Title: Topological insulator of bosons in 3d via dyon condensation and the statistical Witten effect.

Date: Apr 10, 2013 03:30 PM

URL: http://pirsa.org/13040117

Abstract: In
this talk, I will construct a symmetry protected topological phase of bosons in
3d with particle number conservation and time reversal symmetries, which is the
direct bosonic analogue of the familiar electron topological insulator. The
construction employs a parton decomposition of bosons, followed by condensation
of parton-monopole composites. The surface of the resulting state supports a
gapped symmetry respecting phase with intrinsic toric code topological order
where both e and m anyons carry charge 1=2.

It
is well-known that one signature of the 3d electron topological insulator is
the Witten

eect:
if the system is coupled to a compact electromagnetic gauge eld, a monopole in

the
bulk acquires a half-odd-integer polarization charge. I will discuss the
corresponding

phenomenon
for the constructed topological insulator of bosons: a monopole can remain

electrically
neutral, but its statistics are transmuted from bosonic to fermionic. This
\sta-

tistical
Witten eect" guarantees that the surface is either gapless, symmetry
broken or

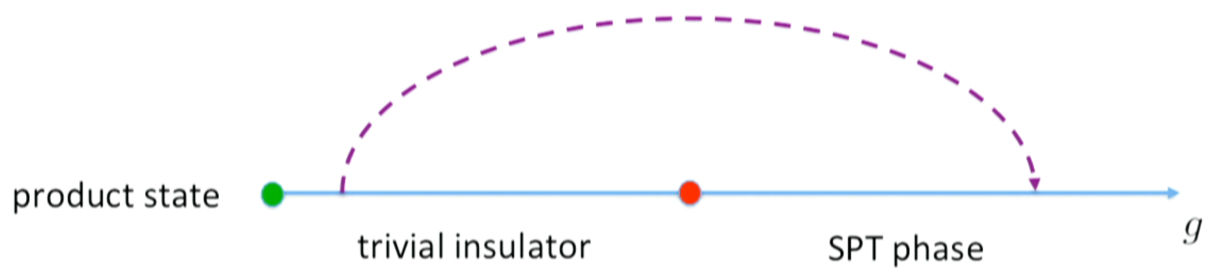
carries

an intrinsic topological order.

Symmetry Protected Topological Phases (SPTs)

- Gapped phases, protected by symmetry group *G*
- In the absence of symmetry – connected to vacuum
  no "intrinsic" topological order:
  - no degeneracy on a torus
  - no excitations with fractional statistics/quantum numbers
  - no long-range entanglement

  (unlike e.g. Fractional Quantum Hall states)



X. Chen, Z.-C. Gu, X.-G. Wen (2011)

SPT's: life on the edge

- SPT's possess protected edge states

Bulk	Edge
1d	gapless
2d	gapless or spontaneously breaks symmetry
3d	gapless, spontaneously breaks symmetry or supports intrinsic 2d topological order

- In all cases, the properties of an edge of a d-dimensional SPT cannot be realized in a strictly d-1 dimensional system.

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SPT examples

- 1d: Spin chain with S = 1 (Haldane phase)

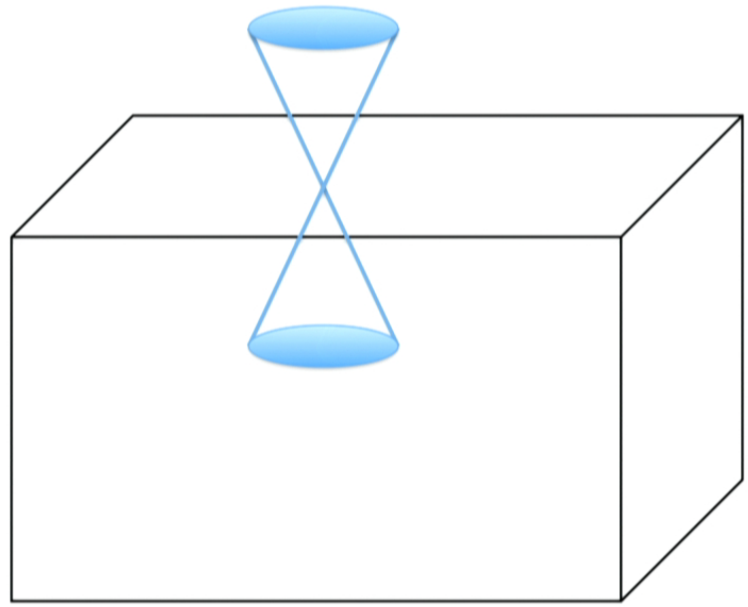
- gapped bulk, excitations carry S = 1
- edge has emergent gapless S = 1/2 excitations

- SPT phase protected by $SO(3)_{spin}$ or time-reversal symmetry

Examples

- 3d Electron topological insulator

  - phase of matter protected by particle number conservation and time-reversal symmetries $U(1) \ltimes Z_2^T$
  - exist already for non-interacting electrons
  - gapped bulk
  - surface: single Dirac cone



L. Fu, C.L. Kane, E.J. Mele (2007)

Recent discoveries in SPT-land

- All phases of non-interacting fermions classified

  A.P.Schnyder, S. Ryu, A. Furusaki and A.W.W.Ludwig (2008)
  A. Kitaev (2009)

- All phases of bosons and fermions in 1d classified using MPS

  L. Fidkowski and A. Kitaev (2010)
  A. M. Turner, F. Pollman and E. Berg (2011)
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  N. Schuch, D. Perez-Garcia and I. Cirac (2011)

- Cohomology classification proposed for all boson phases (and fermion phases with $T^2 = 1$) in any dimension.

  X. Chen, Z.-C. Gu, Z.-X. Liu, X.-G. Wen (2011)
  Z.-C. Gu, X.-G. Wen (2012)

  - exactly solvable Hamiltonians available for each phase
  - edge properties not immediate
  - ways to distinguish phases in the bulk?
  - continuum field theory?

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Non-cohomology approaches to SPTs

- d = 2: multi-component Abelian Chern-Simons theory

  Y. M. Lu, A. Vishwanath (2012)

  - similar to hierarchical construction of fractional quantum Hall
  - reproduces cohomology classification
  - immediate access to the edge properties

- d = 3: surface phases of certain bosonic SPTs inferred

  A. Vishwanath and T. Senthil (2012)

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  - how to distinguish phases in the bulk?

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This talk

- New method for obtaining phases of matter (including SPTs) using parton construction and dyon condensation.
- Construct an SPT phase of bosons with $U(1) \ltimes Z_2^T$ symmetry
  - direct analogue of electron topological insulator.

$$U(1): \quad B \to e^{i\alpha} B, \qquad \mathcal{T}: B \to B$$

- Construction yields the surface properties.
- Can distinguish boson TI in the bulk – "Statistical Witten effect."
  - ensures that the surface is gapless, symmetry broken or carries intrinsic topological order.

Plan

- Introduction to symmetry protected topological phases
- Review of electron topological insulators in 3d,
  the θ-angle and the Witten effect
- Topological insulators of bosons in 3d and the statistical Witten effect
- Bulk-boundary "correspondence"
- Parton construction of the bosonic topological insulator

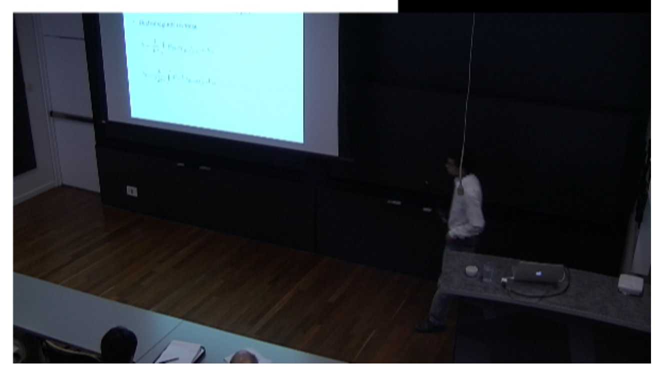
The θ-angle

- 3d insulator of fermions with no intrinsic topological order - excitations are gapped fermions with charge 1.

- Electromagnetic response

$$S = \frac{1}{4e^2} \int d^3x d\tau F_{\mu\nu} F_{\mu\nu} + S_\theta$$

$$S_\theta = \frac{i\theta}{32\pi^2} \int d^3x d\tau \epsilon_{\mu\nu\lambda\sigma} F_{\mu\nu} F_{\lambda\sigma}$$



The θ-angle

$$S_\theta = \frac{i\theta}{32\pi^2}\int d^3x d\tau \epsilon_{\mu\nu\lambda\sigma}F_{\mu\nu}F_{\lambda\sigma}$$

- On a space-time torus T^4, with smooth EM fields

$$S_\theta = i\theta n, \quad n \ - \ \text{integer}, \qquad Z = e^{-S}$$

$$\theta \sim \theta + 2\pi$$

- $\mathcal{T} : \theta \to -\theta$

Time-reversal invariant points: $\theta = 0$ - trivial insulator

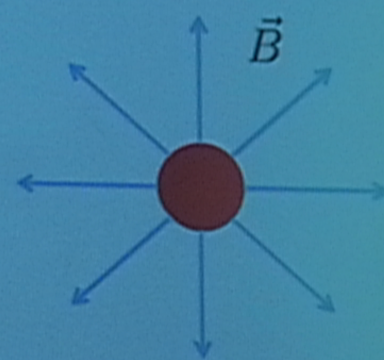
$\theta = \pi$ - electron TI

X. L. Qi, T. L. Hughes and S.-C. Zhang (2008)

The Witten effect

$$S_\theta = \frac{i\theta}{32\pi^2}\int d^3x d\tau \epsilon_{\mu\nu\lambda\sigma}F_{\mu\nu}F_{\lambda\sigma}$$

