Title: From topological insulator to weak scale flavor physics

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Abstract: <span>In this talk, I will start with
briefly introducing some universal physics behind quantum hall and topological insulator , which inspired a BSM flavor model. It intimately relates deconstructed little Higgs to flavor structure: fermion masses, CKM etc.  This new cousin of little Higgs, we call it little flavor, shares a 10-20 Tev cut-off scale with little Higgs, so as to explain flavor structure at surprisingly low scale without rising FCNC problem.</span>

# From Topological insulator to weak scale flavor physics

Sichun Sun University of Washington/KITP

**Spacetime as a topological insulator**

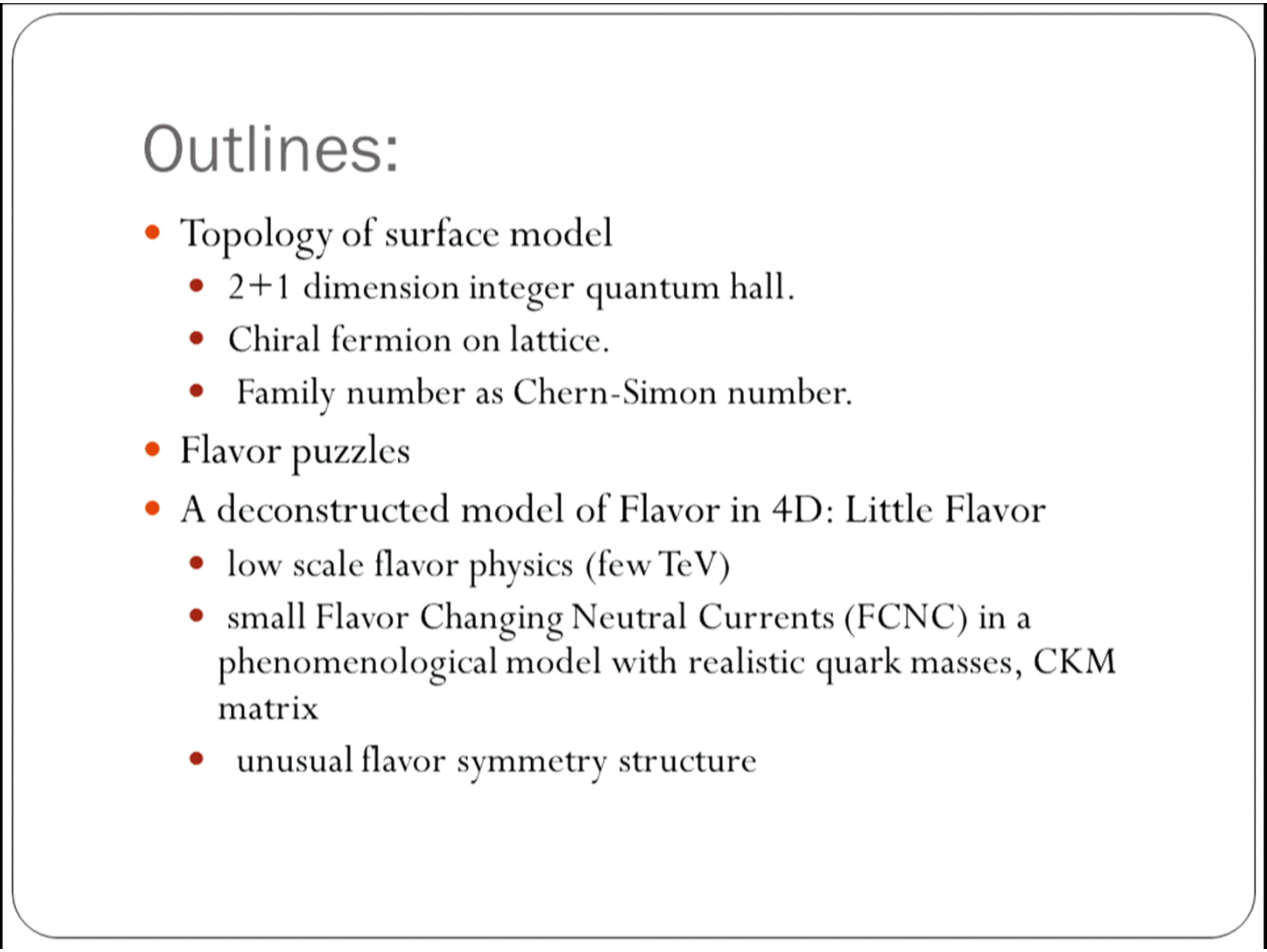
Phys. Rev. Lett. 108 (2012) 181807

David B. Kaplan, Sichun Sun

**Little Flavor**

arXiv:1303.1811

Sichun Sun, David B. Kaplan, Ann E. Nelson



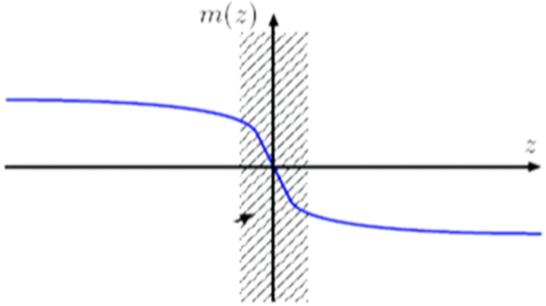
# Outlines:

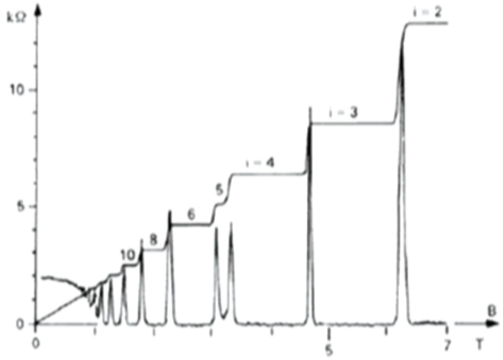
- Topology of surface model
  - 2+1 dimension integer quantum hall.
  - Chiral fermion on lattice.
  - Family number as Chern-Simon number.
- Flavor puzzles
- A deconstructed model of Flavor in 4D: Little Flavor
  - low scale flavor physics (few TeV)
  - small Flavor Changing Neutral Currents (FCNC) in a phenomenological model with realistic quark masses, CKM matrix
  - unusual flavor symmetry structure

# Quantum Hall Effect

- 2+1 bulk mode become gapped under magnetic field.
- 1+1 dimensional edge modes persist
- integer case
- Domain wall model:

R. Jackiw and C. Rebbi, Phys.Rev. D13, 3398 (1976).





$$\sigma_{xy} = n\frac{e^2}{h}$$

# 5D Quantum Hall

- Action:

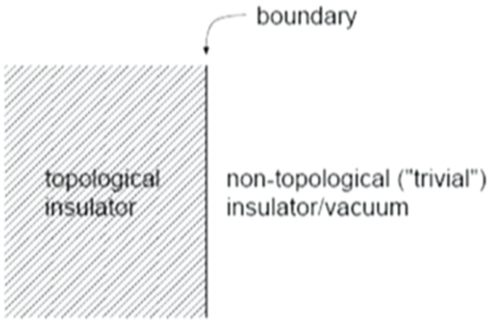
$$S = \int d^4x dx_5 \overline{\psi}(i\partial\!\!\!/_4 + \gamma^5\partial_5 + m\theta(x_5))\psi$$

- To require 4d Dirac equation, for zero modes, left with:

$$(\gamma^5\partial_5 + m\theta(x_5))\psi(x_5) = 0$$

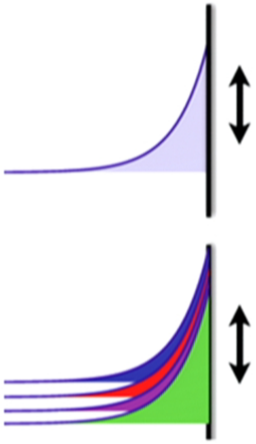
- With solutions, for $\pm\theta(x_5)$ profile:

$$\psi_{L,R}(x_5) = e^{-mx_5}, x_5 > 0$$



# Chiral Fermion on Lattice

- Simulating Chiral fermion on lattice-----Doman Wall fermion D.B Kaplan (1992), What 1/3 of Lattice QCD community are using
- Jansen & Schmaltz (1992):
  - Lattice version will typically have **nf** copies of chiral surface modes
  - nf changes discontinuously when Lagrangian parameters are varied continuously



- **Topology on a lattice**

E.g, d=5 lattice: lattice derivatives

$$\mathcal{L} = \bar{\psi} i \not{\partial} \psi - m\bar{\psi}\psi + w\,\bar{\psi}\partial^2\psi$$


$n_f$: ←0 $1_L$ $4_R$ $6_L$ $4_R$ $1_L$ 0→

# 5D Quantum Hall

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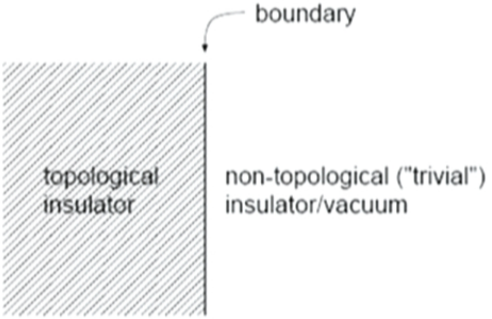

# Magical Chern-Simon number

d=2n+1 dimensions:

- Number of chiral surface modes = cn = integer coefficient of Chern-Simons operator obtained by integrating out heavy bulk fermions Goltermann, Jansen, DBK, PLB301, 219, (1992)

$$c_n = \frac{(-\mathrm{i})^n \epsilon_{\mu_1 \quad \mu_{2n+1}}}{(n+1)(2n+1)!} \int \frac{\mathrm{d}^{2n+1} p}{(2\pi)^{2n+1}} \mathrm{Tr}\{[S(p)\, \partial_{\mu_1} S(p)^{-1}] \ldots [S(p)\, \partial_{\mu_{2n+1}} S(p)^{-1}]\}$$

$$S^{-1}(p) = a(p) + \mathrm{i}\boldsymbol{b}(p)\cdot\gamma = N(p)[\cos|\boldsymbol{\theta}(p)| + \mathrm{i}\hat{\boldsymbol{\theta}}(p)\cdot\gamma \sin|\boldsymbol{\theta}(p)|] \equiv N(p)V(p)$$

- Number of chiral surface modes result of topology of bulk fermion dispersion relation in momentum space

- Cn is also the “integer” in Integer quantum Hall!

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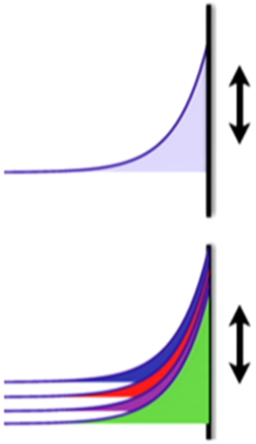

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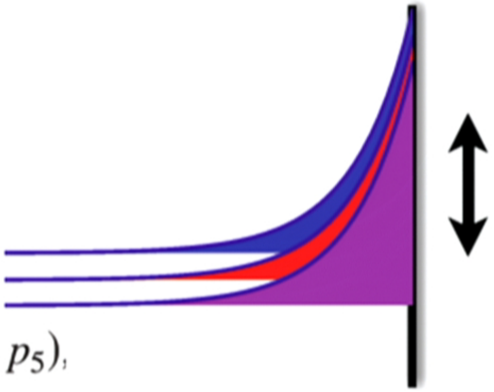
# Now: Flavor physics

- Can 3 families of 4d fermions arise from a single family of 5d fermion through this mechanism?
  *# families determined by coupling constants?!*



Possible to construct model in semi-infinite 5th dimension with 3 families of zeromodes:

Bulk dispersion relation:

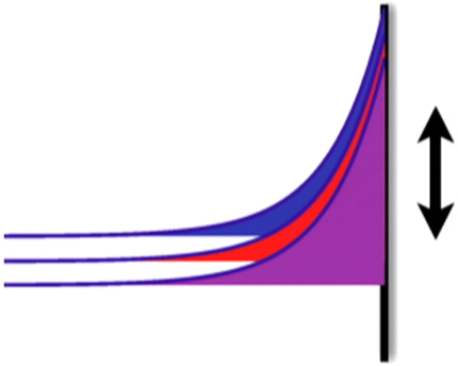
$$iG^{-1}(p_\mu, p_5) = iZ_\mu(p)\gamma^\mu + iZ_5(p_5)\gamma^5 - \Sigma(p, p_5),$$

Chiral zeromodes at boundary for zeros of $G^{-1}$ with imaginary $p_5$

*Normalizability depends on sign of Im[$p_5$]; chirality on sign[$\Sigma(0,p_5)$]*

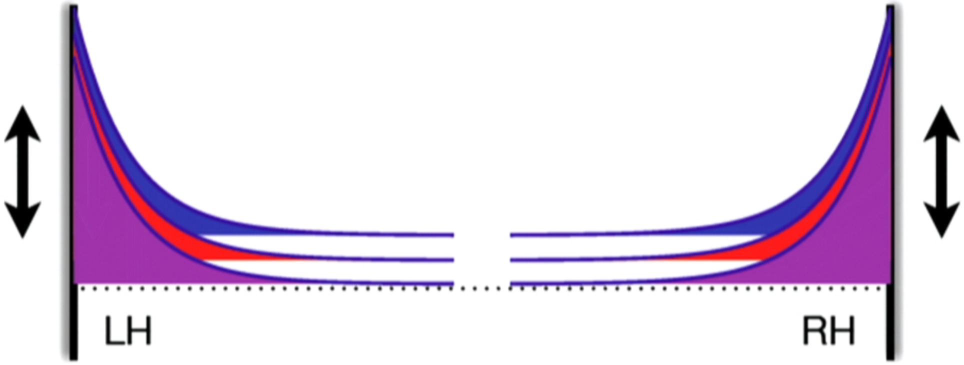
- Possible to construct model in semi-infinite 5th dimension with 3 families of zeromodes; but...

