

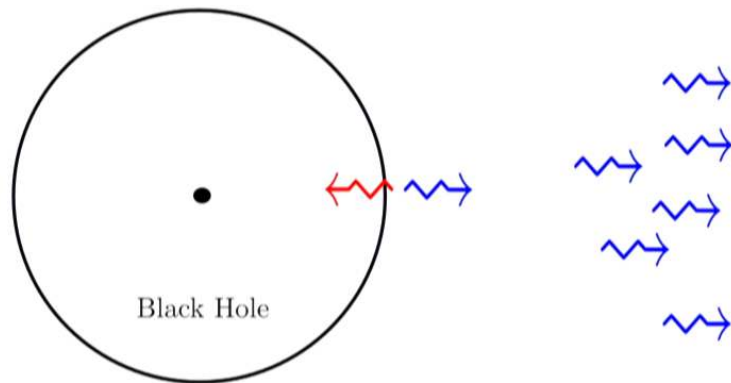
Title: An infalling observer in AdS/CFT and the black hole information paradox

Date: Apr 09, 2013 02:00 PM

URL: <http://pirsa.org/13040118>

Abstract:

Information Paradox



- For unitarity: final state must carry information of initial state
 - (In some sense) Hawking quanta are created near the horizon
 - **If horizon is featureless and we have locality, how is information transferred to outgoing radiation?**
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Information Paradox

We have tension between

- Unitarity
- Locality
- Equivalence Principle (smooth horizon)

CAN SMALL CORRECTIONS RESOLVE THE PARADOX?



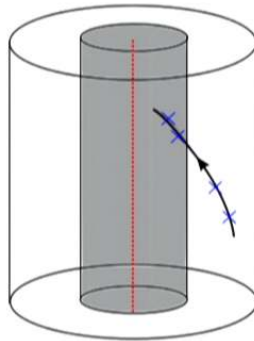
Black Holes in AdS/CFT

Main goals:

- Is the region behind the horizon encoded in the boundary CFT?
 - Understand what happens to an observer falling into a black hole
 - Address the information paradox
-



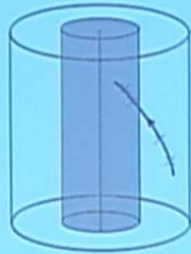
An infalling observer in AdS



- Consider a big black hole in AdS and an observer freely falling towards it
 - The observer performs local experiments
 - We will reconstruct these experiments from the boundary gauge theory
 - We will argue that the results of these experiments are the same as of semi-classical GR
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Reconstructing local observables in empty AdS

Our first goal:

Construct local bulk observables from CFT

(based on earlier works: Banks, Douglas, Horowitz, Martinec, Bena, Balasubramanian, Giddings, Lawrence, Kraus, Trivedi, Susskind, Freivogel Hamilton, Kabat, Lifschytz, Lowe, Heemskerck, Marolf, Polchinski, Sully...)

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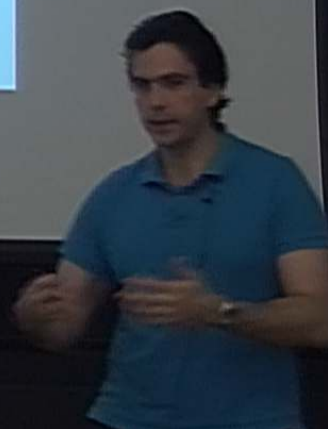
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Reconstructing local observables in empty AdS

- Large N CFTs contain in their spectrum **generalized free fields** i.e. (composite) local operators $\mathcal{O}(x)$ whose correlators factorize

$$\langle \mathcal{O}(x_1) \dots \mathcal{O}(x_{2n}) \rangle = \langle \mathcal{O}(x_1) \mathcal{O}(x_2) \rangle \dots \langle \mathcal{O}(x_{2n-1}) \mathcal{O}(x_{2n}) \rangle + \dots$$

- Factorization \approx “superposition principle”. However, the operators \mathcal{O} **do not satisfy any linear equation of motion in the CFT**.
- Hence, they are not **free fields**, but rather **generalized free fields**
- Excitations created by \mathcal{O} behave like **ordinary free particles** in a higher dimensional AdS spacetime

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Reconstructing local observables in empty AdS