- Monopoles with flux $2\pi m$
  - polarization charge $q = n + \frac{\theta m}{2\pi}$



  - allowed valued of charge periodic under

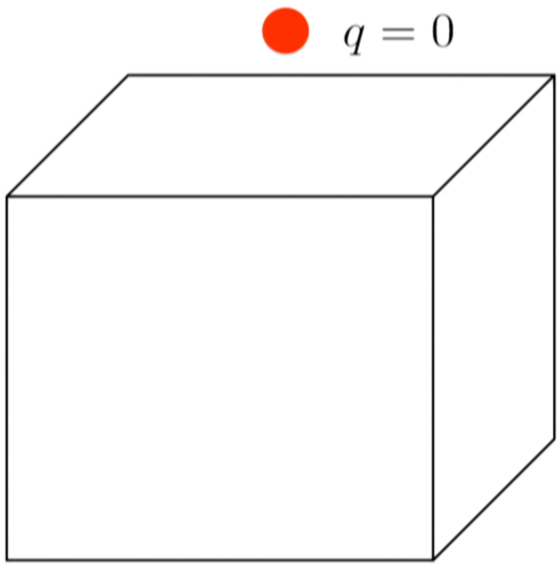
    $\theta \to \theta + 2\pi$

  - single monopoles in electron TI have

    $q = n + 1/2$

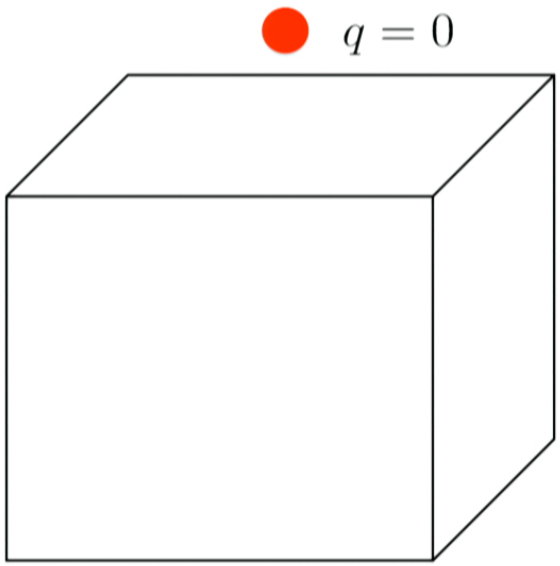
E. Witten (1979)
X. L. Qi, T. L. Hughes and S.-C. Zhang (2008)
G. Rosenberg and M. Franz (2010)

Bulk-boundary "correspondence"



- Charge must appear on the surface as the monopole tunnels through.
  - a strictly 2d system with same properties as the surface cannot exist.

Bulk-boundary "correspondence"



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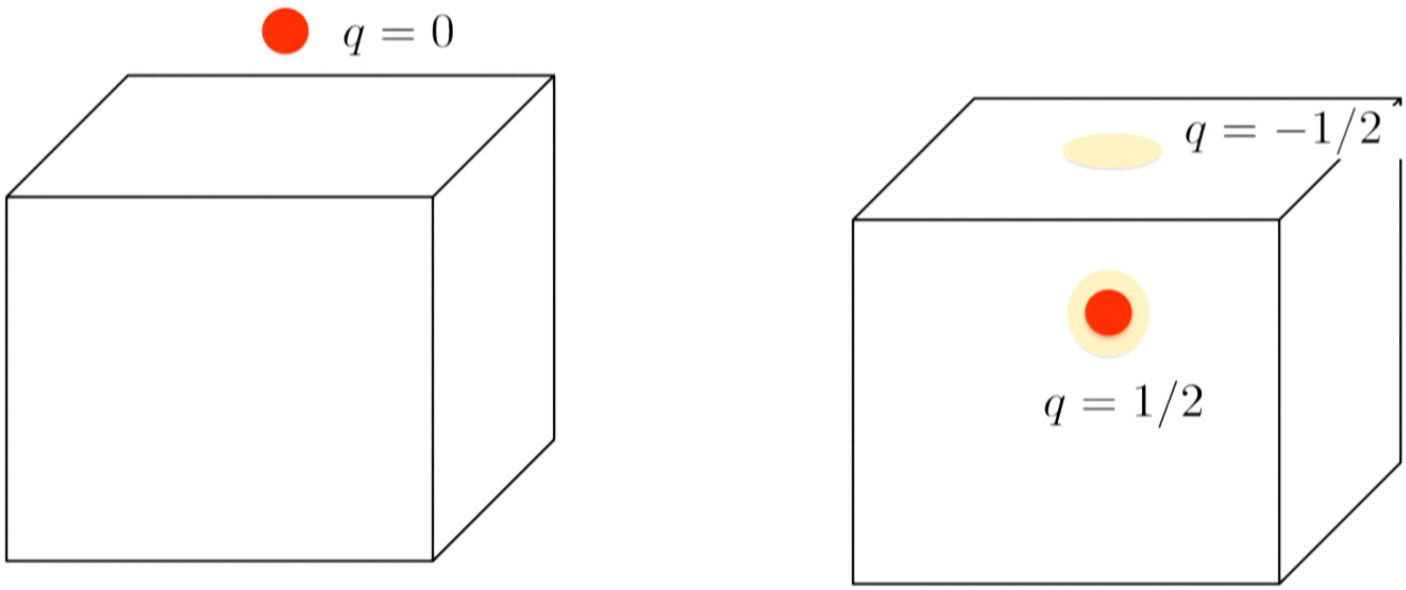

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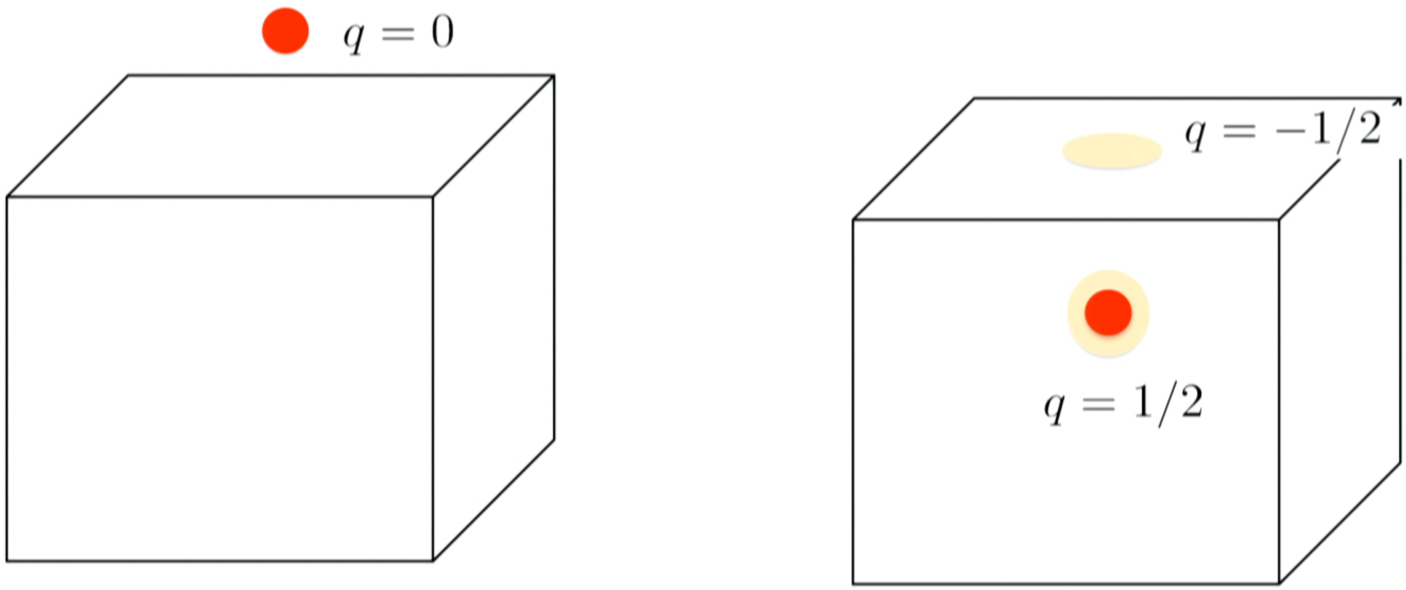

