Title: An infalling observer in AdS/CFT and the black hole information paradox

Date: Apr 09, 2013 02:00 PM

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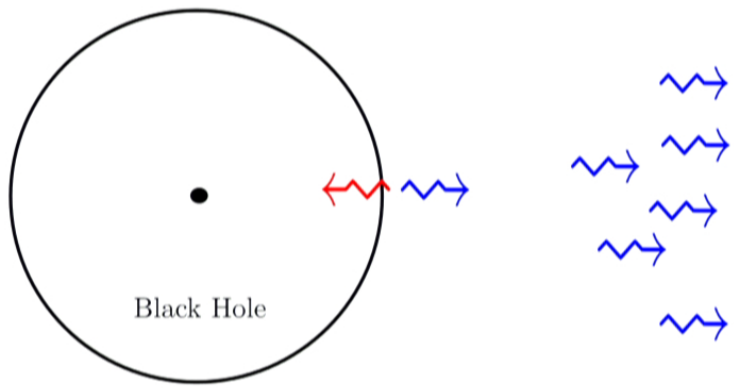
Abstract:

Information Paradox



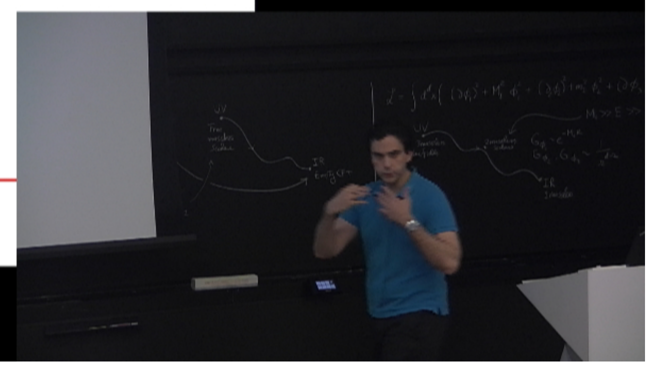
- For unitarity: final state must carry information of initial state
- (In some sense) Hawking quanta are created near the horizon
- **If horizon is featureless and we have locality, how is information transfered to outgoing radiation?**

Information Paradox

We have tension between

- Unitarity
- Locality
- Equivalence Principle (smooth horizon)

CAN SMALL CORRECTIONS RESOLVE THE PARADOX?

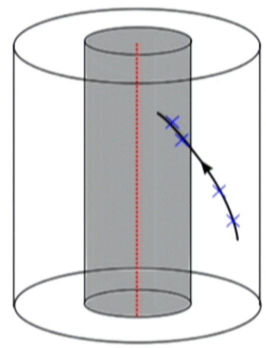


Black Holes in AdS/CFT

Main goals:

- Is the region behind the horizon encoded in the boundary CFT?
- Understand what happens to an observer falling into a black hole
- Address the information paradox

An infalling observer in AdS



- Consider a big black hole in AdS and an observer freely falling towards it
- The observer performs local experiments
- We will reconstruct these experiments from the boundary gauge theory
- We will argue that the results of these experiments are the sar
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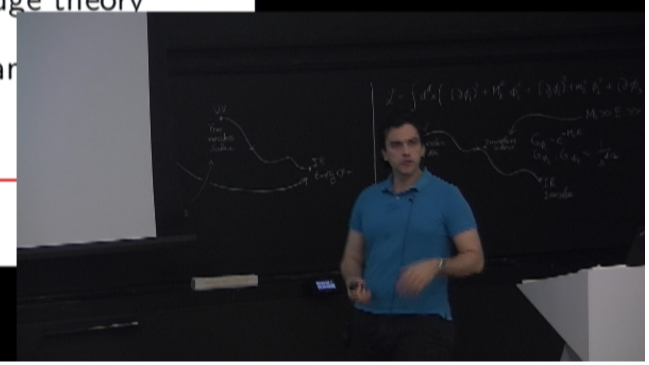

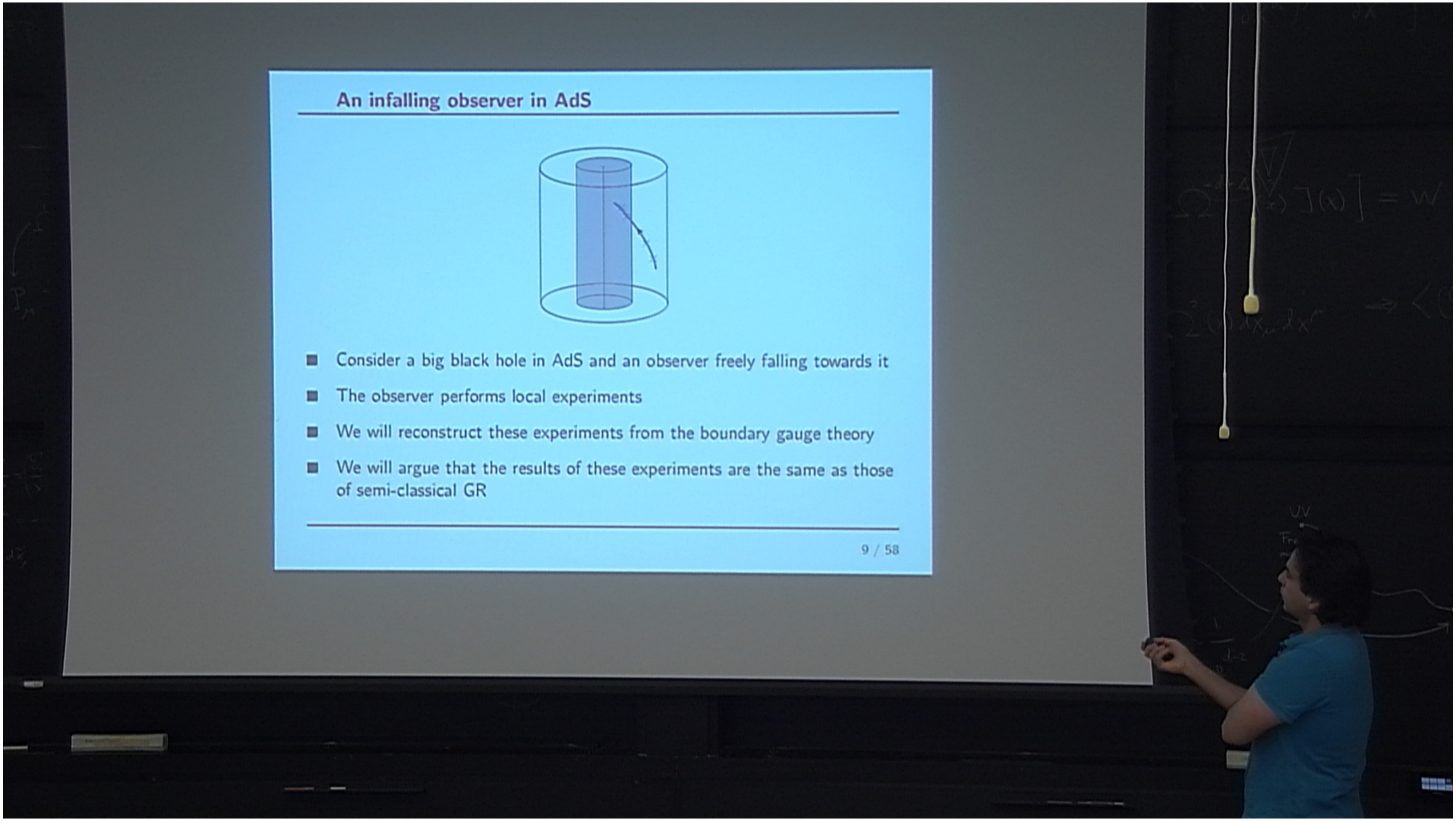
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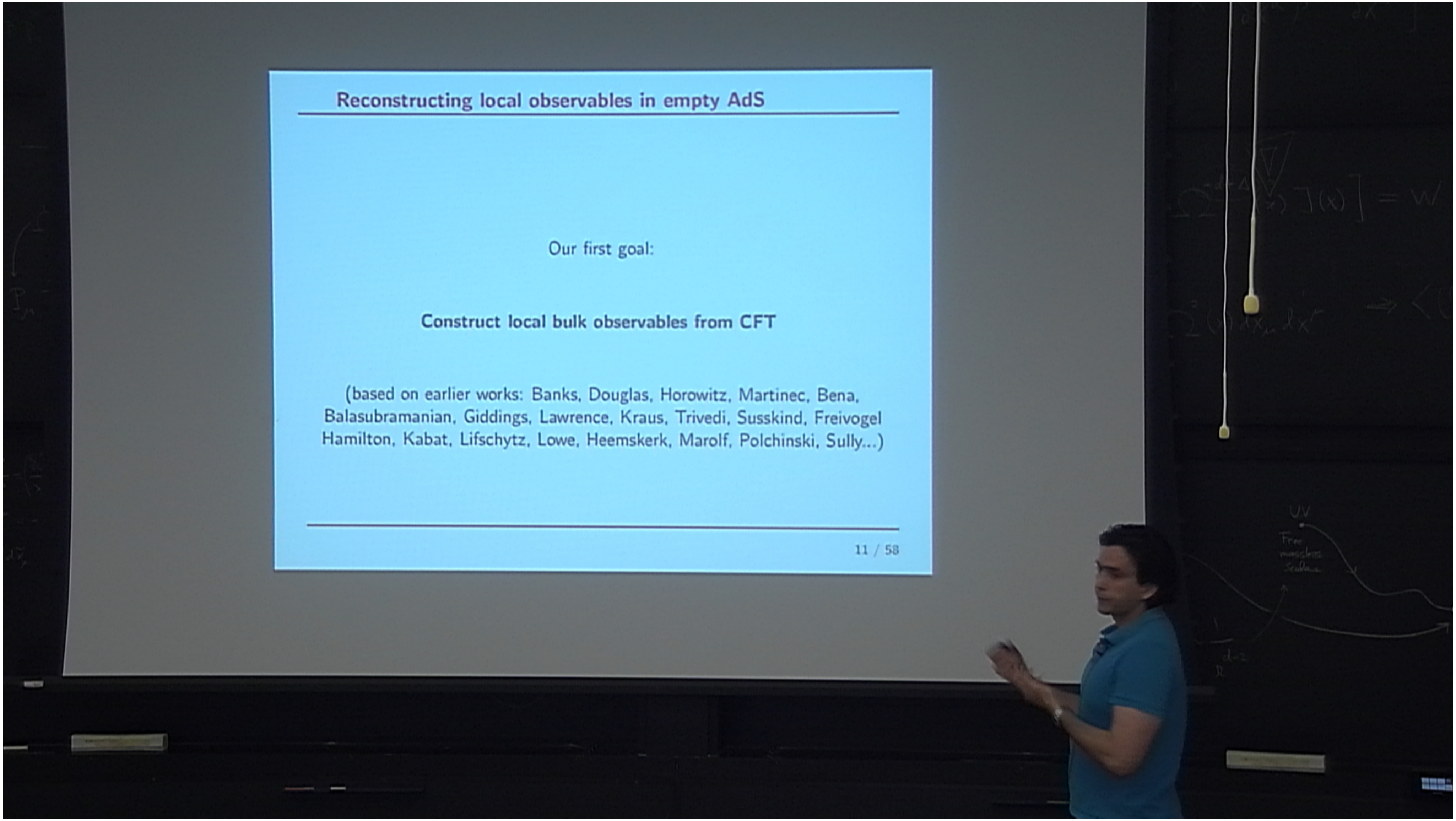
Reconstructing local observables in empty AdS
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Construct local bulk observables from CFT
(based on earlier works: Banks, Douglas, Horowitz, Martinec, Bena,
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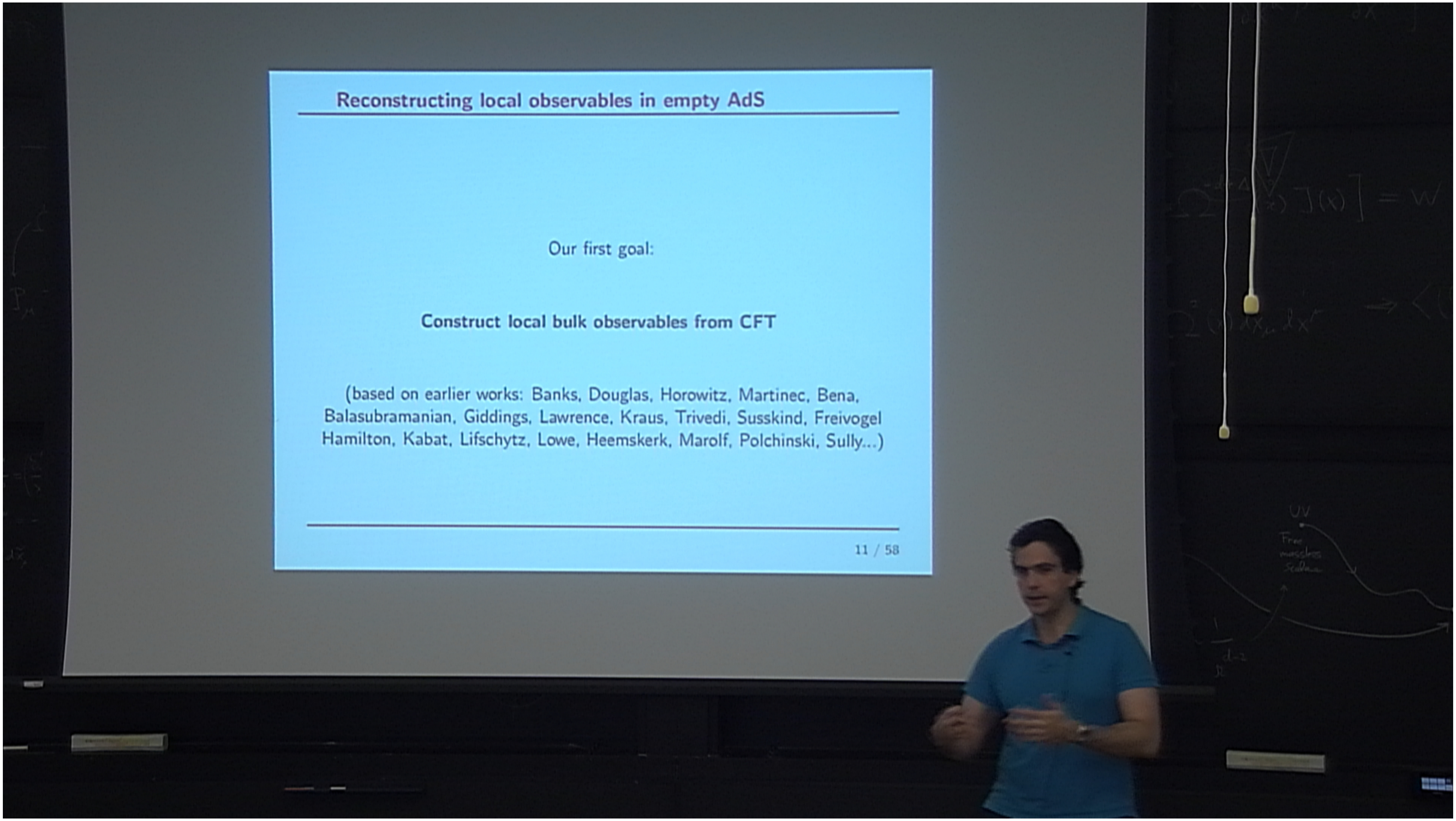
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Reconstructing local observables in empty AdS

- Large N CFTs contain in their spectrum **generalized free fields** i.e. (composite) local operators $\mathcal{O}(x)$ whose correlators factorize

$$\langle \mathcal{O}(x_1)...\mathcal{O}(x_{2n})\rangle = \langle \mathcal{O}(x_1)\mathcal{O}(x_2)\rangle \; ... \; \langle \mathcal{O}(x_{2n-1})\mathcal{O}(x_{2n})\rangle + \ldots$$

- Factorization $\approx$ "superposition principle". However, the operators $\mathcal{O}$ **do not satisfy any linear equation of motion in the CFT**.

- Hence, they are not **free fields**, but rather **generalized free fields**

- Excitations created by $\mathcal{O}$ behave like **ordinary free particles** in a higher dimensional AdS spacetime

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Reconstructing local observables in empty AdS

- From this commutation relation we see that the modes $\mathcal{O}_{\omega,\vec{k}}$ create a **freely generated Fock space** of excitations.
- For an ordinary free field we have dispersion relation $\omega^2 = \vec{k}^2 + m^2$.
- For the generalized free fields, excitations labeled by the **independent** parameters ω and $\vec{k}$.
- $\Rightarrow$ excitations behave like ordinary particles in higher dimensional AdS space

Reconstructing local observables in empty AdS

- Consider AdS in Poincare patch

$$ds^2 = \frac{-dt^2 + d\vec{x}^2 + dz^2}{z^2}$$

- and a scalar field satisfying $\Box\phi = m^2\phi$.

