Title: The imaginary part of the gravitational action and black hole entropy

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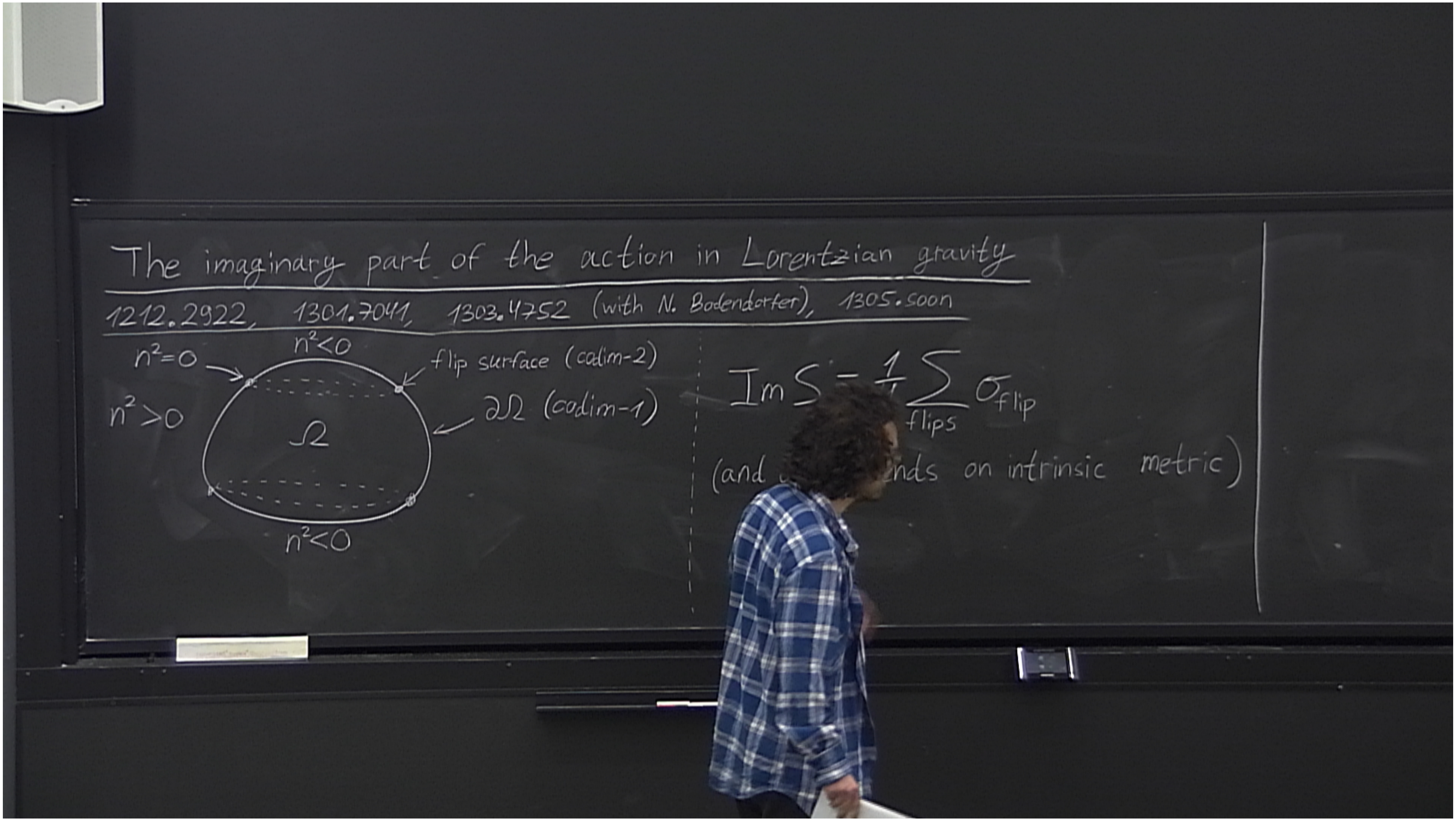
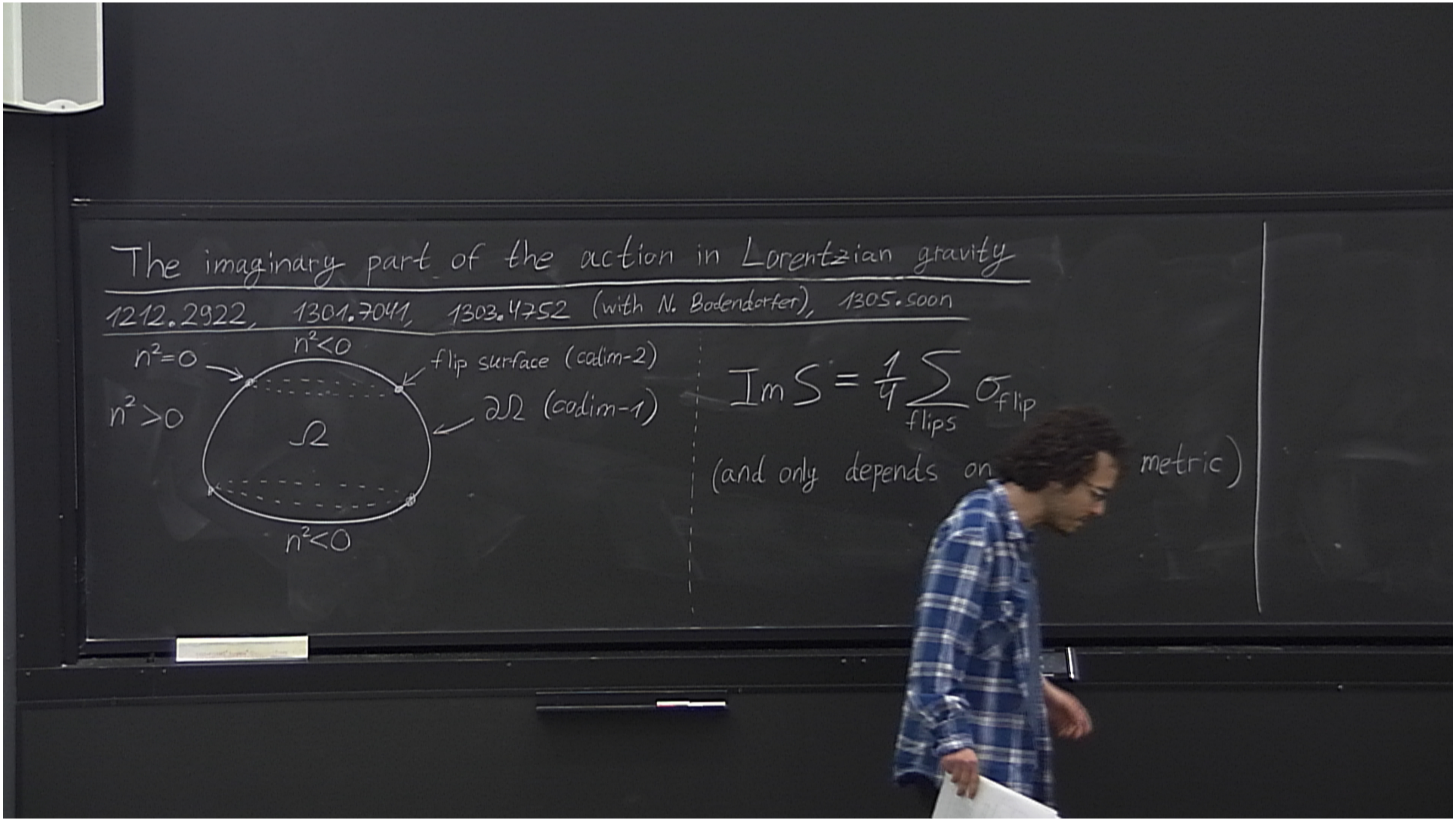
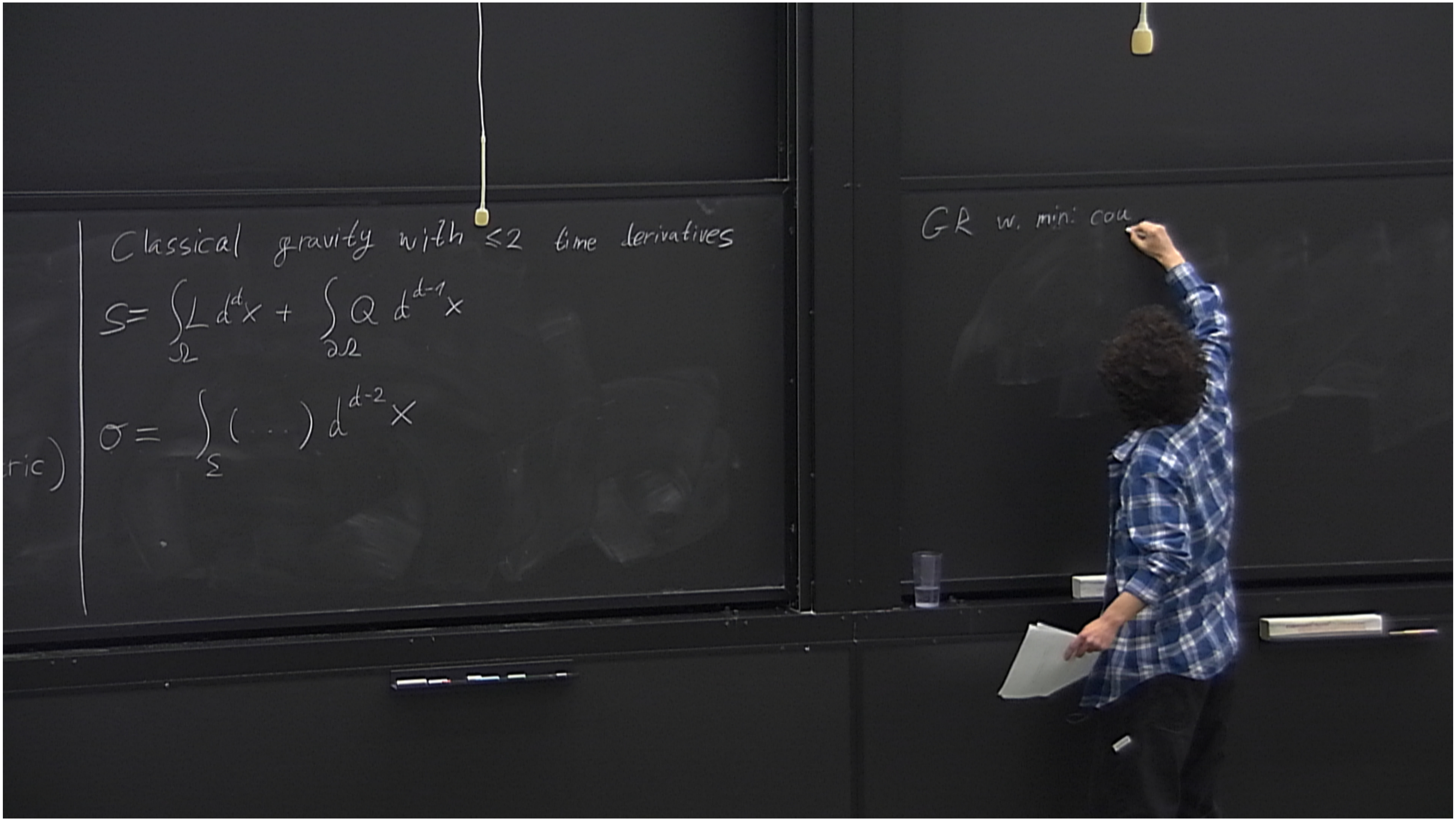
Abstract: I present a candidate for a new derivation of black hole entropy. The key observation is that the action of General Relativity in bounded regions has an imaginary part, arising from the boundary term. The formula for this imaginary part is closely related to the Bekenstein-Hawking entropy formula, and coincides with it for certain classes of regions. This remains true in the presence of matter, and generalizes appropriately to Lovelock gravity. The imaginary part of the action is a versatile notion, requiring neither stationarity nor any knowledge about asymptotic infinity. Thus, it may provide a handle on quantum gravity in finite and dynamical regions. I derive the above results, make connections with standard approaches to black hole entropy, and speculate on the meaning of it all. Implications for loop quantum gravity are also discussed.

