Title: Spinning black-hole binaries as gravitational and cosmological probes

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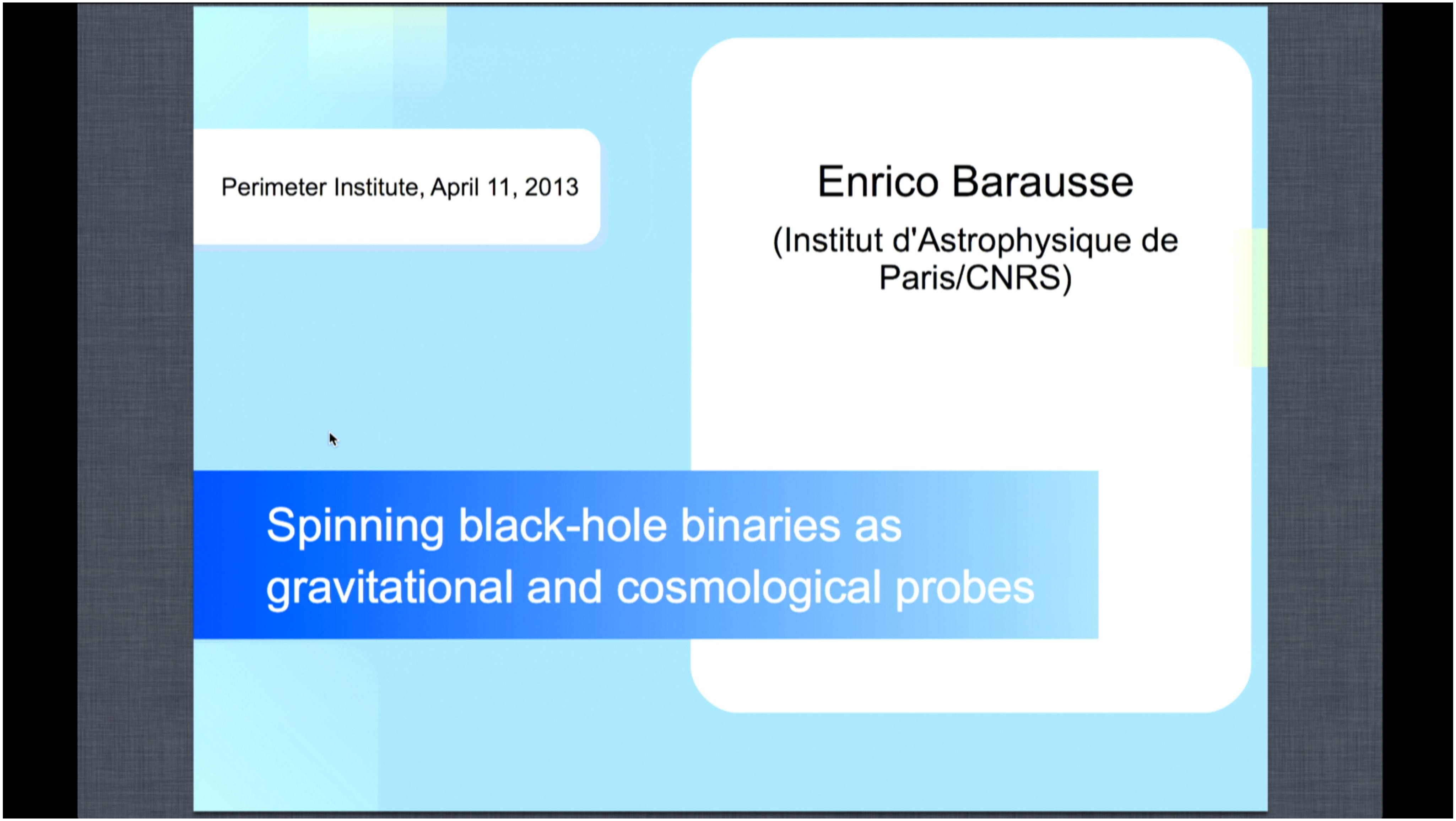
Abstract:



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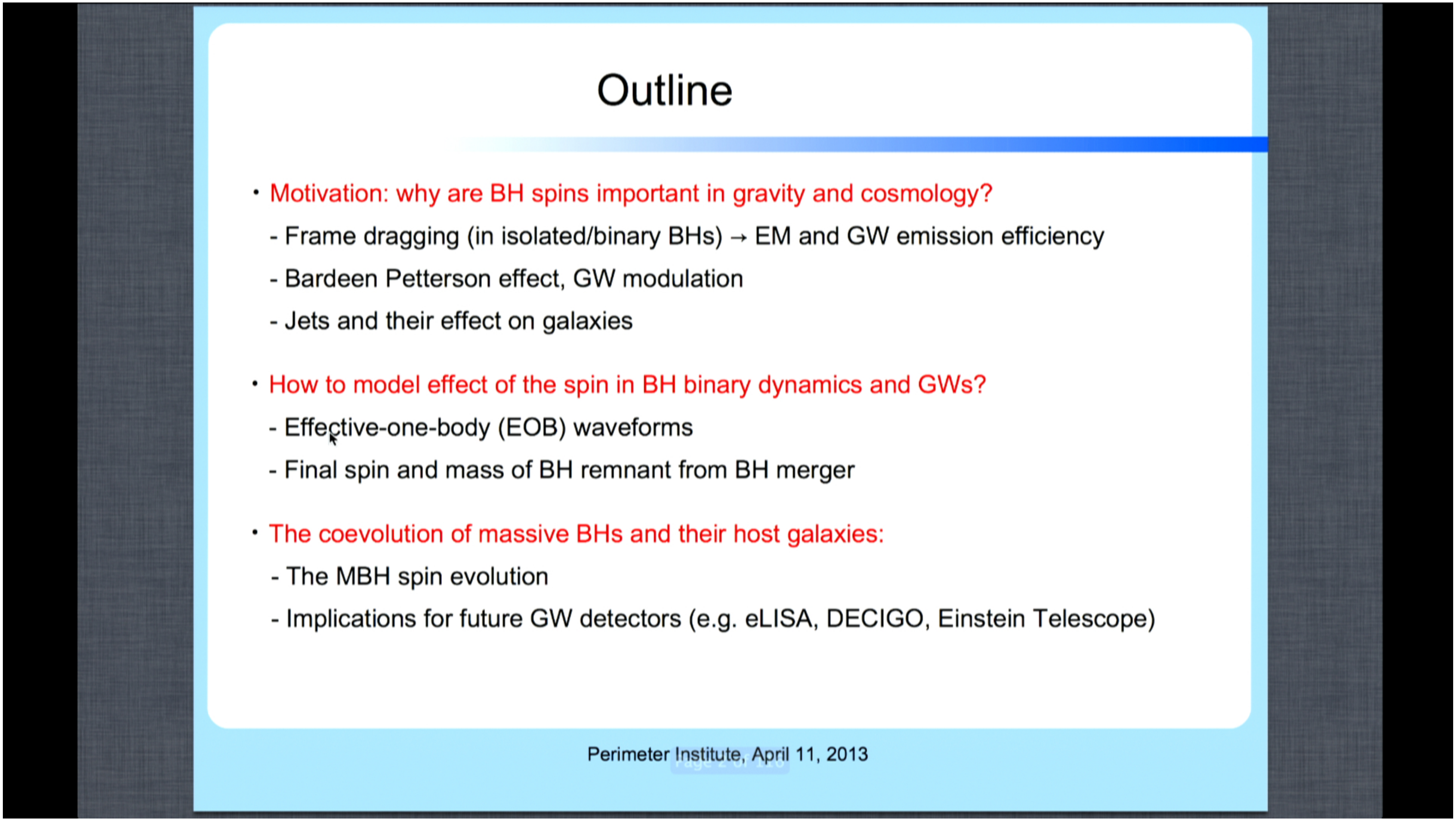
Enrico Barausse

(Institut d'Astrophysique de Paris/CNRS)

Spinning black-hole binaries as gravitational and cosmological probes



Outline

- Motivation: why are BH spins important in gravity and cosmology?
  - Frame dragging (in isolated/binary BHs) → EM and GW emission efficiency
  - Bardeen Petterson effect, GW modulation
  - Jets and their effect on galaxies
- How to model effect of the spin in BH binary dynamics and GWs?
  - Effective-one-body (EOB) waveforms
  - Final spin and mass of BH remnant from BH merger
- The coevolution of massive BHs and their host galaxies:
  - The MBH spin evolution
  - Implications for future GW detectors (e.g. eLISA, DECIGO, Einstein Telescope)

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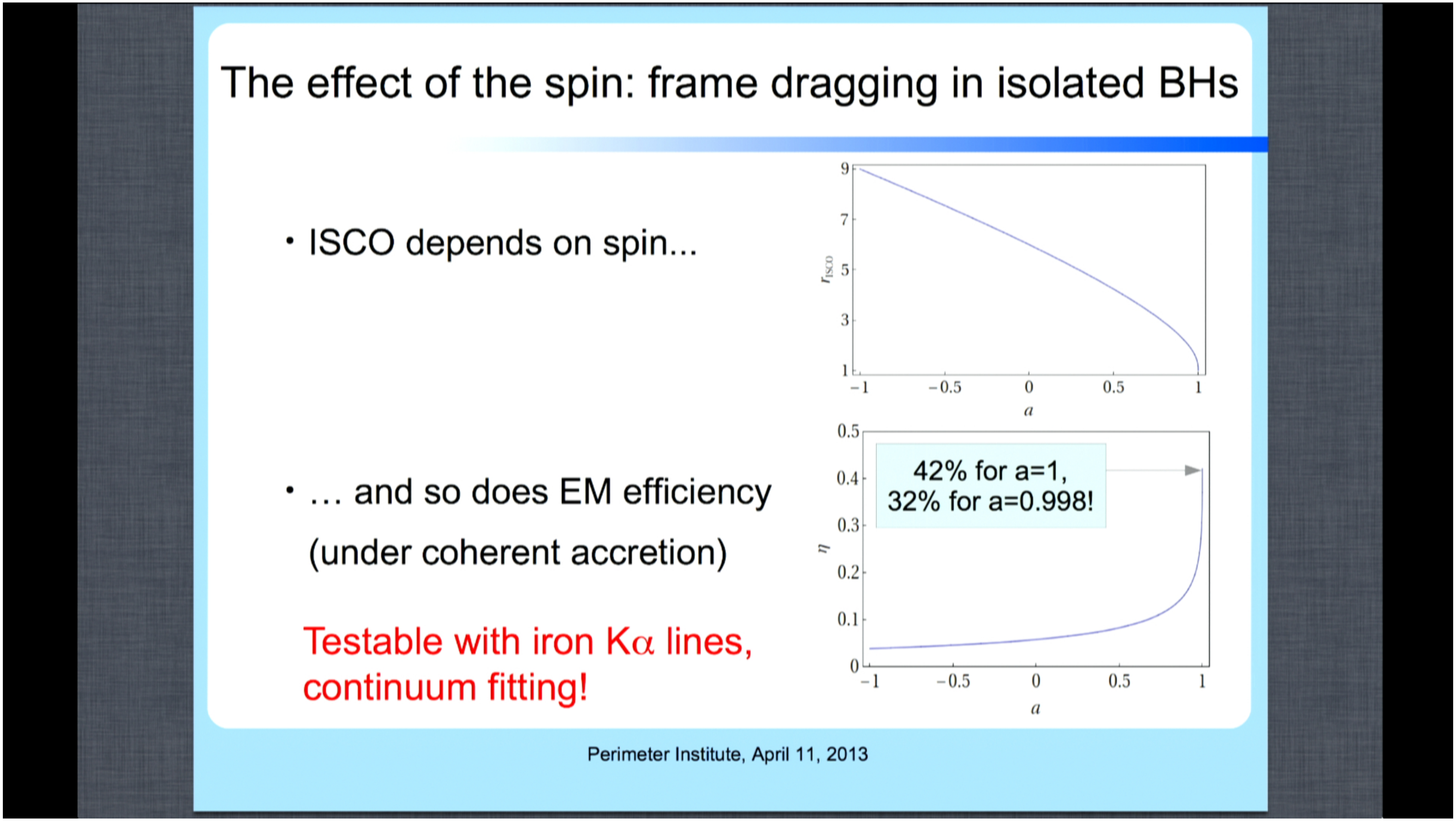

The effect of the spin: frame dragging in isolated BHs

- ISCO depends on spin...

- … and so does EM efficiency (under coherent accretion)

Testable with iron Kα lines, continuum fitting!

42% for a=1,
32% for a=0.998!

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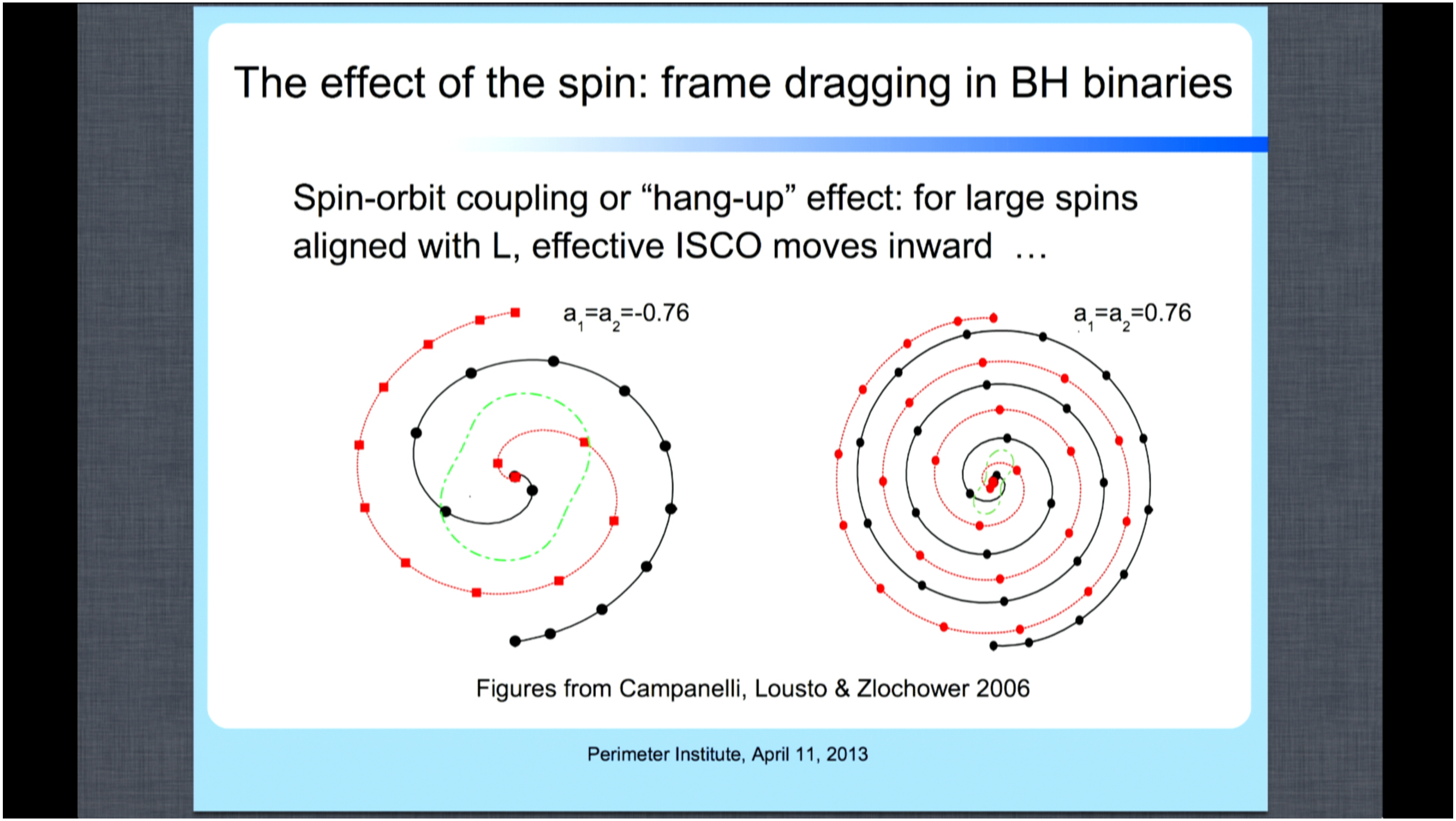

The effect of the spin: frame dragging in BH binaries

Spin-orbit coupling or “hang-up” effect: for large spins aligned with L, effective ISCO moves inward …

Figures from Campanelli, Lousto & Zlochower 2006

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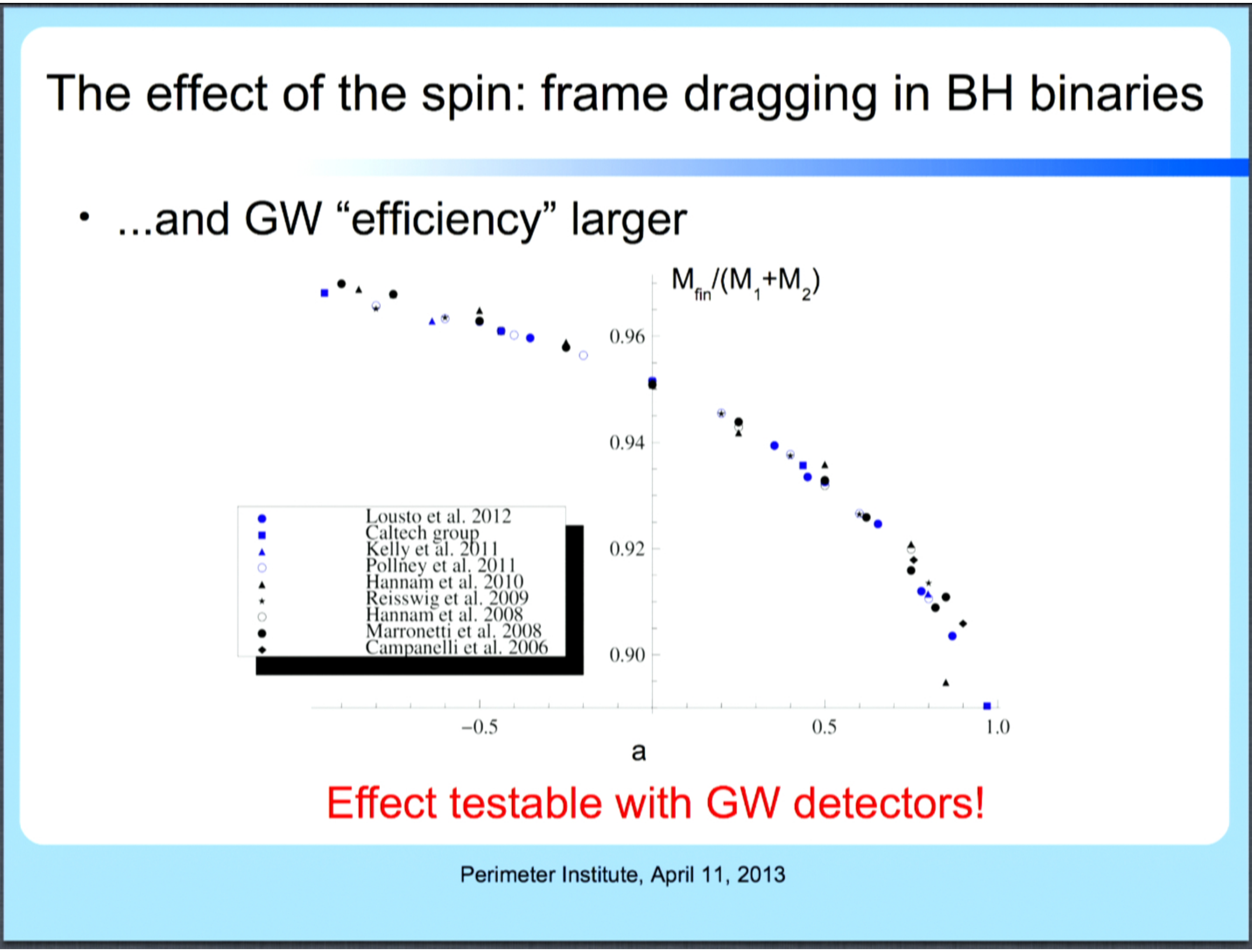

The effect of the spin: frame dragging in BH binaries

- ...and GW "efficiency" larger

Effect testable with GW detectors!

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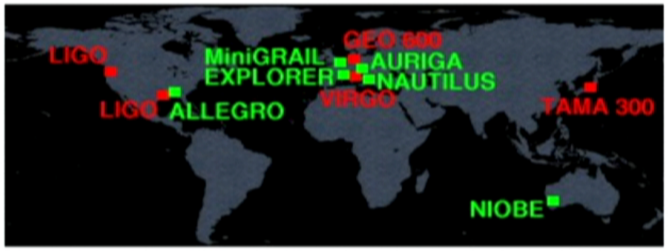
Gravitational waves from binary BHs

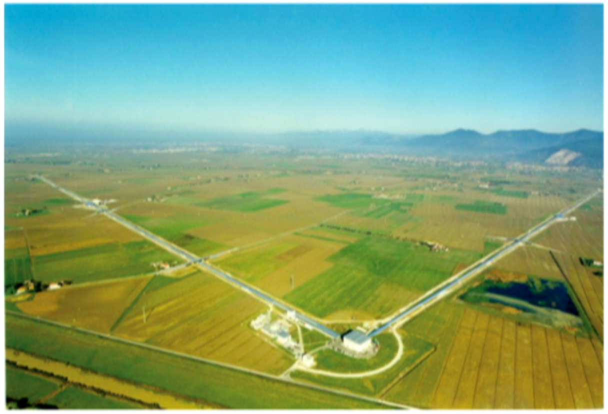
- GW detectors will

  - measure masses to within 0.1% and spins to within 1%

  - tell spin-aligned from precessing binaries thanks to spin induced modulation

- Today: sensitive to stellar-mass BHs (few events per yr at low z)





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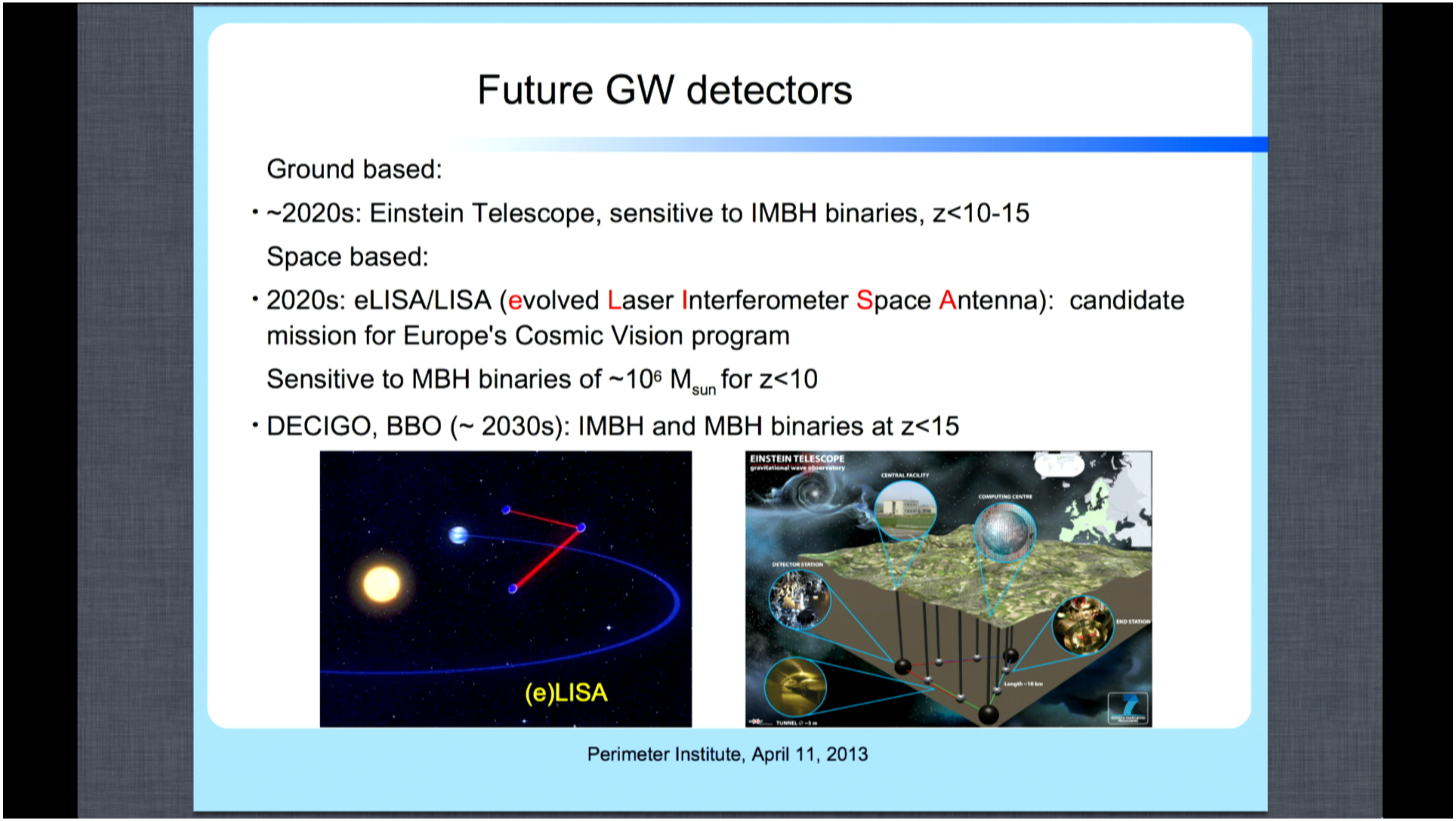


Future GW detectors

Ground based:

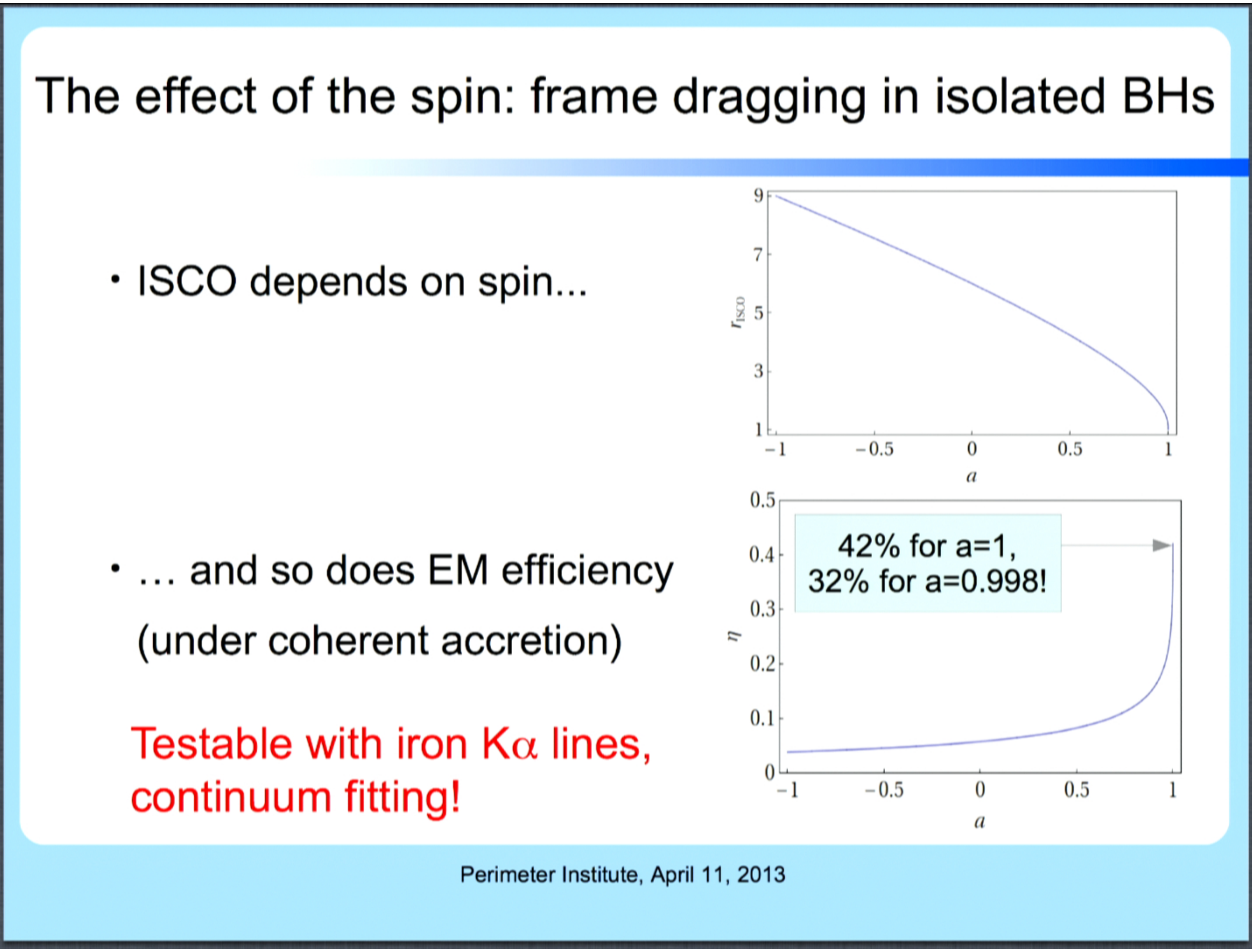
- ~2020s: Einstein Telescope, sensitive to IMBH binaries, $z<10$-15

Space based:

- 2020s: eLISA/LISA (evolved Laser Interferometer Space Antenna): candidate mission for Europe's Cosmic Vision program

  Sensitive to MBH binaries of $\sim 10^6\ M_{sun}$ for $z<10$

- DECIGO, BBO (~ 2030s): IMBH and MBH binaries at $z<15$



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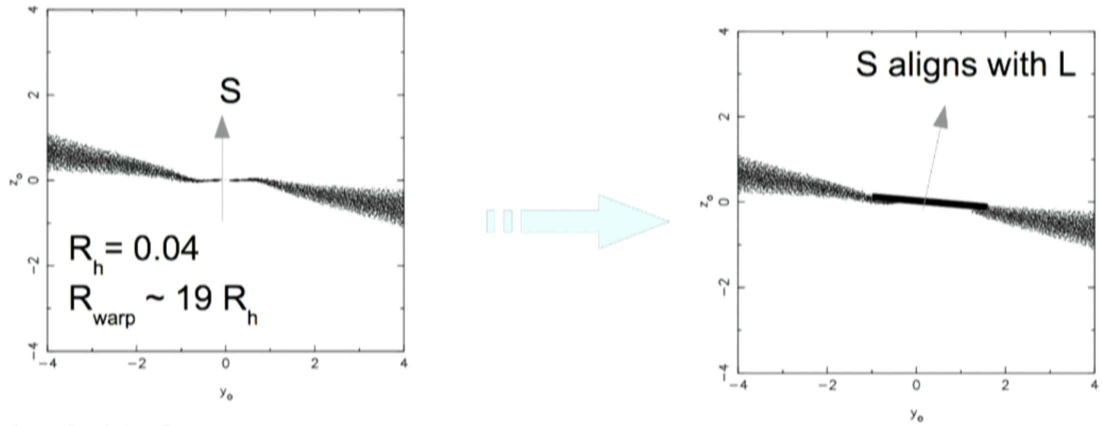
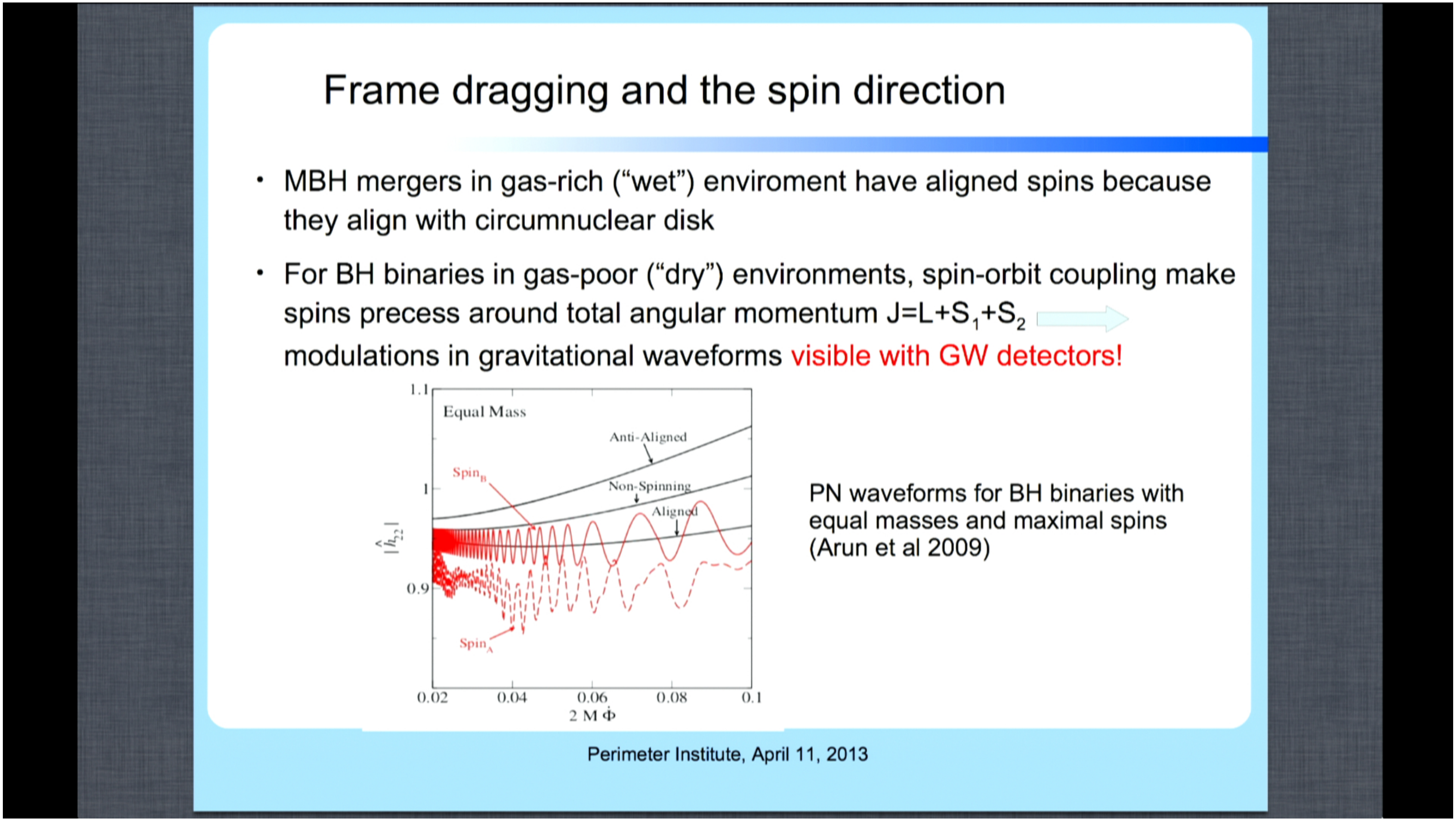
The Bardeen-Petterson effect

- If disk's angular momentum misaligned with BH's spin, spin-orbit coupling and dissipation align (or antialiagn) S and L near BH (Bardeen-Petterson effect)
- Antialignment only happens if disk carries little angular momentum and is initially counterrotating, ie if $\cos(\theta(S,L)) < -L/(2S)$
- On longer timescales (~10^5 yrs for MBHs) warp torques spin and aligns (antialigns) it with L of external disk



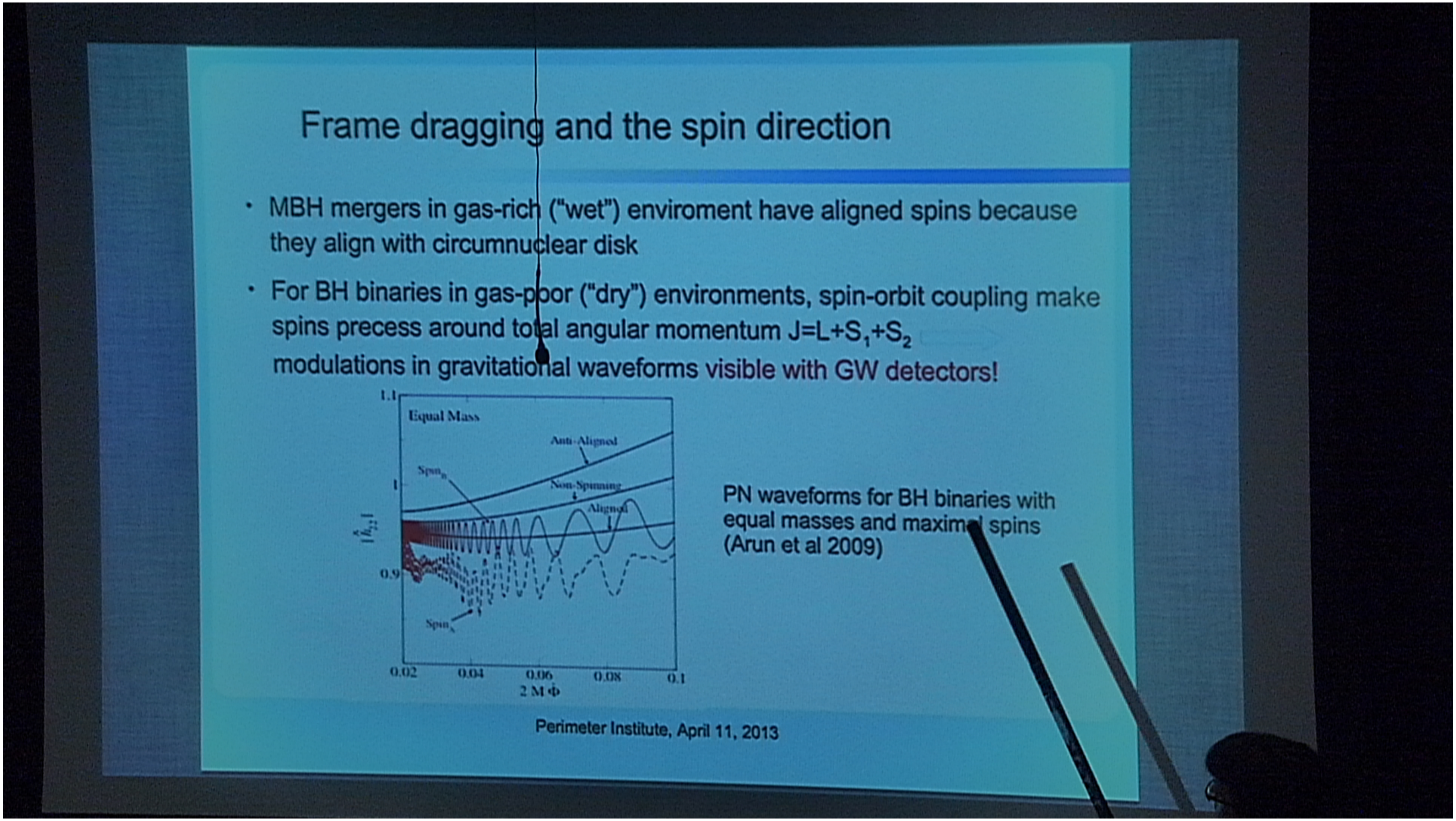
SPH simulation from
Nelson & Papaloizou 2000



Frame dragging and the spin direction

- MBH mergers in gas-rich (“wet”) enviroment have aligned spins because they align with circumnuclear disk
- For BH binaries in gas-poor (“dry”) environments, spin-orbit coupling make spins precess around total angular momentum $J=L+S_1+S_2$ → modulations in gravitational waveforms visible with GW detectors!

PN waveforms for BH binaries with equal masses and maximal spins (Arun et al 2009)



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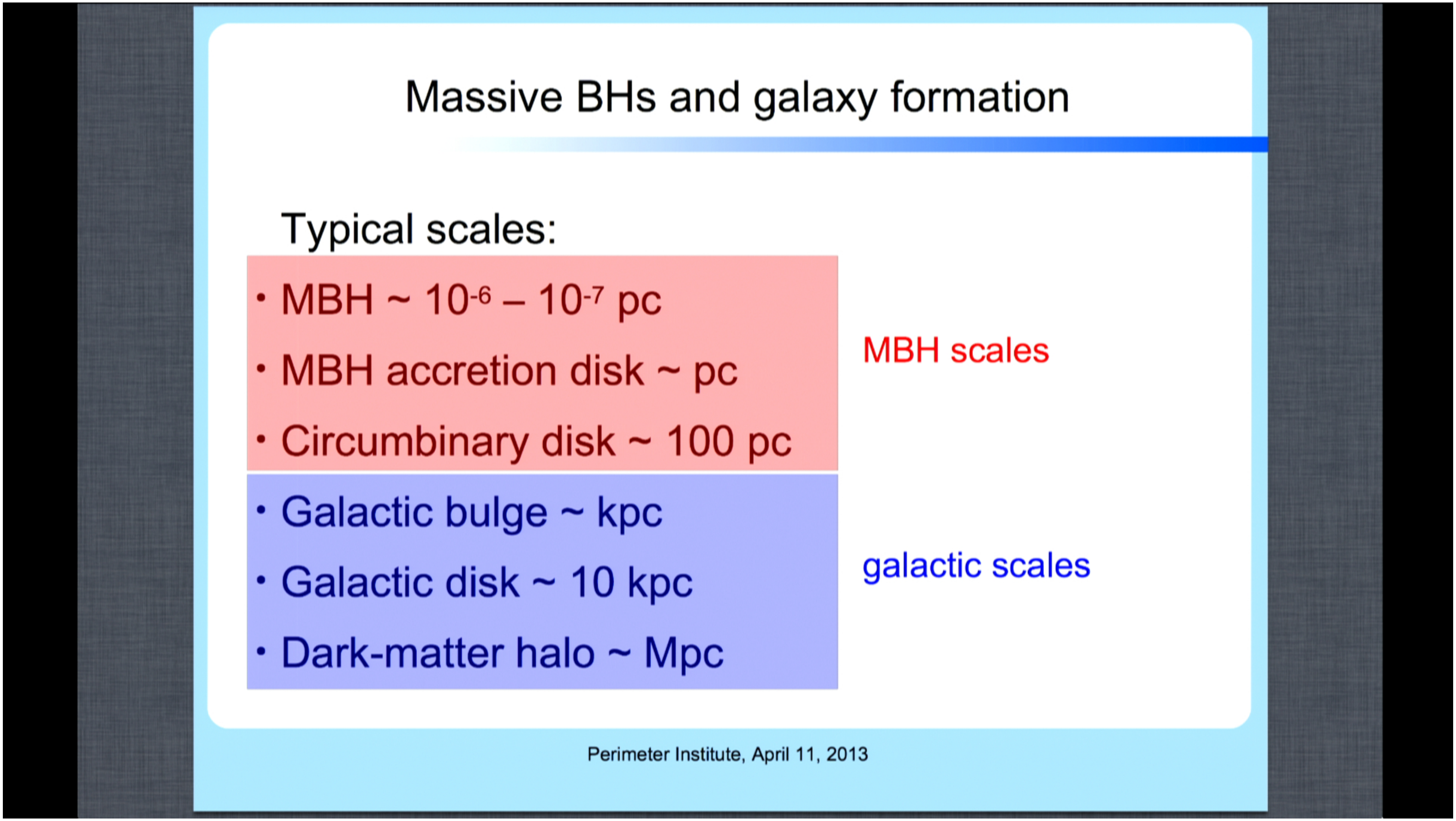

Massive BHs and galaxy formation

Typical scales:

- MBH ~ 10^{-6} – 10^{-7} pc
- MBH accretion disk ~ pc
- Circumbinary disk ~ 100 pc

MBH scales

- Galactic bulge ~ kpc
- Galactic disk ~ 10 kpc
- Dark-matter halo ~ Mpc

galactic scales

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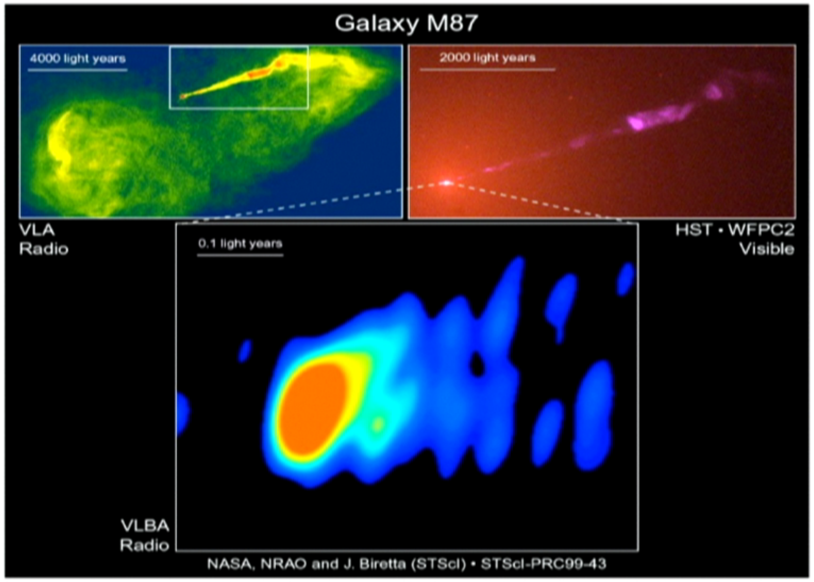
Why does galaxy formation care about MBHs?

- MBHs in AGNs can produce jets that reach far into the galaxy



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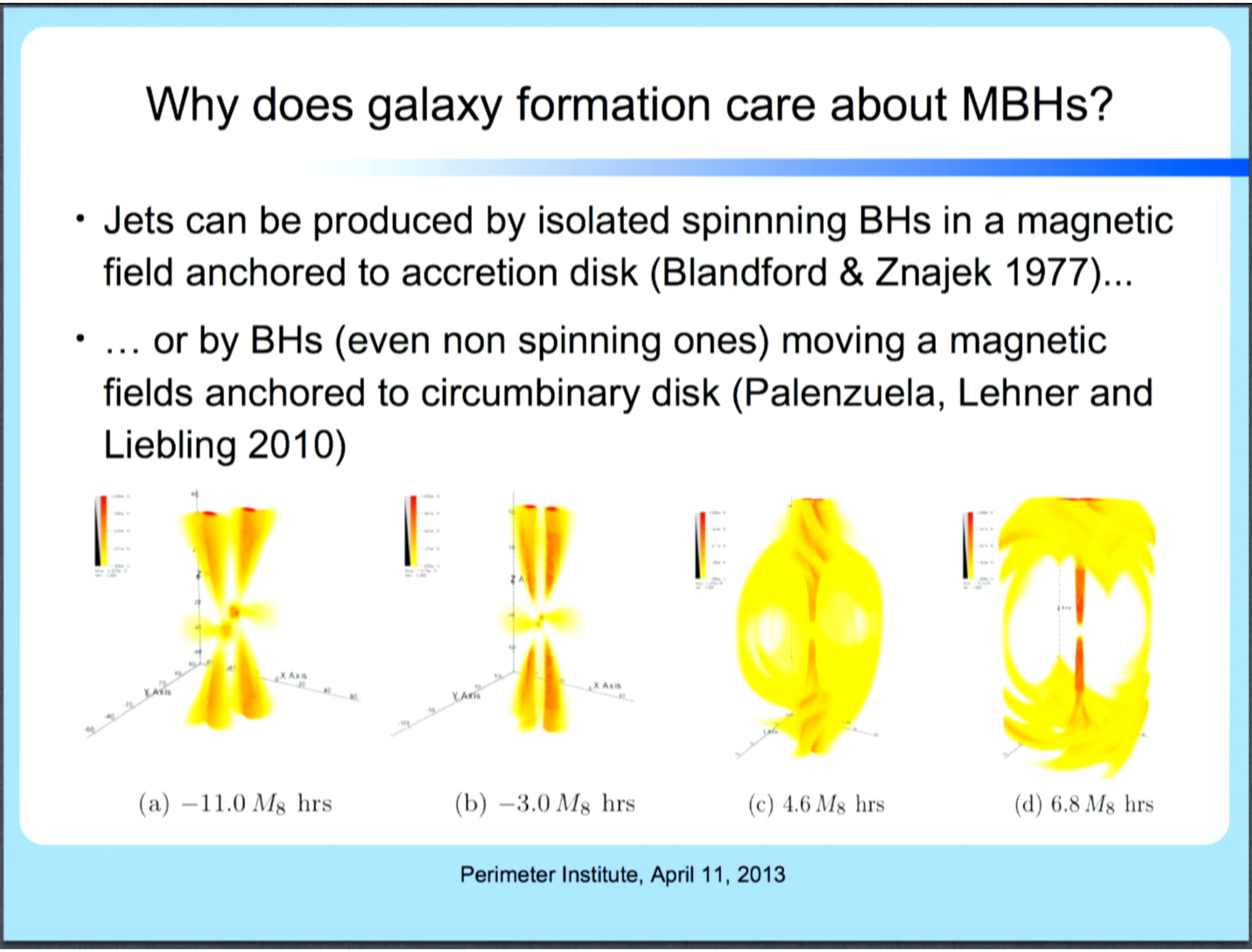
- The kinetic energy of the jets is transferred to the galaxy and keeps it “hot”, quenching star formation (AGN feedback)



Why does galaxy formation care about MBHs?

- Jets can be produced by isolated spinnning BHs in a magnetic field anchored to accretion disk (Blandford & Znajek 1977)...
- … or by BHs (even non spinning ones) moving a magnetic fields anchored to circumbinary disk (Palenzuela, Lehner and Liebling 2010)

(a) $-11.0\,M_8$ hrs (b) $-3.0\,M_8$ hrs (c) $4.6\,M_8$ hrs (d) $6.8\,M_8$ hrs

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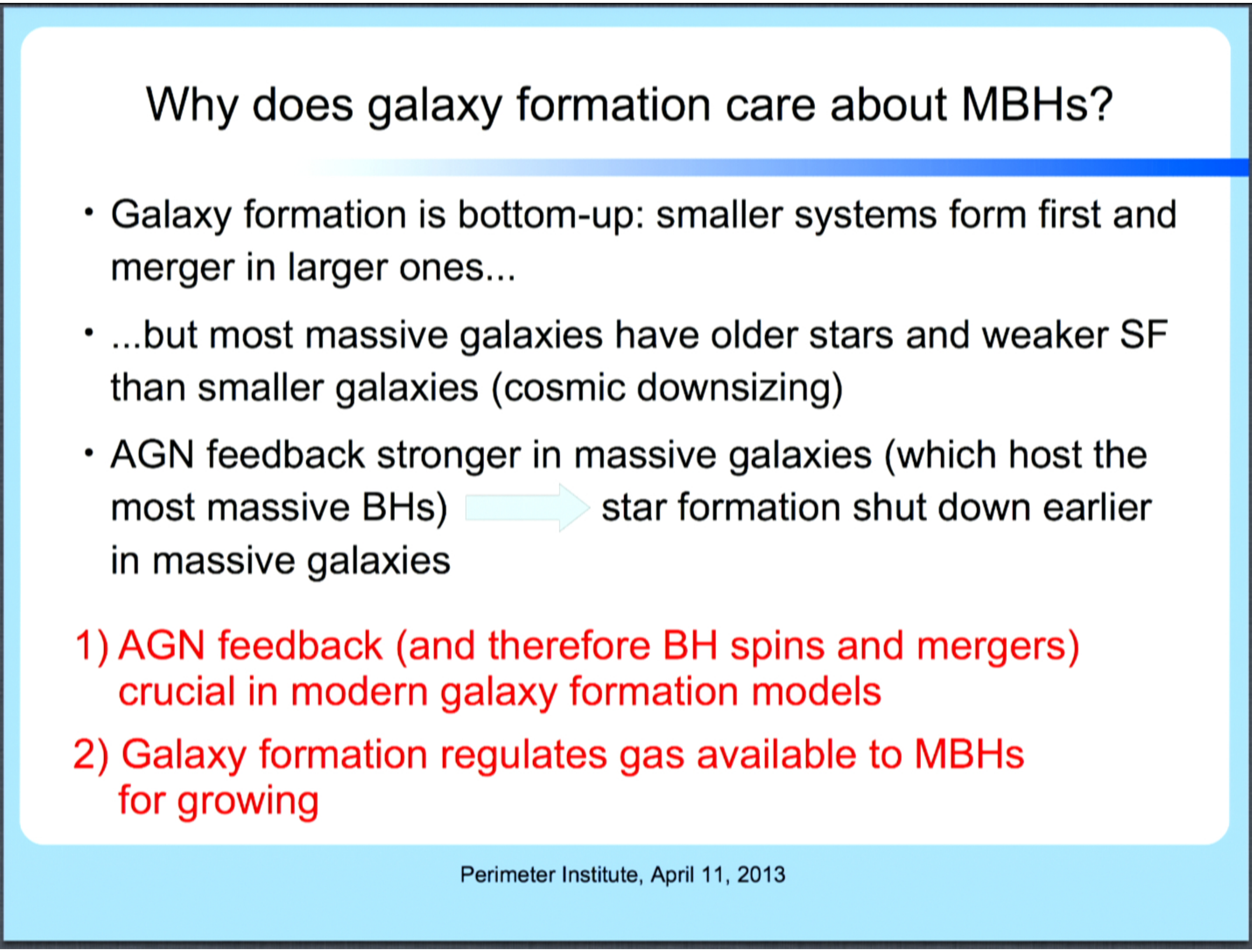

Why does galaxy formation care about MBHs?