- We take m^2 to be related to the conformal dimension Δ of $\mathcal{O}$ by

$$\Delta = \frac{d}{2} + \sqrt{m^2 + d^2/4}$$

- For each value of $\omega, \vec{k}$ we find a solution of the Klein-Gordon equation of the form

$$f_{\omega,\vec{k}}(t, \vec{x}, z) = e^{-i\omega t + i\vec{k}\vec{x}} z^{d/2} J_{\Delta - d/2}(\sqrt{\omega^2 - \vec{k}^2} z)$$

Reconstructing local observables in empty AdS

- We construct non-local CFT operators as

$$\phi_{\rm CFT}(t,\vec{x},z) = \int_{\omega>0} d\omega\, d\vec{k}\, \left(\mathcal{O}_{\omega,\vec{k}}\; f_{\omega,\vec{k}}(t,\vec{x},z) + \text{h.c.}\right)$$

Notice that while these are labeled by the coordinate z, they are really operators in the CFT. They are smeared, nonlocal operators.

- Using the previous results we can show that they satisfy

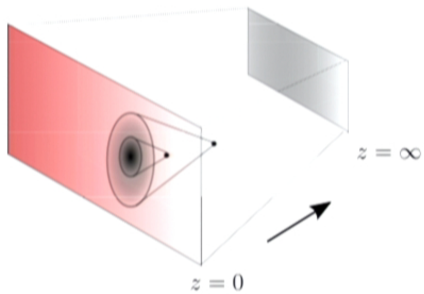
$$\Box_{\rm AdS}\phi_{\rm CFT} = m^2\,\phi_{\rm CFT}$$

and

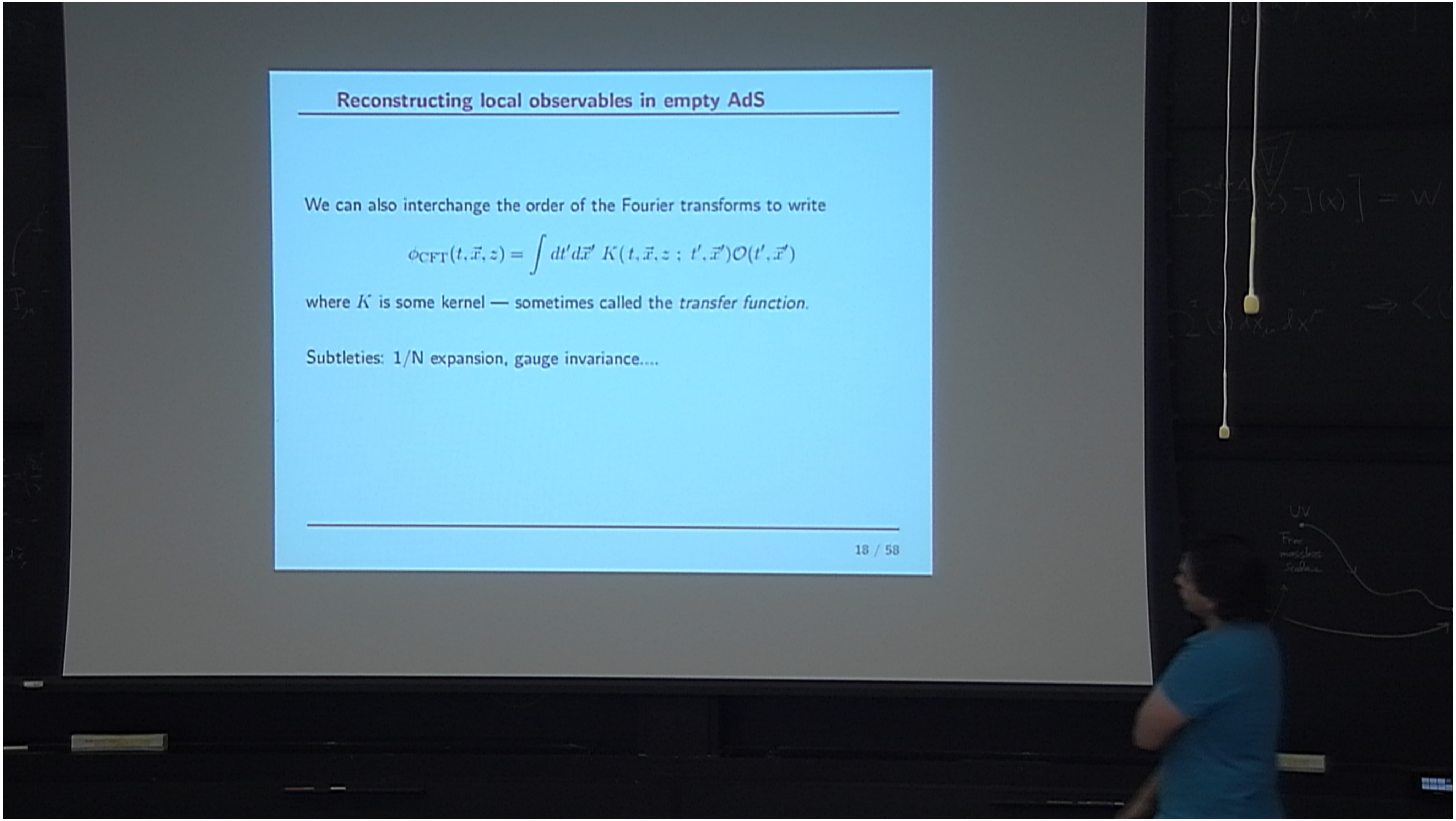
$$[\phi_{\rm CFT}(t,\vec{x},z)\,,\,\phi_{\rm CFT}(t',\vec{x}',z')] = 0$$

for points $(t,\vec{x},z)$ and $(t',\vec{x}',z')$ spacelike **with respect to the AdS metric**.

Reconstructing local observables in empty AdS



- From the point of view of the CFT, coordinate z is an "auxiliary" parameter, which controls the smearing of the operators
- We can explicitly see how AdS space emerges from the lower dimensional CFT, as the combination of the coordinates $t, \vec{x}$ together with the extra parameter z


Reconstructing local observables in empty AdS
We can also interchange the order of the Fourier transforms to write
$\phi_{\text{CFT}}(t,\vec{x},z) = \int dt' d\vec{x}' \; K(t,\vec{x},z \, ; \, t',\vec{x}')\mathcal{O}(t',\vec{x}')$
where K is some kernel — sometimes called the *transfer function*.
Subtleties: 1/N expansion, gauge invariance....
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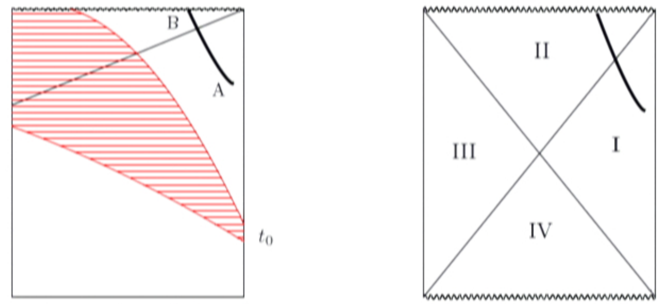
UV

Black Holes in AdS



BH formed by collapse $\approx$ Typical (QGP) pure state $|\Psi\rangle$

Eternal Black Hole in AdS $\approx$ Thermal ensemble in gauge theory

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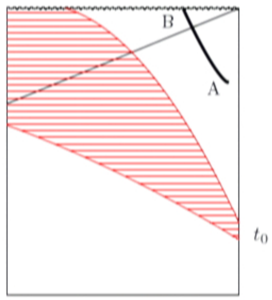

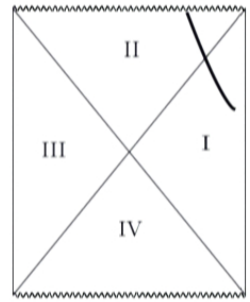

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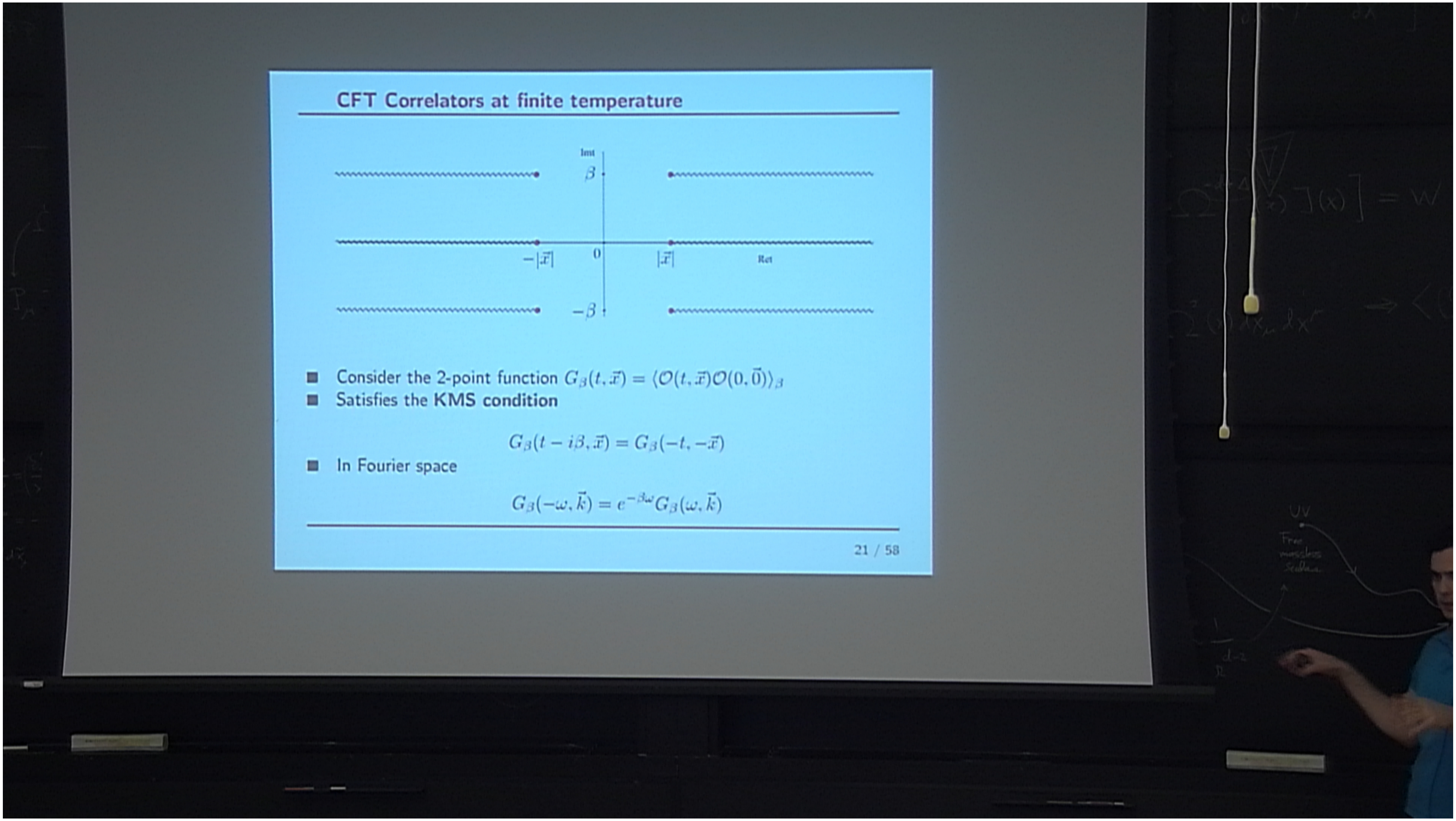
CFT Correlators at finite temperature
Imt
β
$-\beta$
$-|\vec{x}|$
0
$|\vec{x}|$
Ret
Consider the 2-point function $G_\beta(t,\vec{x}) = \langle \mathcal{O}(t,\vec{x})\mathcal{O}(0,\vec{0})\rangle_\beta$
Satisfies the **KMS condition**
$$G_\beta(t-i\beta,\vec{x}) = G_\beta(-t,-\vec{x})$$
In Fourier space
$$G_\beta(-\omega,\vec{k}) = e^{-\beta\omega}G_\beta(\omega,\vec{k})$$
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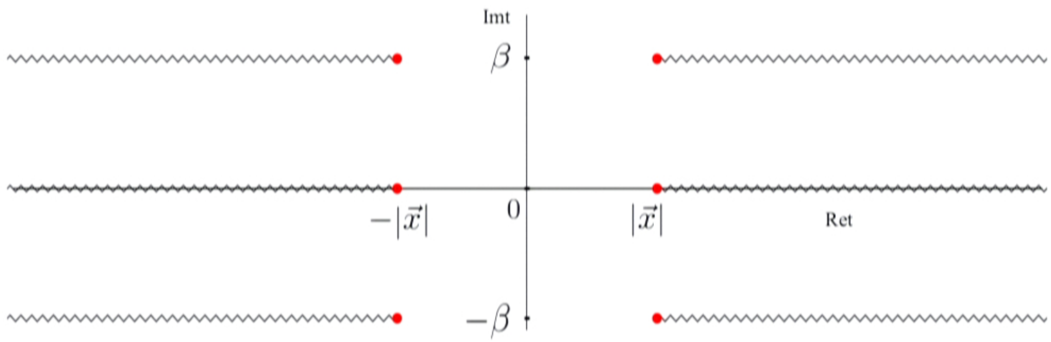

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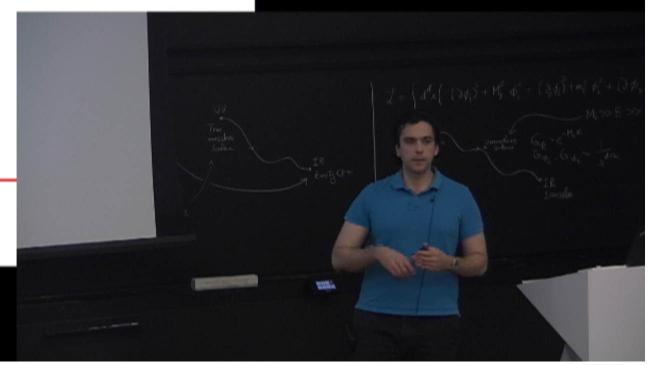

