Title: On memory in exponentially expanding spaces

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Abstract: I will present recent work, done in collaboration with Daniel Roberts, on the global memory of initial conditions that is sometimes, but not always, retained by fluctuating fields on de Sitter space, Euclidean anti de Sitter space, and regular infinite trees. I will discuss applications to the structure of configuration space in de Sitter space and eternal inflation.

What is memory?

Definition: in a system with a Markov update rule, *memory* is the persistence of correlation between the initial state and the state after infinitely many generations.

Broadcasting on a line

Markov system on a line:

$$P_n(u+1) = G_{nm} P_m(u).$$

For example,

$$G = \begin{pmatrix} 1-\gamma & \gamma \\ \gamma & 1-\gamma \end{pmatrix}$$

Broadcasting on a line

This Markov system is the transfer matrix solution of a 1d Ising chain, with the coupling determined by γ.

$$e^{2J} = \frac{P_{noflip}}{P_{flip}} = \frac{1-\gamma}{\gamma}.$$

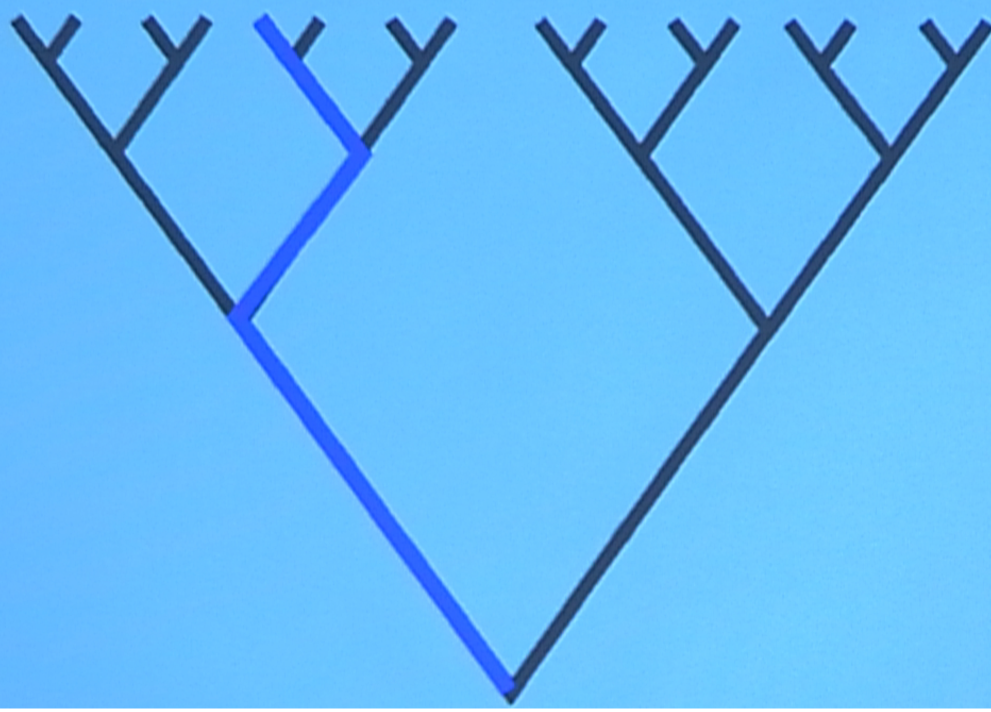
The absence of memory can be rephrased as the decay of the correlation function

$$\langle s(0)s(u)\rangle \sim (1-2\gamma)^u \to 0.$$

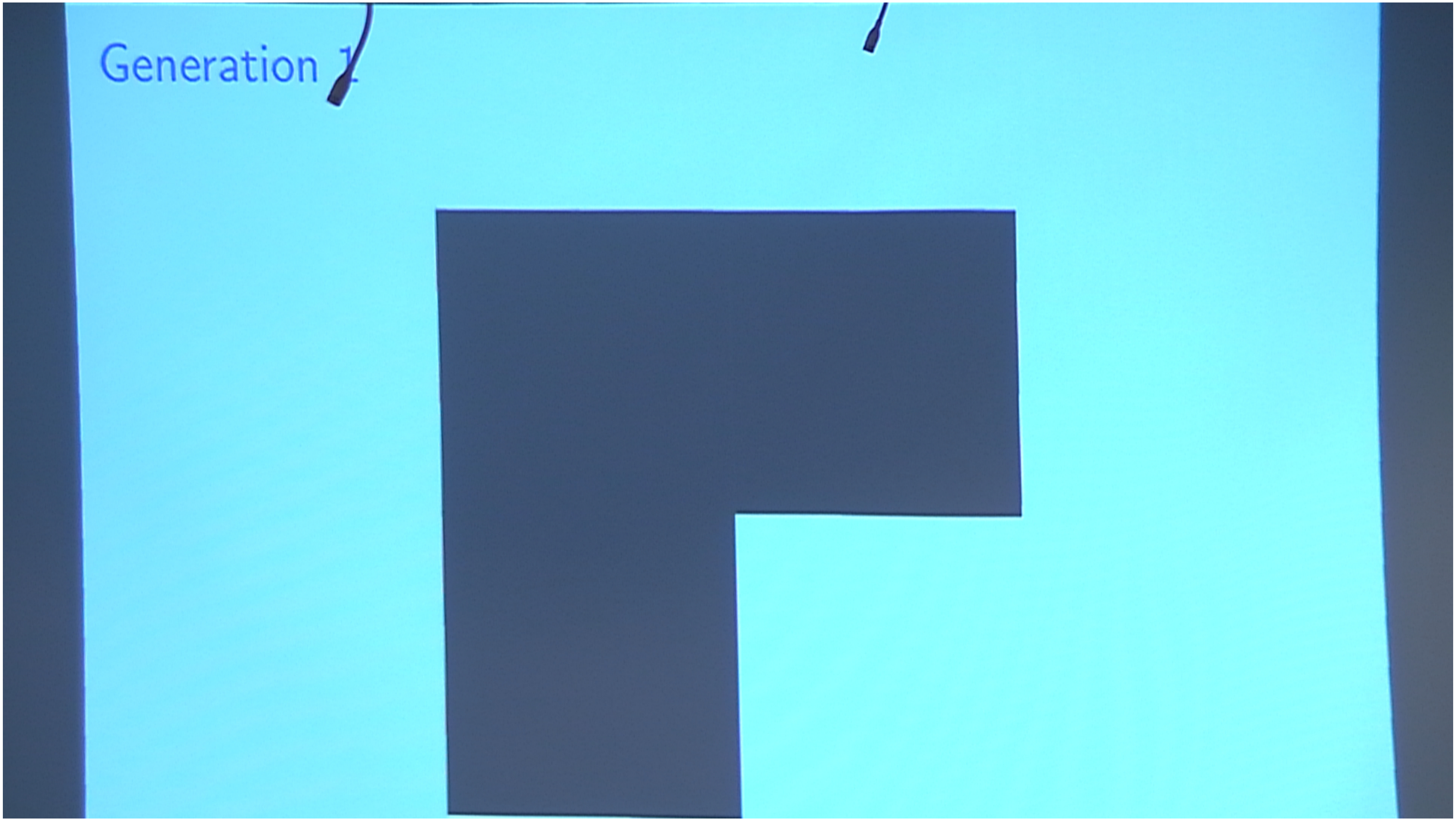
Cluster decomposition $\Longrightarrow$ no memory.