- Galaxy formation is bottom-up: smaller systems form first and merger in larger ones...
- ...but most massive galaxies have older stars and weaker SF than smaller galaxies (cosmic downsizing)
- AGN feedback stronger in massive galaxies (which host the most massive BHs) → star formation shut down earlier in massive galaxies

1) AGN feedback (and therefore BH spins and mergers) crucial in modern galaxy formation models

2) Galaxy formation regulates gas available to MBHs for growing

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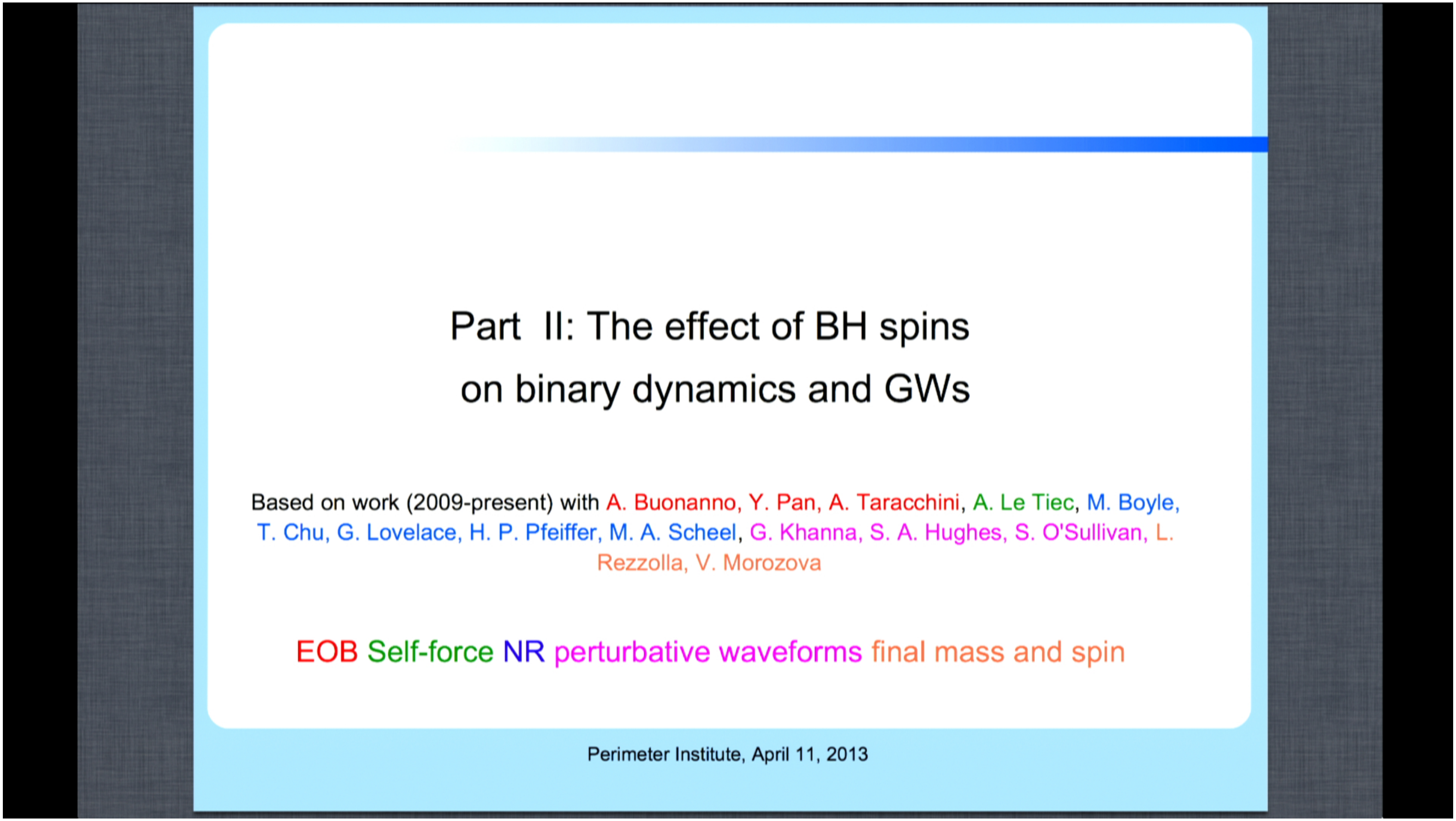

Part II: The effect of BH spins on binary dynamics and GWs

Based on work (2009-present) with A. Buonanno, Y. Pan, A. Taracchini, A. Le Tiec, M. Boyle, T. Chu, G. Lovelace, H. P. Pfeiffer, M. A. Scheel, G. Khanna, S. A. Hughes, S. O'Sullivan, L. Rezzolla, V. Morozova

EOB Self-force NR perturbative waveforms final mass and spin

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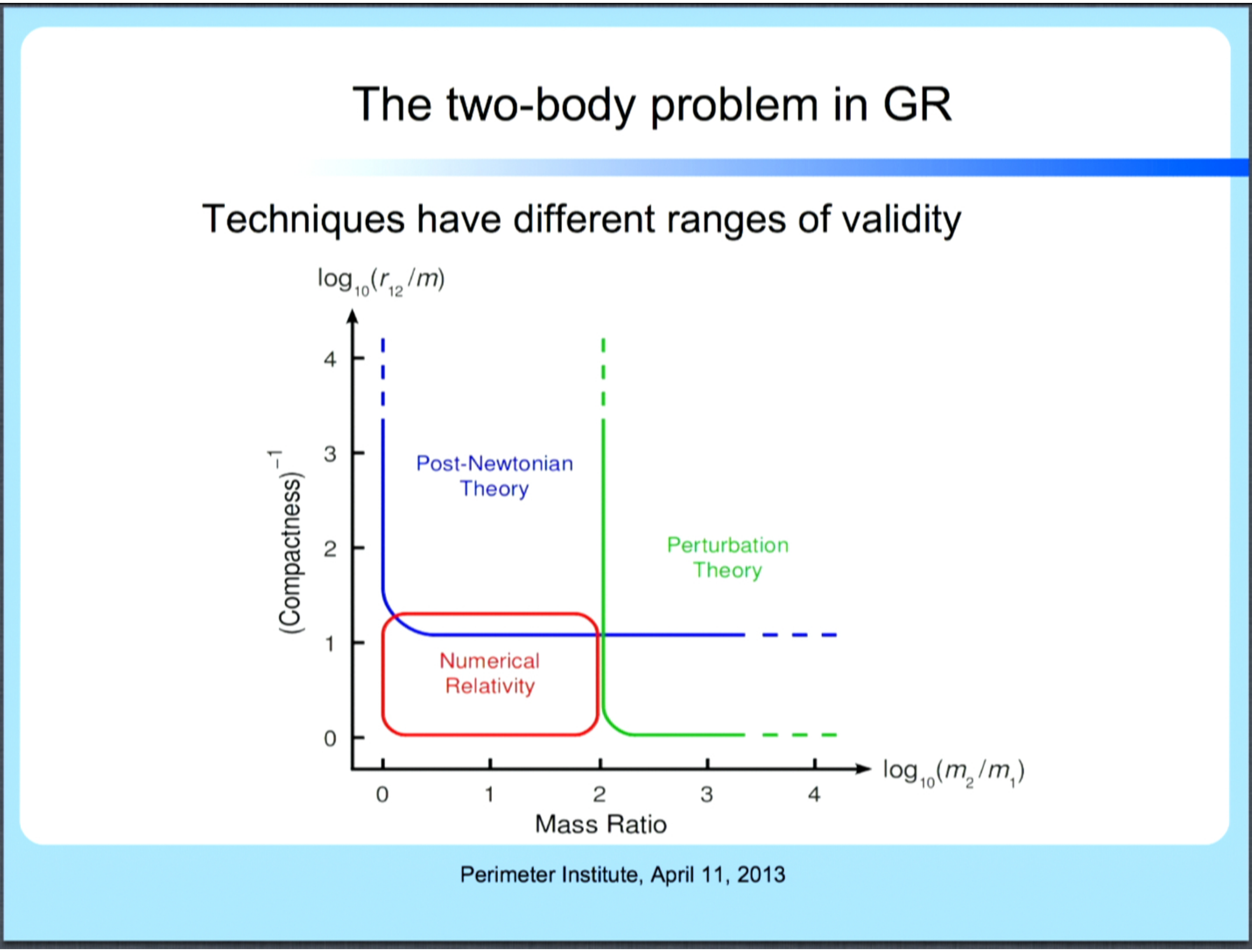

The two-body problem in GR

Techniques have different ranges of validity

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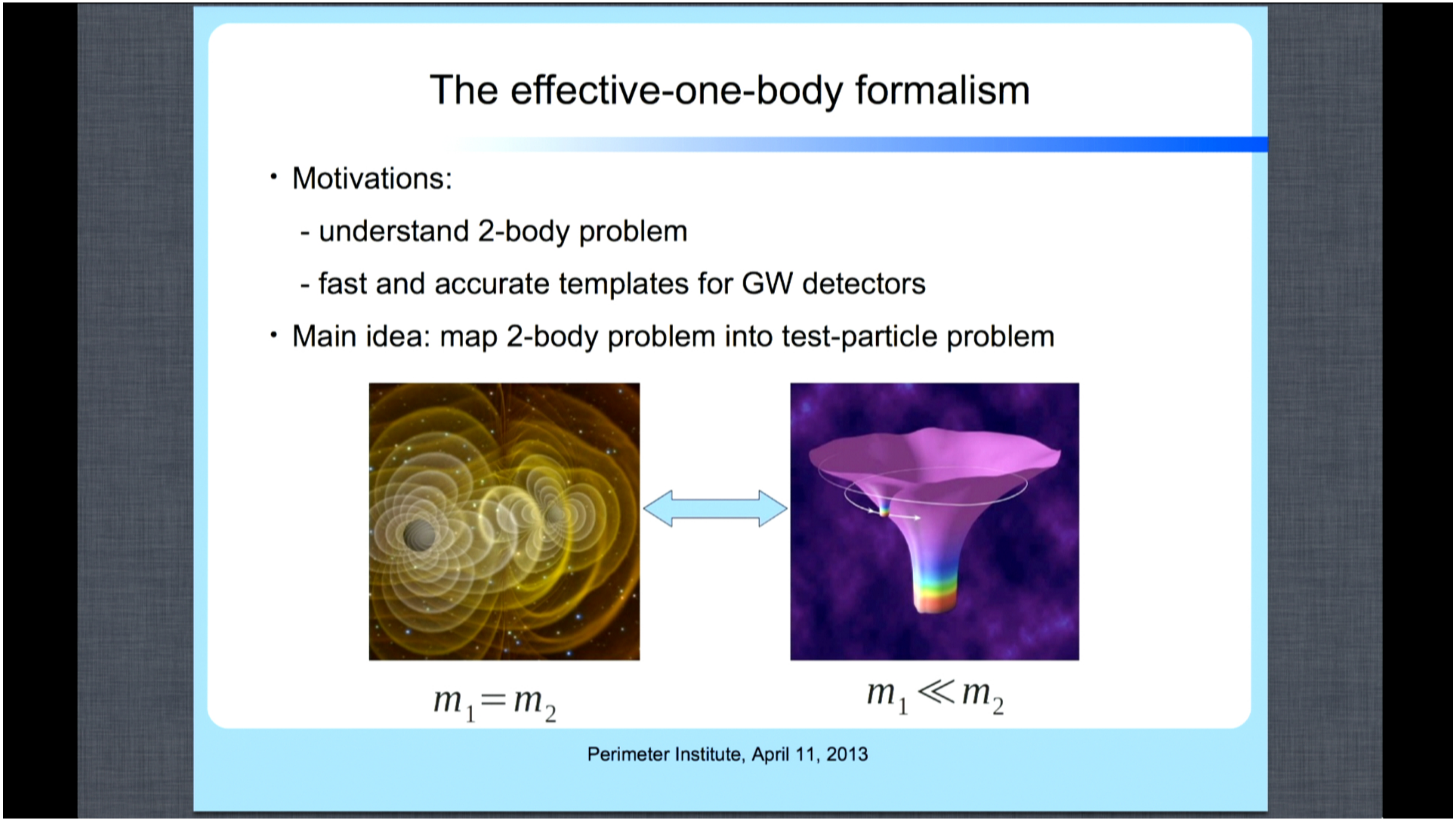

The effective-one-body formalism

- Motivations:
  - understand 2-body problem
  - fast and accurate templates for GW detectors
- Main idea: map 2-body problem into test-particle problem

$m_1 = m_2$ $\longleftrightarrow$ $m_1 \ll m_2$

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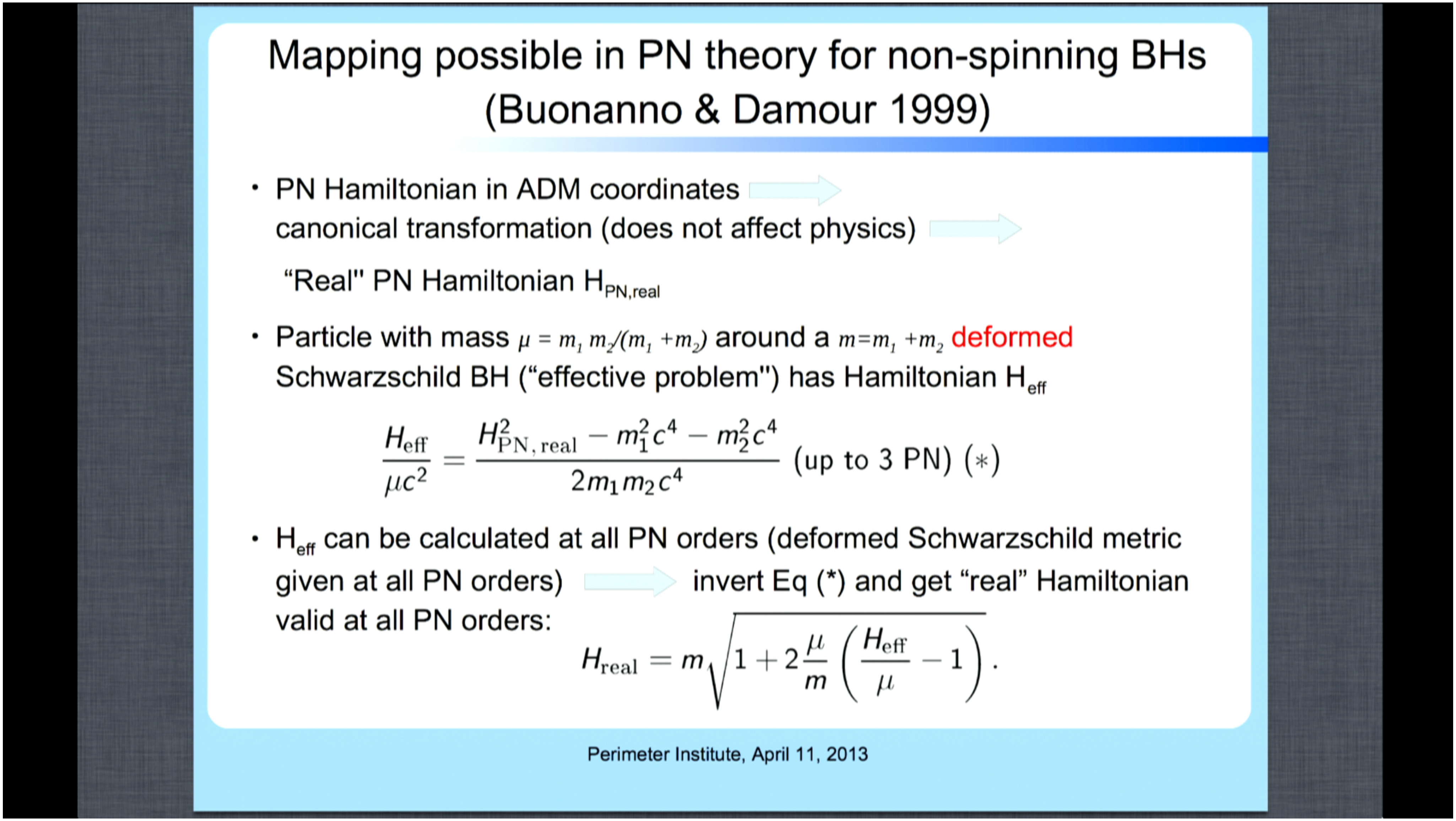
Is this mapping possible?

- Newtonian non-spinning binaries can be mapped to non-spinning test-particle with mass $\mu = m_1 m_2/(m_1 + m_2)$ around mass $m = m_1 + m_2$
- Energy levels of positronium (e^+ - e^-) can be mapped to those of hydrogen through

$$\frac{E_{\mathrm{H}}}{\mu c^2} = \frac{E_{\mathrm{pos}}^2 - m_1^2 c^4 - m_2^2 c^4}{2 m_1 m_2 c^4}$$

$m_1 = m_2$ is the electron/antielectron's mass!



Mapping possible in PN theory for non-spinning BHs (Buonanno & Damour 1999)

- PN Hamiltonian in ADM coordinates → canonical transformation (does not affect physics) →

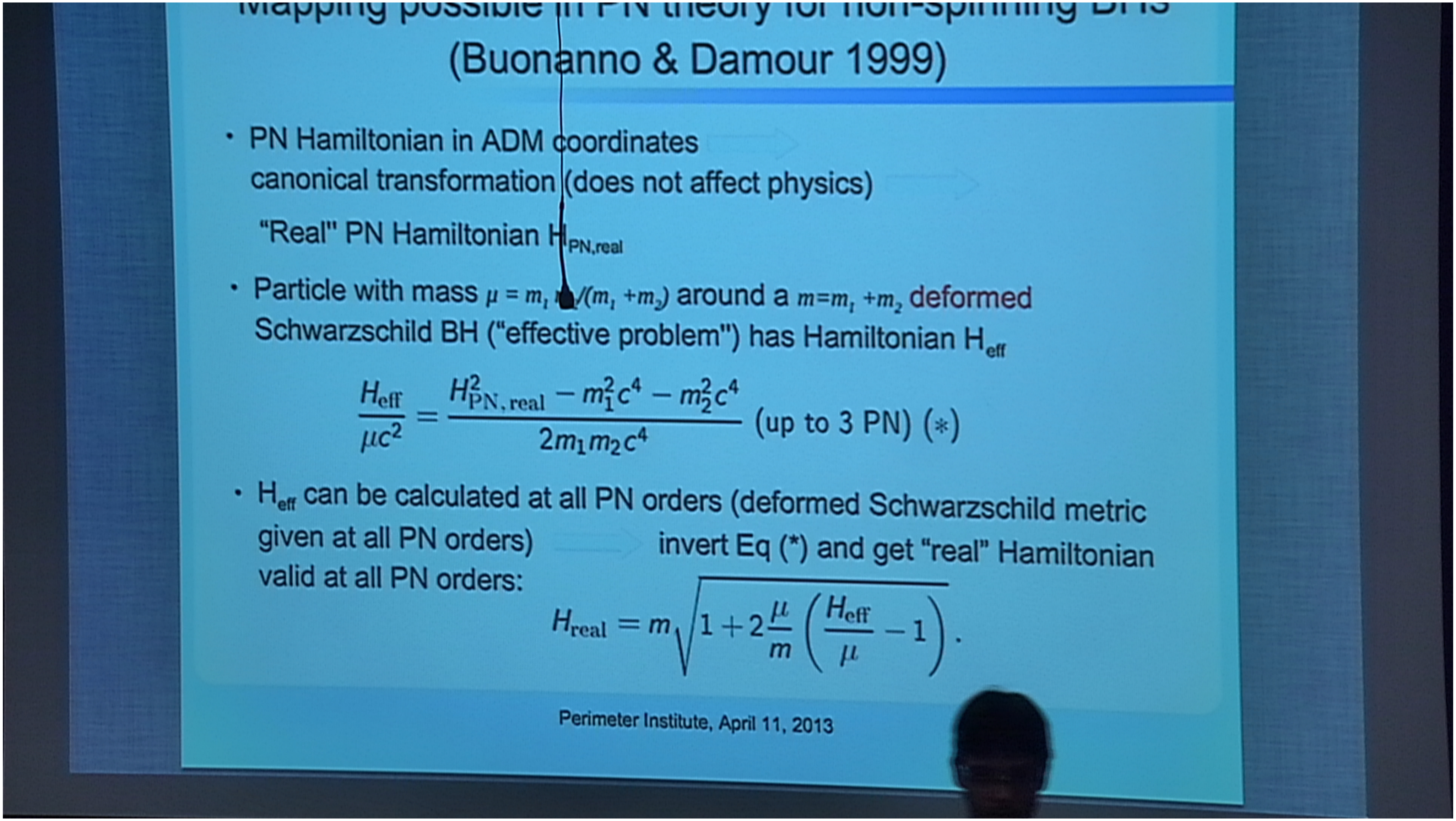
  "Real" PN Hamiltonian $H_{\mathrm{PN,real}}$

- Particle with mass $\mu = m_1 m_2/(m_1 + m_2)$ around a $m = m_1 + m_2$ deformed Schwarzschild BH ("effective problem") has Hamiltonian H_{eff}

$$\frac{H_{\mathrm{eff}}}{\mu c^2} = \frac{H^2_{\mathrm{PN,\,real}} - m_1^2 c^4 - m_2^2 c^4}{2 m_1 m_2 c^4} \quad \text{(up to 3 PN)} \ (*)$$

- H_{eff} can be calculated at all PN orders (deformed Schwarzschild metric given at all PN orders) → invert Eq (*) and get "real" Hamiltonian valid at all PN orders:

$$H_{\mathrm{real}} = m\sqrt{1 + 2\frac{\mu}{m}\left(\frac{H_{\mathrm{eff}}}{\mu} - 1\right)}\,.$$



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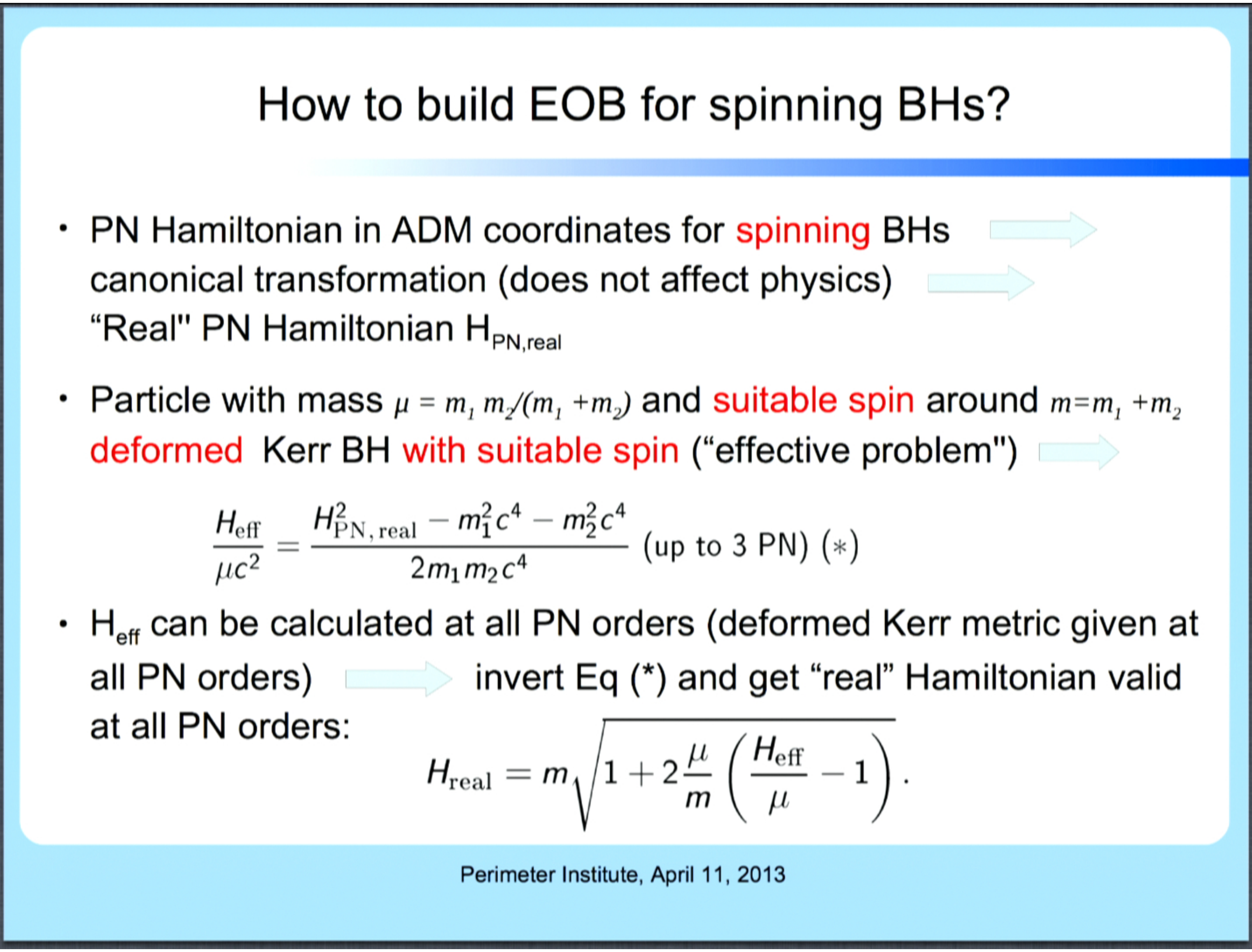

How to build EOB for spinning BHs?

- PN Hamiltonian in ADM coordinates for spinning BHs →
  canonical transformation (does not affect physics) →
  "Real" PN Hamiltonian $H_{\mathrm{PN,real}}$

- Particle with mass $\mu = m_1 m_2/(m_1 + m_2)$ and suitable spin around $m = m_1 + m_2$ deformed Kerr BH with suitable spin ("effective problem") →

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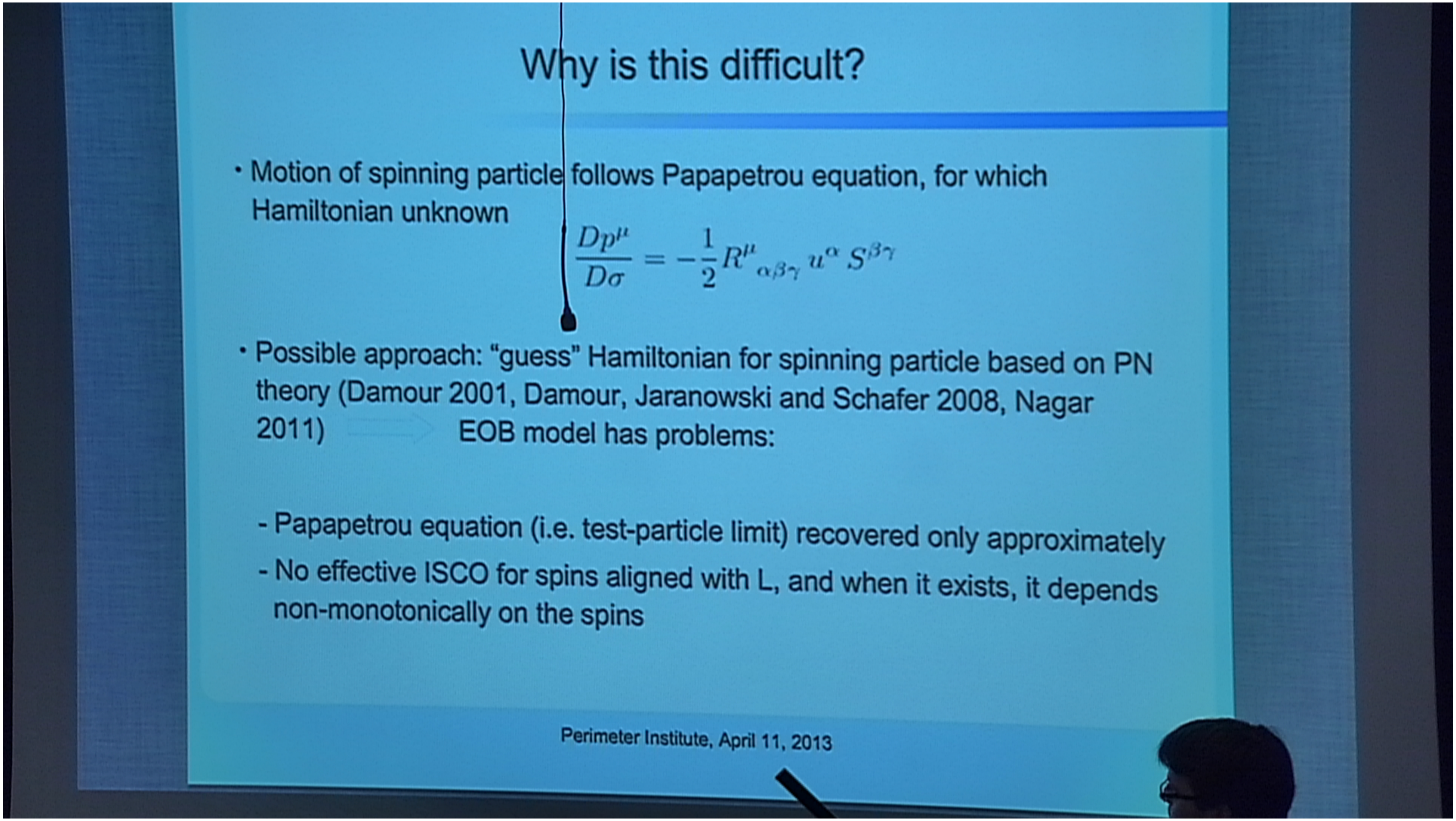

Why is this difficult?

