

Title: Large Scale Bayesian Inference in Cosmology

Date: Apr 11, 2013 11:00 AM

URL: <http://pirsa.org/13040119>

Abstract: <span>Already the last decade has witnessed unprecedented progress in the collection of cosmological data. Presently proposed and designed future cosmological probes and surveys permit us to anticipate the upcoming avalanche of cosmological information during the next decades.<br>The increase of valuable observations needs to be accompanied with the development of efficient and accurate information processing technology in order to analyse and interpret this data. In particular, cosmography projects, aiming at studying the origin and inhomogeneous evolution of<br>the Universe, involve high dimensional inference methods. For example, 3d cosmological density and velocity field inference requires to explore on the order of  $10^7$  or more parameters. Consequently, such projects critically rely on state-of-the-art information processing techniques<br>and, nevertheless, are often on the verge of numerical feasibility with present day computational resources. For this reason, in this talk I will address the problem of high dimensional Bayesian inference from cosmological data sets, subject to a variety of statistical and systematic uncertainties. In particular, I will focus on the discussion of selected Markov Chain Monte Carlo techniques, permitting to efficiently solve inference problems with on the order of  $10^7$  parameters. Furthermore, these methods will be exemplified in various cosmological applications, ranging from 3d non-linear density and photometric redshift inference to 4d physical state inference. These techniques permit us to exploit cosmologically relevant information from<br>observations to unprecedented detail and hence will significantly contribute to the era of precision cosmology.</span>

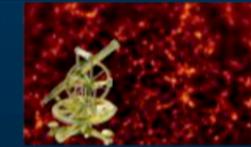


# Large Scale Bayesian Inference in Cosmology

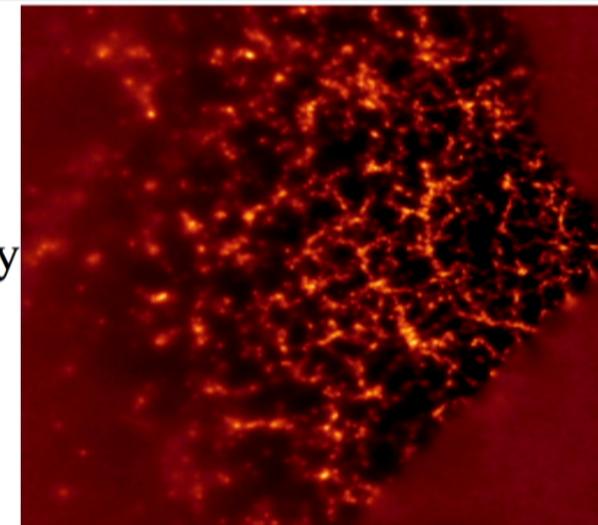
Jens Jasche,  
*Benjamin Wandelt, Emilio Romano Diaz*

*Waterloo, 11 April 2013*

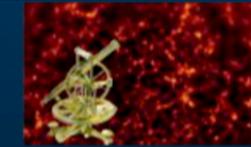
# Introduction



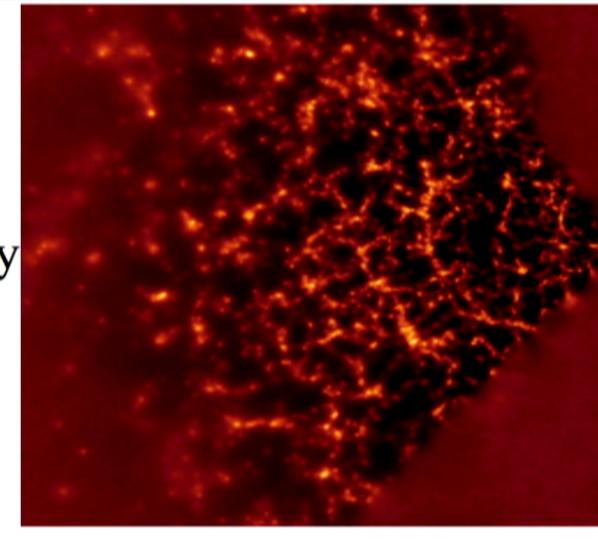
- Cosmography
  - 3D density and velocity fields
  - Power-spectra, bi-spectra
  - Dark Energy, Dark Matter, Gravity
  - Cosmological parameters



# Introduction

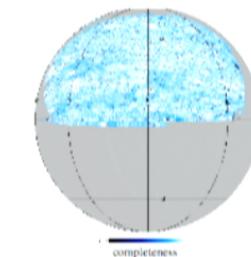


- Cosmography
  - 3D density and velocity fields
  - Power-spectra, bi-spectra
  - Dark Energy, Dark Matter, Gravity
  - Cosmological parameters
  
- Large Scale Bayesian inference
  - High dimensional ( $\sim 10^7$  parameters )
  - State-of-the-art technology
  - On the verge of numerical feasibility



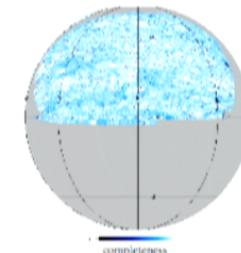
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- Why do we need Bayesian inference?
  - Inference of signals = ill-posed problem
    - Noise
    - Incomplete observations



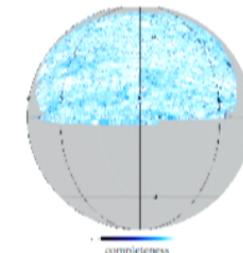
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    - Noise
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→ No unique recovery possible!!!