Broadcasting on a tree

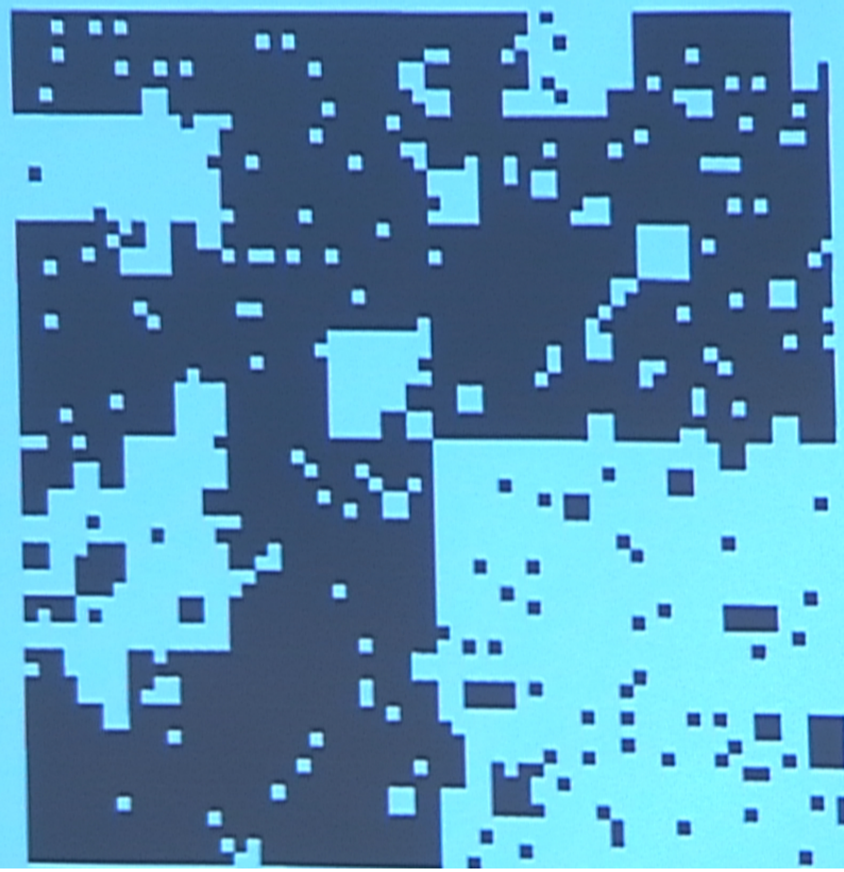
What if we replace the line with a tree graph? Along any branch, the evolution of the system is the same as the 1d version.

