

Title: The theory of composition in physics

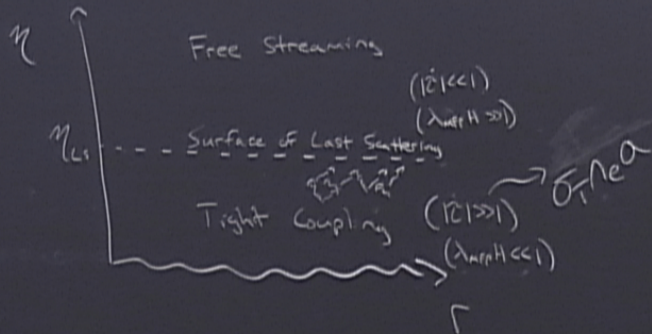
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URL: <http://pirsa.org/13040121>

Abstract: We develop a theory for describing composite objects in physics. These can be static objects, such as tables, or things that happen in spacetime (such as a region of spacetime with fields on it regarded as being composed of smaller such regions joined together). We propose certain fundamental axioms which, it seems, should be satisfied in any theory of composition. A key axiom is the order independence axiom which says we can describe the composition of a composite object in any order. Then we provide a notation for describing composite objects that naturally leads to these axioms being satisfied. In any given physical context we are interested in the value of certain properties for the objects (such as whether the object is possible, what probability it has, how wide it is, and so on). We associate a generalized state with an object. This can be used to calculate the value of those properties we are interested in for for this object. We then propose a certain principle, the composition principle, which says that we can determine the generalized state of a composite object from the generalized states for the components by means of a calculation having the same structure as the description of the generalized state. The composition principle provides a link between description and prediction.



Recap



Tight Coupling

$$\left[\frac{d^2}{d\eta^2} + \frac{\dot{R}}{1+R} \frac{d}{d\eta} + k^2 c_s^2 \right] [\Theta_0 + \Phi] = \frac{k^2}{2} \left[\frac{1}{1+R} \Phi - \Psi \right]$$

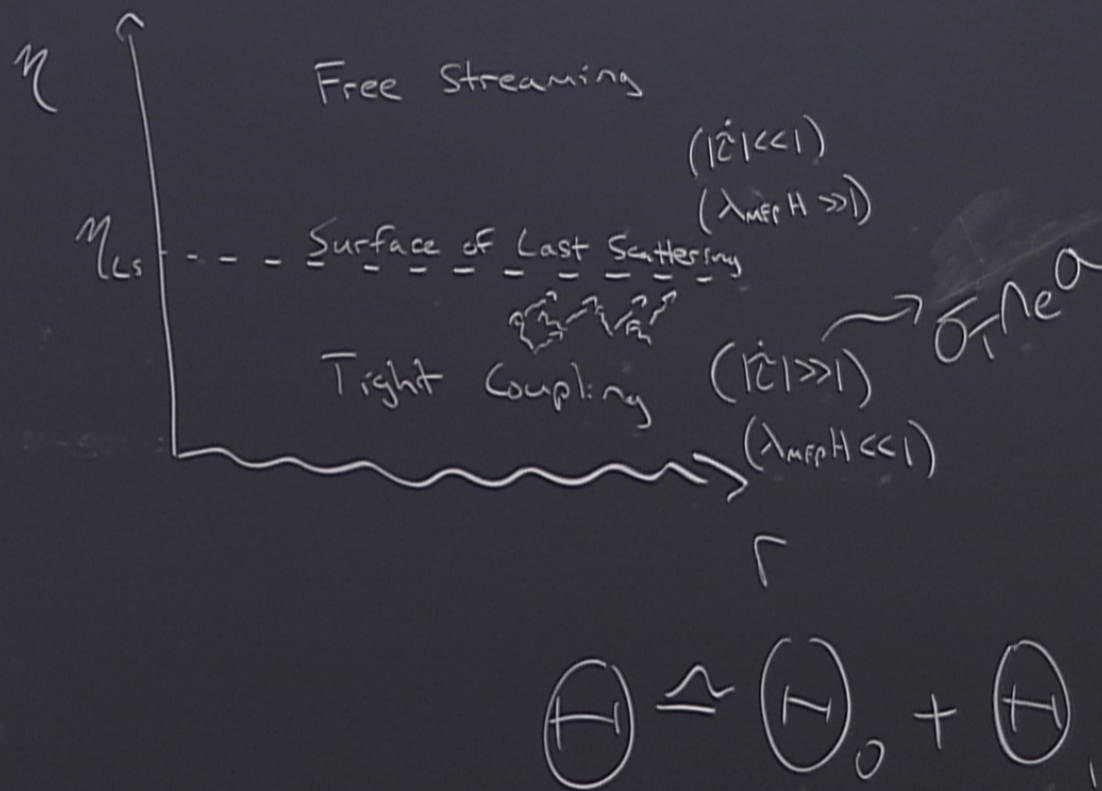
$$c_s = \frac{1}{\sqrt{3(1+R)}}$$

$$\Gamma_S = \int_0^\eta c_s(\eta') d\eta'$$

$$[\Theta_0(\eta) + \Phi(\eta)] = [\Theta_0(0) + \Phi(0)] \cos(k \Gamma_S)$$

$$+ \frac{k}{\sqrt{3}} \int_0^\eta d\eta' [\Phi(\eta') - \Psi(\eta') - \Gamma_S(\eta) - \Gamma_S(\eta')]$$

Recap



Tight coupling

$$\left[\frac{\partial^2}{\partial \eta^2} + \right]$$

$$\left[(-1)_0(\eta) + \dots \right]$$

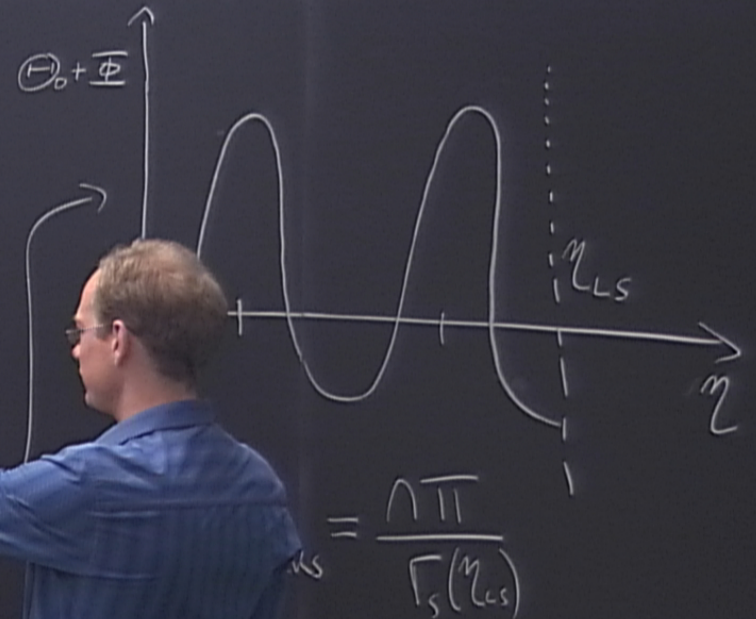
Tight coupling

$$\left[\frac{d^2}{dz^2} + \frac{\tilde{R}}{1+R} \frac{d}{dz} + k^2 c_s^2 \right] [\Theta_0 + \Phi] = \frac{k^2}{3} \left[\frac{1}{1+R} \Phi - \Psi \right]$$

$$c_s = \frac{1}{\sqrt{3(1+R)}}, \quad R = \frac{3\rho_b}{4\rho_s}, \quad \Gamma_s = \int c_s(z) dz$$

$$[\Theta_0(z) + \Phi(z)] = [\Theta_0(0) + \Phi(0)] \cos(k\Gamma_s) \rightarrow \frac{3\rho_b a}{4\rho_s v}$$

$$+ \frac{k}{\sqrt{3}} \int_0^z [\Phi(z') - \Psi(z')] \sin[k(\Gamma_s(z) - \Gamma_s(z'))]$$