# Introduction

**“What are the possible signals compatible with observations?”**

- Object of interest: Signal posterior distribution

$$\mathcal{P}(s|d) = \mathcal{P}(s) \frac{\mathcal{P}(d|s)}{\mathcal{P}(d)}$$



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$$\mathcal{P}(s|d) = \mathcal{P}(s) \frac{\mathcal{P}(d|s)}{\mathcal{P}(d)}$$

- We can do science!
  - Model comparison
  - Parameter studies
  - Report statistical summaries
  - Non-linear, Non-Gaussian error propagation



# Markov Chain Monte Carlo



## □ Problems:

- High dimensional ( $\sim 10^7$  parameter)

# Markov Chain Monte Carlo



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## □ Numerical approximation



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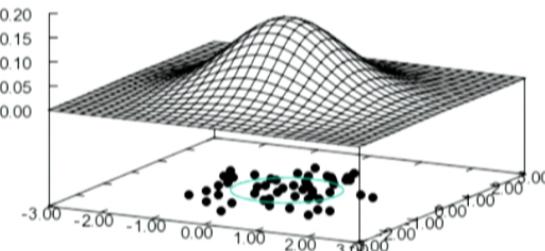
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## □ Numerical approximation

- Dim  $> 4 \rightarrow$  MCMC

$$\mathcal{P}(s|d) \rightarrow \mathcal{P}_N(s|d) = \frac{1}{N} \sum_{i=1}^N \delta^D(s - s_i)$$

- Metropolis-Hastings



# Hamiltonian sampling



- Parameter space exploration via Hamiltonian sampling



# Hamiltonian sampling



- Parameter space exploration via Hamiltonian sampling
  - interpret log-posterior as potential

$$\psi(x) = -\ln(\mathcal{P}(x))$$



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- introduce Gaussian auxiliary “momentum” variable

$$H = \sum_i \sum_j \frac{1}{2} p_i M_{ij}^{-1} p_j + \psi(x)$$



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- resultant joint posterior distribution of  $x$  and  $p$

$$e^{-H} = \mathcal{P}(\{x_i\}) e^{-\frac{1}{2} \sum_i \sum_j p_i M_{ij}^{-1} p_j}$$

➤ separable in  $x$  and  $p$

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- separable in  $x$  and  $p$
- marginalization over  $p$  yields again  $\mathcal{P}(x)$

# Hamiltonian sampling



- IDEA: Use Hamiltonian dynamics to explore  $e^{-H}$



# Hamiltonian sampling

- IDEA: Use Hamiltonian dynamics to explore  $e^{-H}$ 
  - solve Hamiltonian system to obtain new sample

$$\{x^i, p^i\} \rightarrow \boxed{\begin{aligned}\frac{dx_i}{dt} &= \frac{\partial H}{\partial p_i} \\ \frac{dp_i}{dt} &= \frac{\partial H}{\partial x_i} = -\frac{\partial \psi(x)}{\partial x_i}\end{aligned}} \rightarrow \{x^{i+1}, p^{i+1}\}$$

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- Hamiltonian dynamics conserve the Hamiltonian  $H$ 
  - Metropolis acceptance probability is unity

$$\mathcal{P}_A = \min[1, \exp(-(H(\{x'_i\}, \{p'_i\}) - H(\{x_i\}, \{p_i\}))]$$



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# Hamiltonian sampling

- Example: Wiener posterior = multivariate normal distribution

$$\Psi = \frac{1}{2} \sum_{ij} x_i S_{ij}^{-1} x_j + \frac{1}{2} \sum_{ij} (x_i - d_i) N_{ij}^{-1} (x_j - d_j)$$



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EOM:

coupled harmonic oscillator

$$\frac{dx_m}{dt} = \sum_j M_{mj}^{-1} p_j$$

$$\frac{dp_m}{dt} = - \sum_j A_{mj} x_j + B_m$$





# Hamiltonian sampling

- How to set the Mass matrix?
  - Large number of tunable parameter
  - Determines efficiency of sampler

$$\begin{aligned}\frac{d^2x_i}{dt^2} &= - \sum_l M_{il}^{-1} \sum_j A_{lj} x_j + \sum_l M_{il}^{-1} B_l \\ &= - \sum_l M_{il}^{-1} \sum_j A_{lj} x_j + D_m\end{aligned}$$



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- In practice: use diagonal approximation
- The quality of approximation determines sampler efficiency
- Non-Gaussian case: Taylor expand to find Mass matrix

# HMC in action

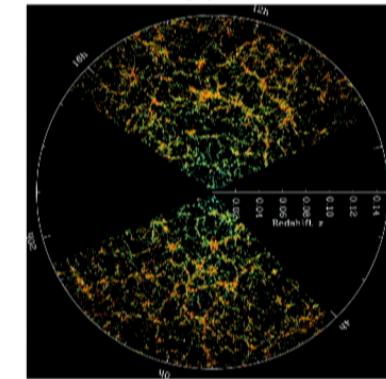
- Inference of non-linear density fields in cosmology
  - Non-linear density field
    - Log-normal prior

See e.g. Coles & Jones (1991), Kayo et al. (2001)

Jasche, Kitaura (2010)

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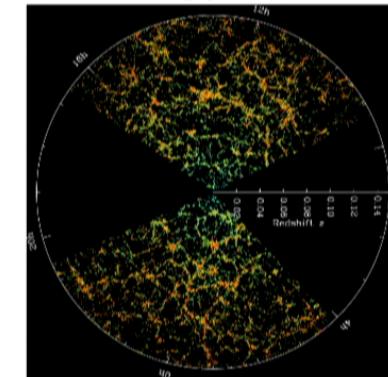
Credit: M. Blanton and the Sloan Digital Sky Survey