- From this commutation relation we see that the modes $\mathcal{O}_{\omega, \vec{k}}$ create a **freely generated Fock space** of excitations.
- For an ordinary free field we have dispersion relation $\omega^2 = \vec{k}^2 + m^2$.
- For the generalized free fields, excitations labeled by the **independent** parameters ω and \vec{k} .
- \Rightarrow excitations behave like ordinary particles in higher dimensional AdS space

Reconstructing local observables in empty AdS

- Consider AdS in Poincare patch

$$ds^2 = \frac{-dt^2 + d\vec{x}^2 + dz^2}{z^2}$$

- and a scalar field satisfying $\square\phi = m^2\phi$.
- We take m^2 to be related to the conformal dimension Δ of \mathcal{O} by

$$\Delta = \frac{d}{2} + \sqrt{m^2 + d^2/4}$$

- For each value of ω, \vec{k} we find a solution of the Klein-Gordon equation of the form

$$f_{\omega, \vec{k}}(t, \vec{x}, z) = e^{-i\omega t + i\vec{k}\vec{x}} z^{d/2} J_{\Delta-d/2}(\sqrt{\omega^2 - \vec{k}^2} z)$$

Reconstructing local observables in empty AdS

- We construct non-local CFT operators as

$$\phi_{\text{CFT}}(t, \vec{x}, z) = \int_{\omega > 0} d\omega d\vec{k} \left(\mathcal{O}_{\omega, \vec{k}} f_{\omega, \vec{k}}(t, \vec{x}, z) + \text{h.c.} \right)$$

Notice that while these are labeled by the coordinate z , they are really operators in the CFT. They are smeared, nonlocal operators.

- Using the previous results we can show that they satisfy

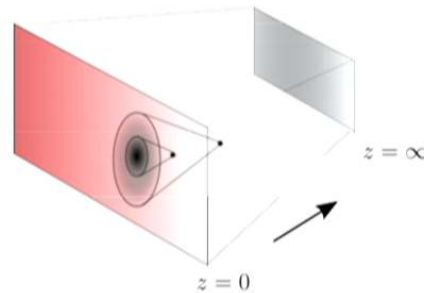
$$\square_{\text{AdS}} \phi_{\text{CFT}} = m^2 \phi_{\text{CFT}}$$

and

$$[\phi_{\text{CFT}}(t, \vec{x}, z), \phi_{\text{CFT}}(t', \vec{x}', z')] = 0$$

for points (t, \vec{x}, z) and (t', \vec{x}', z') spacelike **with respect to the AdS metric**.

Reconstructing local observables in empty AdS



- From the point of view of the CFT, coordinate z is an "auxiliary" parameter, which controls the smearing of the operators
- We can explicitly see how AdS space emerges from the lower dimensional CFT, as the combination of the coordinates t, \vec{x} together with the extra parameter z

Reconstructing local observables in empty AdS

We can also interchange the order of the Fourier transforms to write

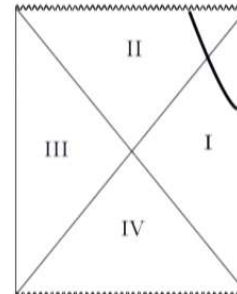
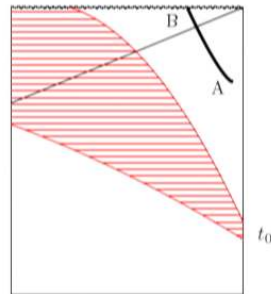
$$\phi_{\text{CFT}}(t, \vec{x}, z) = \int dt' d\vec{x}' K(t, \vec{x}, z; t', \vec{x}') \mathcal{O}(t', \vec{x}')$$

where K is some kernel — sometimes called the *transfer function*.

Subtleties: $1/N$ expansion, gauge invariance....

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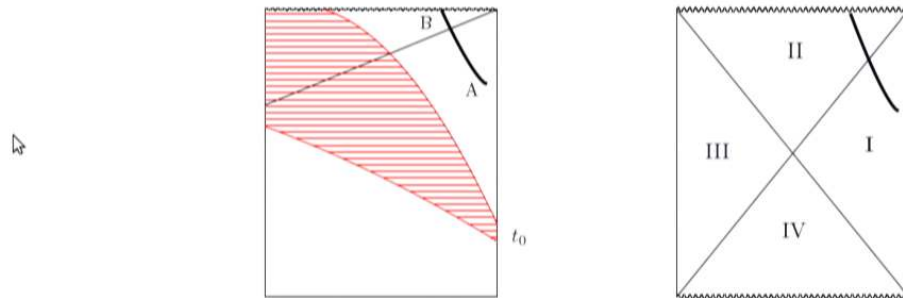
Black Holes in AdS