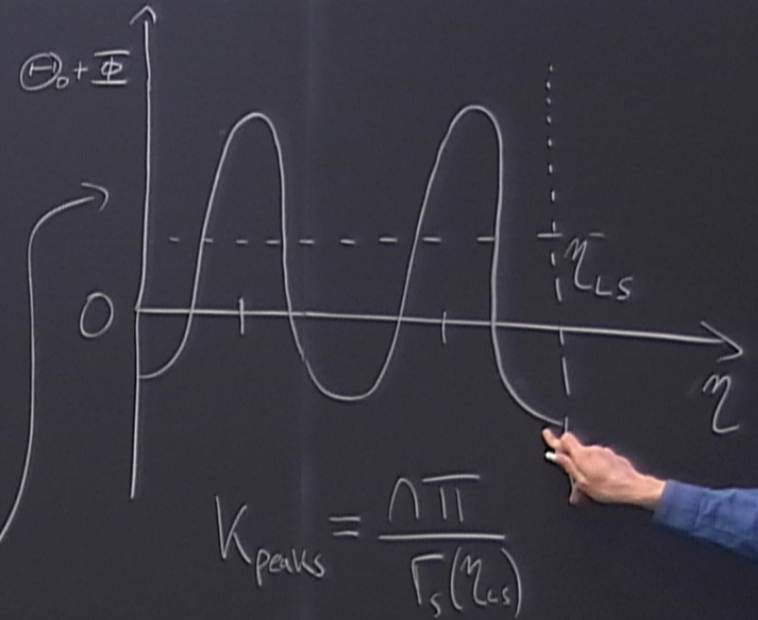
Tight coupling

$$\left[\frac{d^2}{d\eta^2} + \frac{\dot{R}}{1+R} \frac{d}{d\eta} + k^2 c_s^2 \right] [\Theta_0 + \Phi] = \frac{k^2}{3} \left[\frac{1}{1+R} \Phi - \Psi \right]$$

$$c_s = \frac{1}{\sqrt{3(1+R)}}, \quad R = \frac{3\rho_b}{4\rho_r}, \quad \Gamma_S = \int_0^\eta c_s(\eta') d\eta'$$

$$[\Theta_0(\eta) + \Phi(\eta)] = [\Theta_0(0) + \Phi(0)] \cos(k\Gamma_S) \quad \rightarrow \quad \frac{3\rho_b a}{4\rho_r}$$

$$+ \frac{k}{\sqrt{3}} \int_0^\eta d\eta' [\Phi(\eta') - \Psi(\eta')] \sin[k(\Gamma_S(\eta) - \Gamma_S(\eta'))]$$



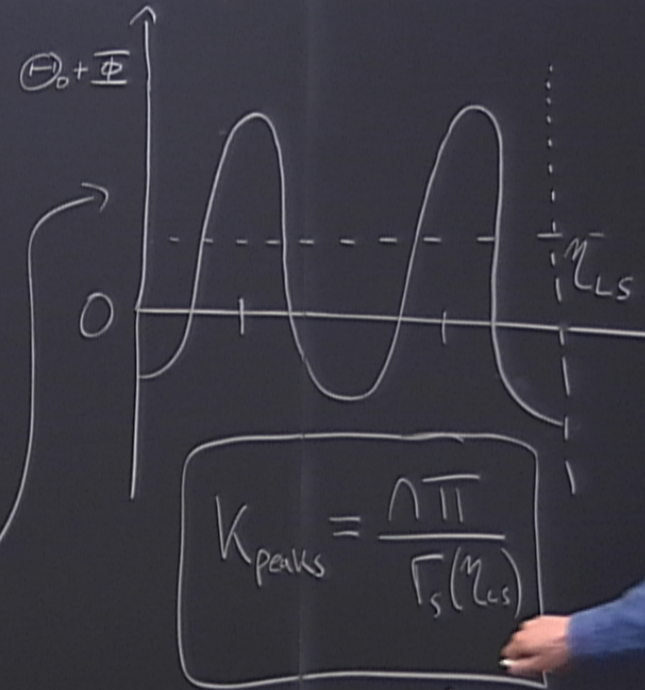
Tight coupling

$$\left[\frac{d^2}{dy^2} + \frac{\dot{R}}{1+R} \frac{d}{dy} + k^2 c_s^2 \right] [\Theta_0 + \Phi] = \frac{k^2}{3} \left[\frac{1}{1+R} \Phi - \Psi \right]$$

$$c_s = \frac{1}{\sqrt{3(1+R)}}, \quad R = \frac{3\rho_b}{4\rho_s}, \quad \Gamma_s = \int_0^{\eta} c_s(y') dy'$$

$$[\Theta_0(\eta) + \Phi(\eta)] = [\Theta_0(0) + \Phi(0)] \cos(k\Gamma_s) \quad \rightarrow \quad \frac{3\rho_b a}{4\rho_s v}$$

$$+ \frac{k}{\sqrt{3}} \int_0^{\eta} dy' [\Phi(y') - \Psi(y')] \sin[k(\Gamma_s(\eta) - \Gamma_s(y'))]$$

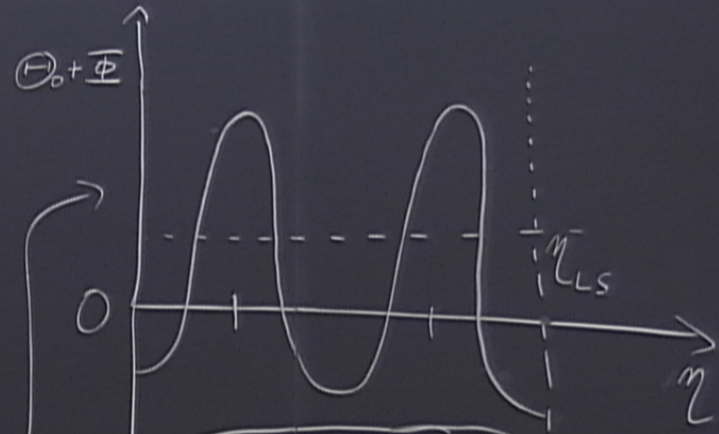


$$\frac{d}{dz} + k^2 C_s^2 \left[\Theta_0 + \Phi \right] = \frac{k^2}{3} \left[\frac{1}{1+R} \Phi - \Psi \right]$$