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Credit: M. Blanton and the  
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Problem: Non-Gaussian sampling in high dimensions



**HADES** (HAmiltonian Density Estimation and Sampling)

Jasche, Kitaura (2010)



## LSS inference with the SDSS

- Application of HADES to SDSS DR7
  - cubic, equidistant box with sidelength 750 Mpc
  - $\sim 3$  Mpc grid resolution
  - $\sim 10^7$  volume elements / parameters

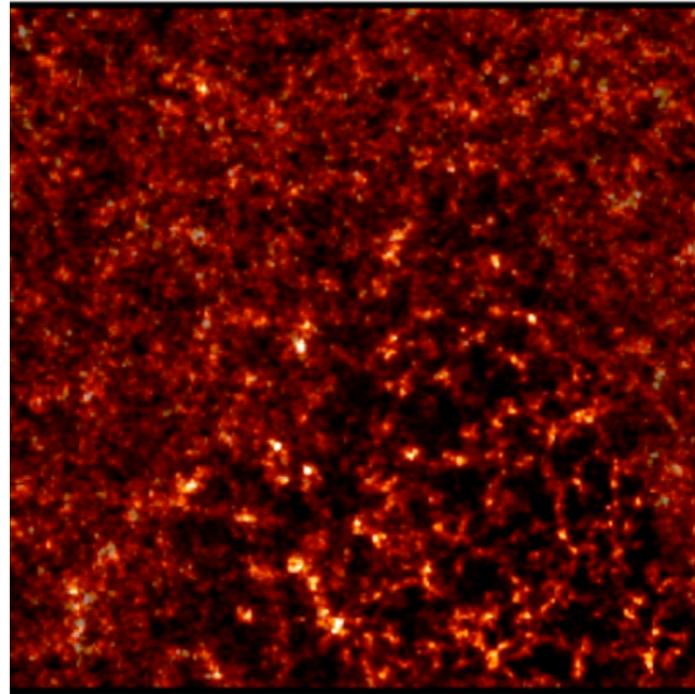
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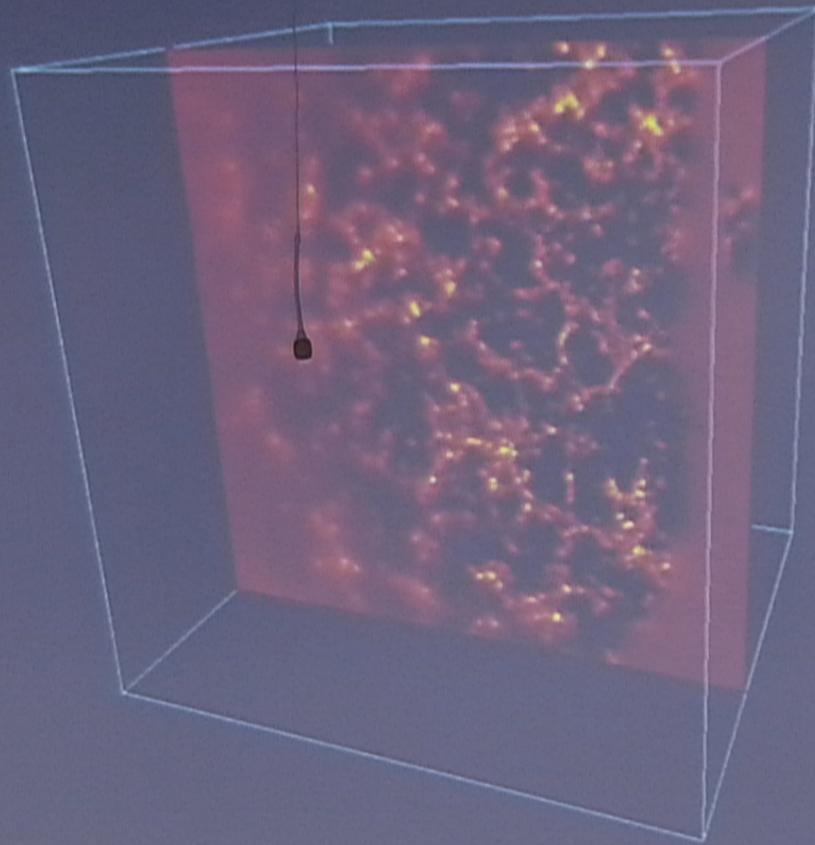
# LSS inference with the SDSS

- Application of HADES to SDSS DR7
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- Goal: provide a representation of the SDSS density posterior
  - to provide 3D cosmographic descriptions
  - to quantify uncertainties of the density distribution

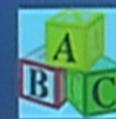
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# LSS inference with the SDSS





# Multiple Block Sampling

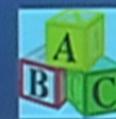


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# Multiple Block Sampling

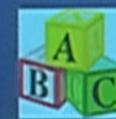


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- Multiple block sampling ( see e.g. Hastings (1997) )

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- Break down into subproblems

$$A^{i+1} \curvearrowleft \mathcal{P}(A|B^i)$$

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Serial processing only!

- simplifies design of conditional proposal distributions
- Average acceptance rate is higher
- Requires serial processing

# Multiple Block Sampling



- Can we “boost” block sampling?

$$\mathcal{P}(A, B) = \int dC \mathcal{P}(A, B, C)$$

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$$C^{i+1} \curvearrowleft \mathcal{P}(C|A^{i+1}, B^{i+1})$$

The slide has a blue header bar with the title "Multiple Block Sampling". Below the header, there is a list of bullet points and some mathematical equations. The equations show the conditional probability relationships between variables A, B, and C at different stages of sampling. The background of the slide is light blue.