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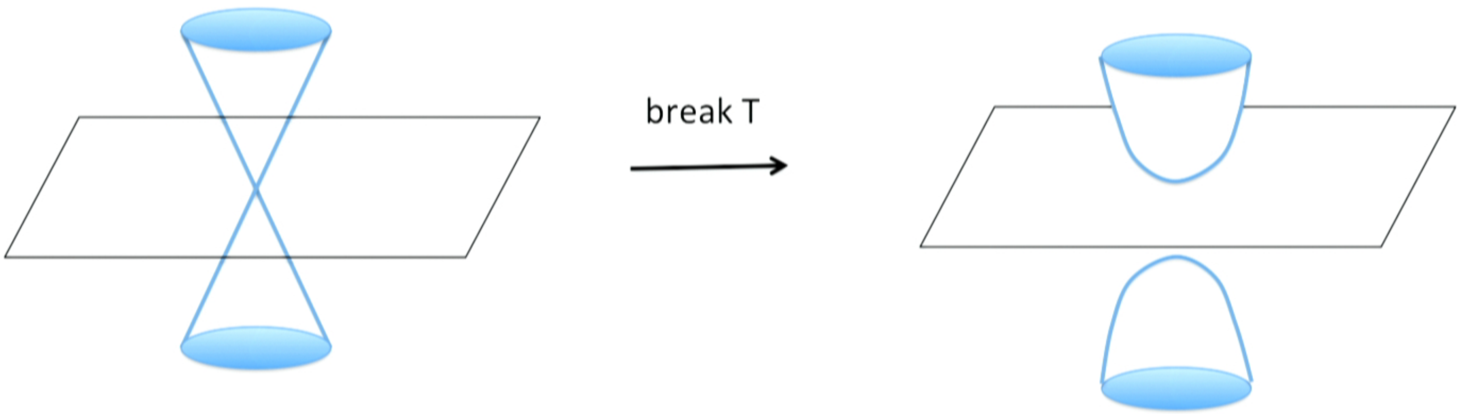
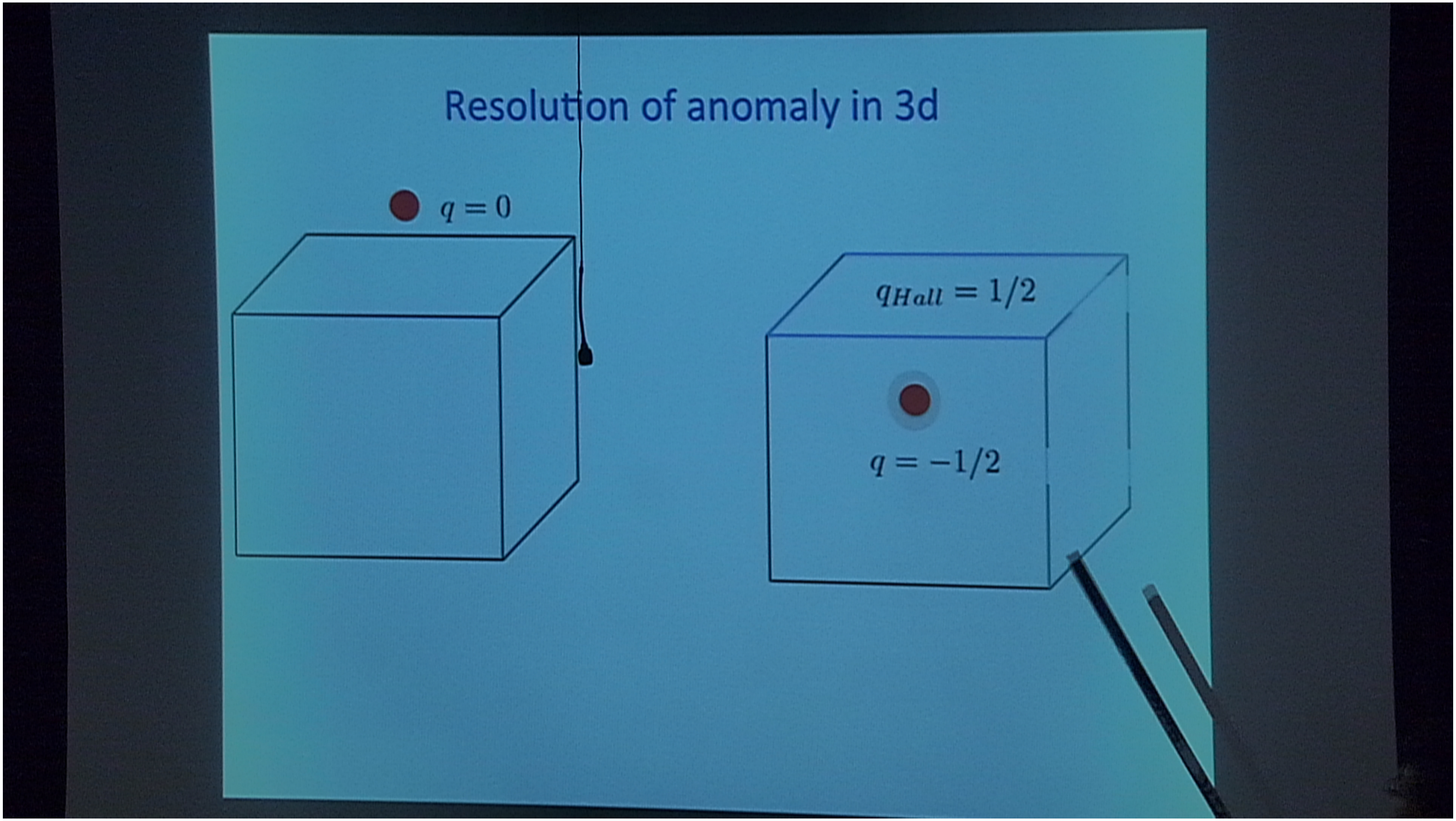
Example of "anomalous surface"

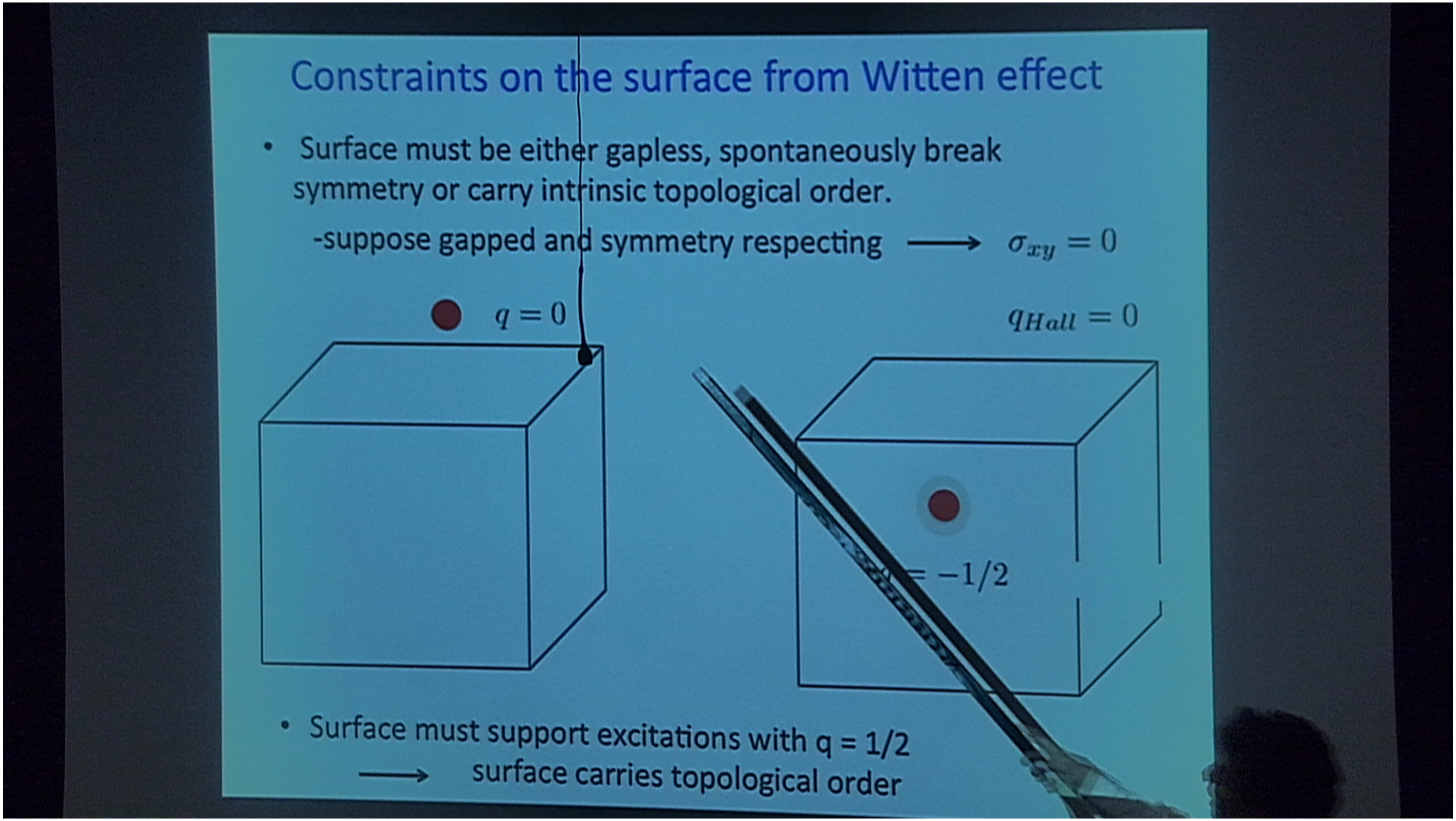


- Fully gapped state with no intrinsic topological order (only electron excitations with q = 1)
- $\sigma_{xy} = \pm 1/2$ $\qquad J_{EM} = J_e + J_{\text{Hall}}$

$$J^{\mu}_{\text{Hall}} = \frac{\sigma_{xy}}{2\pi} \epsilon_{\mu\nu\lambda} \partial_\nu A_\lambda$$

- Cannot be realized strictly in 2d.

