Title: The continuum limit of tensor networks: a path integral representation

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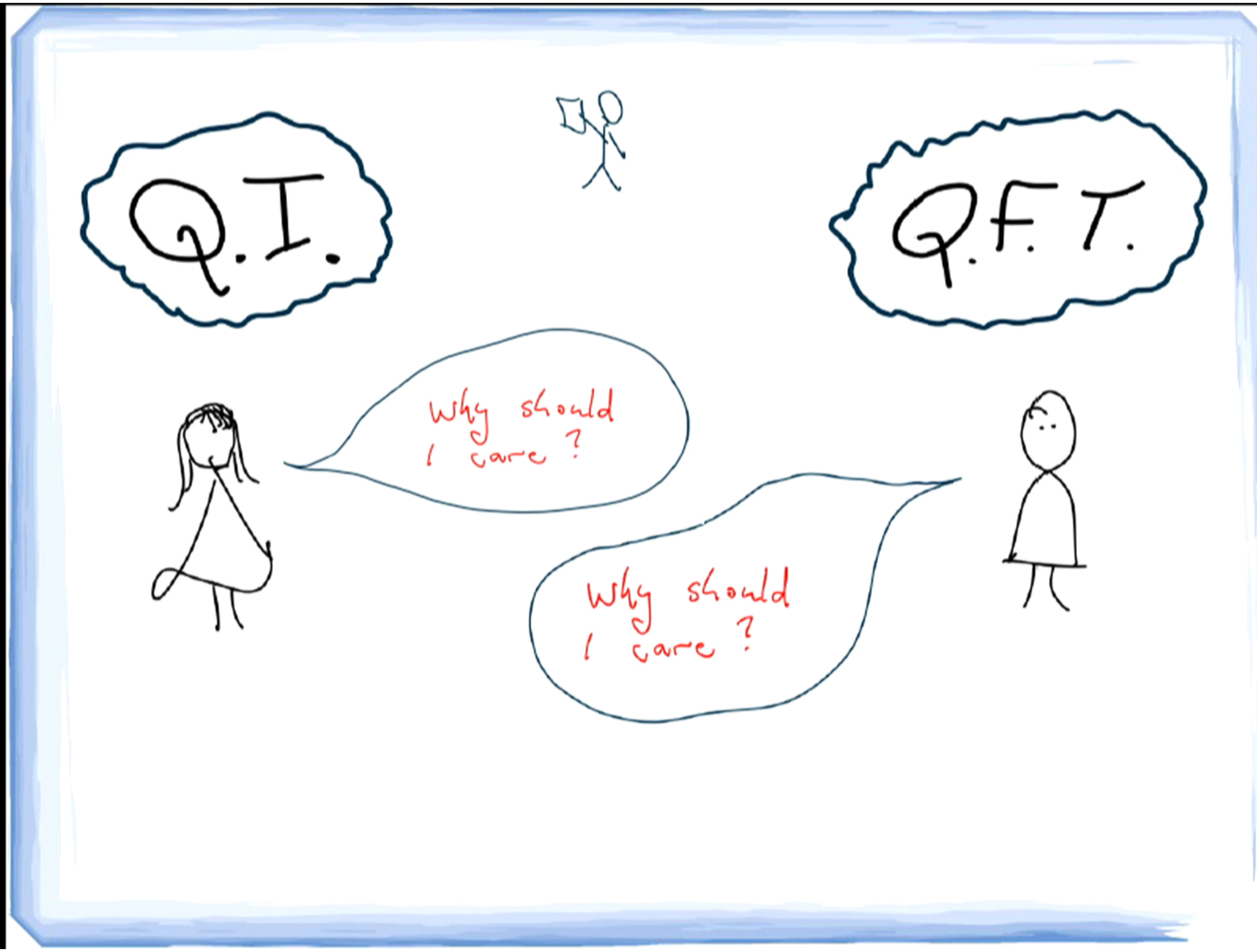
Abstract: I will discuss a path-integral representation of continuum tensor networks that extends the continuous MPS class for 1-D quantum fields to arbitrary spatial dimensions while encoding desirable symmetries. The physical states can be interpreted as arising through a continuous measurement process by a lower dimensional virtual field with Lorentz symmetry. The resultant physical states naturally obey entropy area laws, with the expectation values of observables determined by the dissipative dynamics of the boundary field. The class offers the prospect of powerful new analytical and computational tools to describe the physics of strongly interacting field systems.