- Can we “boost” block sampling?  
 $\mathcal{P}(A, B) = \int dC \mathcal{P}(A, B, C)$
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$$\begin{aligned} &= \mathcal{P}(A|C^i) \\ &= \mathcal{P}(B|C^i) \end{aligned}$$

process in parallel!

- Permits efficient sampling for numerical expensive posteriors

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- Photometric surveys
  - millions of galaxies ( $\sim 10^7 - 10^8$ )

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$$z_N^{i+1} \curvearrowleft \mathcal{P}(z_N|\delta^i, d)$$

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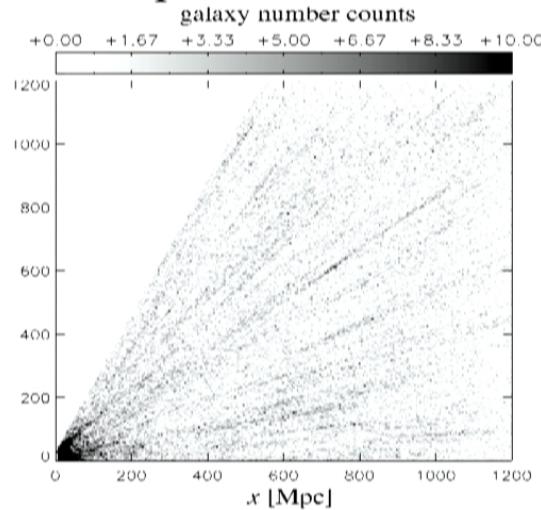
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HMC sampler!

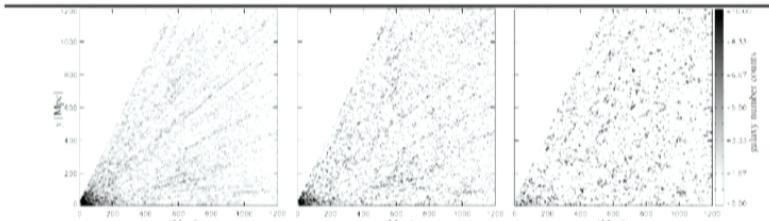
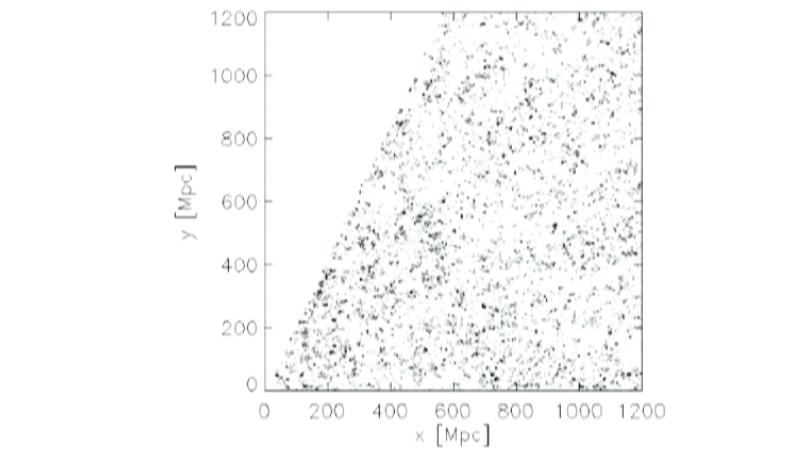
# Photometric redshift sampling

## Application to artificial photometric data



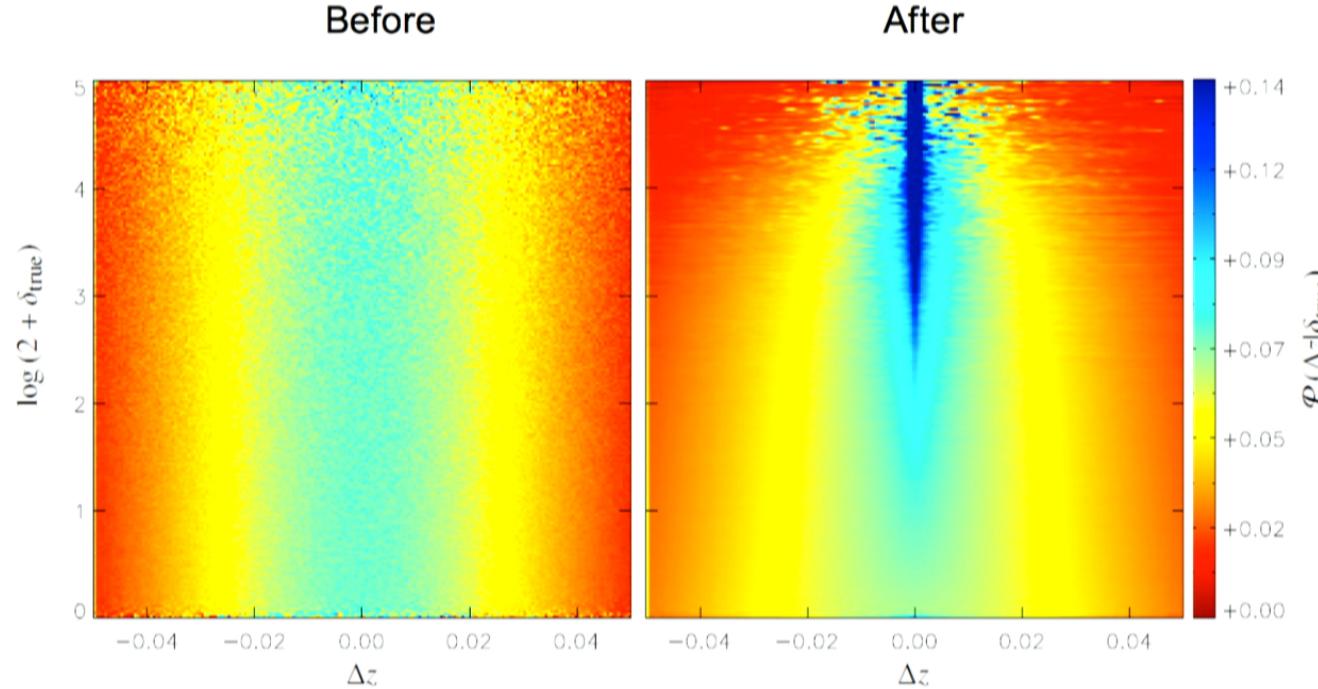
- ~ Noise, Systematics, Position uncertainty ( $\sim 100$  Mpc)
- $\sim 10^7$  density amplitudes /parameters
- $\sim 2 \times 10^7$  radial galaxy positions / parameters