BH formed by collapse \approx Typical (QGP) pure state $|\Psi\rangle$

Eternal Black Hole in AdS \approx Thermal ensemble in gauge theory

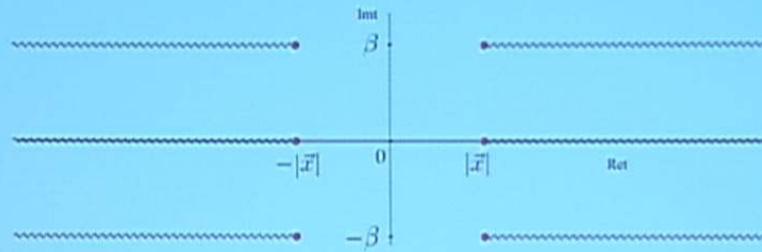
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CFT Correlators at finite temperature



- Consider the 2-point function $G_\beta(t, \vec{x}) = \langle \mathcal{O}(t, \vec{x}) \mathcal{O}(0, \vec{0}) \rangle_\beta$
- Satisfies the KMS condition

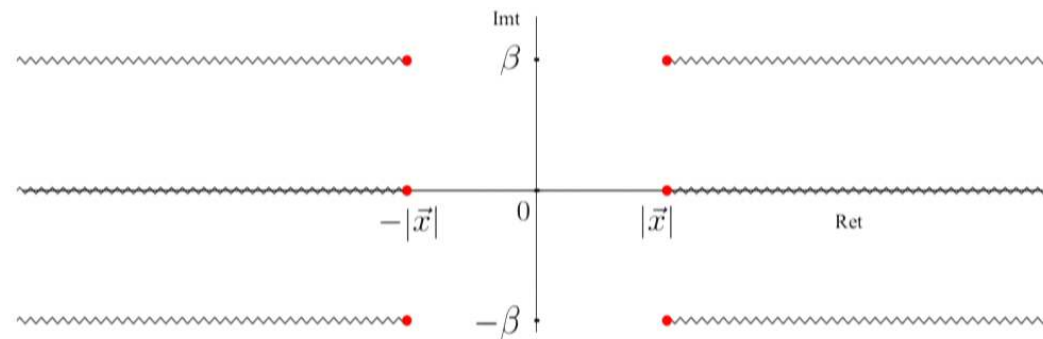
$$G_\beta(t - i\beta, \vec{x}) = G_\beta(-t, -\vec{x})$$

- In Fourier space

$$G_\beta(-\omega, \vec{k}) = e^{-\beta\omega} G_\beta(\omega, \vec{k})$$

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Reconstructing the region outside the black hole

- Consider a black hole in AdS given by the metric

$$ds^2 = \frac{-h(z)dt^2 + dx^2 + h^{-1}(z)dz^2}{z^2}, \quad h(z) = 1 - \frac{z^d}{z_0^d}$$

- Look for solutions of the Klein-Gordon equation of the form

$$f_{\omega, \vec{k}}(t, \vec{x}, z) = e^{-i\omega t + i\vec{k}\vec{x}} \psi_{\omega, \vec{k}}(z)$$

- For every (ω, \vec{k}) there is a unique solution, normalizable at the boundary $z = 0$.
- These are the usual "Schwarzschild modes" that we get when we quantize a scalar field near a black hole. We identify

$$f_{\omega, \vec{k}}(t, \vec{x}, z) \Leftrightarrow \mathcal{O}_{\omega, \vec{k}}$$

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Reconstructing the region outside the black hole

- As before, we can write nonlocal CFT operators

$$\phi_{\text{CFT}}(t, \vec{x}, z) = \int_{\omega > 0} d\omega d\vec{k} \left(\mathcal{O}_{\omega, \vec{k}} f_{\omega, \vec{k}}(t, \vec{x}, z) + \text{h.c.} \right)$$

- which behave like local fields around a black hole

$$(\square - m^2)\phi_{\text{CFT}} = 0$$

$$[\phi_{\text{CFT}}(t, \vec{x}, z), \phi_{\text{CFT}}(t', \vec{x}', z')] = 0 \quad , \quad \text{for spacelike points}$$

