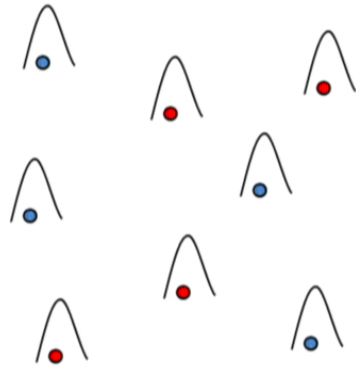
What kinds of electron-lattice interaction do we look at?

We are interested in three representative kinds of electron-lattice interaction:

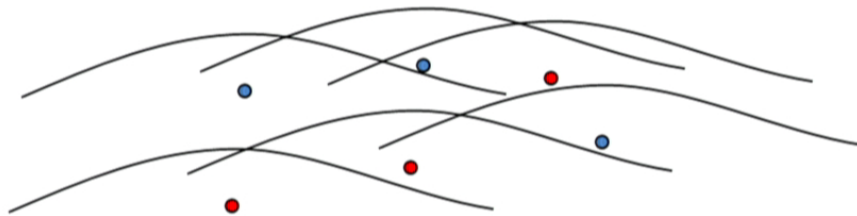
1. Holstein type, modify the onsite energy;
2. Frohlich type (nonlocal Holstein);
3. Su-Schrieffer-Heeger (BLF) type, modify the hopping integral (due to the change in the overlap of wave-functions).



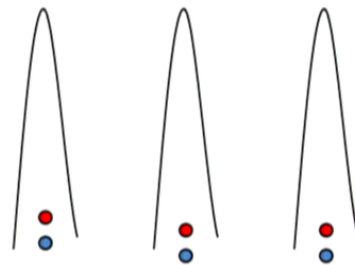
# Phonons' wave function



Weak electron-phonon coupling.  
Small effective mass. (BCS limit)  
Red----spin up, Blue----spin down.

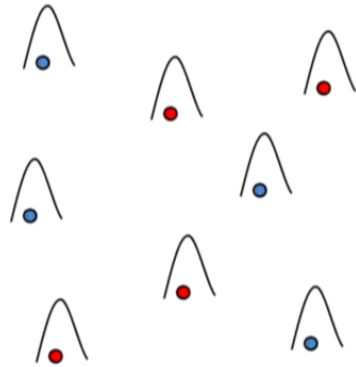


Intermediate coupling.  
Long range wave function.  
Large effective mass.  
-----Large polaron

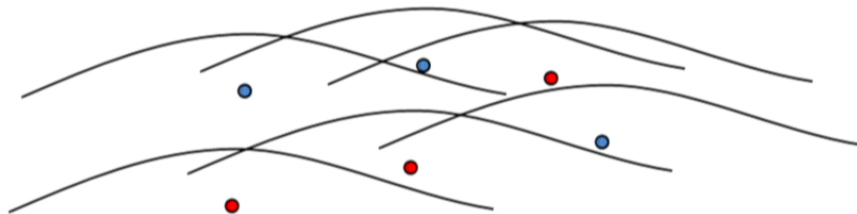


Strong coupling.  
Short range wave function.  
Many phonons on the same site (BEC limit)  
Very large effective mass.  
-----Small polaron or bipolaron

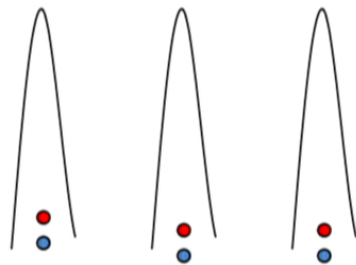
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-----Large polaron



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# Infinite Hilbert Space



1. Quantum computer (e.g. wave-function renormalization group)
2. Ansatz
3. Low energy effective theory

What's the method we are using?  
 -----Trugman's method

$$|\phi_1\rangle = H|\phi_0\rangle - \frac{\langle\phi_0|H|\phi_0\rangle}{\langle\phi_0|\phi_0\rangle}|\phi_0\rangle$$

$$|\phi_2\rangle = H|\phi_1\rangle - \frac{\langle\phi_1|H|\phi_1\rangle}{\langle\phi_1|\phi_1\rangle}|\phi_1\rangle - \frac{\langle\phi_1|\phi_0\rangle}{\langle\phi_0|\phi_0\rangle}|\phi_0\rangle$$

$$|\phi_{n+1}\rangle = H|\phi_n\rangle - a_n|\phi_n\rangle - b_n^2|\phi_{n-1}\rangle$$

$$a_n = \frac{\langle\phi_n|H|\phi_n\rangle}{\langle\phi_n|\phi_n\rangle}, b_n^2 = \frac{\langle\phi_n|\phi_n\rangle}{\langle\phi_{n-1}|\phi_{n-1}\rangle}$$



Tridiagonal matrix ( $10^7 \times 10^7$ )  
 ----Lanczos method

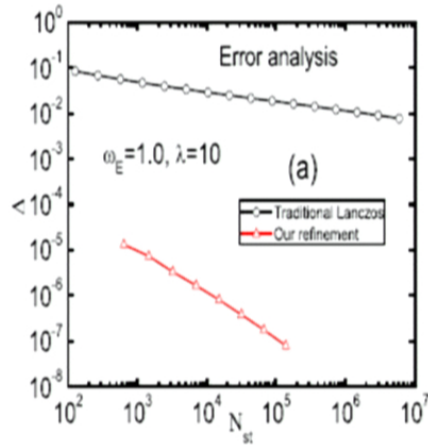
For example, if we start with a Bloch state with translational symmetry

$$|\phi_0\rangle = \sum_j e^{ikR_j} c_j^\dagger |0\rangle$$

$$\begin{aligned} H|\phi_0\rangle &= -t \sum_j e^{ikR_j} (c_{j+1}^\dagger + c_{j-1}^\dagger) |0\rangle - g\omega_E \sum_j e^{ikR_j} c_j^\dagger a_j^\dagger |0\rangle \\ &= -2t \cos k \sum_j e^{ikR_j} c_j^\dagger |0\rangle - g\omega_E \sum_j e^{ikR_j} c_j^\dagger a_j^\dagger |0\rangle \end{aligned}$$