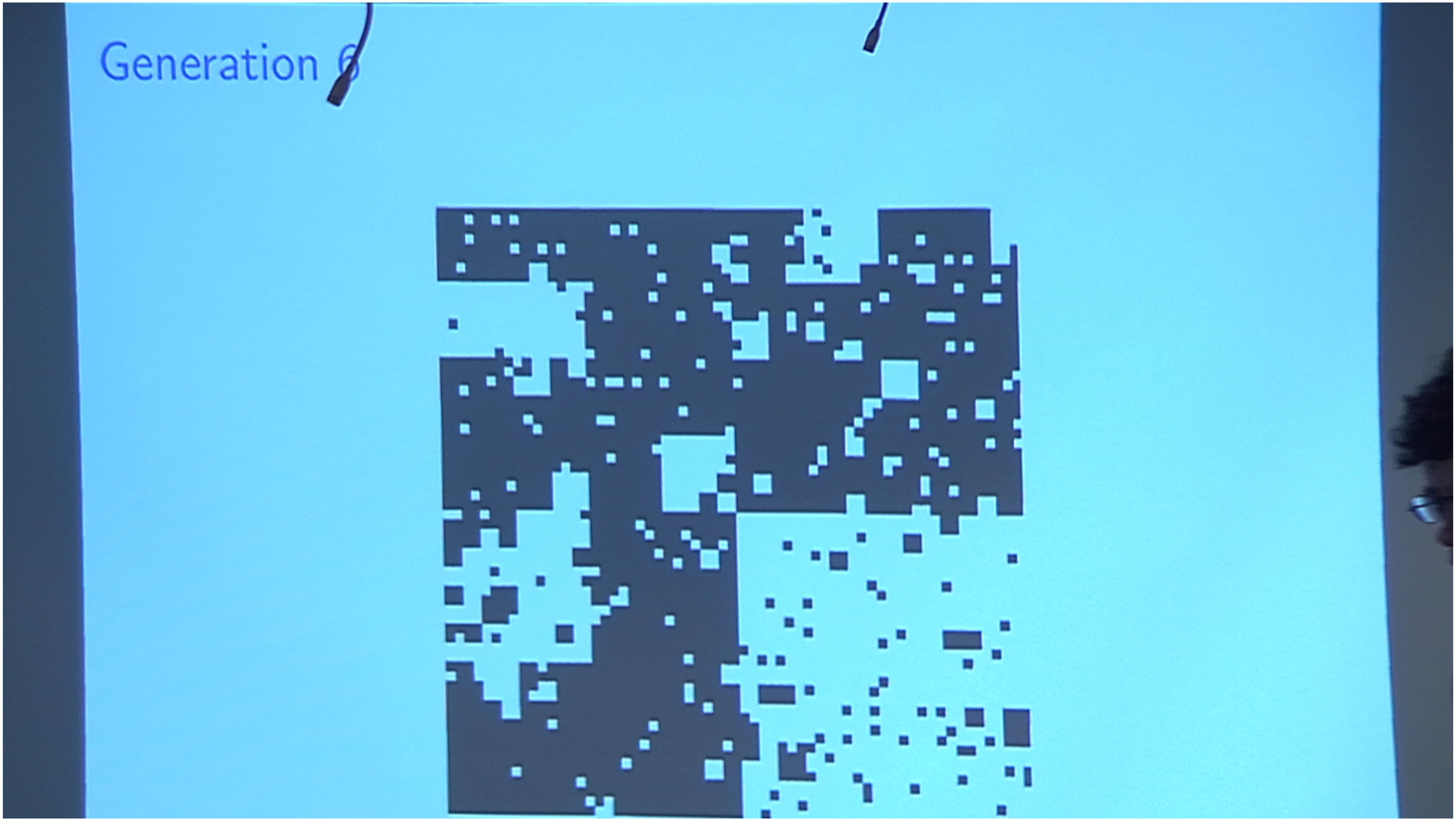


Generation 0

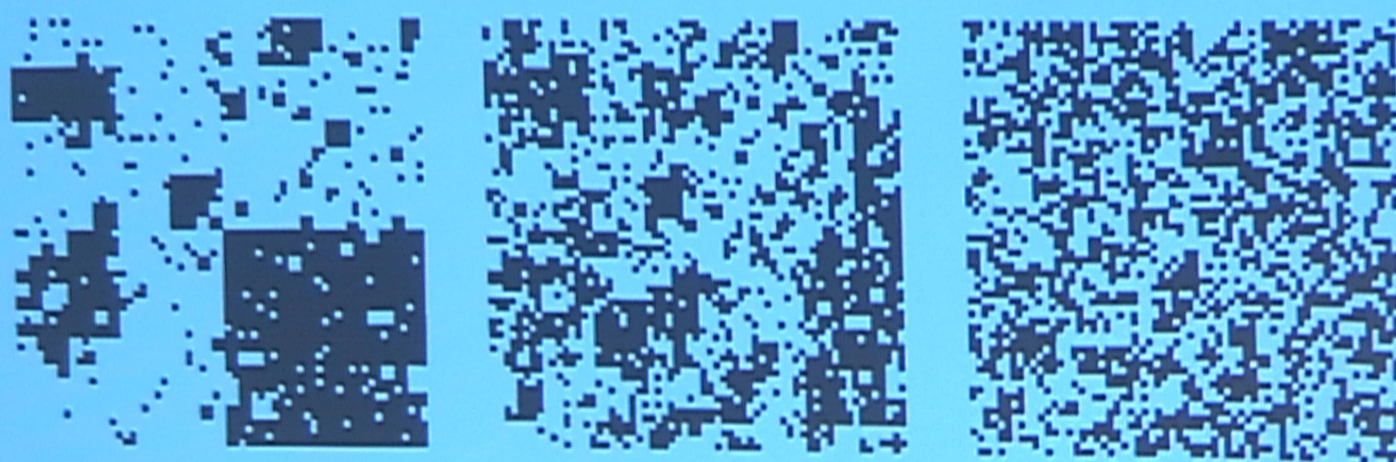

Generation 1

Generation 6




Generation 6

Generation 6, different values of γ



$\gamma = 0.05$ $\gamma = 0.15$ $\gamma = 0.25.$

Heuristic argument for a transition

Majority vote: after u generations, there are p^u variables. Take a census. If the bias in the census grows faster than the variance in the census, then significant correlation remains.

$$\text{bias} = N_{\text{black}} - N_{\text{white}} \sim \lambda^u p^u$$

$$\sqrt{\text{variance in census}} \sim \sqrt{p^u}$$

So if $\lambda > 1/\sqrt{p}$, there is memory! And if $\lambda \leq 1/\sqrt{p}$, there is no majority vote memory.