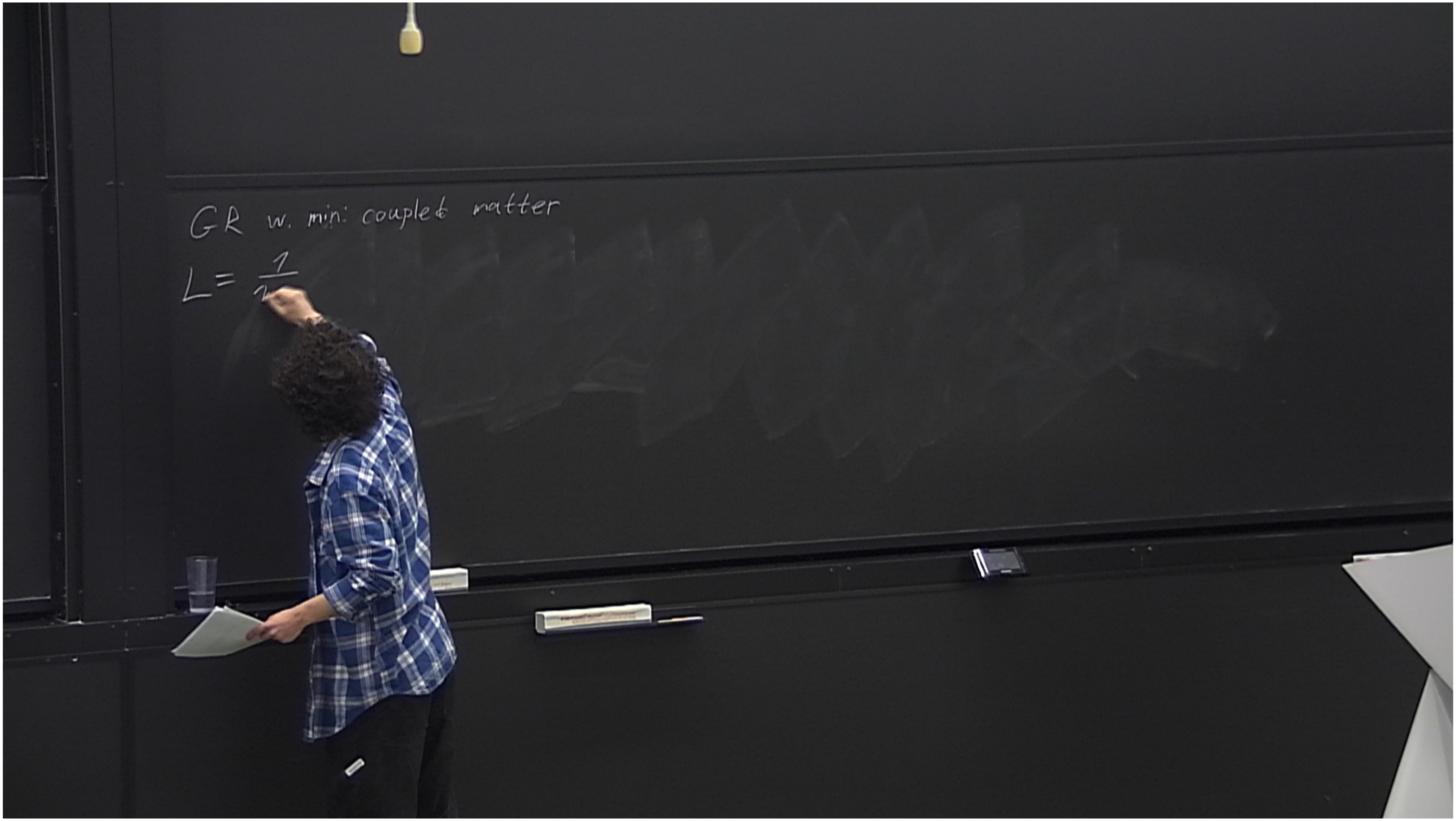


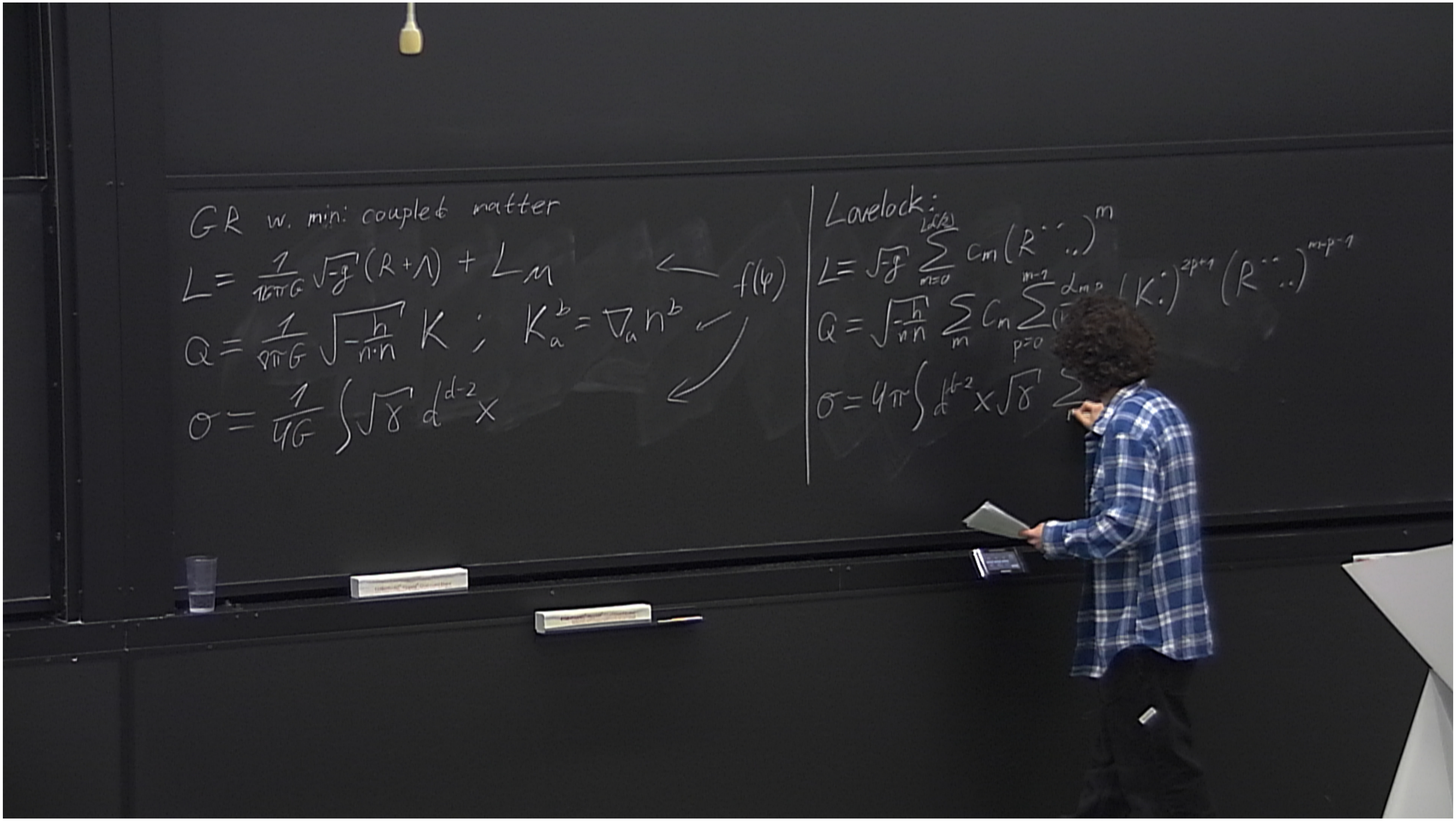
The imaginary part of the action in Lorentzian gravity
1212.2922, 1301.7041, 1303.4752 (with N. Bodendorfer), 1305.soon
$n^2=0$
$n^2<0$
flip surface (codim-2)
$n^2>0$
$\partial\Omega$ (codim-1)
Ω
$n^2<0$
$\mathrm{Im}\, S$
$\sum_{\text{flips}} \sigma_{\text{flip}}$
(and
nds on intrinsic metric)

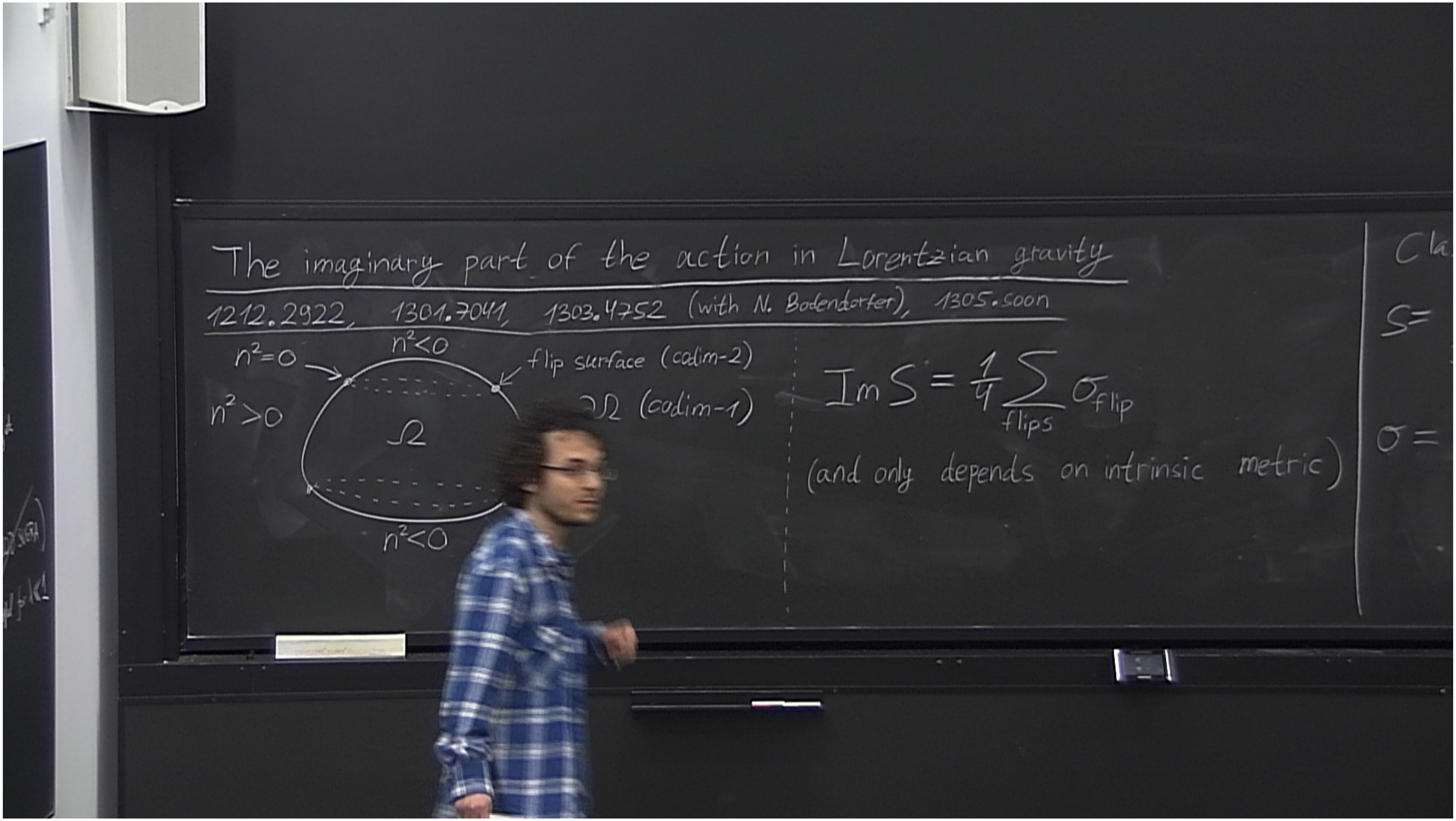

The imaginary part of the action in Lorentzian gravity
1212.2922, 1301.7041, 1303.4752 (with N. Bodendorfer), 1305.soon
n²=0
n²<0
n²>0
flip surface (codim-2)
∂Ω (codim-1)
Ω
n²<0
Im S = 1/4 ∑_flips σ_flip
(and only depends on
metric)



Classical gravity with ≤2 time derivatives
$S = \int_{\Omega} L \, d^d x + \int_{\partial\Omega} Q \, d^{d-1} x$
$\sigma = \int_{\Sigma} (\ldots) \, d^{d-2} x$
ric)
GR w. min. cou