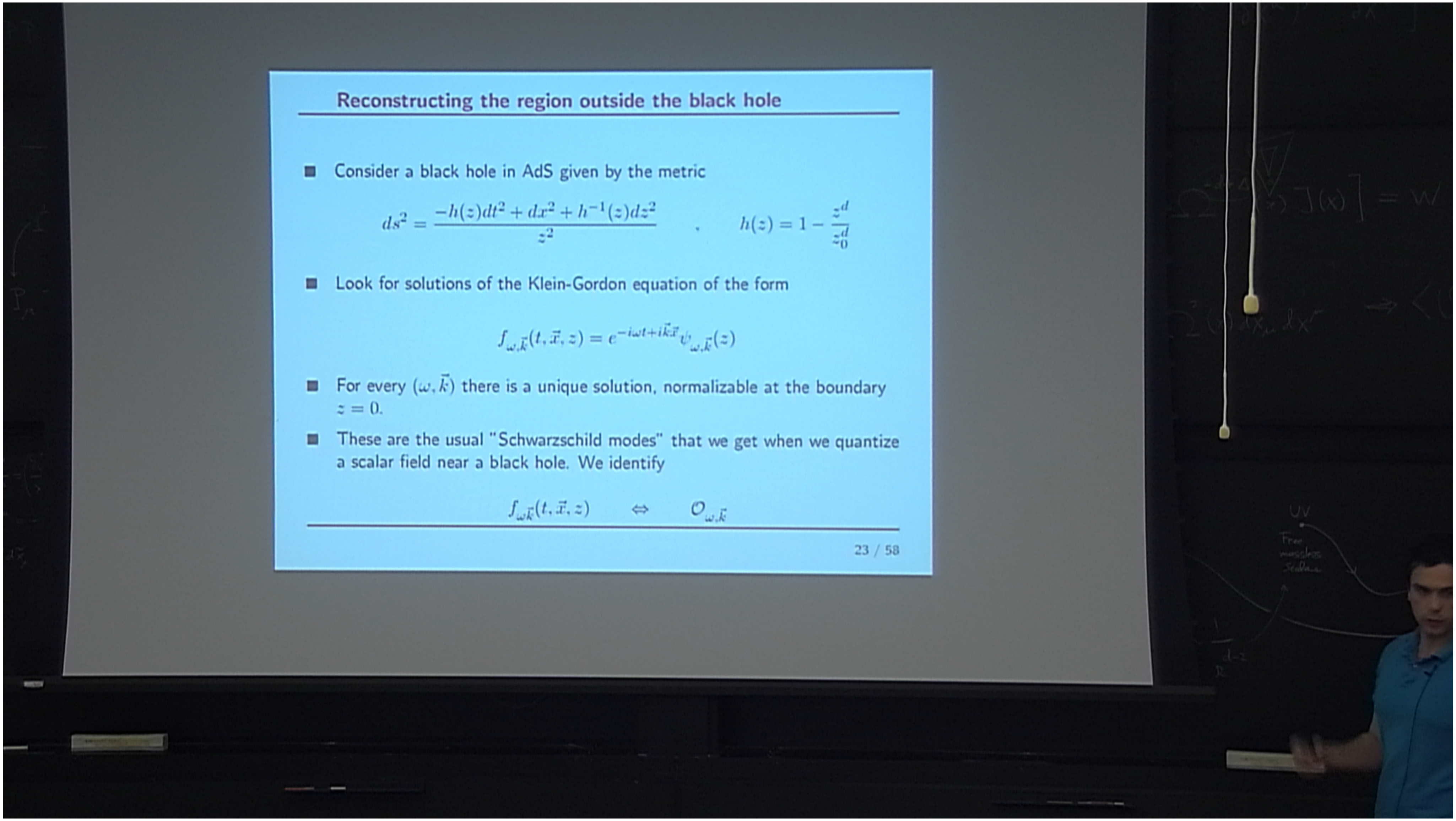
Reconstructing the region outside the black hole
Consider a black hole in AdS given by the metric
$ds^2 = \frac{-h(z)dt^2 + d\vec{x}^2 + h^{-1}(z)dz^2}{z^2}$, $h(z) = 1 - \frac{z^d}{z_0^d}$
Look for solutions of the Klein-Gordon equation of the form
$f_{\omega,\vec{k}}(t,\vec{x},z) = e^{-i\omega t + i\vec{k}\vec{x}} \psi_{\omega,\vec{k}}(z)$
For every $(\omega,\vec{k})$ there is a unique solution, normalizable at the boundary $z = 0$.
These are the usual "Schwarzschild modes" that we get when we quantize a scalar field near a black hole. We identify
$f_{\omega\vec{k}}(t,\vec{x},z) \Leftrightarrow \mathcal{O}_{\omega,\vec{k}}$
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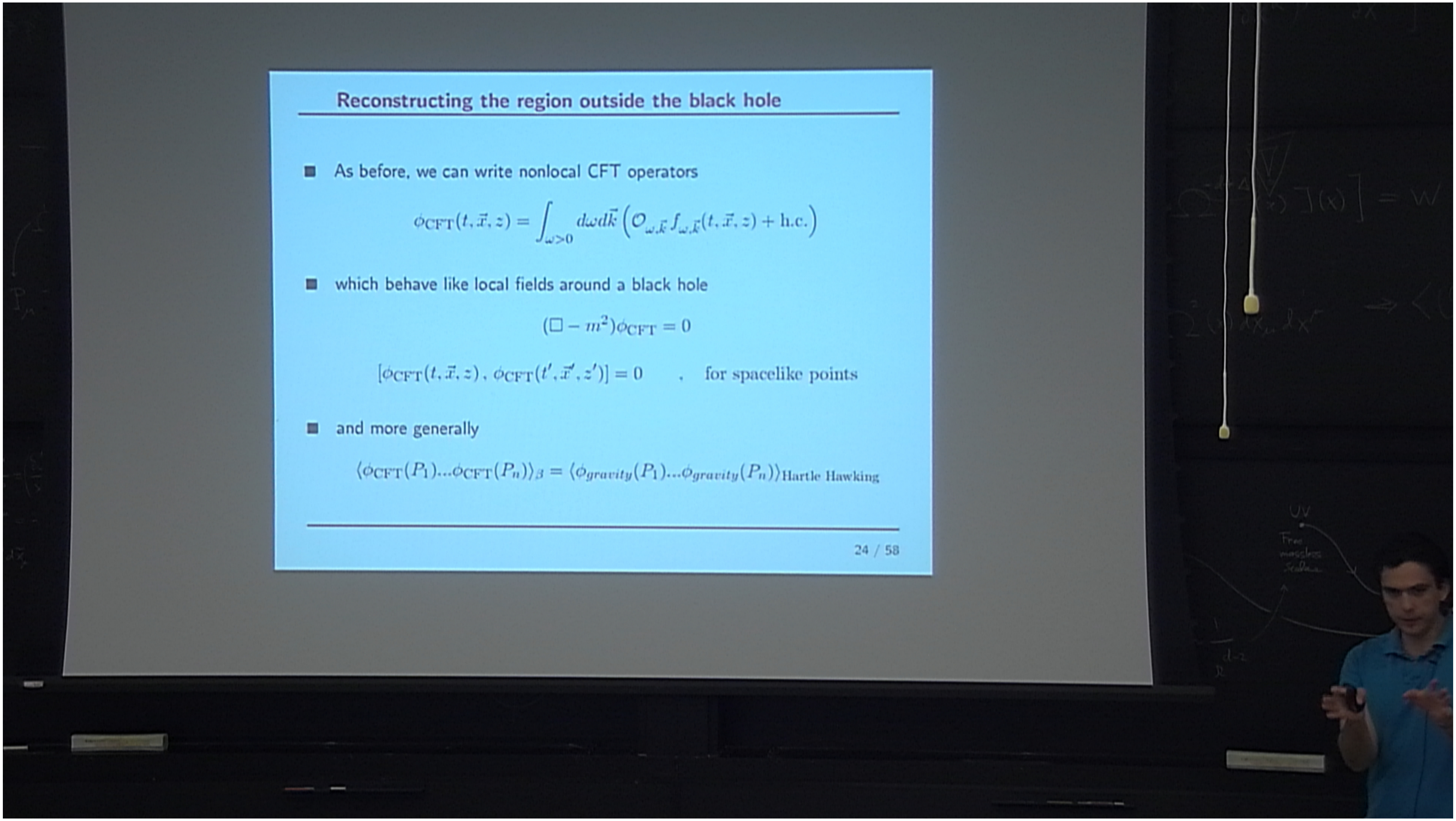
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which behave like local fields around a black hole
$$(\Box - m^2)\phi_{\rm CFT} = 0$$
$$[\phi_{\rm CFT}(t,\vec{x},z)\,,\,\phi_{\rm CFT}(t',\vec{x}',z')] = 0 \qquad , \quad \text{for spacelike points}$$
and more generally
$$\langle \phi_{\rm CFT}(P_1)...\phi_{\rm CFT}(P_n)\rangle_\beta = \langle \phi_{gravity}(P_1)...\phi_{gravity}(P_n)\rangle_{\text{Hartle Hawking}}$$
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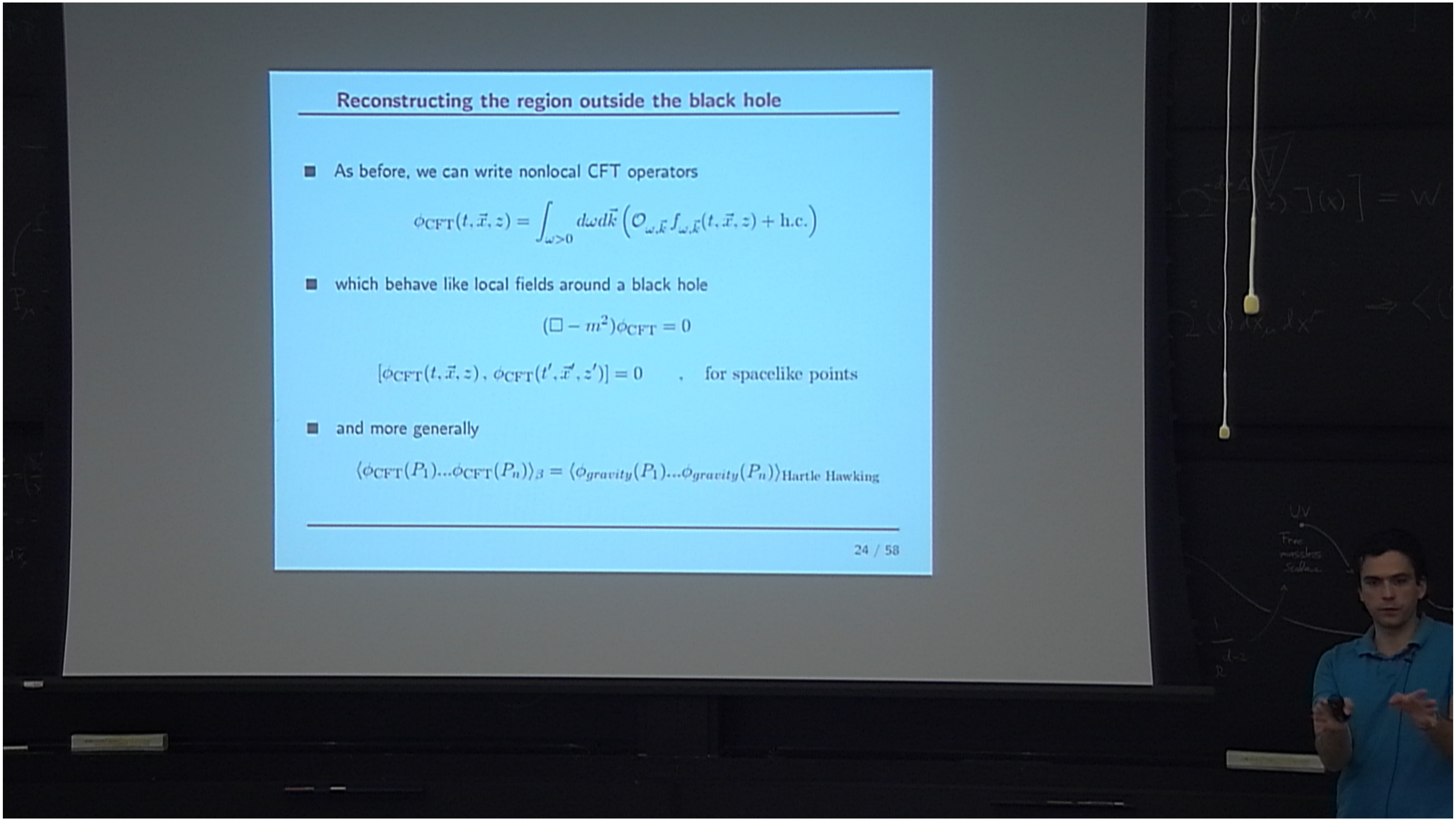
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CFT Correlators at finite temperature

- If we again define the Fourier modes $\mathcal{O}_{\omega,\vec{k}}$ by

$$\mathcal{O}(t,\vec{x}) = \int dt d^{d-1}x \left(\mathcal{O}_{\omega,\vec{k}}\, e^{-i\omega t + i\vec{k}\vec{x}} + \text{h.c.}\right)$$

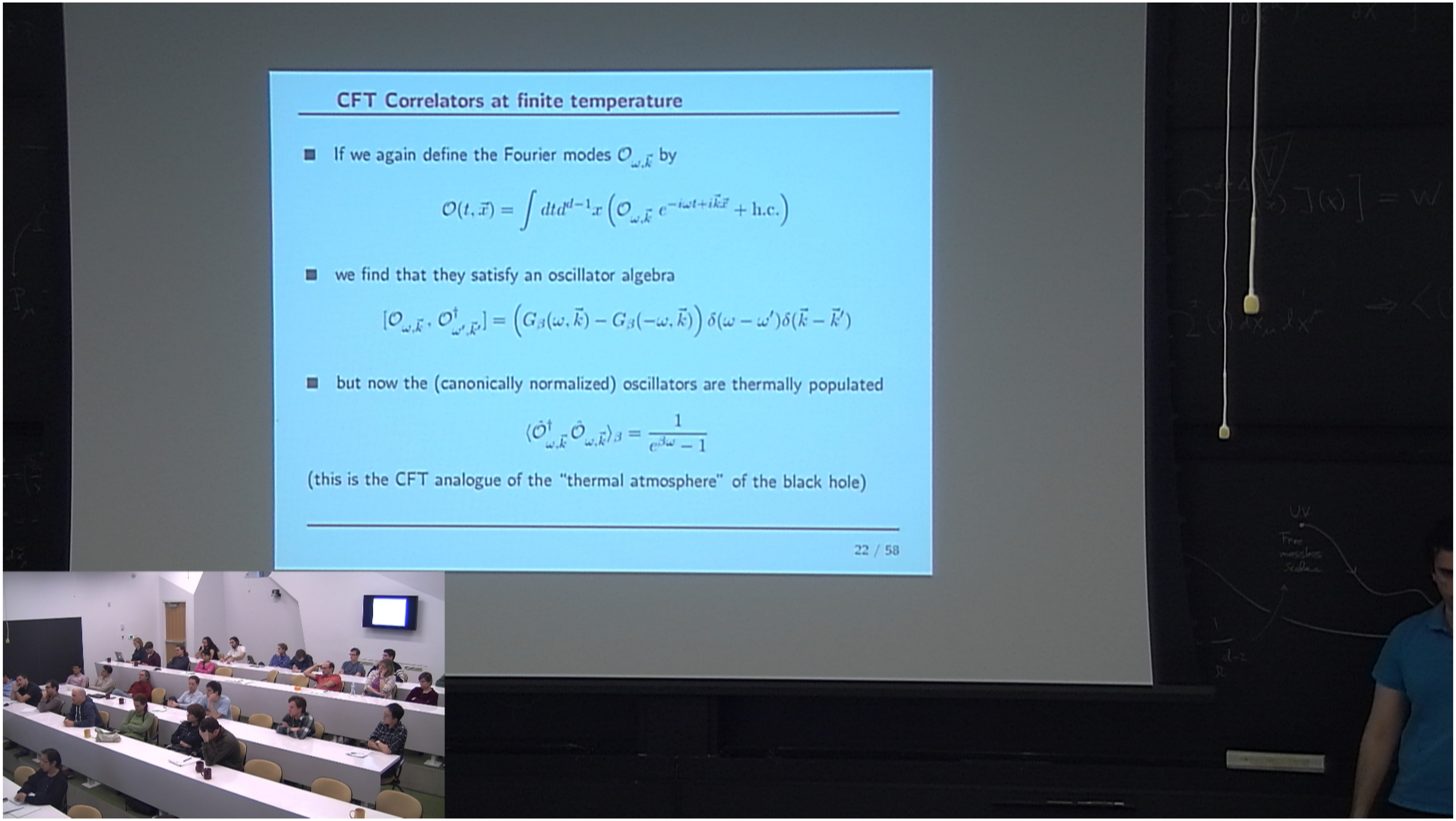
- we find that they satisfy an oscillator algebra

$$[\mathcal{O}_{\omega,\vec{k}}\,,\,\mathcal{O}^{\dagger}_{\omega',\vec{k}'}] = \left(G_\beta(\omega,\vec{k}) - G_\beta(-\omega,\vec{k})\right)\delta(\omega-\omega')\delta(\vec{k}-\vec{k}')$$

- but now the (canonically normalized) oscillators are thermally populated

$$\langle\hat{\mathcal{O}}^{\dagger}_{\omega,\vec{k}}\,\hat{\mathcal{O}}_{\omega,\vec{k}}\rangle_\beta = \frac{1}{e^{\beta\omega}-1}$$