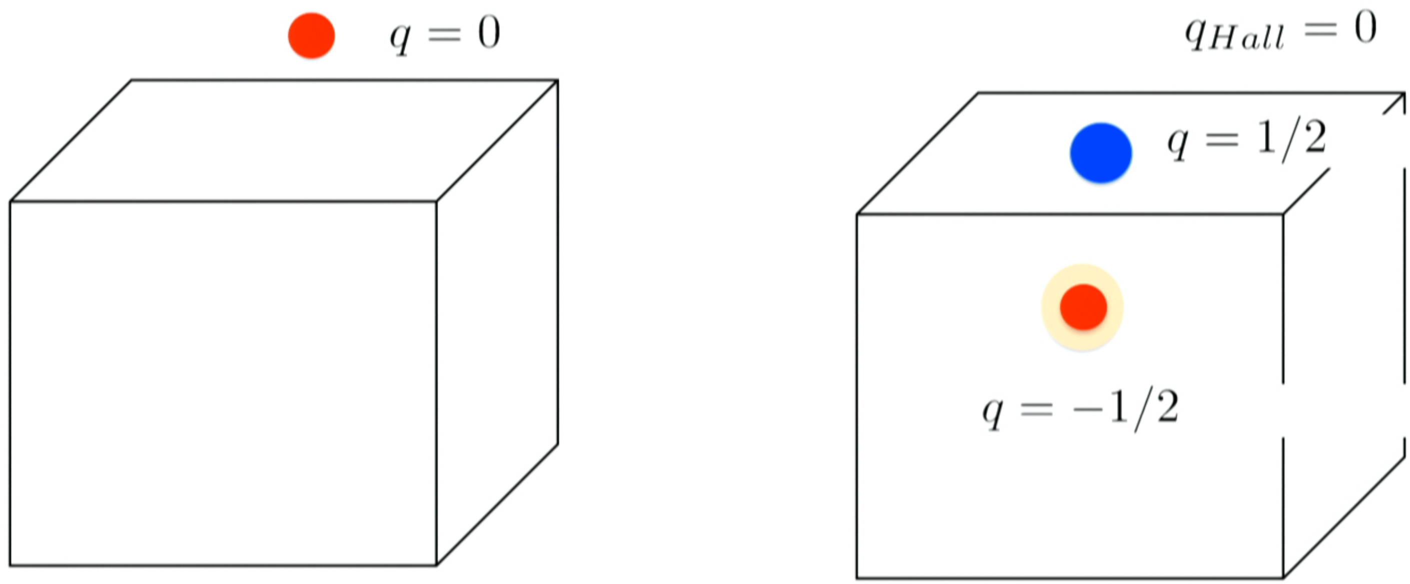

Resolution of anomaly in 3d
$q = 0$
$q_{Hall} = 1/2$
$q = -1/2$


Constraints on the surface from Witten effect
• Surface must be either gapless, spontaneously break symmetry or carry intrinsic topological order.
-suppose gapped and symmetry respecting ⟶ $\sigma_{xy} = 0$
$q = 0$
$q_{Hall} = 0$
$= -1/2$
• Surface must support excitations with q = 1/2
⟶ surface carries topological order

Constraints on the surface from Witten effect

- Surface must be either gapless, spontaneously break symmetry or carry intrinsic topological order.

  -suppose gapped and symmetry respecting $\longrightarrow$ $\sigma_{xy} = 0$



- Surface must support excitations with q = 1/2

  $\longrightarrow$ surface carries topological order

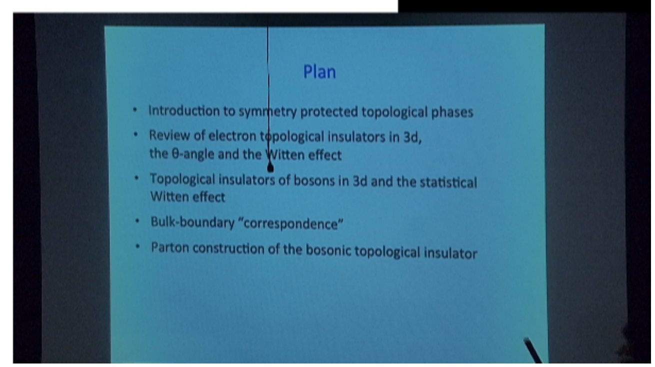
Lessons from electron TI

- Distinguish the phase by the Witten effect in the bulk
- Witten effect guarantees that surface properties cannot be realized strictly in 2d
- Witten effect ensures that the surface is gapless, symmetry broken or topologically ordered
- Example of distinguishing SPTs by "gauging" their global symmetry

  M. Levin and Z.-C. Gu (2012)
  L. Y. Hung and X.G. Wen (2012); X. G. Wen (2013)

Plan

- Introduction to symmetry protected topological phases
- Review of electron topological insulators in 3d,
  the θ-angle and the Witten effect
- Topological insulators of bosons in 3d and the statistical Witten effect
- Bulk-boundary "correspondence"
- Parton construction of the bosonic topological insulator



The θ-angle in bosonic insulators

$$S_\theta = \frac{i\theta}{32\pi^2}\int d^3x d\tau \epsilon_{\mu\nu\lambda\sigma}F_{\mu\nu}F_{\lambda\sigma}$$

- On a space-time torus T^4, with smooth EM fields

$$S_\theta = i\theta n, \quad n \;-\; \text{integer}$$

- Monopoles with flux $2\pi m$ carry charge $q = n + \frac{\theta m}{2\pi}$
- $\mathcal{T}: \theta \to -\theta$
- So is everything identical to fermion insulators?

$$\theta \sim \theta + 2\pi$$

Time-reversal invariant points: $\theta = 0$ - trivial insulator

$\theta = \pi$ - boson TI?

Monopole (dyon) statistics

- Consider vacuum (or trivial bosonic insulator) with $\theta = 0$
- Charges ● are bosons
- Monopoles ● are bosons
- What about bound states of charges and monopoles (dyons)?

(1,1) - fermion

(2,1) - boson

- more generally, a dyon (n,m) with n charges and m monopoles has statistics $(-1)^{nm}$

A. S. Goldhaber (1976)

Monopole (dyon) statistics at finite θ

- Start turning on a finite θ: $q_{tot} = n + \frac{\theta m}{2\pi}$

$$(n,m) \to \left(n + \frac{\theta m}{2\pi}, m\right)$$

- θ-angle does not affect dyon statistics!

A.S. Goldhaber, R. MacKenzie and F. Wilczek (1989)

Monopole (dyon) statistics at finite θ

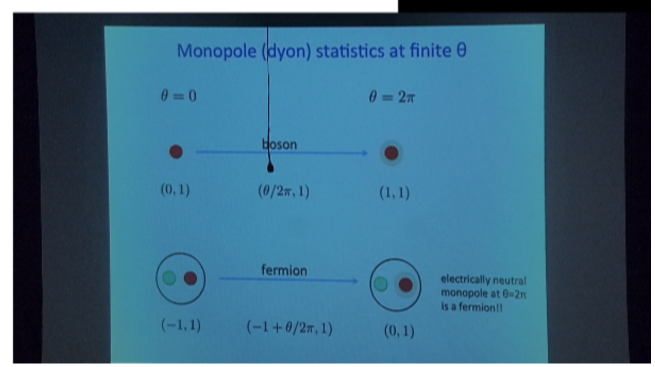
$\theta = 0$ $\theta = 2\pi$

boson

$(0, 1)$ $(\theta/2\pi, 1)$ $(1, 1)$

fermion

$(-1, 1)$ $(-1 + \theta/2\pi, 1)$ $(0, 1)$



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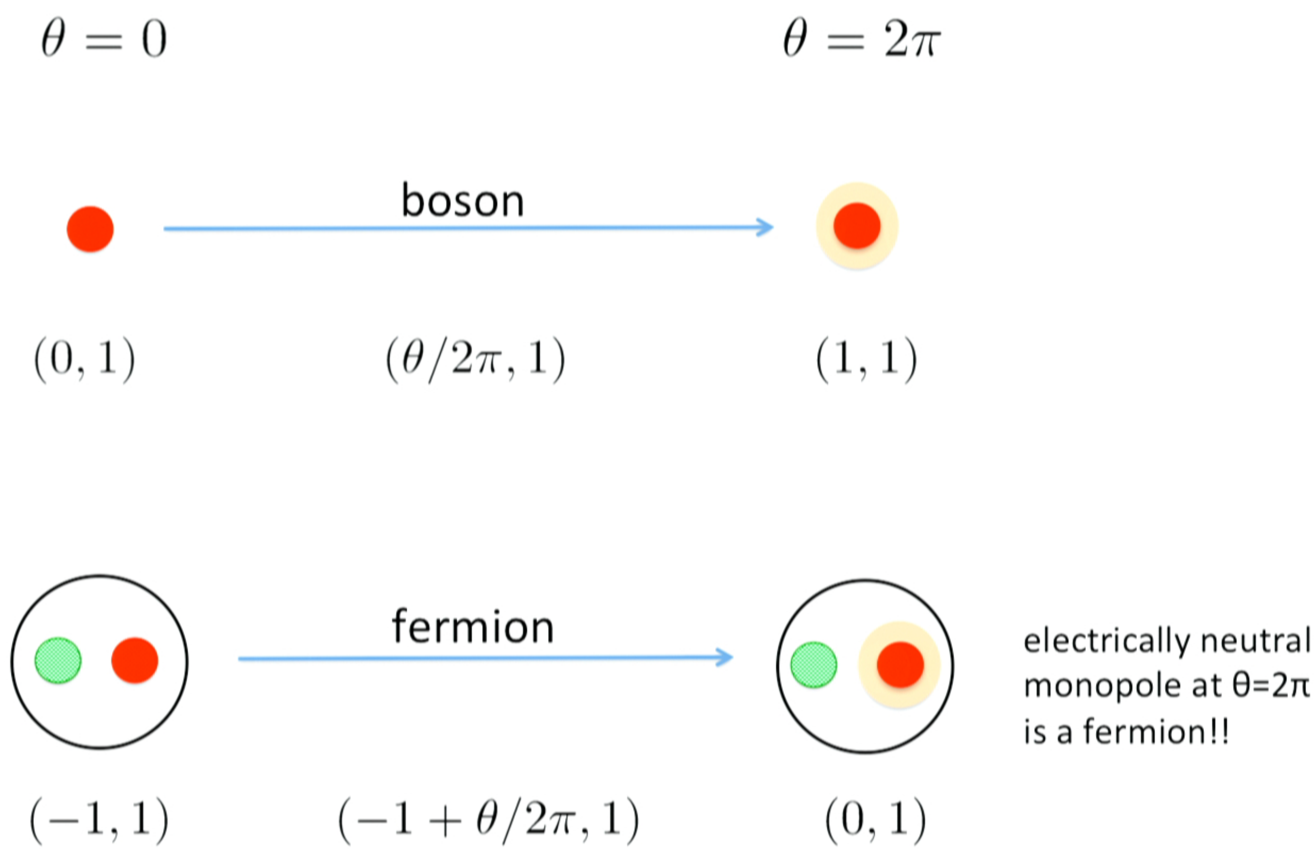
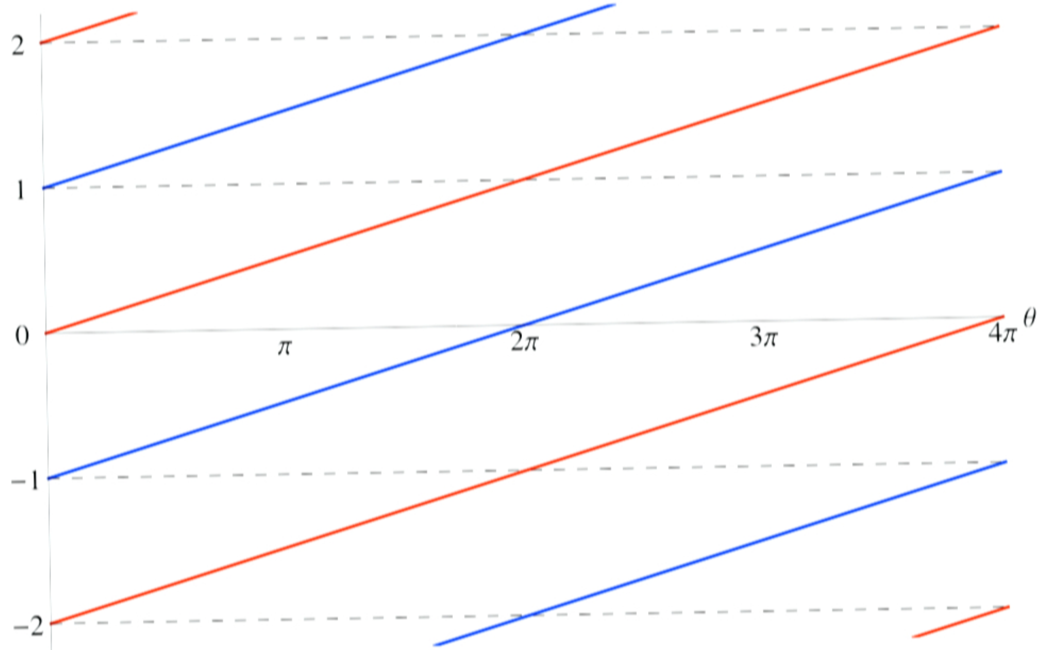