- and more generally

$$\langle \phi_{\text{CFT}}(P_1) \dots \phi_{\text{CFT}}(P_n) \rangle_{\beta} = \langle \phi_{\text{gravity}}(P_1) \dots \phi_{\text{gravity}}(P_n) \rangle_{\text{Hartle-Hawking}}$$

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- we find that they satisfy an oscillator algebra

$$[\mathcal{O}_{\omega, \vec{k}}, \mathcal{O}_{\omega', \vec{k}'}^\dagger] = \left(G_\beta(\omega, \vec{k}) - G_\beta(-\omega, \vec{k}) \right) \delta(\omega - \omega') \delta(\vec{k} - \vec{k}')$$

- but now the (canonically normalized) oscillators are thermally populated

$$\langle \hat{\mathcal{O}}_{\omega, \vec{k}}^\dagger \hat{\mathcal{O}}_{\omega, \vec{k}} \rangle_\beta = \frac{1}{e^{\beta\omega} - 1}$$

(this is the CFT analogue of the “thermal atmosphere” of the black hole)

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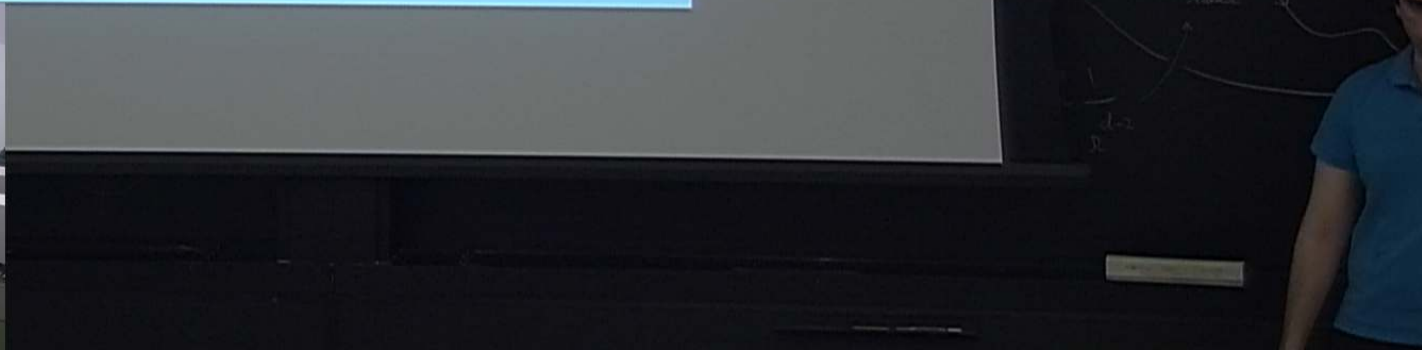
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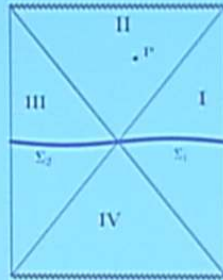
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Falling behind the horizon

- Penrose diagram of (eternal) AdS black hole



- Cauchy slice for points in II is $\Sigma_1 \oplus \Sigma_2$
- To reconstruct local operator at P we need **both** modes on Σ_1 and Σ_2

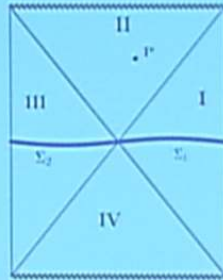
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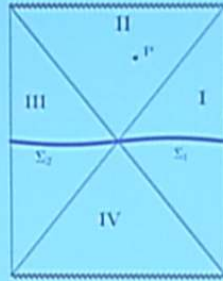


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Falling behind the horizon



- Maldacena: eternal black hole = 2 copies of CFT in entangled state
- In this formalism, modes on Σ_2 are the operators $\tilde{\mathcal{O}}_{\omega, \vec{k}}$ in the second copy of the CFT
- Do we really need the two entangled copies?
- If we work with a single CFT, what is the meaning of the operators $\tilde{\mathcal{O}}_{\omega, \vec{k}}$?