(this is the CFT analogue of the "thermal atmosphere" of the black hole)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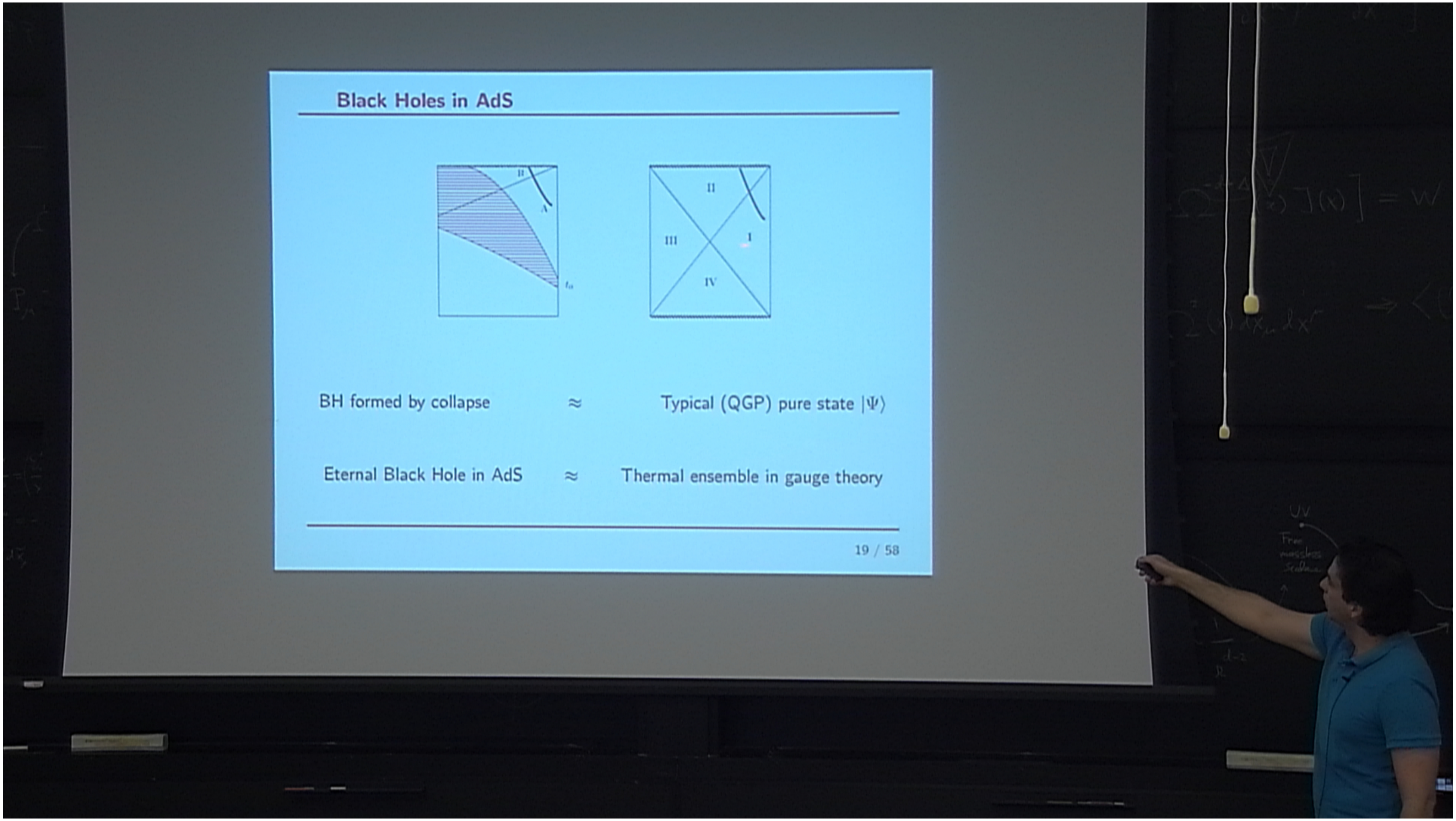
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II
A
III
I
IV
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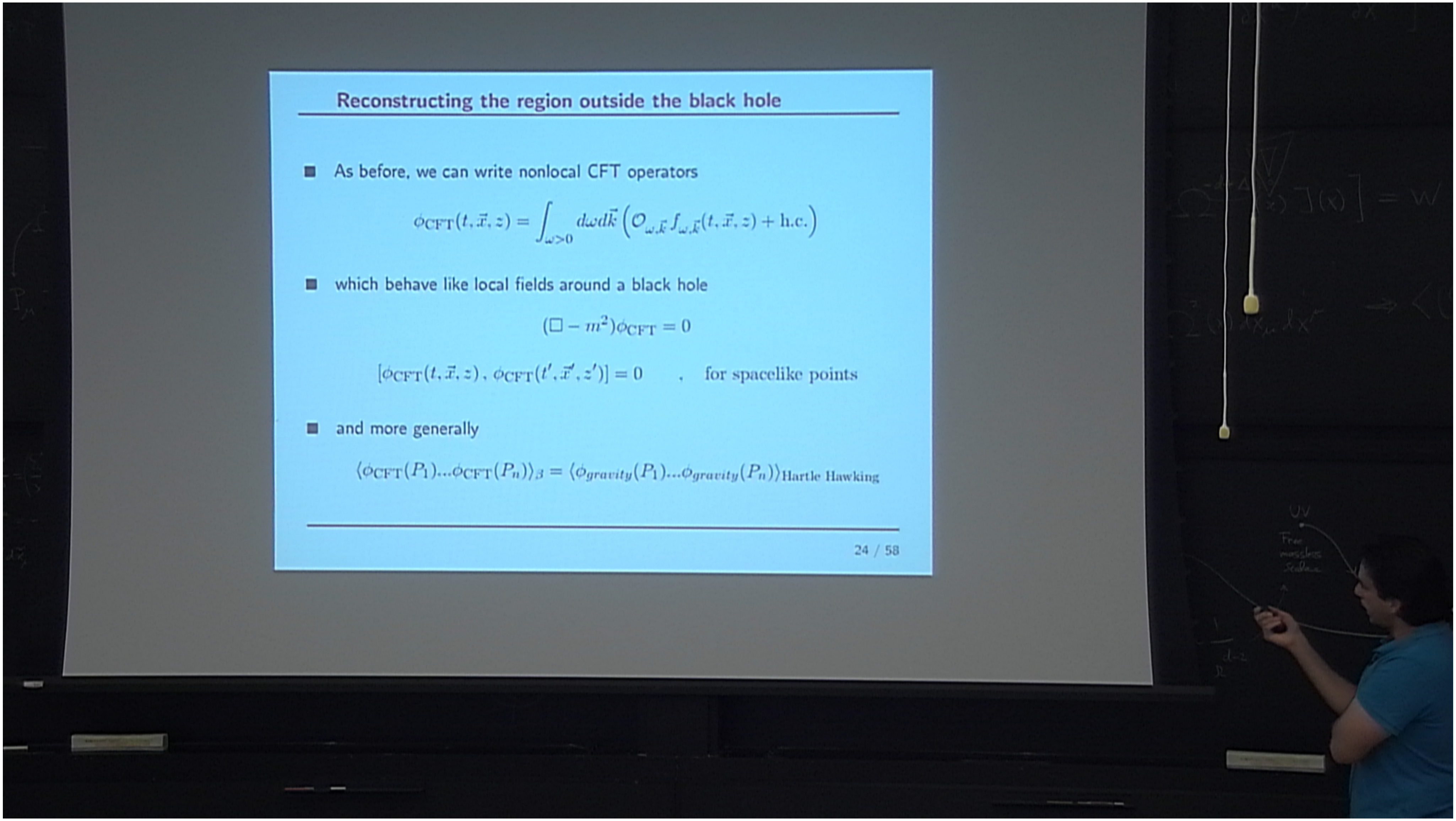
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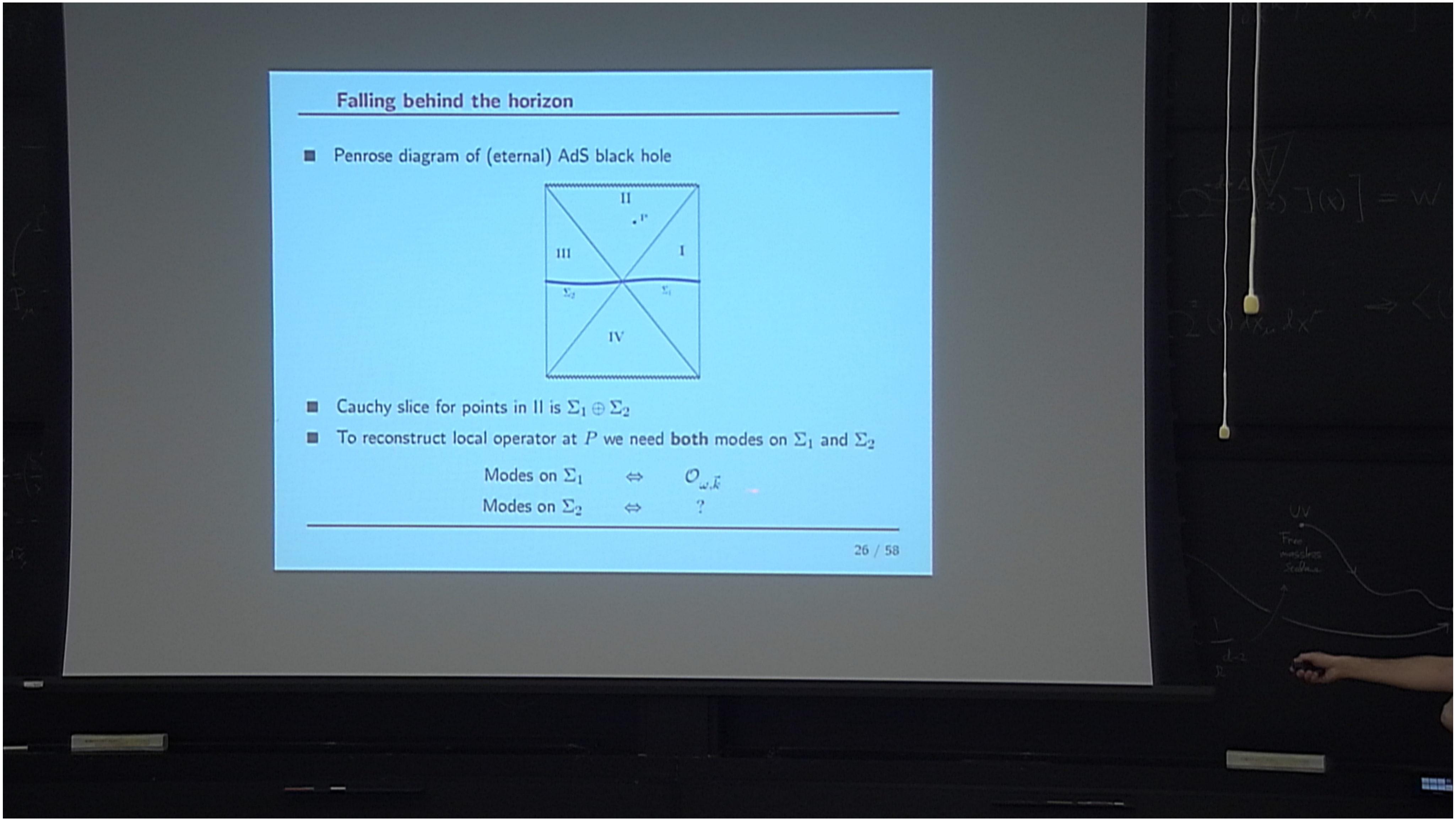
Falling behind the horizon
Penrose diagram of (eternal) AdS black hole
II
P
III
I
Σ_2
Σ_1
IV
Cauchy slice for points in II is $\Sigma_1 \oplus \Sigma_2$
To reconstruct local operator at P we need **both** modes on Σ_1 and Σ_2
Modes on Σ_1 $\Leftrightarrow$ $\mathcal{O}_{\omega,\vec{k}}$
Modes on Σ_2 $\Leftrightarrow$?
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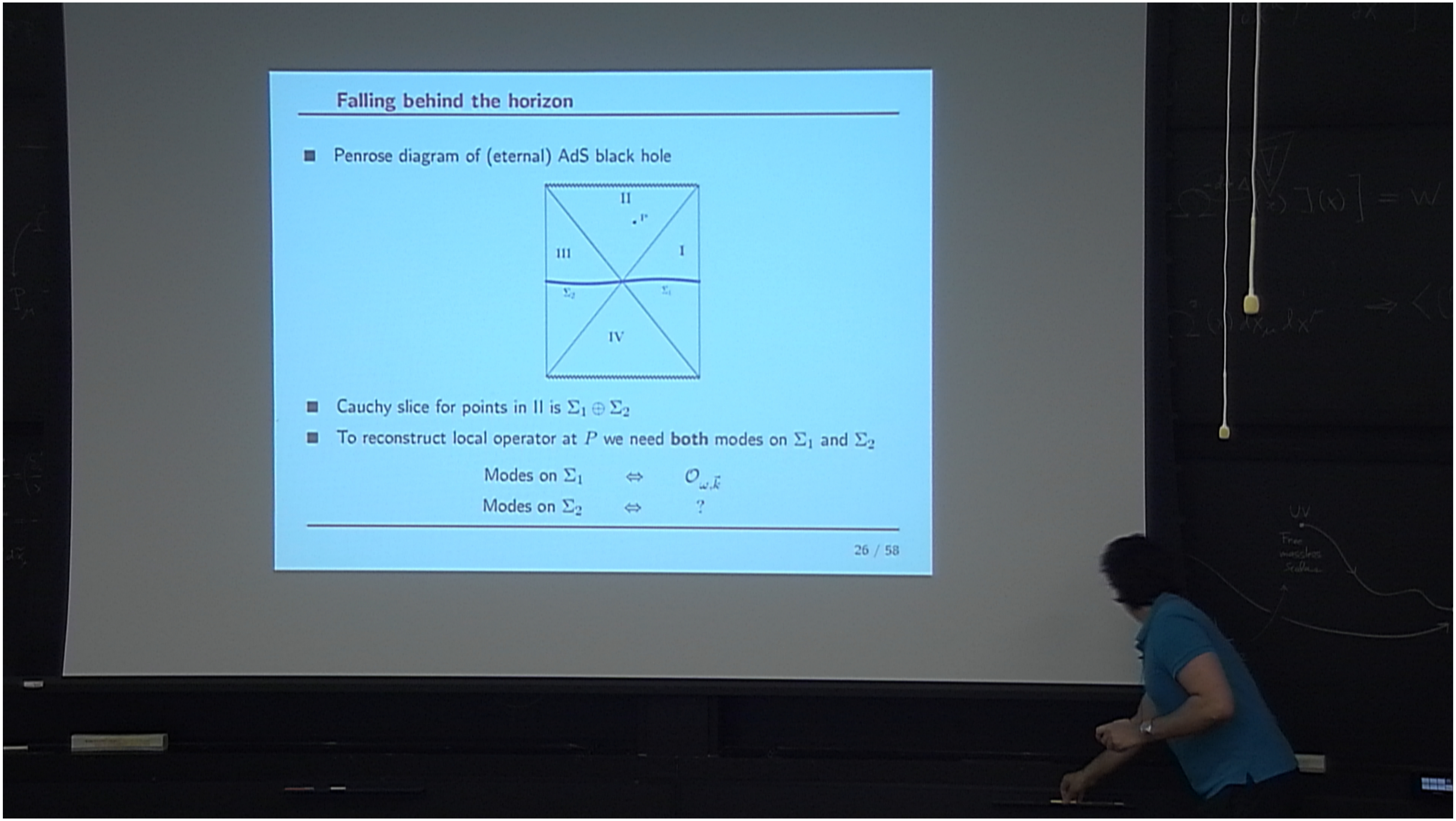
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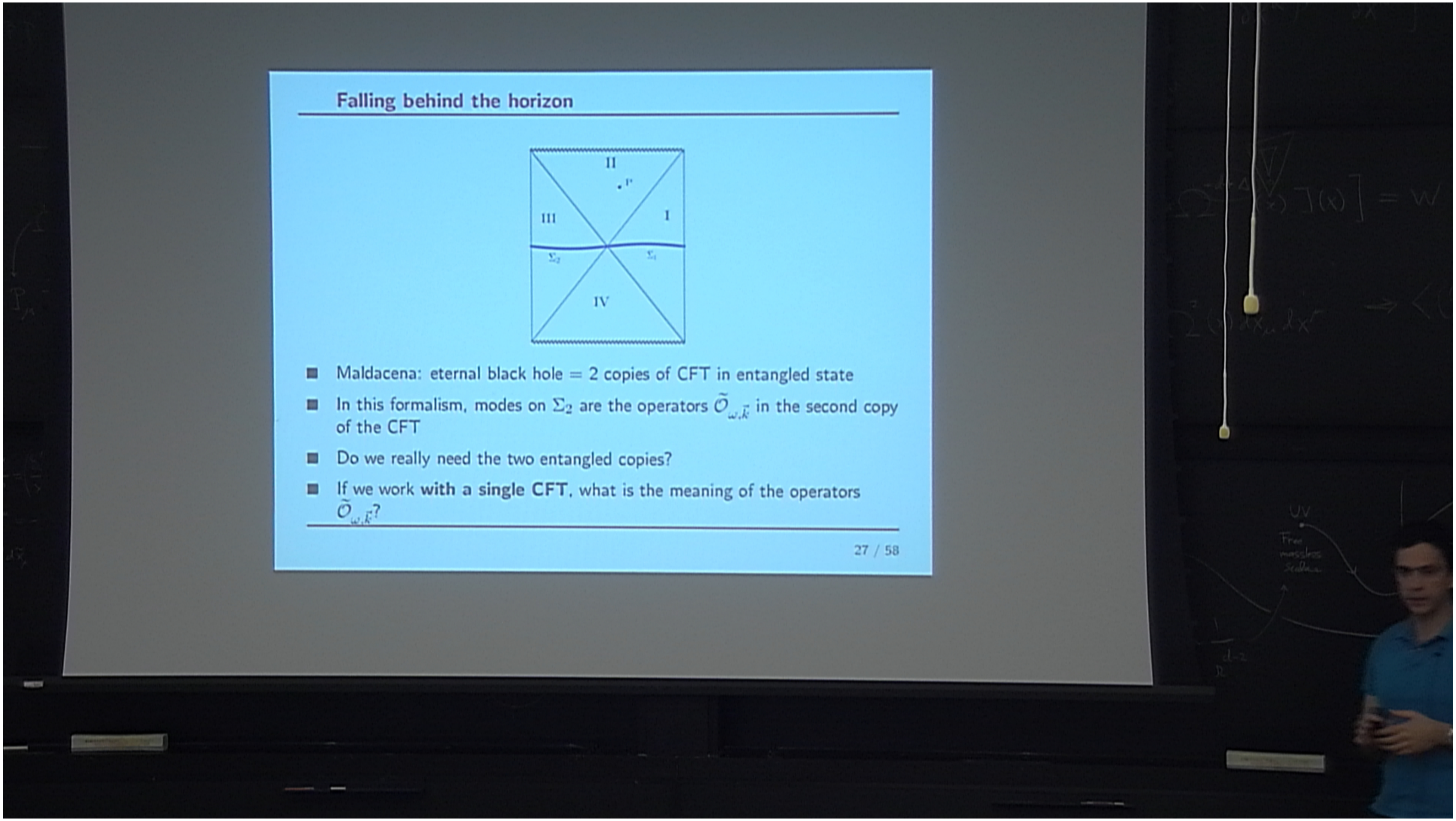