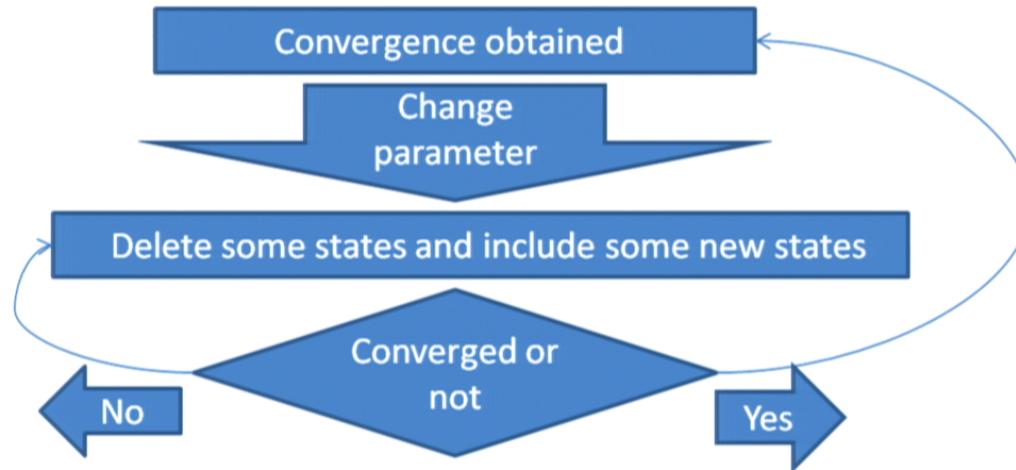
## What have we done to refine this method?



1) Start from the unperturbed state in the strong coupling limit.

$$|\psi\rangle = e^{-g^2/2} \sum_{\ell} e^{ikR_{\ell}} e^{-g\hat{a}_{\ell}^{\dagger}} \hat{c}_{\ell}^{\dagger} |0\rangle$$

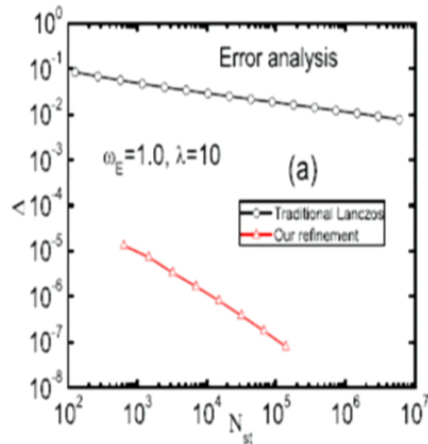
2) Near the adiabatic limit, use the following procedure:



Zhou Li, D. Baillie, C. Blois, F. Marsiglio, Phys. Rev. B 81, 115114 (2010).



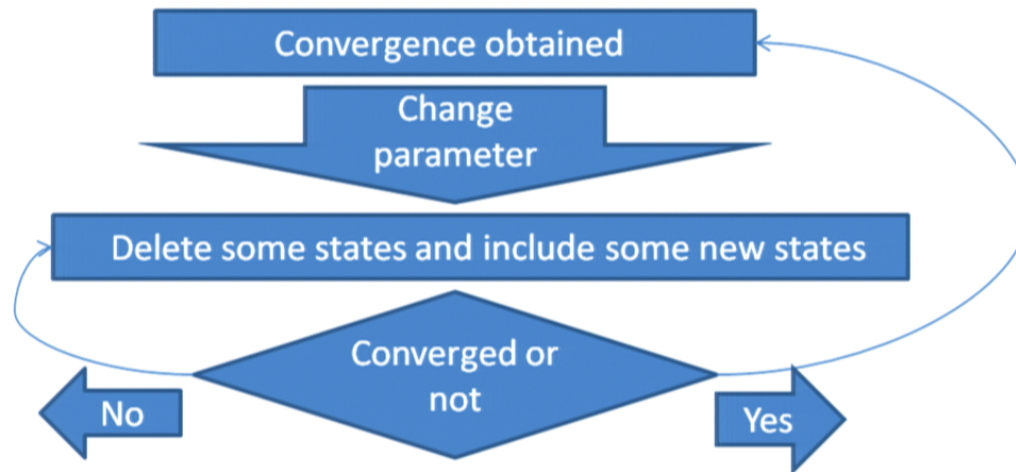
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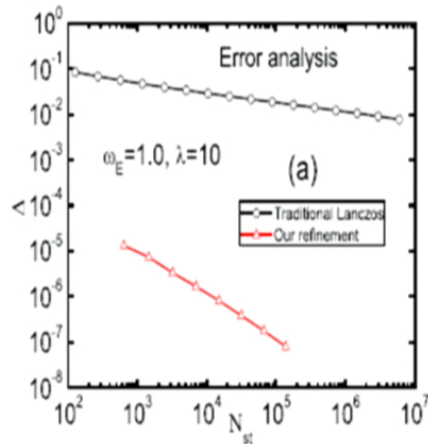
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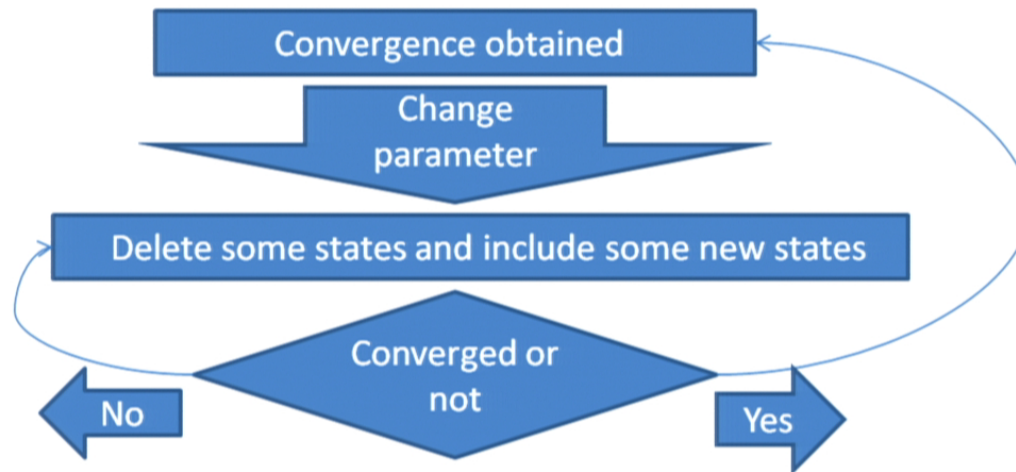
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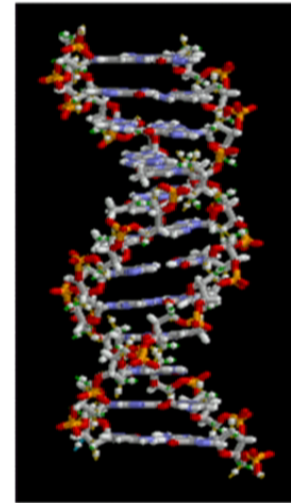
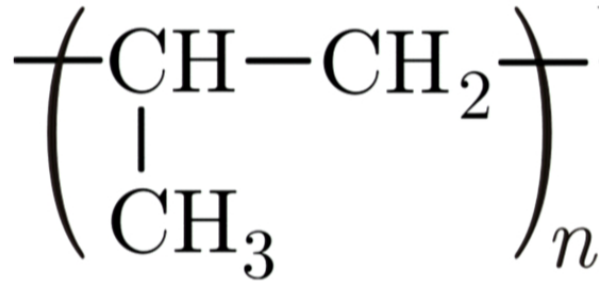
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## Polymers