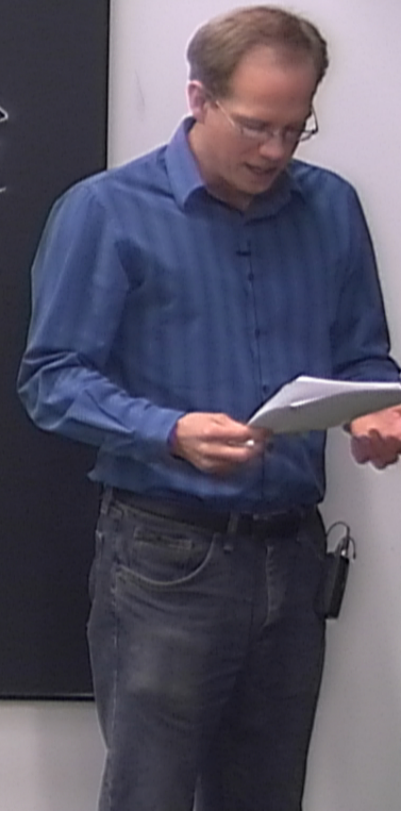
$$C_s = \frac{1}{\sqrt{3(1+R)}}, \quad R = \frac{3\rho_b}{4\rho_s}, \quad \Gamma_s = \int_0^{\eta} C_s(z') dz'$$

$$= \left[\Theta_0(0) + \Phi(0) \right] \cos(k\Gamma_s) \rightarrow \frac{3}{4} \frac{\rho_b}{\rho_s} a$$

$$+ \frac{k}{\sqrt{3}} \int_0^{\eta} dz' \left[\Phi(z') - \Psi(z') \right] \sin \left[k(\Gamma_s(\eta) - \Gamma_s(z')) \right]$$



$$K_{\text{peaks}} = \frac{\Delta \pi}{\Gamma_s(\eta_{LS})}$$



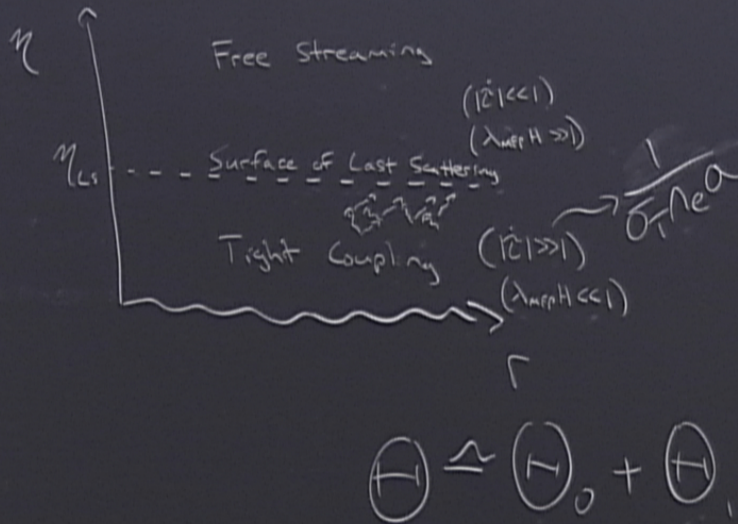
$$\textcircled{1-1} \left(\mu_{row}, \vec{X}_{here}, \hat{P} \right) = \int \frac{\partial \mu}{\partial \mu} \Theta(\mu)$$

$$\textcircled{1-1} () = \int \frac{\partial \mu}{\partial \mu} P_c \Theta(\mu)$$

$$\textcircled{1-1} (\mu_0, K) = -\frac{1}{3} \Phi(\mu_0, K) j_c(\mu(\mu_0 - \mu_c))$$

\uparrow
 $[\Theta_0(\mu_0) + \Phi(\mu_0)]$

Recap



$$\Theta \approx \Theta_0 + \Theta_1$$

Tight coupling

$$\left[\frac{d^2}{d\eta^2} + \frac{R}{1+R} \frac{d}{d\eta} + k^2 c_s^2 \right] [\Theta_0 + \Phi] =$$

$$c_s = \frac{1}{\sqrt{3(1+R)}}, \quad R = \frac{3}{4} \frac{\rho_b}{\rho_r}$$

$$[\Theta_0(\eta) + \Phi(\eta)] = [\Theta_0(0) + \Phi(0)] \cos(k r_s)$$

$$+ \frac{k}{\sqrt{3}} \int_0^\eta [\Phi(\eta') - \Psi(\eta')] S_{\text{eff}} d\eta'$$

$$\textcircled{1} \left(\eta_{\text{low}}, \vec{X}_{\text{high}}, \hat{P} \right) = \left(\frac{\partial^2 \mathcal{L}}{\partial \eta^2} \right) \mathcal{L}(\eta)$$

$$\textcircled{1} \left(\eta \right) = \left(\frac{\partial^2 \mathcal{L}}{\partial \eta^2} \right) \mathcal{L}(\eta)$$

$$\textcircled{1} \left(\eta_0, K \right) = -\frac{1}{3} \Phi \left(\eta_{\text{LS}}, K \right) \left(\eta_{\text{LS}} - \eta_{\text{LS}} \right)$$