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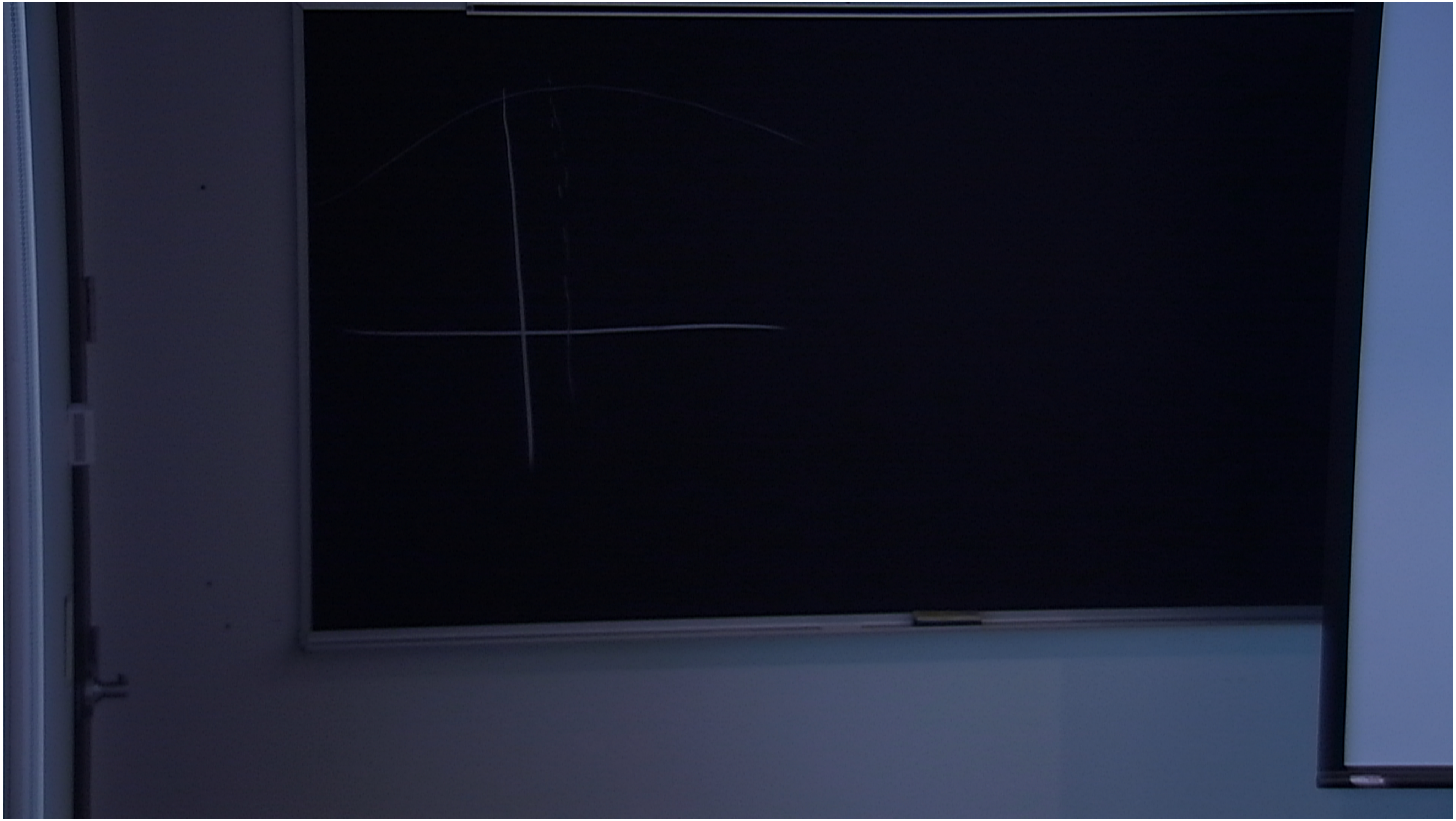
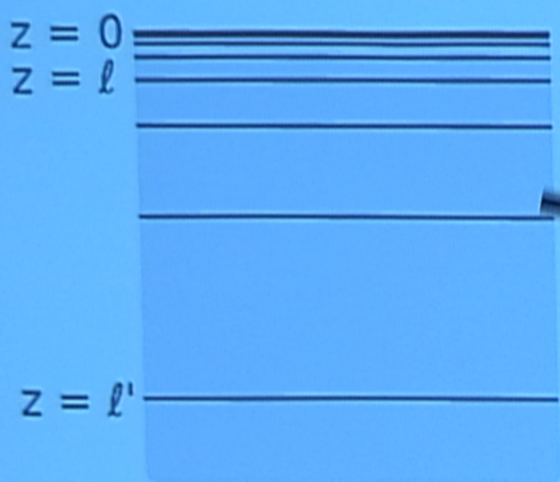


Question

The tree Markov dynamics can be understood as discrete versions of field theory dynamics on EAdS and dS.

Is there a corresponding memory/no-memory transition in real EAdS and dS?

EAdS



The Poincare metric for Euclidean AdS is

$$ds^2 = \frac{dz^2 + dx^i dx^i}{z^2} \qquad i = 1, \ldots, d$$

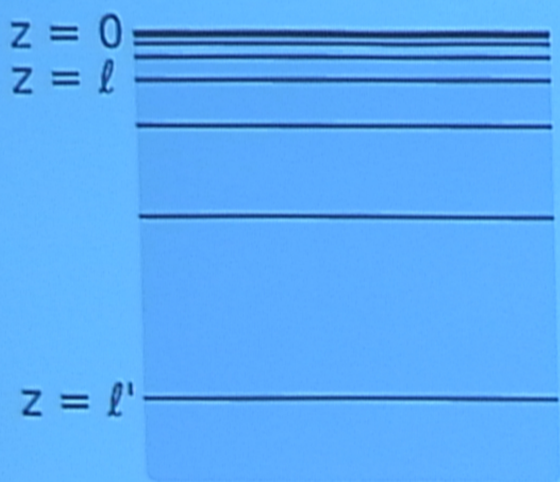
Focus on the zero mode, for which the wave equation

$$\partial_z^2 \varphi - \frac{d-1}{z}\partial_z \varphi - \frac{m^2}{z^2}\varphi = 0$$

has two independent power-law solutions, z^Δ and $z^{d-\Delta}$, where

$$\Delta \equiv \frac{1}{2}\left(d + \sqrt{d^2 + 4m^2}\right).$$

EAdS



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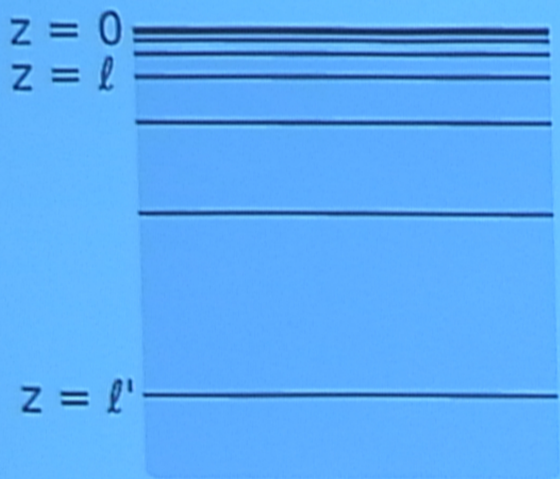