GR w. min. coupled matter
L = 1


GR w. min. coupled matter
L = 1/16πG √-g (R+Λ) + L_M
f(φ)
Q = 1/8πG √(-h/n·n) K ; K_a^b = ∇_a n^b
σ = 1/4G ∫ √γ d^{d-2}x
Lovelock:
L = √-g Σ_{m=0} c_m (R)^m
σ = 4π ∫ d^{d-2}x √γ Σ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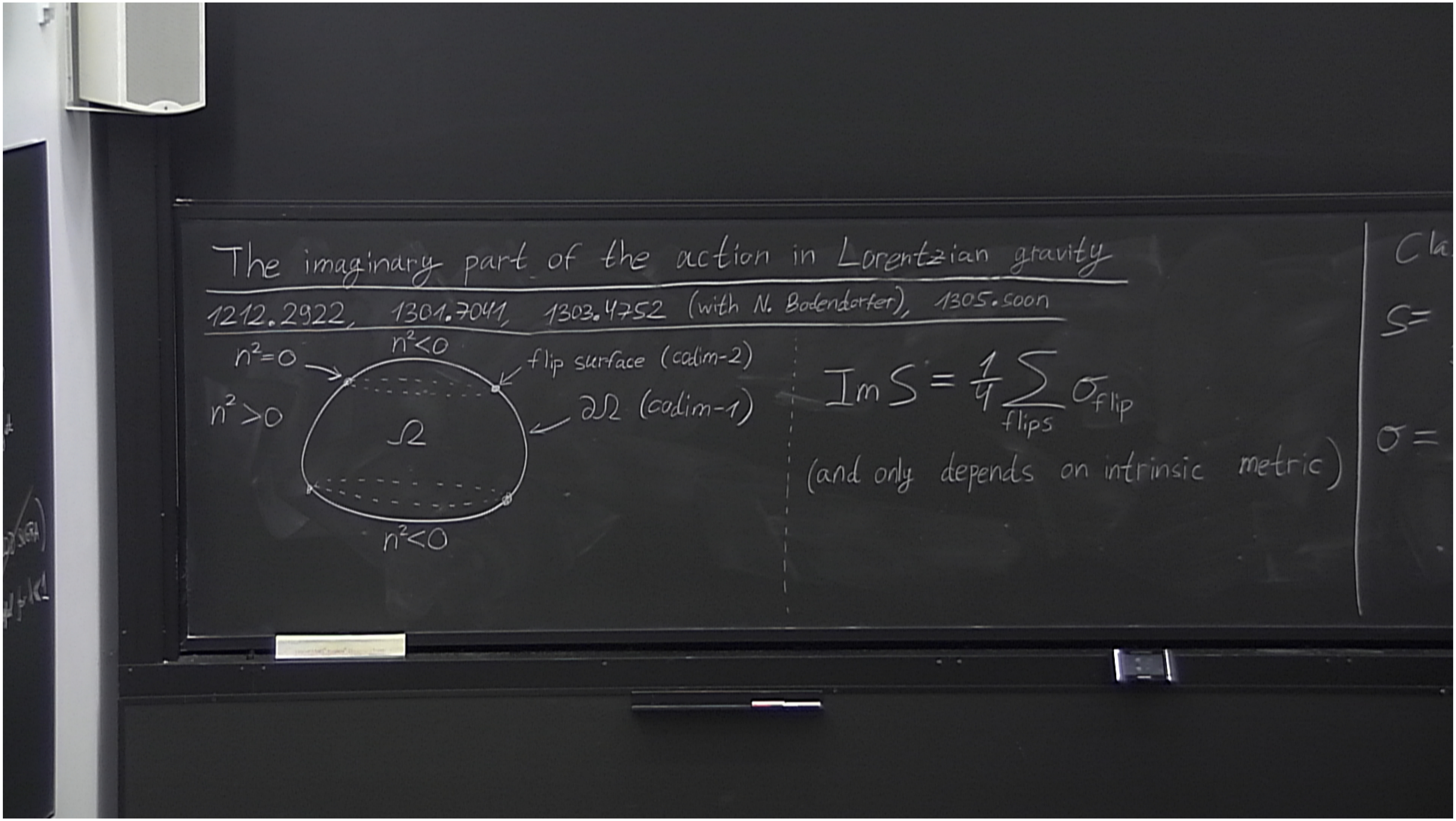
flip surface (codim-2)
$n^2>0$
$\partial\Omega$ (codim-1)
Ω
$n^2<0$
$\mathrm{Im}\, S = \frac{1}{4} \sum_{\text{flips}} \sigma_{\text{flip}}$
(and only depends on intrinsic metric)
$S=$
$\sigma=$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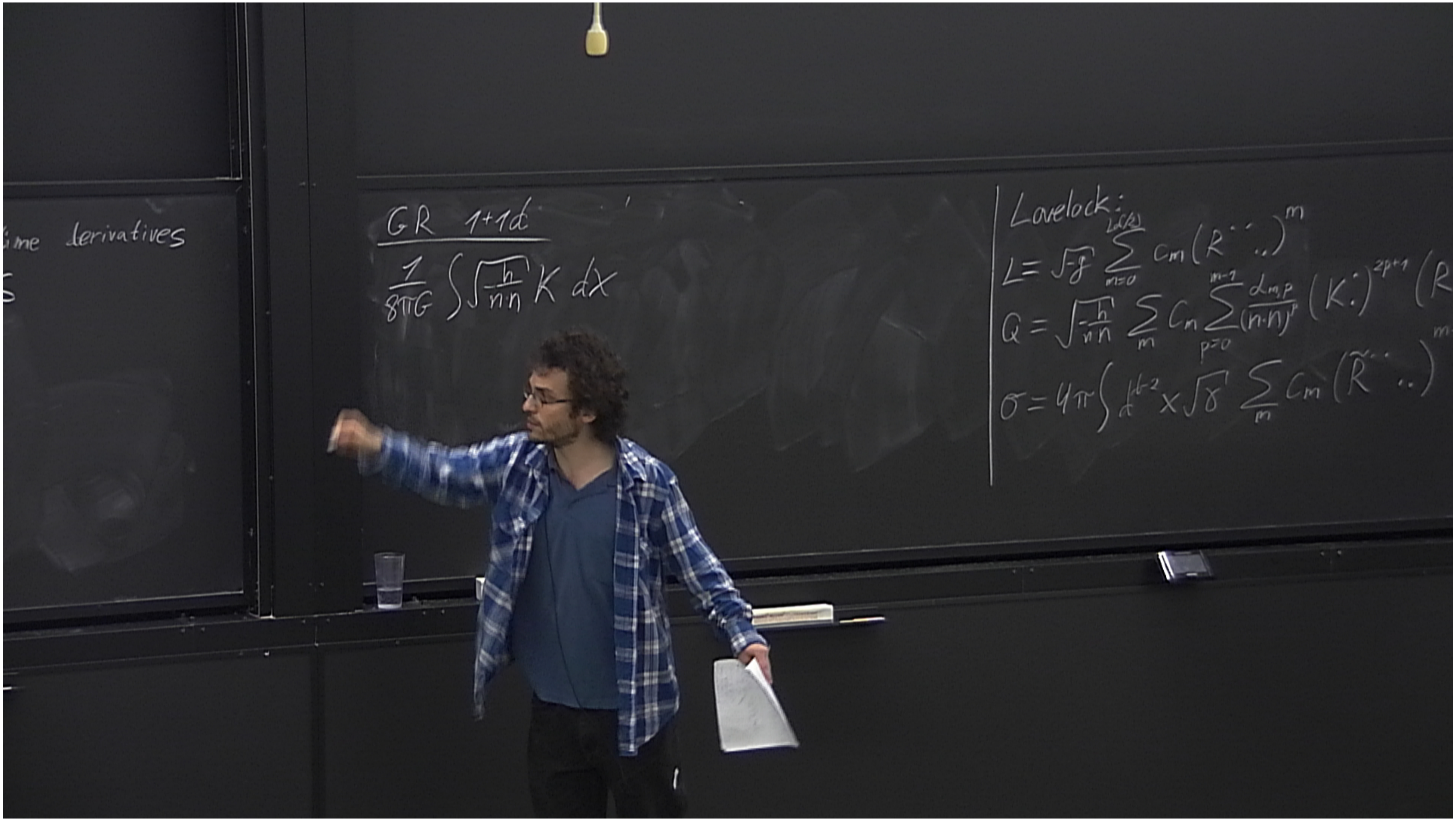
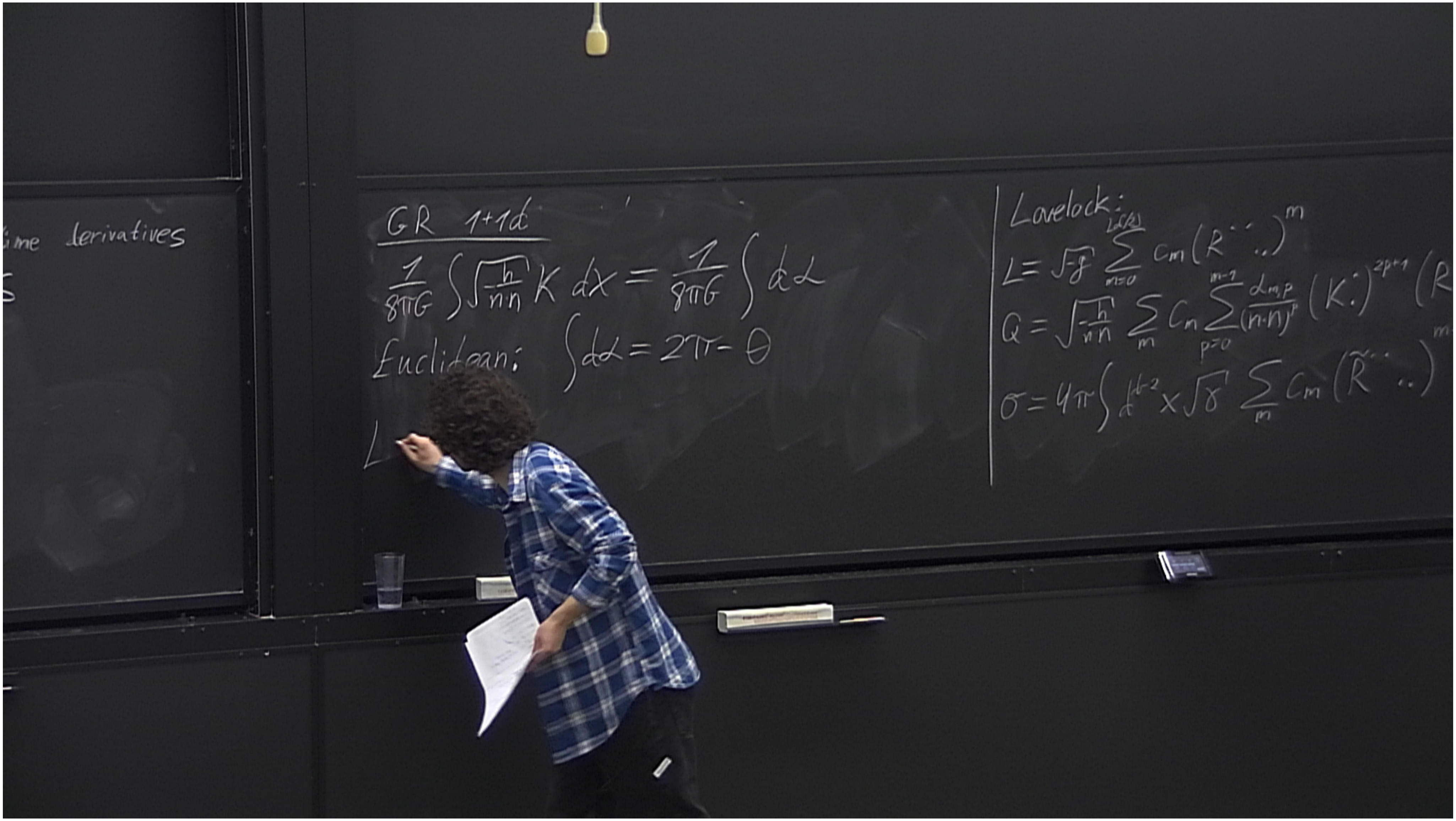
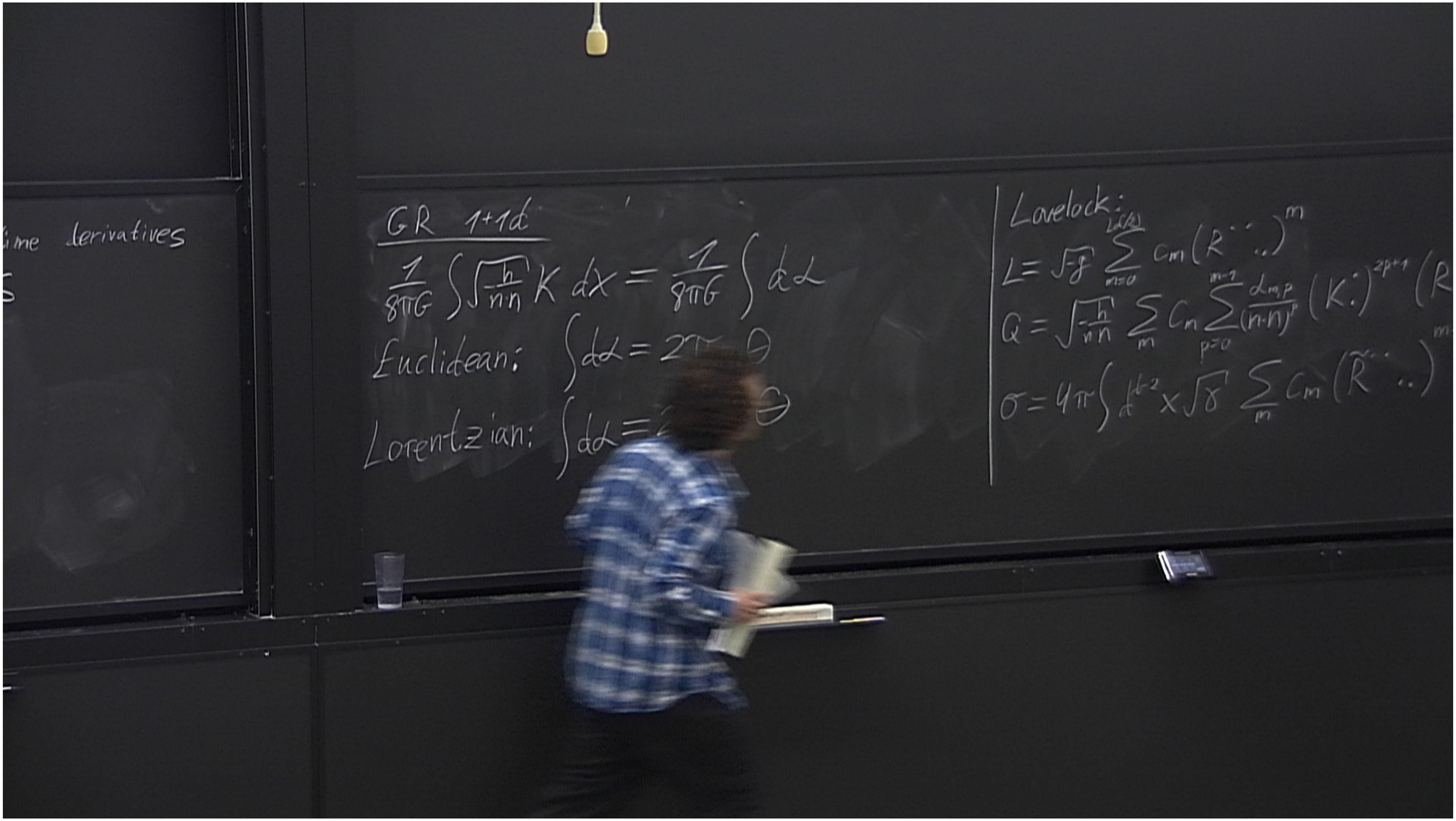
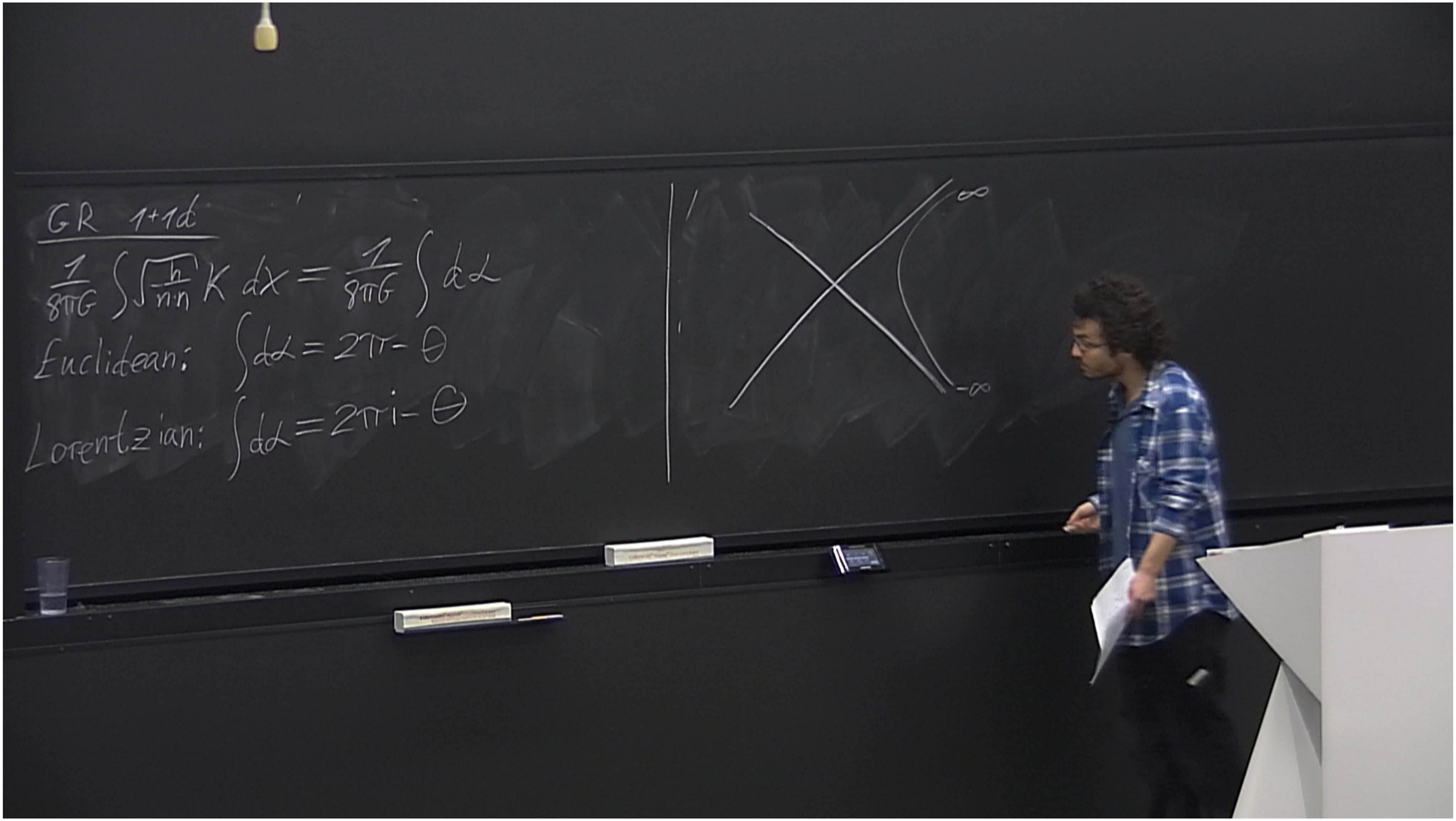
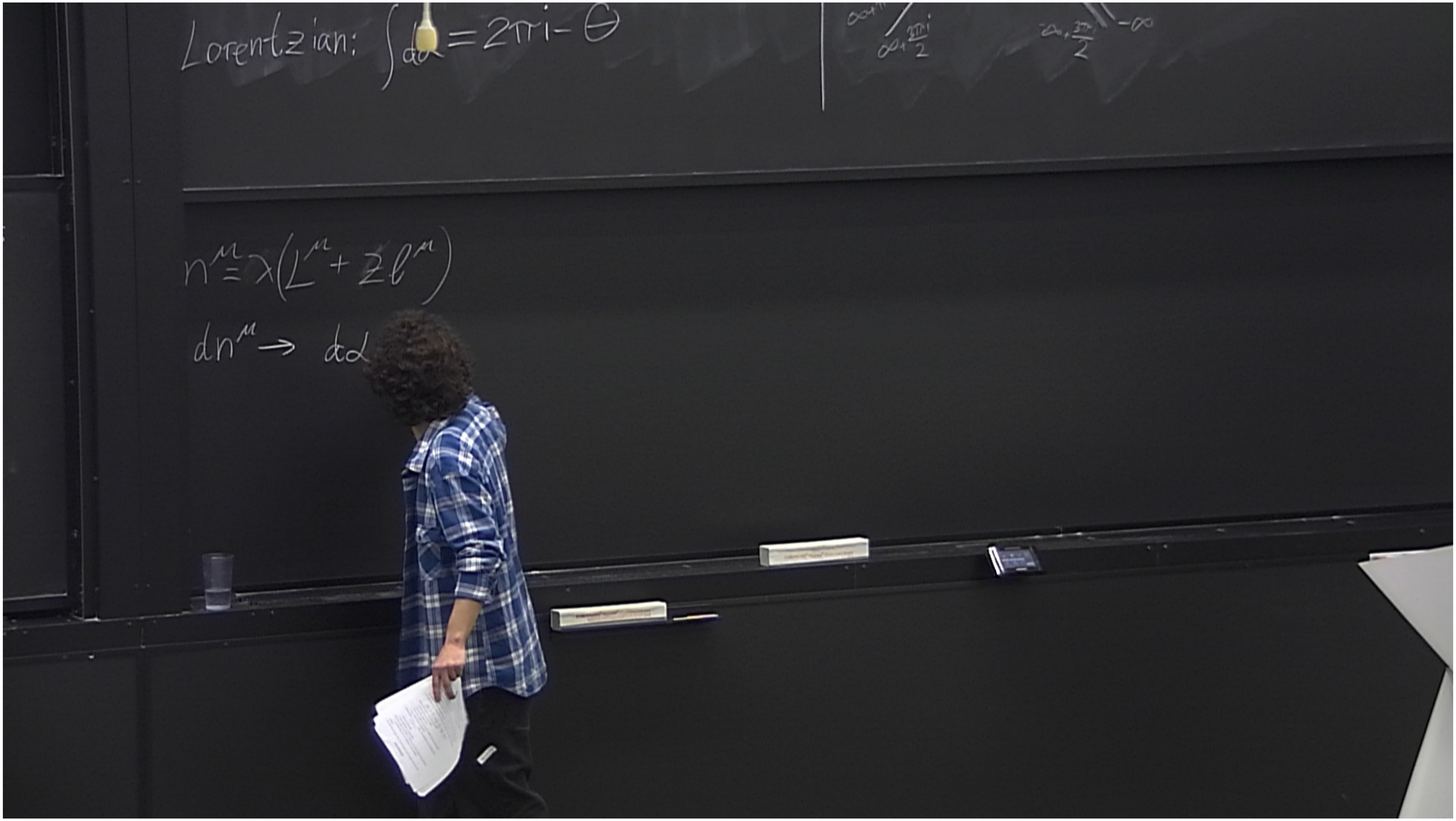
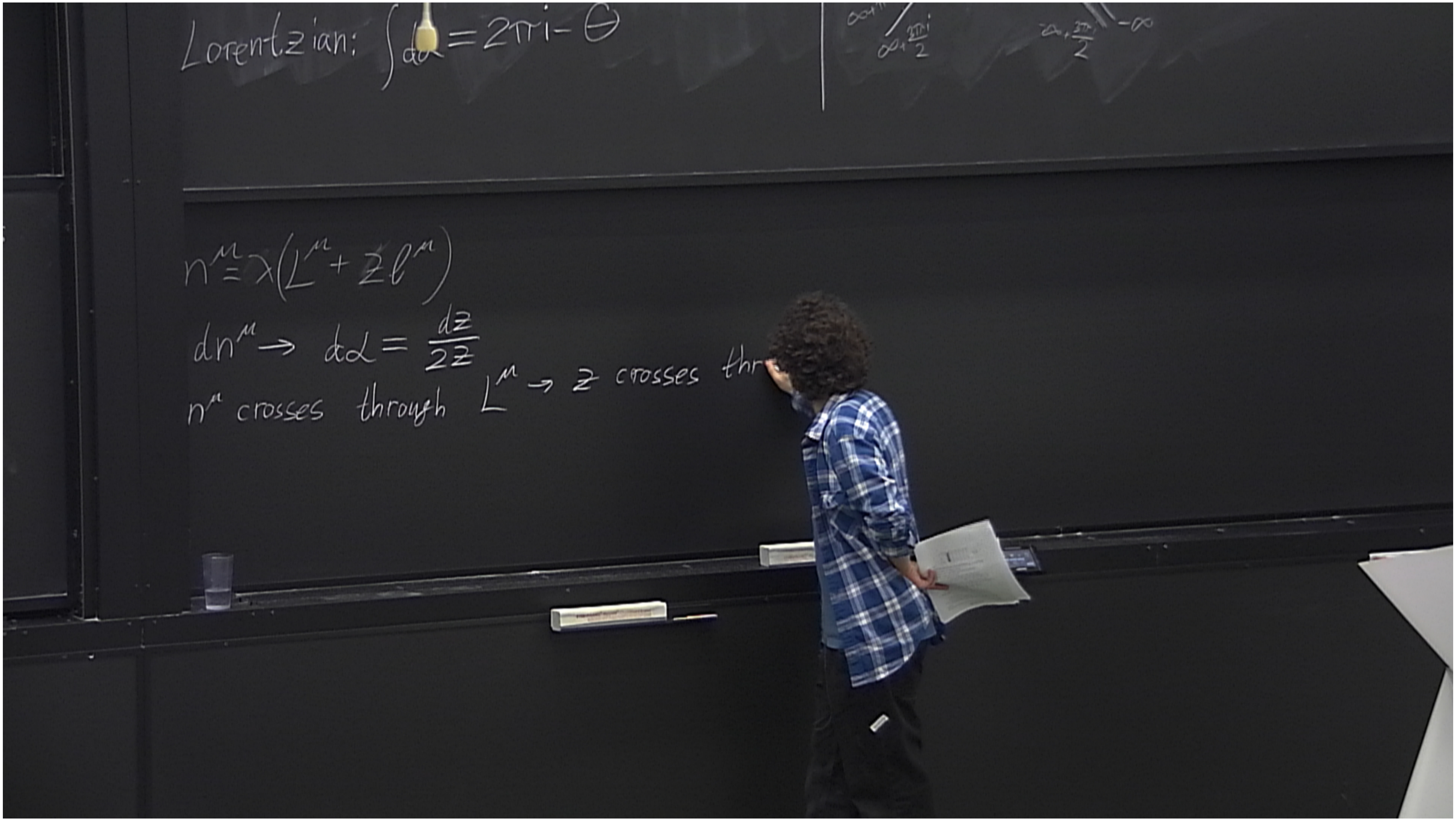

