

Title: Tensor Networks: an Overview

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URL: <http://pirsa.org/13040131>

Abstract: Tensor network algorithms provide highly competitive tools for analyzing ground state properties of quantum lattice models in one and two spatial dimensions. The most notable examples involve matrix product states, projected entangled pair states and multiscale entanglement renormalization ansatz. The key underlying idea of all the approaches is to decompose a quantum many-body state into a carefully chosen network of tensors.
In this talk I will give an introduction to the subject and show how tensor networks can be used to characterize topological order.

Tensor Networks: an overview

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Perimeter Institute

April 25th, 2013

plan

- ▶ introduction to tensor networks
 - ▶ motivation
 - ▶ examples
 - ▶ MPS: efficient description
- ▶ example of application: characterizing topological order emerging from a microscopic 2D Hamiltonian

motivation

- ▶ simulating N quantum systems

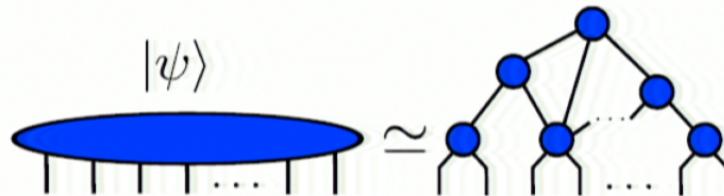


$$\mathcal{H} = \sum_{i=1}^N h^{[i,i+1]}$$

- ▶ exact diagonalization
 - ▶ \mathcal{H} : $d^N \times d^N$ matrix

$$|\psi\rangle = \sum_{i_1, \dots, i_N=1}^d \psi_{i_1, \dots, i_N} |i_1, \dots, i_N\rangle$$

- ▶ $\{\psi_{i_1, \dots, i_N}\}$: d^N coefficients
- ▶ tensor network



motivation

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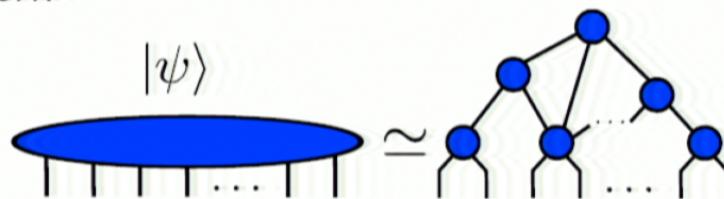


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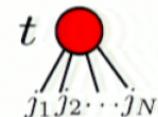


notation

a  a , number

A  A_{ij} , matrix

v  v_j , vector

t  $t_{j_1 j_2 \dots j_N}$, rank N tensor

$$w = v \cdot A$$

multiplication, $w_j = \sum_k v_k A_{kj}$

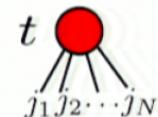
► exercise:

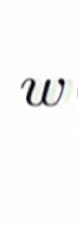
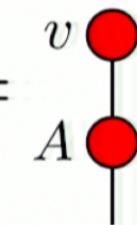
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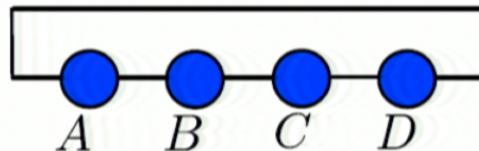
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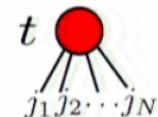


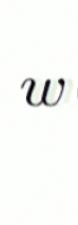
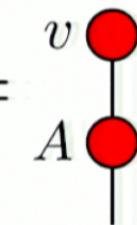
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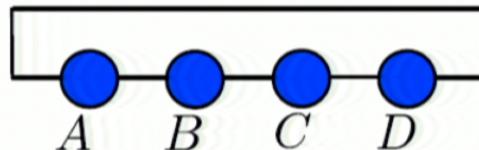
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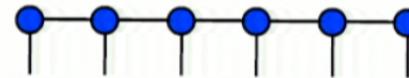
► exercise:



$\text{Tr}(ABCD)$

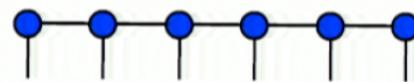
examples, computational costs

► MPS $O(\chi^3)$

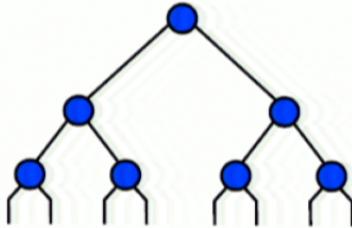


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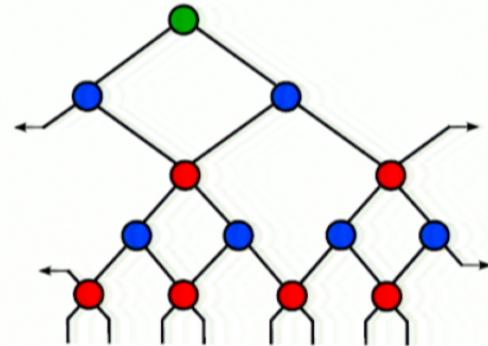
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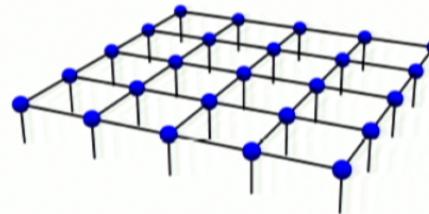
- ▶ TTN $O(\chi^4)$



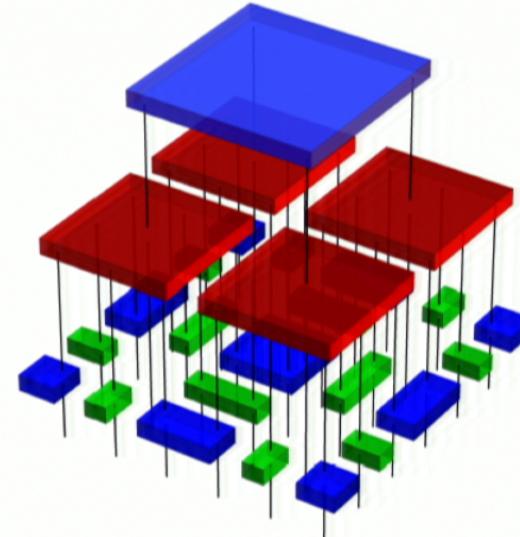
- ▶ 1D MERA $O(\chi^9)$



- ▶ PEPS

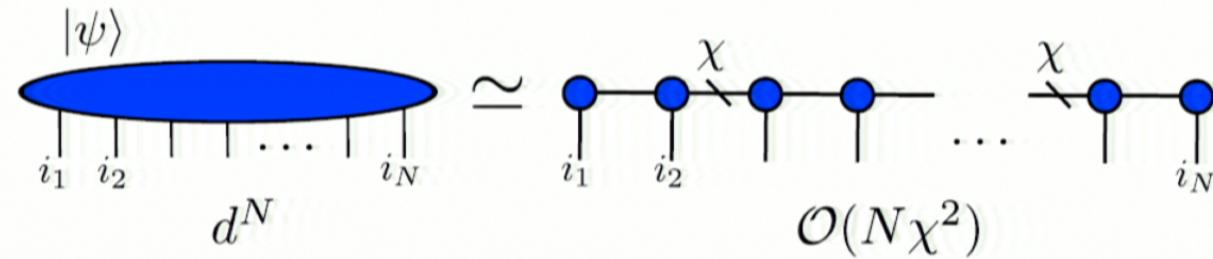


- ▶ 2D MERA $O(\chi^{16})$



MPS: efficiency

- ▶ number of parameters needed to specify wave function



motivation

- ▶ simulating N quantum systems

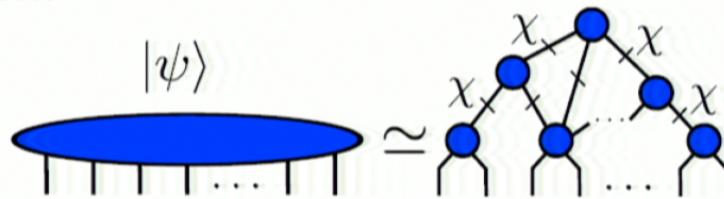


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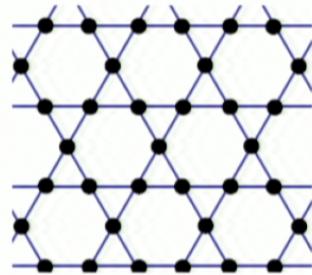
- ▶ $\{\psi_{i_1, \dots, i_N}\}$: d^N coefficients
- ▶ tensor network



application: characterizing topological order (with G. Vidal, PRL 2013)

input:

microscopic 2D lattice
Hamiltonian \mathcal{H}



output:

characterization of emergent anyon model

bulk: anyon model

- ▶ quantum dimensions
- ▶ fusion rules
- ▶ twists
- ▶ central charge

(contained in modular S and U matrices)

