Title: Transporting non-Gaussianity from sub- to super- horizon scales

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Abstract: The non-Gaussian statistics of the primordial density perturbation have become a key test of the inflationary scenario of the very early universe. Currently many techniques are used to calculate the non-Gaussian signatures of a given model of inflation. In particular, simple super-horizon techniques such as the deltaN formalism are often used for models with more than one field, while more technical field theory techniques, referred to as the In-In formalism, are typically used for models where the quantum sub-horizon evolution is important. Recently we have been developing an alternative point of view, called the transport approach. This framework highlights the connections between these other techniques and unifies them. Moreover, since it reduces the problem of calculating the statistics to that of solving a set of coupled ODEs, numerical implementations of the system are extremely simple.

Transporting non-Gaussianity

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arXiv:1302.4636, DJM
arXiv:0909.225, arXiv:1008.3159, arXiv:1205.0024
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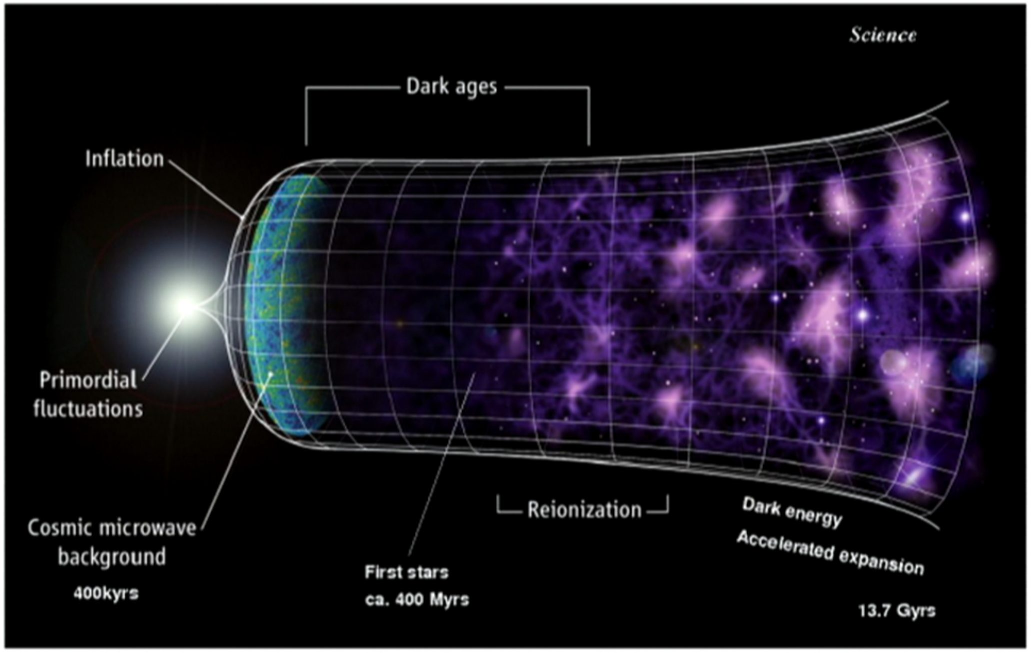
Overview

1. Introduction and motivation
   - Inflation
   - Inflationary perturbations and their statistics
2. The transport approach
   - Classical transport equations
   - Quantum transport equations
   - Transport connections
3. Conclusions and future directions

Evolution of the Universe and Inflation

The cosmological standard model



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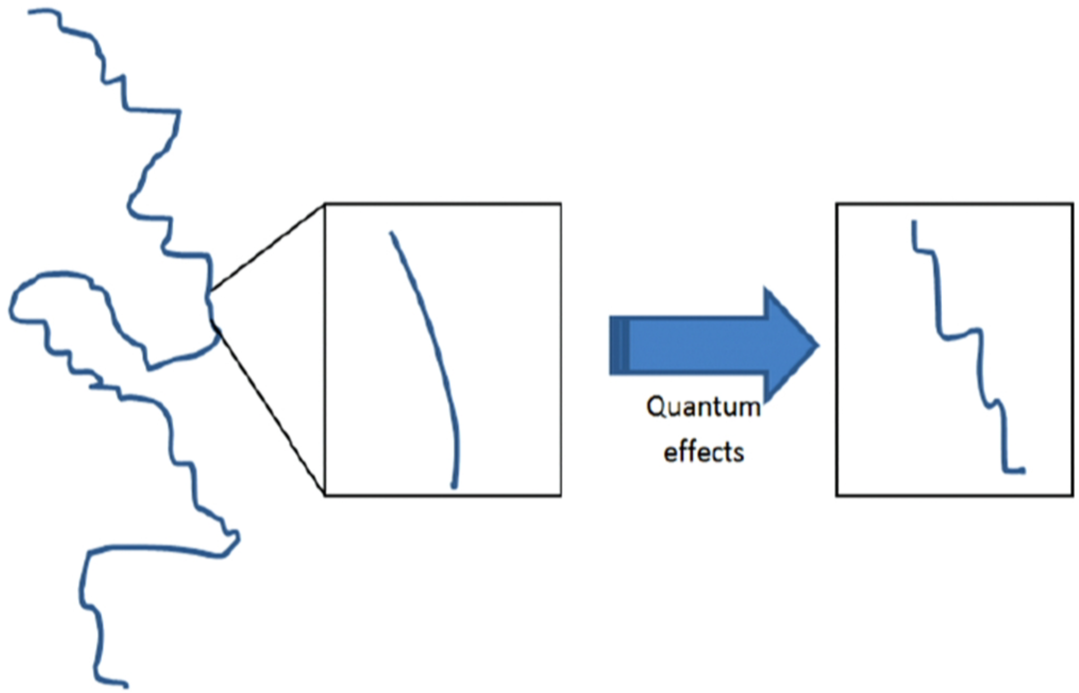