Monopole (dyon) statistics at finite θ

- (Total) charge and statistics of dyons with flux m = 1



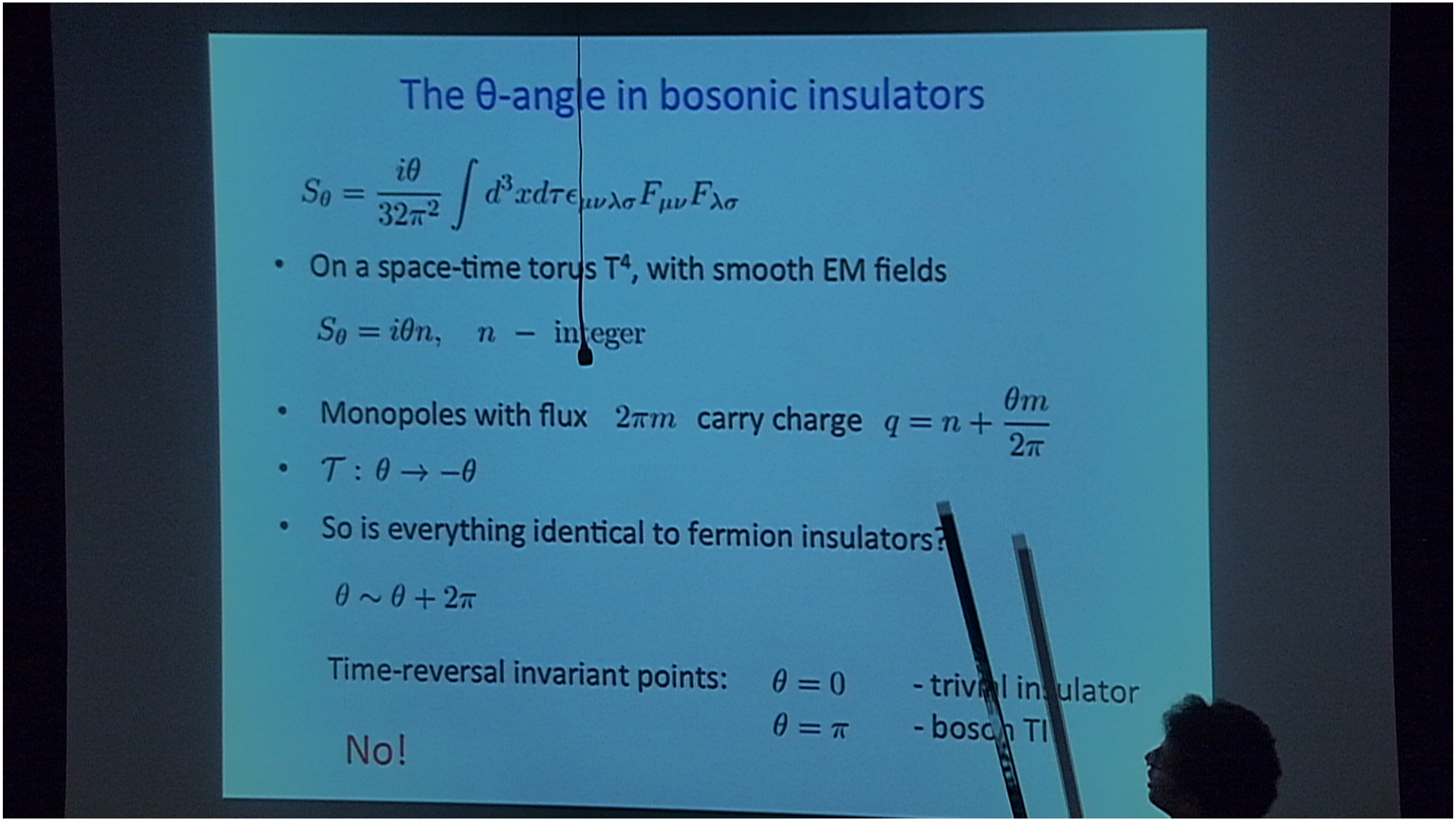
- Allowed charge periodic under $\theta \to \theta + 2\pi$

  but statistics periodic only under $\theta \to \theta + 4\pi$

Bosonic topological insulator

- In a bosonic system, $\theta \sim \theta + 4\pi$
- Time-reversal invariant points: $\theta = 0$ - trivial insulator
  $\theta = 2\pi$ - boson TI
- Boson TI distinguished in the bulk by "statistical Witten effect"
  - electrically neutral monopole is a fermion
  - more generally, statistics of (q, m) dyon is $(-1)^{qm+m}$
- Point $\theta=\pi$ necessarily breaks time-reversal symmetry
  (1/2, 1) and (1/2, -1) dyons have distinct statistics.

MM, C.L. Kane and M.P.A. Fisher (2013); C. Wong and T. Senthil (2013).


The θ-angle in bosonic insulators
$S_\theta = \frac{i\theta}{32\pi^2}\int d^3x d\tau \epsilon_{\mu\nu\lambda\sigma} F_{\mu\nu} F_{\lambda\sigma}$
• On a space-time torus T^4, with smooth EM fields
$S_\theta = i\theta n, \quad n \, - \,$ integer
• Monopoles with flux $2\pi m$ carry charge $q = n + \frac{\theta m}{2\pi}$
• $\mathcal{T}: \theta \to -\theta$
• So is everything identical to fermion insulators?
$\theta \sim \theta + 2\pi$
Time-reversal invariant points: $\theta = 0$ - trivial insulator
$\theta = \pi$ - boson TI
No!


σxy
ρ = σxy Φ/2π

Sanity check for electron insulators

- When θ=0:
  - charges are fermions
  - monopoles are bosons
  - (n, m) dyons have statistics $(-1)^{nm+n}$

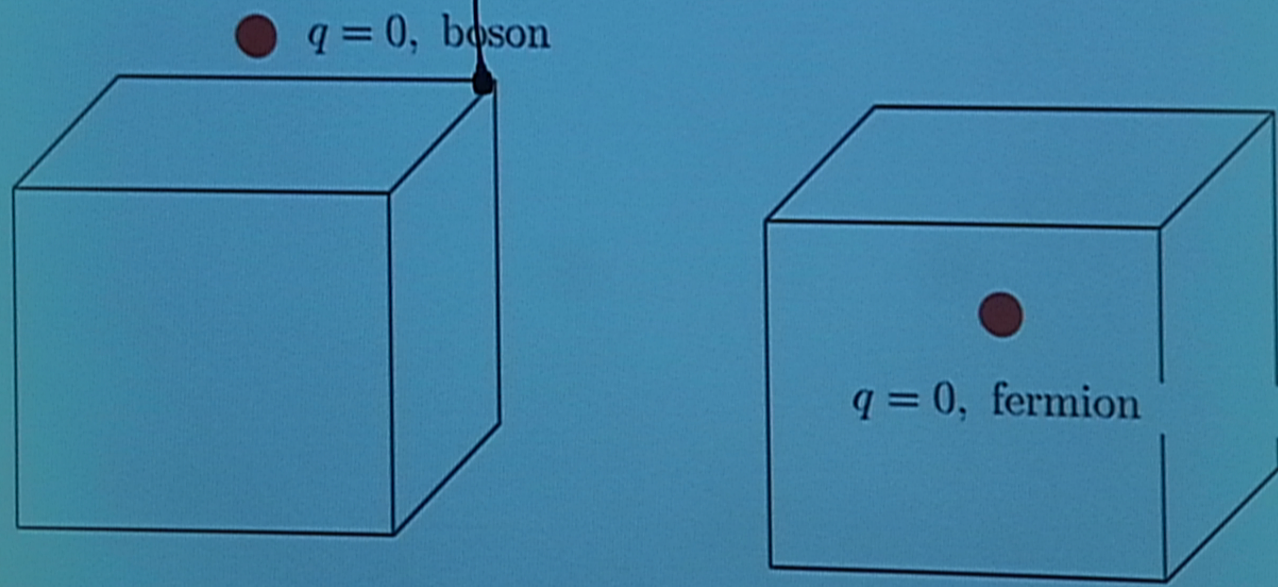
- As θ is tuned $\quad (n, m) \rightarrow \left(n + \frac{\theta m}{2\pi}, m\right)$