"Continuum limit of tensor networks: a path integral representation" *

David Jennings
Quantum Information Theory Group,
Imperial College London,
London SW7 2AZ

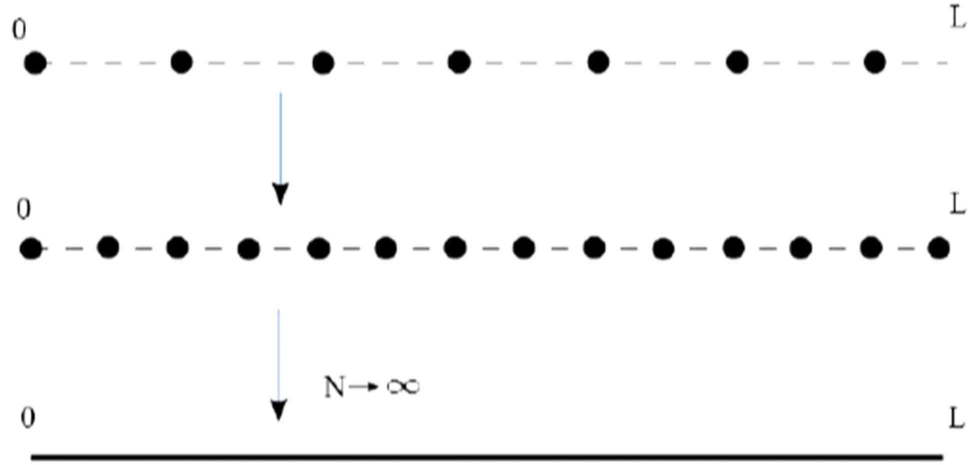
Imperial College
London

*See:
(1) D. Jennings, J. Haegeman, T. Osborne, F. Verstraete, arxiv:1212.3833.
(2) C. Brockt, D. Jennings, J. Haegeman, T. Osborne, F. Verstraete, arxiv:1210.5401
(3) Forth-coming, arxiv:130x.xxxx


Q.I.
Q.F.T.
Why should I care?
Why should I care?

MPS in the continuum limit: cMPS

- Wish to consider the $N \to \infty$ limit of MPS.
- Assume: physical systems = oscillators, $b_x, b_x^\dagger$
- Assume a single D-dim auxiliary system.



$l = N\epsilon$

MPS in the continuum limit: cMPS

- Only special set of matrices will work for limit.

Arbitrary MPS: $\{A^0, A^1, A^2, A^3 \cdots\}$

For $N \to \infty$ we're forced to use: $A^0 = \mathbb{I} + \epsilon Q$

and $A^1 = \sqrt{\epsilon} R \quad A^2 = \sqrt{\frac{\epsilon^2}{2}} R^2 \quad \ldots \quad A^k = \sqrt{\frac{\epsilon^k}{k!}} R^k$

So that $(A^1)(A^1) = A^2$ etc...

MPS in the continuum limit: cMPS

$$|\chi\rangle = \sum_{i_1,i_2,\ldots,i_N} \langle\omega_L|A^{i_1}A^{i_2}\cdots A^{i_N}|\omega_R\rangle|i_1 i_2\cdots i_N\rangle$$

In the limit $N\to\infty$ or $\epsilon\to 0$ gives

$$\boxed{|\chi(K,R)\rangle = \langle\omega_L|\mathcal{T}e^{-i\int_0^l ds K(s)+iR(s)\otimes\widehat{\psi}^\dagger_{\mathcal{A}}}|\omega_R\rangle|\Omega_{\mathcal{A}}\rangle}$$

where we have $K,R\in\mathcal{M}_D$ and $\widehat{\psi}^\dagger(x)\sim\frac{1}{\sqrt{\epsilon}}b_x^\dagger$

$$\mathcal{T}e^{\int_0^l ds\widehat{F}(s)} = \lim_{\epsilon\to 0}[e^{\epsilon\widehat{F}(l)}e^{\epsilon\widehat{F}(l-\epsilon)}\cdots e^{\epsilon\widehat{F}(0)}]$$

*Verstraete, Cirac, Phys.Rev.Lett.104:190405, (2010)