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Maldacena: eternal black hole = 2 copies of CFT in entangled state
In this formalism, modes on Σ_2 are the operators $\widetilde{\mathcal{O}}_{\omega,\vec{k}}$ in the second copy of the CFT
Do we really need the two entangled copies?
If we work with a single CFT, what is the meaning of the operators $\widetilde{\mathcal{O}}_{\omega,\vec{k}}$?
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Coarse-graining and doubling of operators

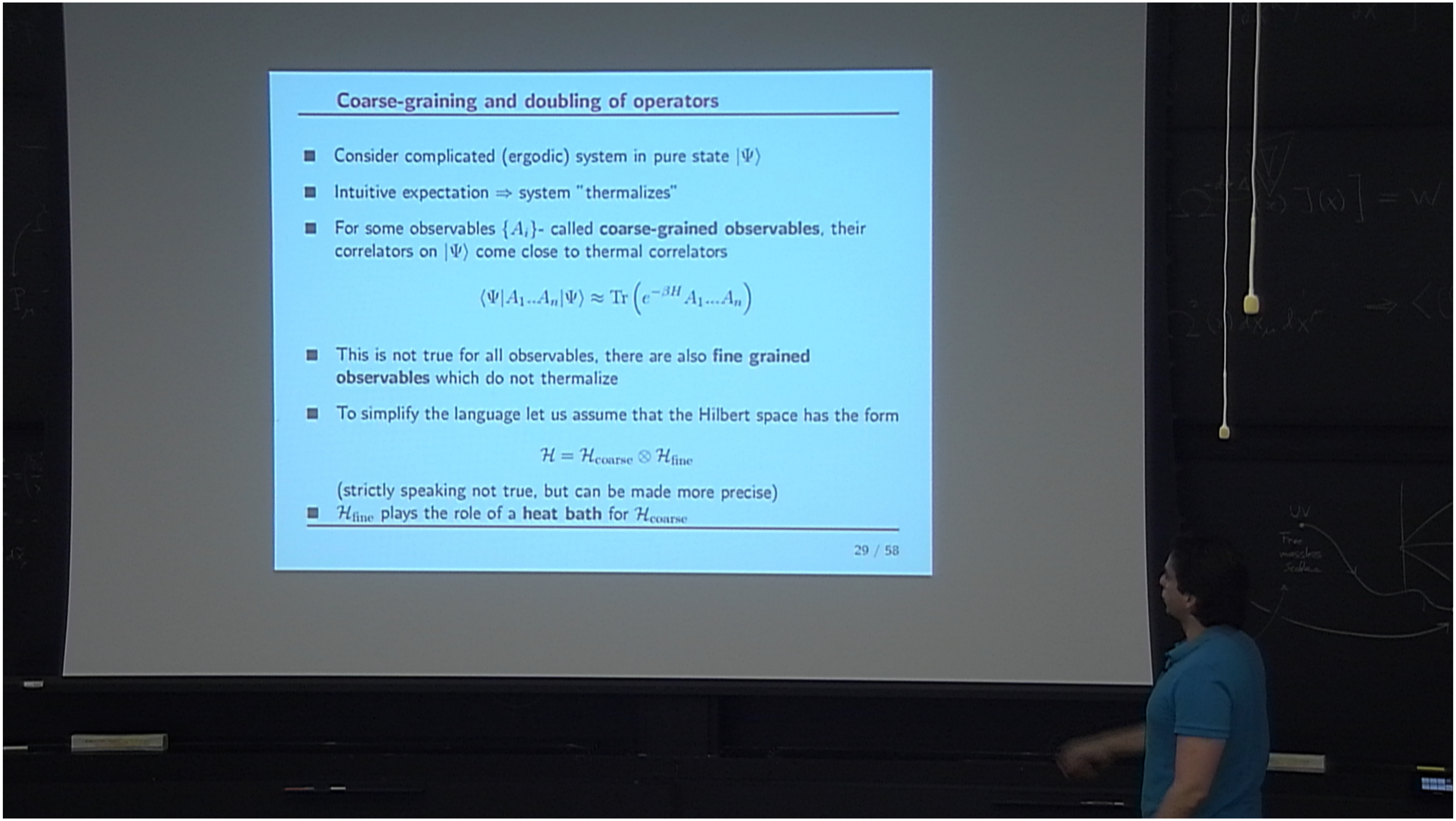
- Consider complicated (ergodic) system in pure state $|\Psi\rangle$
- Intuitive expectation $\Rightarrow$ system "thermalizes"
- For some observables $\{A_i\}$- called **coarse-grained observables**, their correlators on $|\Psi\rangle$ come close to thermal correlators

$$\langle\Psi|A_1..A_n|\Psi\rangle \approx \mathrm{Tr}\left(e^{-\beta H}A_1...A_n\right)$$

- This is not true for all observables, there are also **fine grained observables** which do not thermalize
- To simplify the language let us assume that the Hilbert space has the form

$$\mathcal{H} = \mathcal{H}_{\mathrm{coarse}} \otimes \mathcal{H}_{\mathrm{fine}}$$

(strictly speaking not true, but can be made more precise)
- $\mathcal{H}_{\mathrm{fine}}$ plays the role of a **heat bath** for $\mathcal{H}_{\mathrm{coarse}}$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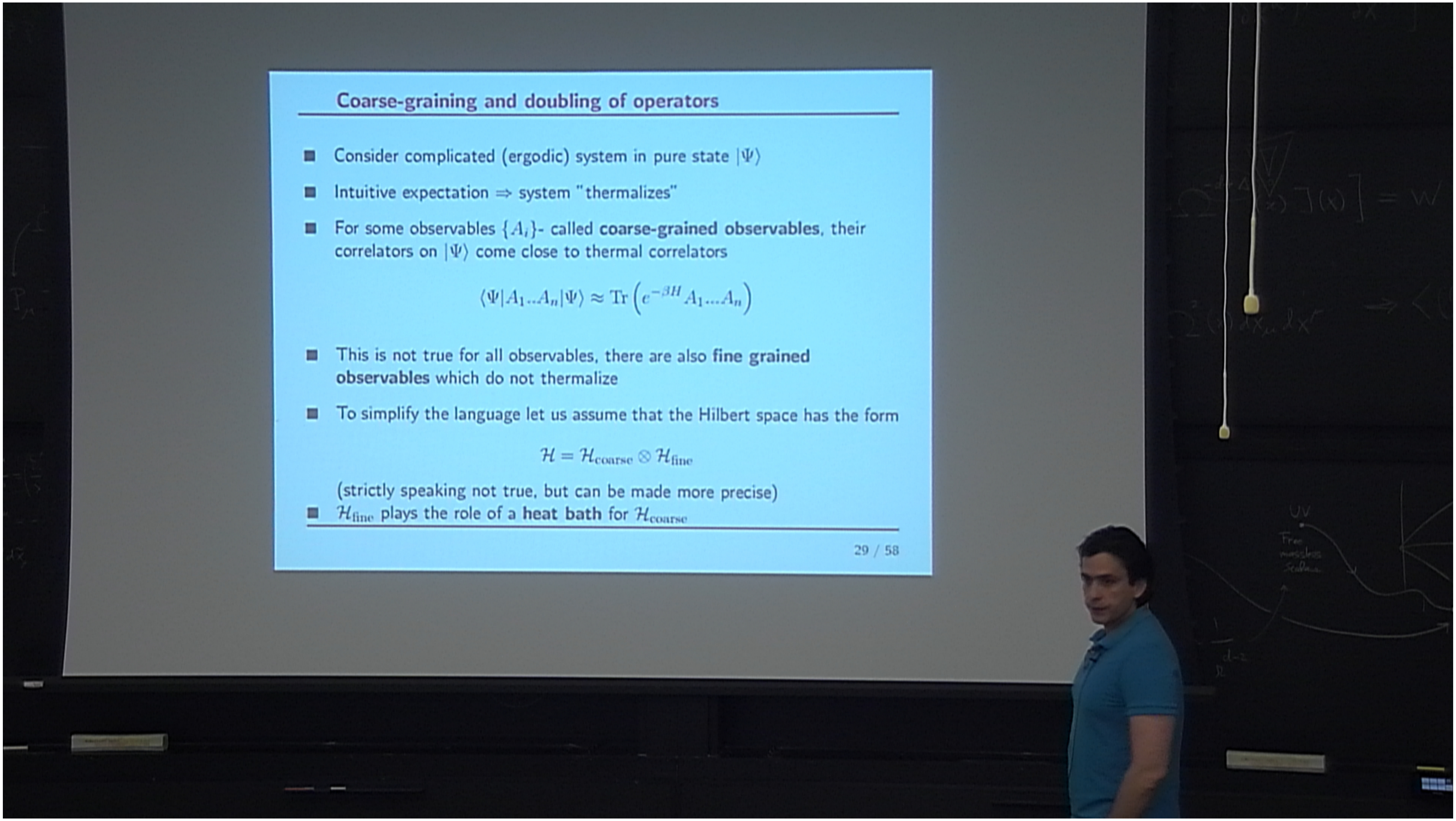
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Coarse graining and doubling of operators
Small system
Large system
(heat bath)
SMALL SUBSYSTEM IS MIRRORED IN HEAT BATH
For us the Quark-Gluon-Pasma is the heat bath
The glueball operators $\mathcal{O}_i$ are the coarse-grained observables
They are mirrored in the QGP, which leads to new operators $\tilde{\mathcal{O}}_i$
This mirroring involves the fine-degrees of freedom
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Coarse graining and doubling of operators

- Every state $|\Psi\rangle$ can be written as

$$|\Psi\rangle = \sum_{ij} c_{ij} |\Psi_i^c\rangle \otimes |\Psi_j^f\rangle$$

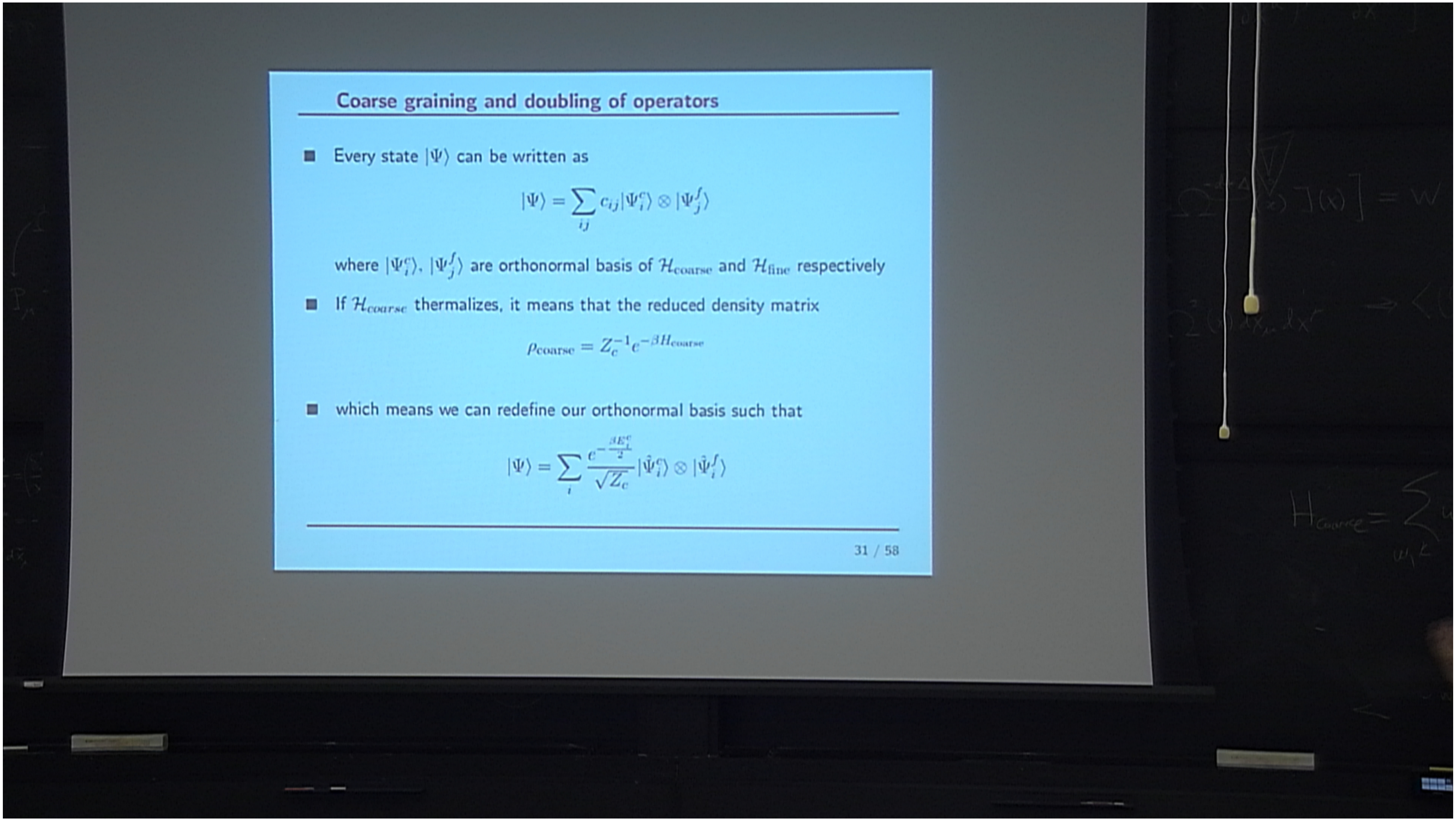
where $|\Psi_i^c\rangle$, $|\Psi_j^f\rangle$ are orthonormal basis of $\mathcal{H}_{\text{coarse}}$ and $\mathcal{H}_{\text{fine}}$ respectively

- If $\mathcal{H}_{coarse}$ thermalizes, it means that the reduced density matrix

$$\rho_{\text{coarse}} = Z_c^{-1} e^{-\beta H_{\text{coarse}}}$$

- which means we can redefine our orthonormal basis such that

$$|\Psi\rangle = \sum_i \frac{e^{-\frac{\beta E_i^c}{2}}}{\sqrt{Z_c}} |\hat{\Psi}_i^c\rangle \otimes |\hat{\Psi}_i^f\rangle$$