Statistics: $\quad (-1)^{nm+n} = (-1)^{(q-\theta m/2\pi)(m+1)}$

- invariant under $\quad \theta \rightarrow \theta + 2\pi$

Bulk-boundary "correspondence"

- What are the implications of the statistical Witten effect for surface of boson TI's?



- Surface cannot be realized strictly in 2d.

Possible surface phases of bosonic TI

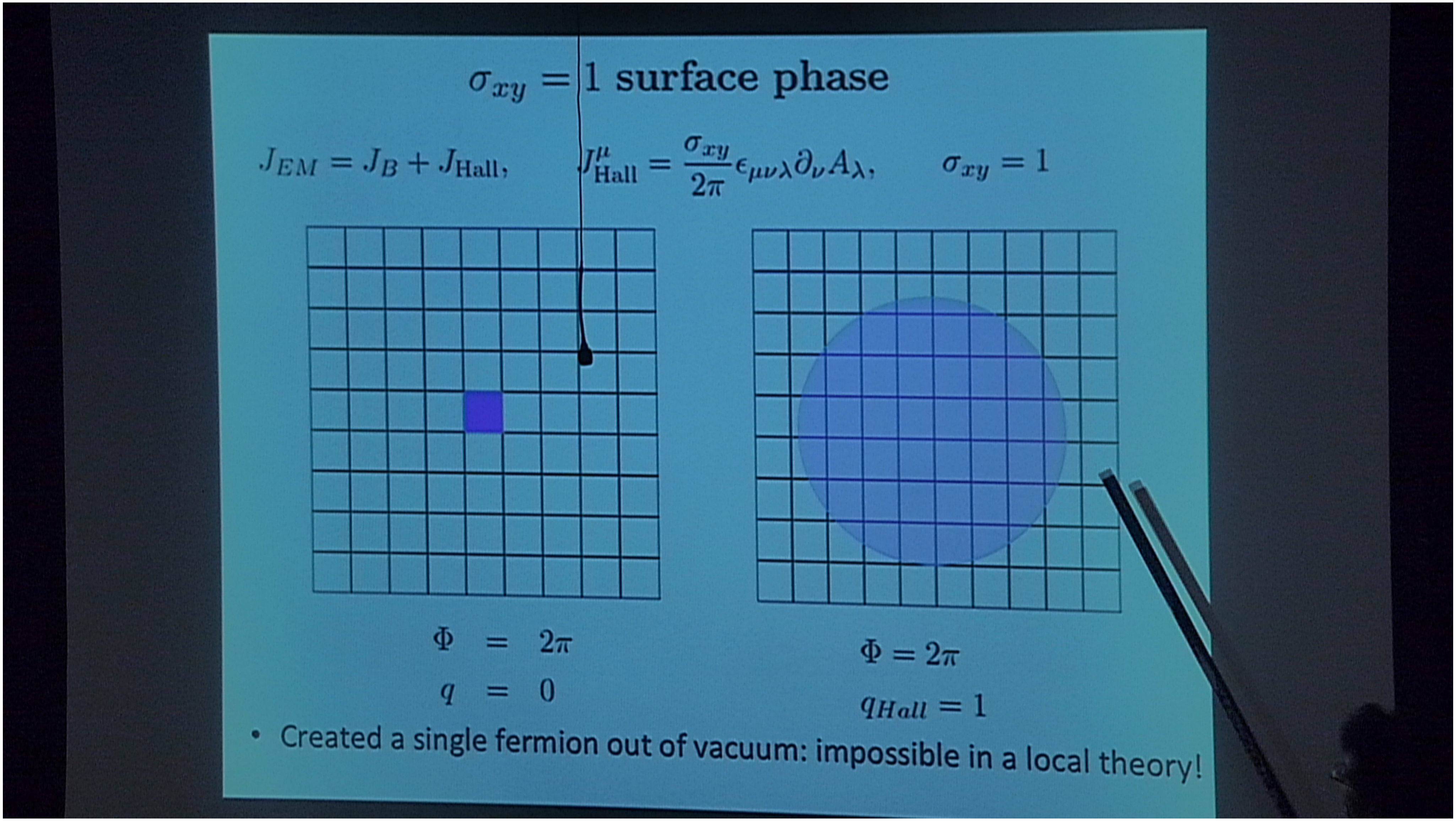
- Initially guessed by A. Vishwanath and T. Senthil (2012)

  i) T-broken phase with $\sigma_{xy} = \pm 1$ and no intrinsic topo. order

  ii) Superfluid phase with fermionic flux-tubes

  iii) Gapped, symmetry preserving phase, with intrinsic toric code topological order and both e and m anyons carrying charge 1/2.

- These phases emerge from our parton construction (below)
- How do these surface phases provide a consistent termination of boson TI?


$\sigma_{xy} = 1$ surface phase
$J_{EM} = J_B + J_{\text{Hall}}, \qquad J^{\mu}_{\text{Hall}} = \frac{\sigma_{xy}}{2\pi}\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda, \qquad \sigma_{xy} = 1$
$\Phi = 2\pi$
$q = 0$
$\Phi = 2\pi$
$q_{Hall} = 1$
• Created a single fermion out of vacuum: impossible in a local theory!

$\sigma_{xy} = 1$ surface phase

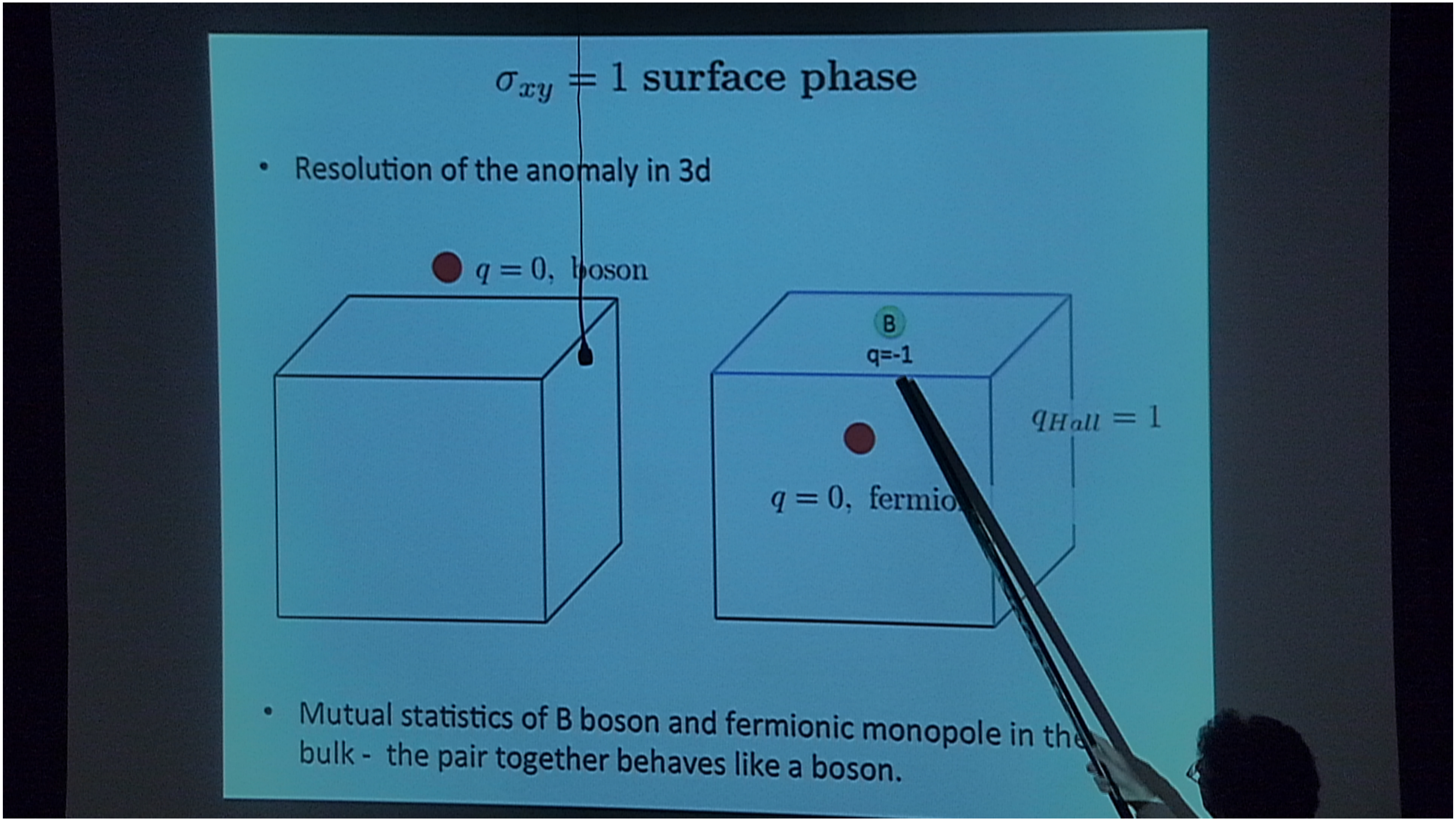
- Bosonic insulators in 2d without topo. order always have

  σ_{xy} – even integer

  - Follows from K-matrix classification of
    Y.M. Lu and A. Vishwanath (2012)
  - General physical argument: T. Senthil and M. Levin (2013)

- Unlike 2d fermion insulators without topo. order which have

  σ_{xy} – any integer



$\sigma_{xy} = 1$ surface phase
• Resolution of the anomaly in 3d
$q = 0,$ boson
B
q=-1
$q_{Hall} = 1$
$q = 0,$ fermio
• Mutual statistics of B boson and fermionic monopole in the bulk - the pair together behaves like a boson.