1. can't have SM gauge fields live in noncompact extra dim
    - ...so compactify
2. on compact manifold, find vector-like fermions instead of chiral
    - can be made chiral with chiral orbifold projection
3. relies on UV physics:
    - topology in x depends on large-x behavior of fields
    - ⇒ topology in p depends on large p behavior of $G^{-1}$
    - Need UV completion to make sense

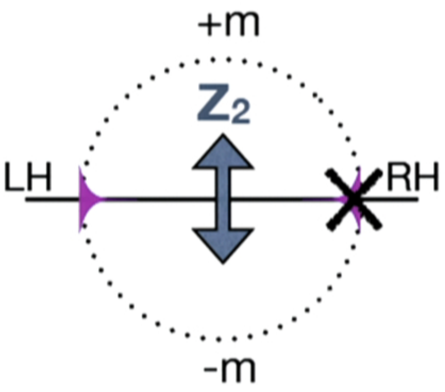


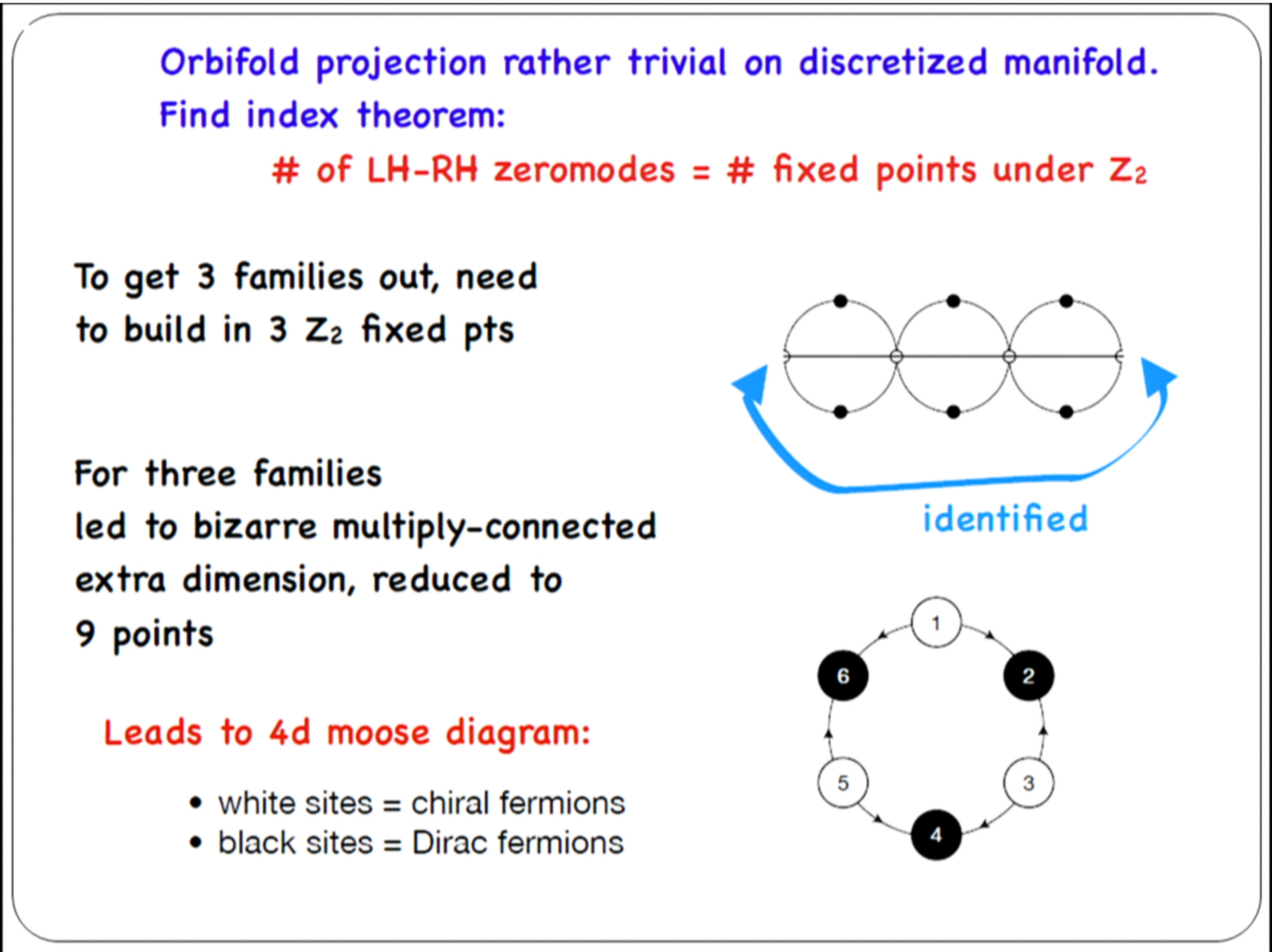
So: compactify & "deconstruct" (discretize extra dimension)

Problem: get Dirac zeromodes



$Z_2$ orbifold projection?





Orbifold projection rather trivial on discretized manifold.

Find index theorem:

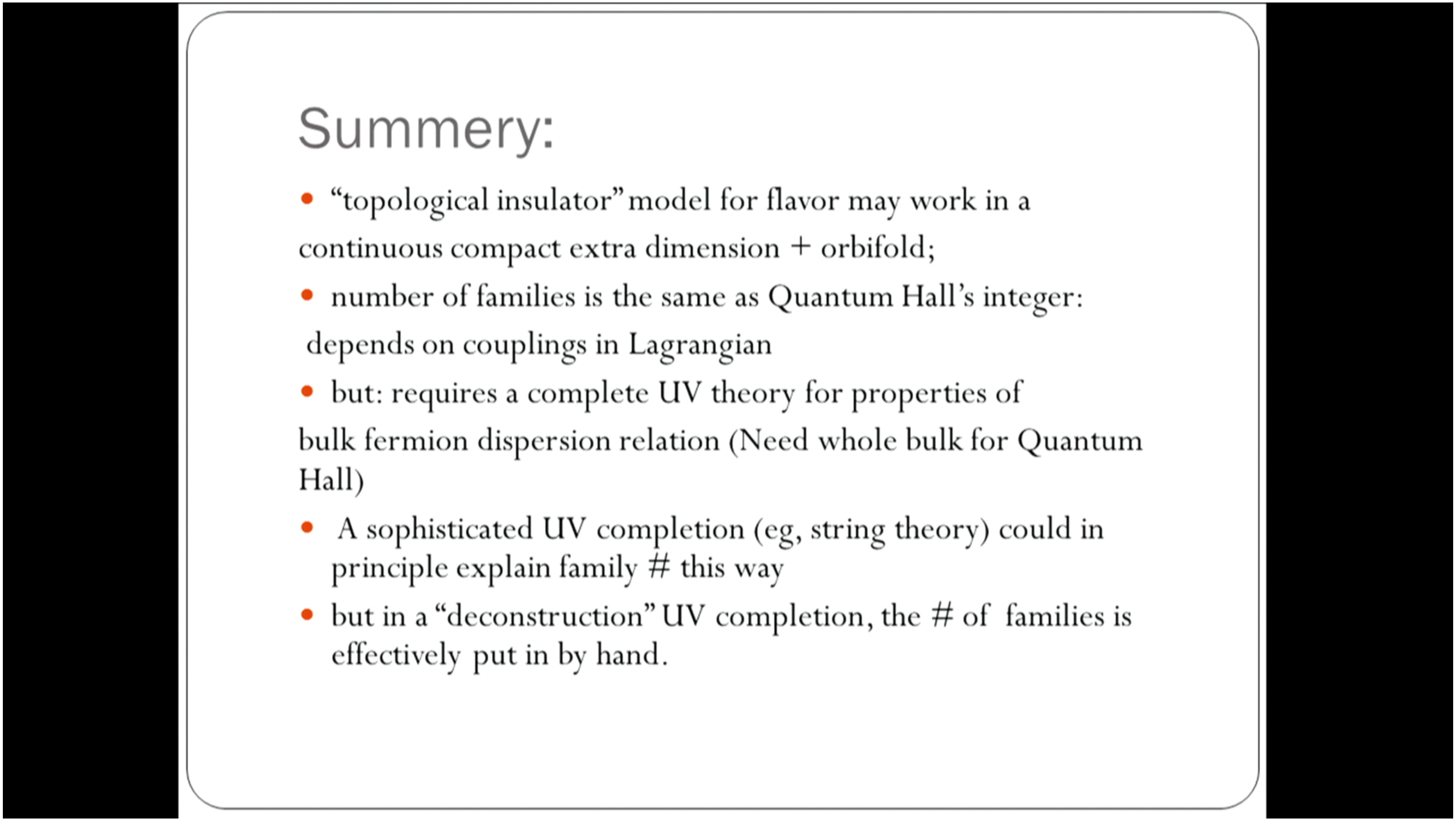
# of LH-RH zeromodes = # fixed points under $Z_2$

To get 3 families out, need to build in 3 $Z_2$ fixed pts

For three families led to bizarre multiply-connected extra dimension, reduced to 9 points

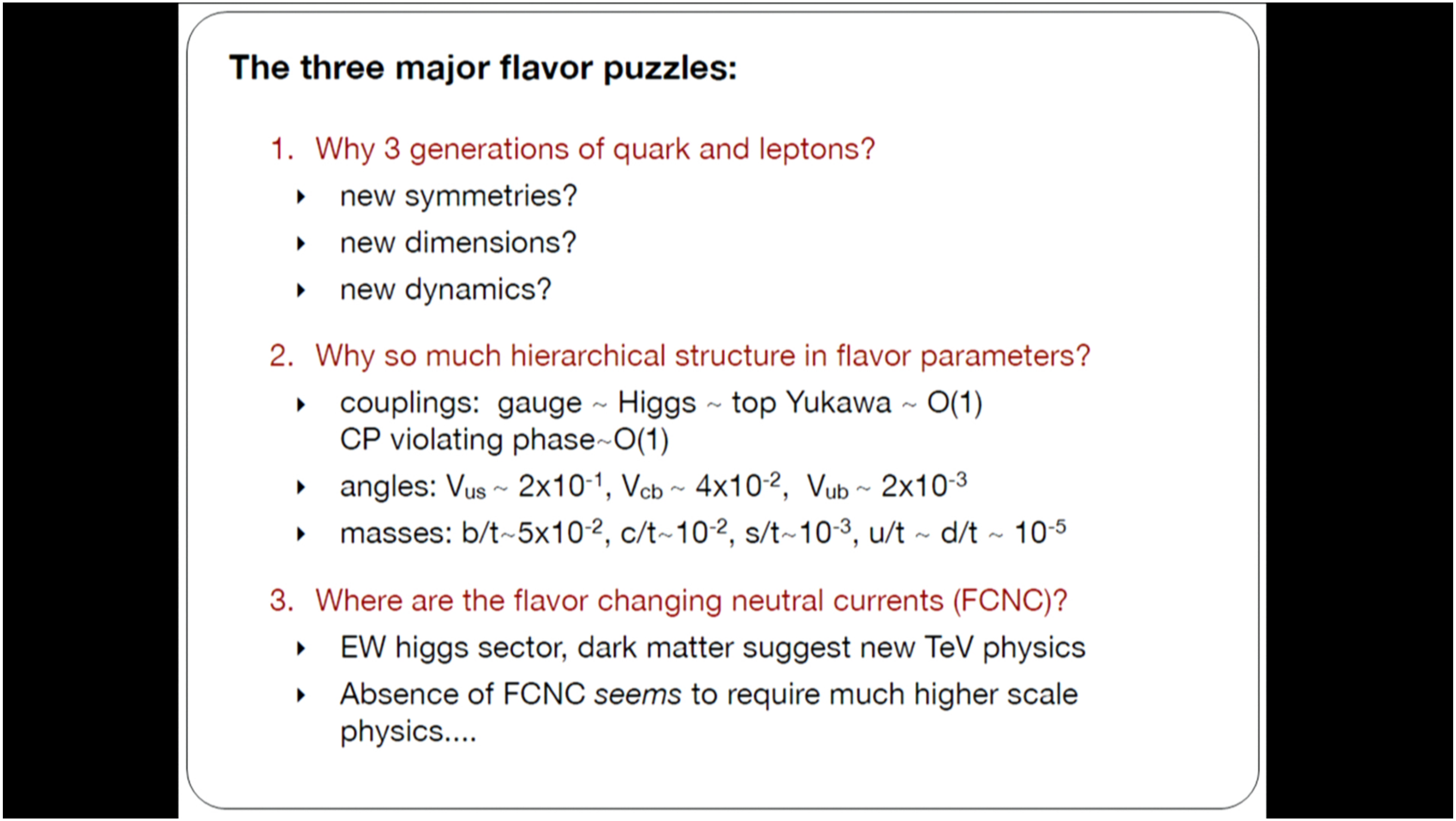
Leads to 4d moose diagram:

- white sites = chiral fermions
- black sites = Dirac fermions



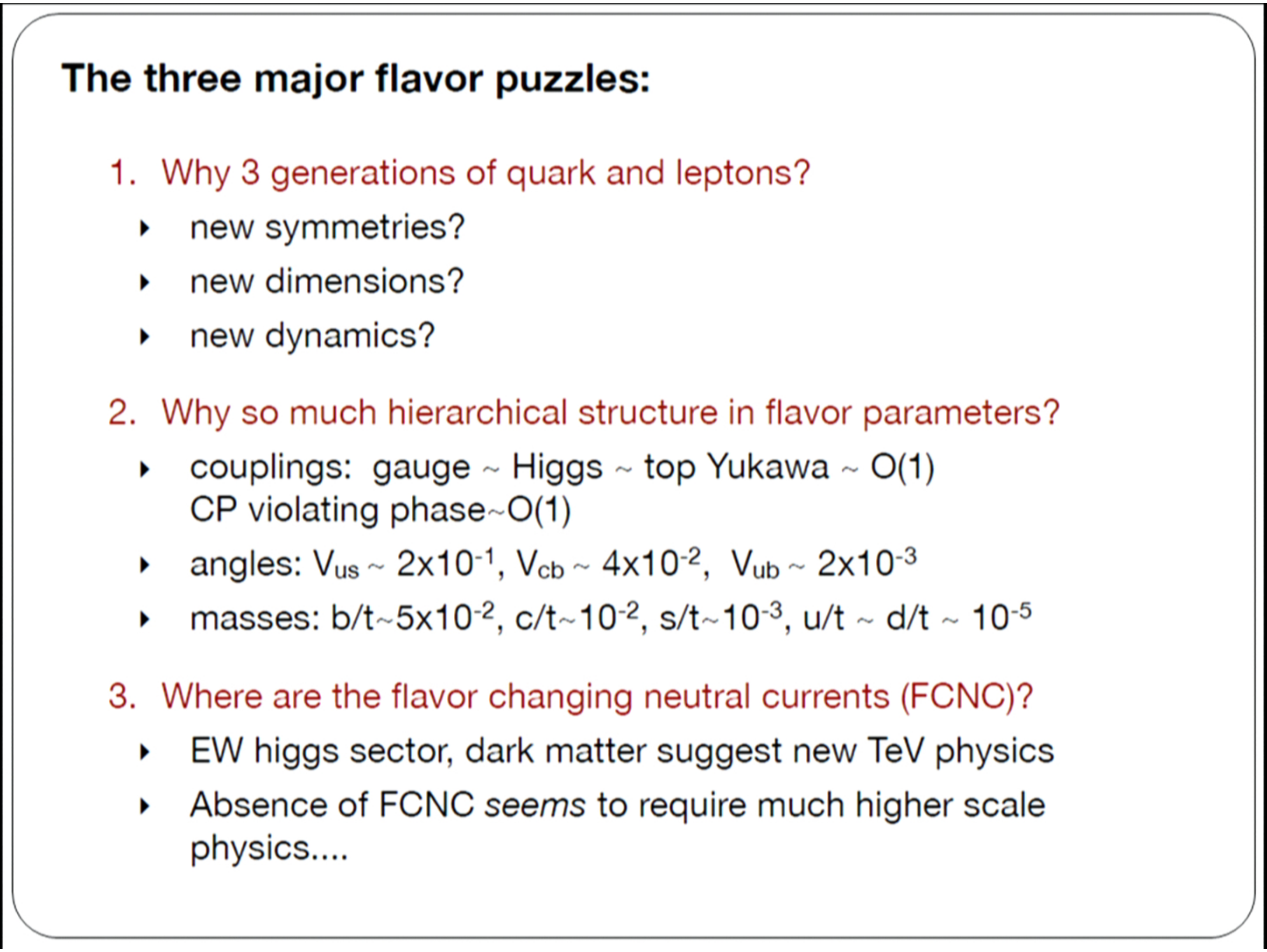
# Summery:

- "topological insulator" model for flavor may work in a continuous compact extra dimension + orbifold;
- number of families is the same as Quantum Hall's integer: depends on couplings in Lagrangian
- but: requires a complete UV theory for properties of bulk fermion dispersion relation (Need whole bulk for Quantum Hall)
- A sophisticated UV completion (eg, string theory) could in principle explain family # this way
- but in a "deconstruction" UV completion, the # of families is effectively put in by hand.



## The three major flavor puzzles:

1. Why 3 generations of quark and leptons?
   - new symmetries?
   - new dimensions?
   - new dynamics?
2. Why so much hierarchical structure in flavor parameters?
   - couplings: gauge ~ Higgs ~ top Yukawa ~ O(1)
     CP violating phase~O(1)
   - angles: $V_{us} \sim 2x10^{-1}$, $V_{cb} \sim 4x10^{-2}$, $V_{ub} \sim 2x10^{-3}$
   - masses: $b/t \sim 5x10^{-2}$, $c/t \sim 10^{-2}$, $s/t \sim 10^{-3}$, $u/t \sim d/t \sim 10^{-5}$
3. Where are the flavor changing neutral currents (FCNC)?
   - EW higgs sector, dark matter suggest new TeV physics
   - Absence of FCNC *seems* to require much higher scale physics....



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## Some allowed dim 6 FCNC operators:

$$\frac{c_{sd}}{\Lambda^2}\left(\bar{s}\gamma_\mu d\right)^2 \qquad \frac{c_{uc}}{\Lambda^2}\left(\bar{u}\gamma_\mu c\right)^2 \qquad \frac{c_{bd}}{\Lambda^2}\left(\bar{b}\gamma_\mu d\right)^2$$

- Im[$c_{sd}$] = O(1) ⇒ $\Lambda > O(10^4)$ TeV... $10^5$ x $M_Z$!
- Re[$c_{sd}$] = O(1) ⇒ $\Lambda > O(10^3)$ TeV
- $c_{uc}$ = O(1) ⇒ $\Lambda > O(10^3)$ TeV
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Absence of s-d transitions key to prediction of charm

*(GIM, Gaillard & Lee)*

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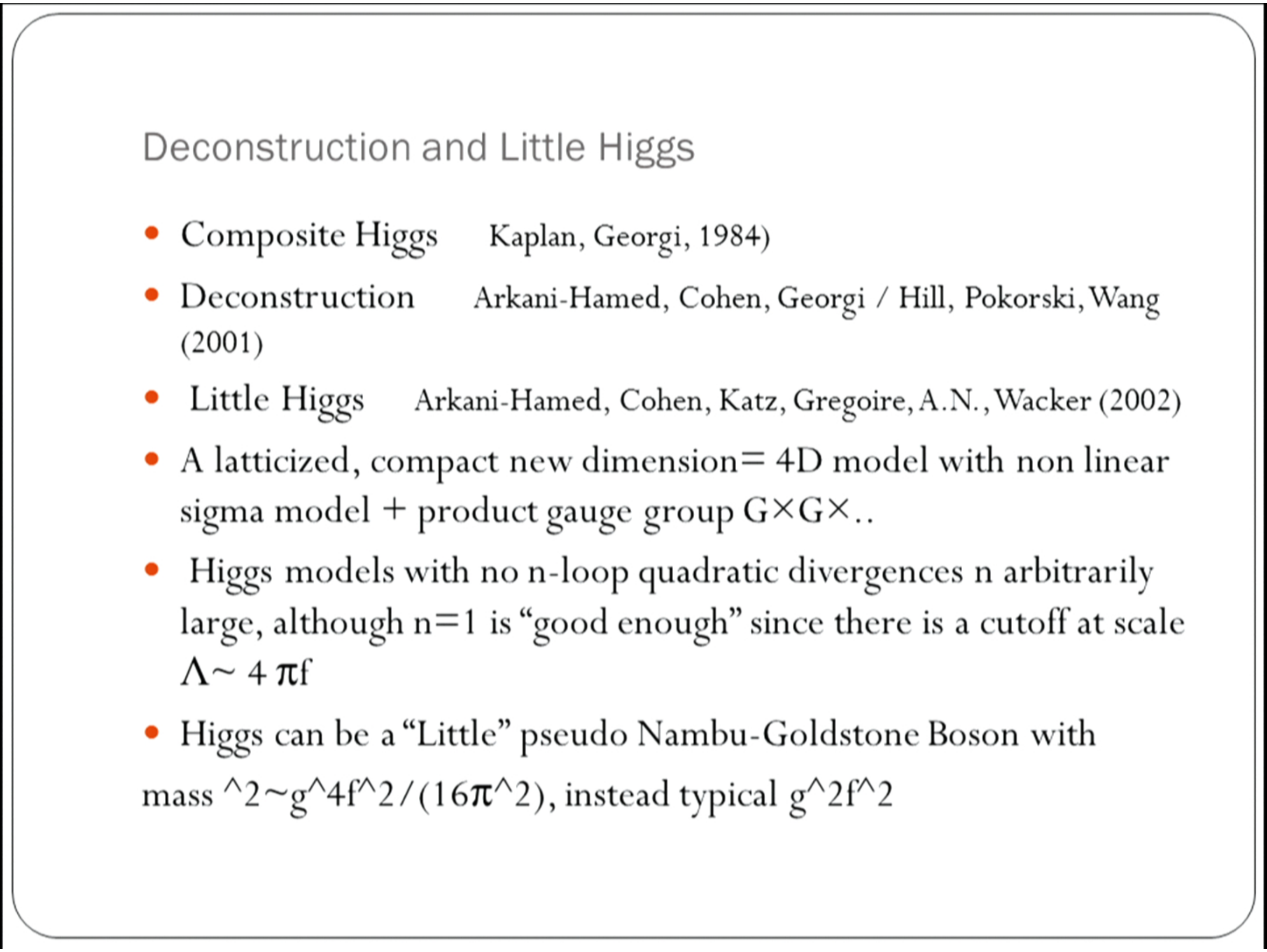


## Deconstruction and Little Higgs

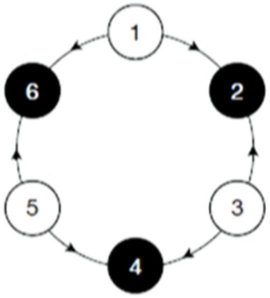
- Composite Higgs Kaplan, Georgi, 1984)
- Deconstruction Arkani-Hamed, Cohen, Georgi / Hill, Pokorski, Wang (2001)
- Little Higgs Arkani-Hamed, Cohen, Katz, Gregoire, A.N., Wacker (2002)
- A latticized, compact new dimension= 4D model with non linear sigma model + product gauge group G×G×..
- Higgs models with no n-loop quadratic divergences n arbitrarily large, although n=1 is "good enough" since there is a cutoff at scale $\Lambda \sim 4\,\pi f$
- Higgs can be a "Little" pseudo Nambu-Goldstone Boson with

mass ^2~g^4f^2/(16π^2), instead typical g^2f^2

# What we learnt from "deconstruction and little higgs"

- Having "extra dimension" without dealing with 5D gravity!
- Novel understanding of new dimensions.
- Mechanism could be totally in 4D: Littlest Higgs
- Alternative solution to hierarchy problem comparing to SUSY: same spin partner

# Combining little higgs with flavor model?



- white sites = chiral fermions
- black sites = Dirac fermions

Little Higgs:

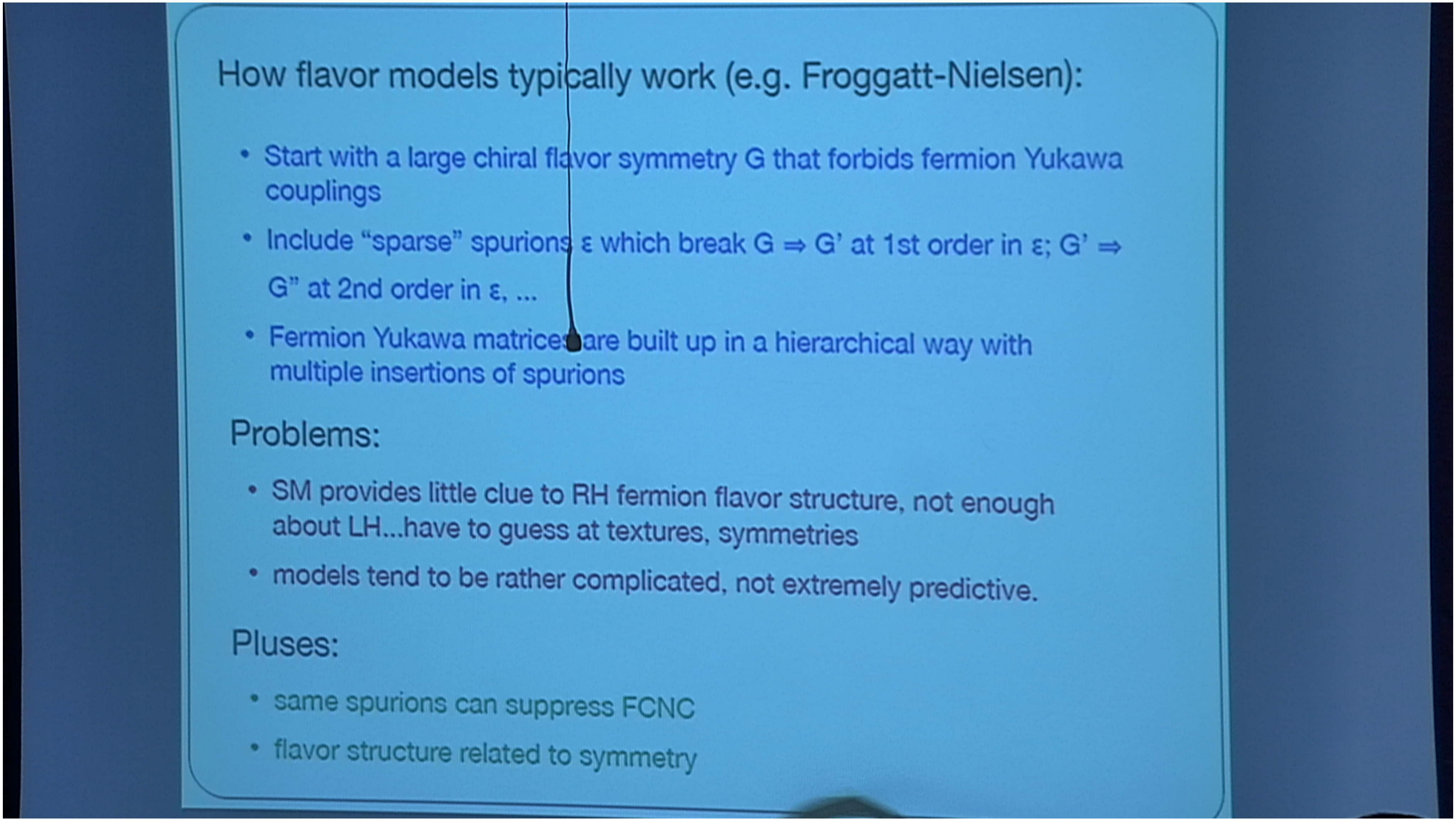
(Arkani-Hamed, Cohen, Georgi (2001); Arkani-Hamed, Cohen, Katz, Nelson (2002) )

large symmetry group + sparse symmetry breaking spurions
= unusually large natural hierarchy between EW scale and UV (eg $1/\alpha^2$)

Flavor models:

(e.g.: Frogatt-Nielsen (1979))

large flavor symmetry group + sparse symmetry breaking spurions
= natural hierarchy between quark masses & mixing angles