- Motion of spinning particle follows Papapetrou equation, for which Hamiltonian unknown

$$\frac{Dp^{\mu}}{D\sigma} = -\frac{1}{2} R^{\mu}{}_{\alpha\beta\gamma}\, u^{\alpha}\, S^{\beta\gamma}$$

- Possible approach: “guess” Hamiltonian for spinning particle based on PN theory (Damour 2001, Damour, Jaranowski and Schafer 2008, Nagar 2011) → EOB model has problems:

- Papapetrou equation (i.e. test-particle limit) recovered only approximately
- No effective ISCO for spins aligned with L, and when it exists, it depends non-monotonically on the spins

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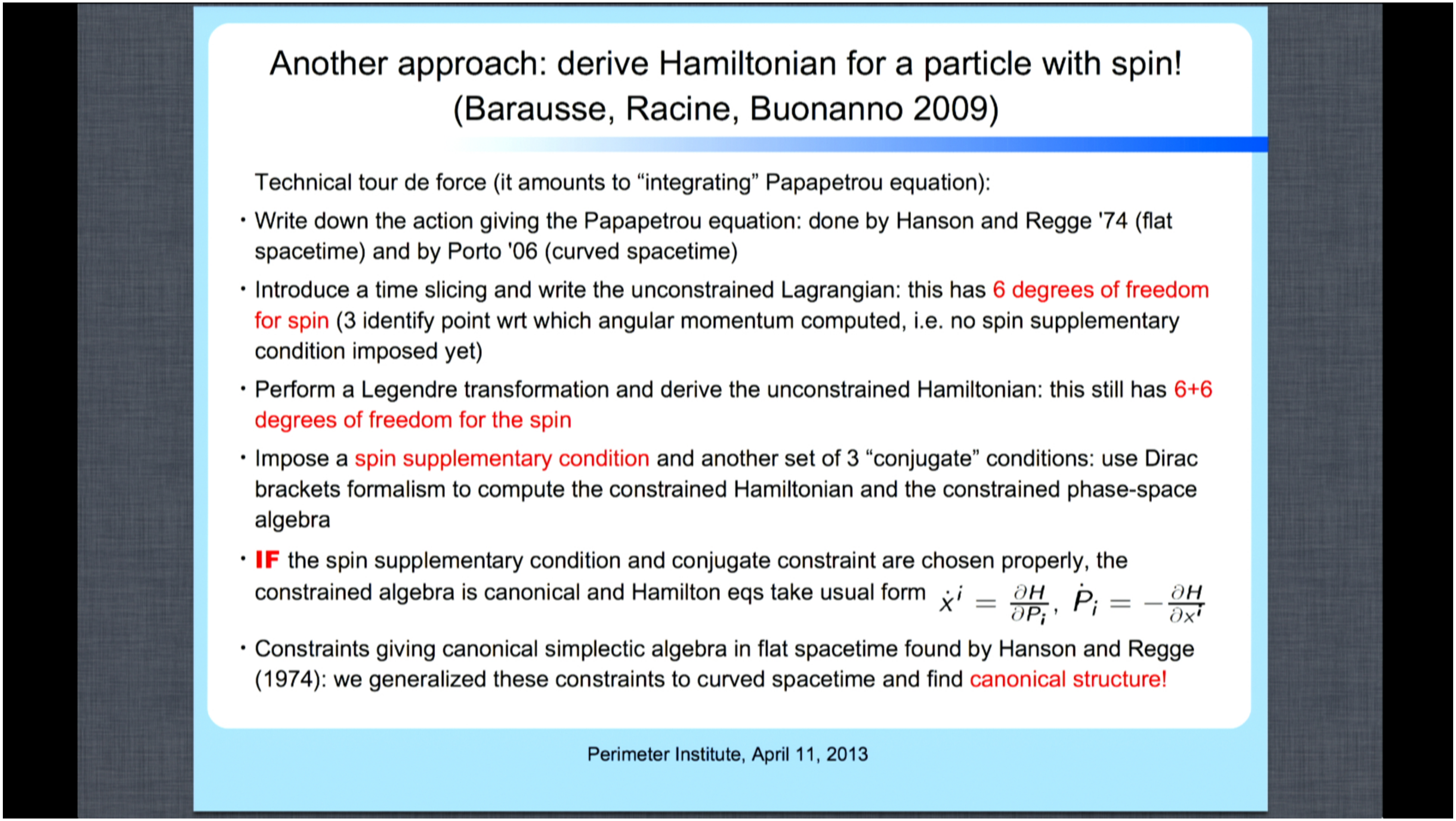

Another approach: derive Hamiltonian for a particle with spin! (Barausse, Racine, Buonanno 2009)

Technical tour de force (it amounts to "integrating" Papapetrou equation):

- Write down the action giving the Papapetrou equation: done by Hanson and Regge '74 (flat spacetime) and by Porto '06 (curved spacetime)
- Introduce a time slicing and write the unconstrained Lagrangian: this has 6 degrees of freedom for spin (3 identify point wrt which angular momentum computed, i.e. no spin supplementary condition imposed yet)
- Perform a Legendre transformation and derive the unconstrained Hamiltonian: this still has 6+6 degrees of freedom for the spin
- Impose a spin supplementary condition and another set of 3 "conjugate" conditions: use Dirac brackets formalism to compute the constrained Hamiltonian and the constrained phase-space algebra
- **IF** the spin supplementary condition and conjugate constraint are chosen properly, the constrained algebra is canonical and Hamilton eqs take usual form $\dot{x}^i = \frac{\partial H}{\partial P_i}, \ \dot{P}_i = -\frac{\partial H}{\partial x^i}$
- Constraints giving canonical simplectic algebra in flat spacetime found by Hanson and Regge (1974): we generalized these constraints to curved spacetime and find canonical structure!

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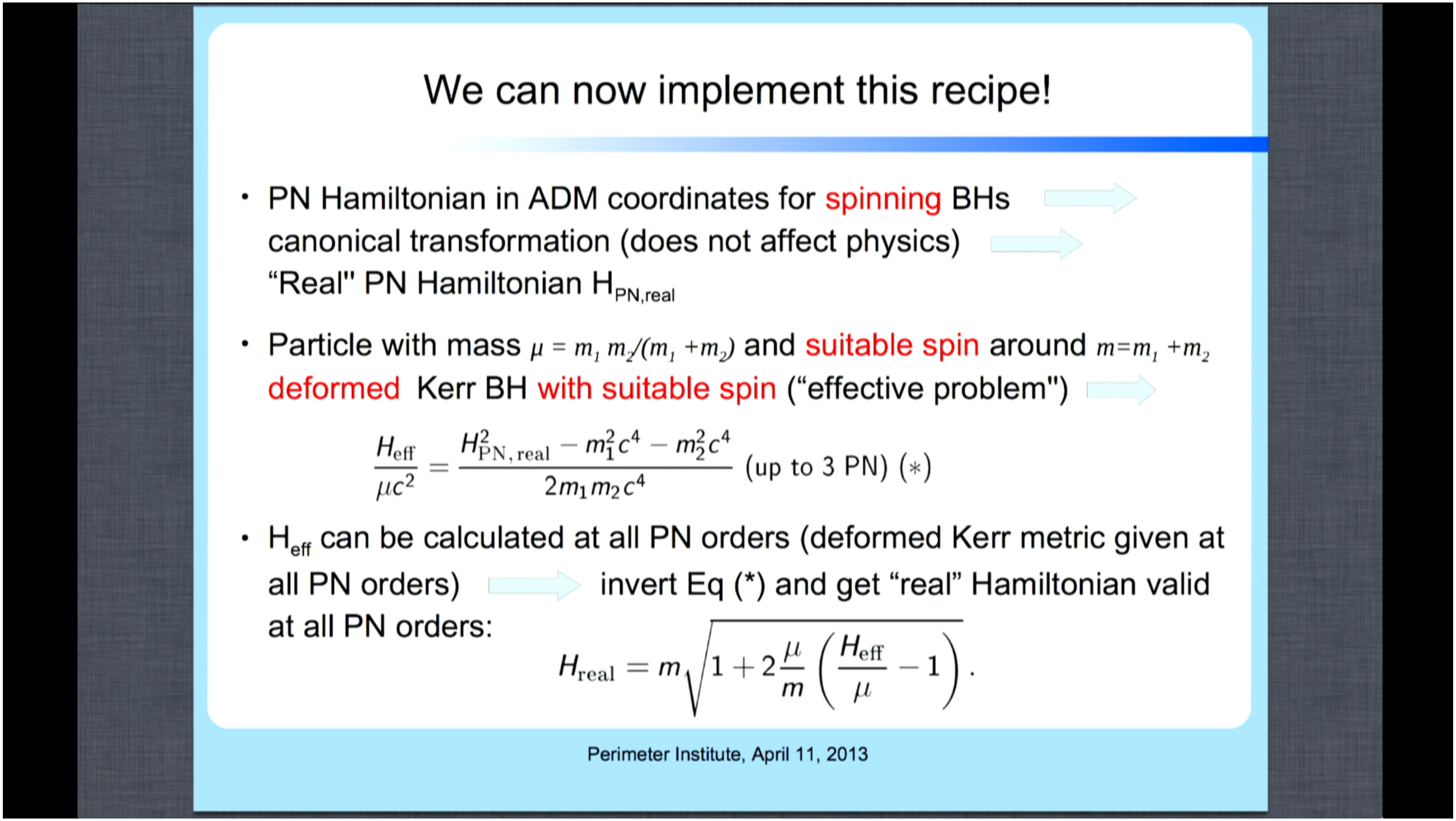
$$S^{\mu\nu}\left(p_\mu + m\,\tilde{e}^{(0)}_\mu\right) = 0$$



We can now implement this recipe!

- PN Hamiltonian in ADM coordinates for spinning BHs → canonical transformation (does not affect physics) → "Real" PN Hamiltonian $H_{PN,real}$
- Particle with mass $\mu = m_1 m_2/(m_1 + m_2)$ and suitable spin around $m = m_1 + m_2$ deformed Kerr BH with suitable spin ("effective problem") →

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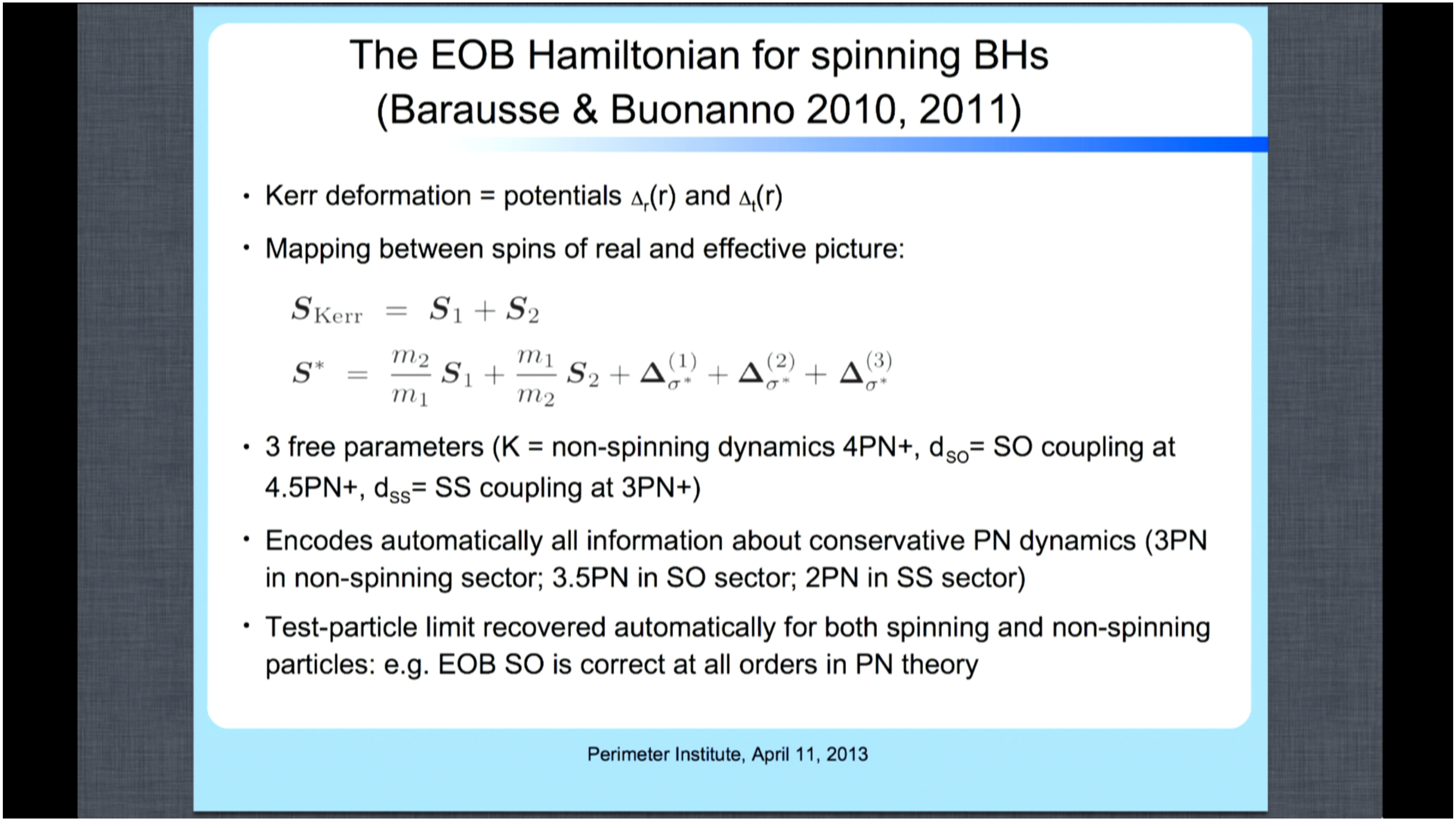
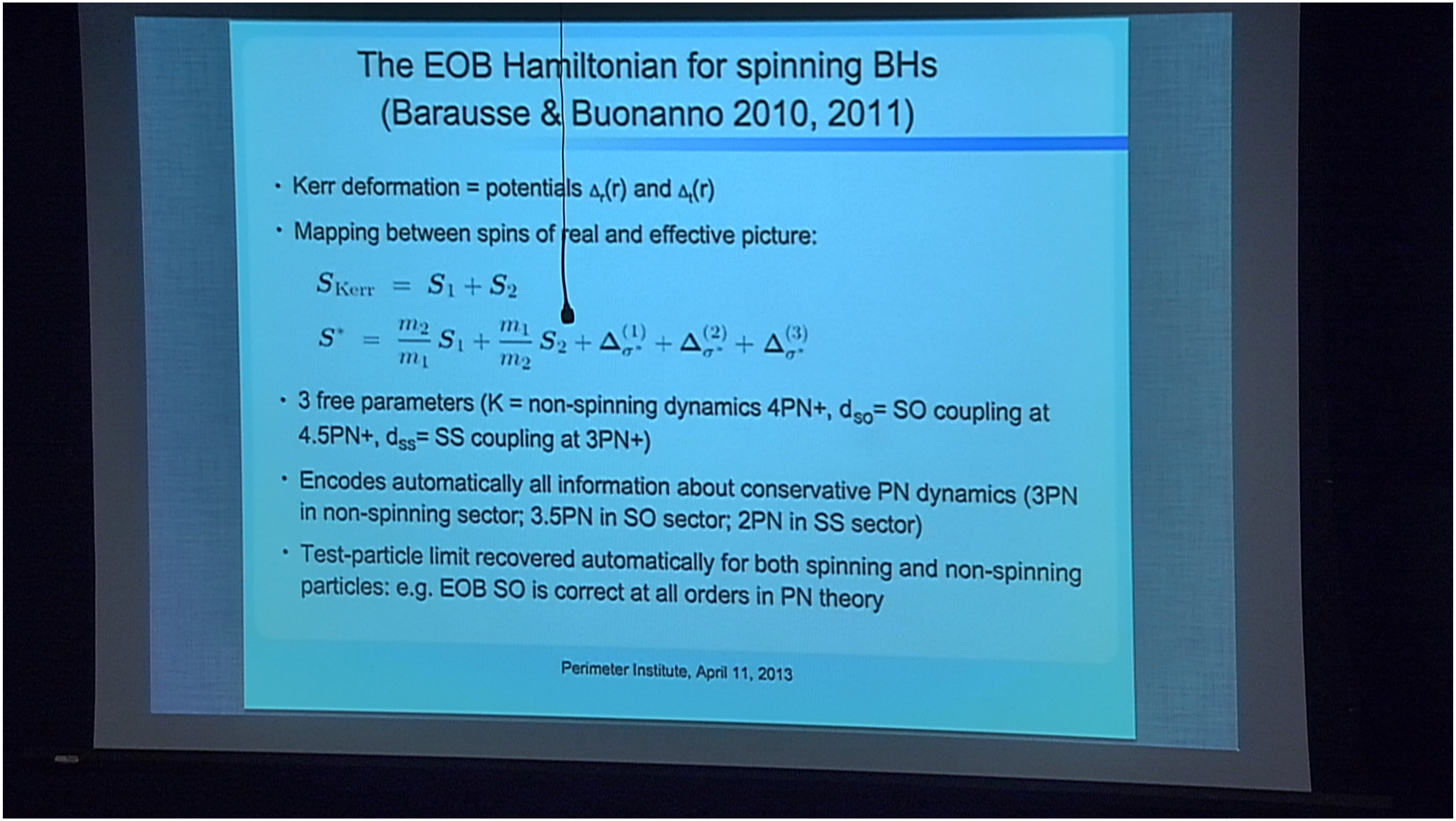


The EOB Hamiltonian for spinning BHs (Barausse & Buonanno 2010, 2011)

- Kerr deformation = potentials $\Delta_r(r)$ and $\Delta_t(r)$
- Mapping between spins of real and effective picture:

$$\begin{aligned} \boldsymbol{S}_{\rm Kerr} &= \boldsymbol{S}_1 + \boldsymbol{S}_2 \\ \boldsymbol{S}^* &= \frac{m_2}{m_1}\boldsymbol{S}_1 + \frac{m_1}{m_2}\boldsymbol{S}_2 + \boldsymbol{\Delta}^{(1)}_{\sigma^*} + \boldsymbol{\Delta}^{(2)}_{\sigma^*} + \boldsymbol{\Delta}^{(3)}_{\sigma^*} \end{aligned}$$

- 3 free parameters (K = non-spinning dynamics 4PN+, d_{SO}= SO coupling at 4.5PN+, d_{SS}= SS coupling at 3PN+)
- Encodes automatically all information about conservative PN dynamics (3PN in non-spinning sector; 3.5PN in SO sector; 2PN in SS sector)
- Test-particle limit recovered automatically for both spinning and non-spinning particles: e.g. EOB SO is correct at all orders in PN theory



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$$S_{\rm Kerr} = S_1 + S_2$$

$$S^* = \frac{m_2}{m_1} S_1 + \frac{m_1}{m_2} S_2 + \Delta^{(1)}_{\sigma^*} + \Delta^{(2)}_{\sigma^*} + \Delta^{(3)}_{\sigma^*}$$

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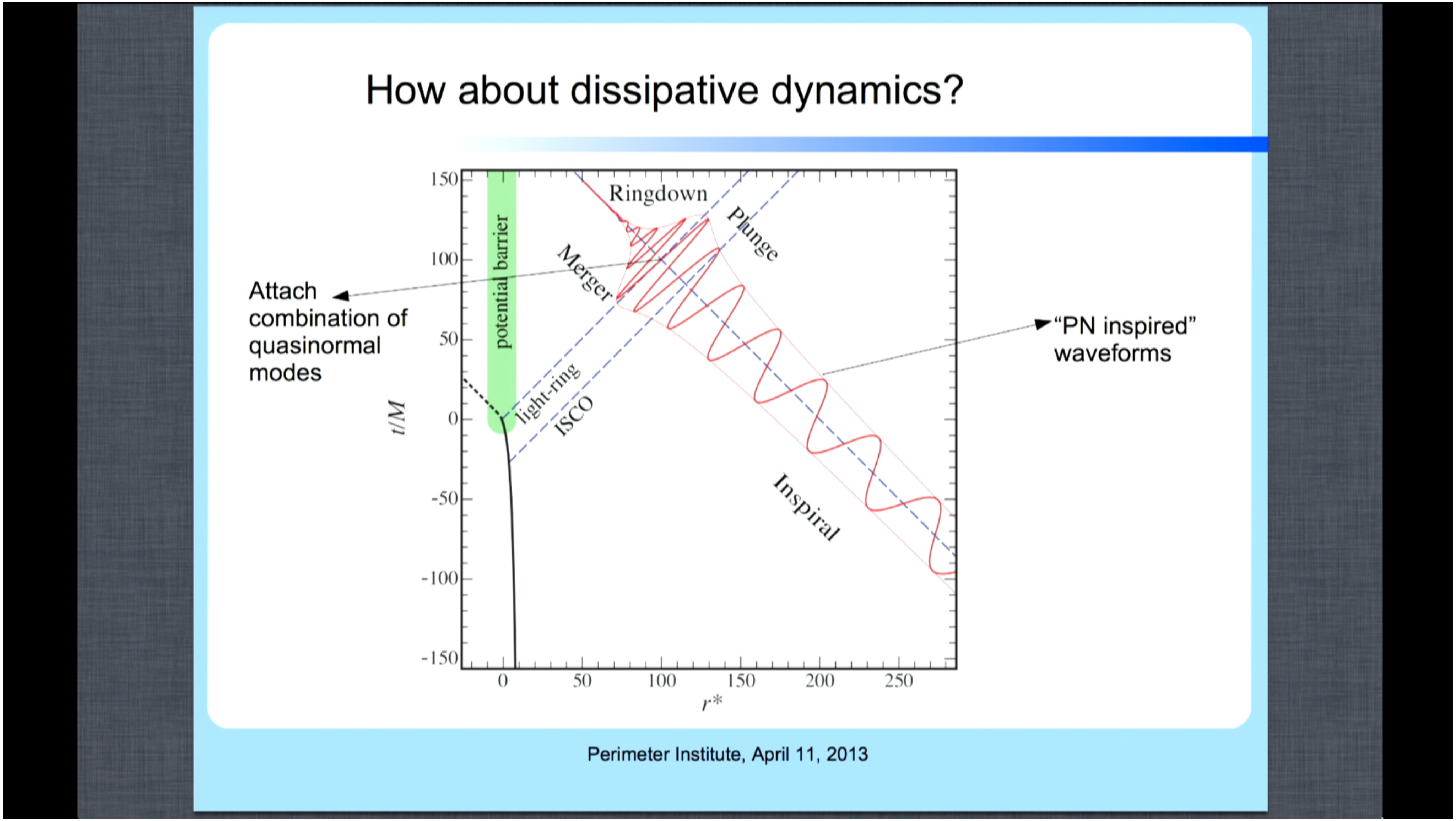
How about dissipative dynamics?