# Photometric redshift sampling



Jasche, Wandelt (2012)

# Deviation from the truth

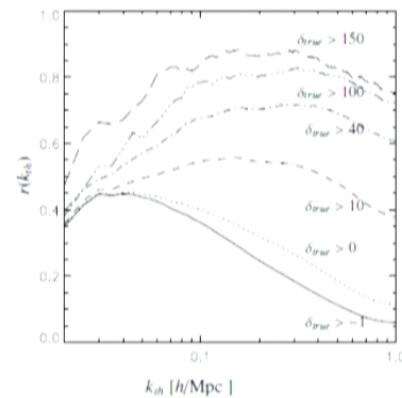


Jasche, Wandelt (2012)

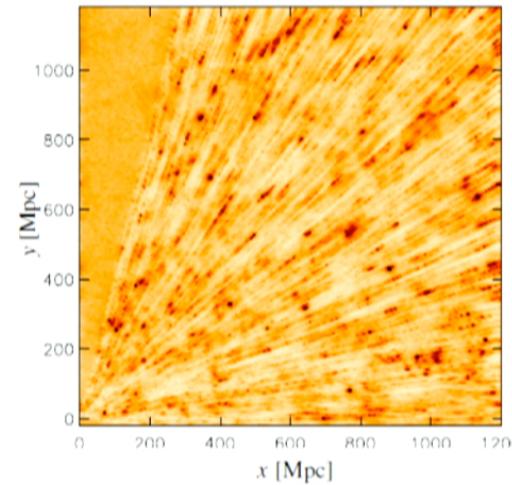
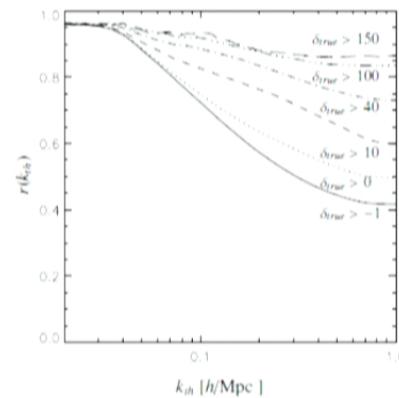
# Deviation from the truth

$$r(k_{th}) = \frac{\langle \delta_{true}^{k_{th}} \langle \delta \rangle^{k_{th}} \rangle}{\sqrt{\langle (\delta_{true}^{k_{th}})^2 \rangle} \sqrt{\langle (\langle \delta \rangle^{k_{th}})^2 \rangle}}$$

Raw data



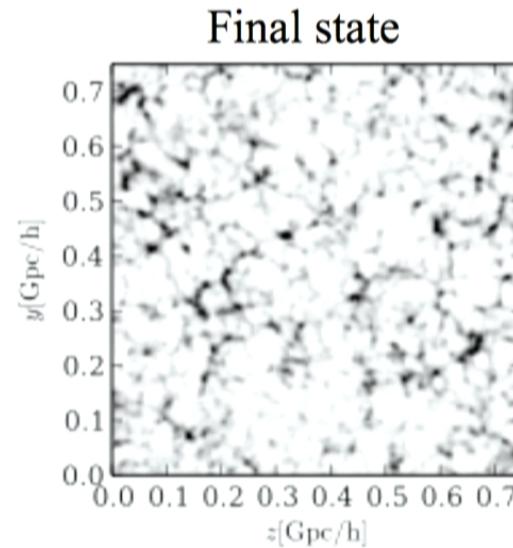
Density estimate



Jasche, Wandelt (2012)

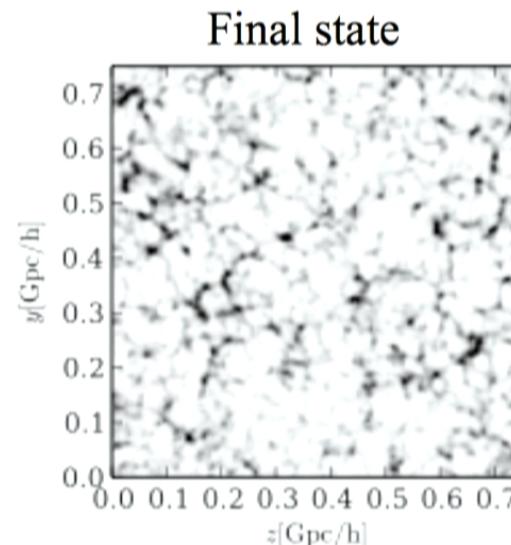
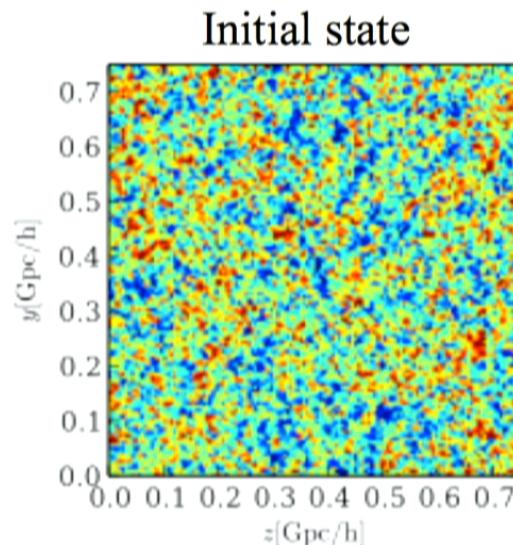
# 4D physical inference