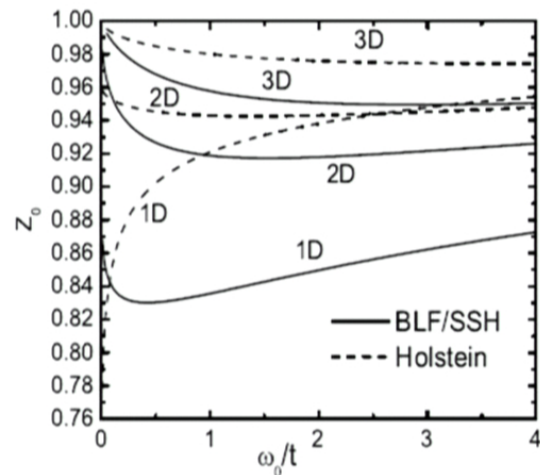
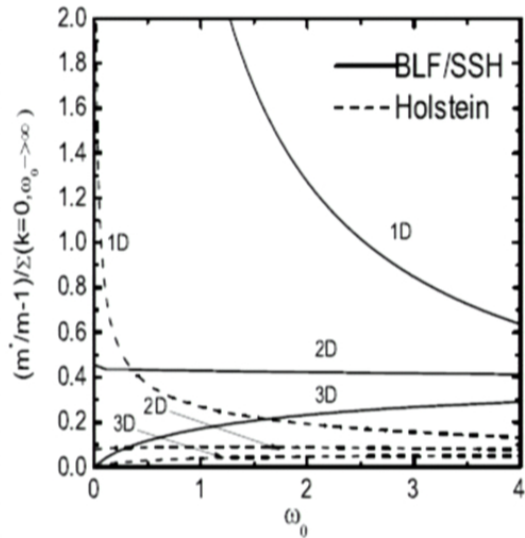


Biopolymer---- structure of DNA

Su-Schrieffer-Heeger (SSH) model is developed to describe electron phonon coupling in polymers,

Barisic-Labbe-Friedel (BLF) model is same as SSH model but came out 10 years early and is used in the area of superconductivity.

The ground state at strong coupling for SSH (BLF) model is still not known almost 40 years after the model is provided.



## SSH (BLF) model Vs. Holstein model

The effective mass will diverge in 1D,  
 SSH (BLF) model: as  $(\omega_0)^{-1}$ ,  
 Holstein model: as  $(\omega_0)^{-0.5}$

The effective mass will be a constant in 2D

The effective mass correction will  
 decrease to zero in 3D.

An unusual trend between effective mass and  
 spectral weight is found, in some parameter  
 regime for SSH (BLF) model:

$$m^*/m \sim z_0$$

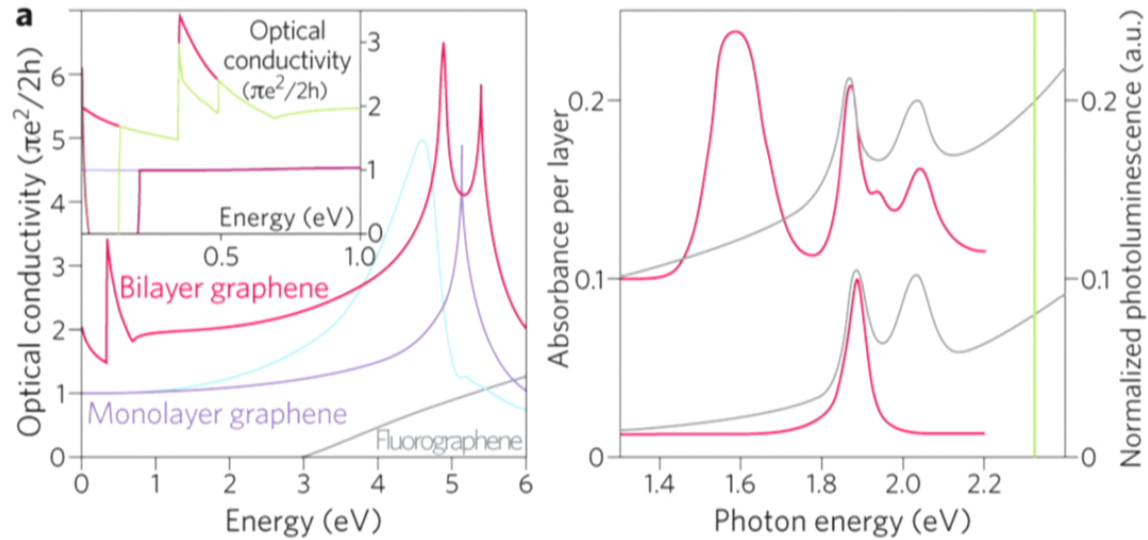
For Holstein model:

$$m^*/m \sim 1/z_0$$

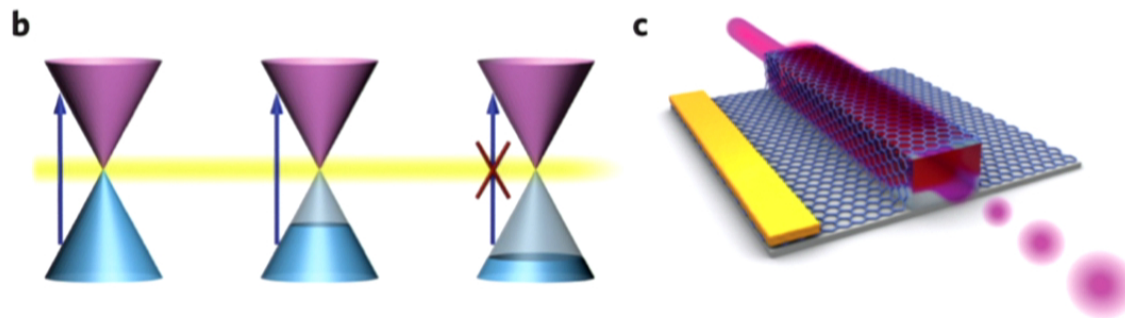
Zhou Li, C. Chandler, F. Marsiglio, Phys. Rev. B 83, 045104 (2011).



# Graphene-based flatband optics



MoS2

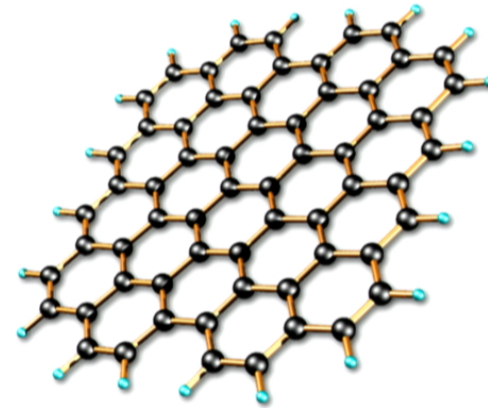
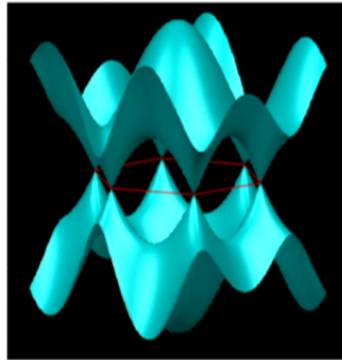


Nature Photonics 6, 749–758 (2012)

# Graphene and MoS2

**Graphene:**

**Gapless, high mobility, Dirac equation, spin valley degenerate**



**MoS2:**

**A large band gap of the order of 1.8 eV,**

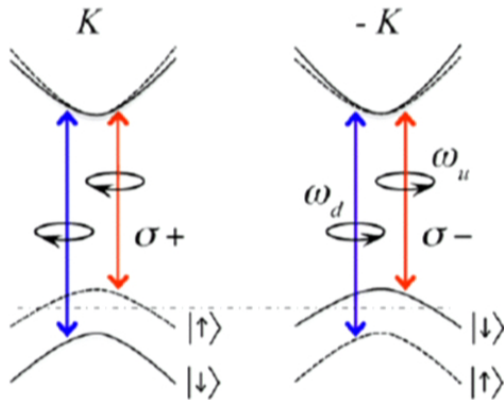
**in the visible light region,**

**good for transistor and sensitive to solar energy,**

**High mobility, gapped Dirac equation, spin valley polarized**



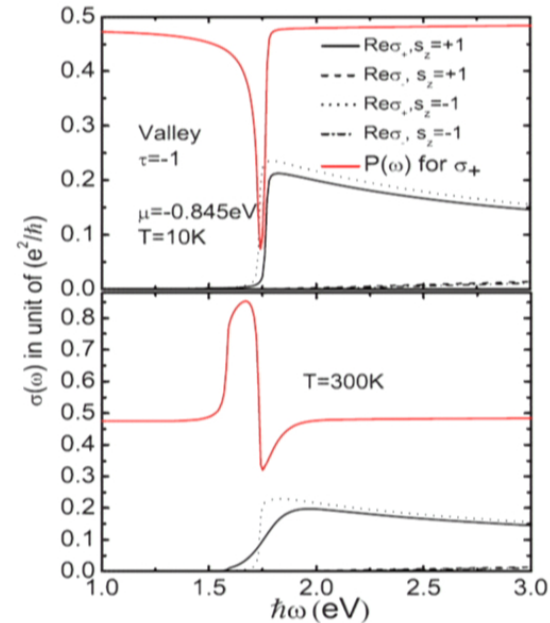
# Optical selection rule for circularly polarized light



**Orbital magnetic moment is opposite!**

$$|\mathcal{P}_{\pm}(k)|^2 = \frac{m_0^2 a^2 t^2}{\hbar^2} \left( 1 \pm \tau \frac{\Delta'}{\sqrt{\Delta'^2 + 4a^2 t^2 k^2}} \right)^2$$

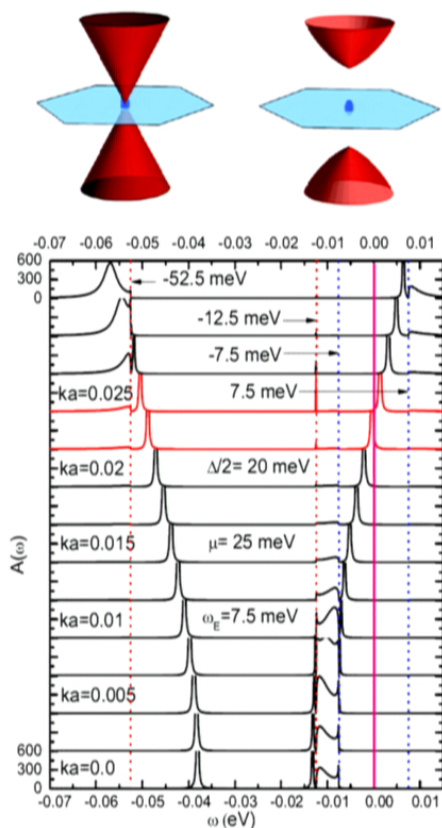
Di Xiao et.al,  
Phys. Rev. Lett. 108, 196802 (2012).