derivatives
GR 1+1d
Lovelock:


ime derivatives
GR 1+1d
Euclidean:
Lovelock:


derivatives
GR 1+1d
Euclidean:
Lorentzian:
Lovelock:


GR 1+1d
1/8πG ∫ √(h/n·n) K dx = 1/8πG ∫ dα
Euclidean: ∫ dα = 2π − θ
Lorentzian: ∫ dα = 2πi − θ
∞
−∞


Lorentzian: ∫dα = 2πi − Θ
nμ = λ(Lμ + z ℓμ)
dnμ → dα


Lorentzian: $\int d\alpha = 2\pi i - \Theta$
$n^\mu = \lambda(L^\mu + z\ell^\mu)$
$dn^\mu \rightarrow d\alpha = \frac{dz}{2z}$
n^μ crosses through $L^\mu \rightarrow z$ crosses thr

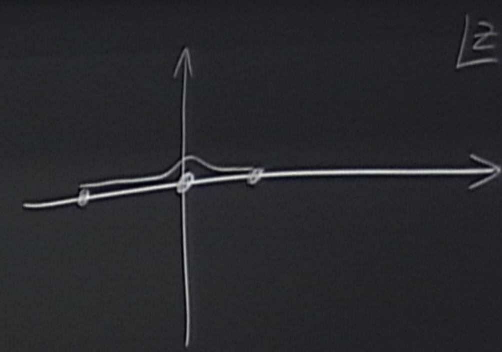
Lorentzian: $\int d\alpha = 2\pi i - \Theta$

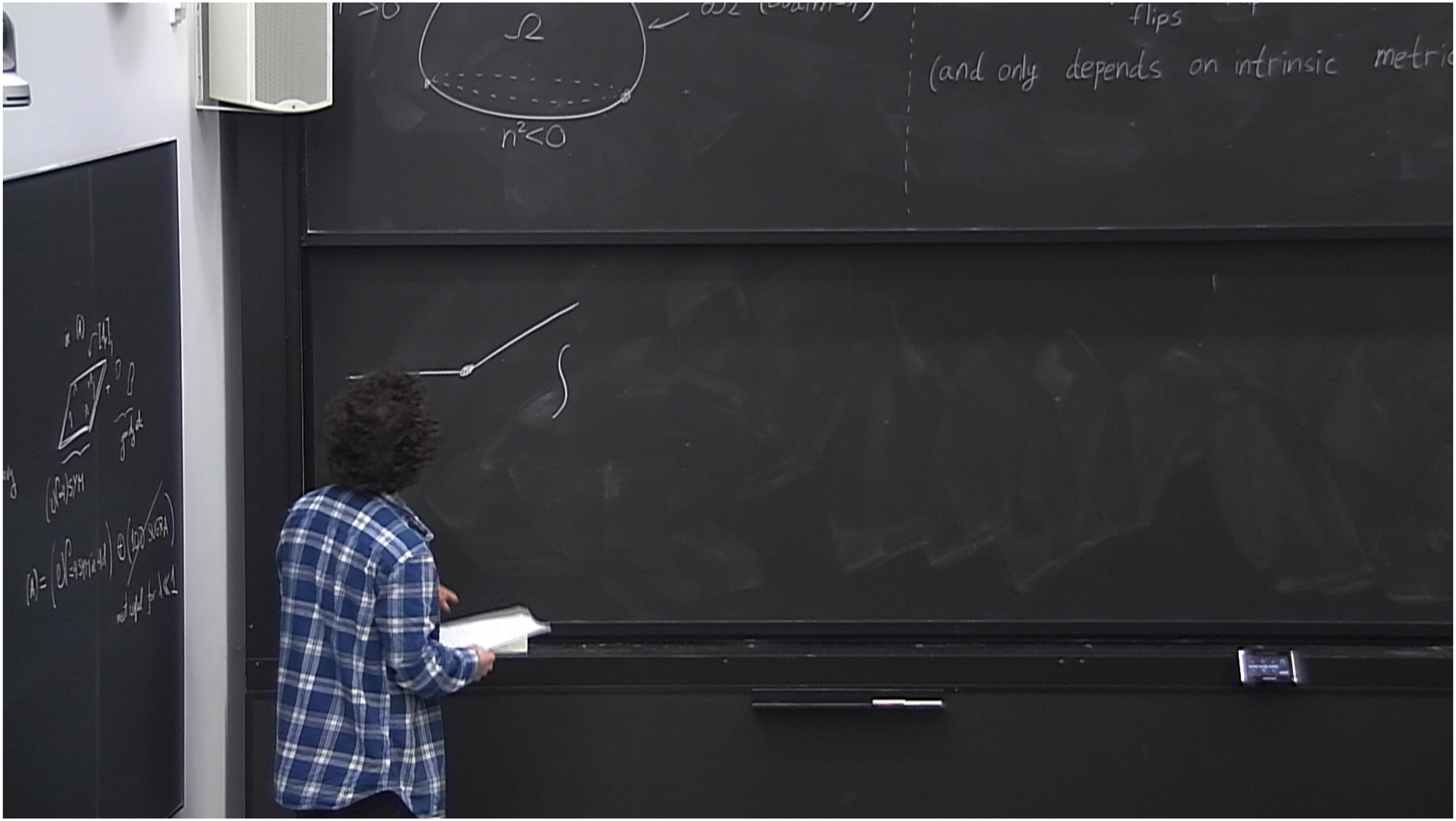
$\infty + \frac{2\pi i}{2}$ $\quad$ $-\infty + \frac{3\pi i}{2}$ $\quad$ $-\infty$