## How flavor models typically work (e.g. Froggatt-Nielsen):

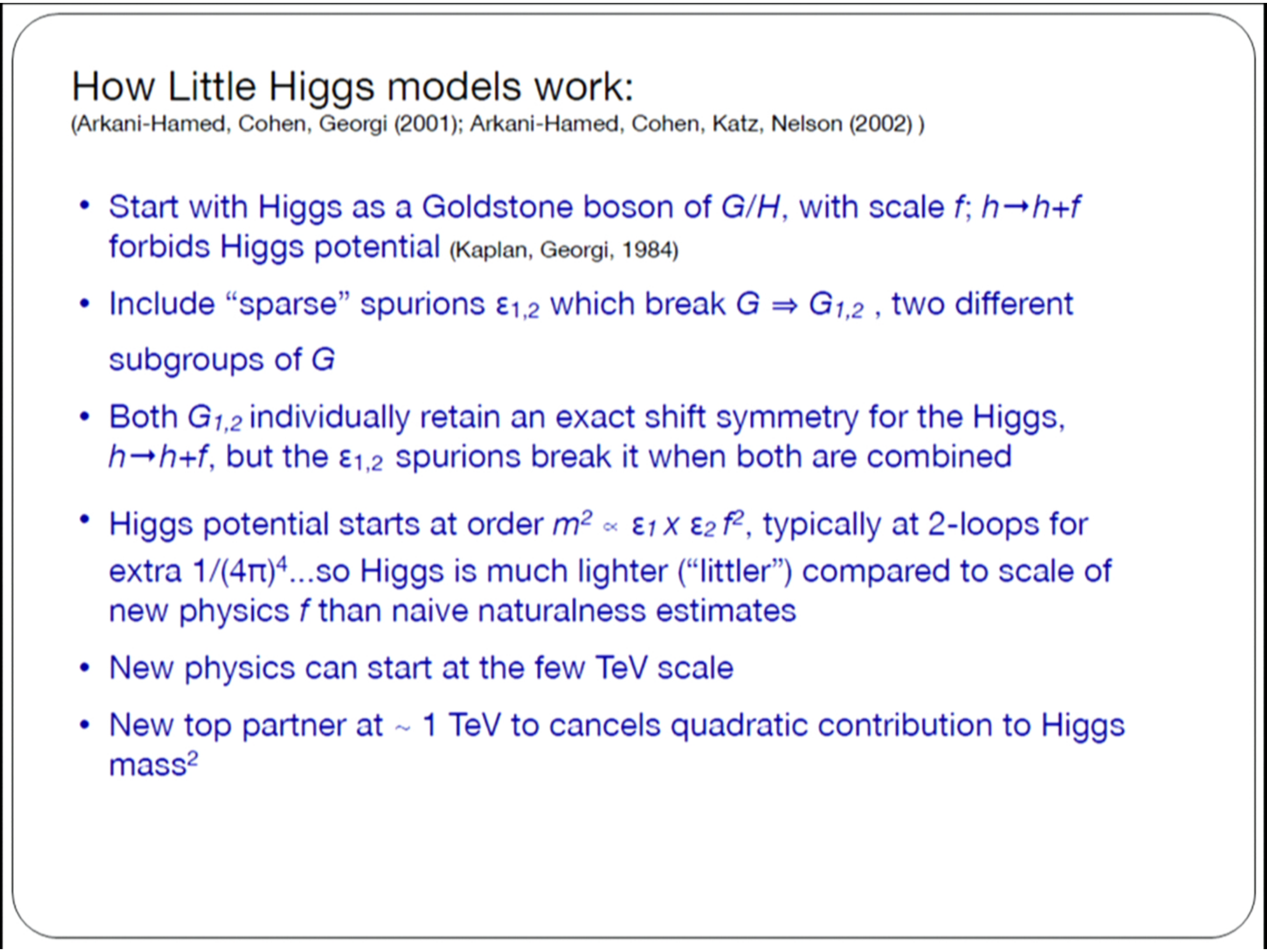
- Start with a large chiral flavor symmetry G that forbids fermion Yukawa couplings
- Include "sparse" spurions ε which break G ⇒ G' at 1st order in ε; G' ⇒ G" at 2nd order in ε, ...
- Fermion Yukawa matrices are built up in a hierarchical way with multiple insertions of spurions

## Problems:

- SM provides little clue to RH fermion flavor structure, not enough about LH...have to guess at textures, symmetries
- models tend to be rather complicated, not extremely predictive.

## Pluses:

- same spurions can suppress FCNC
- flavor structure related to symmetry

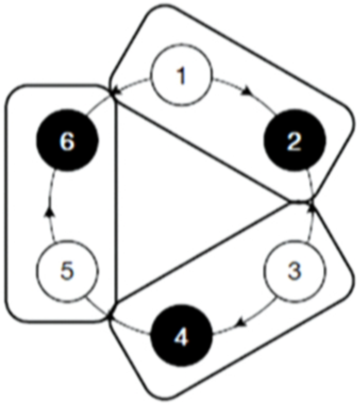


# How Little Higgs models work:

(Arkani-Hamed, Cohen, Georgi (2001); Arkani-Hamed, Cohen, Katz, Nelson (2002) )

- Start with Higgs as a Goldstone boson of $G/H$, with scale $f$; $h \rightarrow h+f$ forbids Higgs potential (Kaplan, Georgi, 1984)
- Include "sparse" spurions $\varepsilon_{1,2}$ which break $G \Rightarrow G_{1,2}$ , two different subgroups of $G$
- Both $G_{1,2}$ individually retain an exact shift symmetry for the Higgs, $h \rightarrow h+f$, but the $\varepsilon_{1,2}$ spurions break it when both are combined
- Higgs potential starts at order $m^2 \propto \varepsilon_1 x\ \varepsilon_2 f^2$, typically at 2-loops for extra $1/(4\pi)^4$...so Higgs is much lighter ("littler") compared to scale of new physics $f$ than naive naturalness estimates
- New physics can start at the few TeV scale
- New top partner at ~ 1 TeV to cancels quadratic contribution to Higgs mass$^2$

# The model (for quarks)



- 3 cells
- on each **black** site:
  - gauge group $G_b = SU(2) \times U(1)$
  - 4 **Dirac** fermions:

$$\Psi = \left(\begin{array}{c} u \\ d \\ \hdashline U \\ D \end{array}\right)$$

SU(2) doublet

SU(2) singlets

- on each **white** site:
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  - 4 **Chiral** fermions:

$$\psi_L = \left(\begin{array}{c} u \\ d \\ \hdashline 0 \\ 0 \end{array}\right)_L \qquad \psi_R = \left(\begin{array}{c} 0 \\ 0 \\ \hdashline U \\ D \end{array}\right)_R$$

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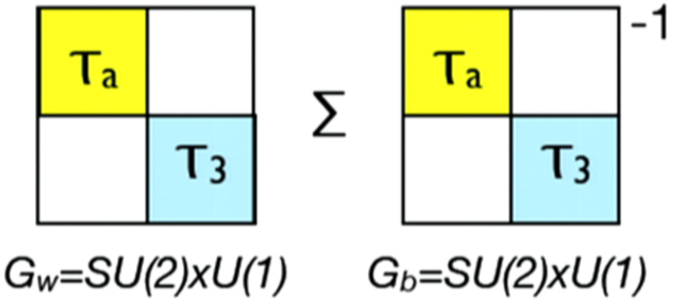
SU(2) singlets

- on each **link**:
  - *SU(4)xSU(4)/SU(4)* nonlinear sigma field

$$\Sigma = \xi \Sigma_H \xi \qquad \Sigma_H = \exp\left[\left(\frac{i\sqrt{2}}{f}\right)\begin{pmatrix}0 & \Phi^\dagger \\ \Phi & 0\end{pmatrix}\right]$$

$$\Phi = \begin{pmatrix}H_u^T \\ H_d^T\end{pmatrix} \qquad \xi = \exp\left[(i/2f)\begin{pmatrix}\vec{\pi}' \cdot \vec{\sigma} + \eta/\sqrt{2} & 0 \\ 0 & \vec{\pi} \cdot \vec{\sigma} - \eta/\sqrt{2}\end{pmatrix}\right]$$

  - $G_w \times G_b = [SU(2) \times U(1)]^2$ gauge group is embedded in *SU(4) x SU(4)*



$G_w=SU(2)xU(1)$ $G_b=SU(2)xU(1)$

  - diagonal *SU(2) x U(1)* will be SM gauge group
  - π'=*SU(2)* triplet
  - $\pi^\pm$, $\pi^0$, η = *SU(2)* singlets
  - $H_u$, $H_d$ = Higgs doublets

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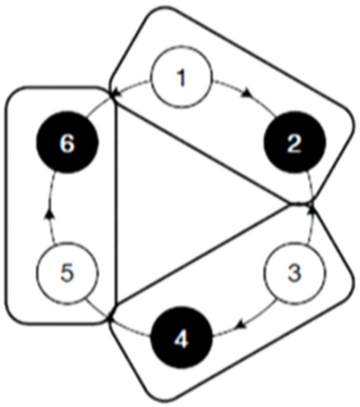

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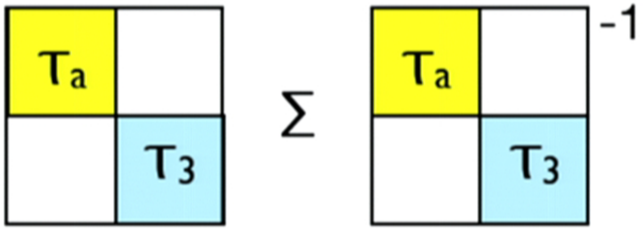
  - *G<sub></sub>*

- $G_w$ x $G_b$ = *[SU(2) x U(1)]*$^2$ gauge group is embedded in *SU(4) x SU(4)*



$G_w$=*SU(2)xU(1)* $G_b$=*SU(2)xU(1)*

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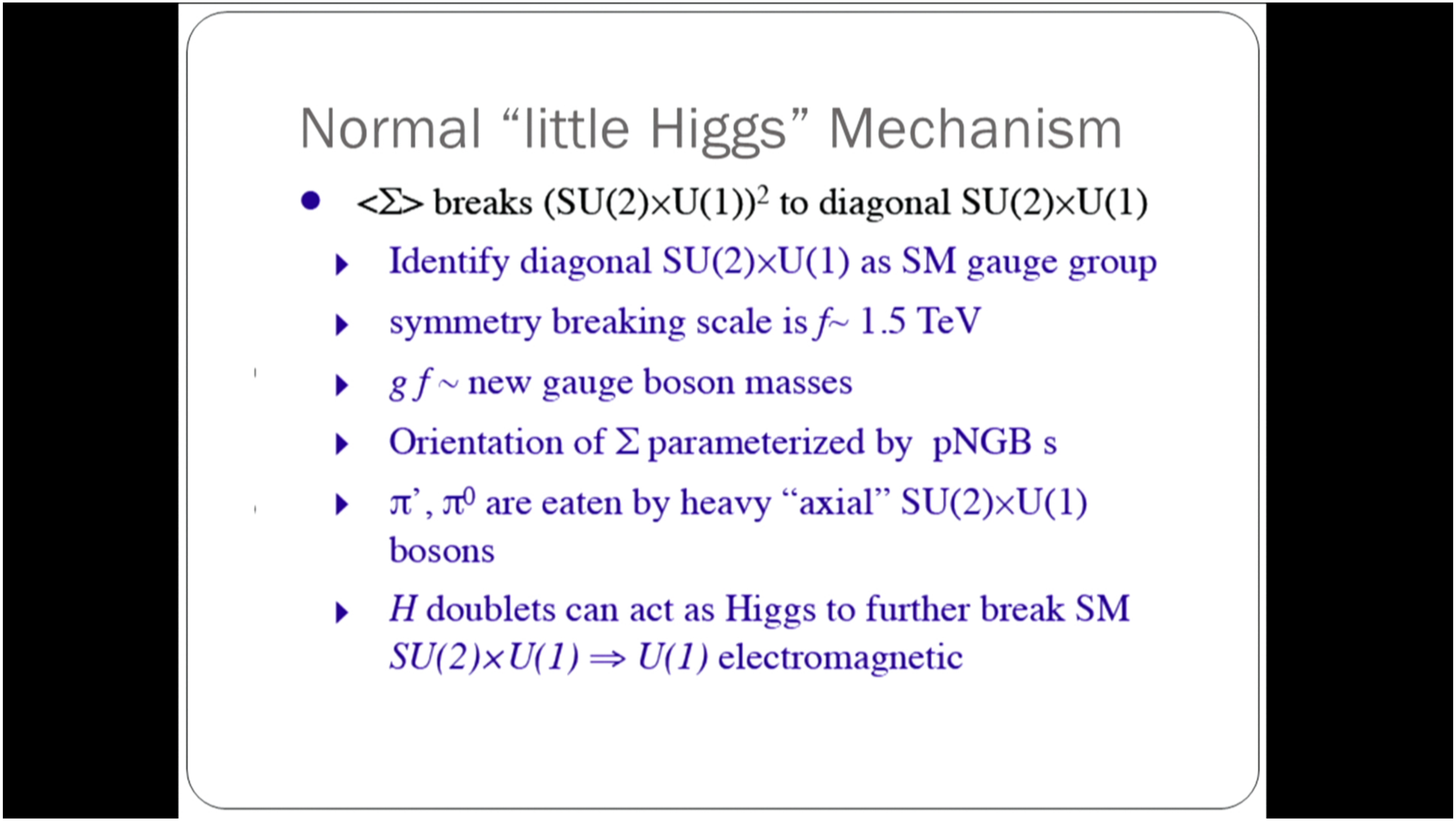

## Normal “little Higgs” Mechanism

- $<\Sigma>$ breaks $(SU(2)\times U(1))^2$ to diagonal $SU(2)\times U(1)$
  - Identify diagonal $SU(2)\times U(1)$ as SM gauge group
  - symmetry breaking scale is $f\sim$ 1.5 TeV
  - $g\, f\sim$ new gauge boson masses
  - Orientation of $\Sigma$ parameterized by pNGB s
  - $\pi'$, $\pi^0$ are eaten by heavy “axial” $SU(2)\times U(1)$ bosons
  - $H$ doublets can act as Higgs to further break SM $SU(2)\times U(1) \Rightarrow U(1)$ electromagnetic