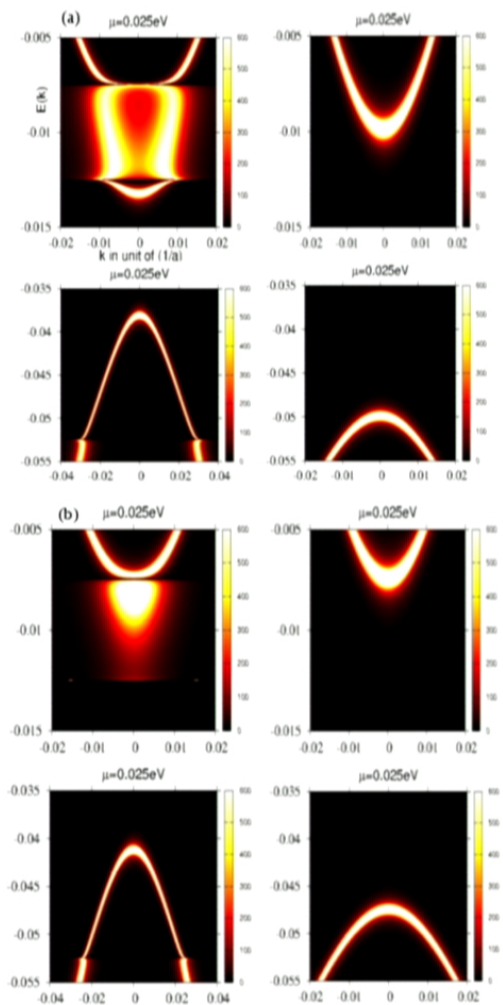
**Optical conductivity for circularly polarized light is drastically different!**

Zhou Li and J. P. Carbotte  
Phys. Rev. B. 86, 205425 (2012).

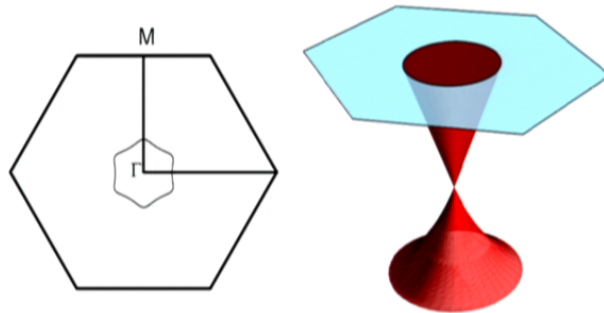
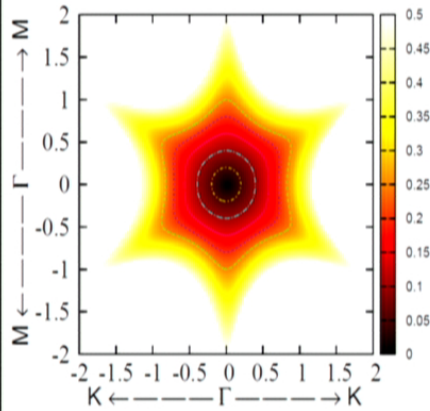
# Phonon structure for massive Dirac fermions



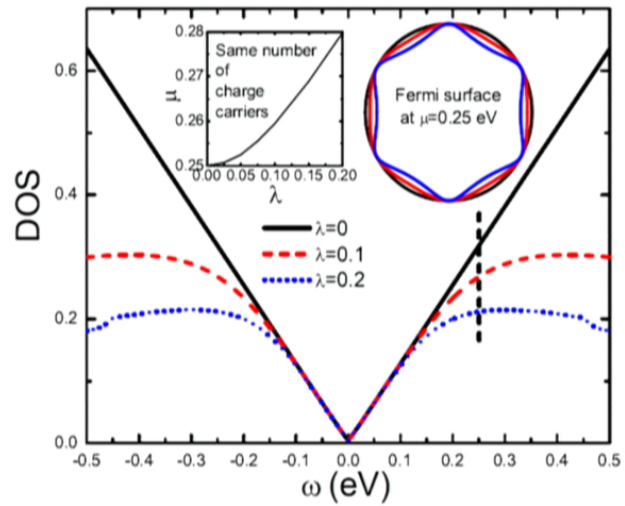
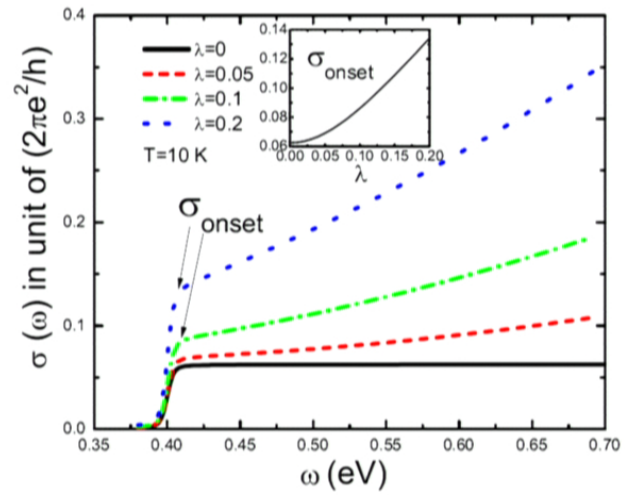
Zhou Li and J. P. Carbotte, submitted.