\uparrow
 $\left[\omega_0(\eta_{\text{LS}}) + \Phi(\eta_{\text{LS}}) \right]$

$$\left(= \frac{1}{\alpha} \frac{\partial \alpha}{\partial \eta} \right)$$

$$\frac{\partial \Phi}{\partial (\eta \alpha)} + \frac{K}{\eta} \Phi$$

Comoving horizon = $\frac{1}{H}$

when $\gg 1$
gradient terms important

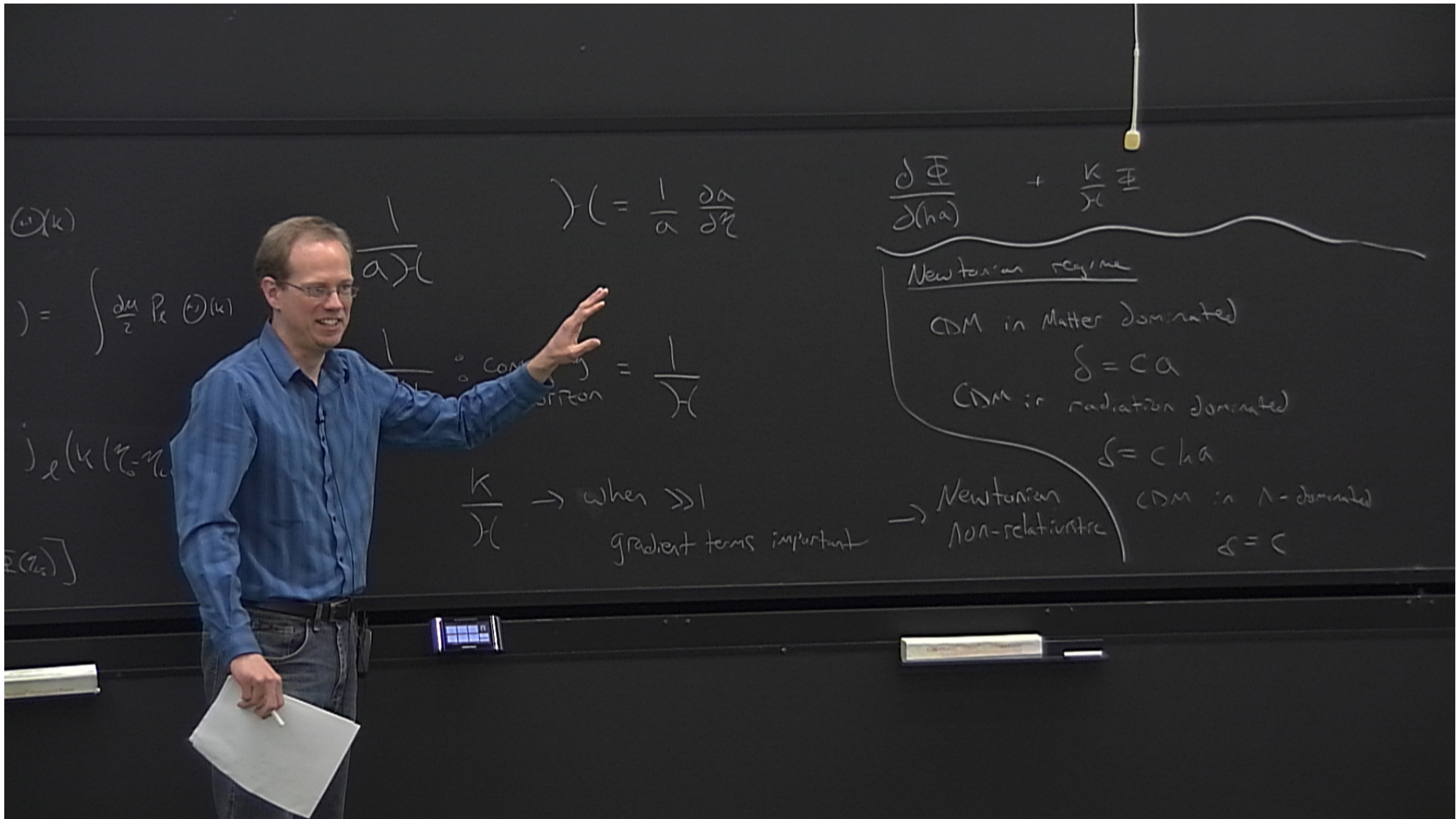
$$\textcircled{-1} \left(\mu_{low}, \vec{X}_{here}, \hat{p} \right) = \left(\frac{\partial^3 \mu}{(\partial \mu)^3} \textcircled{-1}(k) \right)$$

$$\textcircled{-1}_\ell(\) = \int \frac{d\mu}{2} P_\ell \textcircled{-1}(k)$$

$$\textcircled{-1}_\ell(\mu_0, k) = -\frac{1}{3} \overline{\Phi}(\mu_{LS}, k) j_\ell(k(\mu_0 - \mu_{LS}))$$