## Fermion mass and Yukawa interactions:

*U(3) x SU(4) symmetric terms*

$$\Psi = \begin{pmatrix} u \\ d \\ \hdashline U \\ D \end{pmatrix} \quad \psi_L = \begin{pmatrix} u \\ d \\ \hdashline 0 \\ 0 \end{pmatrix}_L \quad \psi_R = \begin{pmatrix} 0 \\ 0 \\ \hdashline U \\ D \end{pmatrix}_R$$

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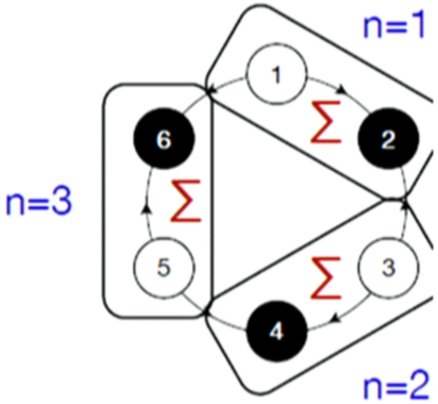
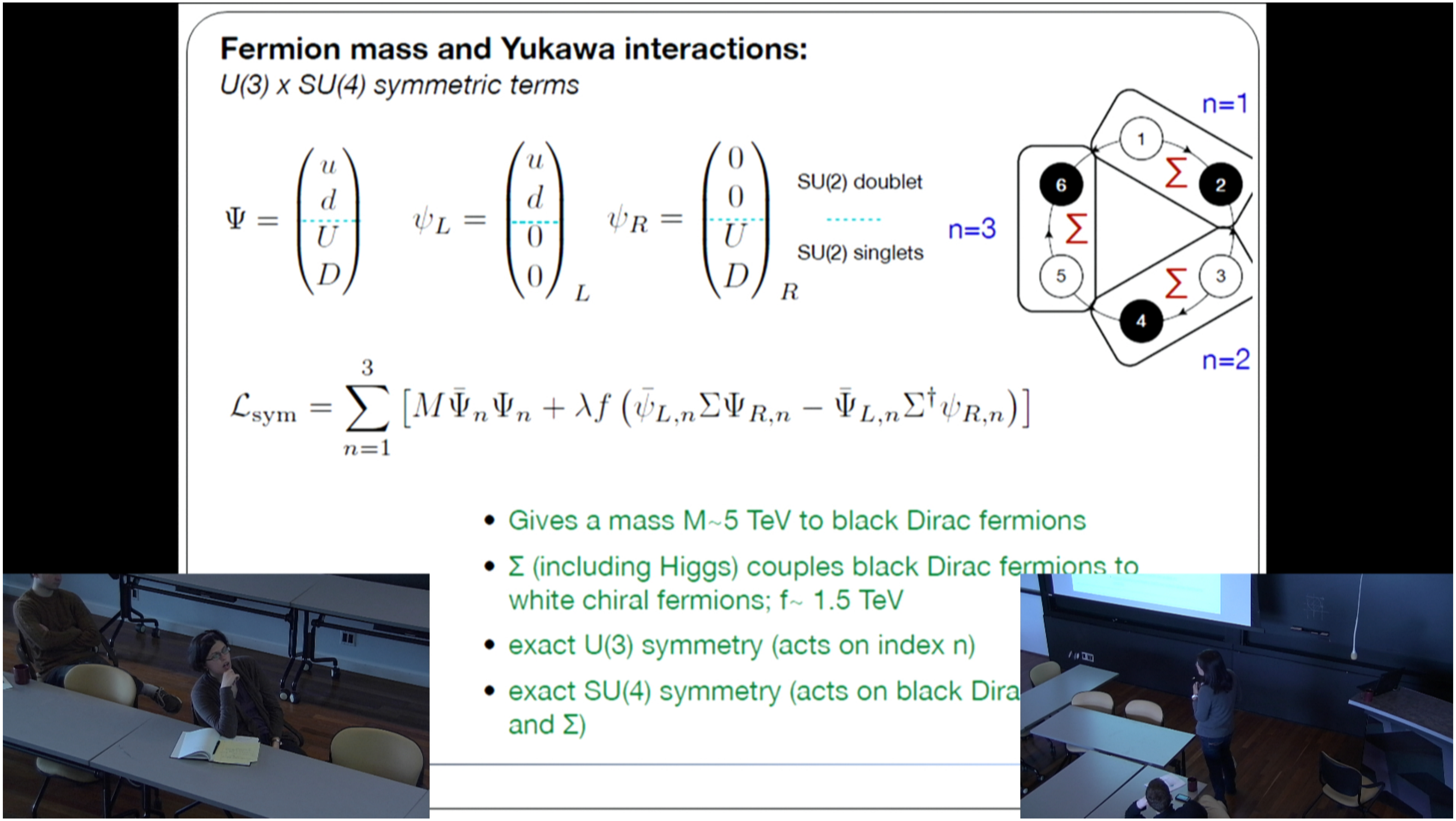
SU(2) singlets



$$\mathcal{L}_{\rm sym} = \sum_{n=1}^{3} \left[ M\bar{\Psi}_n\Psi_n + \lambda f \left( \bar{\psi}_{L,n}\Sigma\Psi_{R,n} - \bar{\Psi}_{L,n}\Sigma^{\dagger}\psi_{R,n} \right) \right]$$

- Gives a mass M~5 TeV to black Dirac fermions
- Σ (including Higgs) couples black Dirac fermions to white chiral fermions; f~ 1.5 TeV
- exact U(3) symmetry (acts on index n)
- exact SU(4) symmetry (acts on black Dirac fermions and Σ)



## Fermion mass and Yukawa interactions:

*U(3) x SU(4) symmetric terms*

$$\Psi = \begin{pmatrix} u \\ d \\ U \\ D \end{pmatrix} \quad \psi_L = \begin{pmatrix} u \\ d \\ 0 \\ 0 \end{pmatrix}_L \quad \psi_R = \begin{pmatrix} 0 \\ 0 \\ U \\ D \end{pmatrix}_R$$

SU(2) doublet

SU(2) singlets

$$\mathcal{L}_{\rm sym} = \sum_{n=1}^{3} \left[ M \bar{\Psi}_n \Psi_n + \lambda f \left( \bar{\psi}_{L,n} \Sigma \Psi_{R,n} - \bar{\Psi}_{L,n} \Sigma^\dagger \psi_{R,n} \right) \right]$$

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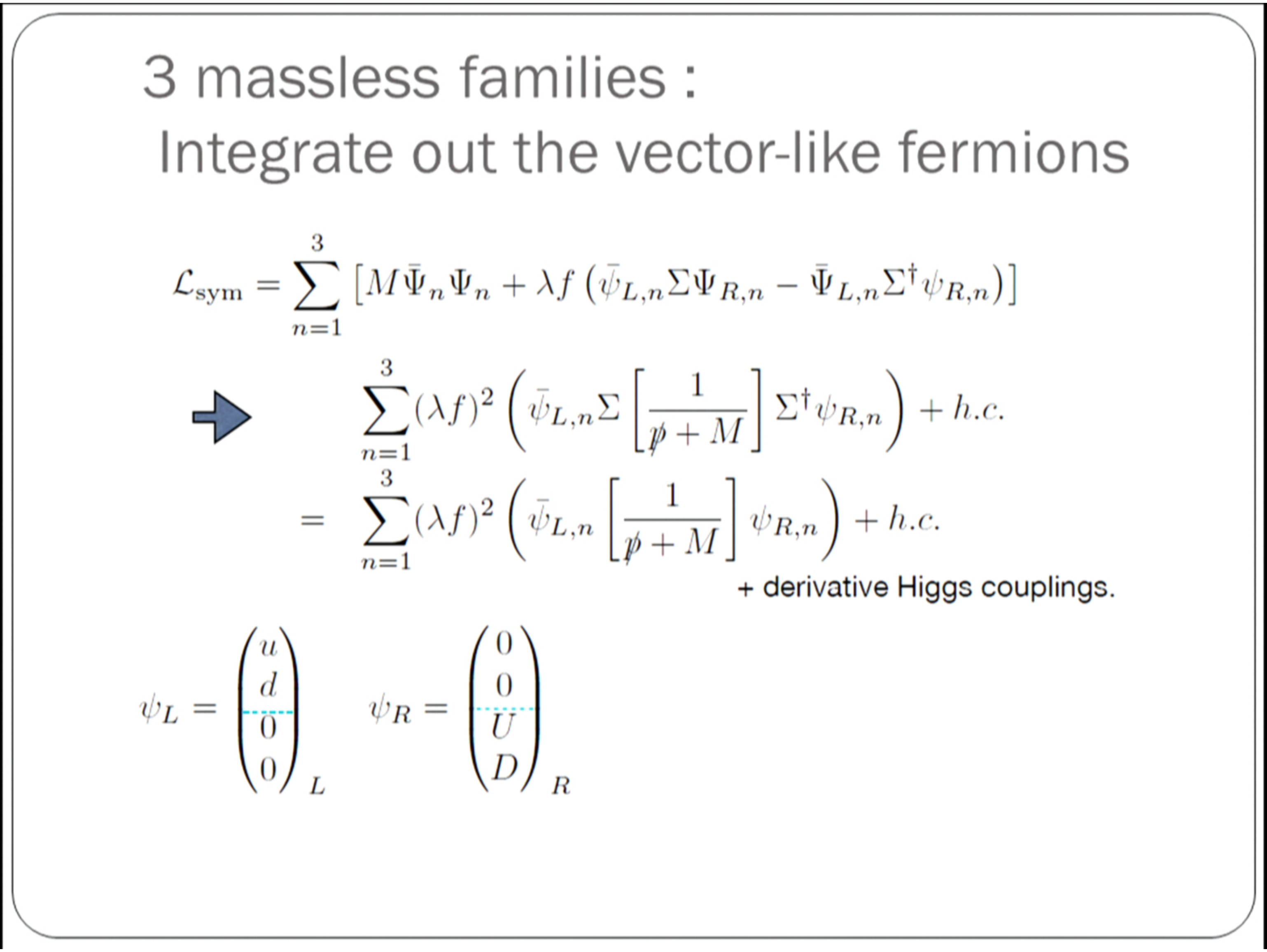
$$\mathcal{L}_{\rm sym} = \sum_{n=1}^{3}\left[M\bar{\Psi}_n\Psi_n + \underline{\lambda f\left(\bar{\psi}_{L,n}\Sigma\Psi_{R,n} - \bar{\Psi}_{L,n}\Sigma^\dagger\psi_{R,n}\right)}\right]$$

Expand to give Higgs couplings:

$$i\sqrt{2}\,\lambda\left[\left((\bar{u}_{w,n},\bar{d}_{w,n})_L\Phi^\dagger\begin{pmatrix}U_{b,n}\\ D_{b,n}\end{pmatrix}_R - (\bar{u}_{b,n},\bar{d}_{b,n})_L\Phi\begin{pmatrix}U_{w,n}\\ D_{w,n}\end{pmatrix}_R\right)\right]$$

$$\Phi^\dagger = (H_u^*, H_d^*)$$

- *Looks* like a Φ (Higgs) vev would give all fermions a mass...
- ...but not true: even with SU(2) x U(1) breaking, still have 3 massless chiral families of quarks + 3 massive Dirac families



# 3 massless families :

# Integrate out the vector-like fermions

$$\mathcal{L}_{\rm sym} = \sum_{n=1}^{3} \left[ M \bar{\Psi}_n \Psi_n + \lambda f \left( \bar{\psi}_{L,n} \Sigma \Psi_{R,n} - \bar{\Psi}_{L,n} \Sigma^\dagger \psi_{R,n} \right) \right]$$

$$\Rightarrow \quad \sum_{n=1}^{3} (\lambda f)^2 \left( \bar{\psi}_{L,n} \Sigma \left[ \frac{1}{\not{p} + M} \right] \Sigma^\dagger \psi_{R,n} \right) + h.c.$$

$$= \sum_{n=1}^{3} (\lambda f)^2 \left( \bar{\psi}_{L,n} \left[ \frac{1}{\not{p} + M} \right] \psi_{R,n} \right) + h.c.$$

+ derivative Higgs couplings.

$$\psi_L = \left( \begin{array}{c} u \\ d \\ \hdashline 0 \\ 0 \end{array} \right)_L \qquad \psi_R = \left( \begin{array}{c} 0 \\ 0 \\ \hdashline U \\ D \end{array} \right)_R$$

## Fermion mass and Yukawa interactions:

*add U(3) x SU(4) symmetry breaking terms*

$$\mathcal{L}_{\text{asym}} = \sum_{m,n=1}^{3} \bar{\Psi}_{m,L} \left(M^u X_u + M^d X_d\right)_{mn} \Psi_{n,R} + h.c.$$

$$\Psi = \begin{pmatrix} u \\ d \\ \hdashline U \\ D \end{pmatrix}$$

SU(2) doublet

SU(2) singlets

- Acts only on black-site Dirac fermions
- $M^u$, $M^d$ break the U(3) symmetry ⇒ *$U(1)_B$* *(particular texture chosen)*

$$M^u = \begin{pmatrix} \mathcal{M}^u_{11} & \mathcal{M}^u_{12} & 0 \\ 0 & \mathcal{M}^u_{22} & 0 \\ \mathcal{M}^u_{31} & 0 & \mathcal{M}^u_{33} \end{pmatrix} \quad , \quad M^d = \begin{pmatrix} \mathcal{M}^d_{11} & 0 & 0 \\ \mathcal{M}^d_{21} & \mathcal{M}^d_{22} & 0 \\ 0 & \mathcal{M}^d_{32} & \mathcal{M}^d_{33} \end{pmatrix}$$

- $X_u$, $X_d$ break the SU(4) symmetry ⇒ different *SU(3)* subgroups

$$X_u = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & -3 & \\ & & & 1 \end{pmatrix} , \quad X_d = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -3 \end{pmatrix}$$

Peculiar symmetry structure ensures Little Higgs mechanism in the fermion sector:

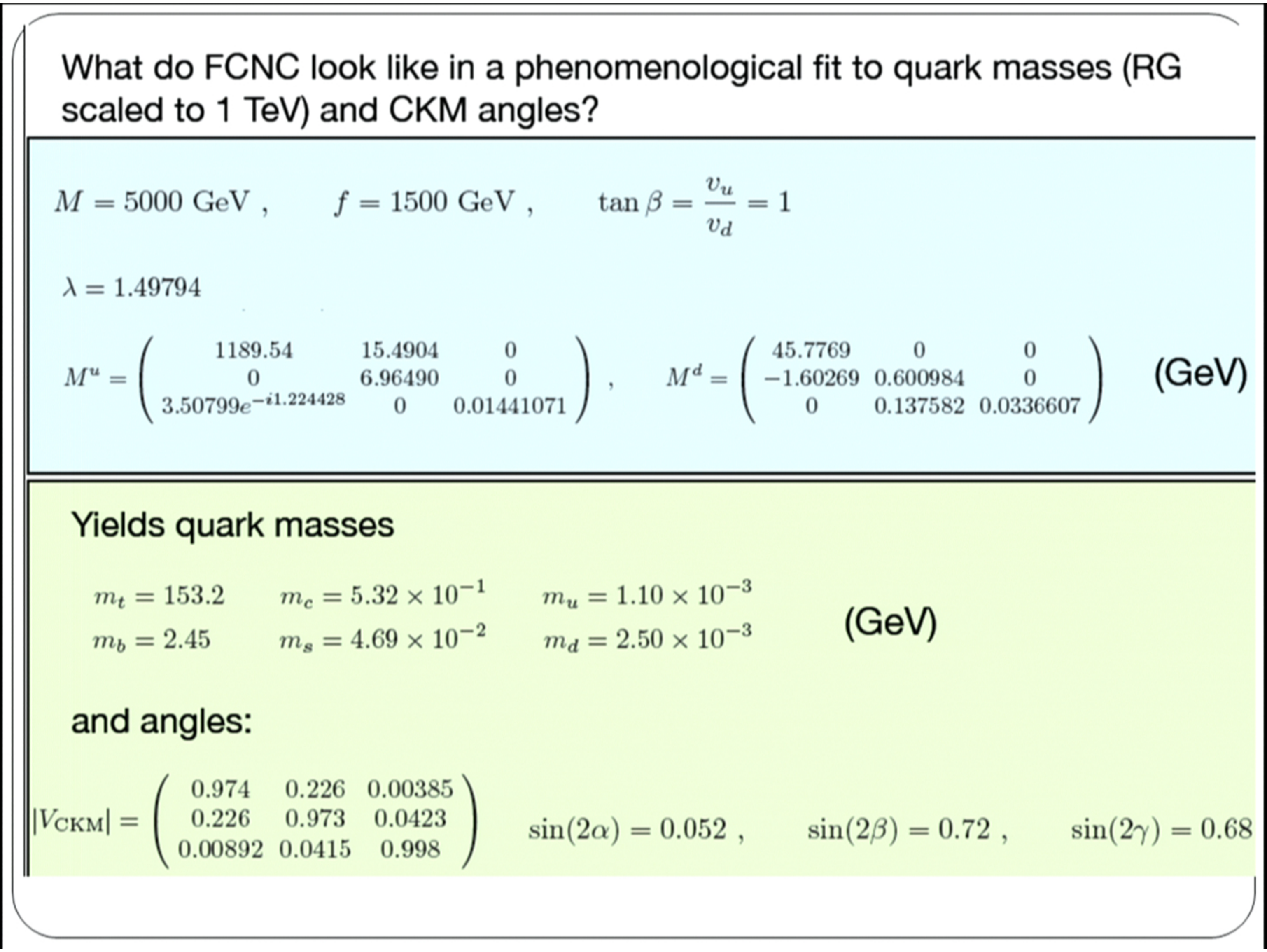
If M is the full fermion mass matrix, then

- Tr $M^{\dagger}M$ is independent of H vevs
- Tr $(M^{\dagger}M)^2$ is independent of H vevs

So there are neither quadratic nor log divergent contributions to the Higgs potential from fermions at one loop

There will be a finite Coleman-Weinberg contribution, Tr $(M^{\dagger}M)^2 \ln(M^{\dagger}M)$. To avoid fine tuning of the Higgs potential, there needs to be a Dirac top-partner at $\sim$ 1 TeV.

*At this level there is a Peccei-Quinn symmetry protecting against flavor violating Higgs couplings...to be softly broken in Higgs potential*



## What do FCNC look like in a phenomenological fit to quark masses (RG scaled to 1 TeV) and CKM angles?

$$M = 5000\ \text{GeV}\ , \qquad f = 1500\ \text{GeV}\ , \qquad \tan\beta = \frac{v_u}{v_d} = 1$$

$$\lambda = 1.49794$$

$$M^u = \begin{pmatrix} 1189.54 & 15.4904 & 0 \\ 0 & 6.96490 & 0 \\ 3.50799e^{-i1.224428} & 0 & 0.01441071 \end{pmatrix}\ , \qquad M^d = \begin{pmatrix} 45.7769 & 0 & 0 \\ -1.60269 & 0.600984 & 0 \\ 0 & 0.137582 & 0.0336607 \end{pmatrix}$$

(GeV)

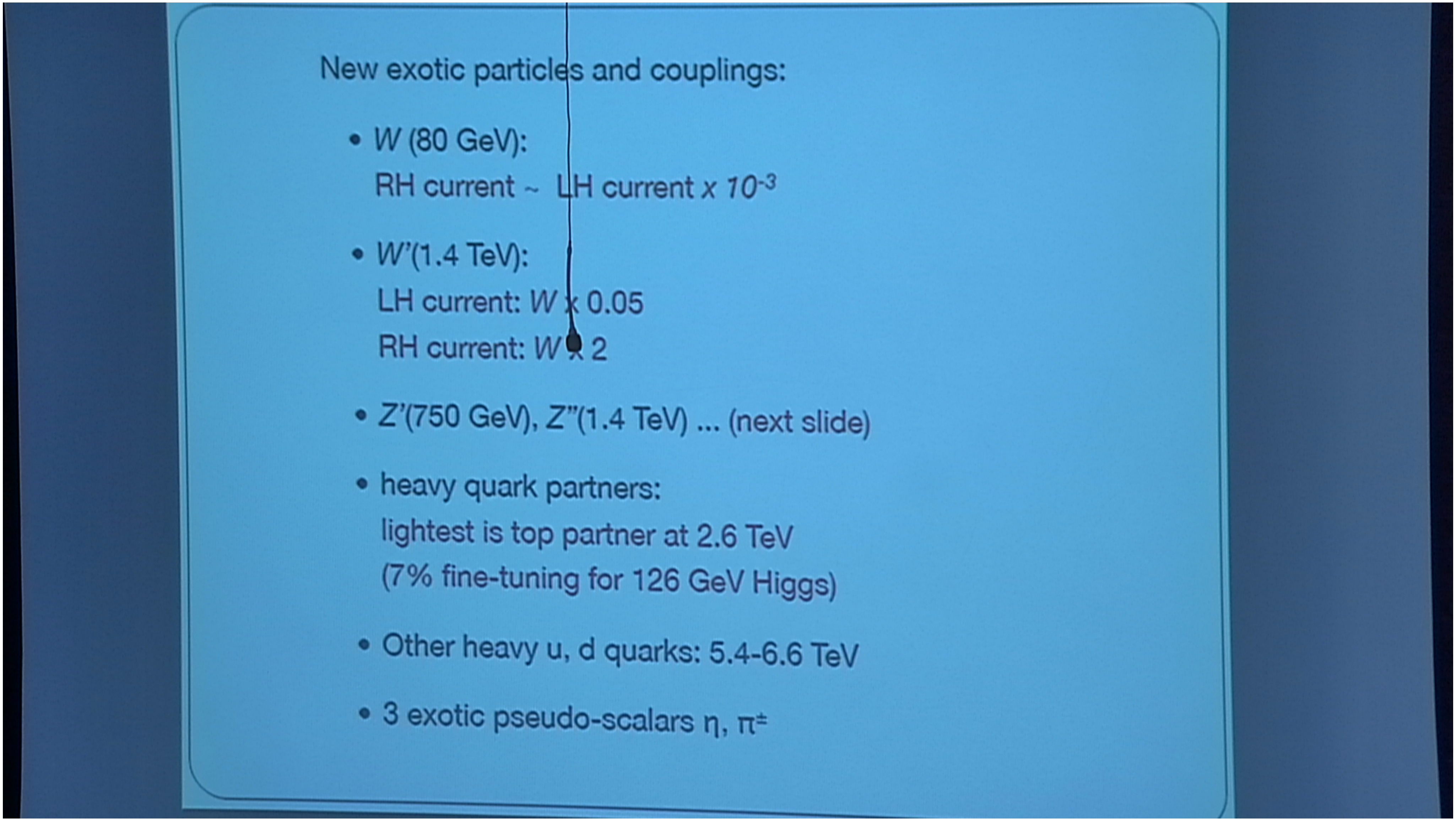
Yields quark masses

$$m_t = 153.2 \qquad m_c = 5.32 \times 10^{-1} \qquad m_u = 1.10 \times 10^{-3}$$
$$m_b = 2.45 \qquad m_s = 4.69 \times 10^{-2} \qquad m_d = 2.50 \times 10^{-3}$$

(GeV)

and angles:

$$|V_{\text{CKM}}| = \begin{pmatrix} 0.974 & 0.226 & 0.00385 \\ 0.226 & 0.973 & 0.0423 \\ 0.00892 & 0.0415 & 0.998 \end{pmatrix} \qquad \sin(2\alpha) = 0.052\ , \qquad \sin(2\beta) = 0.72\ , \qquad \sin(2\gamma) = 0.68$$



New exotic particles and couplings:

- $W$ (80 GeV):
  RH current ~ LH current $x\ 10^{-3}$
- $W'$(1.4 TeV):
  LH current: $W$ x 0.05
  RH current: $W$ x 2
- $Z'$(750 GeV), $Z''$(1.4 TeV) ... (next slide)
- heavy quark partners:
  lightest is top partner at 2.6 TeV
  (7% fine-tuning for 126 GeV Higgs)
- Other heavy u, d quarks: 5.4-6.6 TeV
- 3 exotic pseudo-scalars $\eta$, $\pi^{\pm}$

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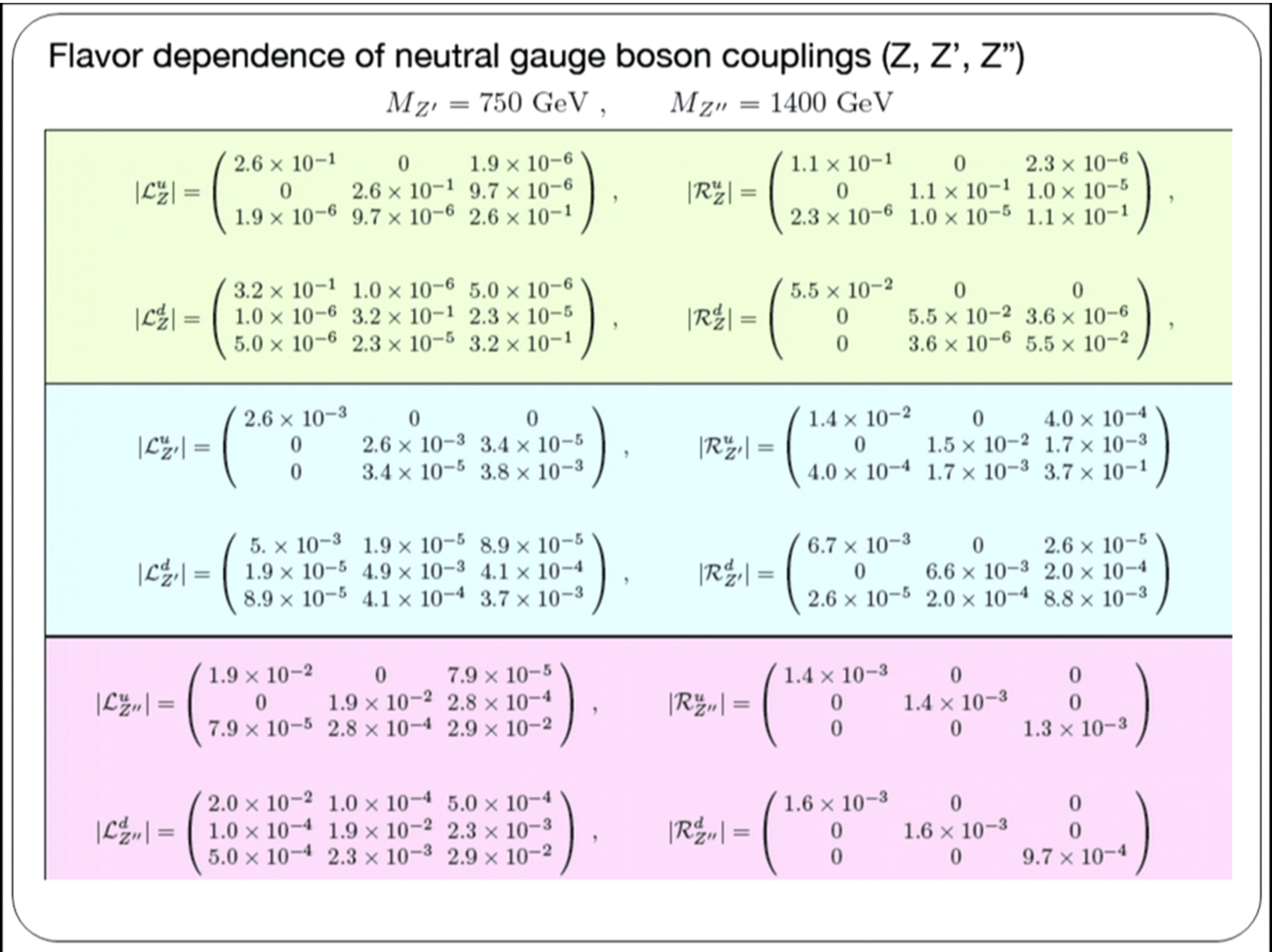