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Coarse-graining and doubling of operators

- Consider complicated (ergodic) system in pure state $|\Psi\rangle$
- Intuitive expectation \Rightarrow system "thermalizes"
- For some observables $\{A_i\}$ - called **coarse-grained observables**, their correlators on $|\Psi\rangle$ come close to thermal correlators

$$\langle \Psi | A_1 \dots A_n | \Psi \rangle \approx \text{Tr} \left(e^{-\beta H} A_1 \dots A_n \right)$$

- This is not true for all observables, there are also **fine grained observables** which do not thermalize
- To simplify the language let us assume that the Hilbert space has the form

$$\mathcal{H} = \mathcal{H}_{\text{coarse}} \otimes \mathcal{H}_{\text{fine}}$$

(strictly speaking not true, but can be made more precise)

- $\mathcal{H}_{\text{fine}}$ plays the role of a **heat bath** for $\mathcal{H}_{\text{coarse}}$
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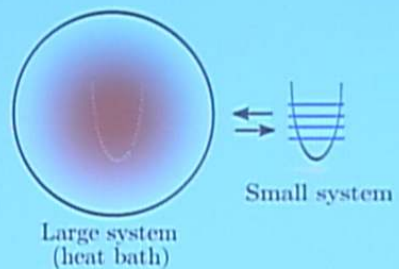
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Coarse graining and doubling of operators



SMALL SUBSYSTEM IS MIRRORED IN HEAT BATH

- For us the Quark-Gluon-Pasma is the heat bath
- The glueball operators \mathcal{O}_i are the coarse-grained observables
- They are mirrored in the QGP, which leads to new operators $\tilde{\mathcal{O}}_i$
- This mirroring involves the fine-degrees of freedom

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Coarse graining and doubling of operators

- Every state $|\Psi\rangle$ can be written as

$$|\Psi\rangle = \sum_{ij} c_{ij} |\Psi_i^c\rangle \otimes |\Psi_j^f\rangle$$

where $|\Psi_i^c\rangle, |\Psi_j^f\rangle$ are orthonormal basis of $\mathcal{H}_{\text{coarse}}$ and $\mathcal{H}_{\text{fine}}$ respectively

- If $\mathcal{H}_{\text{coarse}}$ thermalizes, it means that the reduced density matrix

$$\rho_{\text{coarse}} = Z_c^{-1} e^{-\beta H_{\text{coarse}}}$$

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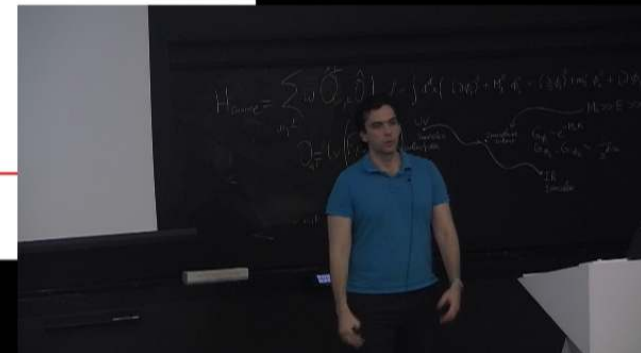
- Consider a **coarse-grained** operator acting on $\mathcal{H}_{\text{coarse}}$ as

$$A = \sum_{ij} a_{ij} |\hat{\Psi}_i^c\rangle \otimes \langle \hat{\Psi}_j^c|$$

- Then we **define** a new operator

$$\tilde{A} = \sum_{ij} a_{ij}^* |\hat{\Psi}_i^f\rangle \otimes \langle \hat{\Psi}_j^f|$$

acting on the fine-grained Hilbert space.



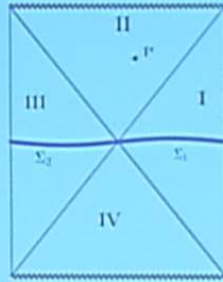
Fine-grained observables

- The “tilde operators” are very special: they are **fine-grained** observables
- They are state-dependent operators, will not “click correctly” with different microstate $|\Psi'\rangle$ (which may be a good thing....)
- Among all possible fine-grained operators, the “tilde operators” are selected/protected via their entanglement with the coarse-grained ones
- They are “very sparse operators”

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$$H_{\text{coarse}} = \sum_{w_i} \dots$$

Falling behind the horizon



Modes on $\Sigma_1 \Leftrightarrow \mathcal{O}_{\omega, \vec{k}}$

Modes on $\Sigma_2 \Leftrightarrow \tilde{\mathcal{O}}_{\omega, \vec{k}}$

where $\tilde{\mathcal{O}}_{\omega, \vec{k}}$ are the Fourier transforms of the mirrored operators $\tilde{\mathcal{O}}$

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Local operators behind the horizon

Using both $\mathcal{O}_{\omega, \vec{k}}$ and $\tilde{\mathcal{O}}_{\omega, \vec{k}}$ we can write local observables behind the horizon of the black hole.