$$\uparrow$$

$$\left[\textcircled{-1}_0(\mu_{LS}) + \overline{\Phi}(\mu_{LS}) \right]$$



$$\delta(\mathbf{k}) = \int \frac{d\mu}{2} P_k(\mu) \delta(\mu)$$

$$\delta(\mathbf{k}) = \frac{1}{a} \frac{\partial \delta a}{\partial \eta}$$

$$\delta(\mathbf{k}) = \frac{1}{a} \frac{\partial \delta a}{\partial \eta}$$

$$\delta(\mathbf{k}) = \frac{1}{a} \frac{\partial \delta a}{\partial \eta}$$

$$\frac{\partial \Phi}{\partial (ha)} + \frac{k}{\mathcal{H}} \Phi$$

Newtonian regime

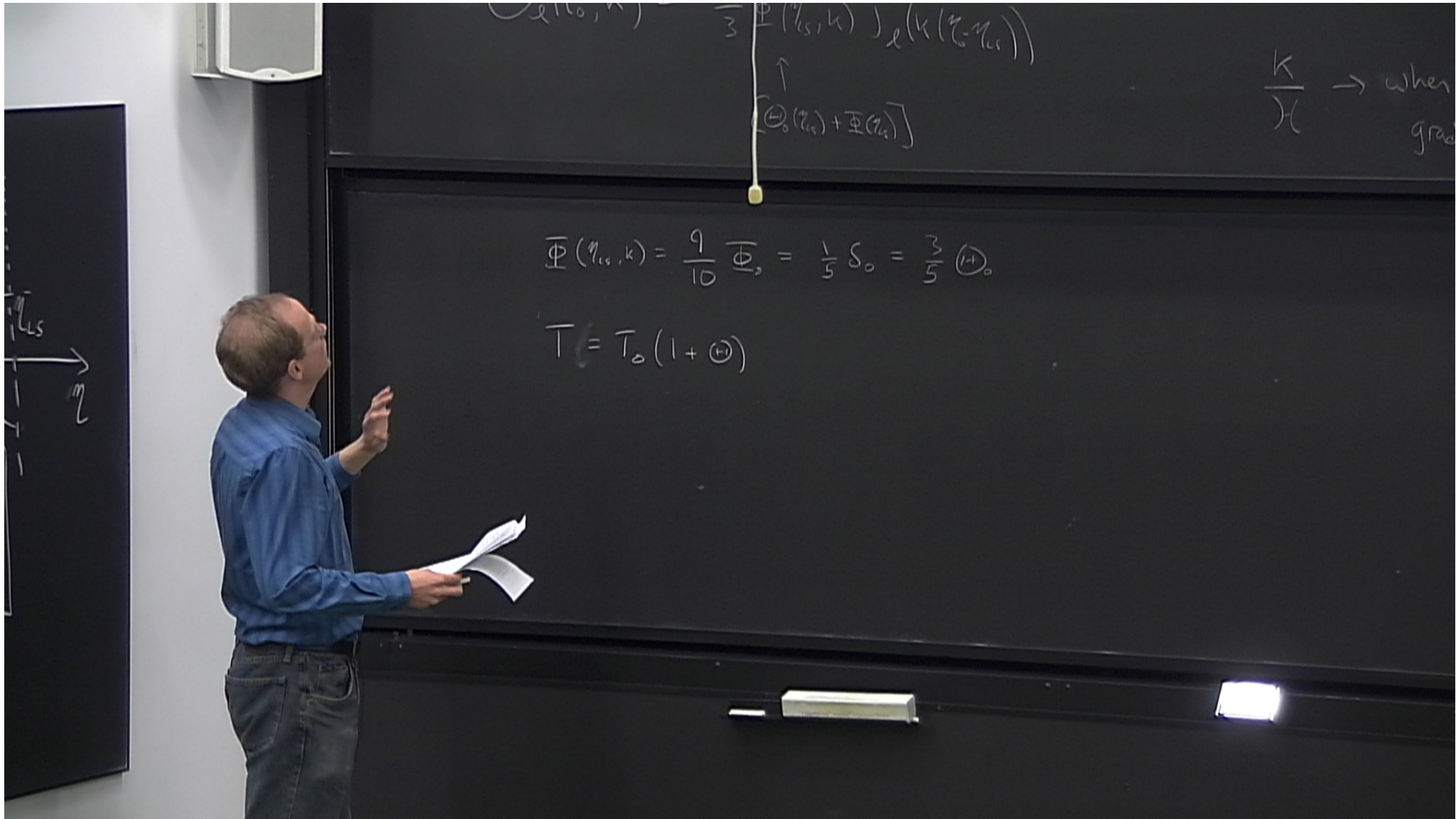
CDM in Matter dominated $\delta = c a$

CDM in radiation dominated $\delta = c h a$

CDM in Λ -dominated $\delta = c$

→ Newtonian non-relativistic

$\frac{k}{\mathcal{H}} \rightarrow$ when $\gg 1$ gradient terms important



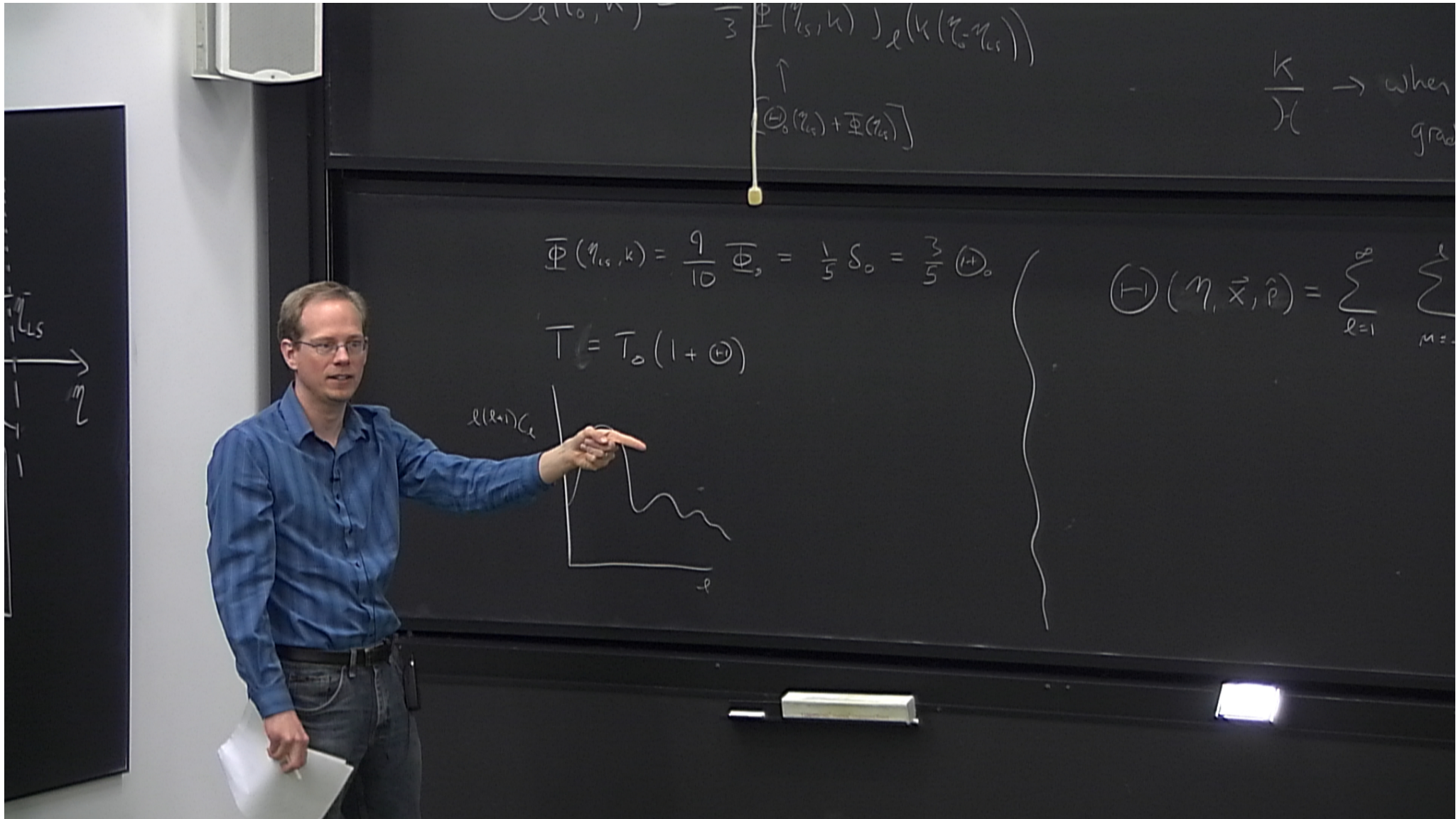
$$\frac{1}{3} \Phi(\eta_{cs}, k) = \frac{1}{3} \left[\Phi_0(\eta_{cs}) + \Phi(\eta_{cs}) \right]$$

$\frac{k}{H} \rightarrow$ when
grad

$$\bar{\Phi}(\eta_{cs}, k) = \frac{9}{10} \Phi_0 = \frac{1}{5} \delta_0 = \frac{3}{5} \Theta_0$$

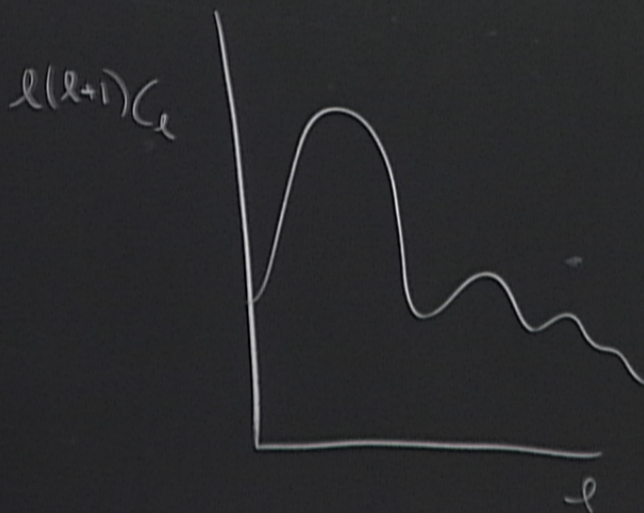
$$\bar{T} = T_0 (1 + \Theta_0)$$





$$\bar{\Phi}(\eta_{15}, k) = \frac{9}{10} \bar{\Phi}_0 = \frac{1}{5} \delta_0 = \frac{3}{5} \textcircled{+}_0$$

$$\bar{T} = T_0 (1 + \textcircled{+})$$



$$\frac{1}{5} S_0 = \frac{3}{5} \textcircled{1+}_0$$

$$\textcircled{1-}(\eta, \vec{x}, \hat{p}) = \sum_{l=1}^{\infty} \sum_{m=-l}^l a_{lm} Y_{lm}(\hat{p})$$