Attach combination of quasinormal modes

"PN inspired" waveforms

The EOB waveforms: the inspiral and the plunge

- Resummed PN waveforms (+ free 4PN non-spinning term $\rho_{22}^{(4)}$ in 22 mode): Damour, Nagar, Iyer, Pan, Buonanno, Racine, Fujita, Tagoshi...
- GW energy flux drives quasi-circular evolution:

$$\frac{dE}{dt} = \frac{\hat{\Omega}^2}{8\pi} \sum_{\ell=2}^{8} \sum_{m=0}^{\ell} m^2 \left| \frac{\mathcal{R}}{M} h_{\ell m} \right|^2$$

- PN waveforms valid for quasi-circular orbits, need corrections when radial motion becomes important

$$h_{\ell m}^{\text{insp-plunge}} = h_{\ell m}^{\text{F}} \, N_{\ell m}$$

$$N_{22} = \left[1 + \left(\frac{p_{r^*}}{r\,\hat{\Omega}} \right)^2 \left(a_1^{h_{22}} + \frac{a_2^{h_{22}}}{r} + \frac{a_3^{h_{22}}}{r^{3/2}} + \frac{a_4^{h_{22}}}{r^2} + \frac{a_5^{h_{22}}}{r^{5/2}} \right) \right] \exp\left[i \frac{p_{r^*}}{r\,\hat{\Omega}} \left(b_1^{h_{22}} + p_{r^*}^2 b_2^{h_{22}} + \frac{p_{r^*}^2}{r^{1/2}} b_3^{h_{22}} + \frac{p_{r^*}^2}{r} b_4^{h_{22}} \right) \right]$$

- “Non-quasicircular coefficients” are functions of the NR amplitude's peak position $\Delta t_{\text{peak}}^{22}$, of amplitude $|h_{22,\text{peak}}^{\text{NR}}|$ and its curvature $M^2 \partial_t^2 |h_{22,\text{peak}}^{\text{NR}}|$ at the peak, and of the frequency $M\omega_{22,\text{peak}}^{\text{NR}}$ and its slope $M^2 \dot{\omega}_{22,\text{peak}}^{\text{NR}}$ at the peak

⇨ NQC coefficients can be easily tabulated or fit from NR waveforms

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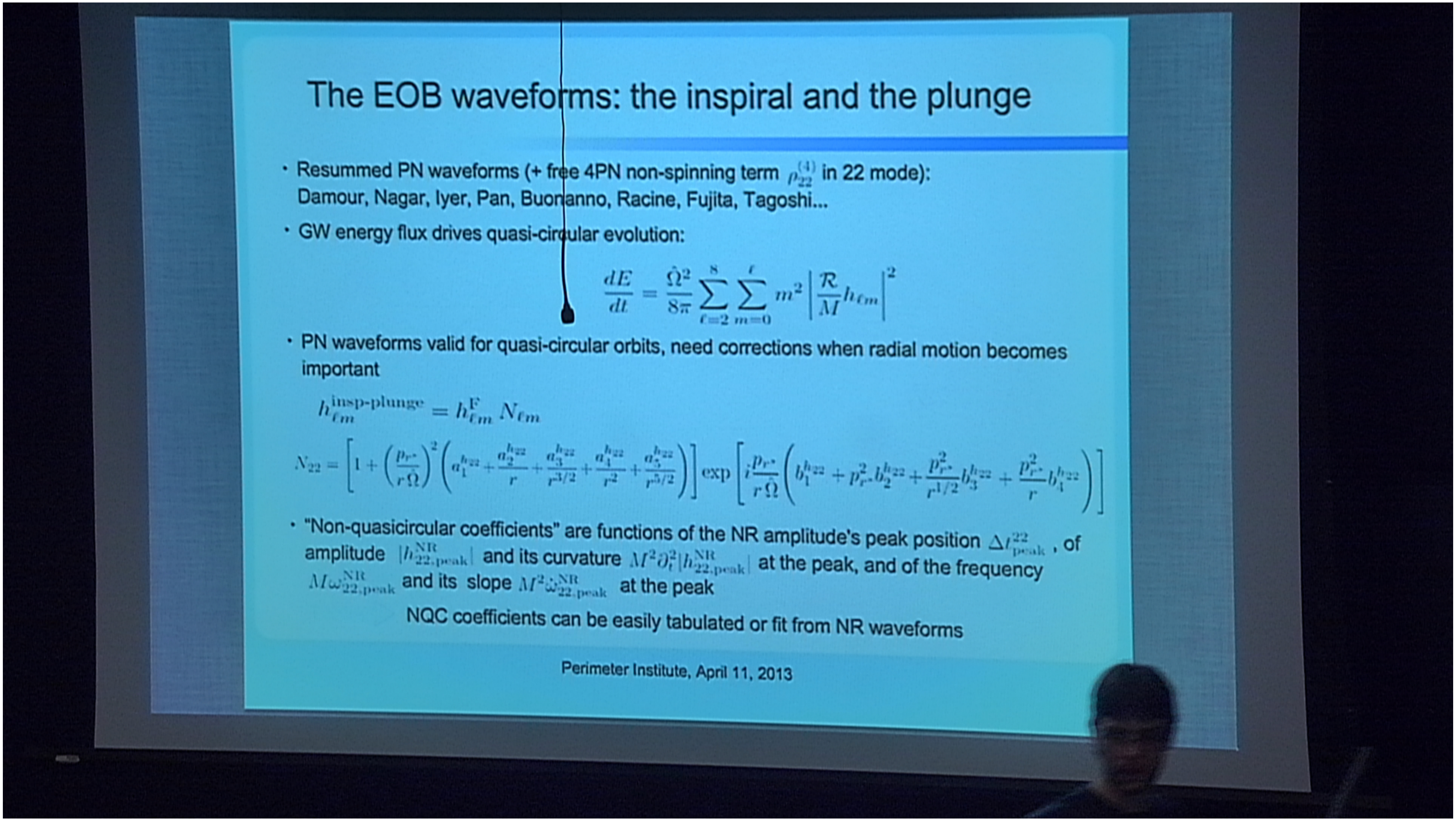

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$$h_{\ell m}^{\text{insp-plunge}} = h_{\ell m}^{\text{F}}\, N_{\ell m}$$

$$N_{22} = \left[1 + \left(\frac{p_{r^*}}{r\hat{\Omega}}\right)^2\left(a_1^{h_{22}} + \frac{a_2^{h_{22}}}{r} + \frac{a_3^{h_{22}}}{r^{3/2}} + \frac{a_4^{h_{22}}}{r^2} + \frac{a_5^{h_{22}}}{r^{5/2}}\right)\right]\exp\left[i\frac{p_{r^*}}{r\hat{\Omega}}\left(b_1^{h_{22}} + p_{r^*}^2 b_2^{h_{22}} + \frac{p_{r^*}^2}{r^{1/2}} b_3^{h_{22}} + \frac{p_{r^*}^2}{r} b_4^{h_{22}}\right)\right]$$

- "Non-quasicircular coefficients" are functions of the NR amplitude's peak position $\Delta t_{\text{peak}}^{22}$, of amplitude $|h_{22,\text{peak}}^{\text{NR}}|$ and its curvature $M^2\partial_t^2|h_{22,\text{peak}}^{\text{NR}}|$ at the peak, and of the frequency $M\omega_{22,\text{peak}}^{\text{NR}}$ and its slope $M^2\dot{\omega}_{22,\text{peak}}^{\text{NR}}$ at the peak

NQC coefficients can be easily tabulated or fit from NR waveforms

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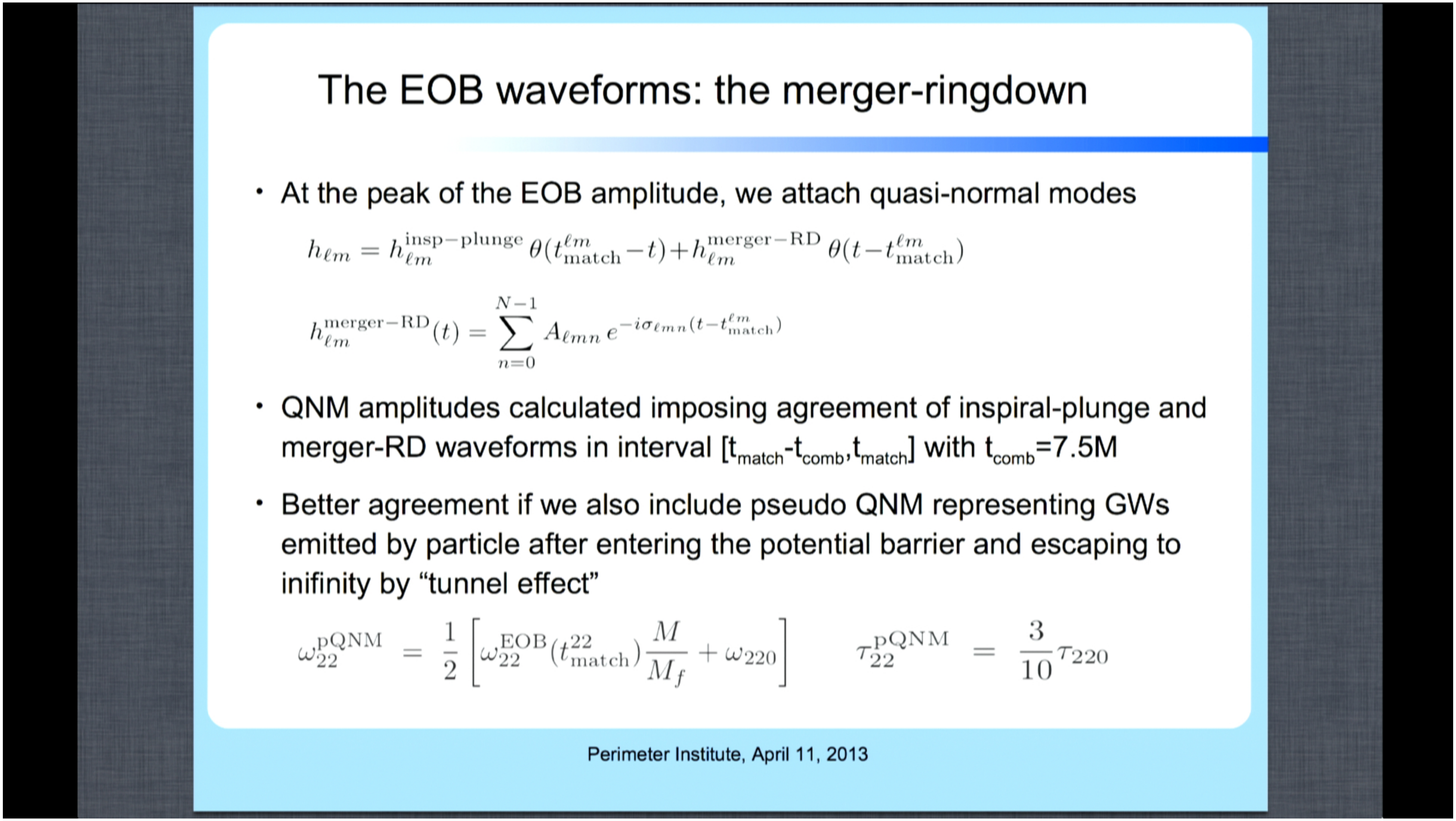

The EOB waveforms: the merger-ringdown

- At the peak of the EOB amplitude, we attach quasi-normal modes

$$h_{\ell m} = h_{\ell m}^{\rm insp-plunge}\,\theta(t^{\ell m}_{\rm match}-t)+h_{\ell m}^{\rm merger-RD}\,\theta(t-t^{\ell m}_{\rm match})$$

$$h_{\ell m}^{\rm merger-RD}(t) = \sum_{n=0}^{N-1} A_{\ell m n}\, e^{-i\sigma_{\ell m n}(t-t^{\ell m}_{\rm match})}$$

- QNM amplitudes calculated imposing agreement of inspiral-plunge and merger-RD waveforms in interval $[t_{\rm match}-t_{\rm comb}, t_{\rm match}]$ with $t_{\rm comb}=7.5M$
- Better agreement if we also include pseudo QNM representing GWs emitted by particle after entering the potential barrier and escaping to inifinity by “tunnel effect”

$$\omega_{22}^{\rm pQNM} = \frac{1}{2}\left[\omega_{22}^{\rm EOB}(t^{22}_{\rm match})\,\frac{M}{M_f} + \omega_{220}\right] \qquad \tau_{22}^{\rm pQNM} = \frac{3}{10}\tau_{220}$$

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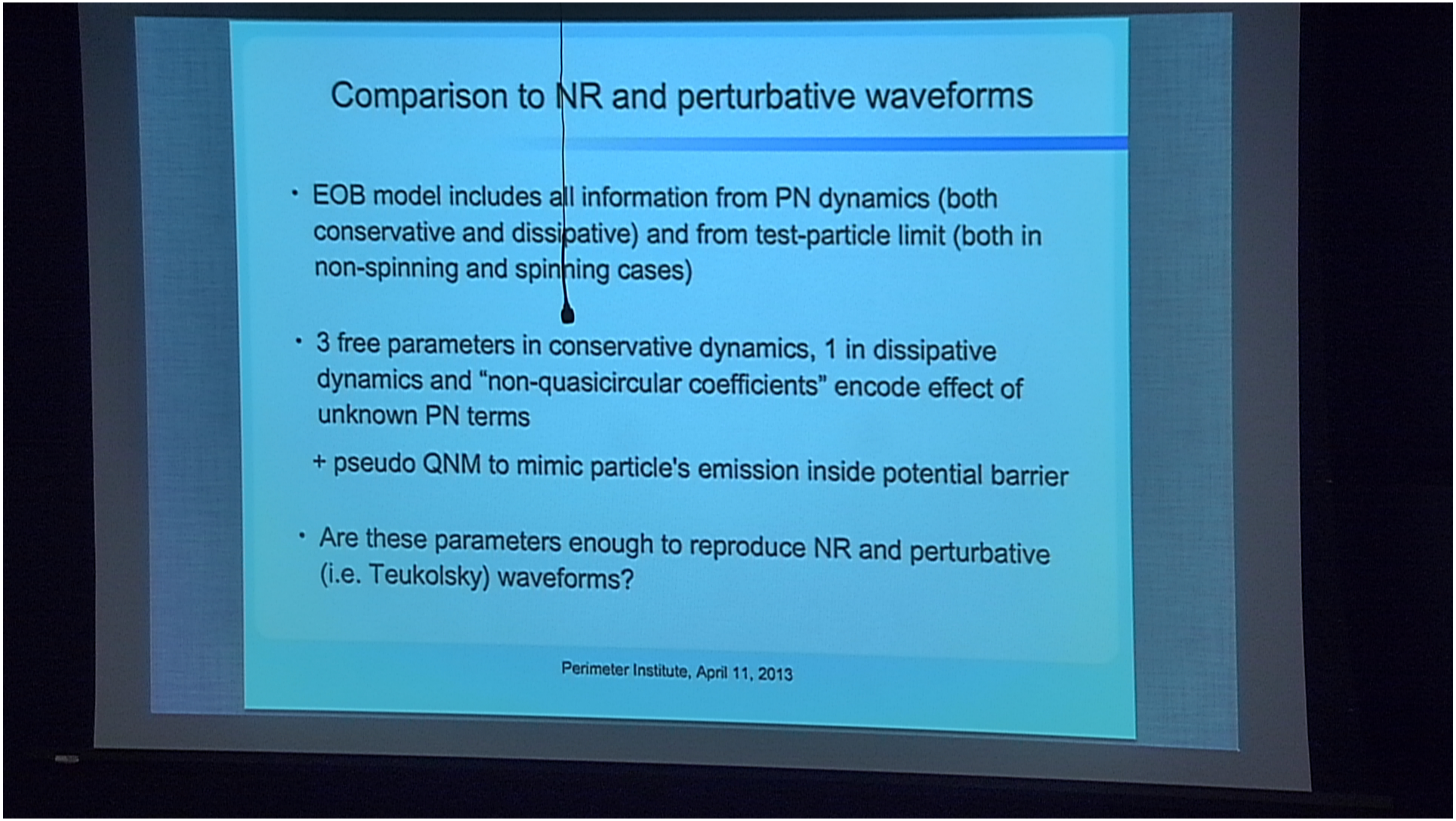

Comparison to NR and perturbative waveforms

- EOB model includes all information from PN dynamics (both conservative and dissipative) and from test-particle limit (both in non-spinning and spinning cases)
- 3 free parameters in conservative dynamics, 1 in dissipative dynamics and “non-quasicircular coefficients” encode effect of unknown PN terms

  + pseudo QNM to mimic particle's emission inside potential barrier
- Are these parameters enough to reproduce NR and perturbative (i.e. Teukolsky) waveforms?

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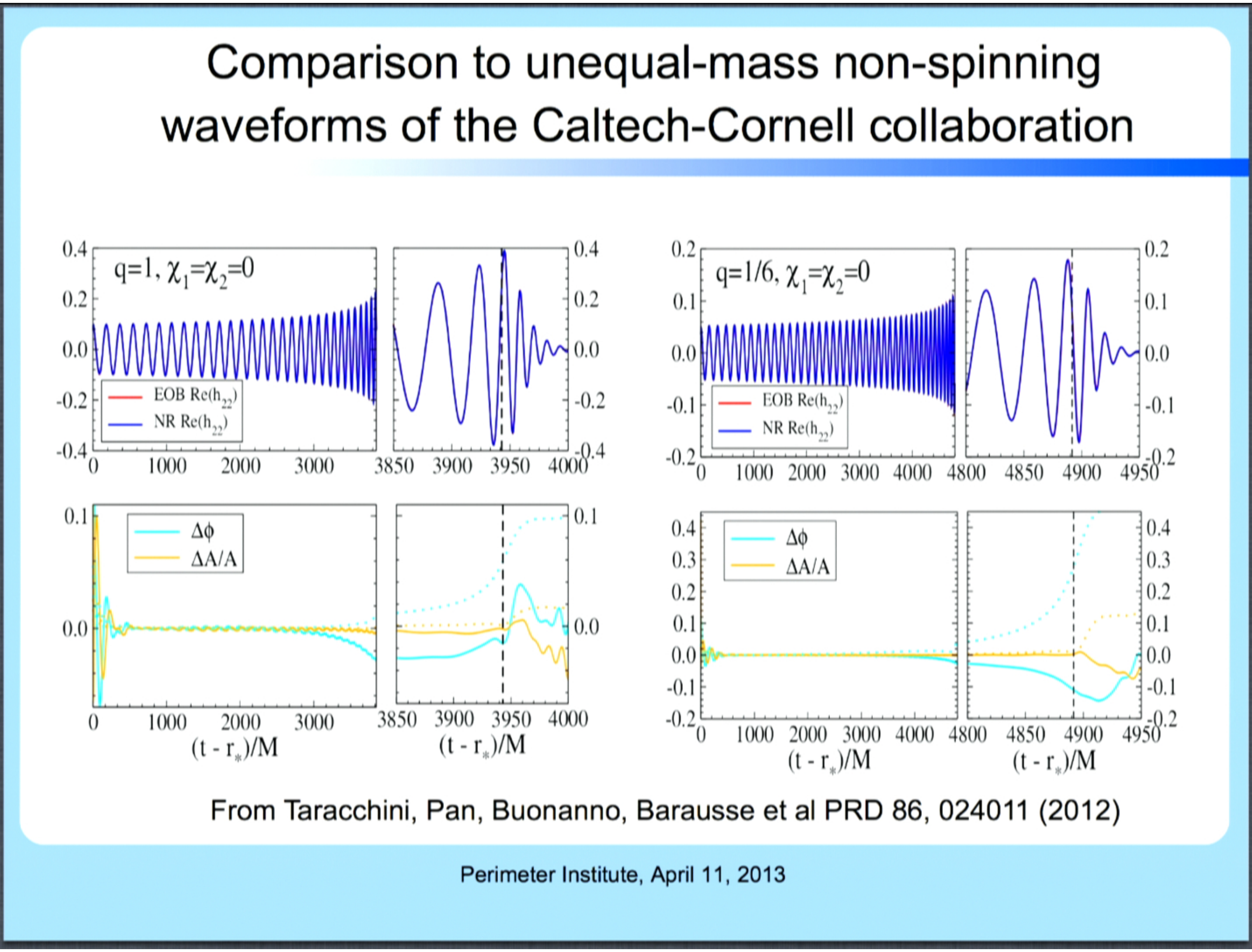

Comparison to unequal-mass non-spinning waveforms of the Caltech-Cornell collaboration

From Taracchini, Pan, Buonanno, Barausse et al PRD 86, 024011 (2012)

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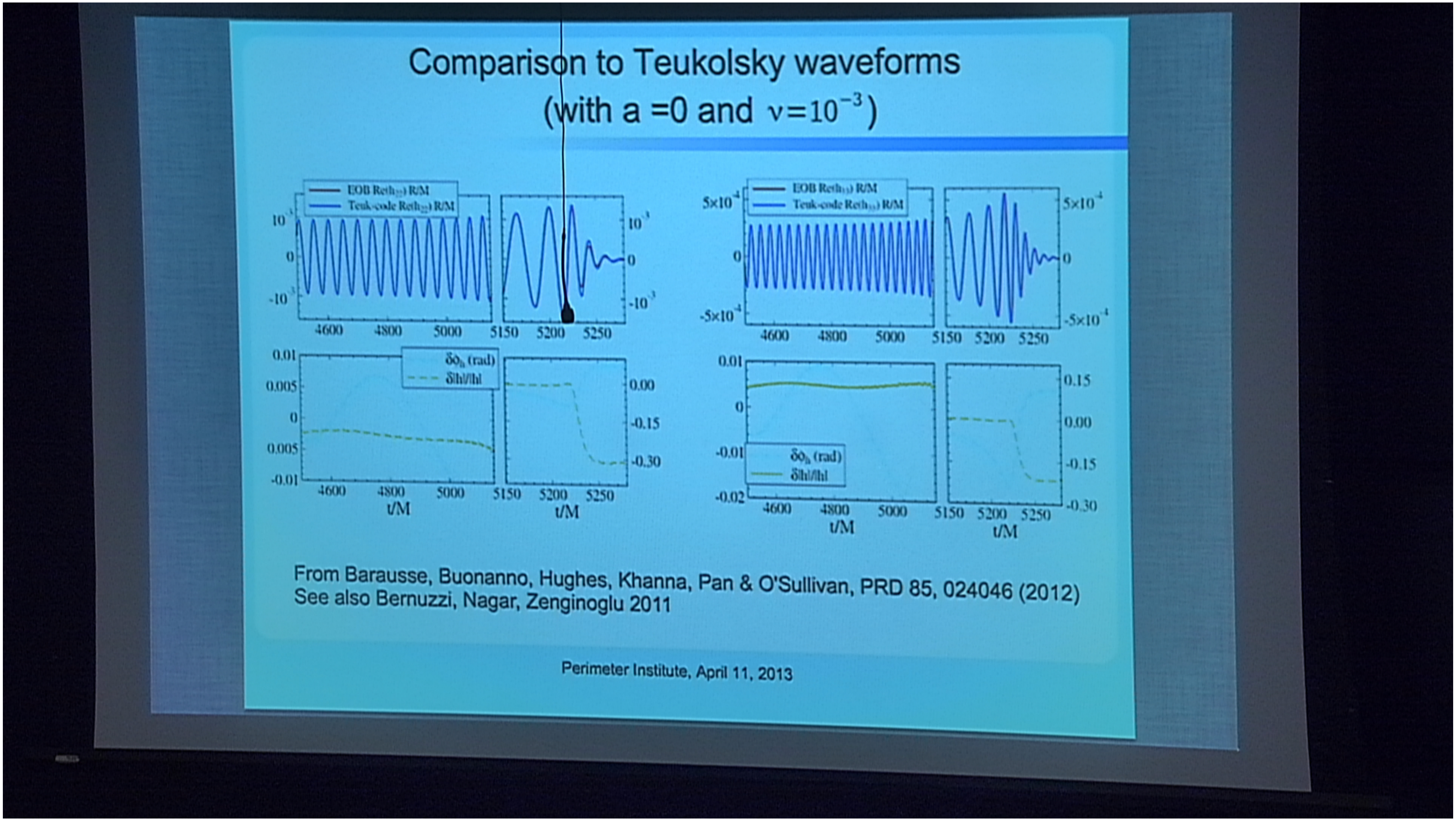

Comparison to Teukolsky waveforms (with a =0 and $\nu=10^{-3}$)

From Barausse, Buonanno, Hughes, Khanna, Pan & O'Sullivan, PRD 85, 024046 (2012)
See also Bernuzzi, Nagar, Zenginoglu 2011

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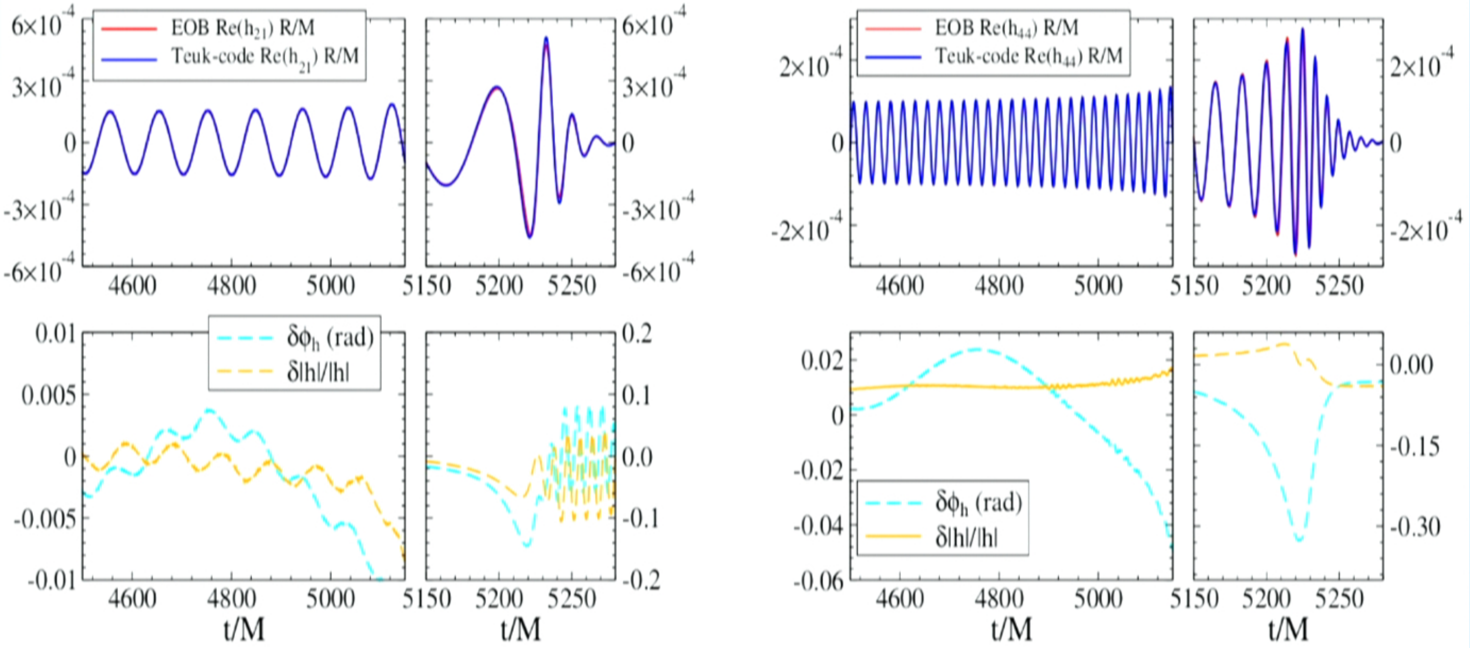
Comparison to Teukolsky waveforms (with a =0 and $\nu=10^{-3}$)



From Barausse, Buonanno, Hughes, Khanna, Pan & O'Sullivan, PRD 85, 024046 (2012)
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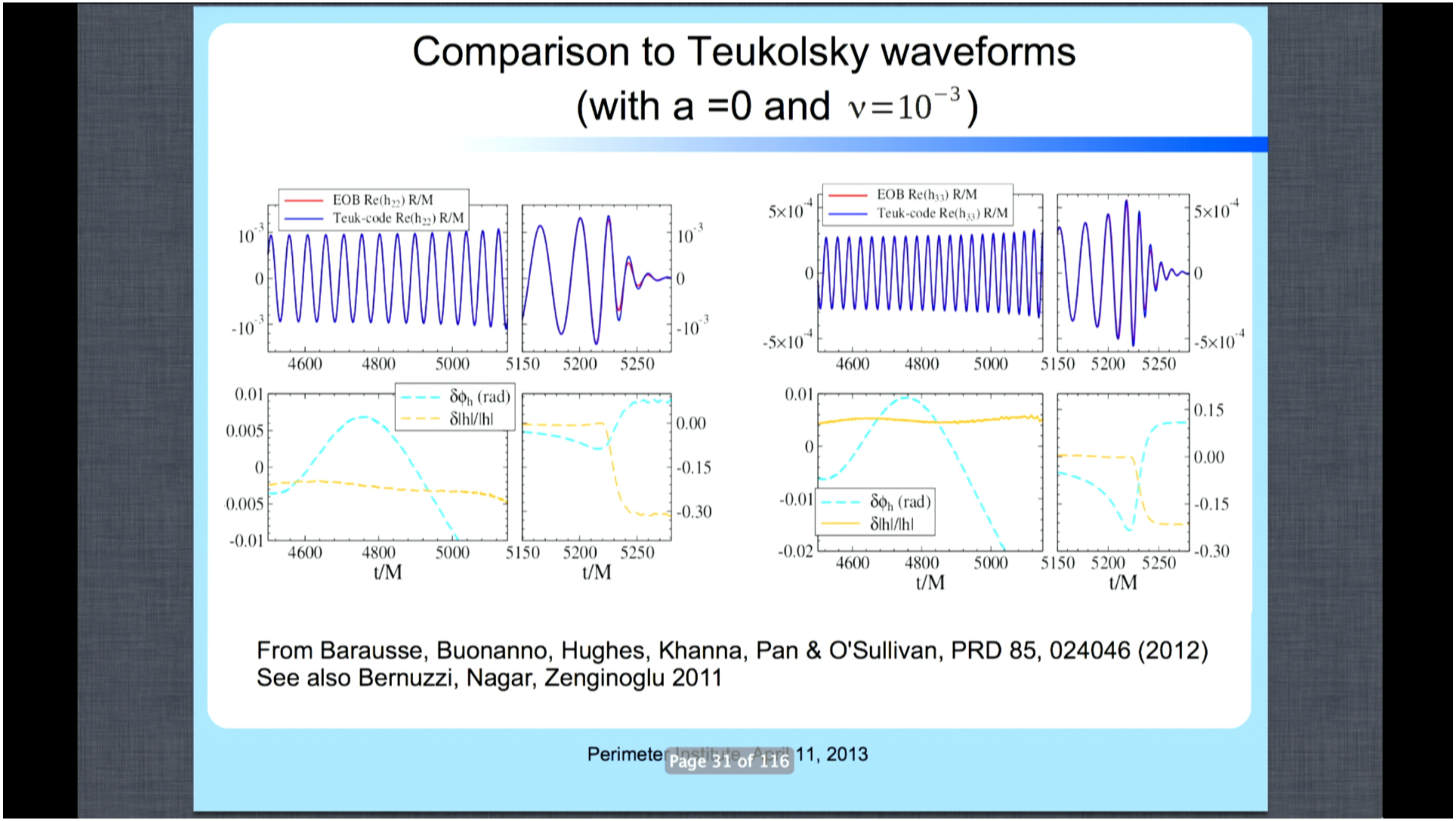