- Physical motivation
  - Complex final state



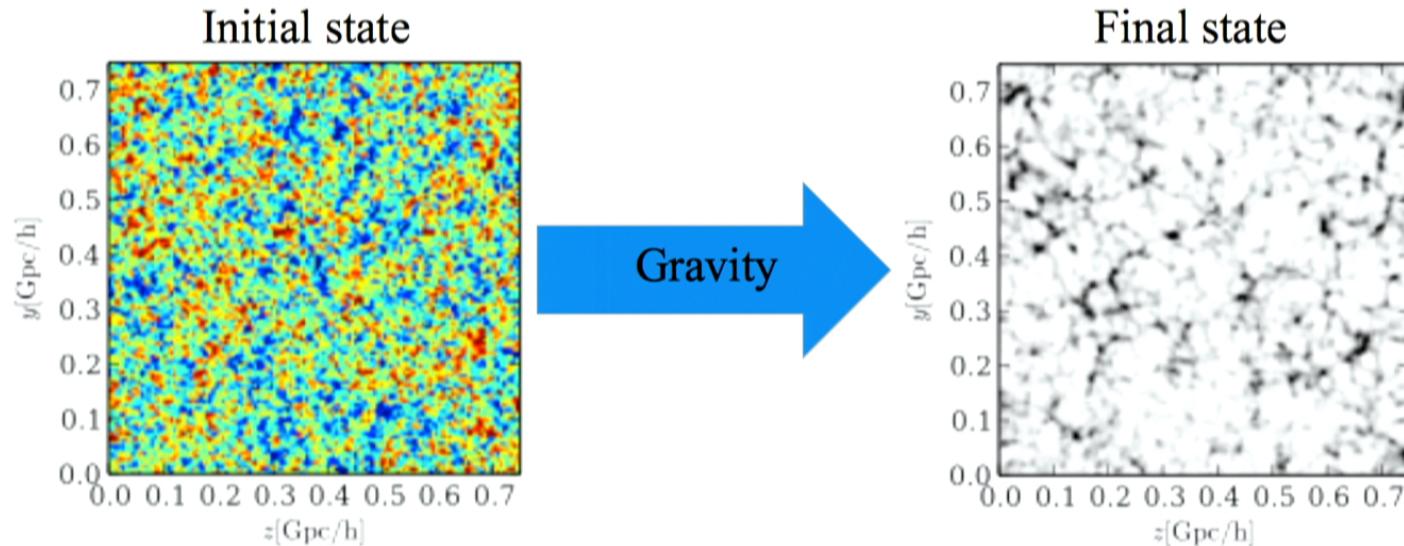
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  - Complex final state
  - Simple initial state



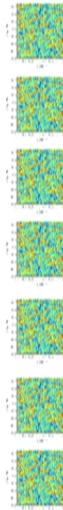
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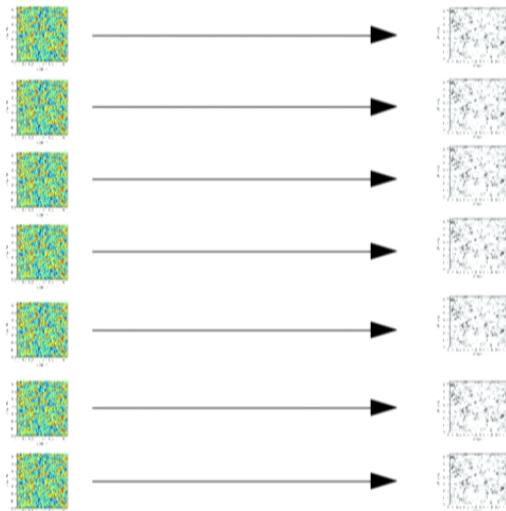
# 4D physical inference

- The ideal scenario:
  - We need a very very very large computer!