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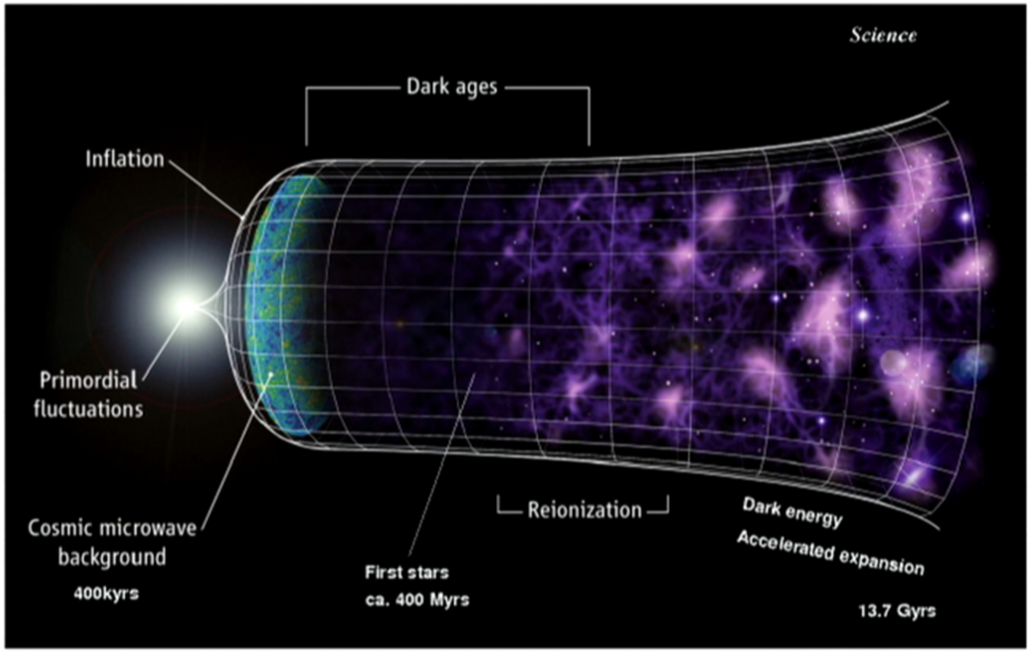
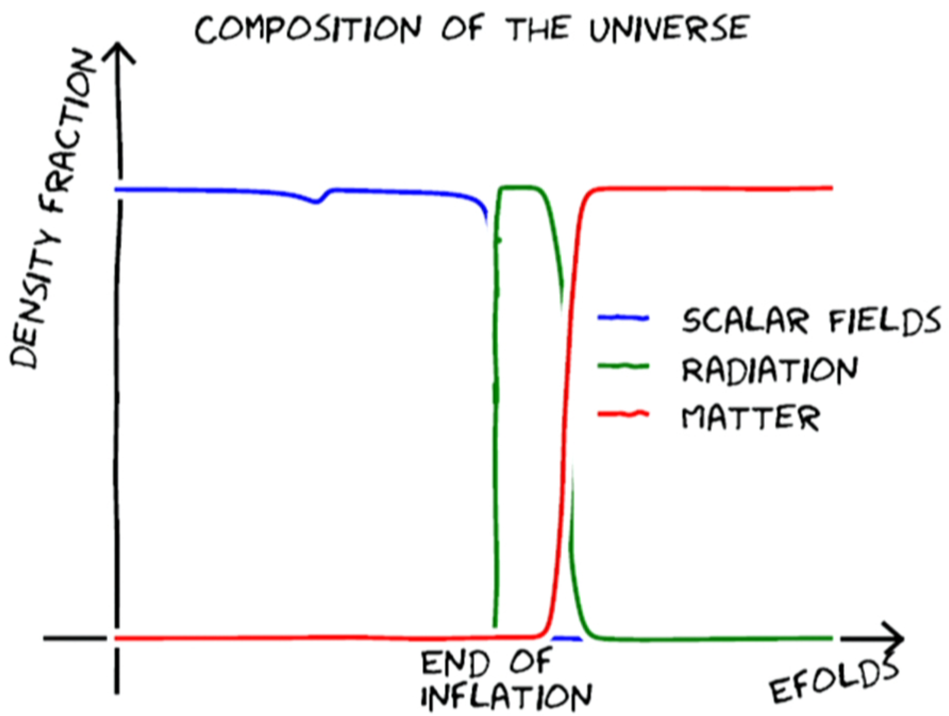


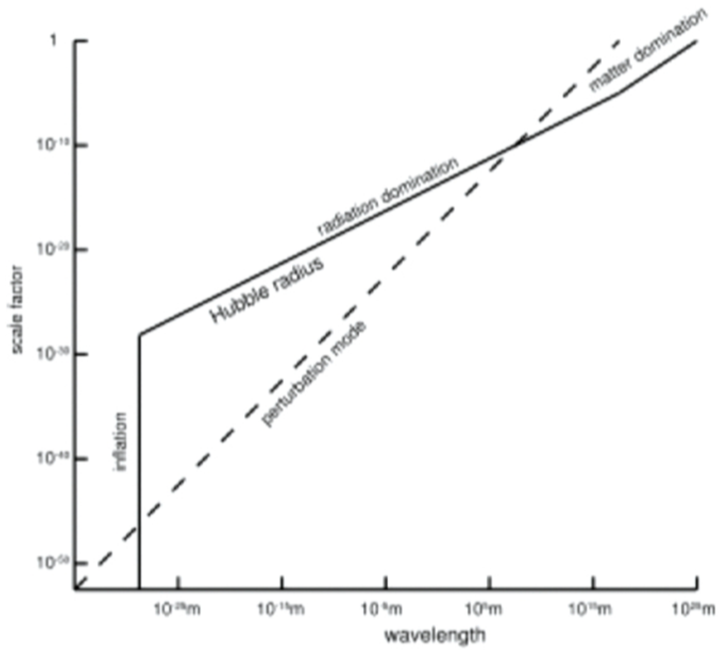
Inflation: Half the history of the universe