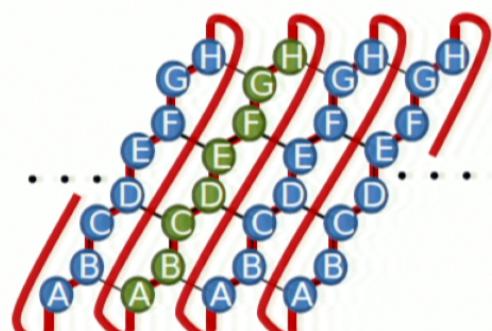
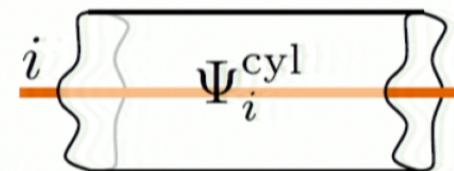


edge: CFT

- ▶ primary fields
- ▶ scaling dimensions

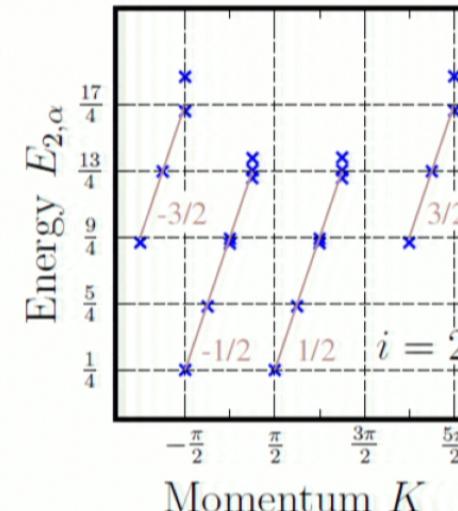
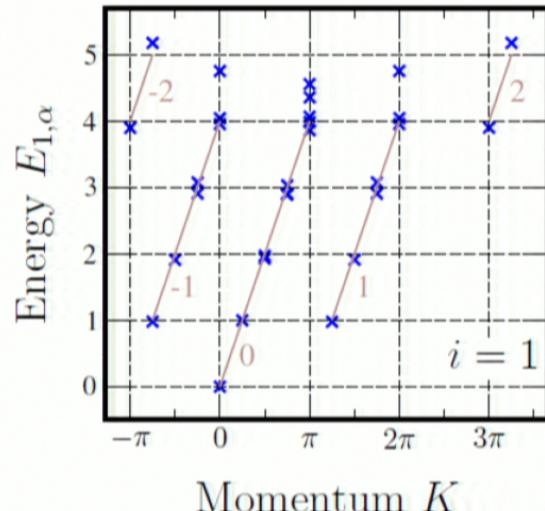
► 1) infinite cylinder

- complete set of ground states $\{|\Psi_i^{\text{cyl}}\rangle\}$
- $|\Psi_i^{\text{cyl}}\rangle$ - well-defined anyon flux inside cylinder
- edge CFT from entanglement spectrum



$$\mathcal{H} \longrightarrow |\Psi_1^{\text{cyl}}\rangle, |\Psi_2^{\text{cyl}}\rangle$$