Coarse graining and doubling of operators
Every state $|\Psi\rangle$ can be written as
$$|\Psi\rangle = \sum_{ij} c_{ij} |\Psi_i^c\rangle \otimes |\Psi_j^f\rangle$$
where $|\Psi_i^c\rangle$, $|\Psi_j^f\rangle$ are orthonormal basis of $\mathcal{H}_{\rm coarse}$ and $\mathcal{H}_{\rm fine}$ respectively
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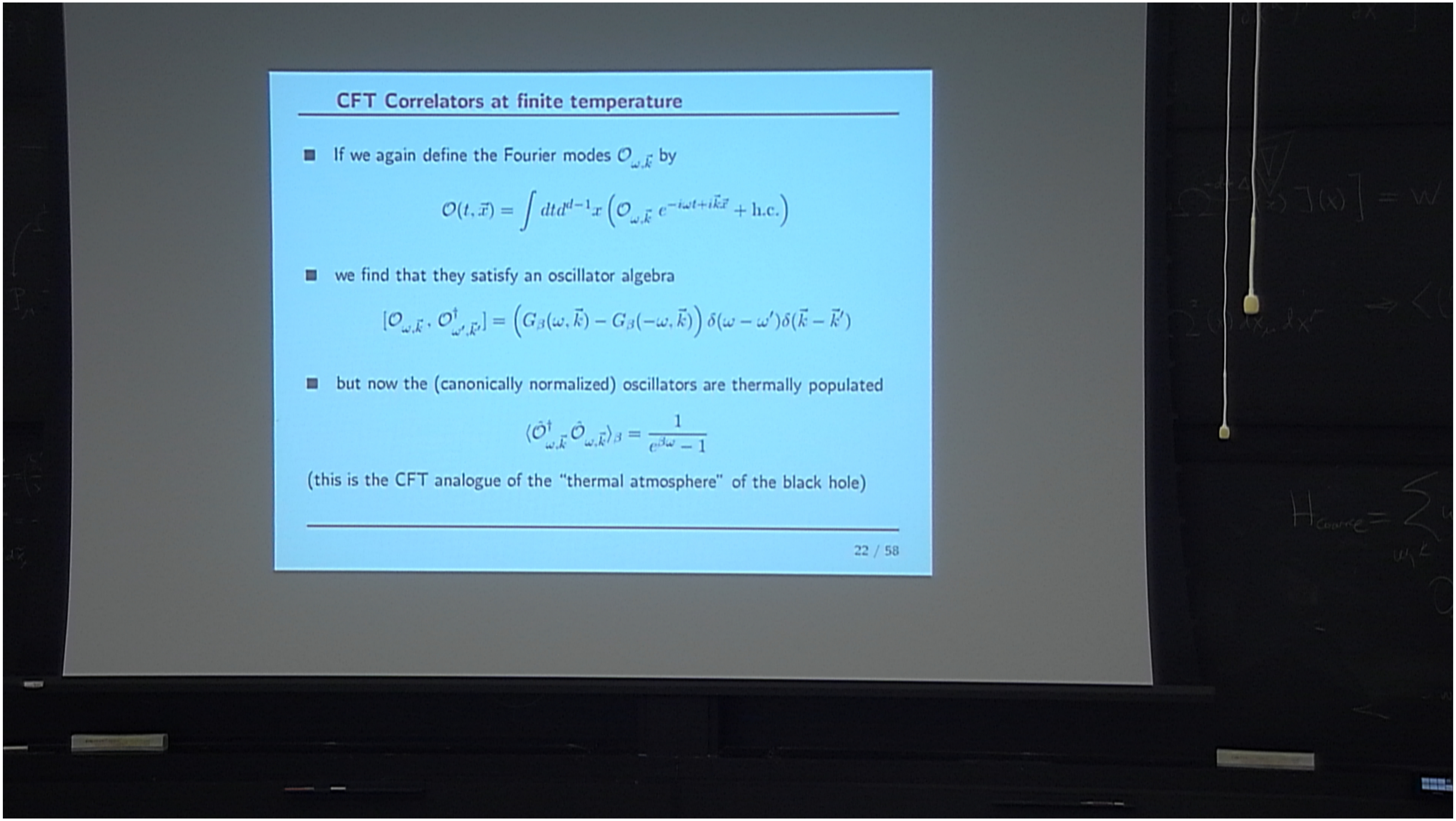
CFT Correlators at finite temperature
If we again define the Fourier modes $\mathcal{O}_{\omega,\vec{k}}$ by
$$\mathcal{O}(t,\vec{x}) = \int dt d^{d-1}x \left(\mathcal{O}_{\omega,\vec{k}}\, e^{-i\omega t + i\vec{k}\vec{x}} + \text{h.c.} \right)$$
we find that they satisfy an oscillator algebra
$$[\mathcal{O}_{\omega,\vec{k}}, \mathcal{O}^{\dagger}_{\omega',\vec{k}'}] = \left(G_{\beta}(\omega,\vec{k}) - G_{\beta}(-\omega,\vec{k}) \right) \delta(\omega-\omega')\delta(\vec{k}-\vec{k}')$$
but now the (canonically normalized) oscillators are thermally populated
$$\langle \hat{\mathcal{O}}^{\dagger}_{\omega,\vec{k}} \hat{\mathcal{O}}_{\omega,\vec{k}} \rangle_{\beta} = \frac{1}{e^{\beta\omega}-1}$$
(this is the CFT analogue of the "thermal atmosphere" of the black hole)
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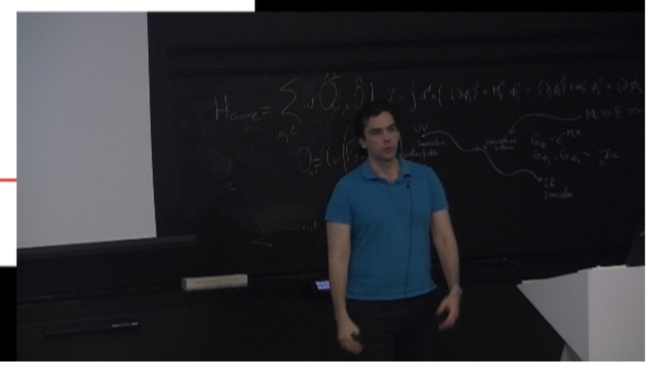
- Consider a **coarse-grained** operator acting on $\mathcal{H}_{\text{coarse}}$ as

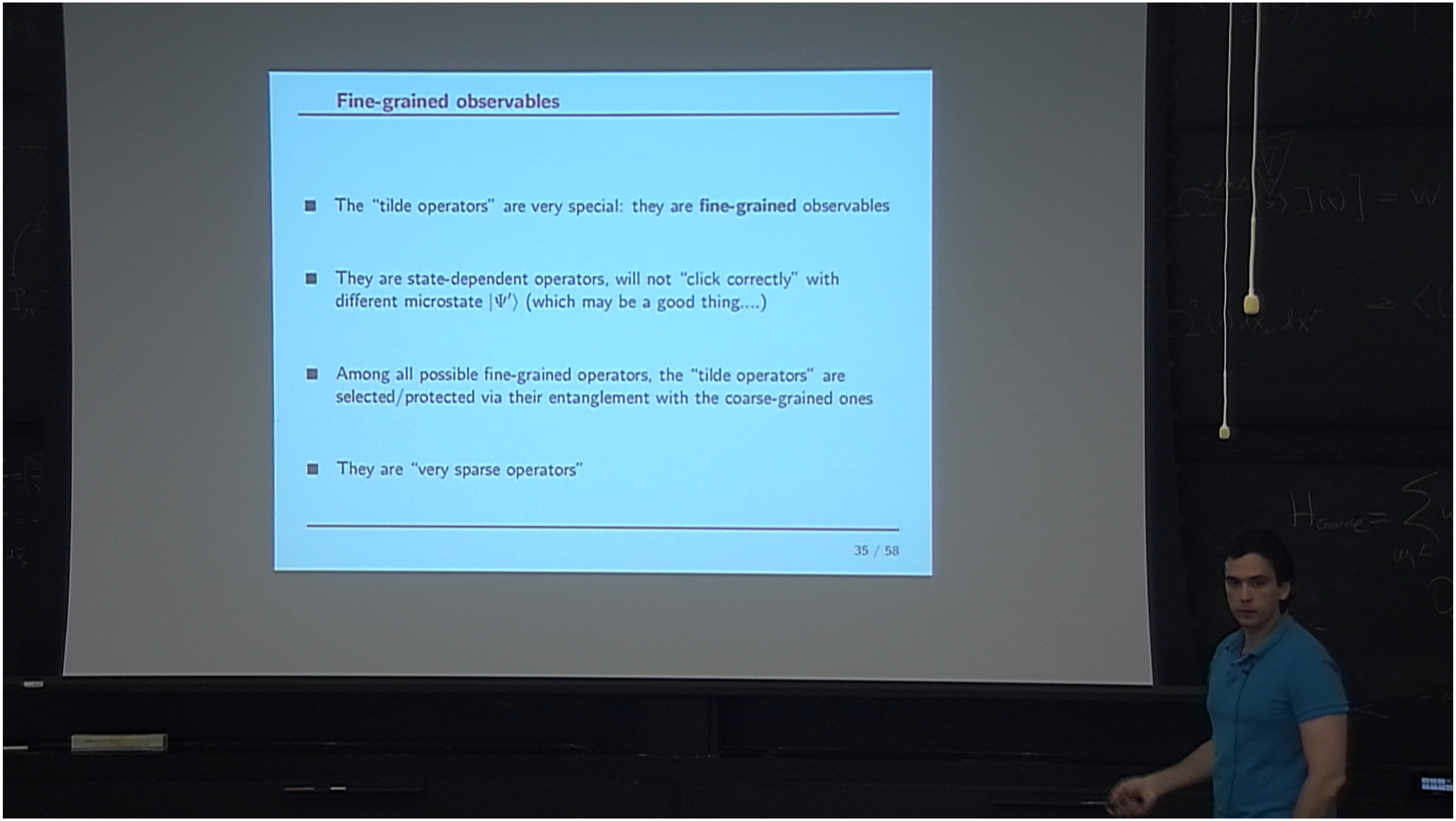
$$A = \sum_{ij} a_{ij} |\hat{\Psi}_i^c\rangle \otimes \langle\hat{\Psi}_j^c|$$

- Then we **define** a new operator

$$\widetilde{A} = \sum_{ij} a_{ij}^* |\hat{\Psi}_i^f\rangle \otimes |\hat{\Psi}_j^f\rangle$$

acting on the fine-grained Hilbert space.




Fine-grained observables
The "tilde operators" are very special: they are fine-grained observables
They are state-dependent operators, will not "click correctly" with different microstate |Ψ′⟩ (which may be a good thing....)
Among all possible fine-grained operators, the "tilde operators" are selected/protected via their entanglement with the coarse-grained ones
They are "very sparse operators"
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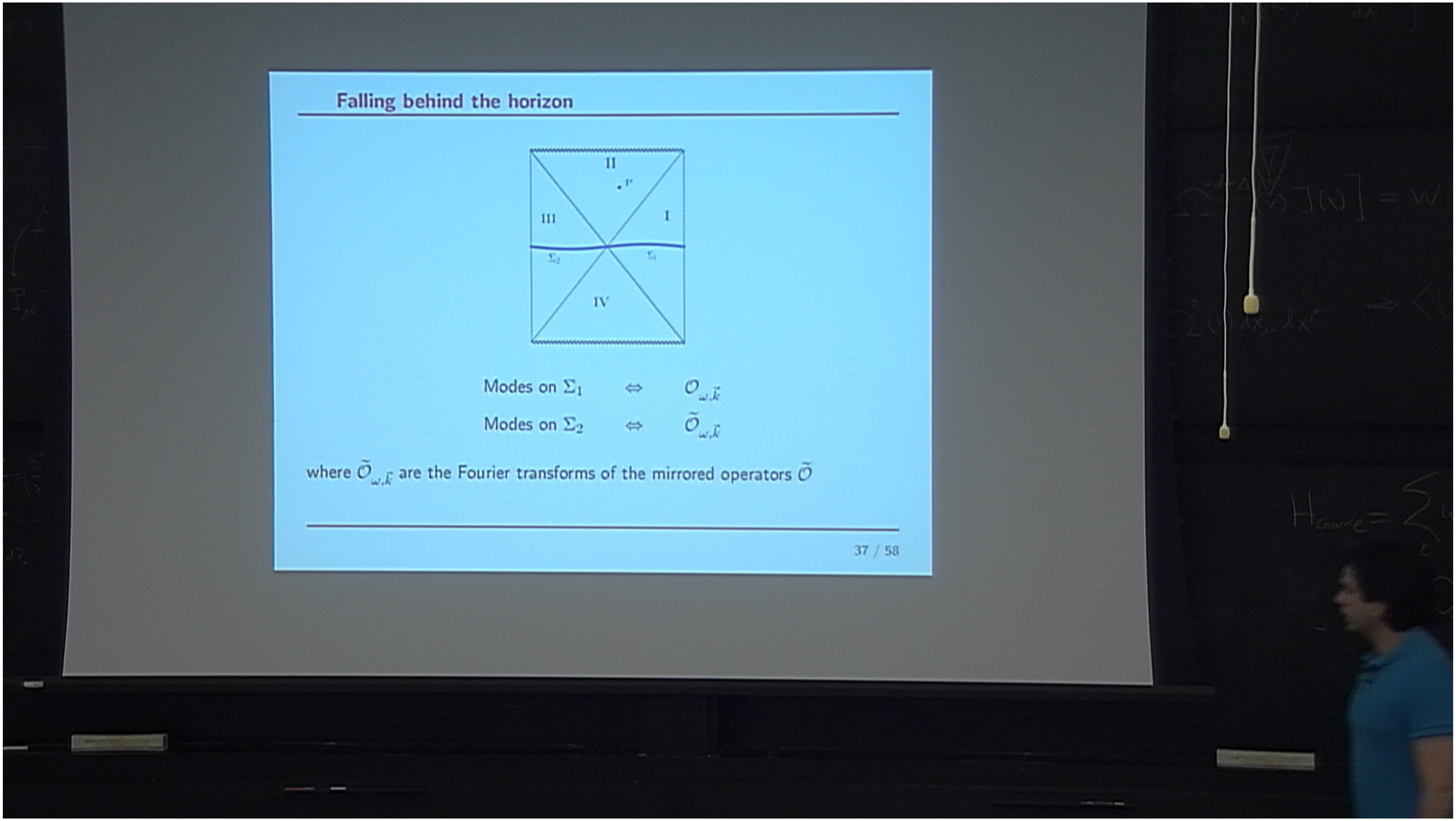
Falling behind the horizon
II
III
I
Σ2
Σ1
IV
Modes on Σ_1 ⇔ $\mathcal{O}_{\omega,\vec{k}}$
Modes on Σ_2 ⇔ $\widetilde{\mathcal{O}}_{\omega,\vec{k}}$
where $\widetilde{\mathcal{O}}_{\omega,\vec{k}}$ are the Fourier transforms of the mirrored operators $\widetilde{\mathcal{O}}$
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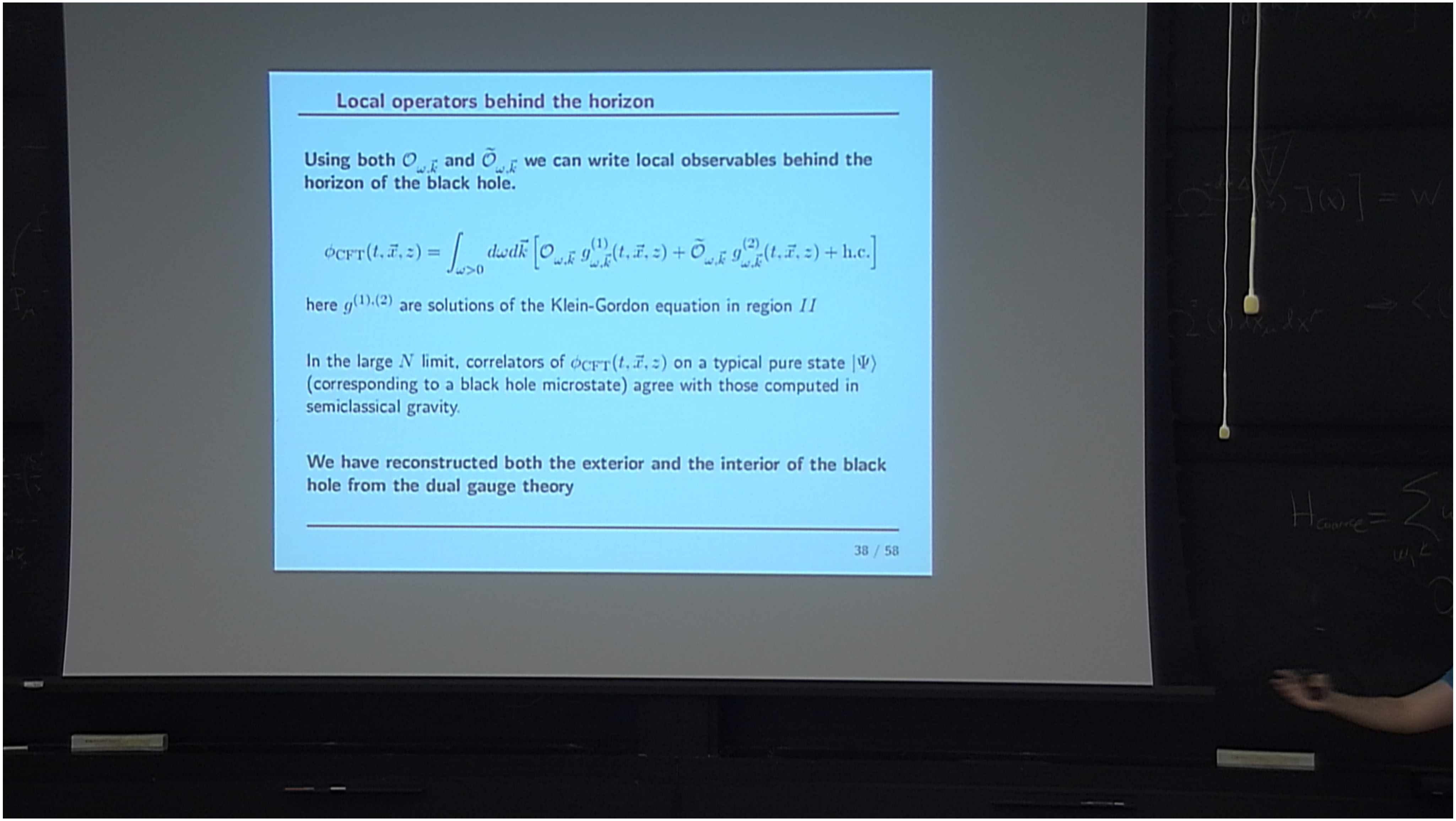
Local operators behind the horizon
Using both $\mathcal{O}_{\omega,\vec{k}}$ and $\widetilde{\mathcal{O}}_{\omega,\vec{k}}$ we can write local observables behind the horizon of the black hole.
$$\phi_{\rm CFT}(t,\vec{x},z) = \int_{\omega>0} d\omega d\vec{k} \left[\mathcal{O}_{\omega,\vec{k}}\, g^{(1)}_{\omega,\vec{k}}(t,\vec{x},z) + \widetilde{\mathcal{O}}_{\omega,\vec{k}}\, g^{(2)}_{\omega,\vec{k}}(t,\vec{x},z) + \text{h.c.}\right]$$
here $g^{(1),(2)}$ are solutions of the Klein-Gordon equation in region II
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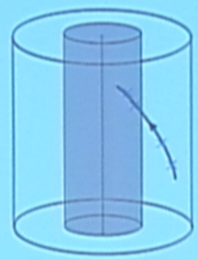
Fate of the infalling observer