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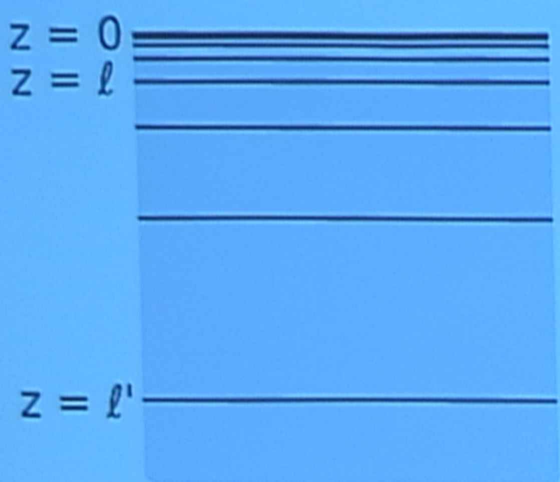

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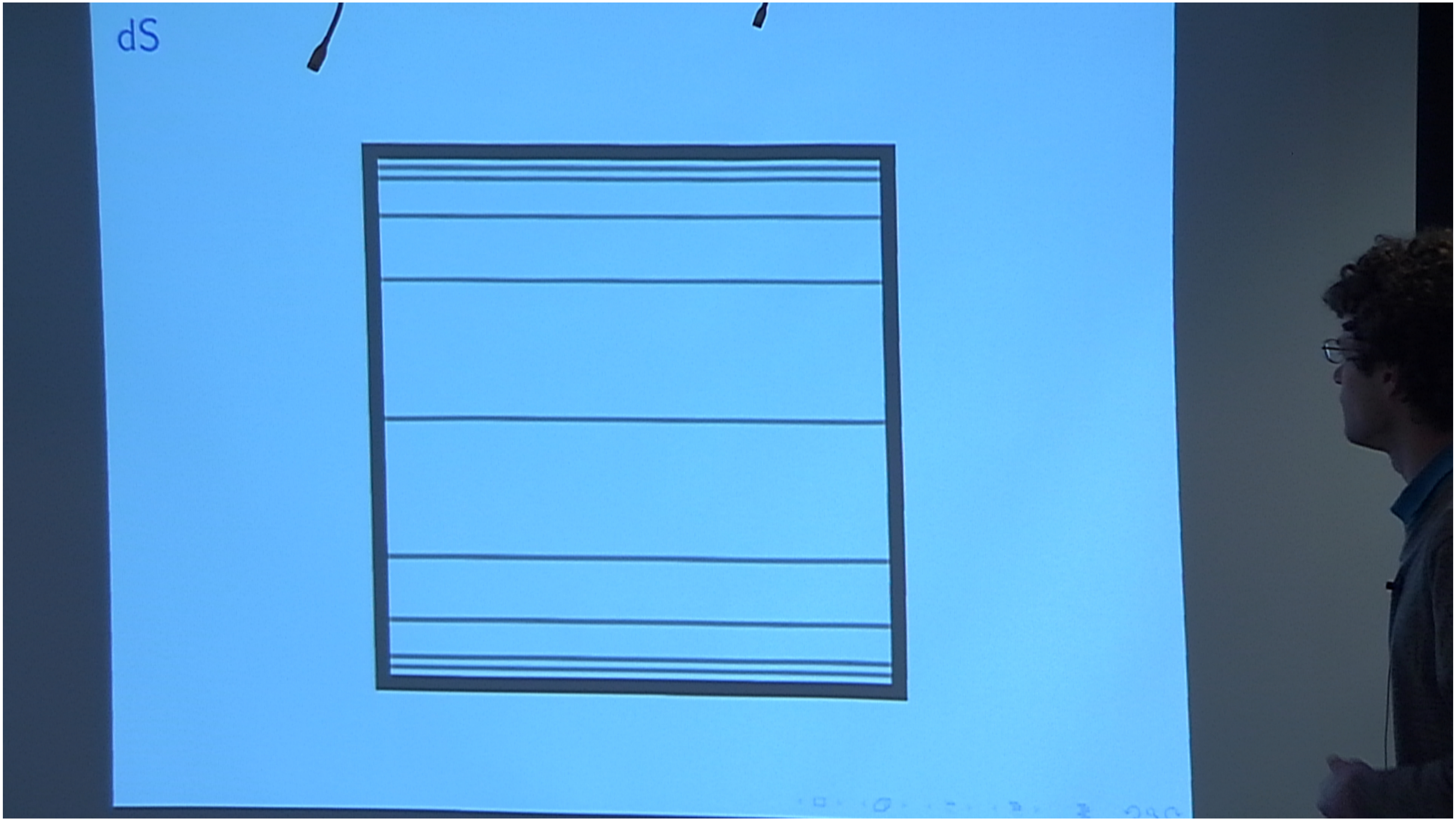

dS

dS

Global metric for de Sitter:

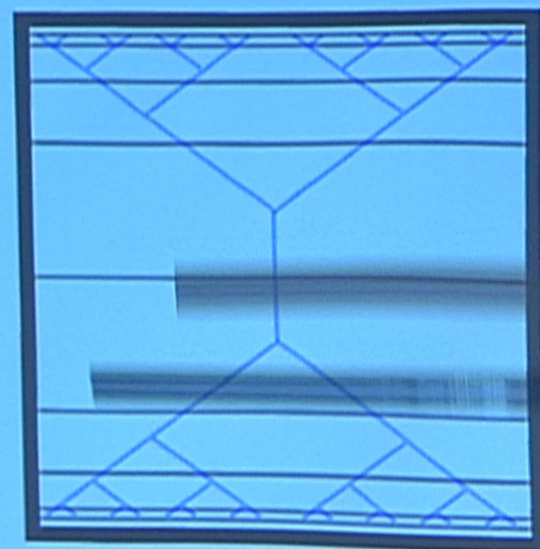
$$ds^2 = -dt^2 + \cosh^2(t)\,d\Omega_d^2.$$

Correlation functions decay like

$$\langle \varphi(0)\varphi(t)\rangle \sim e^{-\Delta_- t}.$$

Spatial volume grows as

$$N_{patches} \sim e^{dt}.$$



Total bias is $N\langle \varphi(0)\varphi(t)\rangle$. This beats $\sqrt{N}$ if $\Delta_- < \frac{d}{2}$.

dS

Unitarity: free fields in Lorentzian signature should always have majority-vote memory.