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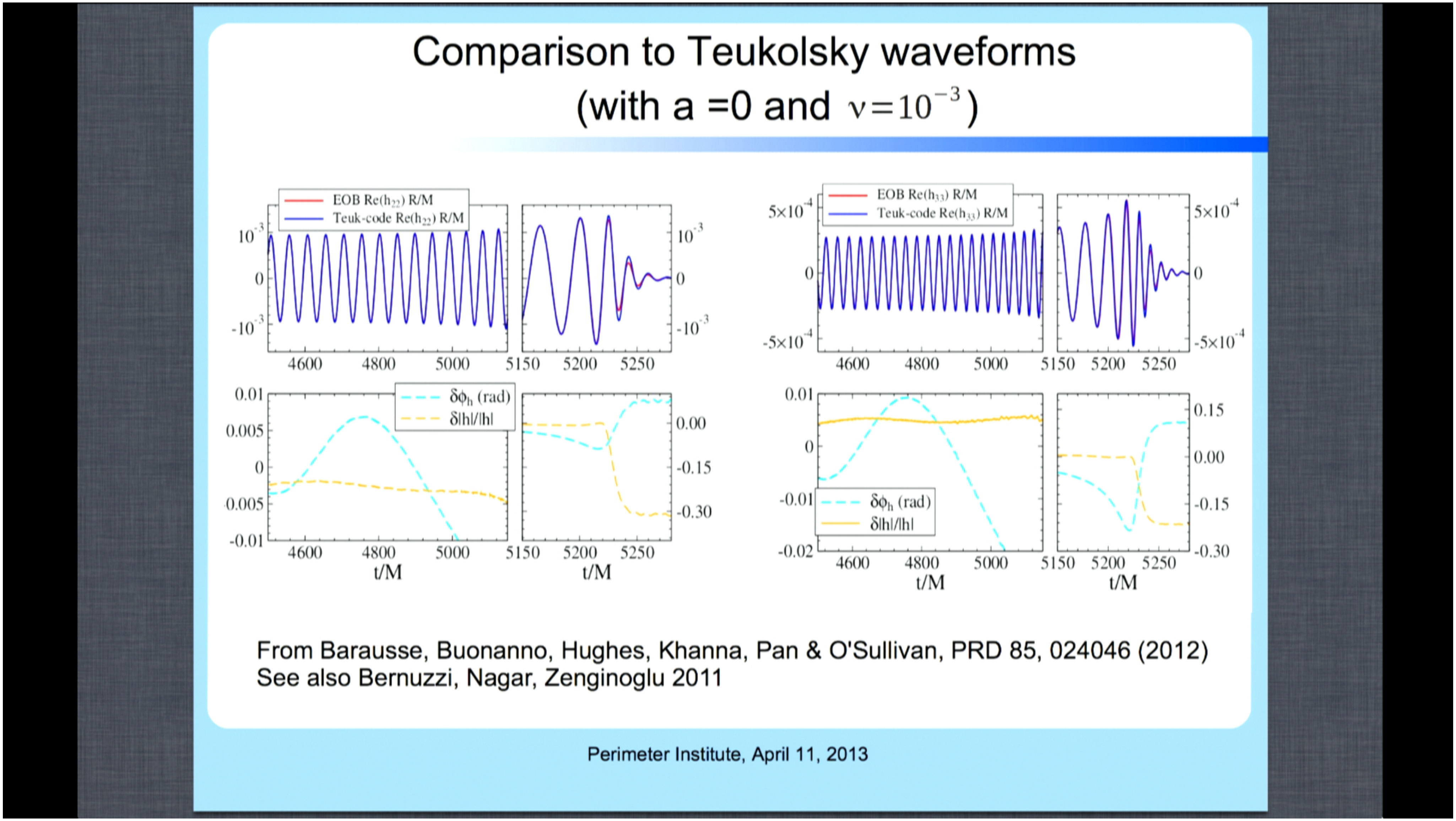

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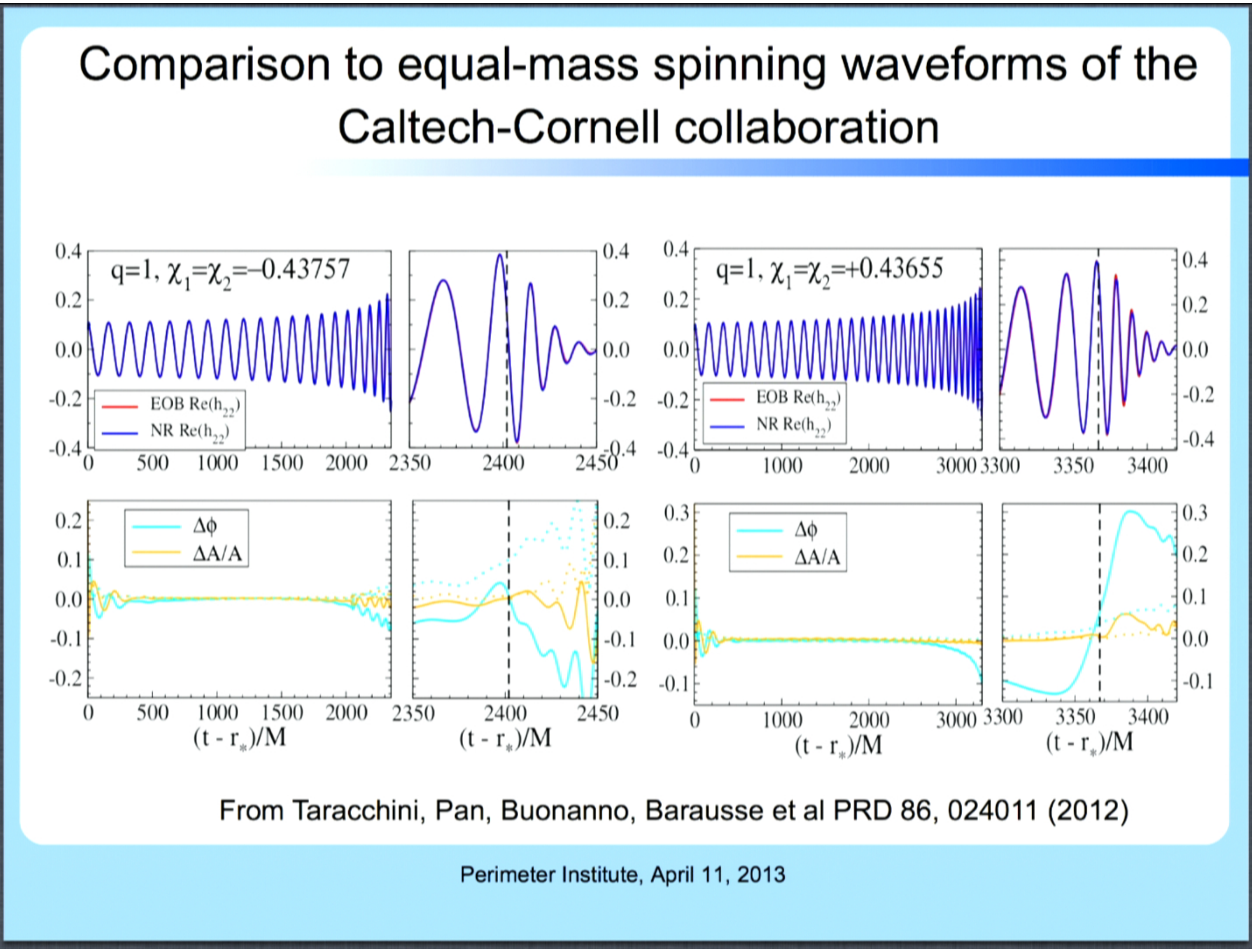

Comparison to equal-mass spinning waveforms of the Caltech-Cornell collaboration

From Taracchini, Pan, Buonanno, Barausse et al PRD 86, 024011 (2012)

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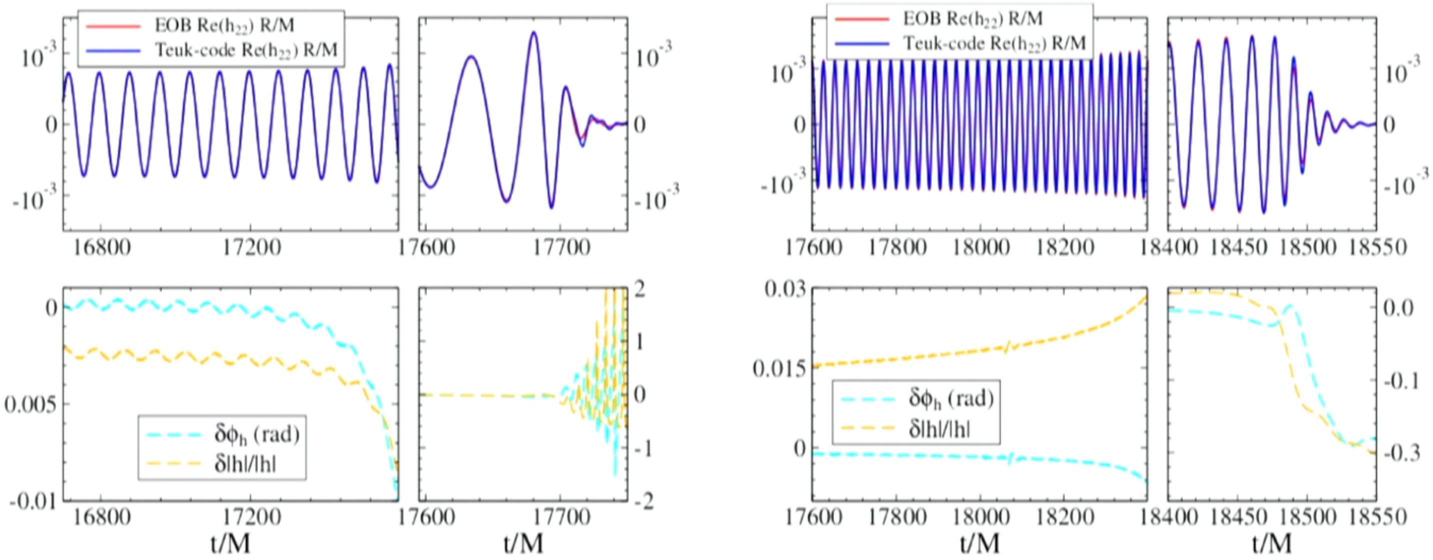
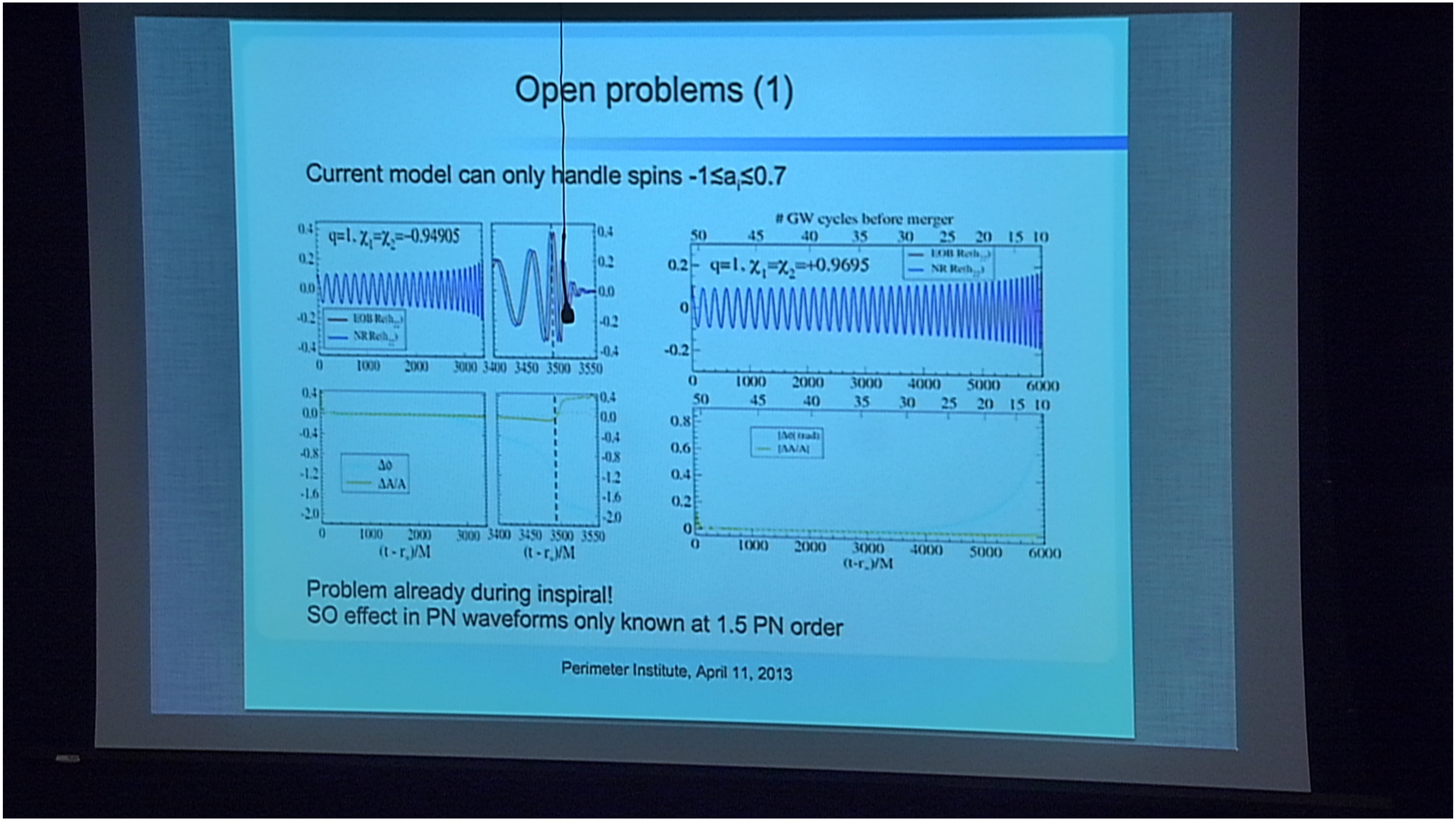
Comparison to Teukolsky waveforms (with $a \neq 0$ and $\nu = 10^{-3}$)



a=-0.9

a=0.7

From Barausse, Buonanno, Hughes, Khanna, Pan & O'Sullivan, PRD 85, 024046 (2012)



Open problems (1)

Current model can only handle spins $-1 \le a_i \le 0.7$

Problem already during inspiral!

SO effect in PN waveforms only known at 1.5 PN order

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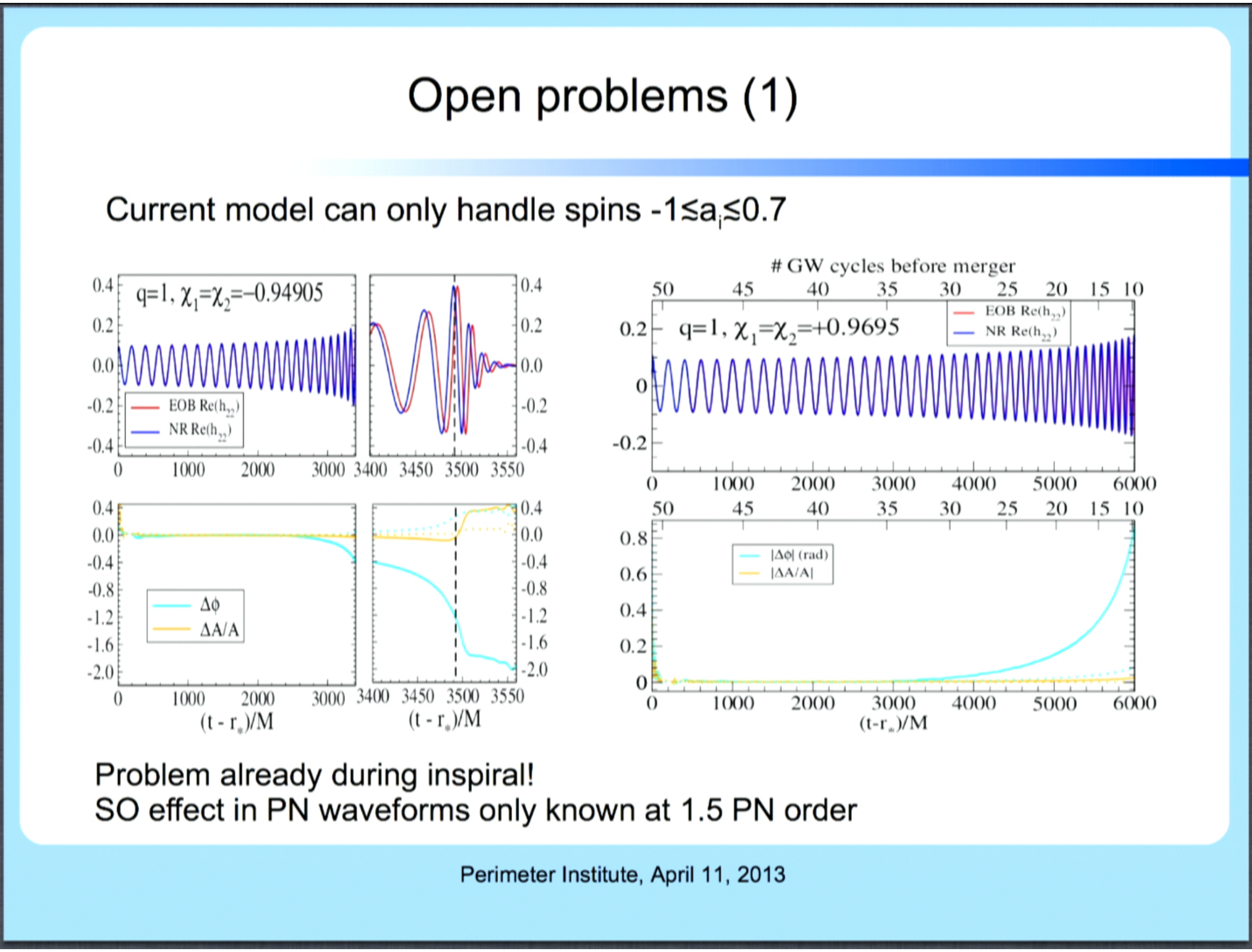

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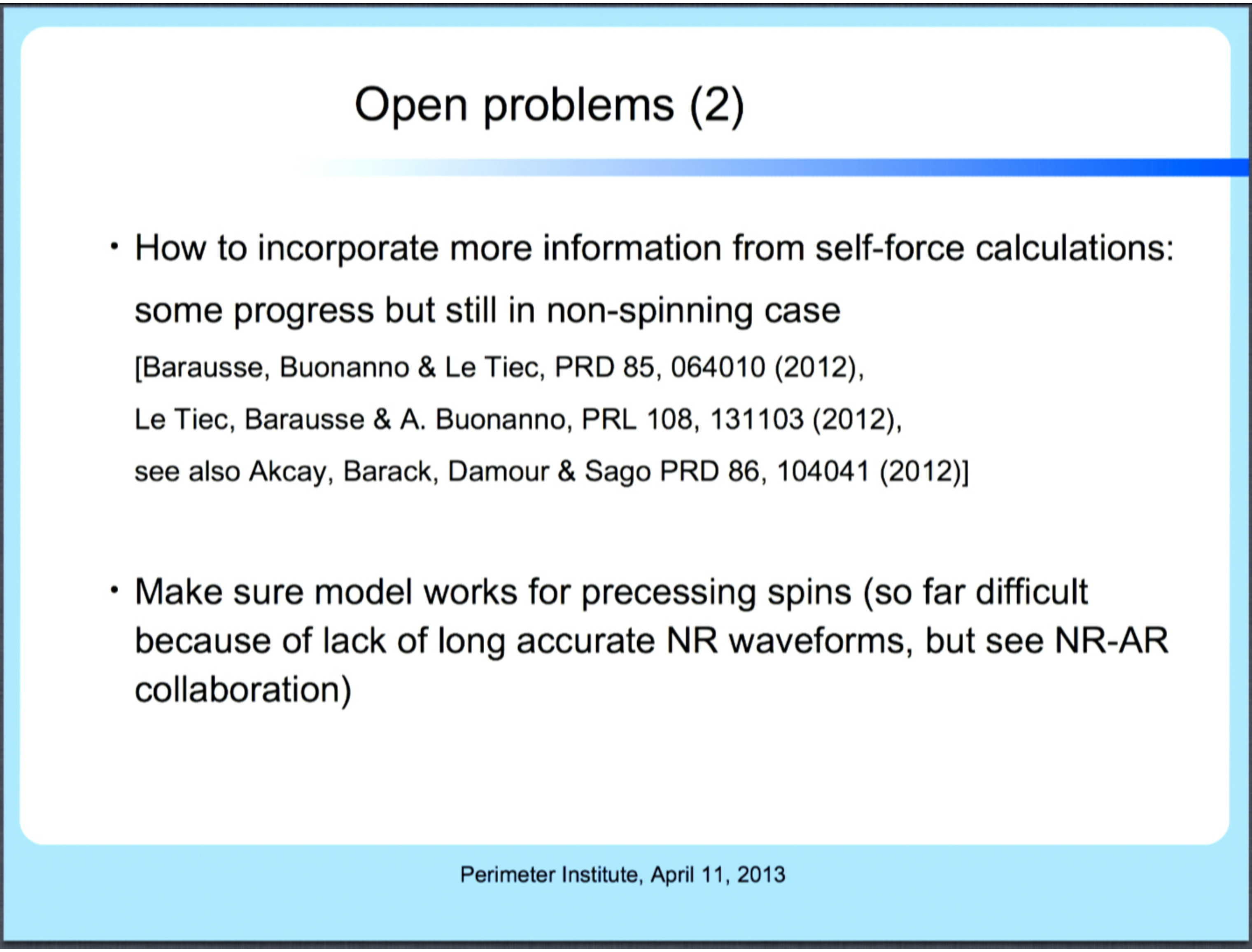

Open problems (2)

- How to incorporate more information from self-force calculations: some progress but still in non-spinning case
  [Barausse, Buonanno & Le Tiec, PRD 85, 064010 (2012),
  Le Tiec, Barausse & A. Buonanno, PRL 108, 131103 (2012),
  see also Akcay, Barack, Damour & Sago PRD 86, 104041 (2012)]

- Make sure model works for precessing spins (so far difficult because of lack of long accurate NR waveforms, but see NR-AR collaboration)

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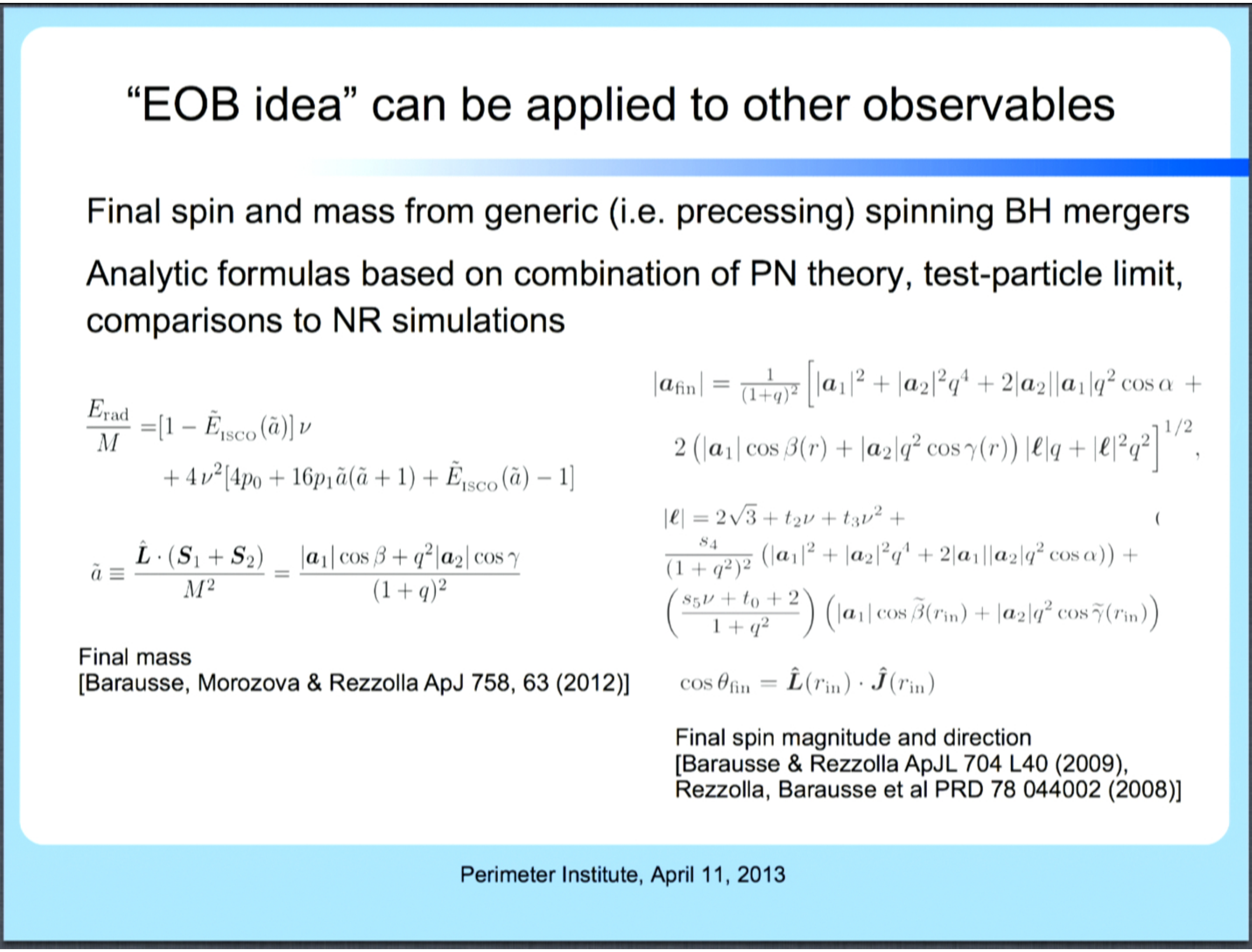