Courtesy of Ian Huston

Inflation: Crossing the horizon

A perturbation $\delta\phi(k)$ 'crossed outside horizon' when $k < aH$ – quantum to classical transition



From universe-galaxies-stars.com

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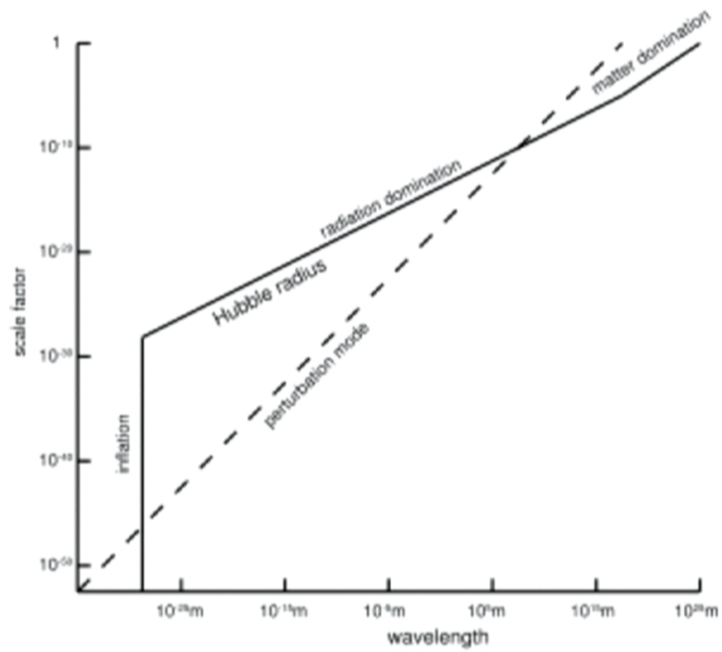

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Inflation: Motivation

So what are the aims of studying inflationary perturbations?

- Improving observations e.g. $f_{\mathsf{NL}}^{\mathrm{loc}} = 38.4 \pm 23.6$ Regan *et al.* (2013) $f_{\mathsf{NL}}^{\mathrm{loc}} = 4.3 \pm 5$ Planck (2013)
- Connect theory with observation – inform fundamental theory
- Immediately – Vanilla single field? Multi-field? Non-canonical?
- With improving observations – rule models in or out *with precision*
- Reconstruct features – $P(\dot{\phi}_1, ...)$, $V(\phi_1, ..)$?
- Requires numerics
- Help to inform what observers should look for

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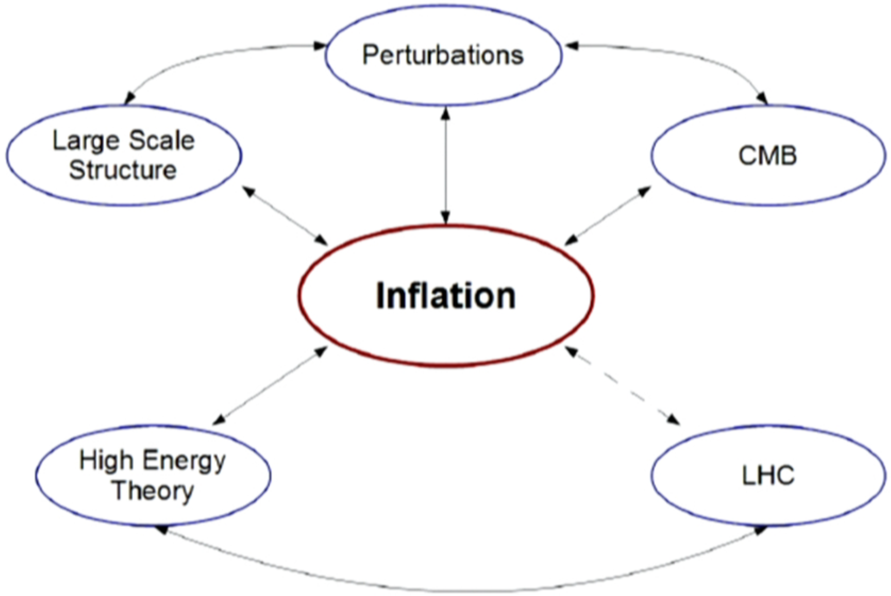

The statistics of inflationary perturbations: A simple view

We care about the statistics of density/curvature/field perturbations – usually finally phrased in terms of $\zeta(\mathbf{x})$. Becomes classical at horizon crossing, can evolve thereafter

Coarse grained on a particular scale, we can consider only one object, ζ:

$$\langle\zeta\zeta\rangle \ , \ \langle\zeta\zeta\zeta\rangle$$

$$f_{\mathsf{NL}}^{\mathrm{loc}} \sim \frac{\langle\zeta\zeta\zeta\rangle}{\langle\zeta\zeta\rangle^2}$$