Possible surface phases of bosonic TI

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- How do these surface phases provide a consistent termination of boson TI

Topologically ordered surface

- Gapped, symmetry preserving phase, with intrinsic toric code topological order and both e and m anyons carrying charge 1/2.

- - e - boson - $q = 1/2$
  - m - boson - $q = 1/2$
  - $f = e\, m^{-1}$ - fermion - $q = 0$

- All particles are mutual semions

- A state with such charge quantum numbers and time-reversal symmetry cannot exist strictly in 2d

Topologically ordered surface

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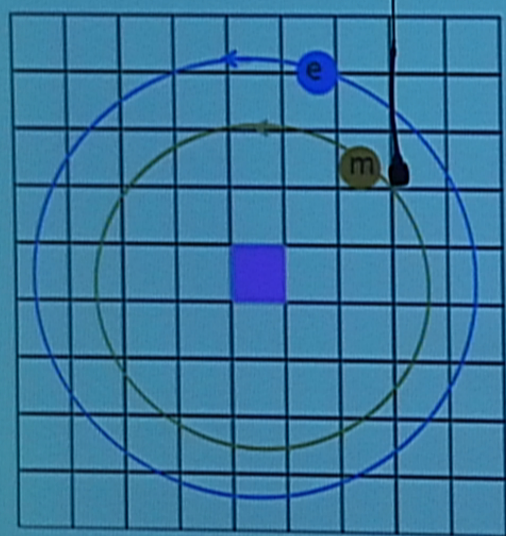
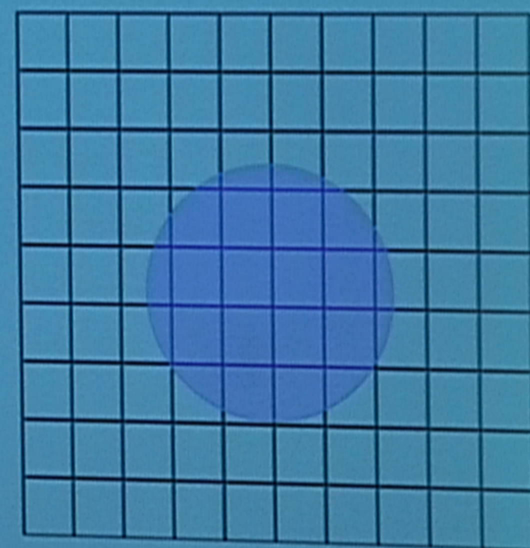
Topologically ordered surface

- $\sigma_{xy} = 0$



$$\Phi = 2\pi$$
$$q = 0$$

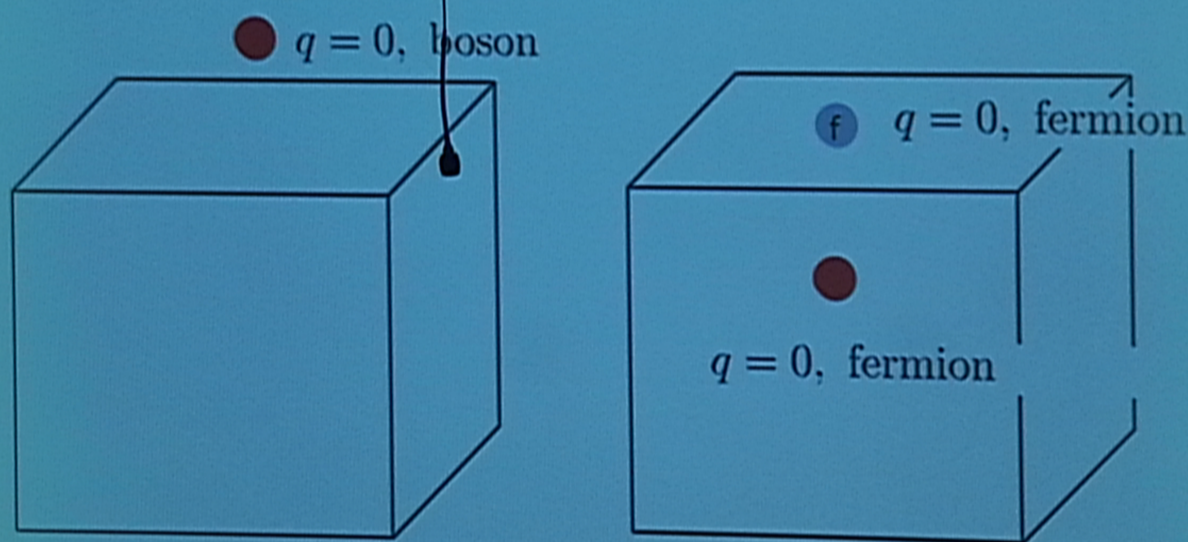


$$\Phi = 2\pi$$
$$q_{Hall} = 0$$

- A neutral fermion f is created in the process! Violates locality in 2d.

Topologically ordered surface

- Resolution in 3d



- More generally if the surface is gapped and symmetry respecting, a local fermion excitation must be created $\longrightarrow$ topo. order.

Boson TI and statistical Witten effect

- Distinguish the phase by the statistical Witten effect in the bulk
- Statistical Witten effect guarantees that surface properties cannot be realized strictly in 2d
- Statistical Witten effect ensures that the surface is gapless, symmetry broken or topologically ordered

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- Parton construction of the bosonic topological insulator

Related work: P. Ye and X.-G. Wen (2013)

Parton construction of boson TI(s)

- Start with microscopic bosons B, e.g.

$$U(1): \quad B \to e^{i\alpha} B, \qquad \mathcal{T}: B \to B$$

- Fractionalize into (bosonic) partons, e.g.

$$B^\dagger = b_+^\dagger b_-$$

- Assign partons quantum numbers, e.g.

$$b_\pm: \quad Q = \pm 1/2$$

$$\mathcal{T}: \qquad b_+ \to b_-^\dagger, \quad b_- \to b_+^\dagger$$

Parton construction of boson TI(s)

- Fractionalize into (bosonic) partons, e.g.

$$B^\dagger = b_+^\dagger b_-$$

- Representation redundant under

$$u(1)_{loc}: \quad b_\pm(x) \to e^{i\alpha(x)} b_\pm(x),$$

- An "internal" $u(1)$ gauge field a_μ emerges.

$$L = \frac{1}{4g}(\partial_\mu a_\nu - \partial_\nu a_\mu)^2 + i(a_\mu \pm \frac{A}{2}) j_\mu^\pm$$

- $\mathcal{T}: \quad b_+ \to b_-^\dagger, \quad b_- \to b_+^\dagger \longrightarrow a_\mu$ transforms oppositely from A_μ

Coulomb phase

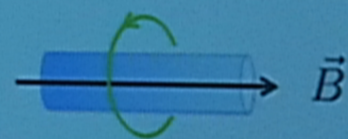
$$L = \frac{1}{4g}(\partial_\mu a_\nu - \partial_\nu a_\mu)^2 + i(a_\mu \pm \frac{A}{2})j_\mu^\pm$$

- Theory can be in deconfined Coulomb phase
  - gapless emergent photon a_μ
  - gapped partons $b_\pm : q = 1,\ Q = \pm 1/2$
  - gapped monopoles of $a_\mu,\ m$
- Symmetry enriched phase with *intrinsic* 3d topological order.
- To obtain an SPT phase need to gap a_μ out and confine the partons.

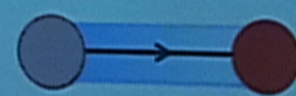
Destroying the Coulomb phase: Higgs

$$L = \frac{1}{4g}(\partial_\mu a_\nu - \partial_\nu a_\mu)^2 + i a_\mu j_\mu$$

- Condense electric charges b (●) : (q, m) = (1,0)
  - Higgs phase
  - Meissner effect: $\Phi_B = 2\pi n$



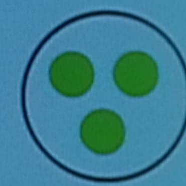
  - Monopoles confined:



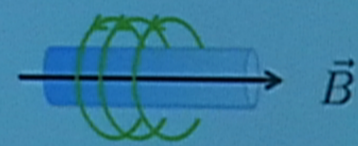
- Fully gapped phase with no intrinsic topological order

Z_k topological order

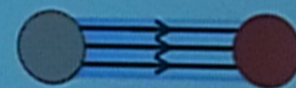
- Condense k electric charges b^k: (q, m) = (k,0)



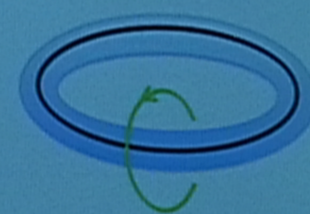
  - Charge k Higgs phase
  - Meissner effect: $\Phi_B = \frac{2\pi n}{k}$



  - Monopoles confined:



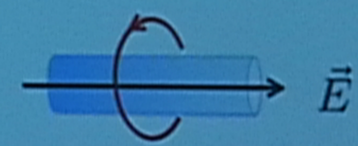
  - single charges deconfined
  - single flux tubes are stable
  - mutual statistics $e^{\frac{2\pi i}{k}}$



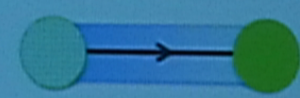
- A gapped phase of matter with intrinsic Z_k topological order.