## Flavor dependence of neutral gauge boson couplings (Z, Z', Z")

$$M_{Z'} = 750\ \text{GeV}\ , \qquad M_{Z''} = 1400\ \text{GeV}$$

$$|\mathcal{L}^u_Z| = \begin{pmatrix} 2.6\times10^{-1} & 0 & 1.9\times10^{-6} \\ 0 & 2.6\times10^{-1} & 9.7\times10^{-6} \\ 1.9\times10^{-6} & 9.7\times10^{-6} & 2.6\times10^{-1} \end{pmatrix}\ , \qquad |\mathcal{R}^u_Z| = \begin{pmatrix} 1.1\times10^{-1} & 0 & 2.3\times10^{-6} \\ 0 & 1.1\times10^{-1} & 1.0\times10^{-5} \\ 2.3\times10^{-6} & 1.0\times10^{-5} & 1.1\times10^{-1} \end{pmatrix}\ ,$$

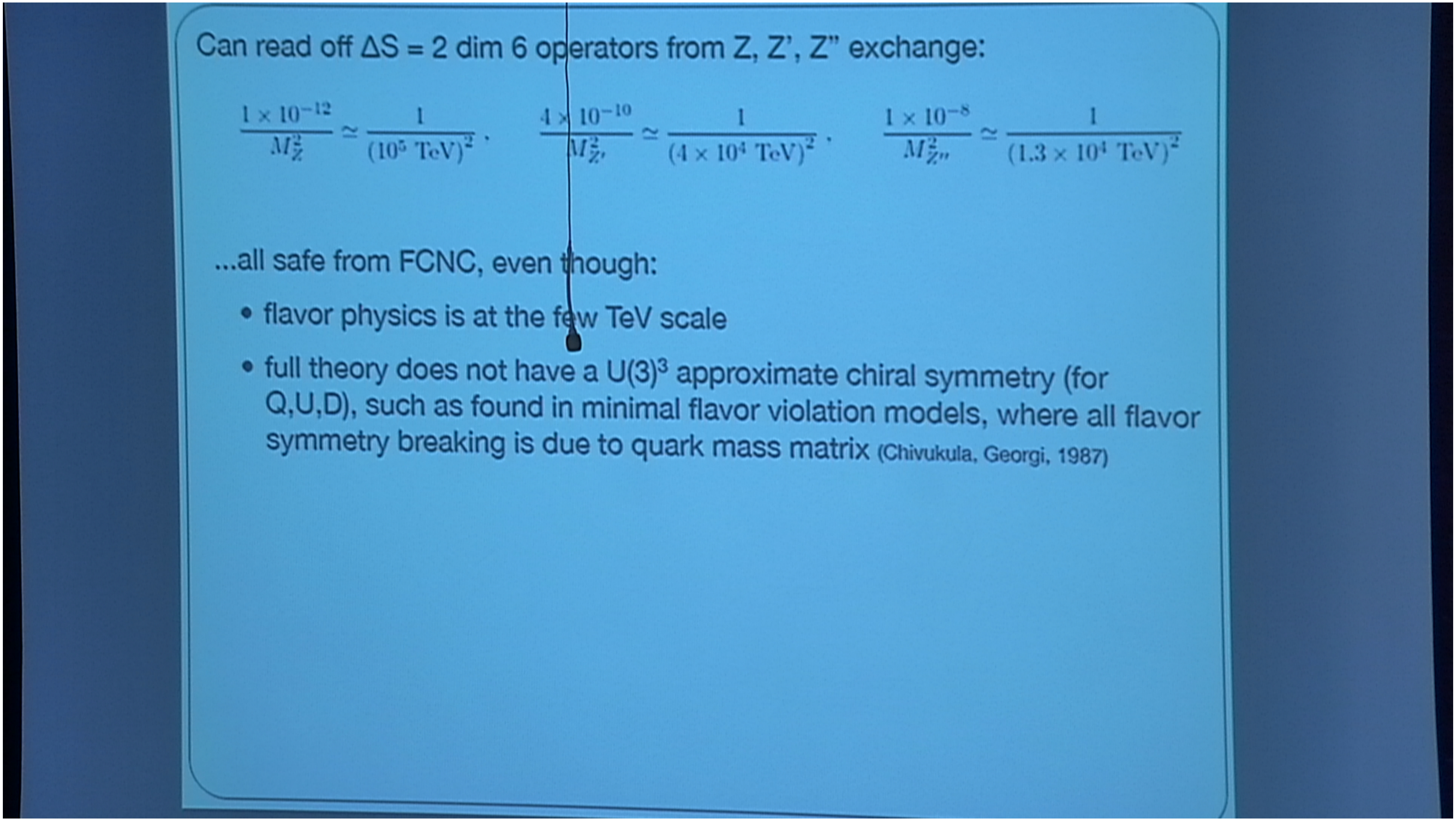
$$|\mathcal{L}^d_Z| = \begin{pmatrix} 3.2\times10^{-1} & 1.0\times10^{-6} & 5.0\times10^{-6} \\ 1.0\times10^{-6} & 3.2\times10^{-1} & 2.3\times10^{-5} \\ 5.0\times10^{-6} & 2.3\times10^{-5} & 3.2\times10^{-1} \end{pmatrix}\ , \qquad |\mathcal{R}^d_Z| = \begin{pmatrix} 5.5\times10^{-2} & 0 & 0 \\ 0 & 5.5\times10^{-2} & 3.6\times10^{-6} \\ 0 & 3.6\times10^{-6} & 5.5\times10^{-2} \end{pmatrix}\ ,$$

$$|\mathcal{L}^u_{Z'}| = \begin{pmatrix} 2.6\times10^{-3} & 0 & 0 \\ 0 & 2.6\times10^{-3} & 3.4\times10^{-5} \\ 0 & 3.4\times10^{-5} & 3.8\times10^{-3} \end{pmatrix}\ , \qquad |\mathcal{R}^u_{Z'}| = \begin{pmatrix} 1.4\times10^{-2} & 0 & 4.0\times10^{-4} \\ 0 & 1.5\times10^{-2} & 1.7\times10^{-3} \\ 4.0\times10^{-4} & 1.7\times10^{-3} & 3.7\times10^{-1} \end{pmatrix}$$

$$|\mathcal{L}^d_{Z'}| = \begin{pmatrix} 5.\times10^{-3} & 1.9\times10^{-5} & 8.9\times10^{-5} \\ 1.9\times10^{-5} & 4.9\times10^{-3} & 4.1\times10^{-4} \\ 8.9\times10^{-5} & 4.1\times10^{-4} & 3.7\times10^{-3} \end{pmatrix}\ , \qquad |\mathcal{R}^d_{Z'}| = \begin{pmatrix} 6.7\times10^{-3} & 0 & 2.6\times10^{-5} \\ 0 & 6.6\times10^{-3} & 2.0\times10^{-4} \\ 2.6\times10^{-5} & 2.0\times10^{-4} & 8.8\times10^{-3} \end{pmatrix}$$

$$|\mathcal{L}^u_{Z''}| = \begin{pmatrix} 1.9\times10^{-2} & 0 & 7.9\times10^{-5} \\ 0 & 1.9\times10^{-2} & 2.8\times10^{-4} \\ 7.9\times10^{-5} & 2.8\times10^{-4} & 2.9\times10^{-2} \end{pmatrix}\ , \qquad |\mathcal{R}^u_{Z''}| = \begin{pmatrix} 1.4\times10^{-3} & 0 & 0 \\ 0 & 1.4\times10^{-3} & 0 \\ 0 & 0 & 1.3\times10^{-3} \end{pmatrix}$$

$$|\mathcal{L}^d_{Z''}| = \begin{pmatrix} 2.0\times10^{-2} & 1.0\times10^{-4} & 5.0\times10^{-4} \\ 1.0\times10^{-4} & 1.9\times10^{-2} & 2.3\times10^{-3} \\ 5.0\times10^{-4} & 2.3\times10^{-3} & 2.9\times10^{-2} \end{pmatrix}\ , \qquad |\mathcal{R}^d_{Z''}| = \begin{pmatrix} 1.6\times10^{-3} & 0 & 0 \\ 0 & 1.6\times10^{-3} & 0 \\ 0 & 0 & 9.7\times10^{-4} \end{pmatrix}$$



Can read off ΔS = 2 dim 6 operators from Z, Z', Z" exchange:

$$\frac{1\times10^{-12}}{M_Z^2}\simeq\frac{1}{(10^5\ \mathrm{TeV})^2}\,,\qquad\frac{4\times10^{-10}}{M_{Z'}^2}\simeq\frac{1}{(4\times10^4\ \mathrm{TeV})^2}\,,\qquad\frac{1\times10^{-8}}{M_{Z''}^2}\simeq\frac{1}{(1.3\times10^4\ \mathrm{TeV})^2}$$

...all safe from FCNC, even though:

- flavor physics is at the few TeV scale
- full theory does not have a $U(3)^3$ approximate chiral symmetry (for Q,U,D), such as found in minimal flavor violation models, where all flavor symmetry breaking is due to quark mass matrix (Chivukula, Georgi, 1987)

# Flavor dependence of neutral gauge boson couplings (Z, Z’, Z”)

$$M_{Z'} = 750 \text{ GeV} , \qquad M_{Z''} = 1400 \text{ GeV}$$

$$|\mathcal{L}_Z^u| = \begin{pmatrix} 2.6\times10^{-1} & 0 & 1.9\times10^{-6} \\ 0 & 2.6\times10^{-1} & 9.7\times10^{-6} \\ 1.9\times10^{-6} & 9.7\times10^{-6} & 2.6\times10^{-1} \end{pmatrix} , \qquad |\mathcal{R}_Z^u| = \begin{pmatrix} 1.1\times10^{-1} & 0 & 2.3\times10^{-6} \\ 0 & 1.1\times10^{-1} & 1.0\times10^{-5} \\ 2.3\times10^{-6} & 1.0\times10^{-5} & 1.1\times10^{-1} \end{pmatrix} ,$$

$$|\mathcal{L}_Z^d| = \begin{pmatrix} 3.2\times10^{-1} & 1.0\times10^{-6} & 5.0\times10^{-6} \\ 1.0\times10^{-6} & 3.2\times10^{-1} & 2.3\times10^{-5} \\ 5.0\times10^{-6} & 2.3\times10^{-5} & 3.2\times10^{-1} \end{pmatrix} , \qquad |\mathcal{R}_Z^d| = \begin{pmatrix} 5.5\times10^{-2} & 0 & 0 \\ 0 & 5.5\times10^{-2} & 3.6\times10^{-6} \\ 0 & 3.6\times10^{-6} & 5.5\times10^{-2} \end{pmatrix} ,$$

$$|\mathcal{L}_{Z'}^u| = \begin{pmatrix} 2.6\times10^{-3} & 0 & 0 \\ 0 & 2.6\times10^{-3} & 3.4\times10^{-5} \\ 0 & 3.4\times10^{-5} & 3.8\times10^{-3} \end{pmatrix} , \qquad |\mathcal{R}_{Z'}^u| = \begin{pmatrix} 1.4\times10^{-2} & 0 & 4.0\times10^{-4} \\ 0 & 1.5\times10^{-2} & 1.7\times10^{-3} \\ 4.0\times10^{-4} & 1.7\times10^{-3} & 3.7\times10^{-1} \end{pmatrix}$$

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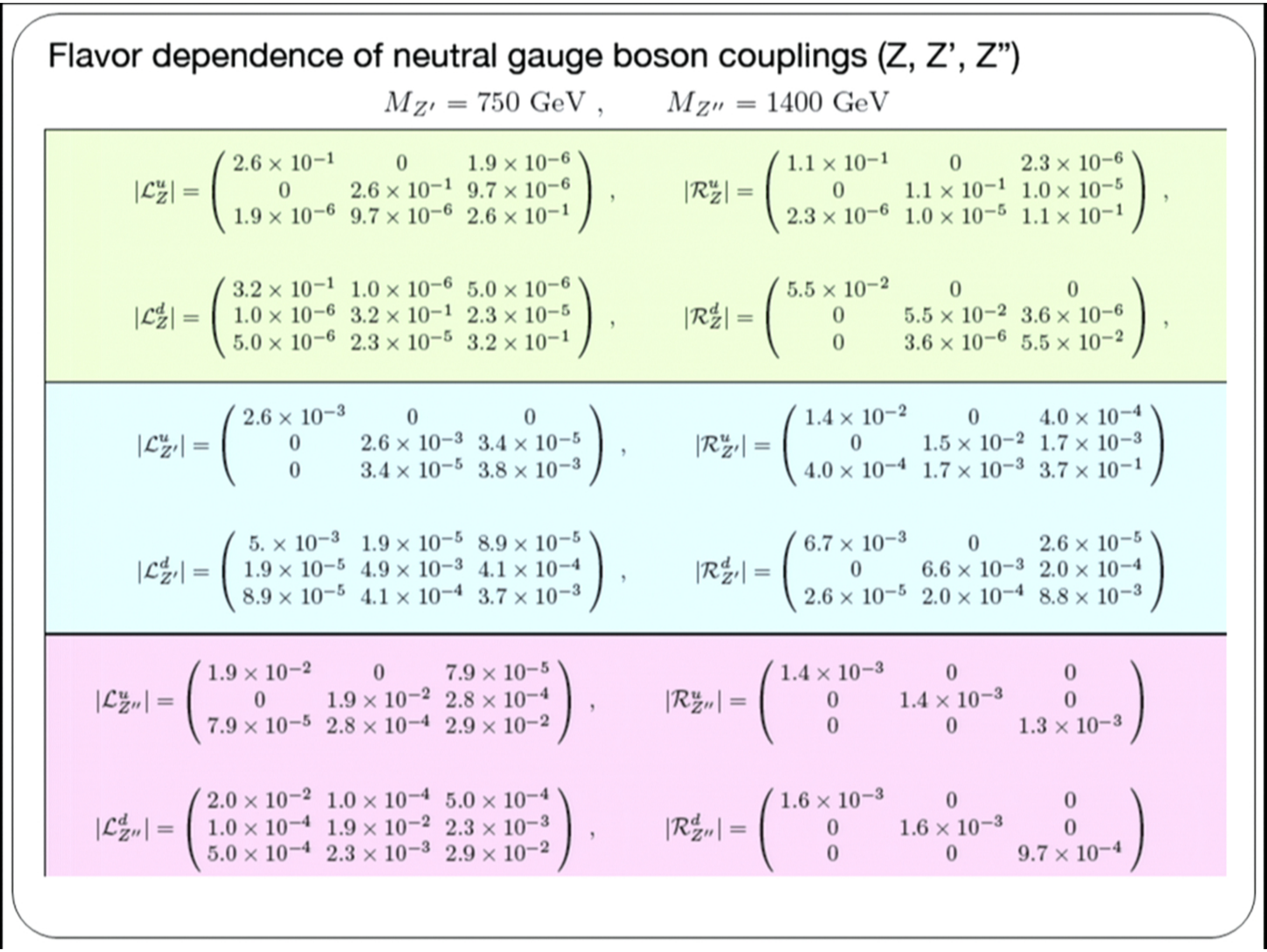
$$|\mathcal{L}_{Z''}^d| = \begin{pmatrix} 2.0\times10^{-2} & 1.0\times10^{-4} & 5.0\times10^{-4} \\ 1.0\times10^{-4} & 1.9\times10^{-2} & 2.3\times10^{-3} \\ 5.0\times10^{-4} & 2.3\times10^{-3} & 2.9\times10^{-2} \end{pmatrix} , \qquad |\mathcal{R}_{Z''}^d| = \begin{pmatrix} 1.6\times10^{-3} & 0 & 0 \\ 0 & 1.6\times10^{-3} & 0 \\ 0 & 0 & 9.7\times10^{-4} \end{pmatrix}$$