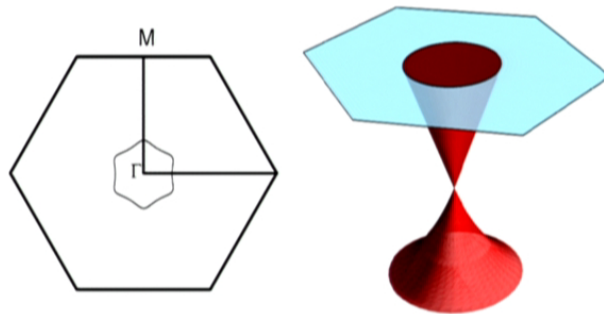
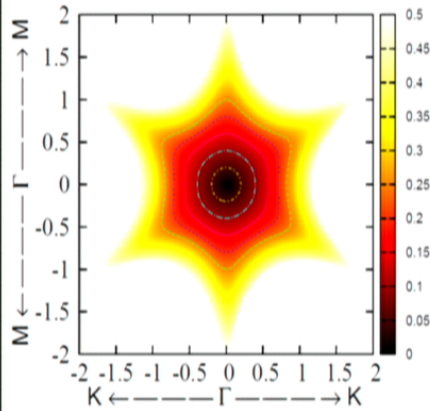
# Surface conductivity of 3D Topological Insulator



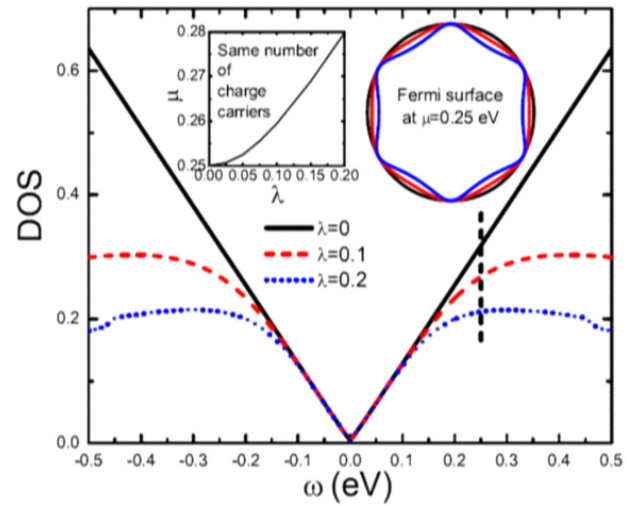
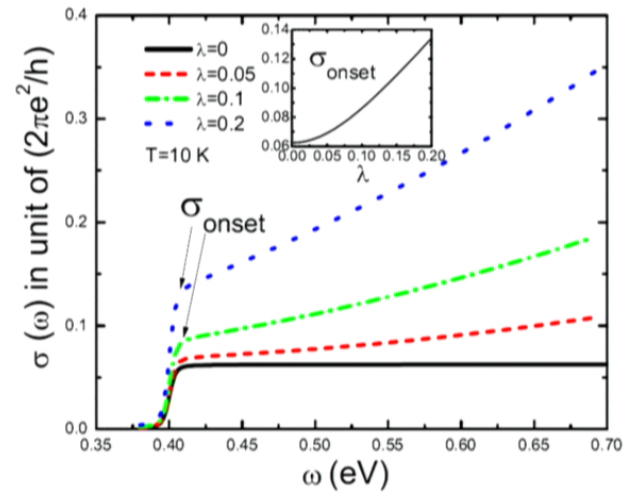
Zhou Li and J. P. Carbotte,  
Phys. Rev. B 87, 155416 (2013).



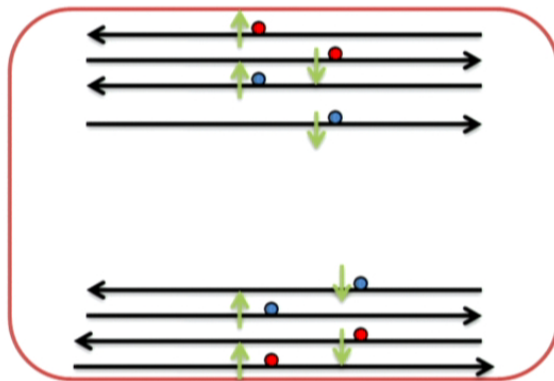
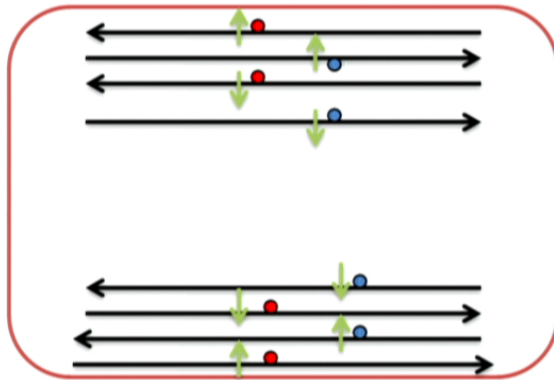
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Zhou Li and J. P. Carbotte,  
Phys. Rev. B 87, 155416 (2013).



# Edge transport of 2D Topological Insulator



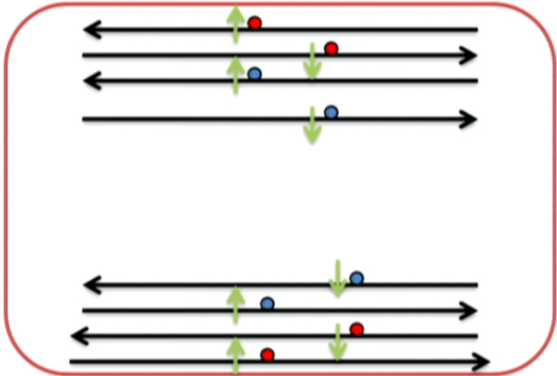
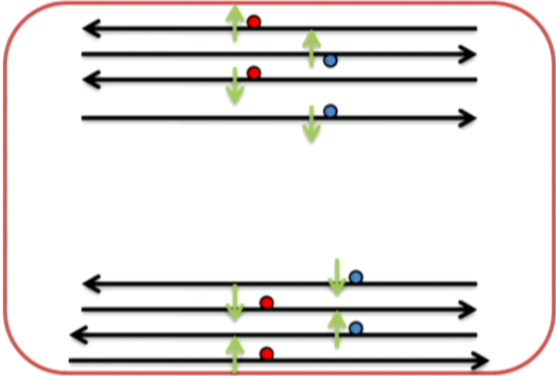
- Electron from valley K
- Electron from valley K'

Zero spin Hall conductivity.  
Nonzero valley Hall conductivity.  
Band insulator.



Nonzero spin Hall conductivity.  
Zero valley Hall conductivity.  
Topological insulator.