Using the operators ϕ_{CFT} we can reconstruct the experiments of the infalling observer



MAIN CONCLUSION: For a big AdS black hole, an infalling semi-classical observer does not notice anything special when crossing the horizon

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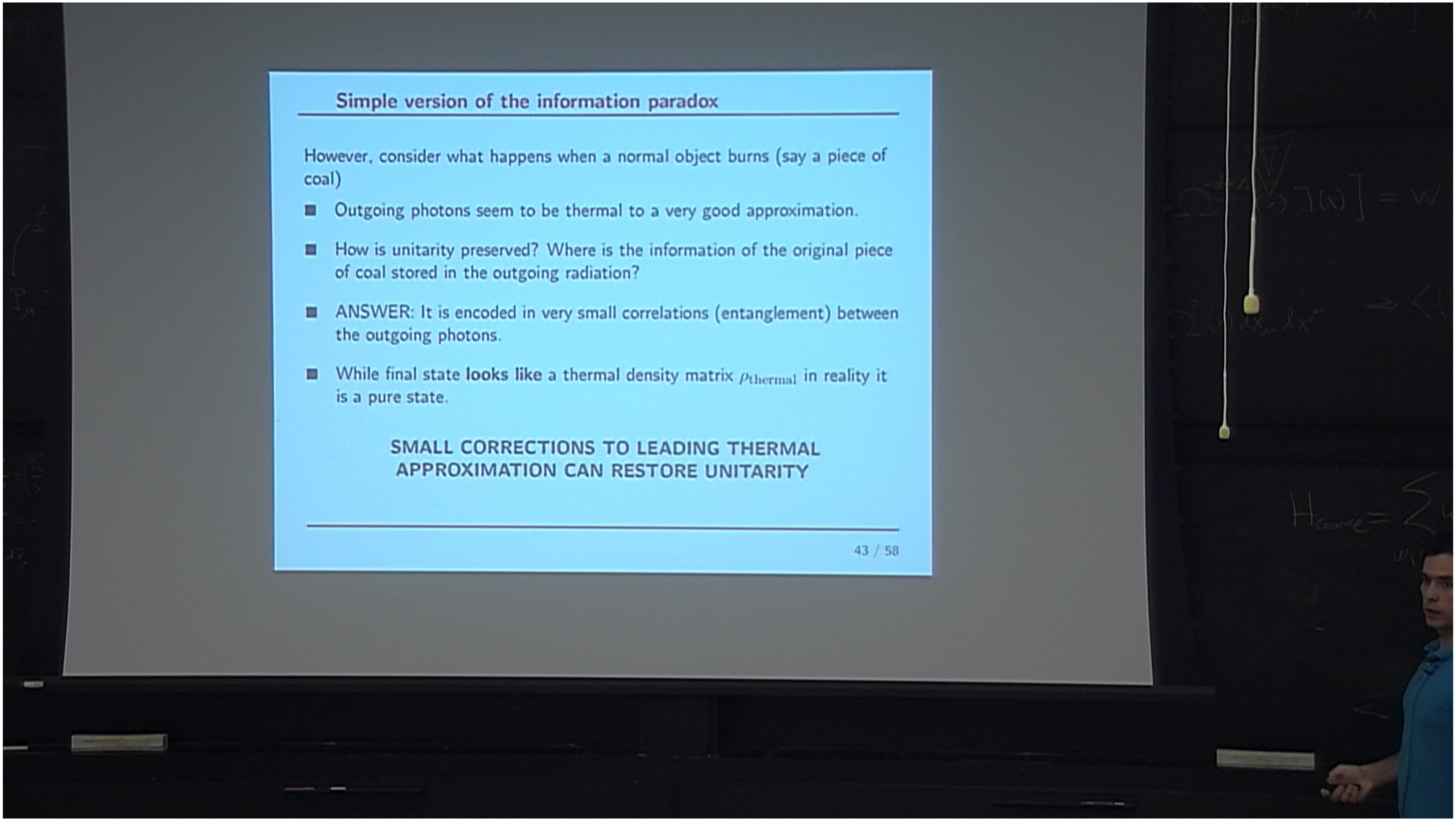
Simple version of the information paradox
However, consider what happens when a normal object burns (say a piece of coal)
Outgoing photons seem to be thermal to a very good approximation.
How is unitarity preserved? Where is the information of the original piece of coal stored in the outgoing radiation?
ANSWER: It is encoded in very small correlations (entanglement) between the outgoing photons.
While final state looks like a thermal density matrix $\rho_{\rm thermal}$ in reality it is a pure state.
SMALL CORRECTIONS TO LEADING THERMAL APPROXIMATION CAN RESTORE UNITARITY
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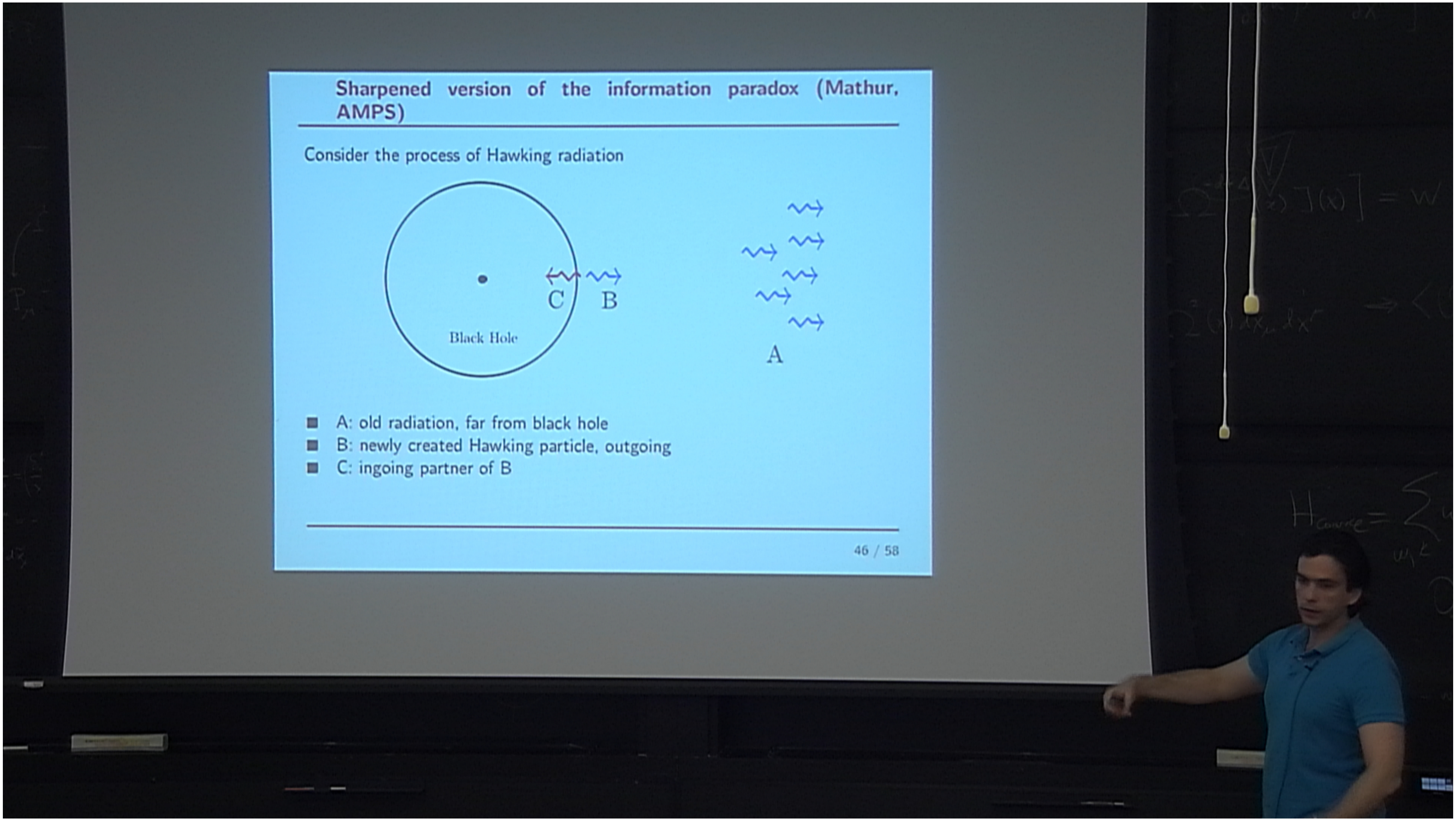
Sharpened version of the information paradox (Mathur, AMPS)
Consider the process of Hawking radiation
C
B
Black Hole
A
A: old radiation, far from black hole
B: newly created Hawking particle, outgoing
C: ingoing partner of B
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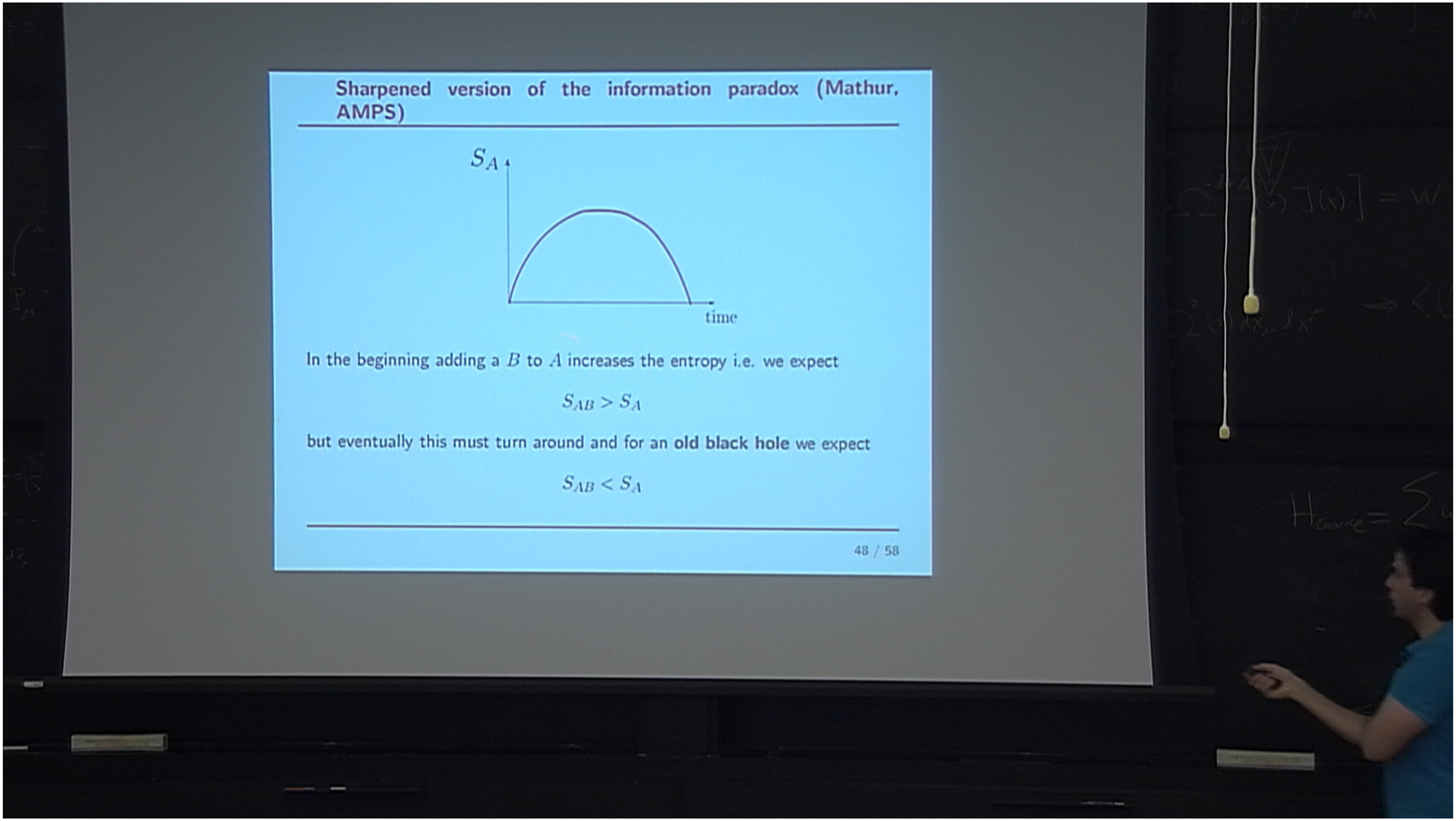
Sharpened version of the information paradox (Mathur, AMPS)
S_A
time
In the beginning adding a B to A increases the entropy i.e. we expect
$S_{AB} > S_A$
but eventually this must turn around and for an **old black hole** we expect
$S_{AB} < S_A$
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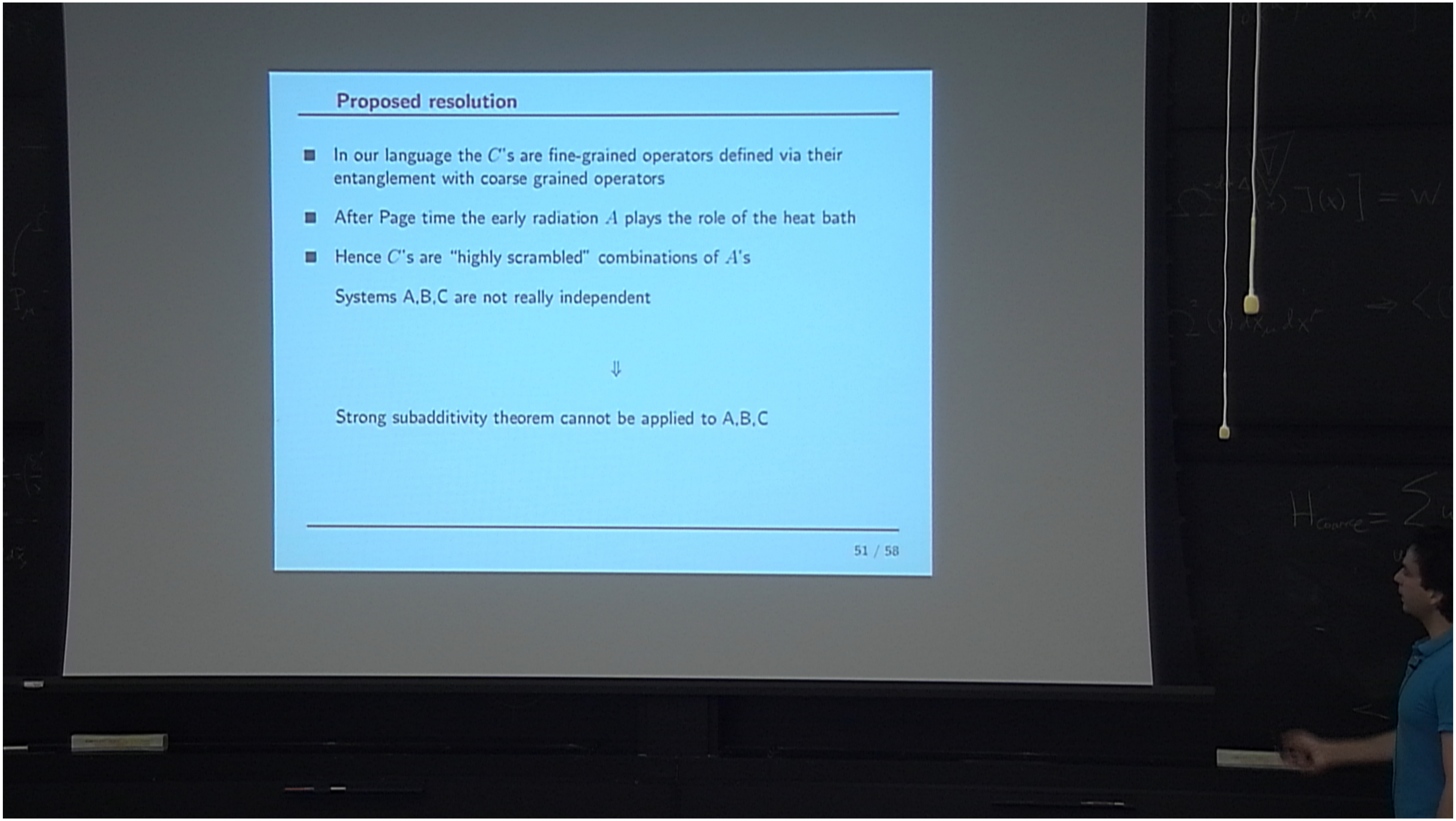