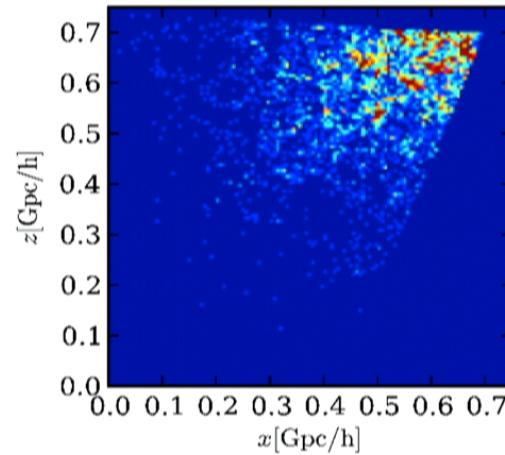
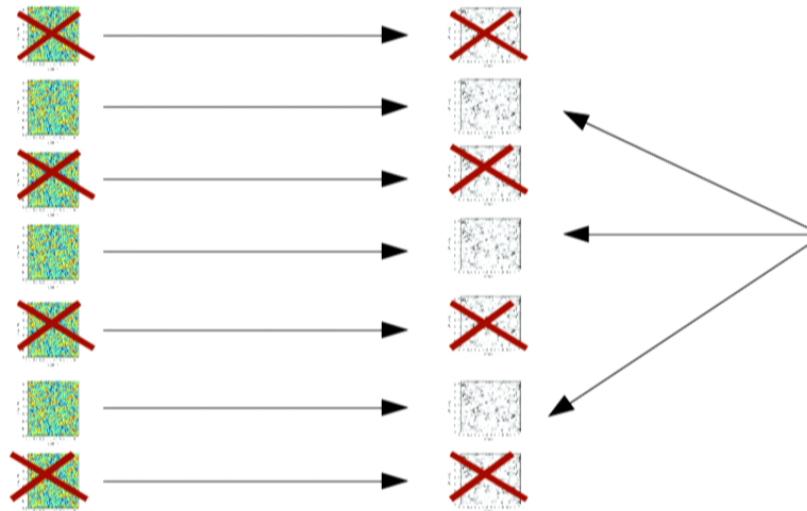
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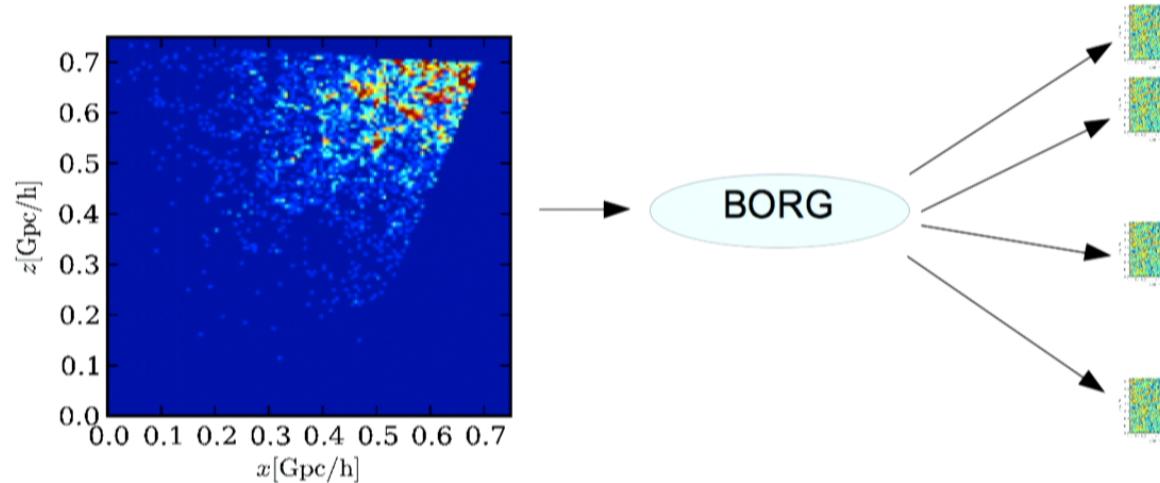
# 4D physical inference

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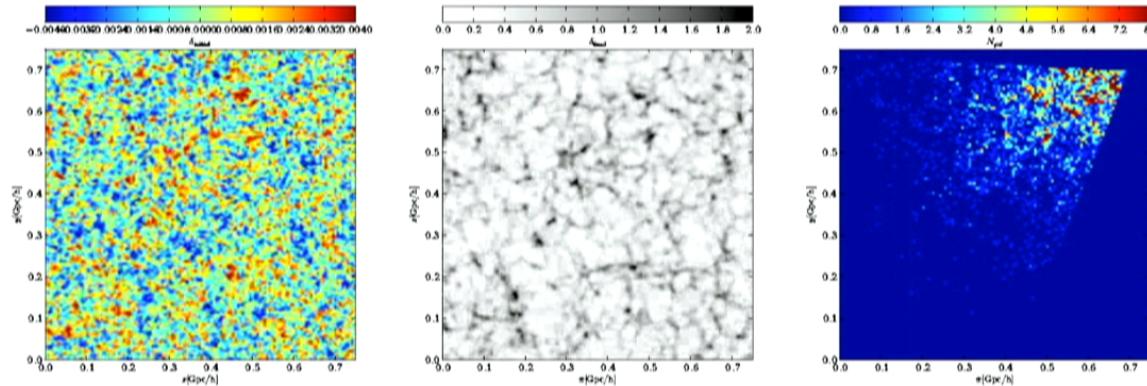
# 4D physical inference

- BORG (Bayesian Origin Reconstruction from Galaxies)
  - HMC
  - Second order Lagrangian perturbation theory

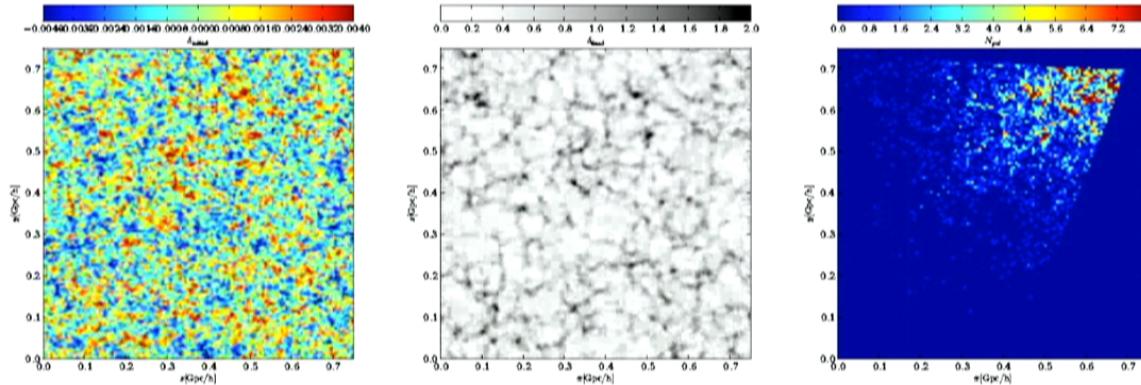


Jasche, Wandelt (2012)