Path Integrals: A brief reminder

- Recall: Amplitude → "sum over trajectories"

$$\langle\psi(x_f)|U(t_f)|\psi(x_0)\rangle = \sum_{\text{all paths}} e^{iS[x(t)]}$$

- Basic construction is

$$\langle\psi(x_f)|U(t_f)|\psi(x_0)\rangle = \langle\psi(x_f)|e^{i\epsilon H(t_N)}e^{i\epsilon H(t_{N-1})}\ldots e^{i\epsilon H(t_1)}|\psi(x_0)\rangle$$

- The use complete set of states: $\mathbb{I} = \sum_x |x\rangle\langle x|$

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Path Integrals: A brief reminder

We obtain:

$$\langle \psi(x_f) | U(t_f) | \psi(x_i) \rangle = \sum_{(x_1, x_2, \ldots, x_N)} e^{i \sum_k S[x_k]}$$

- In the limit $N \to \infty$ we write this as:

$$\langle \psi(x_f) | U(t_f) | \psi(x_i) \rangle = \int \mathcal{D}x(t) e^{iS[x(t)]}$$

where, for example

$$S[x(t)] = \int dt \left(\frac{1}{2}\dot{x}^2 - V(x) \right)$$

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1-D cMPS Path Integral

$$|\chi(K,R)\rangle = \langle\omega_L|\mathcal{T}e^{-i\int ds K(s)+iR(s)\widehat{\psi}^\dagger_{\mathcal{A}}}|\omega_R\rangle|\Omega_{\mathcal{A}}\rangle$$

- $\rightarrow$ Rewrite $\langle\omega_L|\mathcal{T}e^{-i\int ds\widehat{F}(s)}|\omega_R\rangle|\Omega_{\mathcal{A}}\rangle$ as a path integral.
- But auxiliary system is discrete!?
- Resolution: Embed into the 1-particle sector of Fock space.

1-D cMPS Path Integral

- The result takes the form

$$|\chi\rangle = \int \mathcal{D}^2\phi \exp[iS(\phi,\phi^\dagger)]|\Phi\rangle$$

- with action for <u>discrete</u> D-dim quantum system

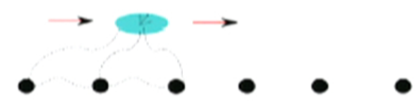
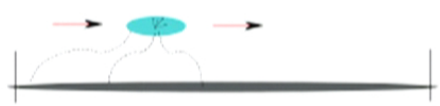
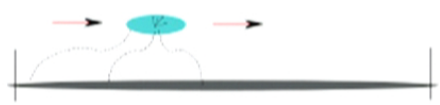
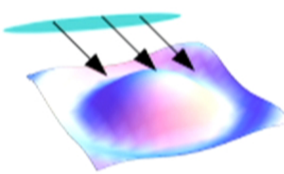
$$S(\phi,\phi^\dagger) = \int ds(i\phi^\dagger\partial_s\phi - \phi^\dagger K\phi)$$

and with a superposition of coherent physical field states

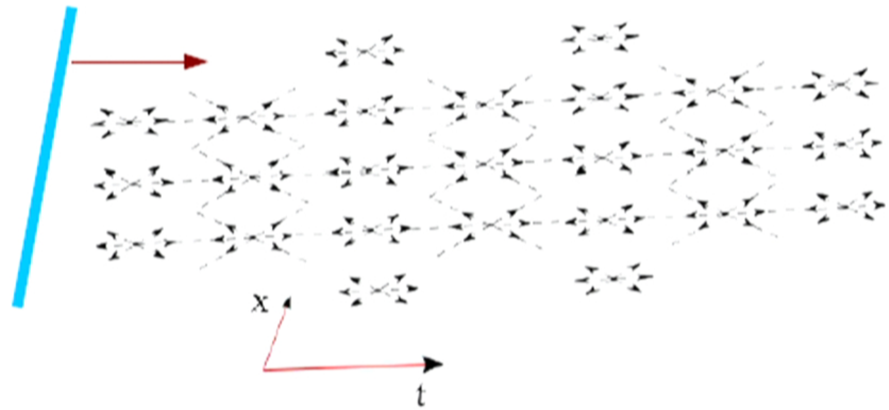
$$|\Phi\rangle = \exp[\int ds(\Phi\widehat{\psi}^\dagger_{\mathcal{A}} - \Phi^*\widehat{\psi}_{\mathcal{A}})]|\Omega_{\mathcal{A}}\rangle$$