The statistics of inflationary perturbations: A simple view

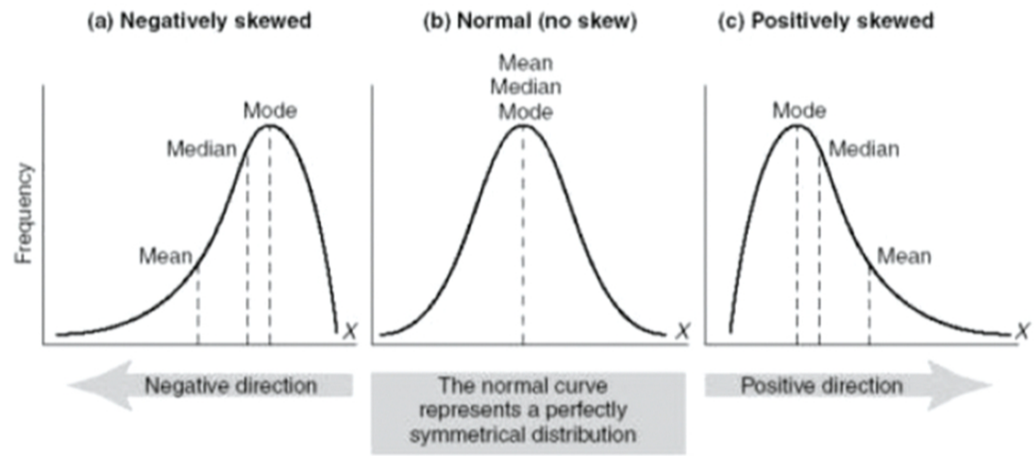

FIGURE 15.6 Examples of normal and skewed distributions

The statistics of inflationary perturbations: More detail

For most applications we are interested in the moments

$$\langle\zeta(k_1)\zeta(k_2)\rangle = (2\pi)^3 P(k_1)\delta^3(\mathbf{k_1}+\mathbf{k_2})$$
$$\langle\zeta(k_1)\zeta(k_2)\zeta(k_3)\rangle = (2\pi)^3 B(k_1,k_2,k_3)\delta^3(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3})$$

where for inflation

$$P(k) \underset{\sim}{\propto} k^{-3}$$

and for ‘vanilla’ inflation

$$f_{\mathsf{NL}}(k_1,k_2,k_3) \sim \text{slow roll parameters}$$

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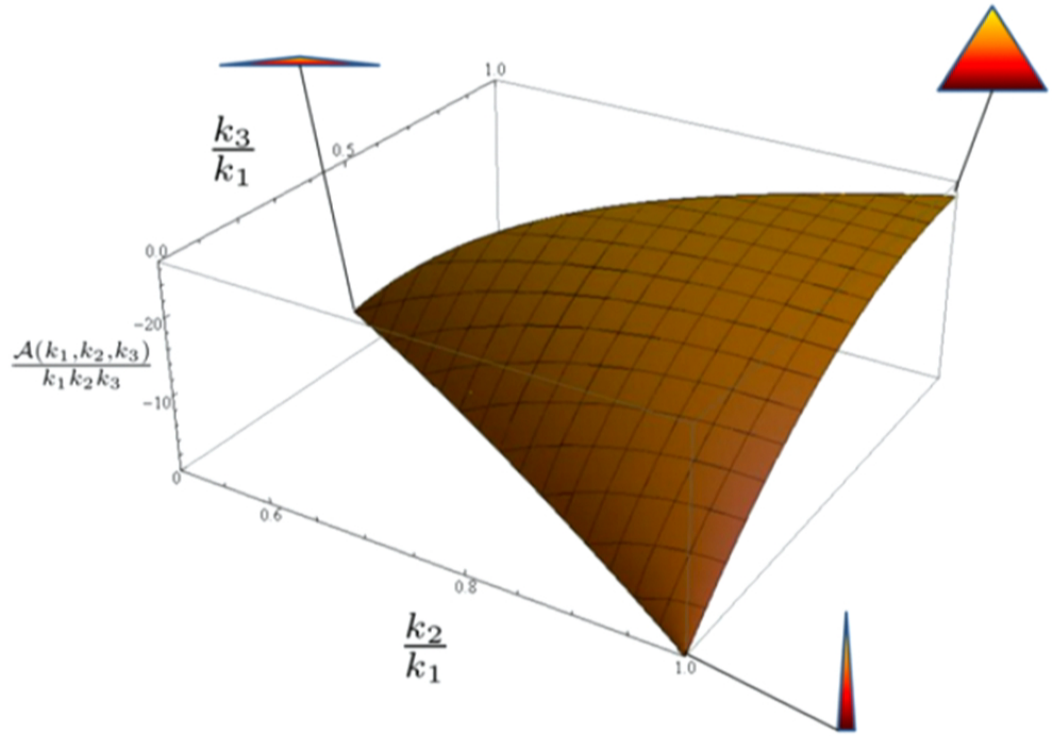

Bispectrum of power law 3-form inflation, Mulryne, Noller and Nunes (2012)