“EOB idea” can be applied to other observables

Final spin and mass from generic (i.e. precessing) spinning BH mergers

Analytic formulas based on combination of PN theory, test-particle limit, comparisons to NR simulations

$$\frac{E_{\rm rad}}{M} = [1-\tilde{E}_{\rm ISCO}(\tilde{a})]\,\nu + 4\,\nu^2[4p_0+16p_1\tilde{a}(\tilde{a}+1)+\tilde{E}_{\rm ISCO}(\tilde{a})-1]$$

$$\tilde{a} \equiv \frac{\hat{\boldsymbol{L}}\cdot(\boldsymbol{S}_1+\boldsymbol{S}_2)}{M^2} = \frac{|\boldsymbol{a}_1|\cos\beta + q^2|\boldsymbol{a}_2|\cos\gamma}{(1+q)^2}$$

Final mass
[Barausse, Morozova & Rezzolla ApJ 758, 63 (2012)]

$$|\boldsymbol{a}_{\rm fin}| = \tfrac{1}{(1+q)^2}\Big[|\boldsymbol{a}_1|^2 + |\boldsymbol{a}_2|^2q^4 + 2|\boldsymbol{a}_2||\boldsymbol{a}_1|q^2\cos\alpha\; + 2\left(|\boldsymbol{a}_1|\cos\beta(r) + |\boldsymbol{a}_2|q^2\cos\gamma(r)\right)|\boldsymbol{\ell}|q + |\boldsymbol{\ell}|^2q^2\Big]^{1/2},$$

$$|\boldsymbol{\ell}| = 2\sqrt{3} + t_2\nu + t_3\nu^2 + \qquad (\\ \frac{s_4}{(1+q^2)^2}\left(|\boldsymbol{a}_1|^2 + |\boldsymbol{a}_2|^2q^4 + 2|\boldsymbol{a}_1||\boldsymbol{a}_2|q^2\cos\alpha\right)) + \\ \left(\frac{s_5\nu+t_0+2}{1+q^2}\right)\left(|\boldsymbol{a}_1|\cos\widetilde{\beta}(r_{\rm in}) + |\boldsymbol{a}_2|q^2\cos\widetilde{\gamma}(r_{\rm in})\right)$$

$$\cos\theta_{\rm fin} = \hat{\boldsymbol{L}}(r_{\rm in})\cdot\hat{\boldsymbol{J}}(r_{\rm in})$$

Final spin magnitude and direction
[Barausse & Rezzolla ApJL 704 L40 (2009),
Rezzolla, Barausse et al PRD 78 044002 (2008)]

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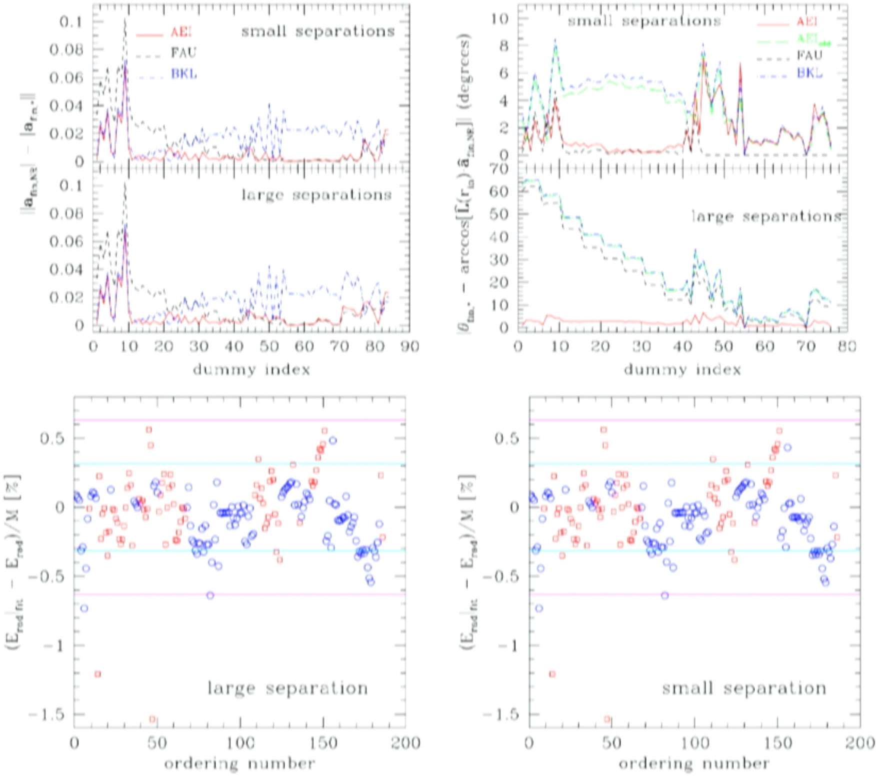
Formulas agree with NR data!



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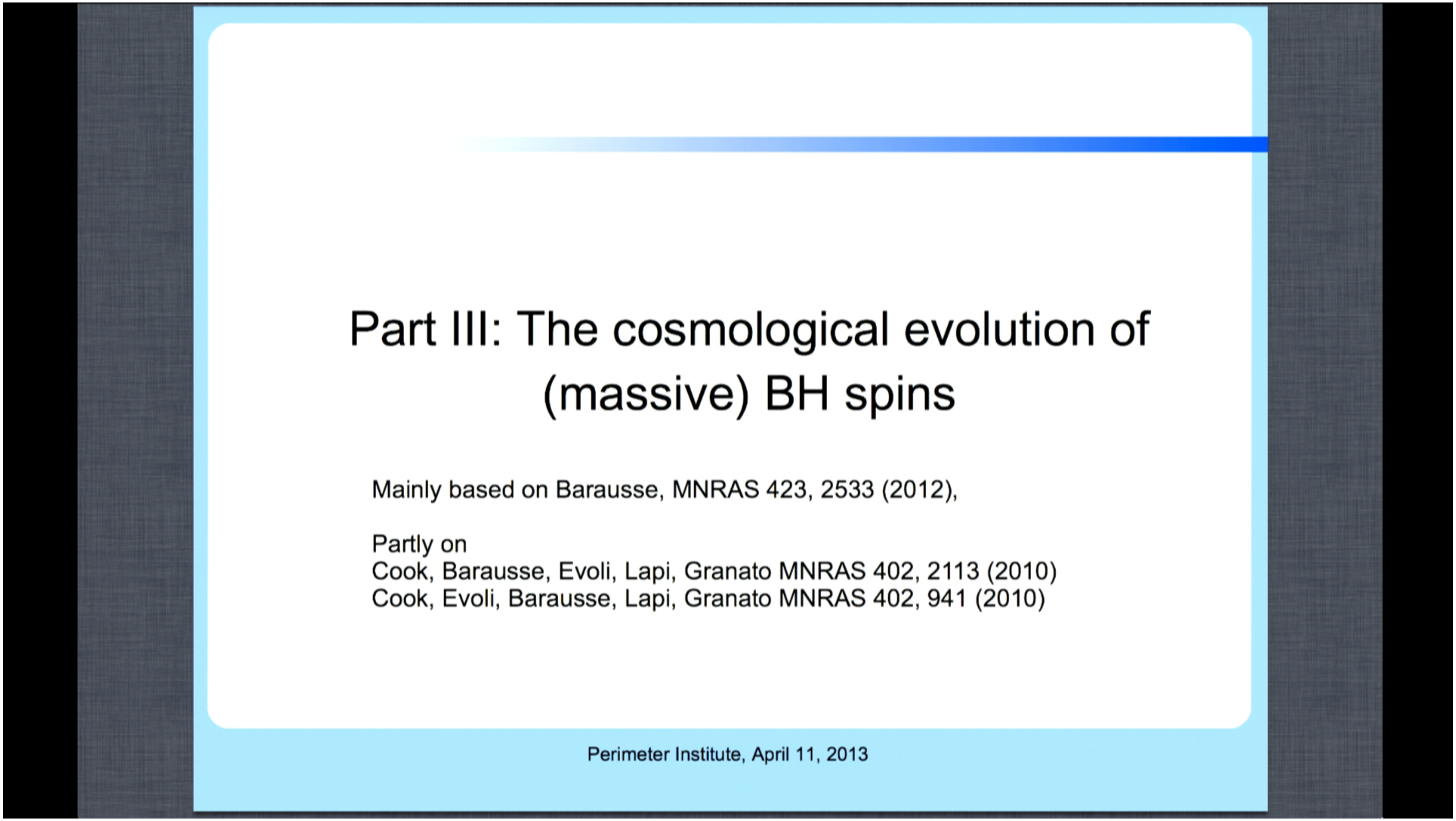

Part III: The cosmological evolution of (massive) BH spins

Mainly based on Barausse, MNRAS 423, 2533 (2012),

Partly on
Cook, Barausse, Evoli, Lapi, Granato MNRAS 402, 2113 (2010)
Cook, Evoli, Barausse, Lapi, Granato MNRAS 402, 941 (2010)

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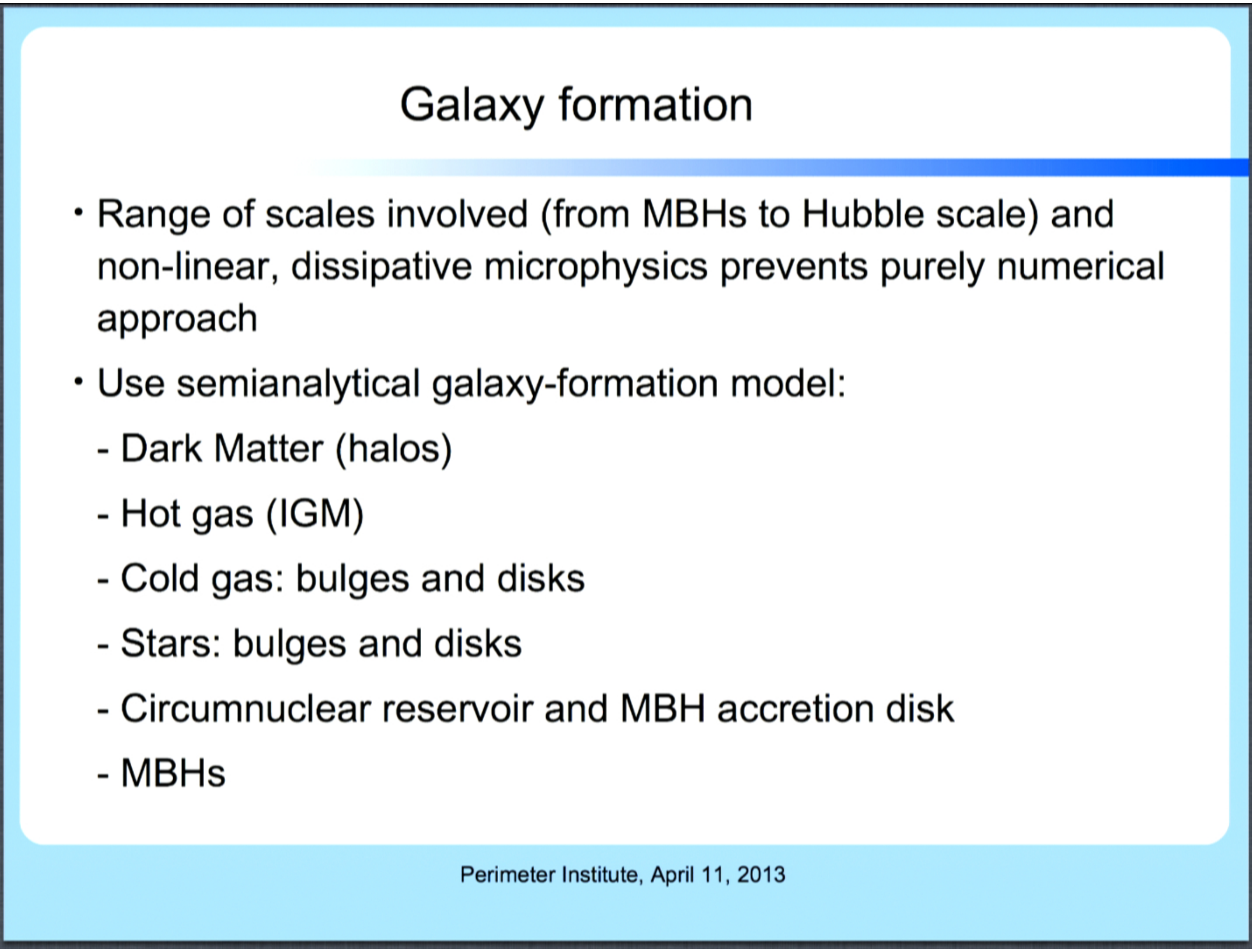

Galaxy formation

- Range of scales involved (from MBHs to Hubble scale) and non-linear, dissipative microphysics prevents purely numerical approach
- Use semianalytical galaxy-formation model:
  - Dark Matter (halos)
  - Hot gas (IGM)
  - Cold gas: bulges and disks
  - Stars: bulges and disks
  - Circumnuclear reservoir and MBH accretion disk
  - MBHs

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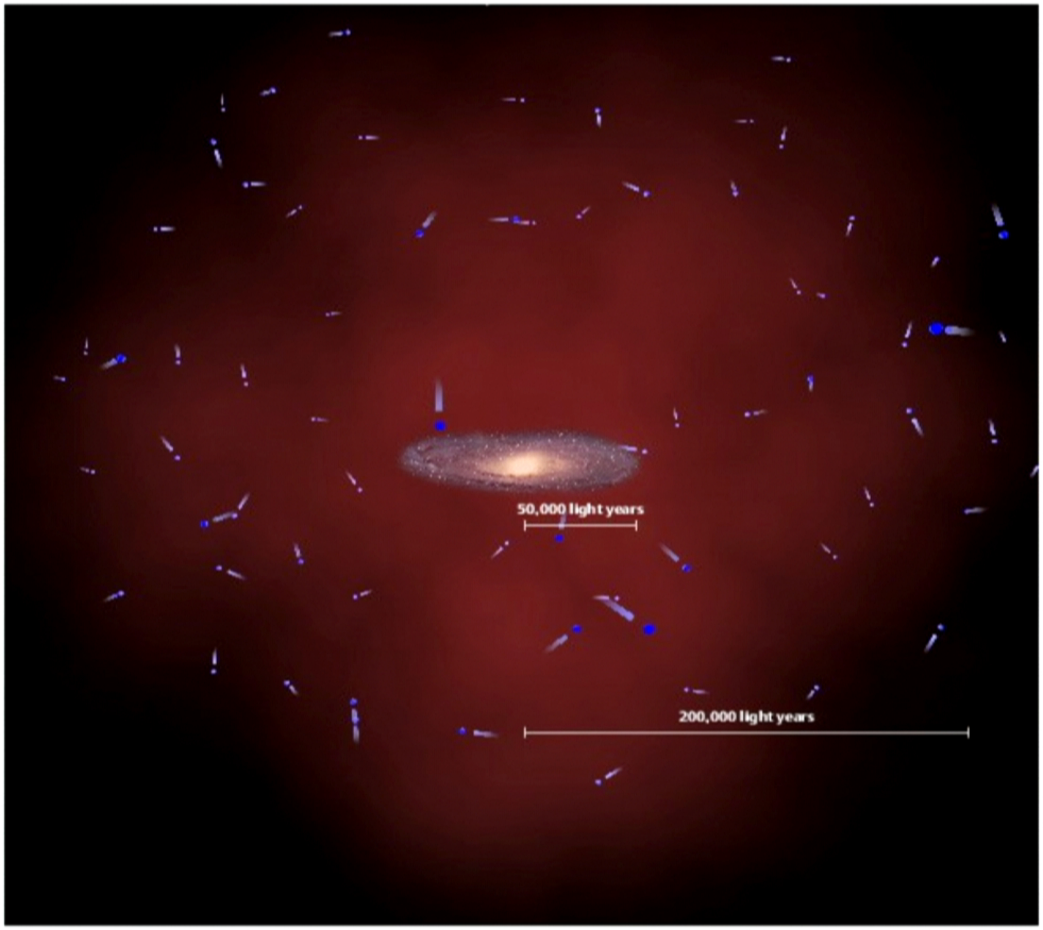

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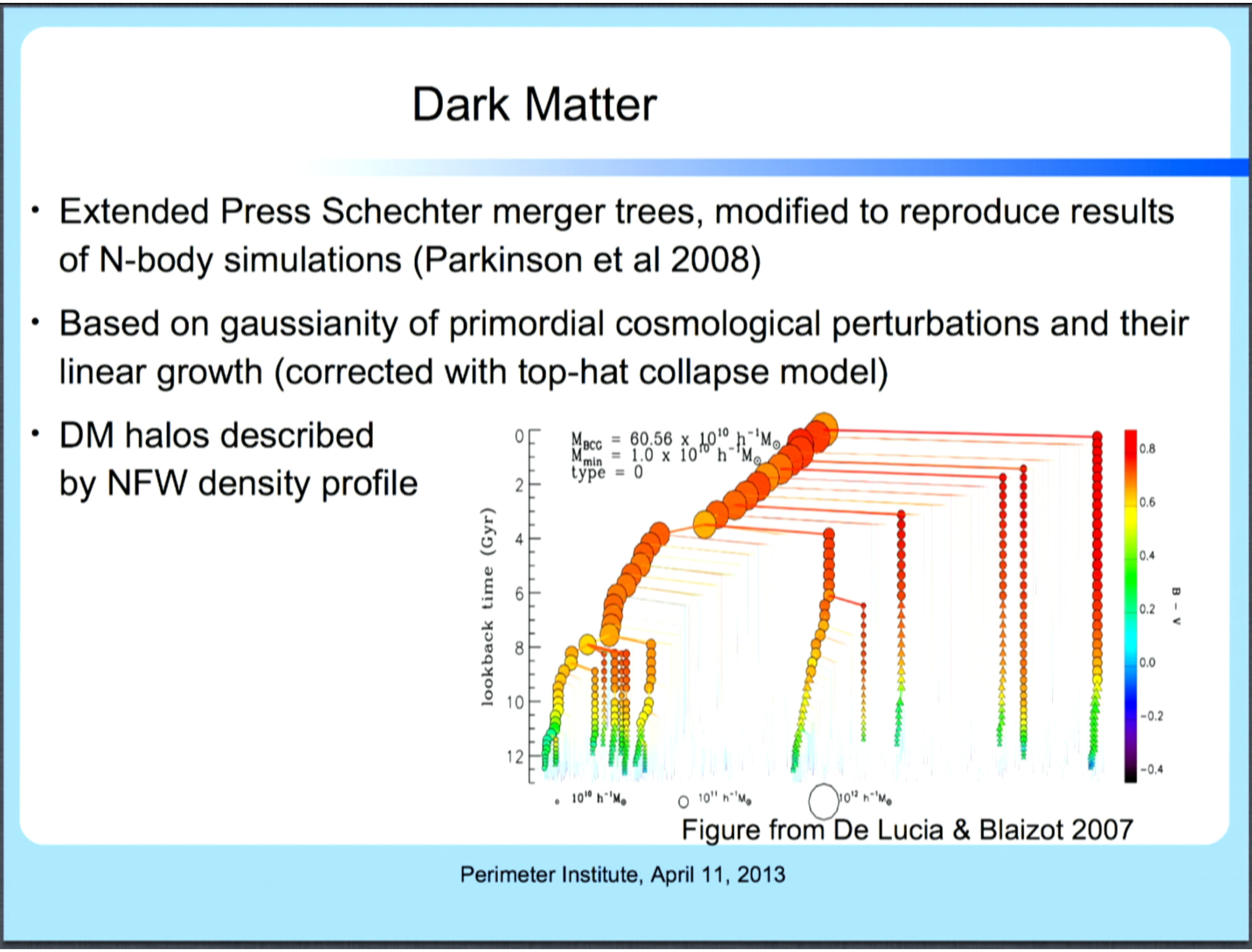

Dark Matter

- Extended Press Schechter merger trees, modified to reproduce results of N-body simulations (Parkinson et al 2008)
- Based on gaussianity of primordial cosmological perturbations and their linear growth (corrected with top-hat collapse model)
- DM halos described by NFW density profile

Figure from De Lucia & Blaizot 2007

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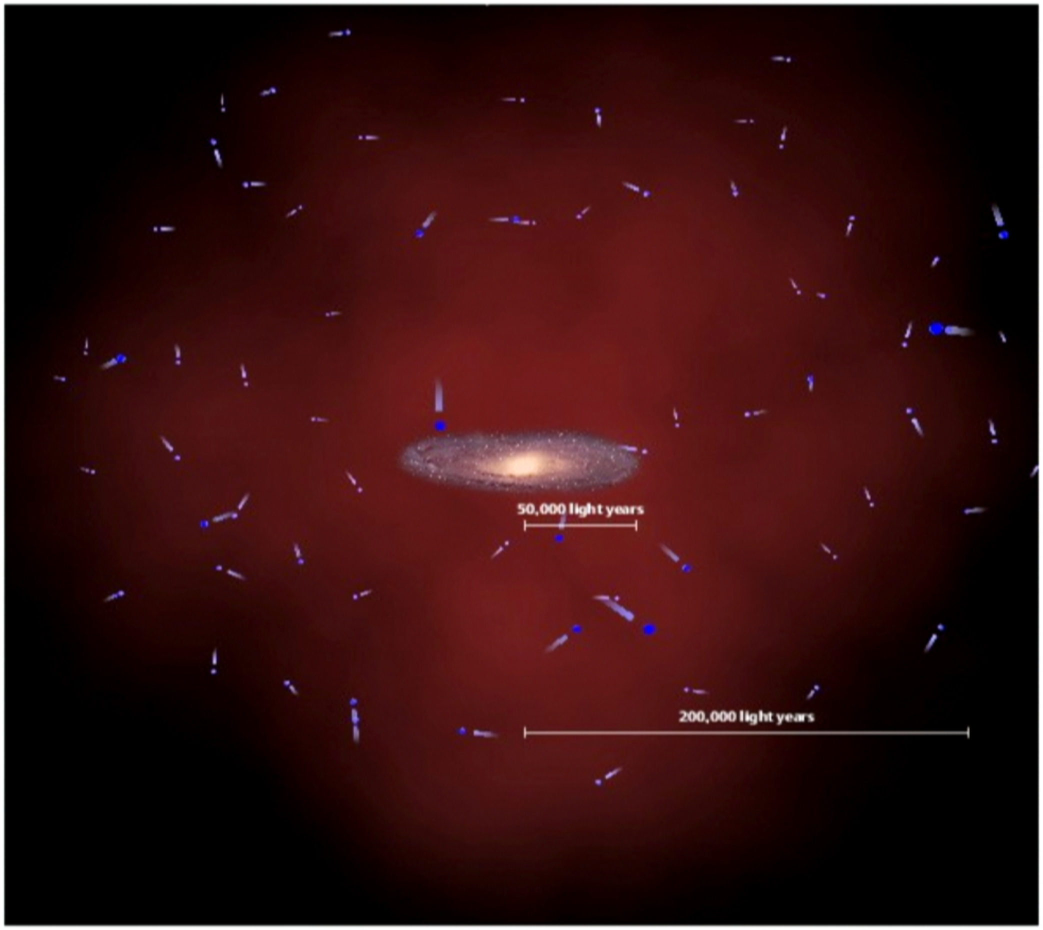

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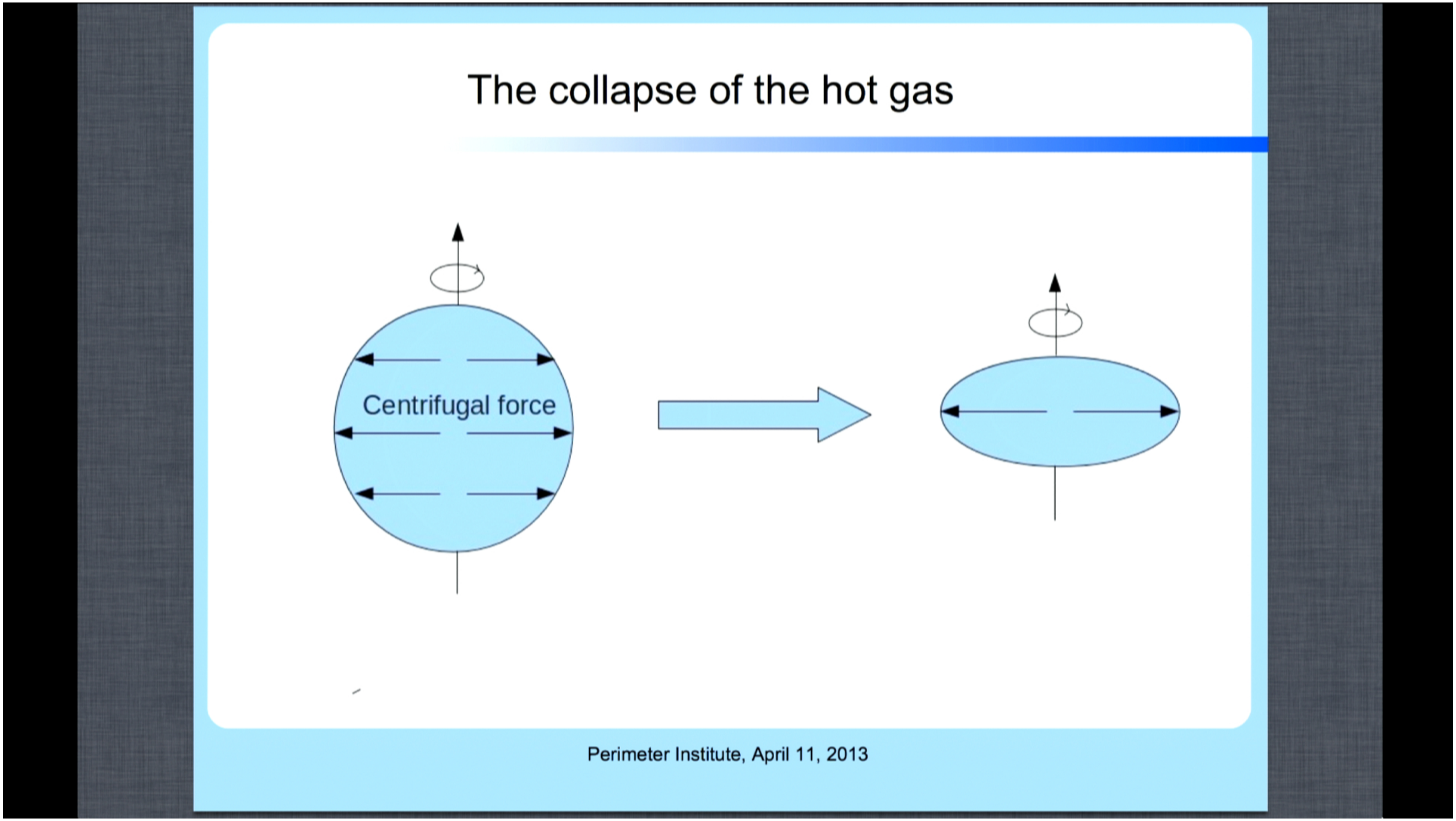
The baryonic components: the hot gas

- Hot gas: primordial metallicity, brought in by DM accreting on halos between mergers

$$\dot{M}_{hot} = f_b \, \dot{M}_{DM}$$

with baryon fraction $f_b \leqslant \Omega_b / \Omega_{DM}$
including effect of UV background

- Hot gas shock-heated to virial T, unless in low-mass halos at high z, where it streams in on dynamical time (cold accretion flows)
- Hot gas collapses in gaseous disks on dynamical timescale (if it cools “rapidly”) or on cooling timescale (if it cools “slowly”).