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①

$$\textcircled{1} (\eta, \vec{x}, \hat{p}) = \sum_{l=1}^{\infty} \sum_{m=-l}^l a_{lm} Y_{lm}(\hat{p})$$

$$a_{lm}(\vec{x}, \eta) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} \int d\Omega Y_{lm}^*(\hat{p}) \textcircled{1}(\vec{k}, \hat{p}, \eta)$$

$$\langle a_{lm} a_{l'm'}^* \rangle = \delta_{l'l} \delta_{m'm} \zeta_l$$

$$\textcircled{1} \langle \vec{k}, \hat{p}, \eta \rangle \propto \Phi_0(\vec{k})$$

$$\langle \Phi_0(\vec{k}) \Phi_0(\vec{k}') \rangle = (2\pi)^3 \delta(\vec{k} - \vec{k}') P_{\vec{k}}(k)$$

$$\langle \Phi_0(x)^2 \rangle = \int \frac{d^3k}{(2\pi)^3} P_{\vec{k}}(k) = \int d(\ln k) \frac{k^3 P_{\vec{k}}(k)}{2\pi^2}$$

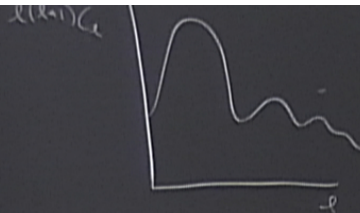
$$\langle a_{\vec{k}m} a_{\vec{k}'m'}^\dagger \rangle = \int \int$$

$$\langle \Theta(\vec{k}, \hat{p}, \eta) \Theta(\vec{k}', \hat{p}', \eta) \rangle$$

$$\propto \langle \Phi_0(\omega) \Phi_0(\omega') \rangle$$

$$\vec{k}' \cdot \underline{P}(\underline{k})$$

$$(\underline{k}) \frac{V^3 P_{\vec{k}}(\omega)}{2\pi^2}$$



$$\langle a_{lm} a_{lm}^* \rangle = \delta_{ll} \delta_{mm} C_l$$

$$\textcircled{1-1} \langle \psi(\vec{r}, \hat{p}, \eta) \rangle \propto \Phi_0(\vec{r})$$

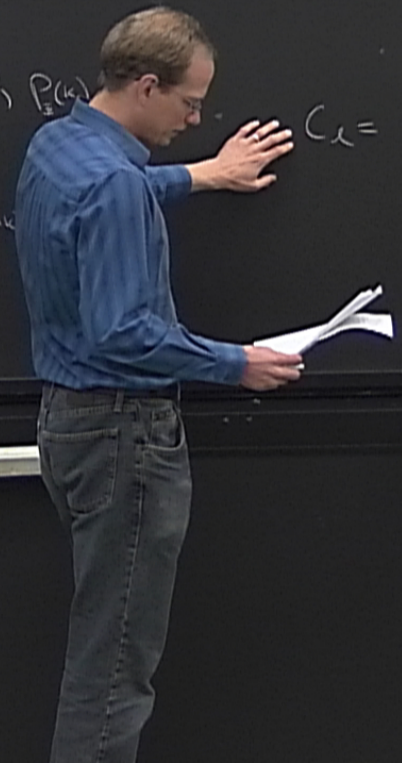
$$\langle a_{lm} a_{lm}^* \rangle = \int \int$$

$$\langle \Theta(\vec{r}, \hat{p}, \eta) \Theta(\vec{r}', \hat{p}', \eta') \rangle \propto \langle \Phi_0(\vec{r}) \Phi_0(\vec{r}') \rangle$$

$$\langle \Phi_0(\vec{r}) \Phi_0(\vec{r}') \rangle = (2\pi)^3 \delta(\vec{r} - \vec{r}') P_{\frac{1}{2}}(k)$$

$$C_l = \frac{2}{\pi} \int_0^\infty dk k^2 P_{\frac{1}{2}}(k) \left| \frac{\psi_l}{\Phi_0} \right|^2$$

$$\langle \Phi_0(x^2) \rangle = \int \frac{d^3k}{(2\pi)^3} P_{\frac{1}{2}}(k) = \int d(\ln k)$$



$$\langle a_{\ell m} a_{\ell' m'}^+ \rangle = \left(\begin{array}{c} \\ \end{array} \right)$$

$$\langle \Theta(\vec{k}, \hat{p}, \eta) \Theta(\vec{k}', \hat{p}', \eta) \rangle$$

$$\rightarrow \propto \langle \bar{\Phi}_0(k) \bar{\Phi}_0(k') \rangle$$

$$C_\ell = \frac{2}{\pi} \int_0^\infty dk k^2 P_{\mathbb{F}}(k) \left| \frac{\Theta_\ell}{\bar{\Phi}_0} \right|^2$$

Sachs-Wolfe limit

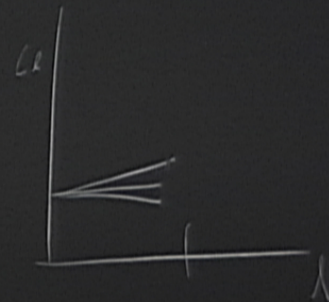
largest scales (Scales that are superhorizon at LS)

$$\Theta_{\ell} = -\frac{3}{10} \Phi_{\ell}^{(0)} j_{\ell}[k(\eta - \eta_{LS})]$$

$$C_{\ell} = \frac{24}{1100} \int_0^{\infty} dk k^2 j_{\ell}[k(\eta - \eta_{LS})]^2 \frac{P(k)}{k^3}$$

$$P_{\frac{1}{4}}(k) = A k^{n_s - 1}$$

$$\Rightarrow C_{\ell} = () A \frac{\Gamma(\ell + \frac{n_s}{2} - 1) \Gamma(4 - n_s)}{\Gamma(\ell + \frac{7}{2} - \frac{n_s}{2}) \Gamma(\frac{5}{2} - \frac{n_s}{2})}$$



Sachs-Wolfe limit

largest scales (Scales that are
superhorizon at
LS)

$$\Theta_l = -\frac{3}{10} \Phi_0^{(k)} j_l [k(\eta - \eta_{LS})]$$

$$C_l = \frac{9}{50\pi} \int_0^\infty dk k^2 j_l [k(\eta - \eta_{LS})]^2 \frac{P(k)}{k^3}$$

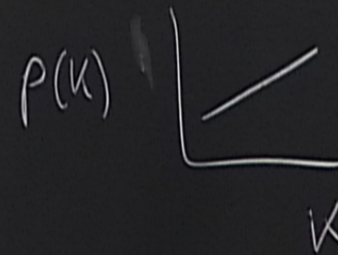
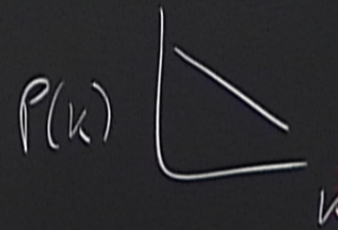
$\alpha_s = 1$ Scale invariant

$$\frac{4 - \alpha_s}{\frac{5}{2} - \frac{\alpha_s}{2}}$$

$$P(k) = A$$

$\alpha_s < 1$ red tilt

$\alpha_s > 1$ blue tilt

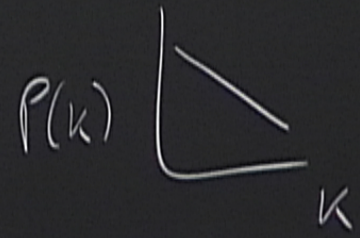


$n_s = 1$ Scale invariant (Harrison-Zeldovich)