- Indeed, for free fields, the dimension

$$\Delta_- = \frac{1}{2}(d - \sqrt{d^2 - 4m^2}) \leq \frac{d}{2}$$

  always has real part less than or equal to $d/2$.
- But for interacting fields, Δ_- can be greater than $\frac{d}{2}$ [Marolf/Morrison, Jatkar/Leblond/Rajaraman].

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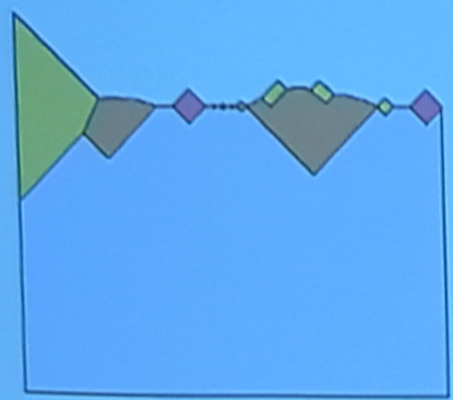
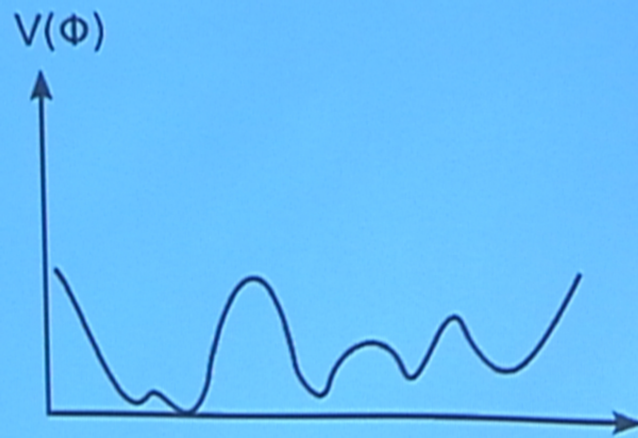
Application: eternal inflation





Bubble nucleation is represented by a large number of fields with small dimension

$$\Delta_- \sim e^{-S_{CDL}}.$$

So, even though, locally, the system reaches equilibrium, globally there is an almost perfect record of the entire history.

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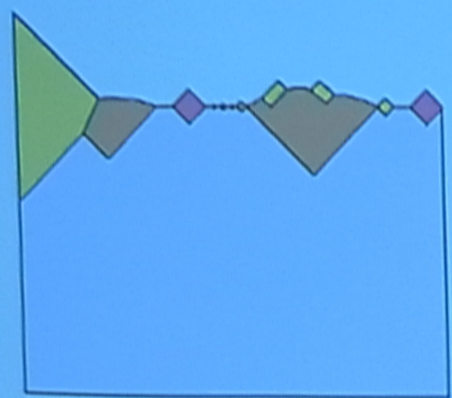

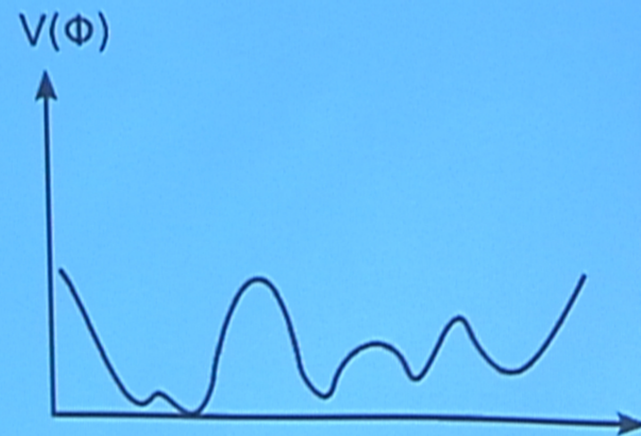

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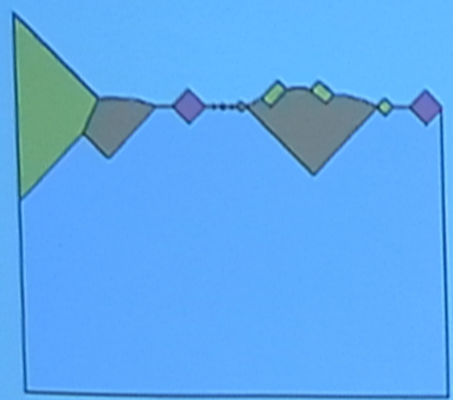

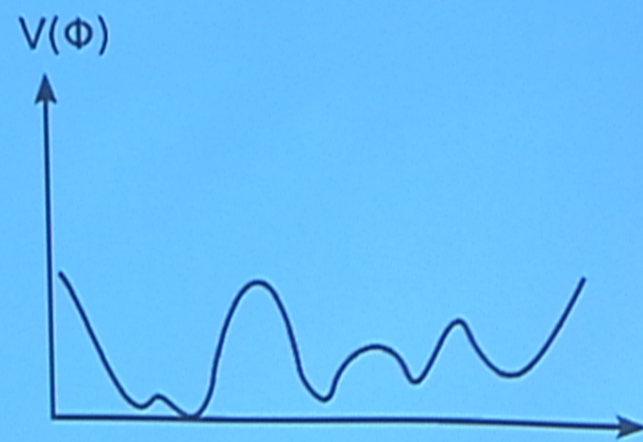

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Extreme states

Tail field: collection of events that don't depend on any finite number of variables. E.g. $M_\infty > 0$, where