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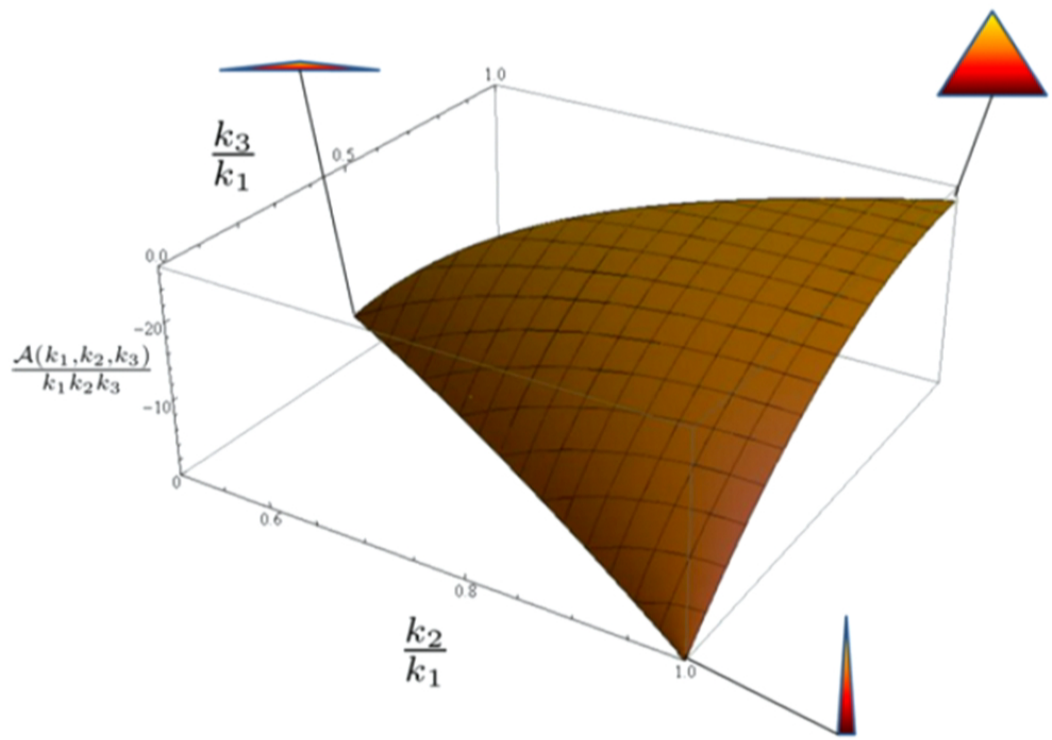
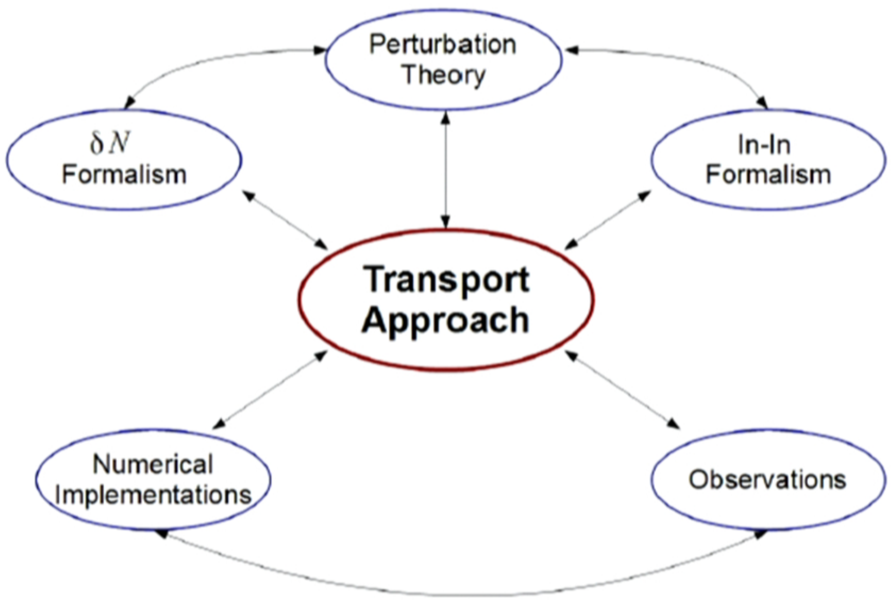


Bispectrum of power law 3-form inflation, Mulryne, Noller and Nunes (2012)

Transport approach: Motivation

- Complicated models (many fields, ICs and parameters)
- Can give rise to a large non-Gaussianity
  e.g. Lyth and Wands (2002), Byrnes, Choi, Hall (2008), Kim, Liddle and Seery (2010), Elliston, Mulryne, Seery and Tavakol (2011)
- Many methods exist to calculate NG, how are they related?
  e.g. δN, Lyth and Rodriguez (2005), 'Backwards formalism', Yokoyama et al. (2007), In-In formalism, Maldacena (2003), Seery and Lidsey (2005), Chen *et al.* (2006), Perturbation theory
- For probing large parameter space, or for precise predictions → numerics and ODE methods
- Is there a unified approach – not splitting sub and super-horizon dynamics?

Transport approach: Motivation



Transport approach: Some details

Consider inflation sourced by M scalar fields, perturbations $\delta\varphi_a$, which together with $\delta\dot{\varphi}_b$ form a complete phase space, x_α

Aim. Evolution equations for cummulants/correlation functions of statistical distribution of fluctuations

Could start from equations for full probability distribution $P(\boldsymbol{x}, N)$, which satisfies

$$\frac{\partial P(\boldsymbol{x}, N)}{\partial N} + \frac{\partial}{\partial x_\alpha}\left[u_\alpha P(\boldsymbol{x}, N)\right] = 0$$

Itself useful if full probability distribution is required

eg primordial black holes, Hidalgo (2005), Byrnes, Copeland and Green (2012)

This can be decomposed into ODEs for moments/cumulants/correlation functions of the perturbations

Transport approach: Some details

One way to do this is to make an expansion of the full distribution

$$P(\boldsymbol{x}) = P_g(\boldsymbol{x}) \left(1 + \frac{\alpha_{\alpha\beta\gamma} H_{\alpha\beta\gamma}}{6} + ...\right) \quad , \quad \alpha_{\alpha\beta\gamma} \equiv \langle x_\alpha x_\beta x_\gamma \rangle$$

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Or we can use the simple principal

$$\mathrm{d}\langle \mathcal{O} \rangle / \mathrm{d}N = \langle \mathrm{d}\mathcal{O} / \mathrm{d}N \rangle$$

Lets use the second route, move to Fourier space, and see what we get

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In compact notation cosmological perturbation theory leads to equations of form

$$\frac{\mathrm{d}x_{\alpha'}}{\mathrm{dN}} = u_{\alpha'\beta'}x_{\beta'} + \frac{1}{2!}u_{\alpha'\beta'\gamma'}\Big(x_{\beta'}x_{\gamma'} - \langle x_{\beta'}x_{\gamma'}\rangle\Big) + \ldots$$