Proposed resolution
In our language the C's are fine-grained operators defined via their entanglement with coarse grained operators
After Page time the early radiation A plays the role of the heat bath
Hence C's are "highly scrambled" combinations of A's
Systems A,B,C are not really independent
⇓
Strong subadditivity theorem cannot be applied to A,B,C
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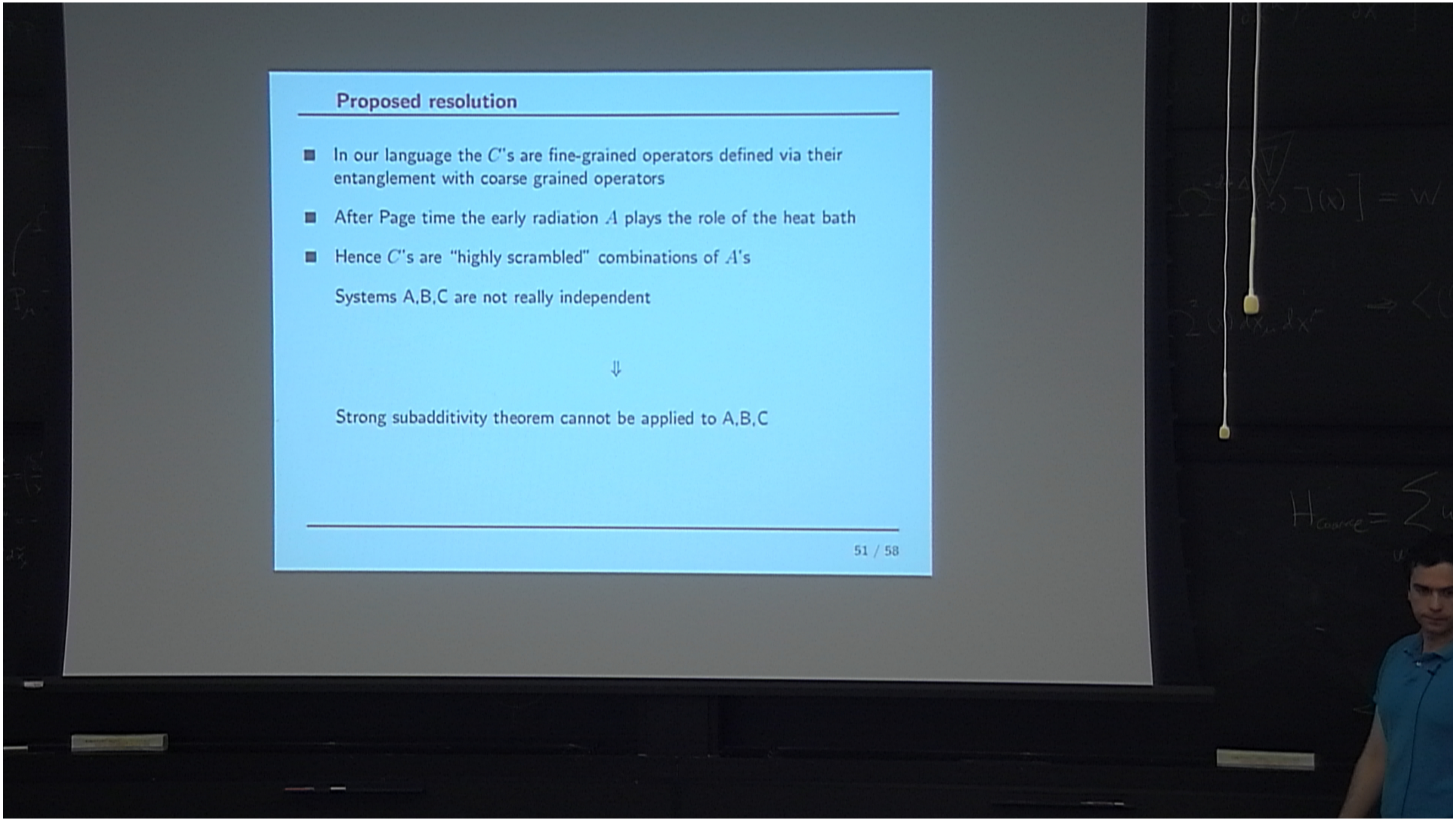
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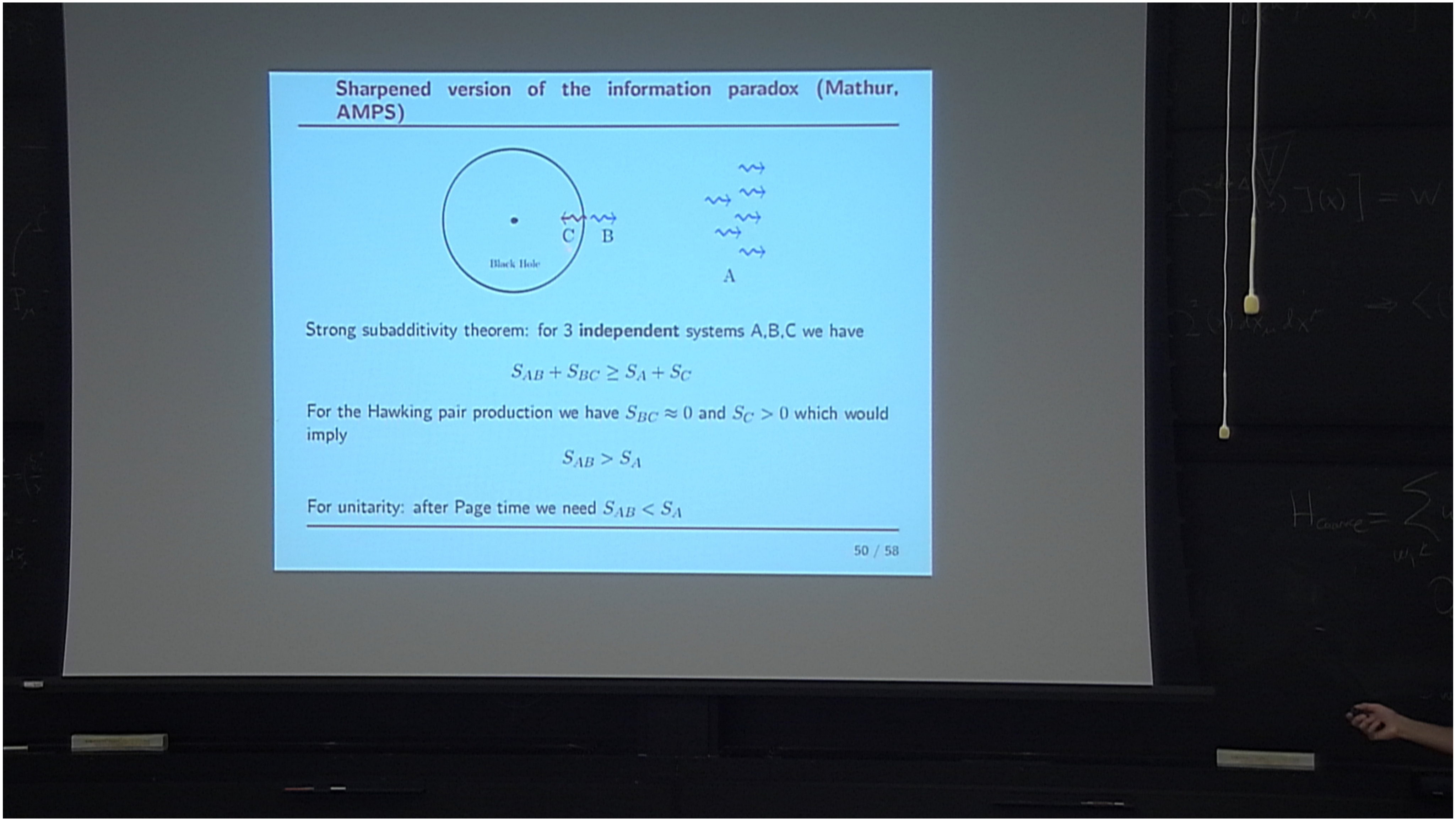
Sharpened version of the information paradox (Mathur, AMPS)
C
B
A
Black Hole
Strong subadditivity theorem: for 3 **independent** systems A,B,C we have
$$S_{AB} + S_{BC} \geq S_A + S_C$$
For the Hawking pair production we have $S_{BC} \approx 0$ and $S_C > 0$ which would imply
$$S_{AB} > S_A$$
For unitarity: after Page time we need $S_{AB} < S_A$
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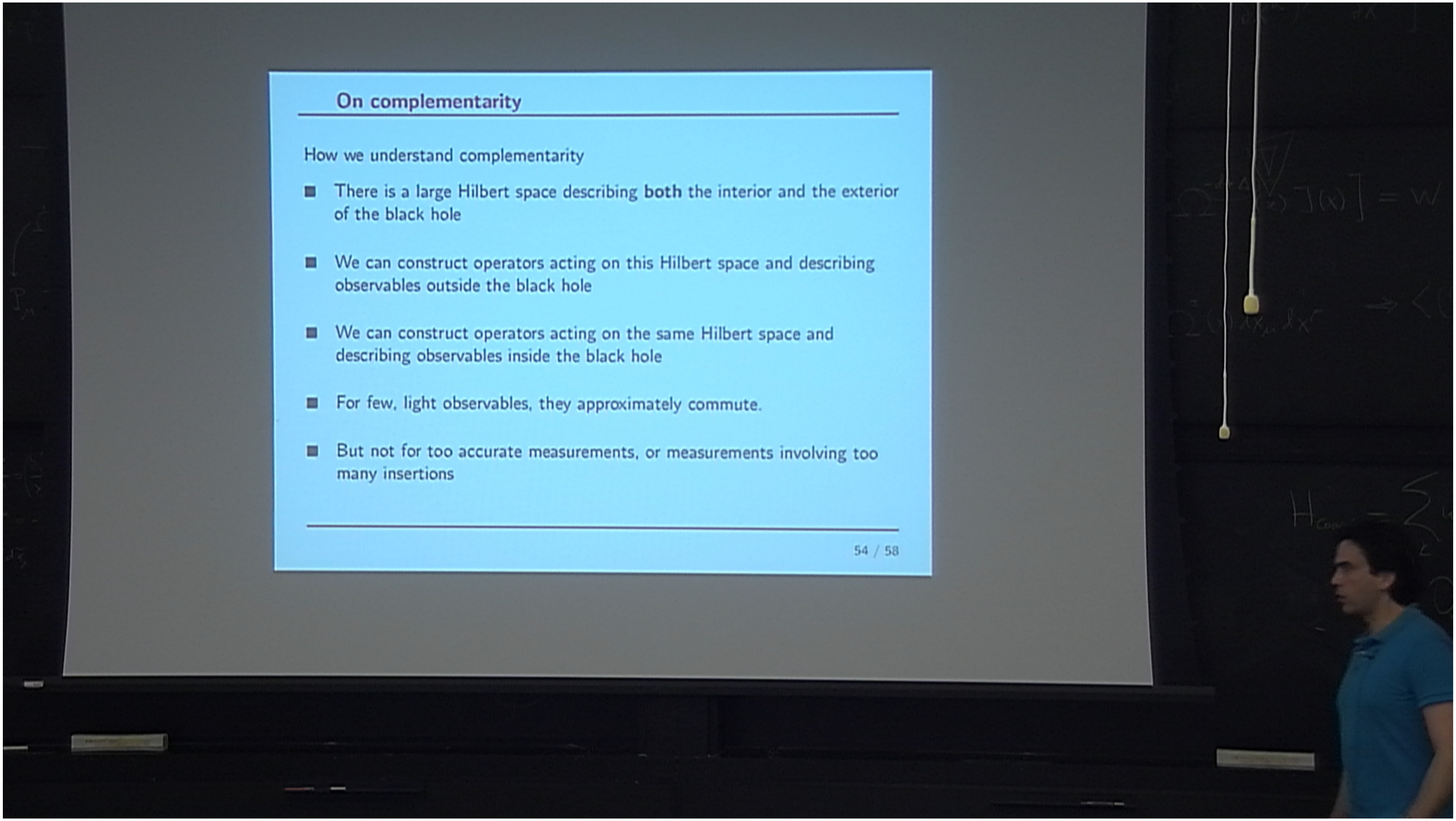
On complementarity
How we understand complementarity
There is a large Hilbert space describing both the interior and the exterior of the black hole
We can construct operators acting on this Hilbert space and describing observables outside the black hole
We can construct operators acting on the same Hilbert space and describing observables inside the black hole
For few, light observables, they approximately commute.
But not for too accurate measurements, or measurements involving too many insertions
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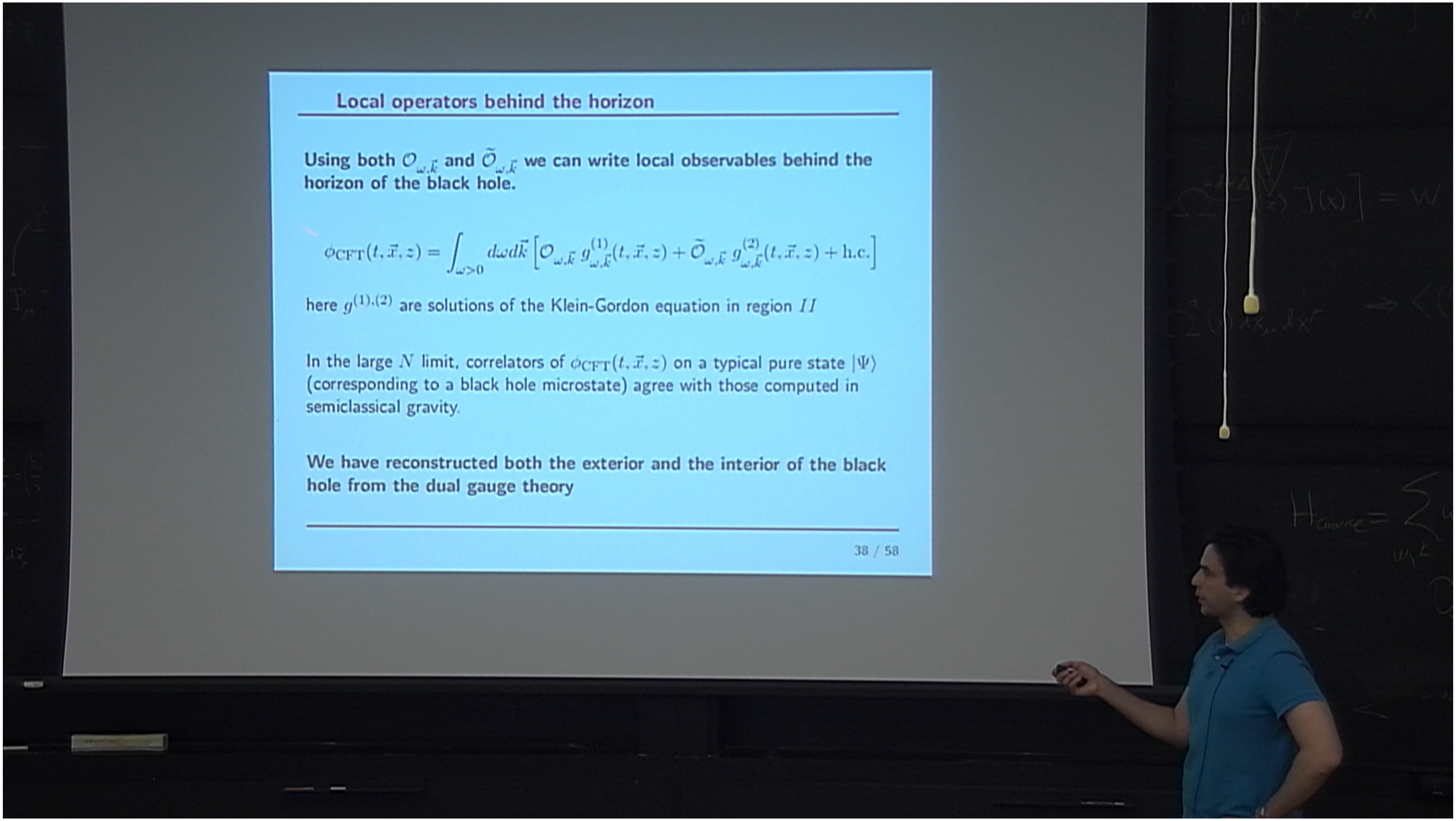
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