# 4D physical inference



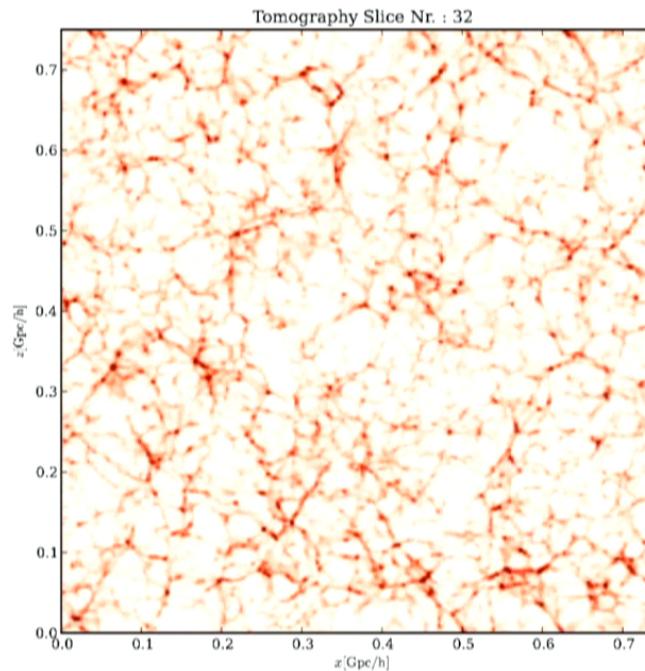
# 4D physical inference



- Cosmological applications:
  - Higher order statistics → primordial non-Gaussianity
  - 4D dynamic states → Dark Energy, ISW, kSZ
  - Physically joint analysis of data at different cosmic Epochs

# Constrained simulations

- Constrained simulation of the Sloan Volume ([Preliminary work](#))

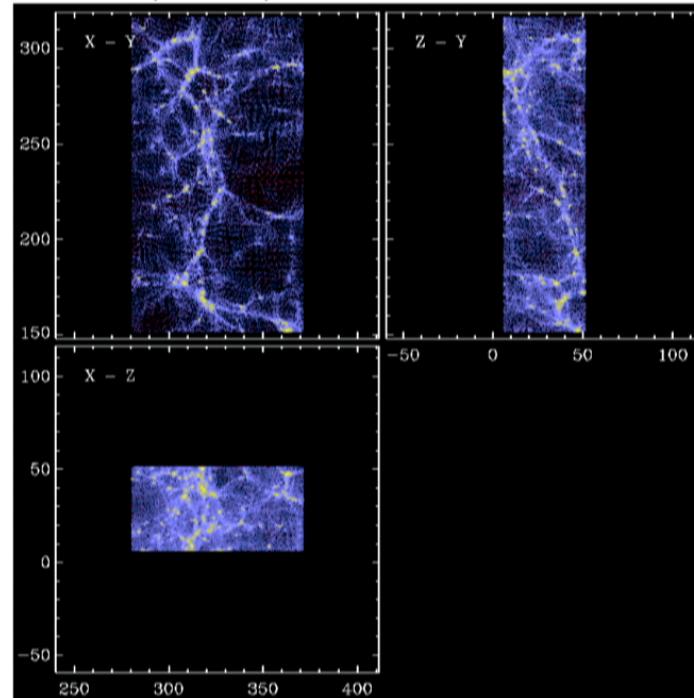


Simulation by Emilio Romano Diaz

# Resimulating the SGW

## Preliminary Work

- Sloan Great Wall (SGW)

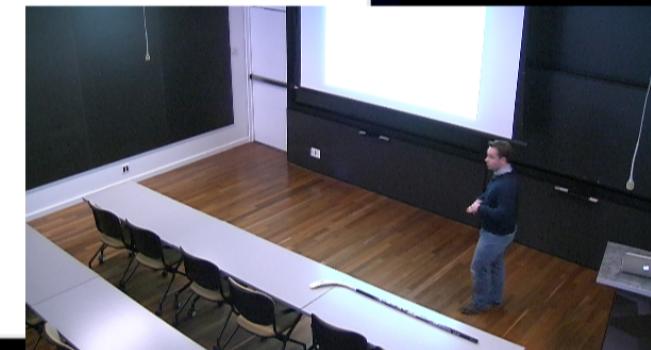


Simulation by Emilio Romano Diaz



# Summary & Conclusion

- Large scale Bayesian inference
  - Inference in high dimensions from incomplete observations
    - Noise, systematic effects, survey geometry, selection effects, biases
  - Need to quantify uncertainties → **explore posterior distribution**
    - Markov Chain Monte Carlo methods
    - Hamiltonian sampling (exploit symmetries, decouple system)
    - Multiple block sampling (break down into subproblems)
- 3 high dimensional examples ( $>10^7$  parameter)
  - Nonlinear density inference
  - Photometric redshift and density inference
  - 4D physical inference



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