$$M_\infty = \lim_{u \to \infty} \frac{M_u}{(\lambda p)^u}.$$

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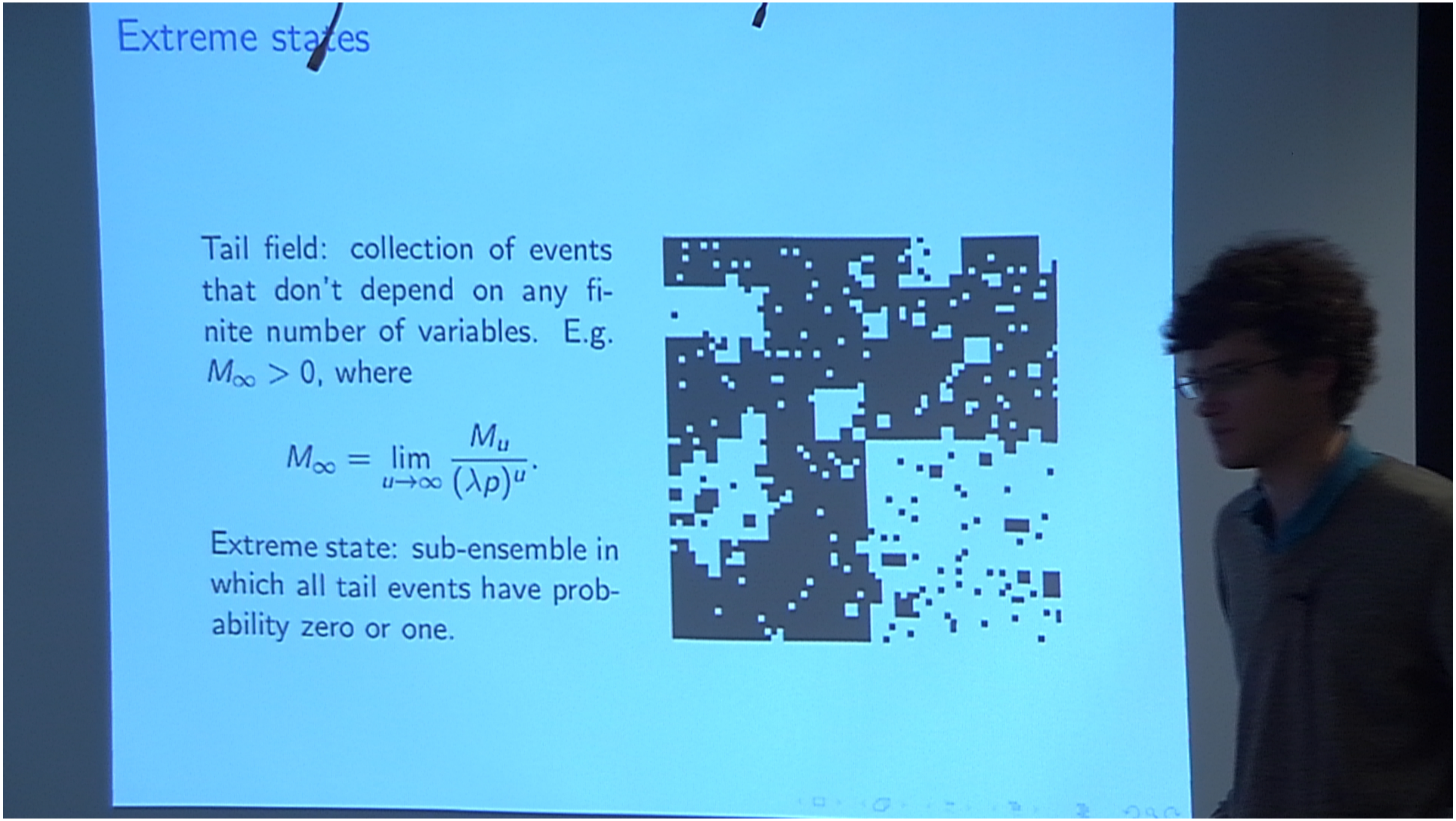
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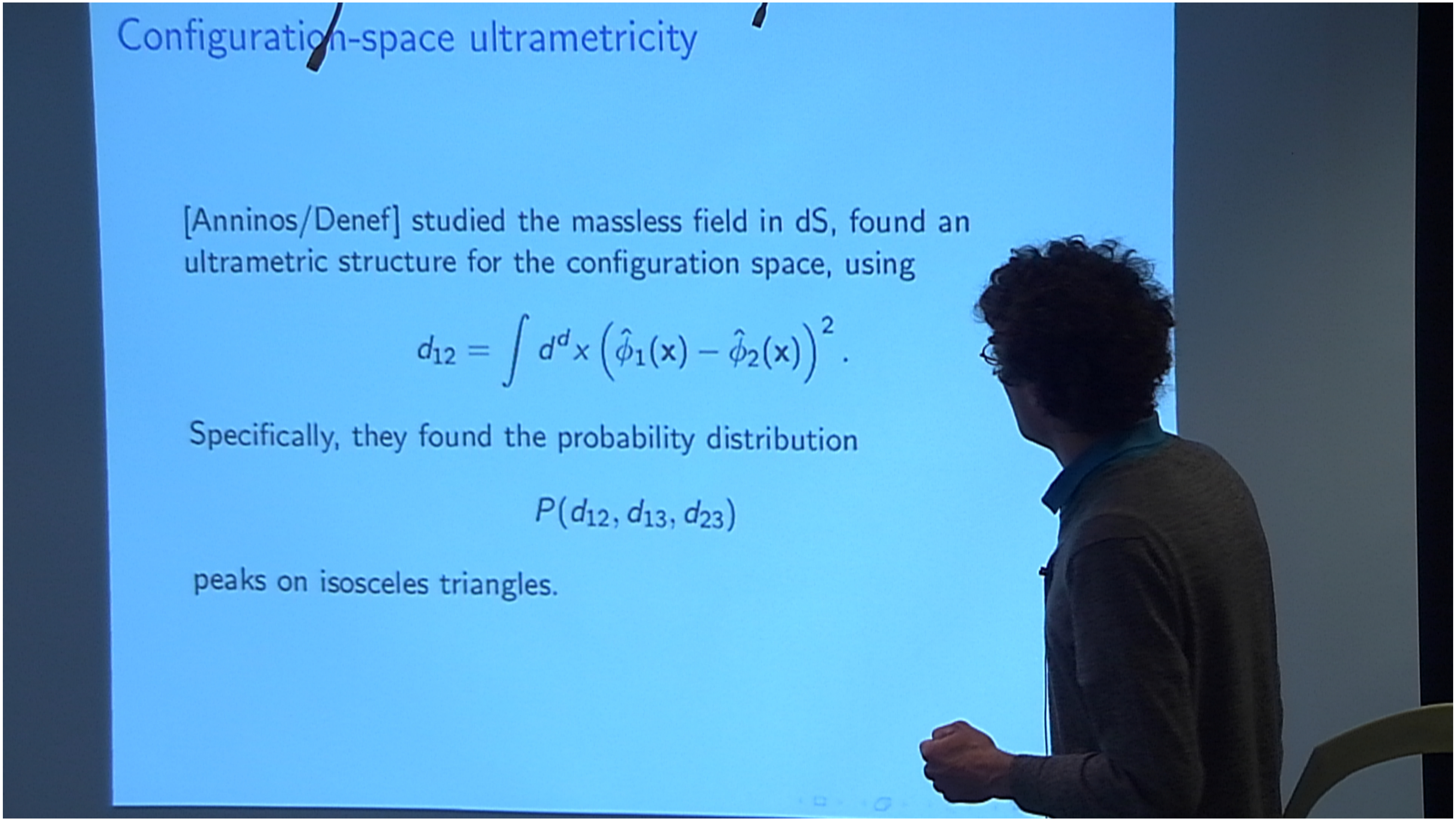
Configuration-space ultrametricity
[Anninos/Denef] studied the massless field in dS, found an ultrametric structure for the configuration space, using
$$d_{12} = \int d^d x \left(\hat{\phi}_1(\mathbf{x}) - \hat{\phi}_2(\mathbf{x})\right)^2 .$$
Specifically, they found the probability distribution
$$P(d_{12}, d_{13}, d_{23})$$
peaks on isosceles triangles.