$$\phi_{\text{CFT}}(t, \vec{x}, z) = \int_{\omega > 0} d\omega d\vec{k} \left[\mathcal{O}_{\omega, \vec{k}} g_{\omega, \vec{k}}^{(1)}(t, \vec{x}, z) + \tilde{\mathcal{O}}_{\omega, \vec{k}} g_{\omega, \vec{k}}^{(2)}(t, \vec{x}, z) + \text{h.c.} \right]$$

here $g^{(1),(2)}$ are solutions of the Klein-Gordon equation in region II

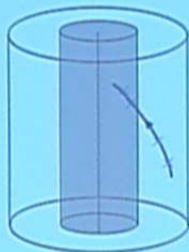
In the large N limit, correlators of $\phi_{\text{CFT}}(t, \vec{x}, z)$ on a typical pure state $|\Psi\rangle$ (corresponding to a black hole microstate) agree with those computed in semiclassical gravity.

We have reconstructed both the exterior and the interior of the black hole from the dual gauge theory

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Fate of the infalling observer

Using the operators ϕ_{CFT} we can reconstruct the experiments of the infalling observer



MAIN CONCLUSION: For a big AdS black hole, an infalling semi-classical observer does not notice anything special when crossing the horizon

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Simple version of the information paradox

However, consider what happens when a normal object burns (say a piece of coal)

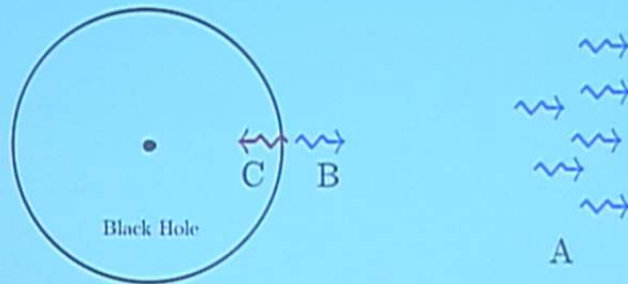
- Outgoing photons seem to be thermal to a very good approximation.
- How is unitarity preserved? Where is the information of the original piece of coal stored in the outgoing radiation?
- ANSWER: It is encoded in very small correlations (entanglement) between the outgoing photons.
- While final state looks like a thermal density matrix ρ_{thermal} in reality it is a pure state.

SMALL CORRECTIONS TO LEADING THERMAL APPROXIMATION CAN RESTORE UNITARITY

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Sharpened version of the information paradox (Mathur, AMPS)

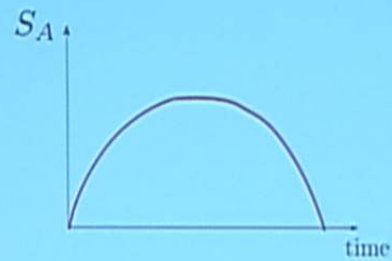
Consider the process of Hawking radiation



- A: old radiation, far from black hole
- B: newly created Hawking particle, outgoing
- C: ingoing partner of B

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Sharpened version of the information paradox (Mathur, AMPS)



In the beginning adding a B to A increases the entropy i.e. we expect

$$S_{AB} > S_A$$

but eventually this must turn around and for an **old black hole** we expect

$$S_{AB} < S_A$$

Proposed resolution

- In our language the C 's are fine-grained operators defined via their entanglement with coarse grained operators
- After Page time the early radiation A plays the role of the heat bath
- Hence C 's are "highly scrambled" combinations of A 's

Systems A,B,C are not really independent

⇓

Strong subadditivity theorem cannot be applied to A,B,C

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Sharpened version of the information paradox (Mathur, AMPS)



Strong subadditivity theorem: for 3 independent systems A,B,C we have

$$S_{AB} + S_{BC} \geq S_A + S_C$$

For the Hawking pair production we have $S_{BC} \approx 0$ and $S_C > 0$ which would imply

$$S_{AB} > S_A$$

For unitarity: after Page time we need $S_{AB} < S_A$

On complementarity

How we understand complementarity

- There is a large Hilbert space describing **both** the interior and the exterior of the black hole
- We can construct operators acting on this Hilbert space and describing observables outside the black hole
- We can construct operators acting on the same Hilbert space and describing observables inside the black hole
- For few, light observables, they approximately commute.
- But not for too accurate measurements, or measurements involving too many insertions

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