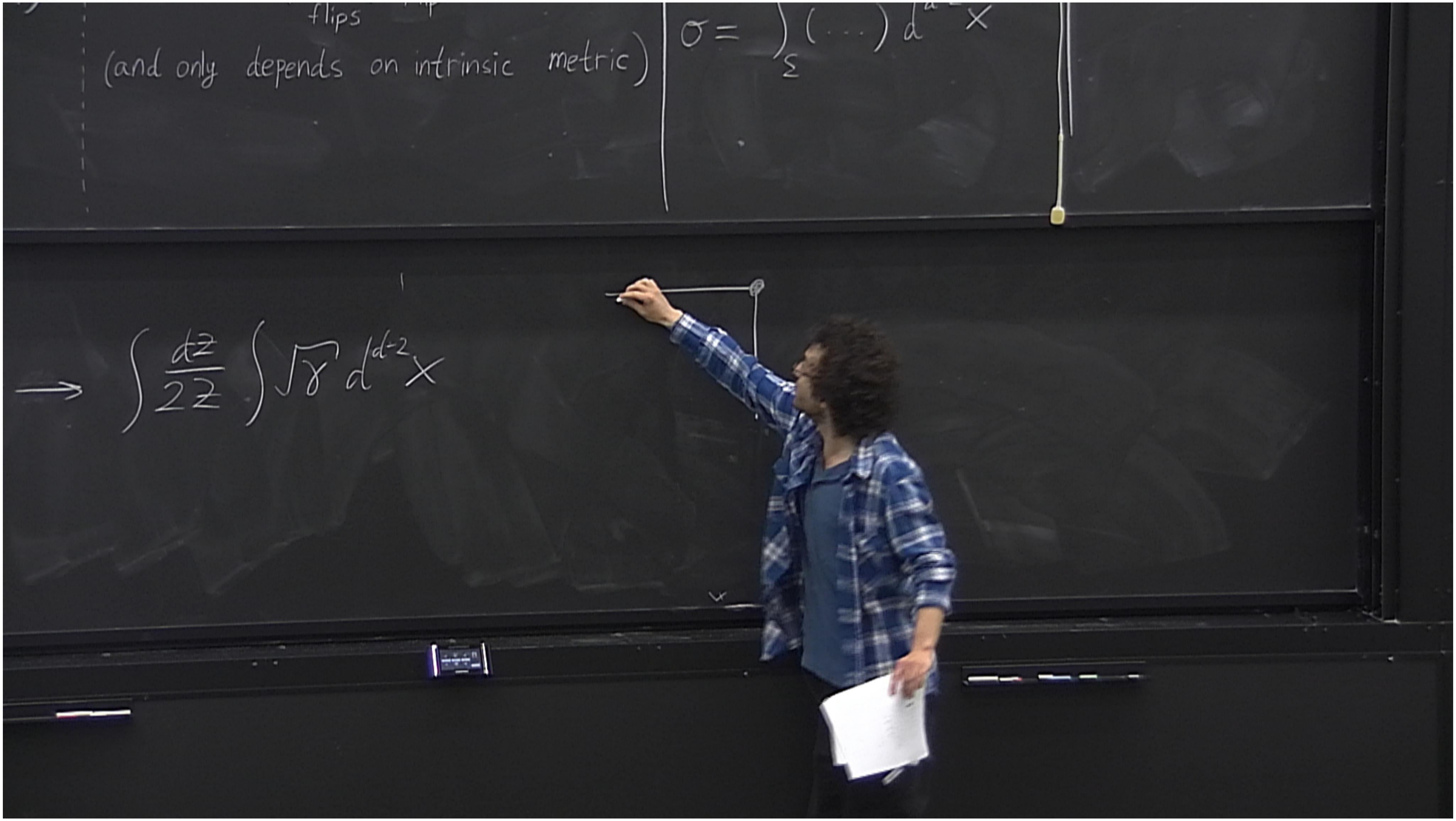
$n^\mu = \lambda(L^\mu + z\,\ell^\mu)$

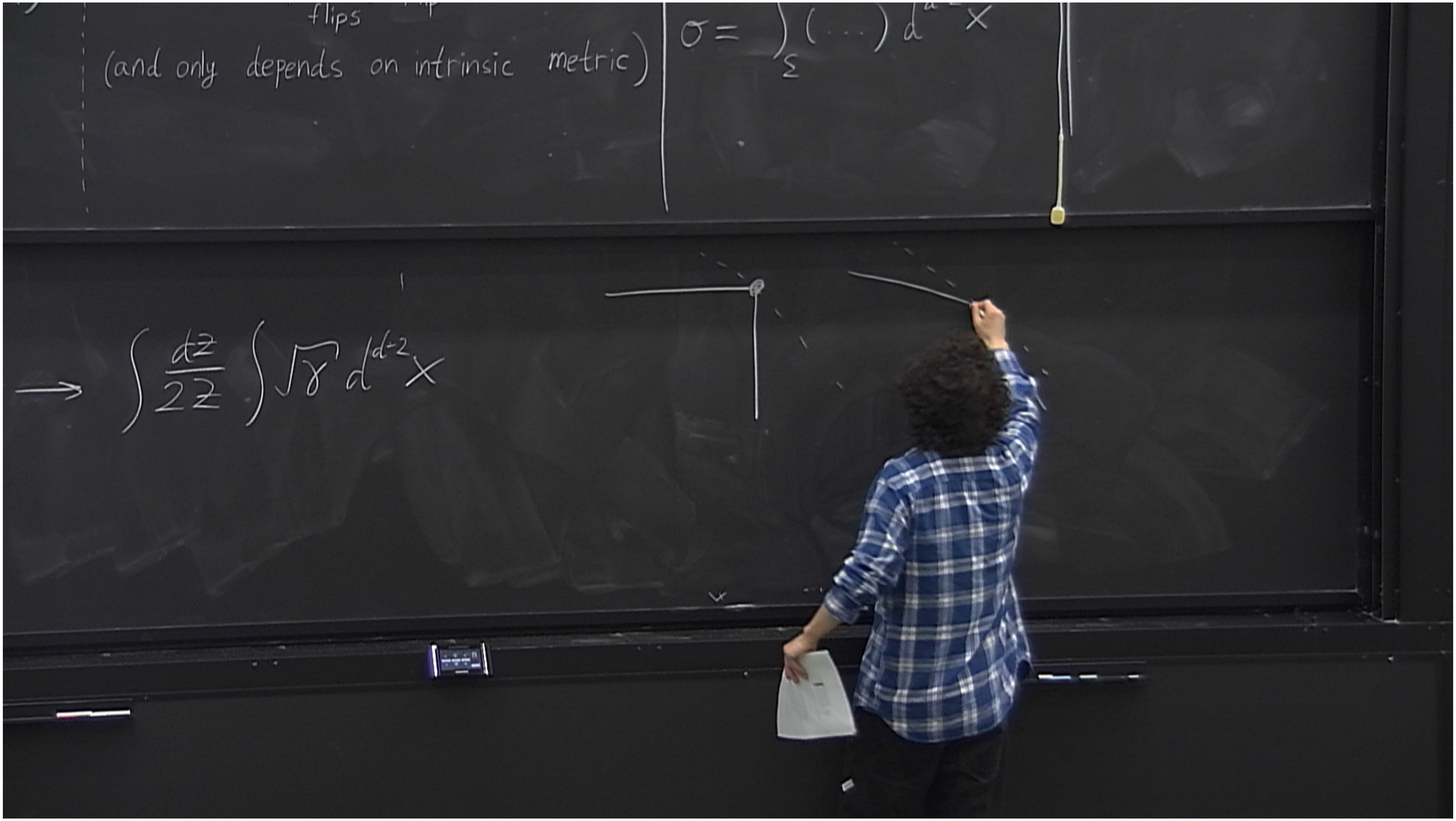
$dn^\mu \to d\alpha = \frac{dz}{2z}$

n^μ crosses through $L^\mu \to z$ crosses through 0.




flips
(and only depends on intrinsic
Ω
$n^2<0$


flips
(and only depends on intrinsic metric)
σ = ∫_Σ (. . .) d^{d-2}x
→ ∫ dz/2z ∫ √γ d^{d-2}x

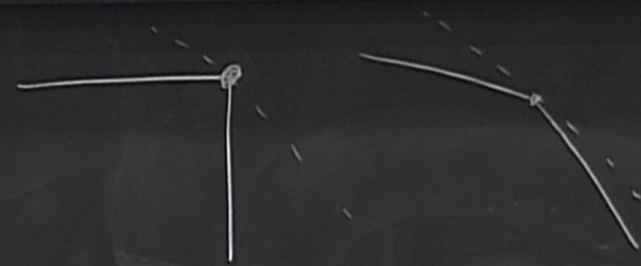

flips
(and only depends on intrinsic metric)
$\sigma = \int_\Sigma (\dots)\, d^{d-2}x$
$\rightarrow \int \frac{dz}{2z} \int \sqrt{\gamma}\, d^{d-2}x$

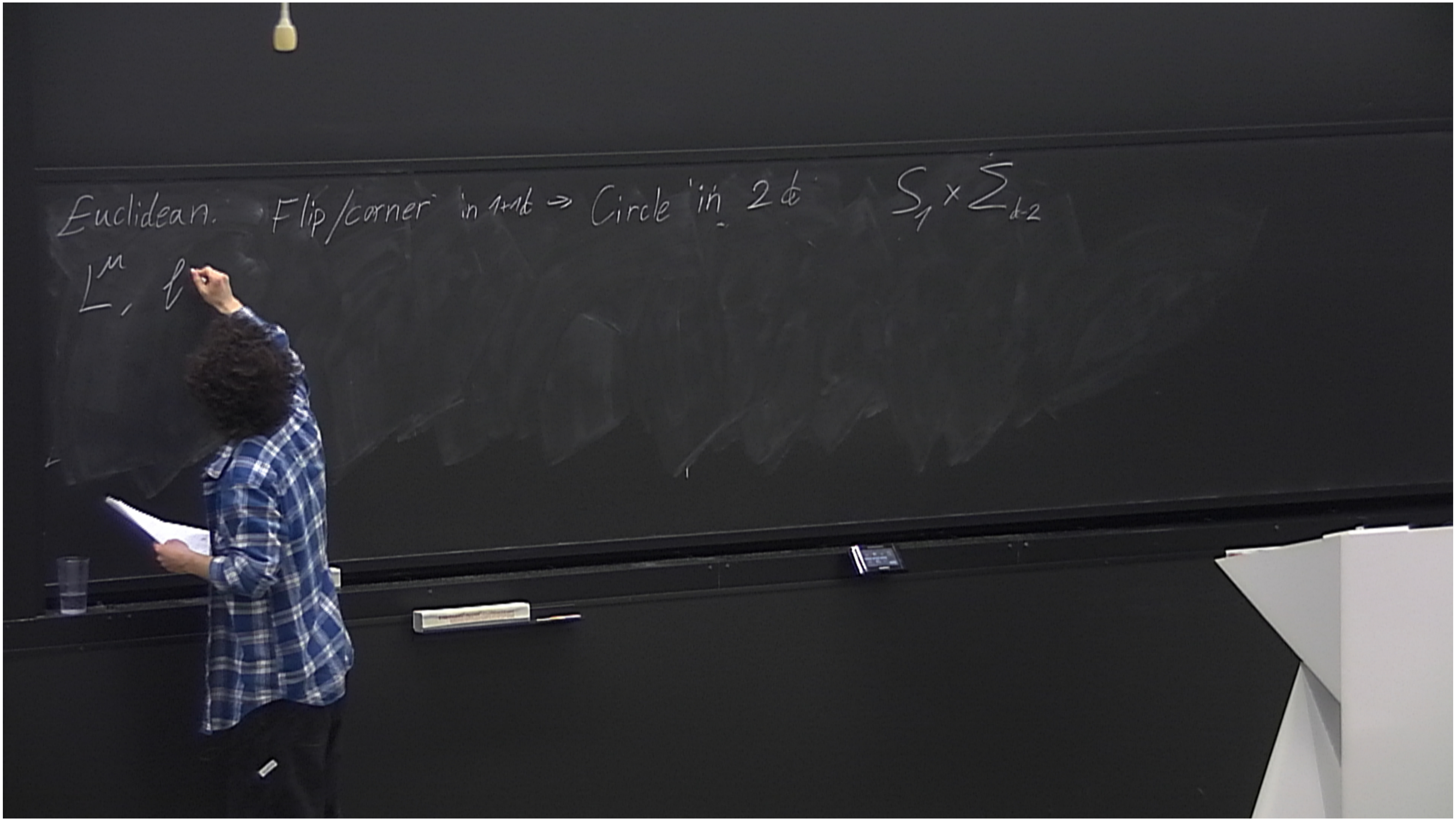
flips

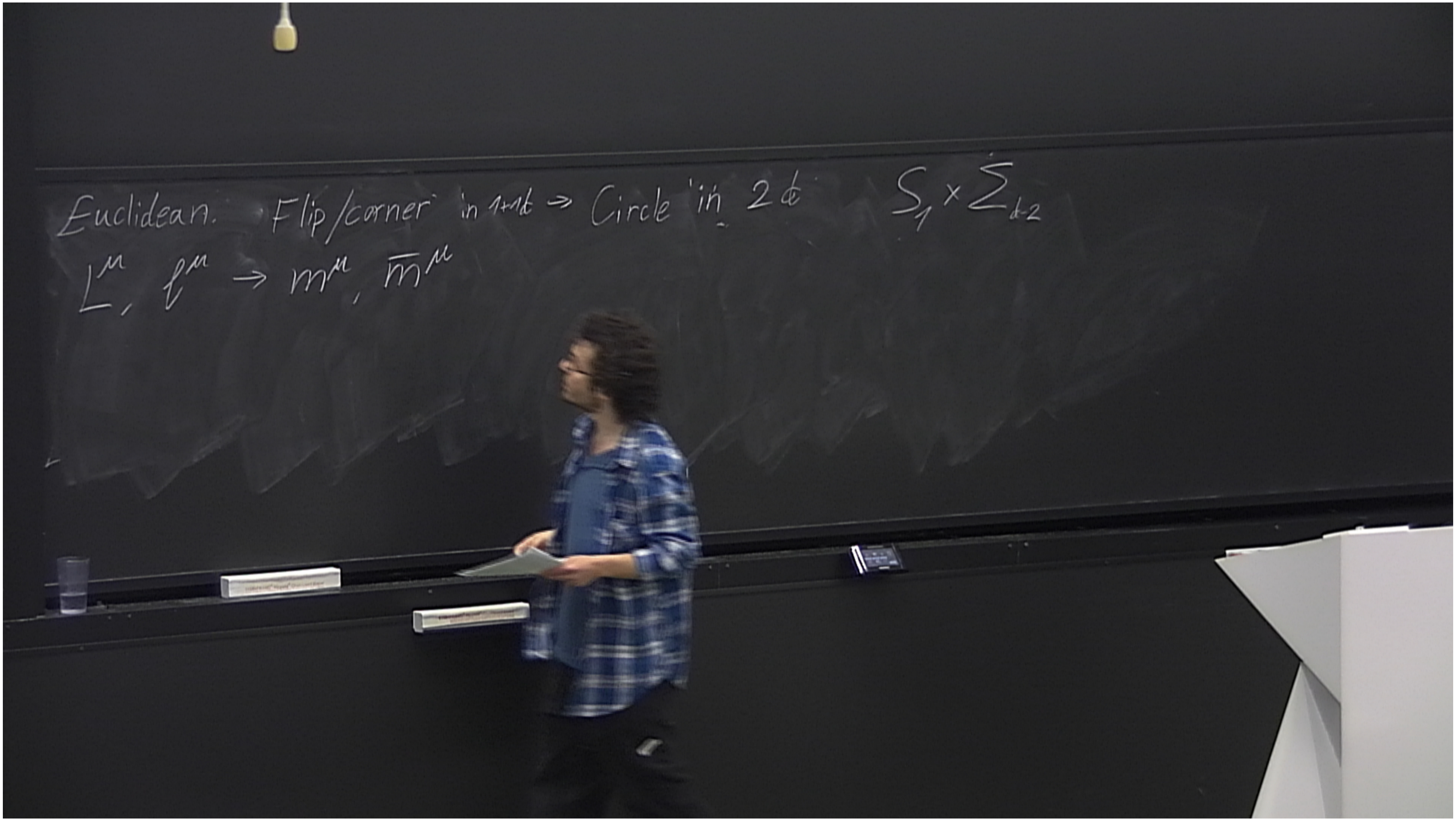
(and only depends on intrinsic metric)