For example, $\delta\varphi_{1'} = \delta\varphi_1(k_1)$,

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$$u_{\alpha'\beta'} = (2\pi)^3 u_{\alpha\beta}(k_\alpha)\delta(\mathbf{k}_\alpha - \mathbf{k}_\beta)\ ,\ \ u_{\alpha'\beta'\gamma'} = (2\pi)^3 u_{\alpha\beta\gamma}(k_\alpha, k_\beta, k_\gamma)\delta(\mathbf{k}_\alpha - \mathbf{k}_\beta - \mathbf{k}_\gamma)$$

u coefficients depend on background evolution $P(\dot{\phi}_1, ...)$ and $V(\phi_1, ...)$, and

$$u_{\alpha'\beta'}x_{\beta'} = \frac{1}{(2\pi)^3}\int \mathrm{d}^3k_\beta (2\pi)^3 u_{\alpha\beta}(k_\alpha)\delta(\mathbf{k}_\alpha - \mathbf{k}_\beta)x_\beta(\mathbf{k}_\beta) = u_{\alpha\beta}(k_\alpha)x_\beta(\mathbf{k}_\alpha)$$

And schematically, nonlinear terms lead to $\mathrm{d}x_\alpha(k_\alpha)/\mathrm{d}N \supseteq u_{\alpha\beta\gamma}x_\beta * x_\gamma(k_\alpha)$

Transport approach: Some details

With $\langle \mathcal{O} \rangle = \langle x_{\alpha'} x_{\beta'} \rangle = \Sigma_{\alpha'\beta'}$ and similarly for three point function $\alpha_{\alpha'\beta'\gamma'}$

Mulryne, Seery, Wesley (2009 and 2010), Anderson, Mulryne, Seery (2012)

$$\frac{\mathrm{d}\Sigma_{\alpha'\beta'}}{\mathrm{d}N} = u_{\alpha'\gamma'}\Sigma_{\gamma'\beta'} + u_{\beta'\gamma'}\Sigma_{\gamma'\alpha'}$$

$$\frac{\mathrm{d}\alpha_{\alpha'\beta'\gamma'}}{\mathrm{d}N} = u_{\alpha'\lambda'}\alpha_{\lambda'\beta'\gamma'} + u_{\alpha'\lambda'\mu'}\Sigma_{\lambda'\beta'}\Sigma_{\mu'\gamma'} + \text{cyclic}$$

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These equations are **simpler** than raw perturbation equations (no convolutions) since

$$\Sigma_{\alpha'\beta'} = (2\pi)^3\delta(\mathbf{k}_\alpha + \mathbf{k}_\beta)\Sigma_{\alpha\beta}(k_\alpha)$$

$$\alpha_{\alpha'\beta'\gamma'} = (2\pi)^3\delta(\mathbf{k}_\alpha + \mathbf{k}_\beta + \mathbf{k}_\gamma)\alpha_{\alpha\beta\gamma}(k_\alpha, k_\beta, k_\gamma)$$

$$\Rightarrow \quad \frac{\mathrm{d}\Sigma_{\alpha\beta}(k_\alpha)}{\mathrm{dN}} = u_{\alpha\gamma}(k_\alpha)\Sigma_{\gamma\beta}(k_\alpha) + u_{\beta\gamma}(k_\alpha)\Sigma_{\gamma\alpha}(k_\alpha)$$

$$\frac{\mathrm{d}\alpha_{\alpha\beta\gamma}(k_\alpha, k_\beta, k_\gamma)}{\mathrm{d}N} = u_{\alpha\lambda}(k_\alpha)\alpha_{\lambda\beta\gamma}(k_\alpha, k_\beta, k_\gamma) + u_{\alpha\lambda\mu}(k_\alpha, k_\beta, k_\gamma)\Sigma_{\lambda\beta}(k_\beta)\Sigma_{\mu\gamma}(k_\gamma) + \text{cyclic}$$

Transport approach: Further properties

Many further attractive properties, for example

- Easy to convert to the statistics of ζ
- Evolution can be decomposed into equations for 'shapes' such as local f_{NL}, τ_{NL} and g_{NL}
  Anderson, Mulryne and Seery (2012)
- A geometrical decomposition and interpretation
  Seery, Mulryne, Frazer and Ribeiro (2012)
- Method can be easily extended to quantum sub-horizon regime
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- Moreover, integral solutions to transport hierarchy in terms of 'Γ' matrices are possible – connect to the **integral** solutions of the In-In formalism
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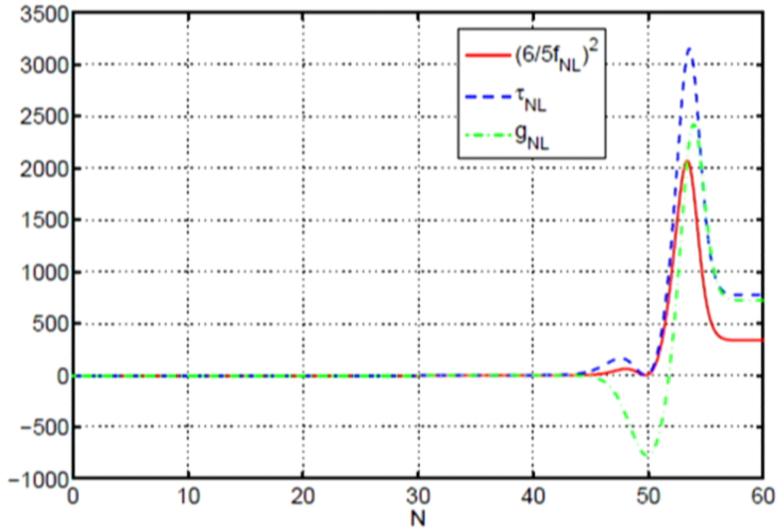
Transport approach: Applications