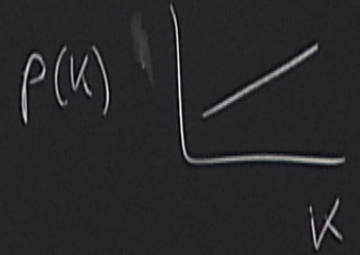
$$\frac{4 - n_s}{\frac{5}{2} - \frac{n_s}{2}}$$

$$P(k) = A$$

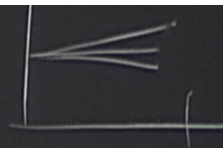
$n_s < 1$ \circ red tilt



$n_s > 1$ \circ Blue tilt



$$C_l = \frac{9}{50\pi} \int_0^{\infty} dk k^2 j_l[k(\eta - \eta_{LS})] \frac{P(k)}{k^3}$$



Subhorizon Scales

$$\bar{\Phi}(\eta_{LS}, \vec{k}) \neq \frac{9}{10} \bar{\Phi}_0(\eta_0, \vec{k})$$

On smaller scales, $\bar{\Phi}(\eta_{LS}, \vec{k})$

Peaks for $k_1 = \frac{\pi \Lambda}{\Gamma_S(\eta_{LS})}$

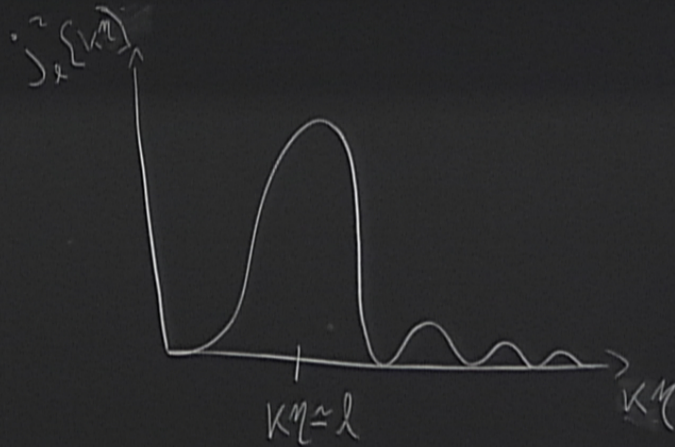
$$C_l = \frac{9}{50\pi} \int_0^{\infty} dk k^2 j_l[k(\eta - \eta_{LS})] \frac{P(k)}{k^3}$$

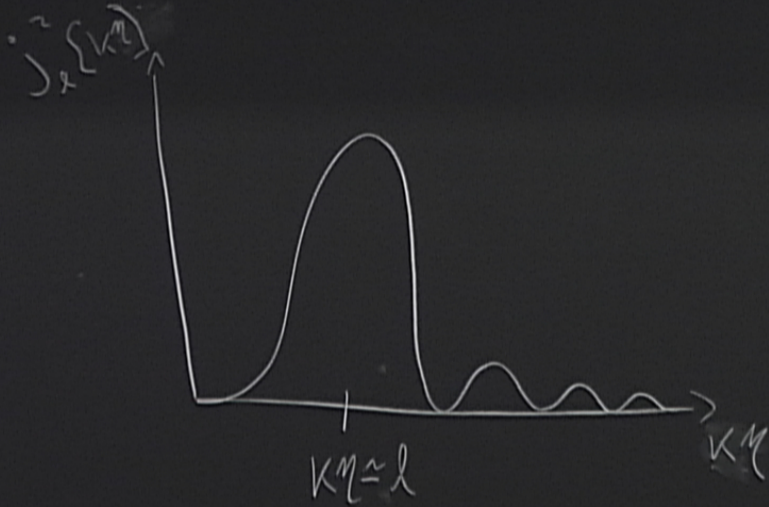
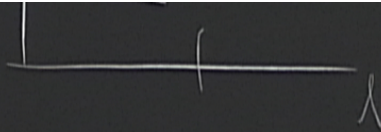
Subhorizon Scales

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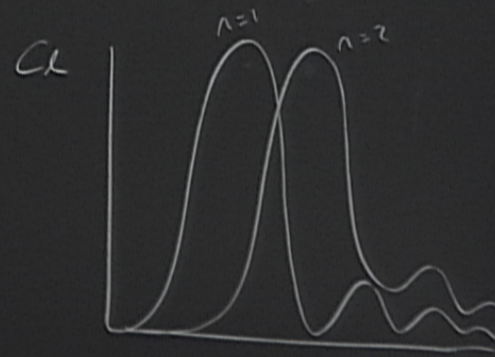
Peaks for $k_1 = \frac{\pi n}{\Gamma_S(\eta_{LS})}$





Extra power on scales

$$l \approx \frac{n\pi}{r_s} (M_0 - M_{LS})$$



Null joins in tensorial notation

Using R -enabled composition locality and the null joins refinement axiom we can show

$$(A \cdot B^{a_1} C_{a_1})_0 = A^{b_2 c_3} B_{b_2}^{a_1} C_{a_1 c_3} = B^{a_1 b_2^R} A_{b_2^R}^{c_3} C_{a_1 c_3} = (B \cdot A^{c_3} C_{c_3})_{\alpha} = ((A \cdot C)_{\beta} \cdot B)_{\alpha^R}$$

Using the null joins refinement axiom we have,

$$(A \cdot B^{a_1} C_{a_1})_0 = A^{0_2 0_3} B_{0_2}^{a_1} C_{a_1 0_3}$$

We can notate this simply by omitting the null joins. Thus, we can write

$$(A \cdot B^{a_1} C_{a_1})_0 = AB^{a_1} C_{a_1}$$

Null joins in tensorial notation

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Using the null joins refinement axiom we have,

$$(A \cdot B^{a_1} C_{a_1})_0 = A^{0^0 a_1} B_{0^0}^{a_1} C_{a_1 0^0}$$

We can notate this simply by omitting the null joins. Thus, we can write

$$(A \cdot B^{a_1} C_{a_1})_0 = AB^{a_1} C_{a_1}$$

Position of peaks
related to Γ_s and $(M_0 - M_{LS})$

\downarrow
 $\frac{P_b}{P_r}$

\downarrow
 P_m, P_n

$$\textcircled{1-1} \ell(k, \eta_0) = \int_0^{\eta_0} d\eta \, g(\eta) \left[\textcircled{1+}_0(k, \eta) + \Psi(k, \eta) \right] J_\ell[k(\eta_0 - \eta)] \quad \text{Sachs Wolfe}$$

$$g(\eta) = -\dot{\tau} e^{-\dot{\tau}}$$

$$- \int d\eta \, g(\eta) \frac{v_L(k, \eta)}{k} \frac{\partial}{\partial \eta} \left[J_\ell[k(\eta_0 - \eta)] \right] \quad \text{dop}$$

visibility $\xi \approx \eta$

$$\int_0^{\eta} d\eta \, g(\eta) = 1$$

Previous limit - $g(\eta) = \delta(\eta - \eta_{LS})$

$$= \int_0^{\eta_0} d\eta g(\eta) \left[\bar{\Phi}_0(k, \eta) + \Psi(k, \eta) \right] j_\ell [k(\eta_0 - \eta)] \quad \text{Sachs-Wolfe}$$

$$- \int d\eta g(\eta) \frac{iv_L(k, \eta)}{k} \frac{\partial}{\partial \eta} \left[j_\ell [k(\eta_0 - \eta)] \right] \quad \text{Doppler } (v_b \propto \bar{\Phi}_1)$$

$$+ \int d\eta e^{-\hat{t}} \left[\dot{\Psi} - \dot{\bar{\Phi}} \right] j_\ell [k(\eta - \eta_0)] \quad \text{Integrated-Sachs-Wolfe}$$

$$g(\eta) = \int (\eta - \eta_{LS})$$

My ambitions for the composition principle

- Ambition 1 All physics is consistent with the composition principle.
- Ambition 2 All physics ought be formulated in such a way that it is clear the composition principle holds.
- Ambition 3 The composition principle will be a powerful tool in formulating new physical theories such as quantum gravity.