# Edge transport of 2D Topological Insulator



- Electron from valley K
- Electron from valley K'

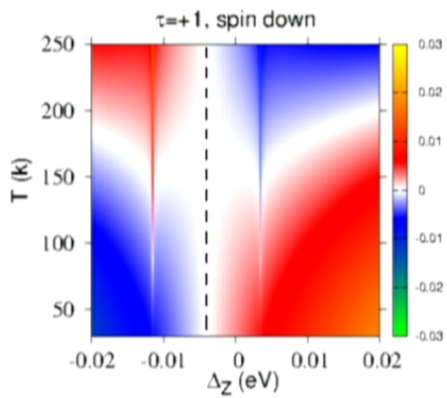
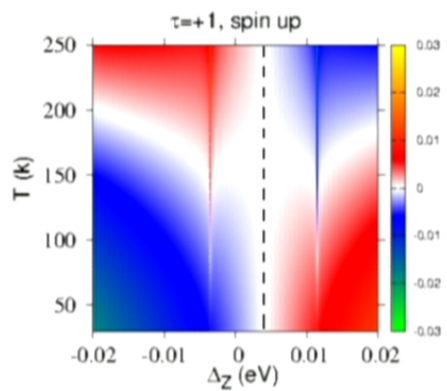
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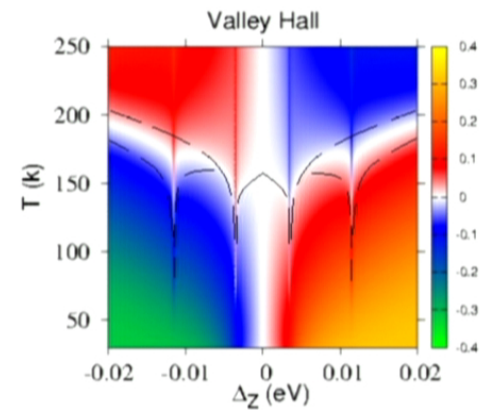
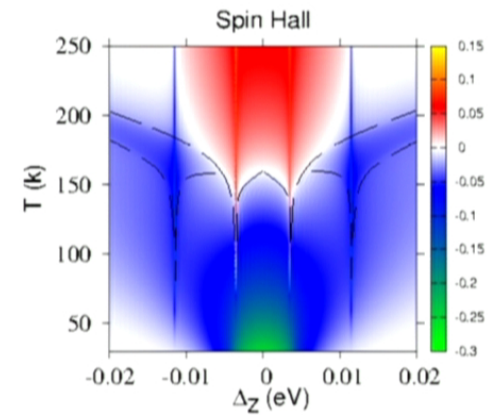
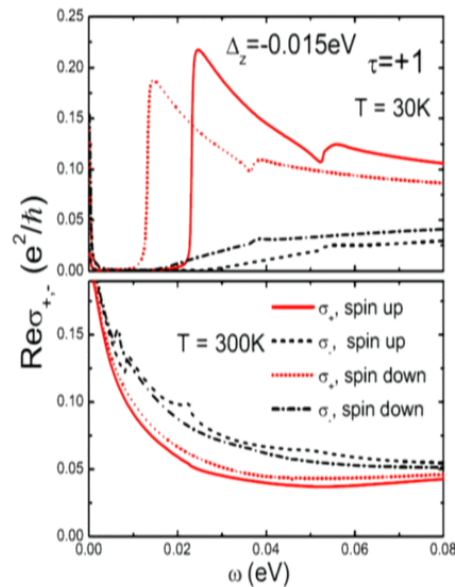
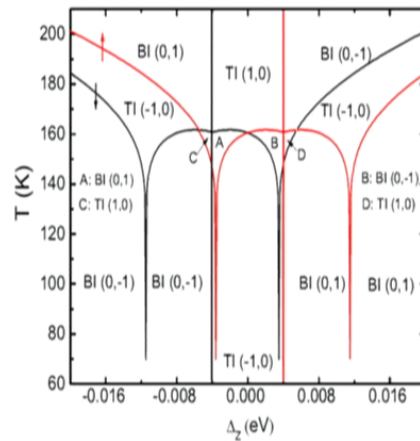
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# Topological crossovers due to phonons



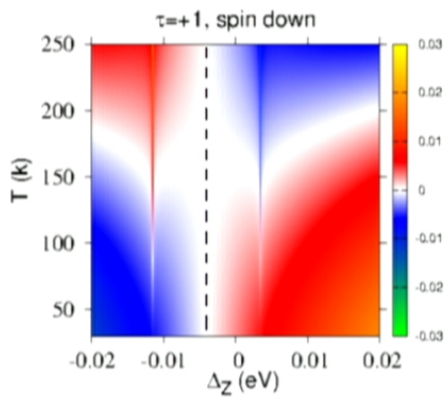
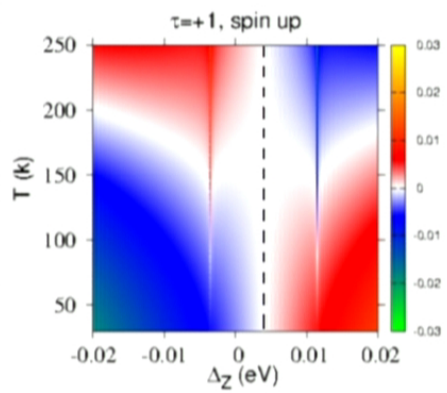
$$C = \frac{1}{4} \sum_{\tau, s_z} \tau [\text{sgn}(\tilde{\Delta}_{s_z}^{\tau})]$$