These methods have been used in a number of studies. In particular where very many initial conditions or parameter choices need to be probed rapidly

Frazer and Liddle (2011), Dias, Frazer and Liddle (2012)

A simple example (Mulryne, Orani and Rajantie (2011))

$$V = M^4 \left[\frac{1}{2} m^2 \phi_1^2 + \frac{1}{2} g^2 \phi_1^2 \phi_2^2 + \frac{\lambda}{4} \left(\phi_2^2 - v^2 \right)^2 \right]$$



Quantum transport equations

Transport equations can be extended to quantum regime, for example using using Ehrenfest's theorem Mulryne (2013)

$$\frac{\mathrm{d}\langle\widehat{\mathcal{O}}\rangle}{\mathrm{d}N} = -\frac{i}{\hbar}\left\langle[\widehat{\mathcal{O}}, \widehat{H}]\right\rangle$$

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Given Heisenberg picture equations

$$\frac{\mathrm{d}\mathcal{O}}{\mathrm{d}N} = -\frac{i}{\hbar}\left[\widehat{\mathcal{O}},\widehat{H}\right]$$

this can be seen as to be equivalent of simple principal above

For us:

$$\frac{\mathrm{d}\widehat{x}_{\alpha'}}{\mathrm{d}N} = u_{\alpha'\beta'}\widehat{x}_{\beta'} + \frac{1}{2}u_{\alpha'\beta'\gamma'}(\widehat{x}_{\beta'}\widehat{x}_{\gamma'} - \langle\widehat{x}_{\beta'}\widehat{x}_{\gamma'}\rangle)$$

We arrive at similar equations to before – only difference is ordering – integral solutions equivalent to those of In-In

Quantum transport equations

Forming symmetrised correlation functions from:

$$\Sigma_{\alpha'\beta'} = \langle \widehat{x}_{\alpha'} \widehat{x}_{\beta'} \rangle$$

such that

$$\Sigma_{(\alpha'\beta')} \equiv \Sigma^{w}_{\alpha'\beta'}$$

and similarly the fully symmetric ordered three-point function as $\alpha^{w}_{\alpha'\beta'\gamma'}$,

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where $\Sigma^{i}_{\alpha'\beta'}$ is imaginary part

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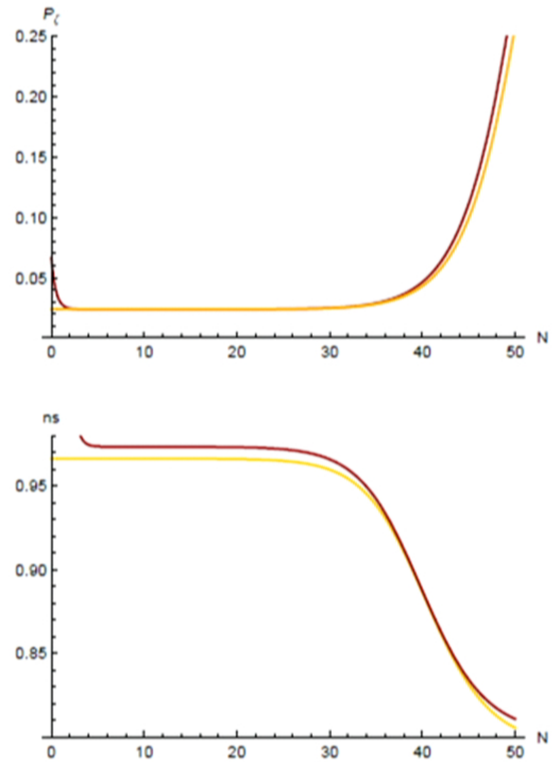
Quantum Transport equations: Applications

Preliminary stages, but as a simple example



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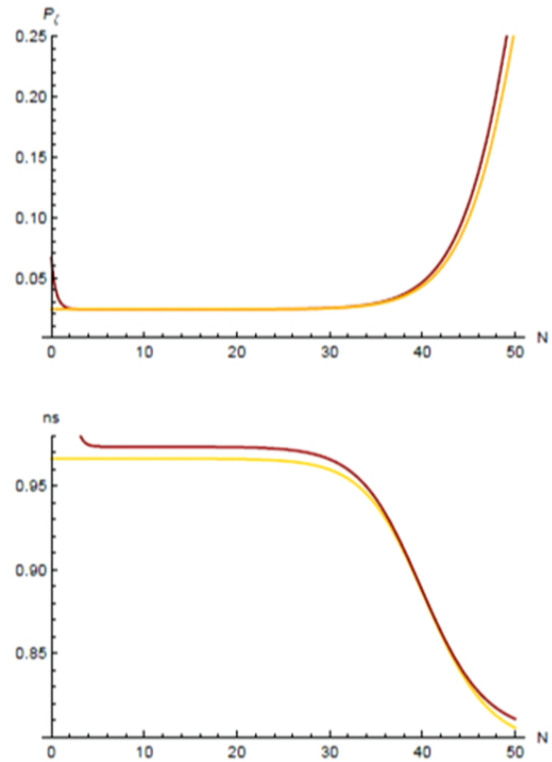

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Transport: Connection to δN

Consider a formal solution of form

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Substituting into EOM one finds