Will actually give just two examples.

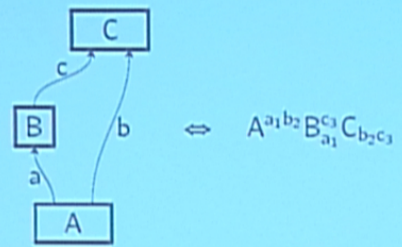
My ambitions for the composition principle

- Ambition 1 All physics is consistent with the composition principle.
- Ambition 2 All physics ought be formulated in such a way that it is clear the composition principle holds.
- Ambition 3 The composition principle will be a powerful tool in formulating new physical theories such as quantum gravity.

Will actually give just two examples.

Example 1 - Circuits

In the case where we have *tomographic locality* the composition principle holds for calculating probabilities. To calculate the probability of the circuit



we can write

$$\text{Prob}(A^{a_1 b_2} B_{a_1}^{c_3} C_{b_2 c_3}) = A^{a_1 b_2} B_{a_1}^{c_3} C_{b_2 c_3}$$

where $A^{a_1 b_2}$, $B_{a_1}^{c_3}$, and $C_{b_2 c_3}$ are tensors.

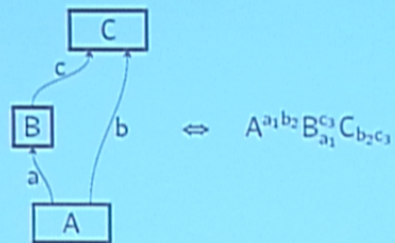
In quantum theory can replace these tensors by operator tensors

$$\text{Prob}(A^{a_1 b_2} B_{a_1}^{c_3} C_{b_2 c_3}) = \bar{A}^{a_1 b_2} \bar{B}_{a_1}^{c_3} \bar{C}_{b_2 c_3}$$

In the notation on the right, the repeated label indicates partial trace over the appropriate part of the operator space.

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In the notation on the right, the repeated label indicates partial trace over the appropriate part of the operator

Example 2 - Labeled tiles

We consider square tiles of unit length that are labeled $n = 1, 2, \dots$. Let the n th such tile be $T[n]$. A complete set of join types is $\{x, y, 0\}$. The join type x corresponds to placing one immediately to the right of the other as follows

$$T^{x_1}[m]T^{x_2}[n] \Leftrightarrow \boxed{m} \boxed{n}$$

The join type y corresponds to placing one tile immediately above the other

$$T^{y_1}[m]T^{y_2}[n] \Leftrightarrow \begin{array}{c} \boxed{n} \\ \boxed{m} \end{array}$$

The third join type is the null join where we simply consider the two tiles as part of the same picture without specifying their relationship any further.

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$$T^{y_1}[m]T^{y_2}[n] \Leftrightarrow \begin{array}{|c|c|} \hline & 5 \\ \hline 1 & 4 \\ \hline 2 & 3 \\ \hline \end{array} \Leftrightarrow \begin{array}{|c|c|} \hline 6 & 7 \\ \hline & n \\ \hline & m \\ \hline \end{array}$$

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$$T[m]T[n] \Leftrightarrow \begin{array}{|c|} \hline m \\ \hline \end{array} \quad \begin{array}{|c|} \hline n \\ \hline \end{array}$$

Example 2 - Labeled tiles

We are interested in calculating geometric properties having to do with the relative displacement of tiles (where defined). Hence, we define the following generalized state for objects that have no disjoint parts

$$A = \{((m, n), (\Delta x, \Delta y)) : \text{for all tile labels } m, n \text{ in } A\}$$

where Δx is the horizontal displacement from tile m to tile n and Δy is the vertical displacement between these tiles (so $(\Delta x, \Delta y)$ is the displacement between these two tiles). We can obtain the generalized state for objects that do have disjoint parts by using the null join. Corresponding to the object, AB we have

$$AB = A \cup B$$

(recall that we suppress the 0 but we could write $A^{0_1}B_{0_1}$ for this object). For the composite object D on previous slide we would get all the displacements in each of the two disjoint parts. However, no displacements would be specified between the two disjoint parts since these are not defined.

Example 2 - Labeled tiles

We can join two composite objects by an x join by specifying the tiles in each object where this join is to occur. So,

$$C = A^{x_1}[u]B_{x_1}[v]$$

means that we join A at tile u to B at tile v by an x type join. The associated generalized state is given by

$$C = A^{x_1}[u]B_{x_1}[v] = A \cup B \cup \mathcal{X}(A[u], B[v])$$

where

$$\begin{aligned} \mathcal{X}(A[u], B[v]) = \\ \{((m, n), \Delta_{(m,u)} + (1, 0) + \Delta_{(v,n)}) : \forall (m, u) \in A \text{ and } (v, n) \in B\} \end{aligned}$$

and $\Delta_{(m,u)}$ is the displacement between tile m and tile u . This is the set of displacements between A and B that are established by this new join.

Example 2 - Labeled tiles

If we just have a single tile, $T[n]$, then the corresponding generalized state is

$$T[n] = \{((n, n), (0, 0))\}$$

It is now easy to see that, in building up the generalized state corresponding to any composite object, we get the same answer no matter what order we do the calculation.

Further, it is clear that geometric properties are all given by appropriate functions of the resulting generalized state.

The composition principle clearly holds since we obtain generalized states by means of a calculation having the same structure as that of the description of the composite object.

Conclusions

- We have developed the tensorial notation for *describing* composite objects and explored the assumptions going into this notation using the more primitive bipartite notation.
- The composition principle suggests that there is a correspondence between objects and certain mathematical objects (generalized states) that allow us to make *predictions* of properties of the objects by means of equations that have the same structure as the description of the composition of those objects.
- We have given a few simple examples where the composition principle can be seen to hold.
- We make the stronger claim that any reasonable physical theory can be formulated in a way that this principle holds.
- Further, we make the moral claim that all physical theories *ought* to be formulated in a way that makes it clear that the composition principle holds.
- This is a potentially very useful principle in constructing new physical theories, such as a theory of quantum gravity.
- This compositional approach also may provide a *deeper* approach to reconstructing quantum theory from reasonable axioms.

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