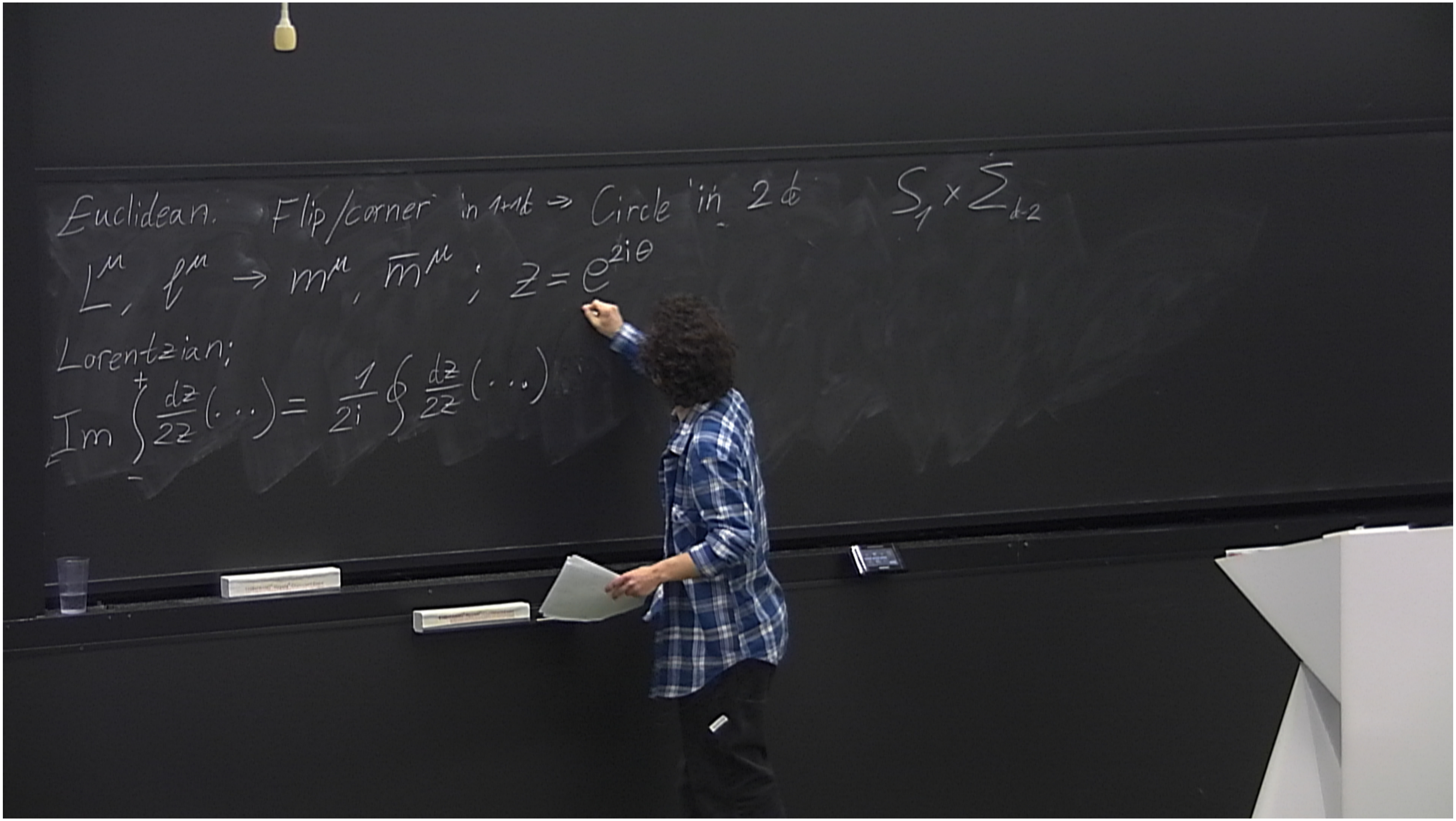
$$\sigma = \int_\Sigma (\dots)\, d^{d-2}x$$

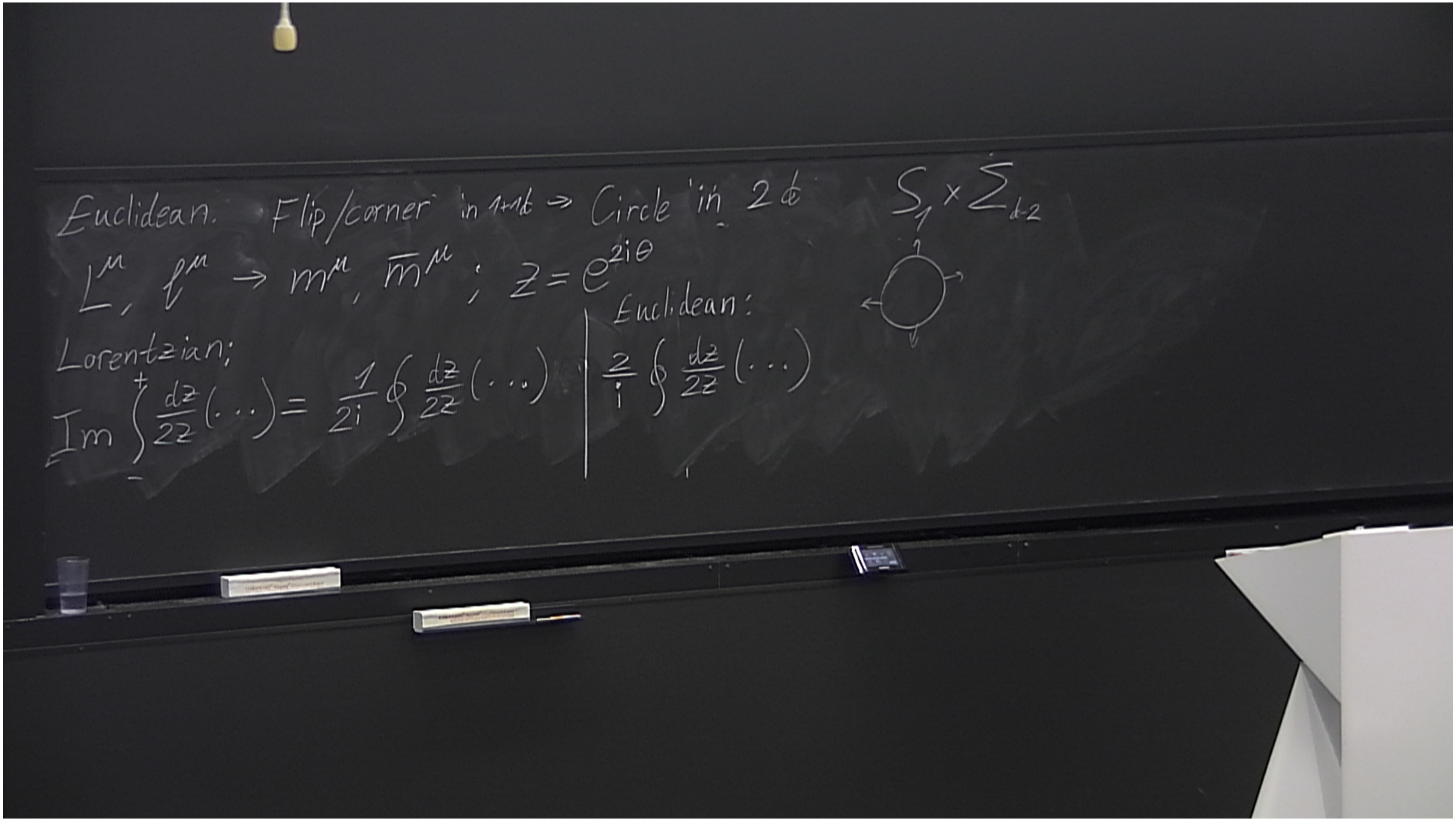
$$\longrightarrow \int \frac{dz}{2z} \int \sqrt{\gamma}\, d^{d-2}x$$

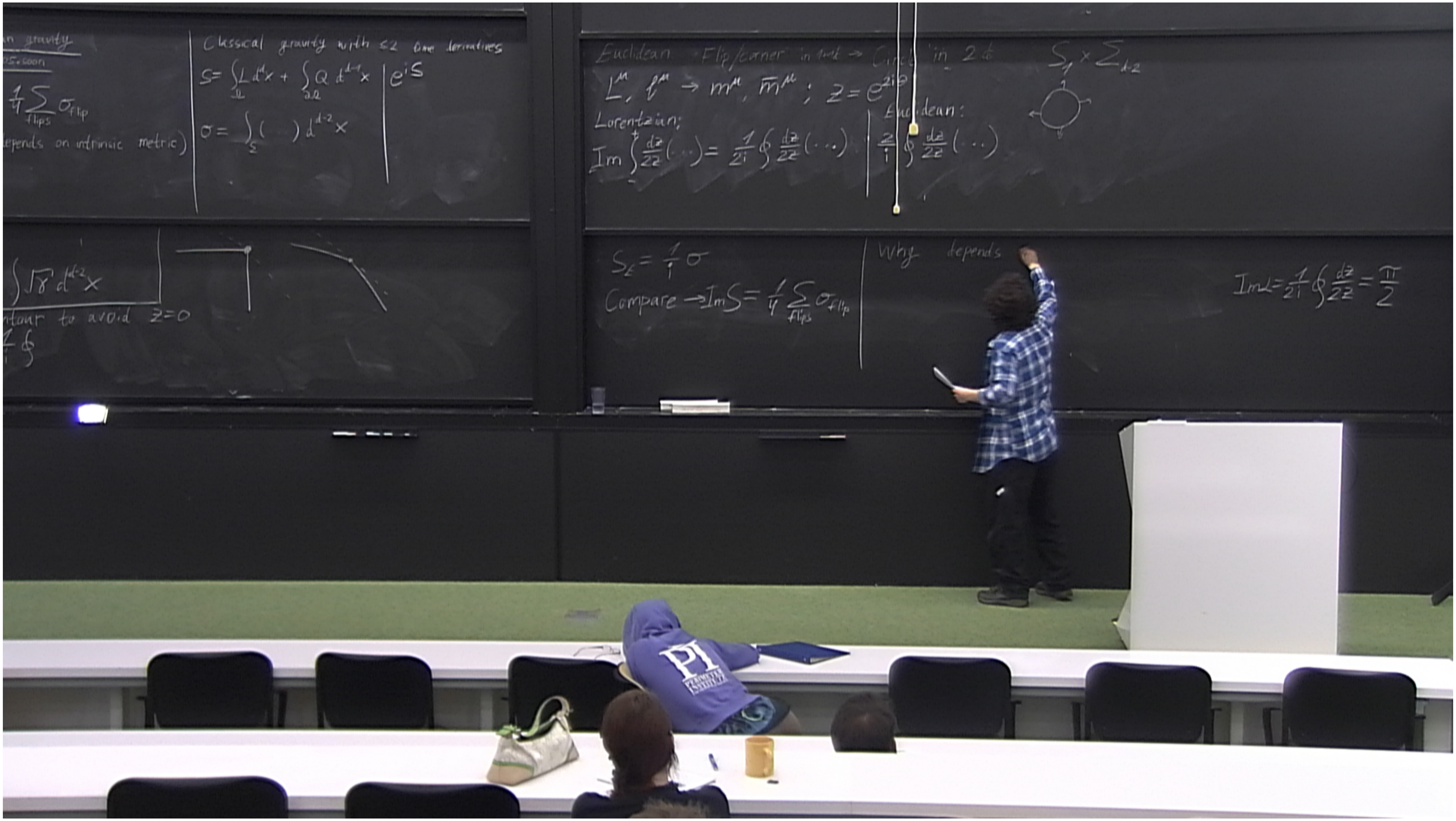


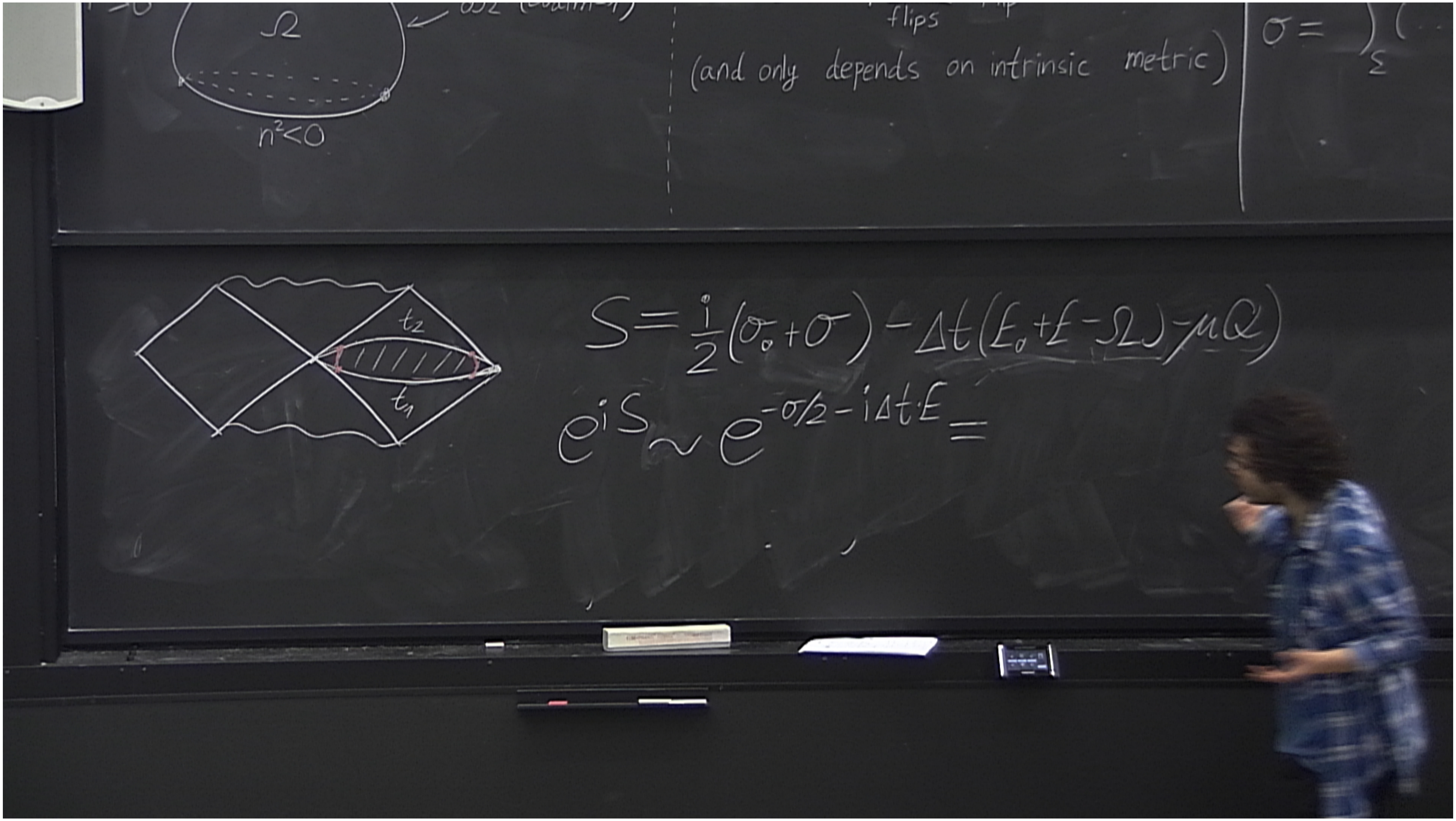

Euclidean. Flip/corner in 1+1d ⇒ Circle in 2d $S_1 \times \Sigma_{d-2}$
L^μ, ℓ


Euclidean. Flip/corner in 1+1d ⇒ Circle in 2d $S_1 \times \Sigma_{d-2}$
$L^\mu, \ell^\mu \to m^\mu, \bar{m}^\mu$