Configuration-space ultrametricity

For massive fields, we generalize the definition of distance to

$$d_{12} = e^{2\Delta_- t} \int d^d x \left(\hat{\phi}_1(\mathbf{x}, t) - \hat{\phi}_2(\mathbf{x}, t) \right)^2$$

$$\delta_{12} = d_{12} - \langle d_{12} \rangle .$$

Following [Anninos/Denef], we compute the generating function

$$\langle e^{-s d_{12}} \rangle \sim \prod_{\mathbf{k} \neq 0} \int d\phi_{\mathbf{k}}^{(1)} d\phi_{\mathbf{k}}^{(2)} e^{-\beta_{\mathbf{k}}(t)(|\phi_{\mathbf{k}}^{(1)}|^2 + |\phi_{\mathbf{k}}^{(2)}|^2) - s e^{2\Delta_- t} |\phi_{\mathbf{k}}^{(1)} - \phi_{\mathbf{k}}^{(2)}|^2}$$

$$\langle e^{-s \delta_{12}} \rangle \sim \prod_{\mathbf{k} \neq 0} \frac{e^{s/k^{d-2\Delta_-}}}{1 + s/k^{d-2\Delta_-}} \qquad \beta_{\mathbf{k}}(t) \sim k^{d-2\Delta_-} e^{2\Delta_- t}$$

and get the probability distribution $P(\delta)$ by Laplace transform.

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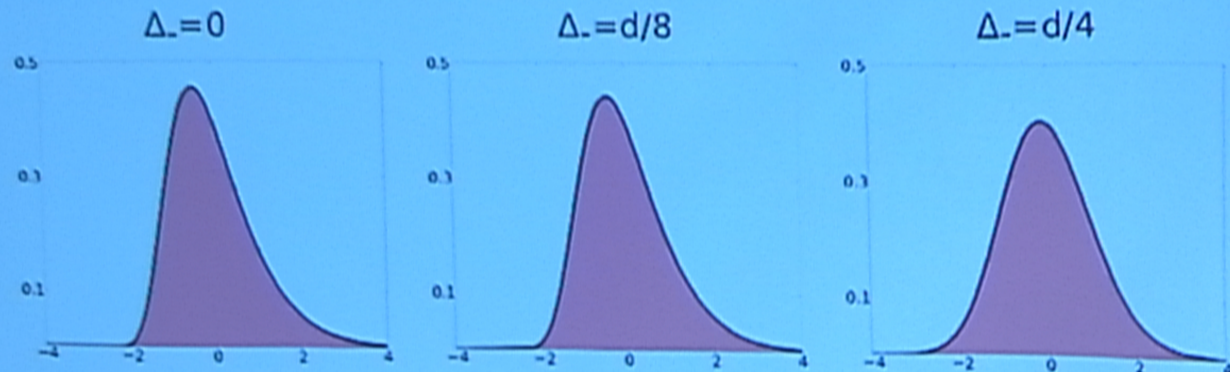
Configuration-space ultrametricity

Plots of $P(\delta)$:



Gumbel tends to a Gaussian as Δ increases to $d/4$.

Conclusion

- There is a well-studied global memory/no memory transition in the context of statistical mechanics on trees.
- There is a corresponding transition in field theory in Euclidean anti de Sitter, correlated to the choice of standard vs. alternate boundary conditions.
- In de Sitter, free fields always have global memory, although interacting fields may not.
- Memory implies a hierarchical decomposition of the configuration space into extreme states. If $\Delta < d/4$, this is visible in Anninos-Denef ultrametricity.