$$\Phi(s) := \phi^\dagger R\phi$$

Road Map

1. Discrete MPS	$\vert\psi\rangle = \sum_{\mathbf{j},\boldsymbol{\alpha}} e^{iS[\boldsymbol{\alpha},\boldsymbol{j}]]} \vert\boldsymbol{j}\rangle$ $S[\boldsymbol{\alpha},\boldsymbol{j}_{\cdot} = \sum_{k=1}^{N} -i \log A_{\alpha_{k-1},j_k,\alpha_k}$	
2. Continuum 1-D cMPS	$\langle\omega_L\vert\mathcal{T} e^{-i\int ds K(s)+iR(s)\hat{\psi}^{\dagger}_{A}}\vert\omega_R\rangle\vert\Omega_A\rangle$	
3. Path integral form 1-D cMPS	$\int \mathcal{D}^2\phi e^{iS[\phi]}\vert\Phi\rangle$ $S[\phi] = \int ds(i\phi^{\dagger}\partial_s\phi - \phi^{\dagger}K\phi)$	
4. Symmetric 2-D cMPS	$\int \mathcal{D}\Psi\mathcal{D}\Psi^{\dagger} e^{iS[\Psi,\Psi^{\dagger}]}\vert\Phi\rangle$ $S[\Psi,\Psi^{\dagger}] = \int dxdy(i\Psi^{\dagger}B^{\mu}\partial_{\mu}\Psi - \Psi^{\dagger}K\Psi)$	

2-D Discrete to Continuum



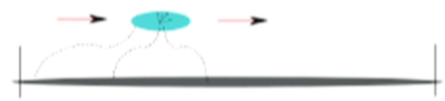
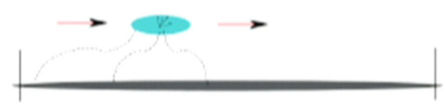
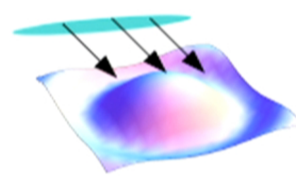
- Nielsen-Ninomiya theorem: fermionic doubling
- Doubling only occurs on virtual d.o.f.

  Bond dimension D $\rightarrow$ 2D

What we've learned

(1) MPS extended to continuum for arbitrary dimensions.
(2) Well-behaved continuum limit of PEPS.
(3) MPS = Path Integral.
(4) Complete class for Fock space.
(5) States are natural sums over coherent field states.

Summary

1. Discrete MPS	$\langle \omega_L \| A^{i_1} \cdots A^{i_N} \| \omega_R \rangle \| i_1 \ldots i_N \rangle$	
2. Continuum 1-D cMPS	$\langle \omega_L \| \mathcal{T} e^{-i \int ds K(s) + iR(s) \hat{\psi}^\dagger_A} \| \omega_R \rangle \| \Omega_A \rangle$	
3. Path integral form 1-D cMPS	$\int \mathcal{D}^2 \phi e^{iS[\phi]} \| \Phi \rangle$ $S[\phi] = \int ds (i \phi^\dagger \partial_s \phi - \phi^\dagger K \phi)$	
4. Symmetric 2-D cMPS	$\int \mathcal{D}\Psi \mathcal{D}\Psi^\dagger e^{iS[\Psi, \Psi^\dagger]} \| \Phi \rangle$ $S[\Psi, \Psi^\dagger] = \int dx dy (i \Psi^\dagger B^\mu \partial_\mu \Psi - \Psi^\dagger K \Psi)$	

Continuum limit

- Obtain rotationally invariant 2-D state obtained from a discrete PEPS model

$$|\chi\rangle = \int \mathcal{D}\Psi\mathcal{D}\bar{\Psi} e^{-S_E[\Psi,\bar{\Psi}]}|\Phi\rangle$$

- State automatically obeys an area law.
- State depends on DxD matrices m^{jk} and R^{jk}
- Variational class: $\{|\chi(R,m)\rangle : R, m \in \mathcal{M}_D\}$