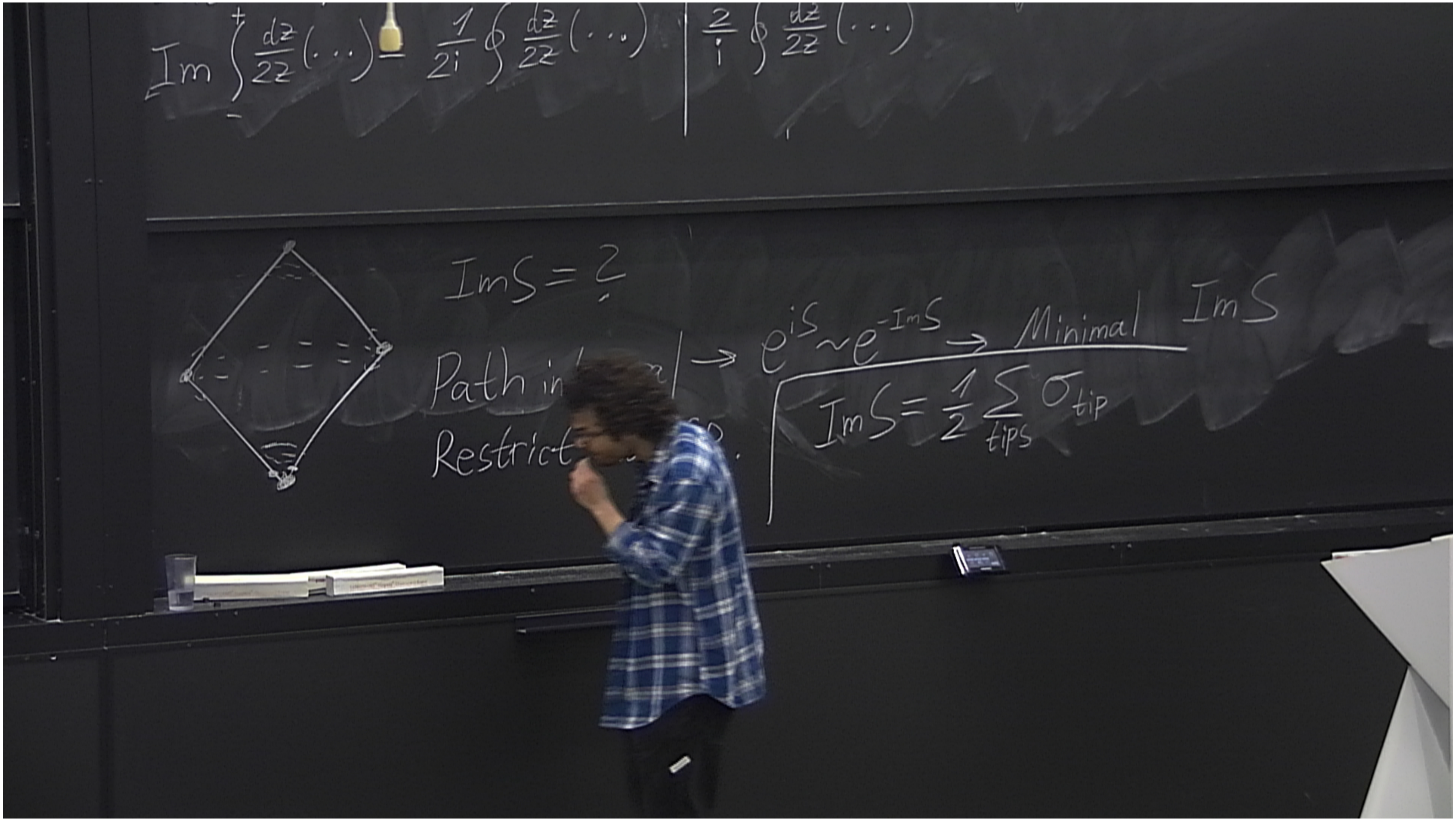

Euclidean. Flip/corner in 1+1d $\Rightarrow$ Circle in 2d $S_1 \times \Sigma_{d-2}$
$L^\mu, \ell^\mu \to m^\mu, \bar{m}^\mu$; $z = e^{2i\theta}$
Lorentzian;
$\mathrm{Im} \int_-^+ \frac{dz}{2z}(\ldots) = \frac{1}{2i} \oint \frac{dz}{2z}(\ldots)$


Euclidean. Flip/corner in 1+1d → Circle in 2d
$S_1 \times \Sigma_{d-2}$
$L^\mu, \ell^\mu \rightarrow m^\mu, \bar{m}^\mu$; $z = e^{2i\theta}$
Lorentzian;
$\mathrm{Im} \int_{-}^{+} \frac{dz}{2z}(\dots) = \frac{1}{2i} \oint \frac{dz}{2z}(\dots)$
Euclidean:
$\frac{2}{i} \oint \frac{dz}{2z}(\dots)$

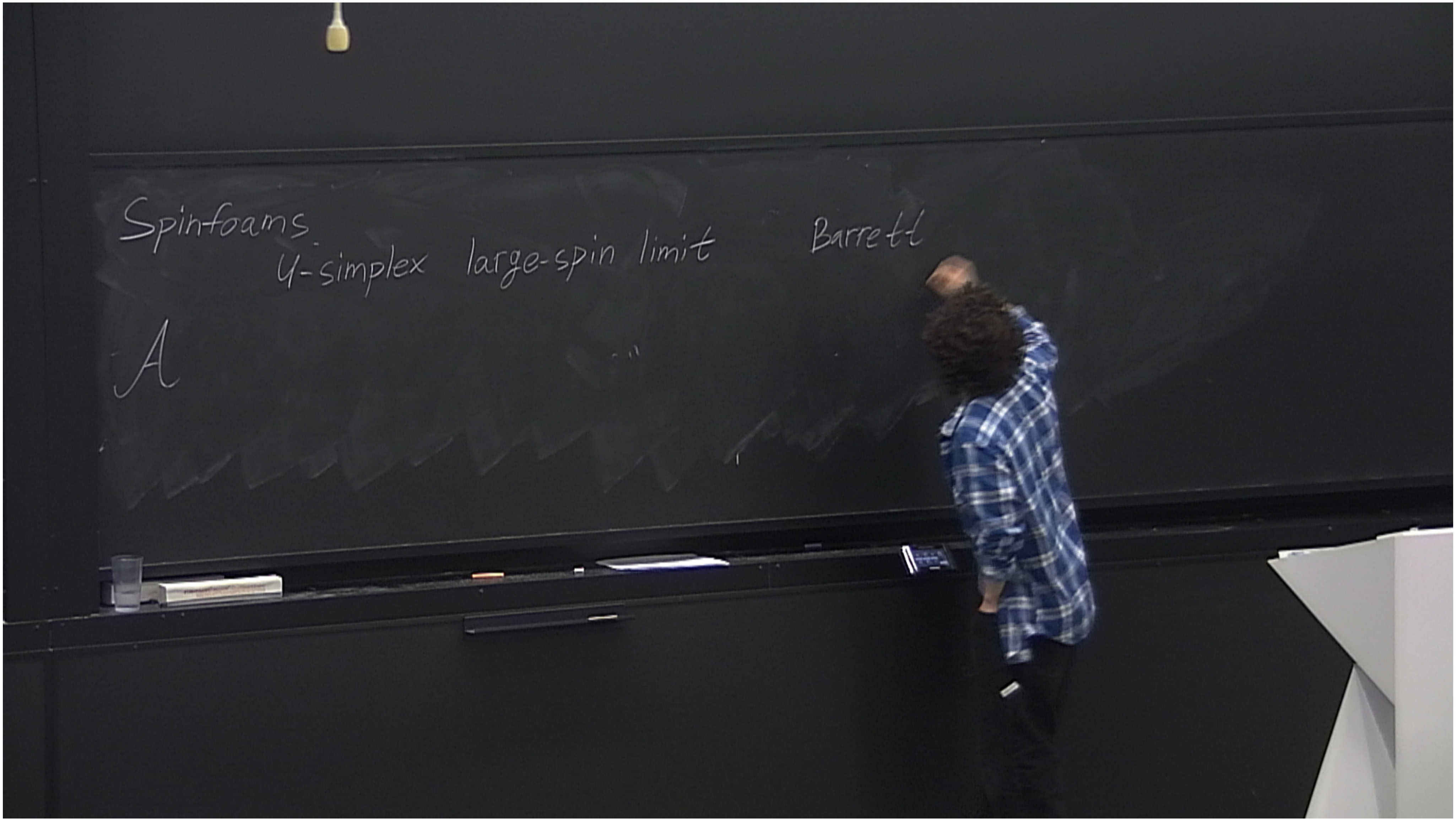

Classical gravity with ≤2 time derivatives
depends on intrinsic metric)
Euclidean
Lorentzian;
Euclidean:
Compare → Im S = 1/4 Σ σ_flip
Why depends

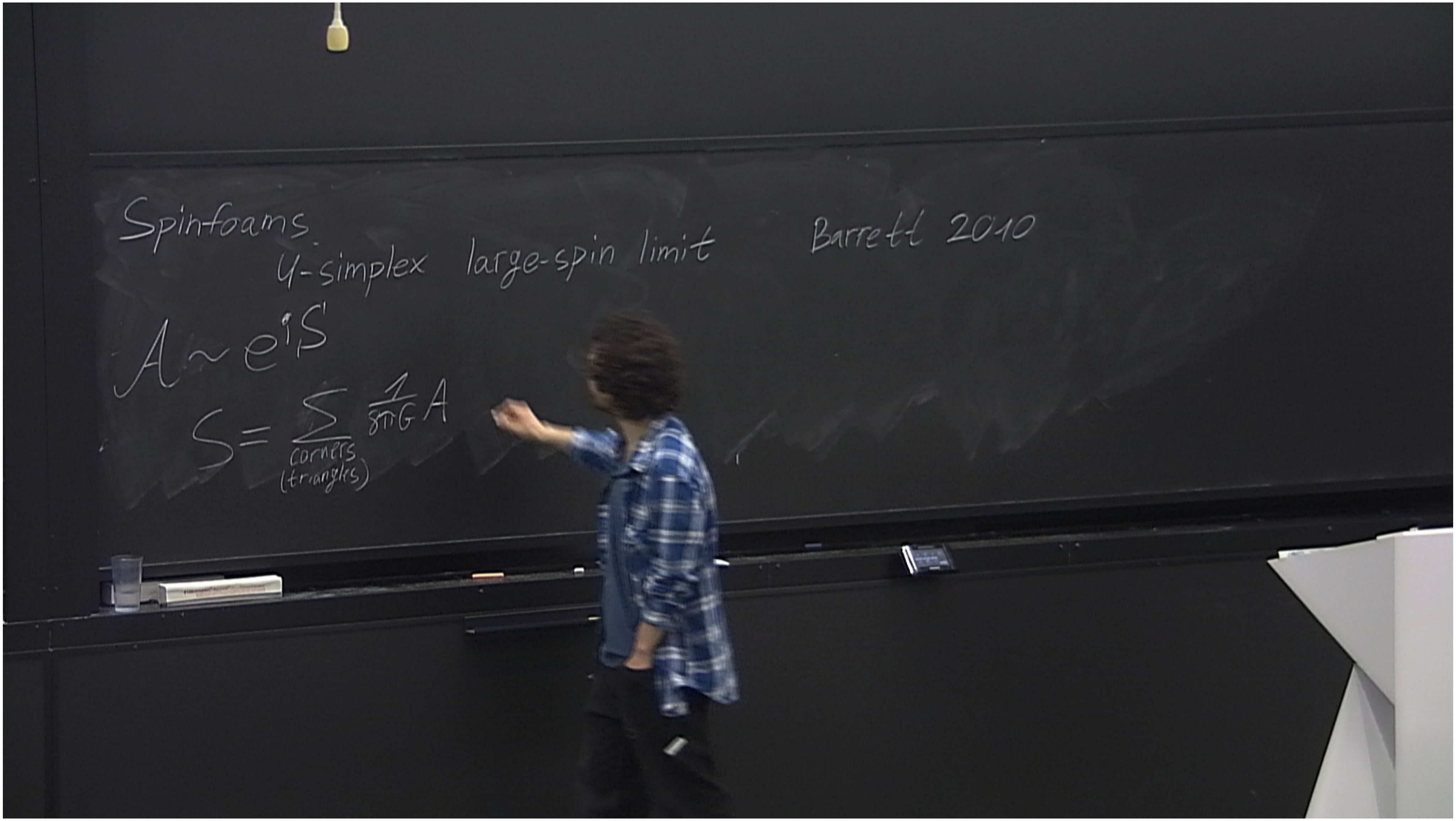


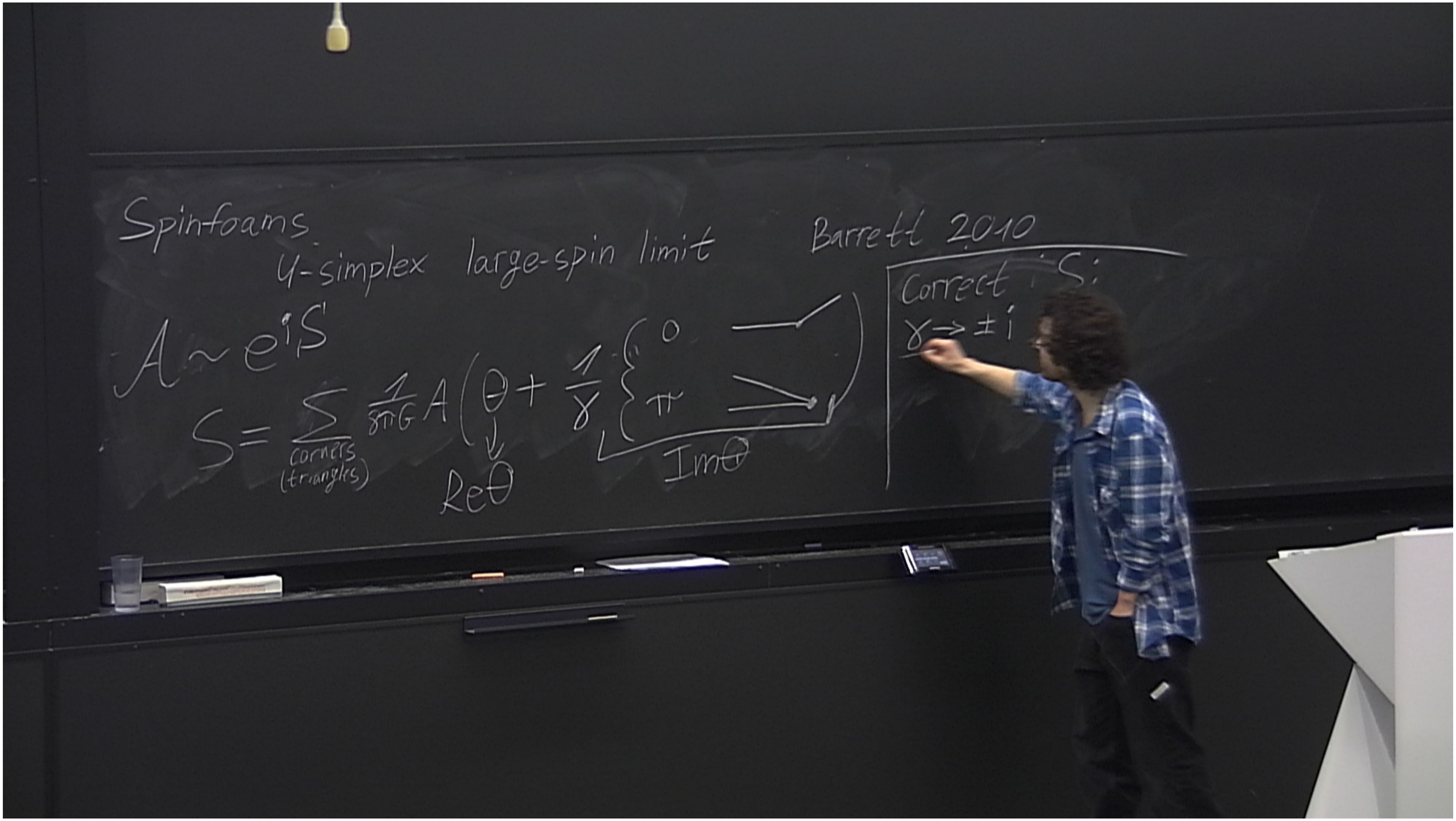
Ω
$n^2<0$
flips
(and only depends on intrinsic metric)
$\sigma = \int_\Sigma ($
t_2
t_1
$S = \frac{i}{2}(\sigma_0 + \sigma) - \Delta t(E_0 + E - \Omega J - \mu Q)$
$e^{iS} \sim e^{-\sigma/2 - i\Delta t \cdot E} =$