entanglement spectrum: edge chiral CFT



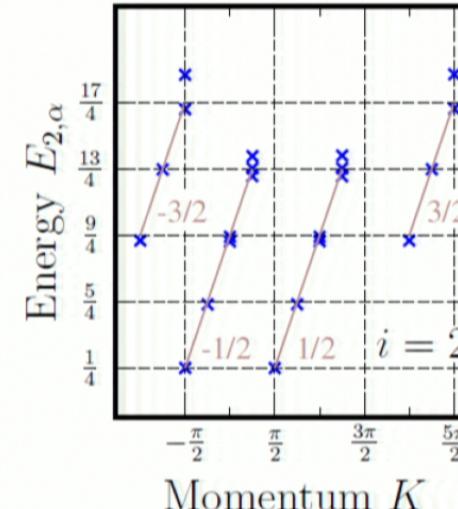
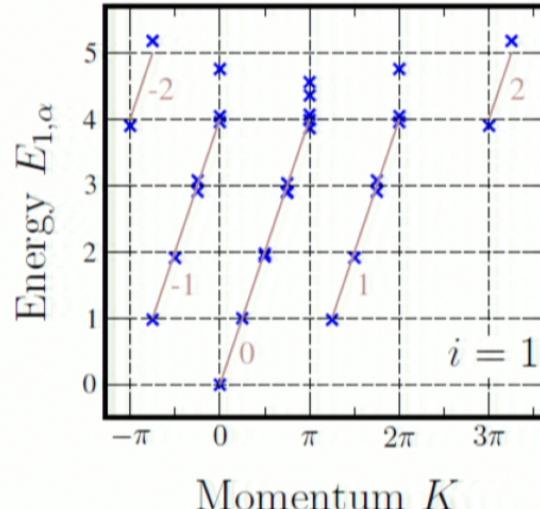
L_0	m					$su(2)$ decomposition
	-2	-1	0	1	2	
0			1			(0)
1		1	1	1		(2)
2		1	2	1		(2)+(0)
3		2	3	2		2(2)+(0)
4	1	3	5	3	1	(4)+2(2)+2(0)
5	1	5	7	5	1	(4)+4(2)+2(0)
6	2	7	11	7	2	2(4)+5(2)+4(0)

L_0	-2	-1	0	1	2	3	$su(2)$ decomposition
$\frac{1}{4}$			1	1			(1)
$\frac{5}{4}$			1	1			(1)
$\frac{9}{4}$			1	2	2	1	(3)+(1)
$\frac{13}{4}$			1	3	3	1	(3)+2(1)
$\frac{17}{4}$			2	5	5	2	2(3)+3(1)
$\frac{21}{4}$			3	7	7	3	3(3)+4(1)
$\frac{25}{4}$	1	5	11	11	5	1	(5)+4(3)+6(1)



P. Di Francesco, P. Mathieu, D. Sénéchal, *Conformal Field Theory*, 1997

entanglement spectrum: edge chiral CFT



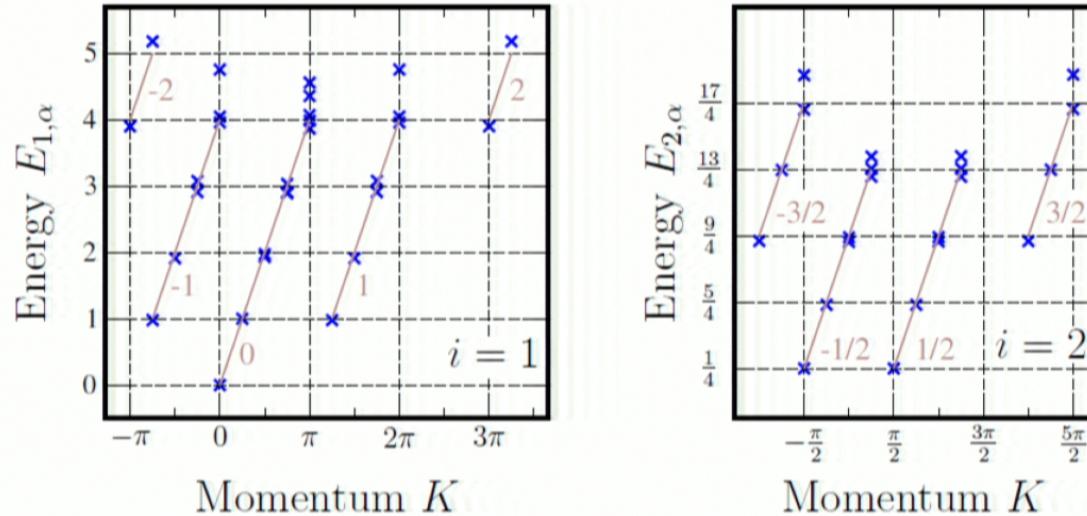
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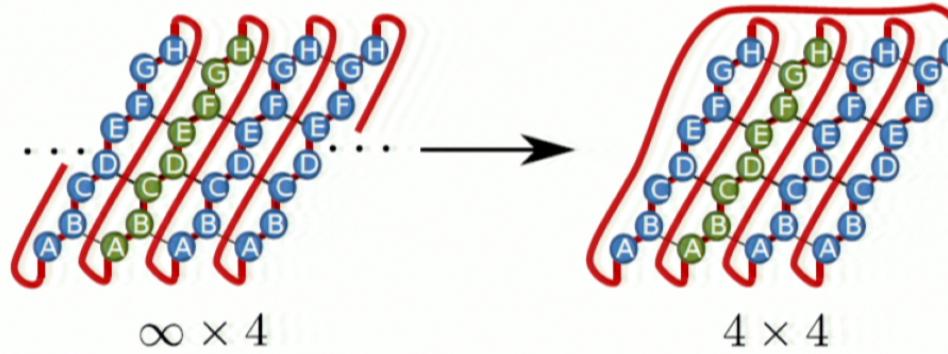
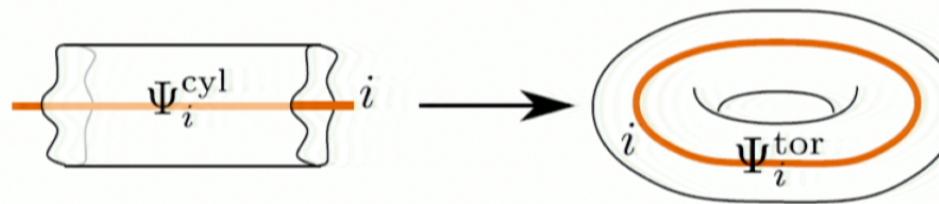
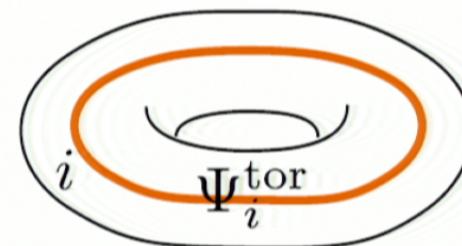