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Transport: Connection to δN

Hence, using

$$\Gamma_{\alpha' i'} = \Gamma_{\alpha i}(k_\alpha)\delta^3(\boldsymbol{k_\alpha} - \boldsymbol{k_i})$$

we can identify

$$\delta\varphi_\alpha = \Gamma_{\alpha i}\delta\varphi_i + \frac{1}{2!}\Gamma_{\alpha ij}\delta\varphi_i\delta\varphi_j + \frac{1}{3!}\Gamma_{\alpha ijk}\delta\phi_i\delta\varphi_j\delta\varphi_k + \cdots$$

and hence

$$\Gamma_{\alpha i} = \frac{\partial\varphi_\alpha(N)}{\partial\varphi_i(N_0)}$$

$$\Gamma_{\alpha ij} = \frac{\partial^2\varphi_\alpha(N)}{\partial\varphi_i(N_0)\varphi_j(N_0)}$$

Transport: Connection to In-In

A neat aspect is that the Γ matrices allow formal integral solutions

$$\Gamma_{\alpha i}(k_\alpha) = \mathcal{P}\exp\left(\int_{N_0}^{N_f} \mathrm{dN'}\ \mathrm{u}_{\tilde{\alpha}\tilde{\gamma}}(\mathrm{k}_\alpha)\right)\delta_{\tilde{\gamma}i}$$

$$\Gamma_{\alpha ij}(k_\alpha, k_i, k_j) = \int_{N_0}^{N_f} \mathrm{dN'}\Gamma_{\alpha\tilde{\mu}}(\mathrm{k}_\alpha)\mathrm{u}_{\tilde{\mu}\tilde{\nu}\tilde{\sigma}}(\mathrm{k}_\alpha, \mathrm{k_i}, \mathrm{k_j})\Gamma_{\tilde{\mu}\mathrm{i}}(\mathrm{k_i})\Gamma_{\tilde{\sigma}\mathrm{j}}(\mathrm{k_j})$$

Transport: Connection to In-In

A neat aspect is that the Γ matrices allow formal integral solutions

$$\Gamma_{\alpha i}(k_\alpha) = \mathcal{P}\exp\left(\int_{N_0}^{N_f} \mathrm{dN'}\ \mathrm{u}_{\tilde{\alpha}\tilde{\gamma}}(\mathrm{k}_\alpha)\right)\delta_{\tilde{\gamma}i}$$

$$\Gamma_{\alpha ij}(k_\alpha, k_i, k_j) = \int_{N_0}^{N_f} \mathrm{dN'}\Gamma_{\alpha\tilde{\mu}}(\mathrm{k}_\alpha)\mathrm{u}_{\tilde{\mu}\tilde{\nu}\tilde{\sigma}}(\mathrm{k}_\alpha, \mathrm{k}_\mathrm{i}, \mathrm{k}_\mathrm{j})\Gamma_{\tilde{\mu}\mathrm{i}}(\mathrm{k}_\mathrm{i})\Gamma_{\tilde{\sigma}\mathrm{j}}(\mathrm{k}_\mathrm{j})$$

and so

$$\begin{aligned} \alpha_{\alpha\beta\gamma} &= \Gamma_{\alpha i}(k_\alpha)\Gamma_{\beta j}(k_\beta)\Gamma_{\gamma k}(k_\gamma)\alpha_{ijk}(k_\alpha, k_\beta, k_\gamma) \\ &+ \left[\int_{N_0}^{N_f} \mathrm{d\tilde{N}}\Gamma_{\alpha\tilde{\mu}}(\mathrm{k}_\alpha)\mathrm{u}_{\tilde{\mu}\tilde{\nu}\tilde{\sigma}}(\mathrm{k}_\alpha, \mathrm{k}_\beta, \mathrm{k}_\gamma)\Sigma_{\beta\tilde{\nu}}(\mathrm{k}_\beta)\Sigma_{\gamma\tilde{\sigma}}(\mathrm{k}_\gamma)\right] \\ &+ \text{cyclic} \end{aligned}$$

Transport: Connection to In-In

A neat aspect is that the Γ matrices allow formal integral solutions

$$\Gamma_{\alpha i}(k_\alpha) = \mathcal{P}\exp\left(\int_{N_0}^{N_f} \mathrm{dN}'\, \mathrm{u}_{\tilde{\alpha}\tilde{\gamma}}(\mathrm{k}_\alpha)\right)\delta_{\tilde{\gamma}i}$$

$$\Gamma_{\alpha ij}(k_\alpha, k_i, k_j) = \int_{N_0}^{N_f} \mathrm{dN}'\Gamma_{\alpha\tilde{\mu}}(\mathrm{k}_\alpha)\mathrm{u}_{\tilde{\mu}\tilde{\nu}\tilde{\sigma}}(\mathrm{k}_\alpha, \mathrm{k}_\mathrm{i}, \mathrm{k}_\mathrm{j})\Gamma_{\tilde{\mu}\mathrm{i}}(\mathrm{k}_\mathrm{i})\Gamma_{\tilde{\sigma}\mathrm{j}}(\mathrm{k}_\mathrm{j})$$

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c.f. $\langle \hat{x}_{\alpha'}\hat{x}_{\beta'}\hat{x}_{\gamma'}\rangle = \alpha_{\alpha'\beta'\gamma'} = -i\int_{N_0}^{N_f} \mathrm{d}\tilde{\mathrm{N}}\langle\left[\hat{\mathrm{x}}_{\alpha'}\hat{\mathrm{x}}_{\beta'}\hat{\mathrm{x}}_{\gamma'}, \hat{\mathcal{H}}_{\mathrm{int}}(\tilde{\mathrm{N}})\right]\rangle$

Maldacena (2003), Chen *et al.* (2006) etc

Conclusions and future directions

Transport framework provides a complete approach to calculating non-Gaussianity, unifying other methods, and providing a suite of tools

Particularly useful in multiple field models of inflation

Numerically simple to implement

Future Directions:

- To produce a unified code implementation for the community
- To explore implications of full distribution – (in quantum arena?)

  e.g. structure formation and primordial black holes