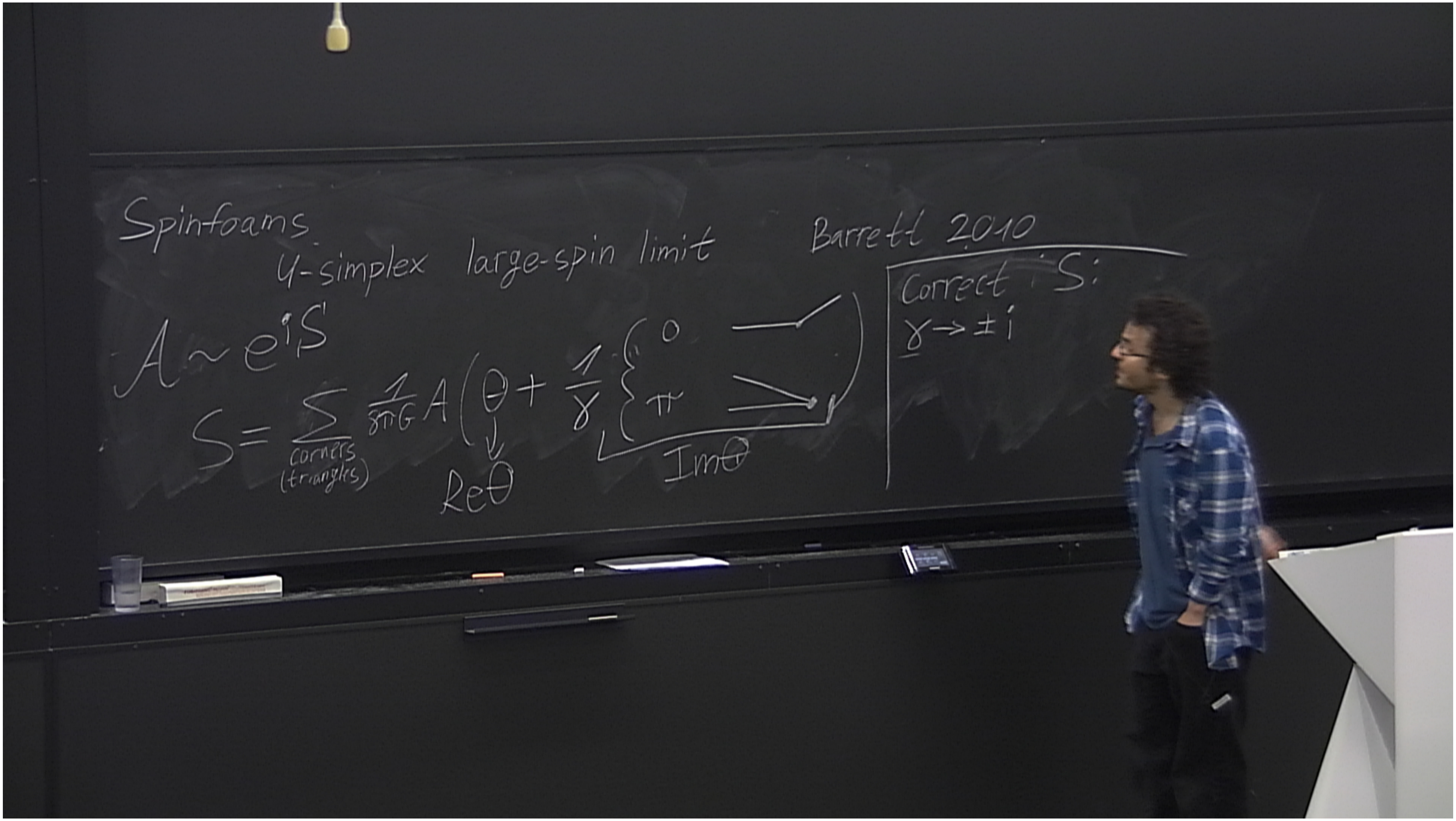


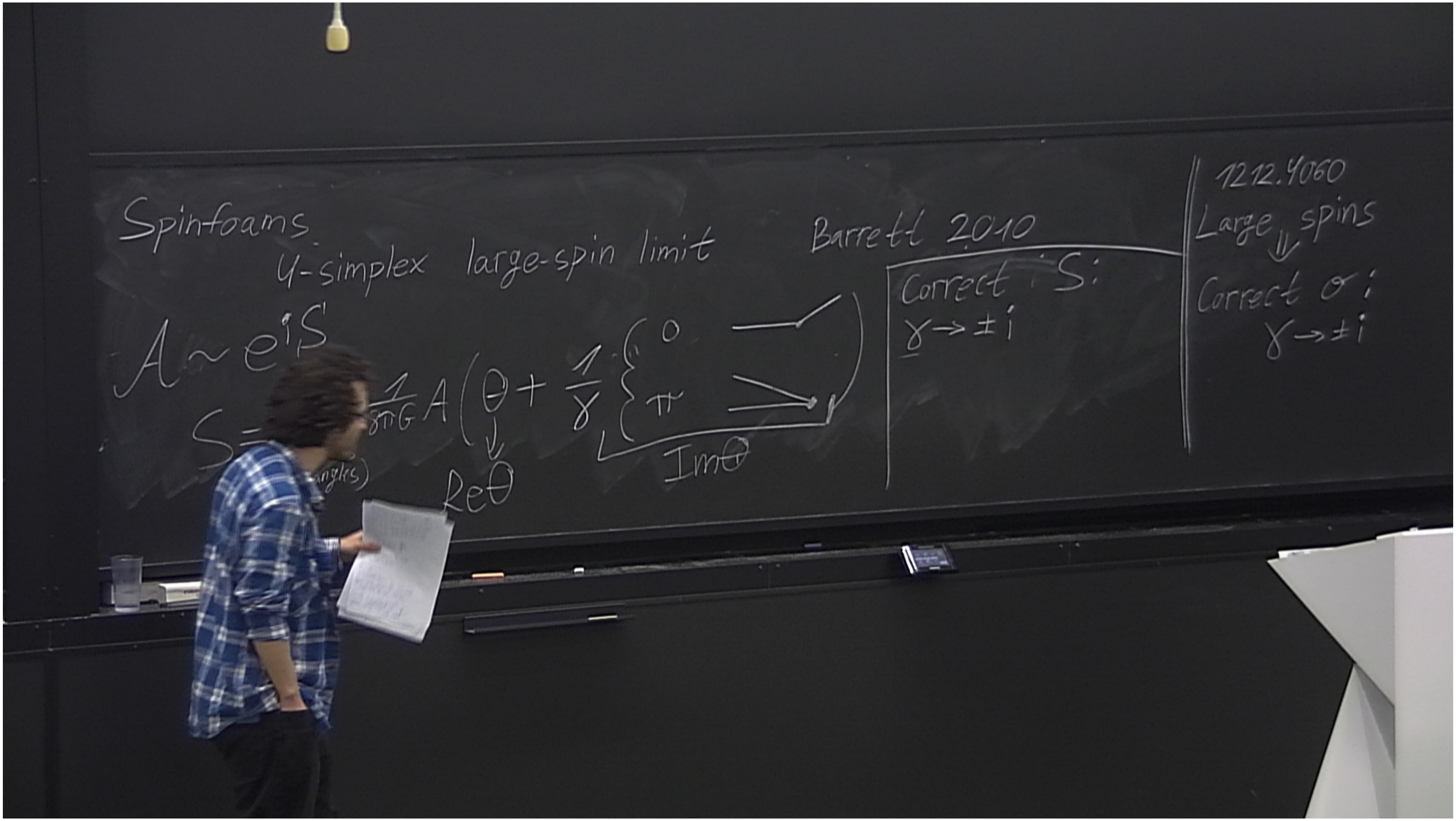
Im ∫ dz/2z (...) = 1/2i ∮ dz/2z (...)
2/i ∮ dz/2z (...)
ImS = ?
Path in
→ e^iS ~ e^-ImS → Minimal ImS
Restrict
ImS = 1/2 Σ tips σ tip

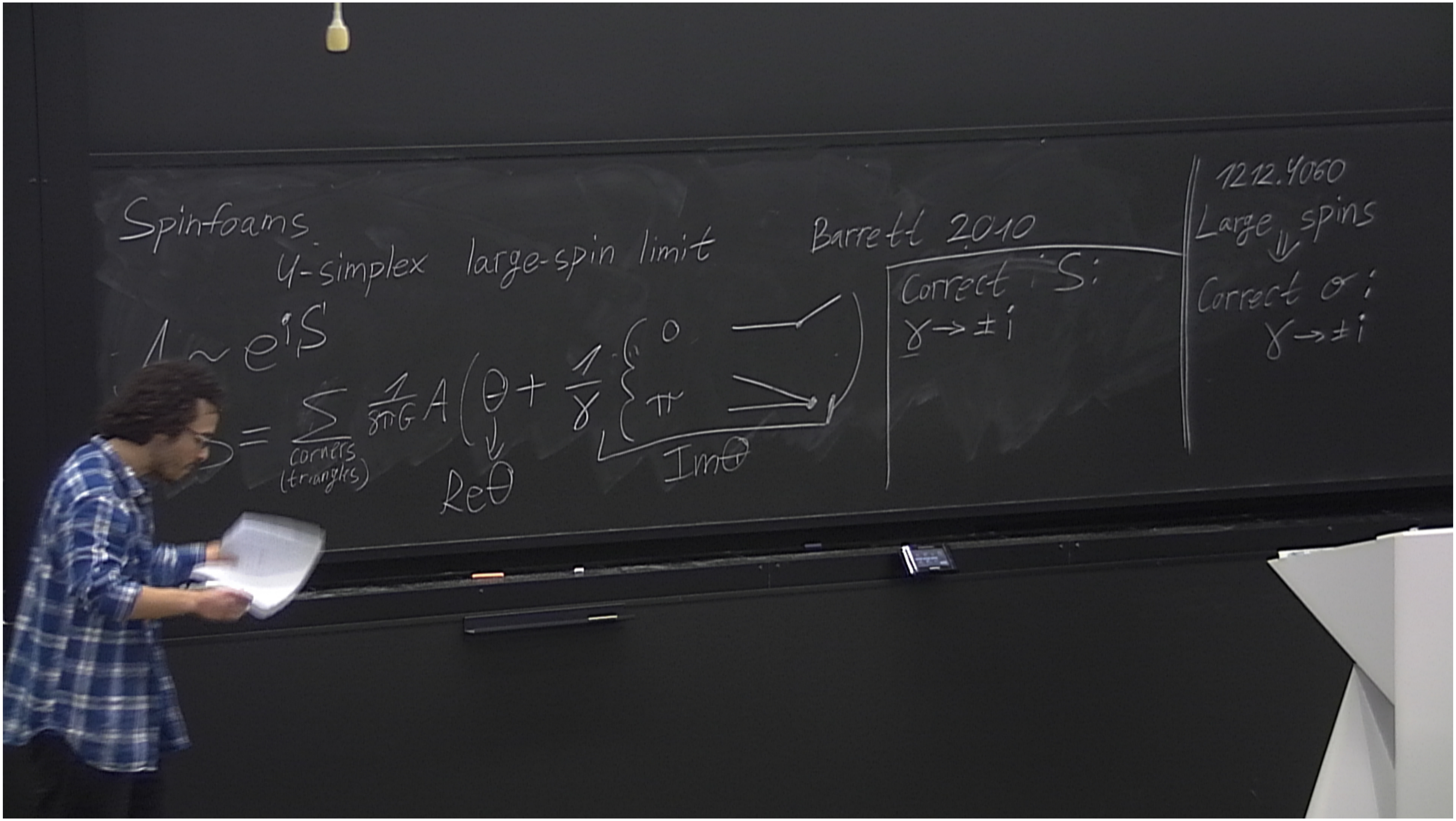

Spinfoams
4-simplex large-spin limit
Barrett
A


Spinfoams
4-simplex large-spin limit
Barrett 2010
$A \sim e^{iS}$
$S = \sum_{\substack{\text{corners} \\ \text{(triangles)}}} \frac{1}{8\pi G} A$


Spinfoams
4-simplex large-spin limit
Barrett 2010
Correct S:
$\gamma \to \pm i$
$A \sim e^{iS}$
$S = \sum_{\substack{\text{corners}\\ \text{(triangles)}}} \frac{1}{8\pi G} A \left(\theta + \frac{1}{\gamma} \begin{cases} 0 \\ \pi \end{cases} \right)$
$\mathrm{Re}\,\theta$
$\mathrm{Im}\,\theta$


Spinfoams
4-simplex large-spin limit
Barrett 2010
Correct S:
γ → ±i
A ~ e^{iS}
S = Σ 1/8πG A (θ + 1/γ {0, π
corners
(triangles)
Reθ
Imθ


Spinfoams
4-simplex large-spin limit
Barrett 2010
A ~ e^{iS}
S = 1/8πG A (θ + 1/γ {0, π}
(angles)
Re θ
Im θ
Correct S:
γ → ±i
1212.4060
Large spins
Correct σ:
γ → ±i


Spinfoams
4-simplex large-spin limit
Barrett 2010
Correct S:
γ → ±i
1212.4060
Large spins
Correct σ:
γ → ±i
A ~ e^{iS}
= Σ corners (triangles) 1/8πG A(θ + 1/γ {0, π})
Reθ
Imθ