chiral $SU(2)_1$ Wess-Zumino-Witten CFT

- ▶ primary fields
 - ▶ $|\Psi_1^{\text{cyl}}\rangle \longleftrightarrow$ identity, singlet
 - ▶ $|\Psi_2^{\text{cyl}}\rangle \longleftrightarrow$ chiral vertex boson operator $e^{i\phi/\sqrt{2}}$, doublet
- ▶ Kac-Moody-Virasoro descendants

► 2) infinite cylinder \rightarrow finite torus

- quasi-degenerate ground subspace on a torus
- modular S and U matrices for the bulk anyon model



S and U from modular transformations

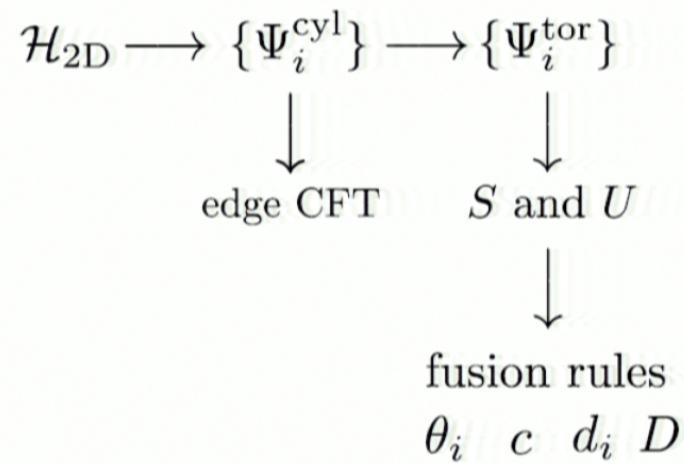
- $L_y = 6$, $6 \times 6 \times 2 = 72$ sites, Monte Carlo sampling:

$$S = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} + \frac{10^{-3}}{\sqrt{2}} \begin{bmatrix} -1.4 & 0.2 \\ -1.4 & 4+4i \end{bmatrix}$$
$$U = e^{-i\frac{2\pi}{24}} \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \times \left(e^{i\frac{2\pi}{24}0.01} \begin{bmatrix} 1 & 0 \\ 0 & e^{-i0.007} \end{bmatrix} \right)$$

chiral semion *deviation*

summary

- ▶ tensor networks
 - ▶ efficient representation of a many body wave function
- ▶ characterizing topological order



► 2) infinite cylinder \rightarrow finite torus

- quasi-degenerate ground subspace on a torus
- modular S and U matrices for the bulk anyon model