## Direct detection of Z’(750 GeV) or Z”(1.4 TeV)?

- Production rate of Z’ is down by $10^{-3}$ compared to Z-like couplings
- Production rate of Z” is down by 5 x$10^{-3}$ compared to Z-like couplings
- Leptonic partial width not computable in this model (no leptons!)
- Likely not ruled out presently



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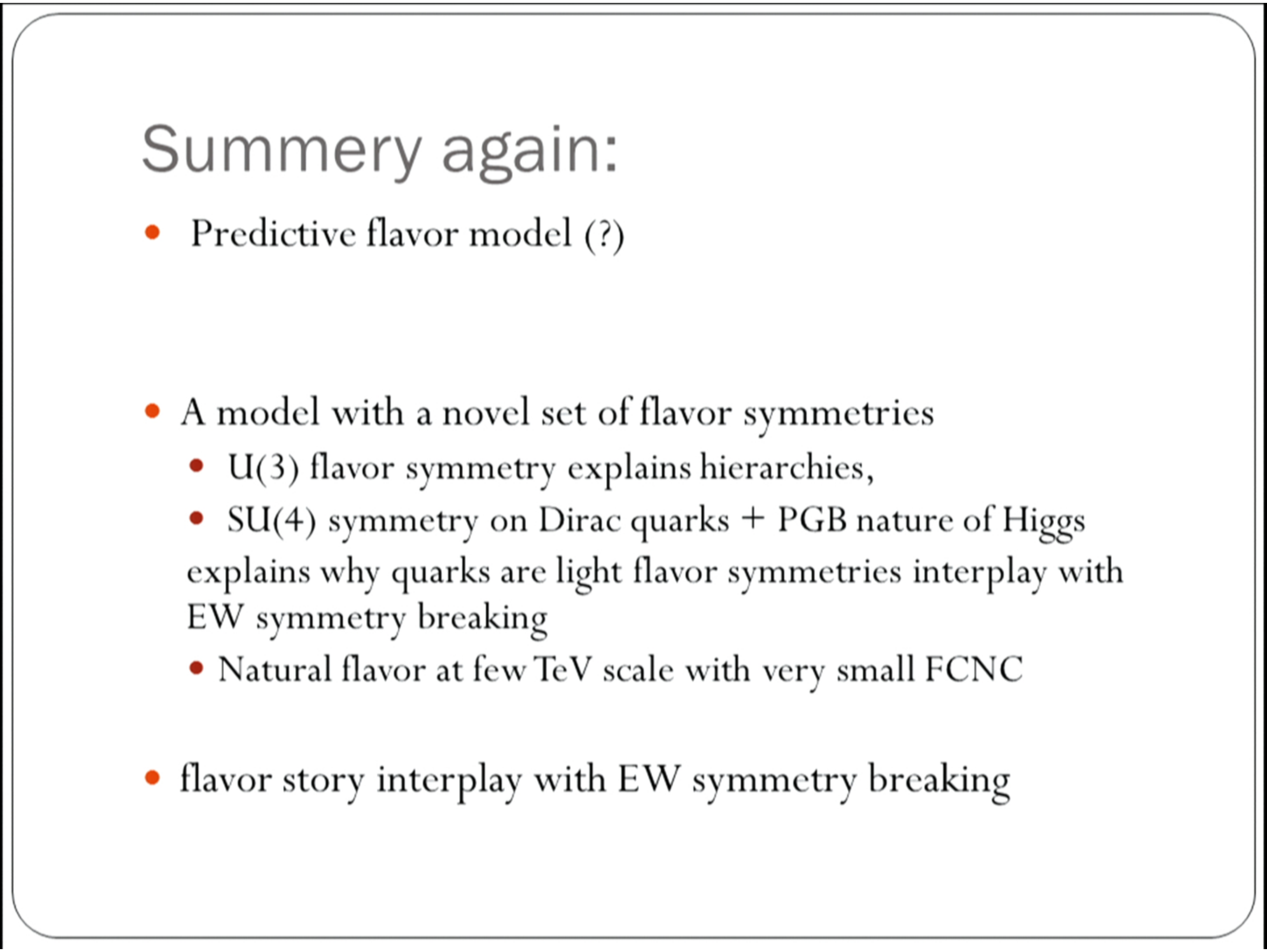
# Summery again:

- Predictive flavor model (?)

- A model with a novel set of flavor symmetries
  - U(3) flavor symmetry explains hierarchies,
  - SU(4) symmetry on Dirac quarks + PGB nature of Higgs explains why quarks are light flavor symmetries interplay with EW symmetry breaking
  - Natural flavor at few TeV scale with very small FCNC

- flavor story interplay with EW symmetry breaking



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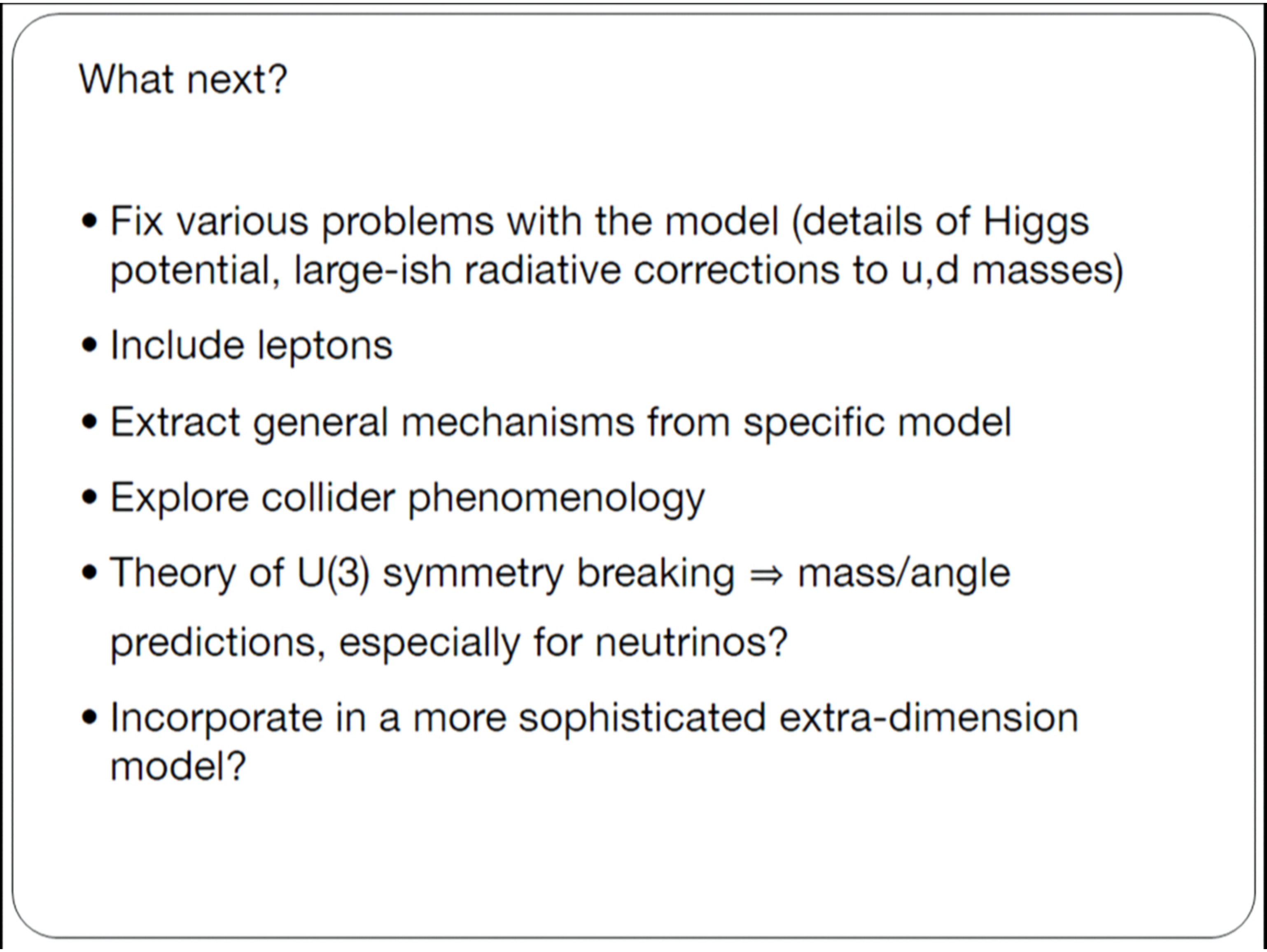

## What next?

- Fix various problems with the model (details of Higgs potential, large-ish radiative corrections to u,d masses)
- Include leptons
- Extract general mechanisms from specific model
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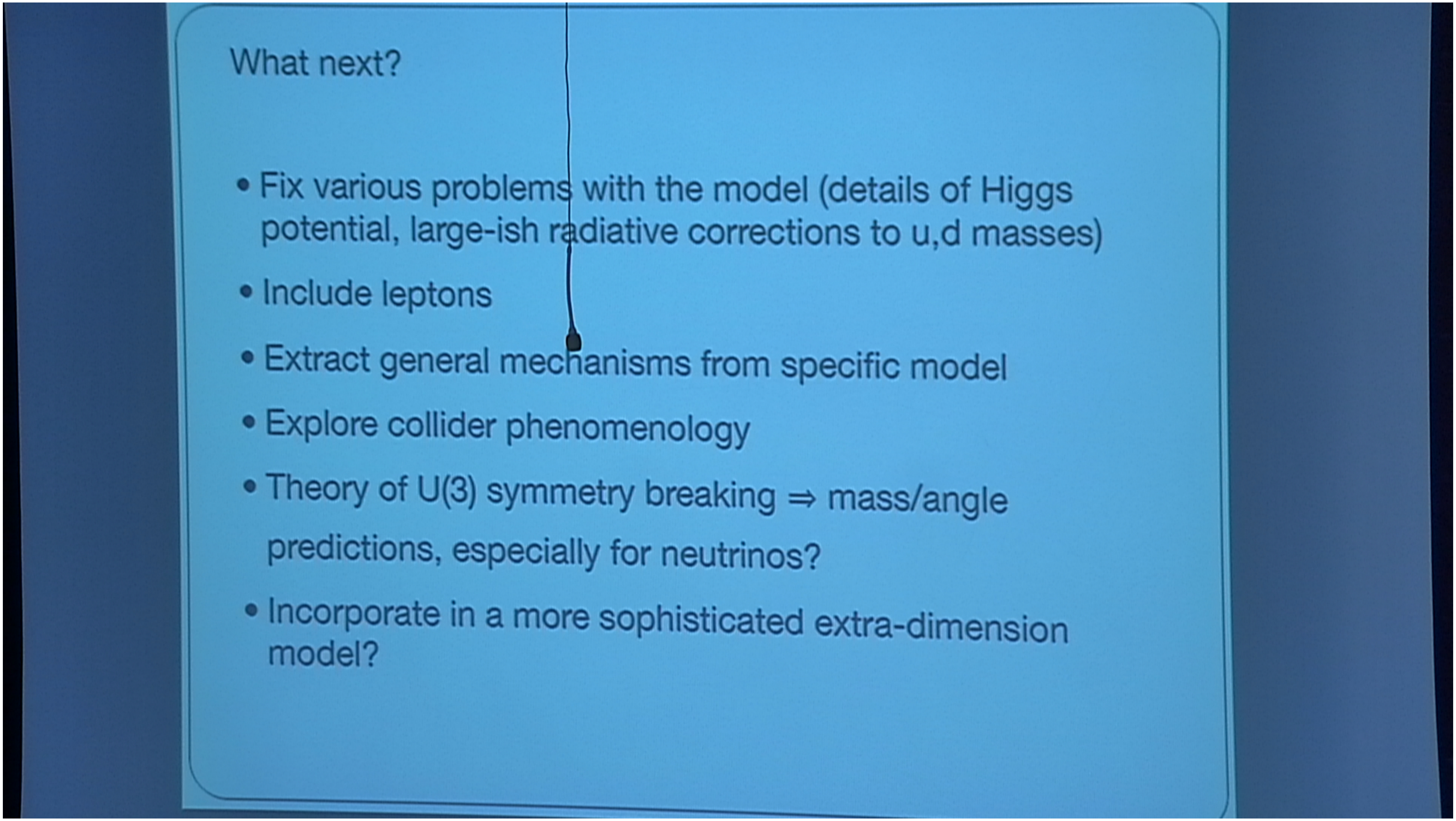
- Theory of U(3) symmetry breaking ⇒ mass/angle predictions, especially for neutrinos?
- Incorporate in a more sophisticated extra-dimension model?



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