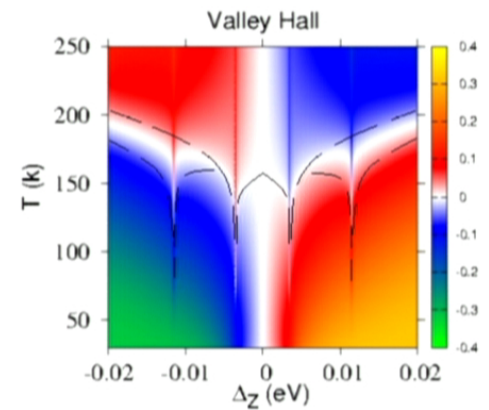
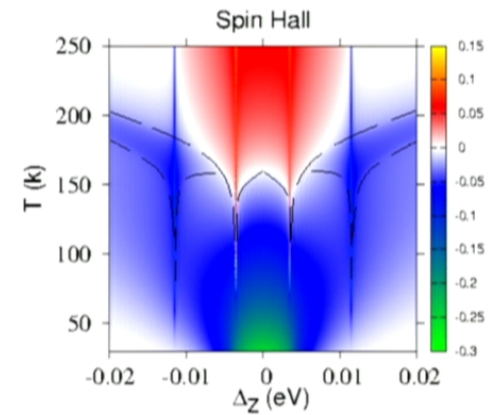
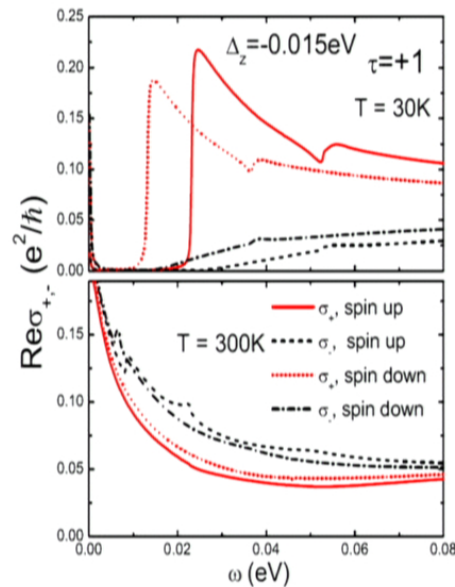
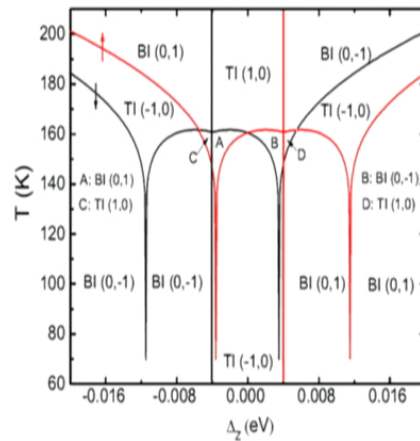
Zhou Li and J. P. Carbotte,  
In preparation.



# Topological crossovers due to phonons

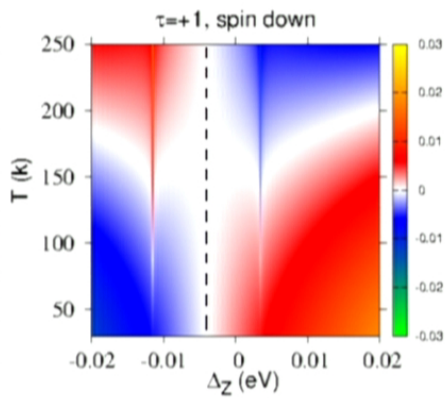
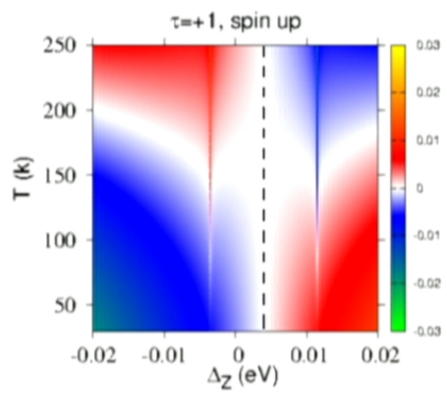


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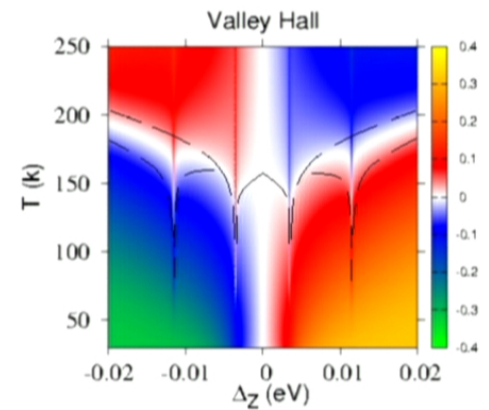
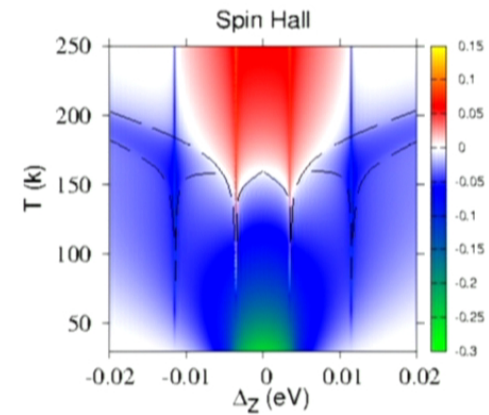
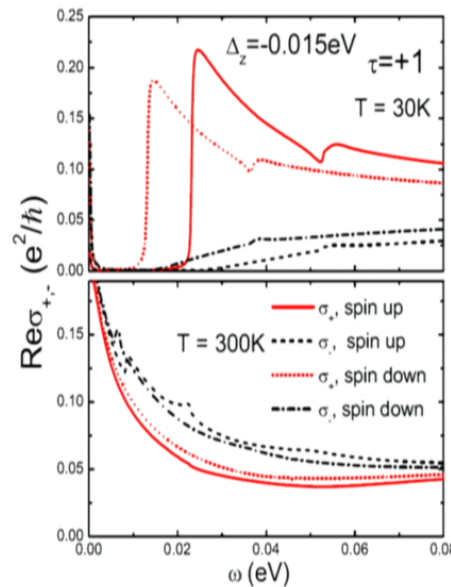
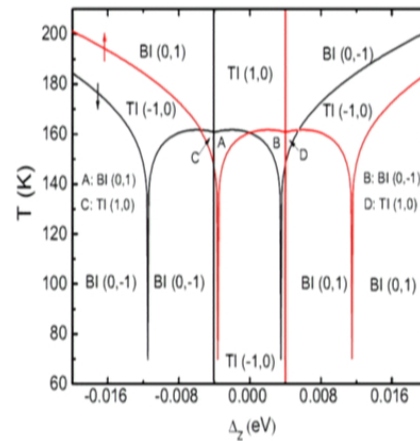


Zhou Li and J. P. Carbotte,  
In preparation.

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