Confined phase

- Condense monopoles (●) : (q, m) = (0,1)
  - Confined phase
  - Meissner effect: $\Phi_E = n$



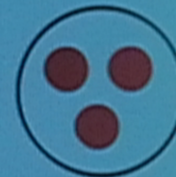
  - Charges confined:



- Fully gapped phase with no intrinsic topological order
  - in the absence of symmetry identical to charge 1 Higgs phase

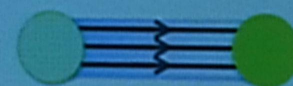
Z_k topological order

- Condense k monopoles: (q, m) = (0,k)



  - Meissner effect: $\Phi_E = \frac{n}{k}$

  - Charges confined:



  - single monopoles ● deconfined
  - single electric flux tubes are stable
  - mutual statistics $e^{\frac{-2\pi i}{k}}$

- A gapped phase of matter with intrinsic Z_k topological order.
  - identical to charge k Higgs.

Dyon condensation

- Condense a dyon (n, m)
  - Must be a boson: $(-1)^{nm} = 1$
  - Meissner effect:

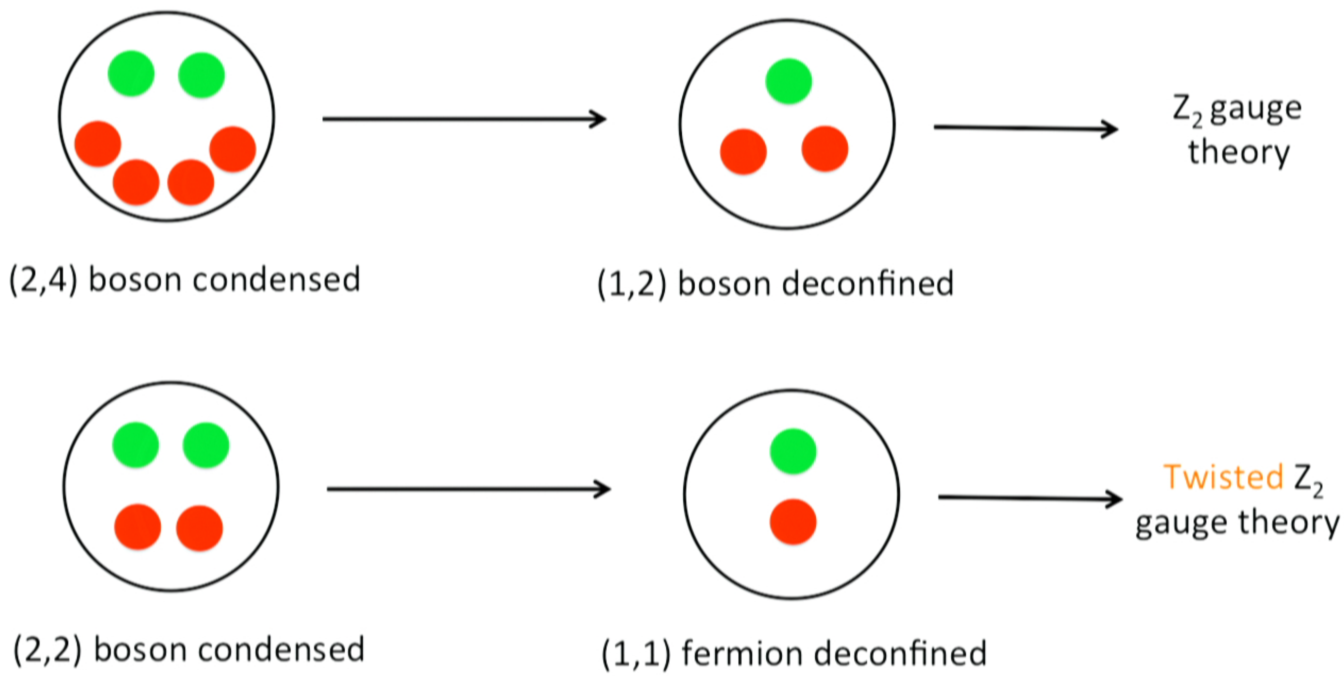
$$m\vec{E} - \frac{n}{2\pi}\dot{B}$$

$$m\Phi_E - \frac{n}{2\pi}\Phi_B \in \mathbb{Z}$$

- If gcd(n,m) = 1
  - all charges and monopoles confined. No stable flux-tubes.
  - phase with no intrinsic topological order (identical to confined)
  - ground state wave-function very similar to Walker-Wang models. K. Walker and Z. Wang (2011)

Dyon condensation

- If gcd(n, m) = k ≠ 1
  - (n,m)/k dyons deconfined. Can be bosons or fermions.
  - phase of matter with intrinsic (twisted) Z_k topological order



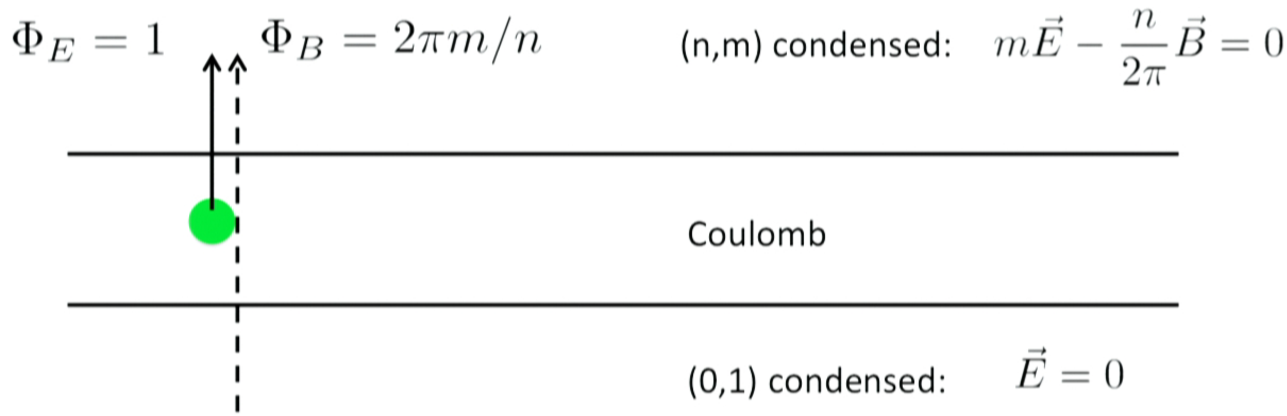
Dyon condensation – surface physics

- What is the interface between (n,m) and (0,1)? (assume gcd(n,m) = 1).

(n,m) condensed: $m\vec{E} - \frac{n}{2\pi}\vec{B} = 0$

(0,1) condensed: $\vec{E} = 0$

- An electric charge attaches a magnetic flux: $\Phi_B = 2\pi m/n$



- Interface described by Chern-Simons theory:

$$L = \frac{ik}{4\pi}\epsilon_{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda + i a_\mu j_\mu, \qquad k = \frac{n}{m}$$

- Interface has 2d topological order; identical to a hierarchical fractional quantum Hall state. Similar to Walker-Wang models.

- Topo. order may be destroyed via a surface phase transition. (Consistent with trivial bulk).

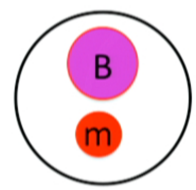
Bosonic TI via dyon condensation

$B^\dagger = b_+^\dagger b_-$ $$L = \frac{1}{4g}(\partial_\mu a_\nu - \partial_\nu a_\mu)^2 + i(a_\mu \pm \frac{A}{2})j_\mu^\pm$$

- Condense a monopole m (●) of a_μ
  - preserves all symmetries
  - partons confined; no topological order
  - monopoles of external gauge field A_μ are bosons

  $\longrightarrow$ trivial insulator

- Condense a bound state of m and B
  - preserves time-reversal
  - preserves particle-number: insulator



$$\tilde{u}(1)_{loc} \times U(1) \to U(1)_{EM}; \qquad Q_{EM} = Q - m$$

Bosonic TI via dyon condensation

$B^\dagger = b_+^\dagger b_-$

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- Condense a monopole m (●) of a_μ
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  ⟶ trivial insulator

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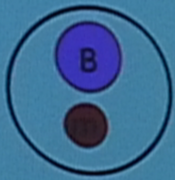

$\tilde{u}(1)_{loc} \times U(1) \to U(1)_{EM}$: $\quad Q_{EM} = Q - m$

Conclusion. Future directions.

- Dyon condensation – a new way to construct phases of matter
  - related to Walker-Wang models?

  F. Burnell, X. Chen, L. Fidkowski and A. Vishwanath (2013).

- Other SPT phases of bosons (and fermions?) with dyons?

  $Z_2^T : \ Z_2^2$

  $U(1) \ltimes Z_2^T : \ Z_2^3$ $\qquad$ $U(1) \times Z_2^T : \ Z_2^4$

- Can we guess realistic Hamiltonians from parton construction?

- Describing general 3d SPT phases by their symmetry preserving, topologically ordered surface.