The collapse of the hot gas

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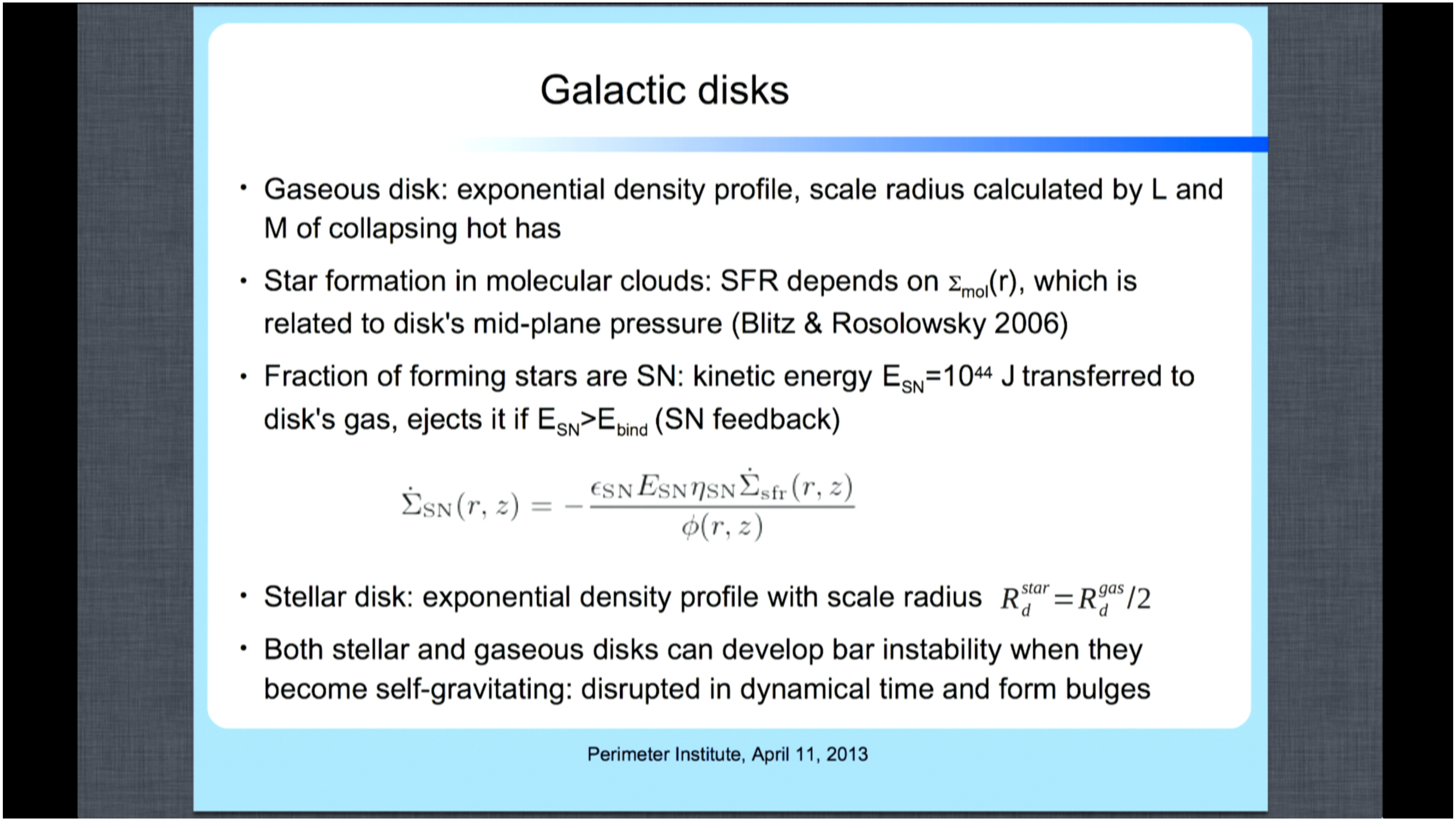

Galactic disks

- Gaseous disk: exponential density profile, scale radius calculated by L and M of collapsing hot has
- Star formation in molecular clouds: SFR depends on $\Sigma_{mol}(r)$, which is related to disk's mid-plane pressure (Blitz & Rosolowsky 2006)
- Fraction of forming stars are SN: kinetic energy $E_{SN}=10^{44}$ J transferred to disk's gas, ejects it if $E_{SN}>E_{bind}$ (SN feedback)

$$\dot{\Sigma}_{\rm SN}(r,z) = -\frac{\epsilon_{\rm SN} E_{\rm SN} \eta_{\rm SN} \dot{\Sigma}_{\rm sfr}(r,z)}{\phi(r,z)}$$

- Stellar disk: exponential density profile with scale radius $R_d^{star} = R_d^{gas}/2$
- Both stellar and gaseous disks can develop bar instability when they become self-gravitating: disrupted in dynamical time and form bulges

The Sombrero galaxy

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A galactic merger from Hubble

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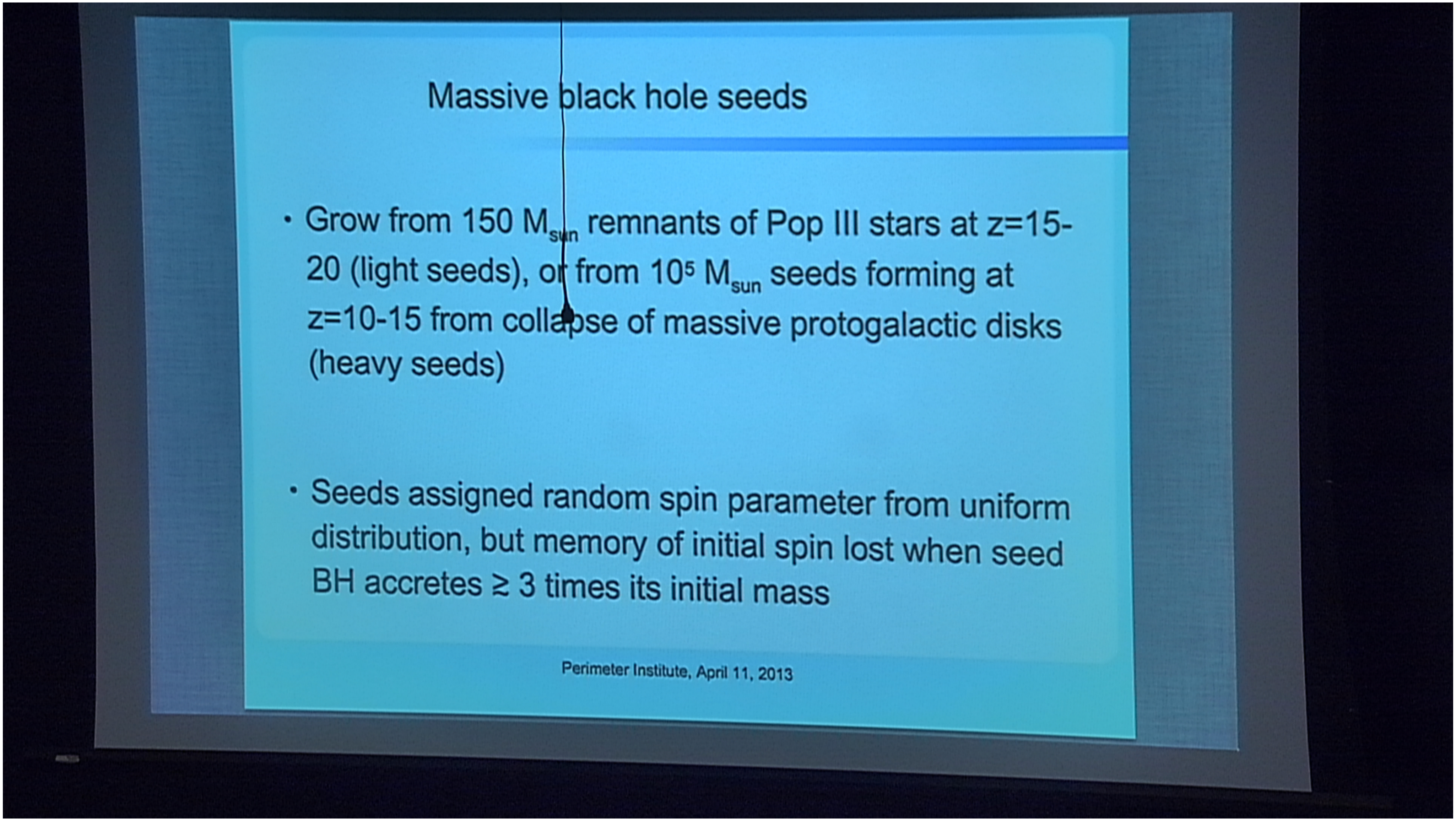
Galaxy mergers

- When two DM halos merge, baryonic structures (the "galaxies") do not merge right away, but are slowly brought together by dynamical friction (~ Gyr)
- During dynamical friction time, satellite galaxy suffers from tidal stripping and evaporation
- When galaxy merge:

  - If $M_{sat}^{disk+bulge} / M_{main}^{disk+bulge} > 0.25$ ("major merger") gasous and stellar disks disrupted and added to stellar and gaseous bulge

  - If $M_{sat}^{disk+bulge} / M_{main}^{disk+bulge} < 0.25$ ("minor merger") disks survive



Massive black hole seeds

- Grow from 150 M_{sun} remnants of Pop III stars at z=15-20 (light seeds), or from 10^5 M_{sun} seeds forming at z=10-15 from collapse of massive protogalactic disks (heavy seeds)
- Seeds assigned random spin parameter from uniform distribution, but memory of initial spin lost when seed BH accretes ≳ 3 times its initial mass

The QSO phase

- When SF happens in bulges (due to disk instabilities or major mergers), radiation drag forces cold gas into circumnuclear reservoir:

$$\dot{M}_{\rm res} = A_{\rm res}\psi_b(t)$$

- Circumnuclear reservoir accretes on MBH with rate

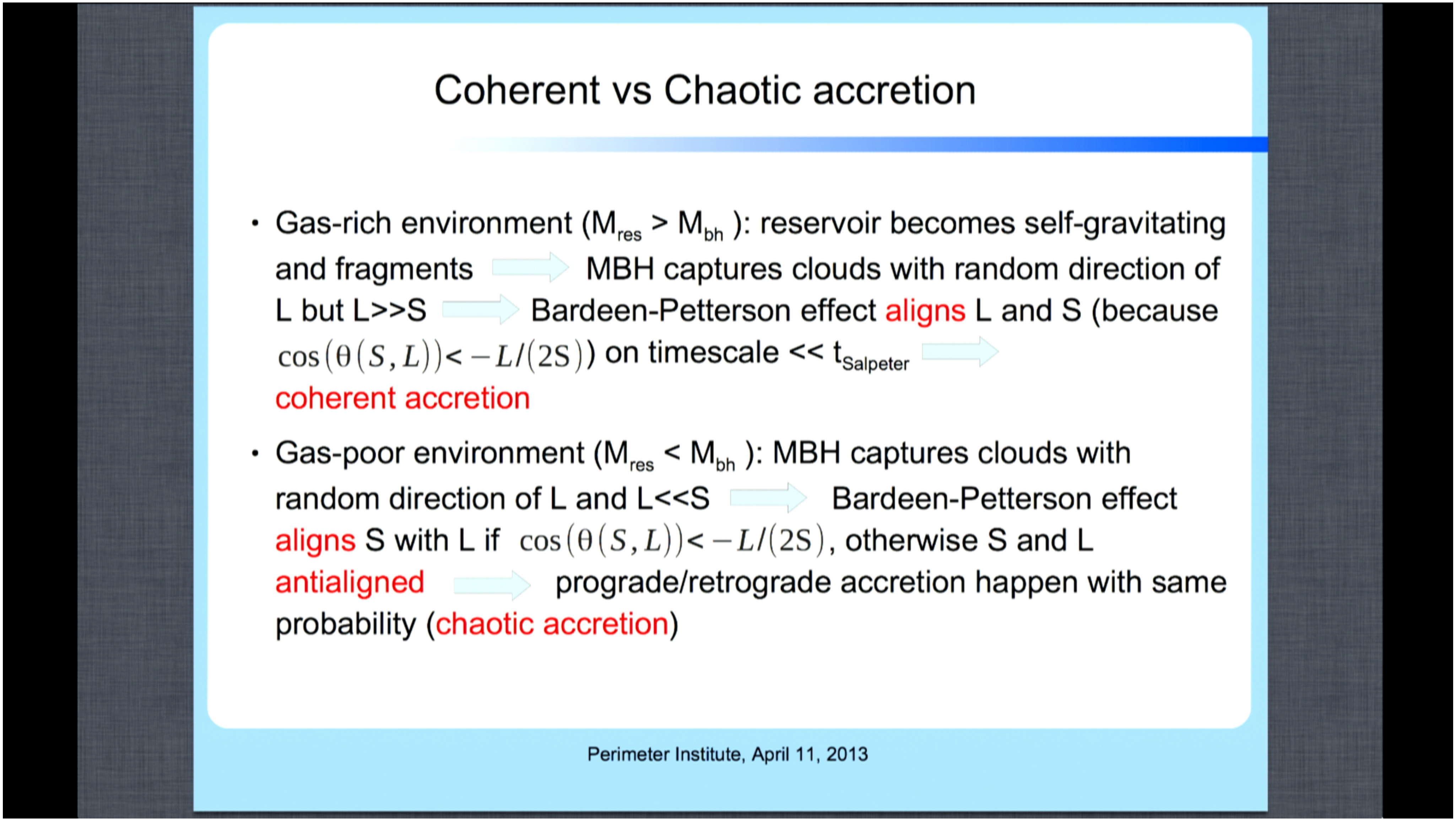
$$\dot{M}_{\rm QSO} = \frac{M_{\rm res}}{t_{\rm accr}}$$ t_{accr} is a free parameter

$$\dot{M}_{\rm bh,QSO} = \dot{M}_{\rm QSO}(1-\eta(a_{\rm bh}))$$

- If $M_{res} > M_{bh}$: coherent coherent (i.e. thin disk)
  If $M_{res} < M_{bh}$: chaotic accretion (i.e. accretion of clouds with random L)

- $\dot{M}$ can be super-Eddington, but luminosity cannot

$$L_{\rm bh,QSO} = \min\left\{\eta(a_{\rm bh})\dot{M}_{\rm QSO}c^2\ ,\ L_{\rm Edd}\left[1+\ln\left(\frac{\eta(a_{\rm bh})\dot{M}_{\rm QSO}c^2}{L_{\rm Edd}}\right)\right]\right\}$$



Coherent vs Chaotic accretion

- Gas-rich environment ($M_{res} > M_{bh}$): reservoir becomes self-gravitating and fragments → MBH captures clouds with random direction of L but L>>S → Bardeen-Petterson effect aligns L and S (because $\cos(\theta(S,L))<-L/(2S)$) on timescale << $t_{Salpeter}$ → coherent accretion
- Gas-poor environment ($M_{res} < M_{bh}$): MBH captures clouds with random direction of L and L<<S → Bardeen-Petterson effect aligns S with L if $\cos(\theta(S,L))<-L/(2S)$, otherwise S and L antialigned → prograde/retrograde accretion happen with same probability (chaotic accretion)

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The MBH spin evolution

- Coherent accretion = prograde (thin disk accretion)

  → spin up

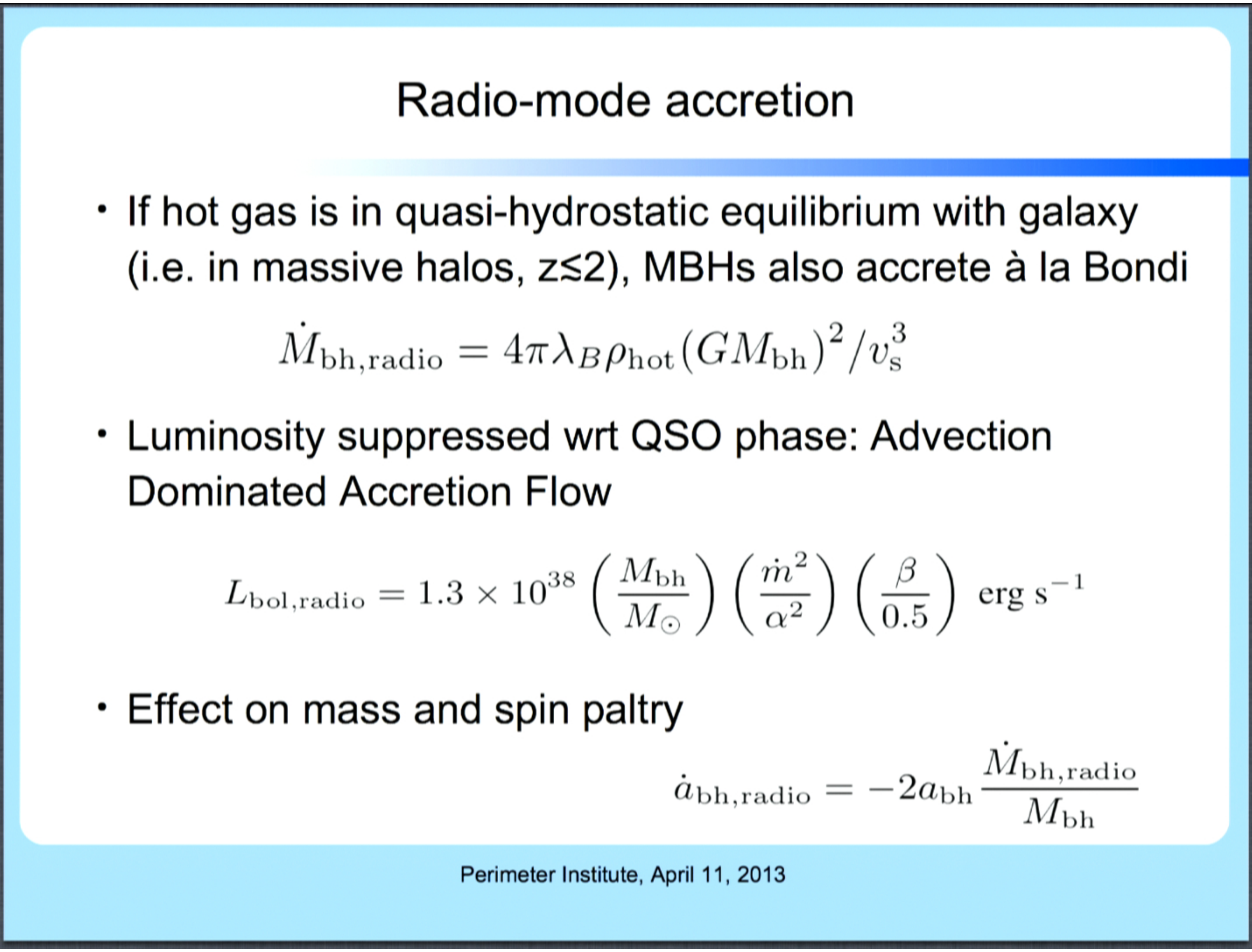
$$\dot{a}^{\rm coherent}_{\rm bh,QSO} = [L_{\rm ISCO}(a_{\rm bh}) - 2a_{\rm bh}E_{\rm ISCO}(a_{\rm bh})]\frac{\dot{M}_{\rm QSO}}{M_{\rm bh}}$$

$$\eta(a_{\rm bh}) = 1 - E_{\rm ISCO}(a_{\rm bh})$$

- Chaotic accretion: half of gas accretes on prograde orbits, half on retrograde orbits → spin down

$$\dot{a}^{\rm chaotic}_{\rm bh,QSO} = \left\{\frac{L_{\rm ISCO}(a_{\rm bh}) + L_{\rm ISCO}(-a_{\rm bh})}{2} - a_{\rm bh}[E_{\rm ISCO}(a_{\rm bh}) + E_{\rm ISCO}(-a_{\rm bh})]\right\}\frac{\dot{M}_{\rm QSO}}{M_{\rm bh}}$$

$$\eta(a_{\rm bh}) = 1 - \frac{E_{\rm ISCO}(a_{\rm bh}) + E_{\rm ISCO}(-a_{\rm bh})}{2}$$



Radio-mode accretion

- If hot gas is in quasi-hydrostatic equilibrium with galaxy (i.e. in massive halos, z≲2), MBHs also accrete à la Bondi

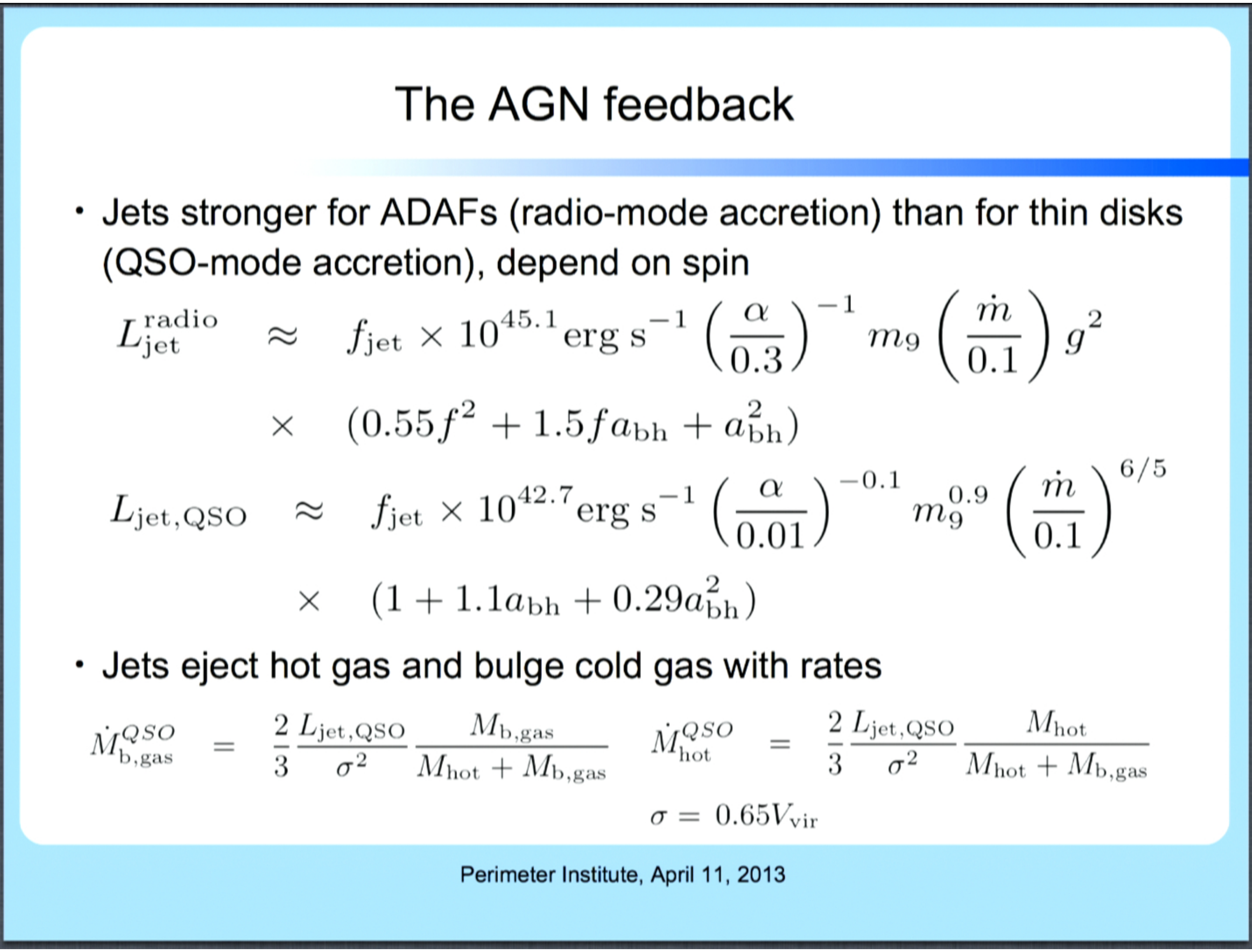
$$\dot{M}_{\rm bh,radio} = 4\pi\lambda_B \rho_{\rm hot} (G M_{\rm bh})^2 / v_{\rm s}^3$$

- Luminosity suppressed wrt QSO phase: Advection Dominated Accretion Flow

$$L_{\rm bol,radio} = 1.3 \times 10^{38} \left(\frac{M_{\rm bh}}{M_\odot}\right) \left(\frac{\dot{m}^2}{\alpha^2}\right) \left(\frac{\beta}{0.5}\right) \ {\rm erg\ s^{-1}}$$

- Effect on mass and spin paltry

$$\dot{a}_{\rm bh,radio} = -2 a_{\rm bh} \frac{\dot{M}_{\rm bh,radio}}{M_{\rm bh}}$$



The AGN feedback

- Jets stronger for ADAFs (radio-mode accretion) than for thin disks (QSO-mode accretion), depend on spin

$$L_{\rm jet}^{\rm radio} \approx f_{\rm jet} \times 10^{45.1} {\rm erg\ s}^{-1} \left(\frac{\alpha}{0.3}\right)^{-1} m_9 \left(\frac{\dot{m}}{0.1}\right) g^2$$
$$\times \quad (0.55 f^2 + 1.5 f a_{\rm bh} + a_{\rm bh}^2)$$
$$L_{\rm jet,QSO} \approx f_{\rm jet} \times 10^{42.7} {\rm erg\ s}^{-1} \left(\frac{\alpha}{0.01}\right)^{-0.1} m_9^{0.9} \left(\frac{\dot{m}}{0.1}\right)^{6/5}$$
$$\times \quad (1 + 1.1 a_{\rm bh} + 0.29 a_{\rm bh}^2)$$

- Jets eject hot gas and bulge cold gas with rates

$$\dot{M}_{\rm b,gas}^{QSO} = \frac{2}{3} \frac{L_{\rm jet,QSO}}{\sigma^2} \frac{M_{\rm b,gas}}{M_{\rm hot} + M_{\rm b,gas}} \qquad \dot{M}_{\rm hot}^{QSO} = \frac{2}{3} \frac{L_{\rm jet,QSO}}{\sigma^2} \frac{M_{\rm hot}}{M_{\rm hot} + M_{\rm b,gas}}$$
$$\sigma = 0.65 V_{\rm vir}$$

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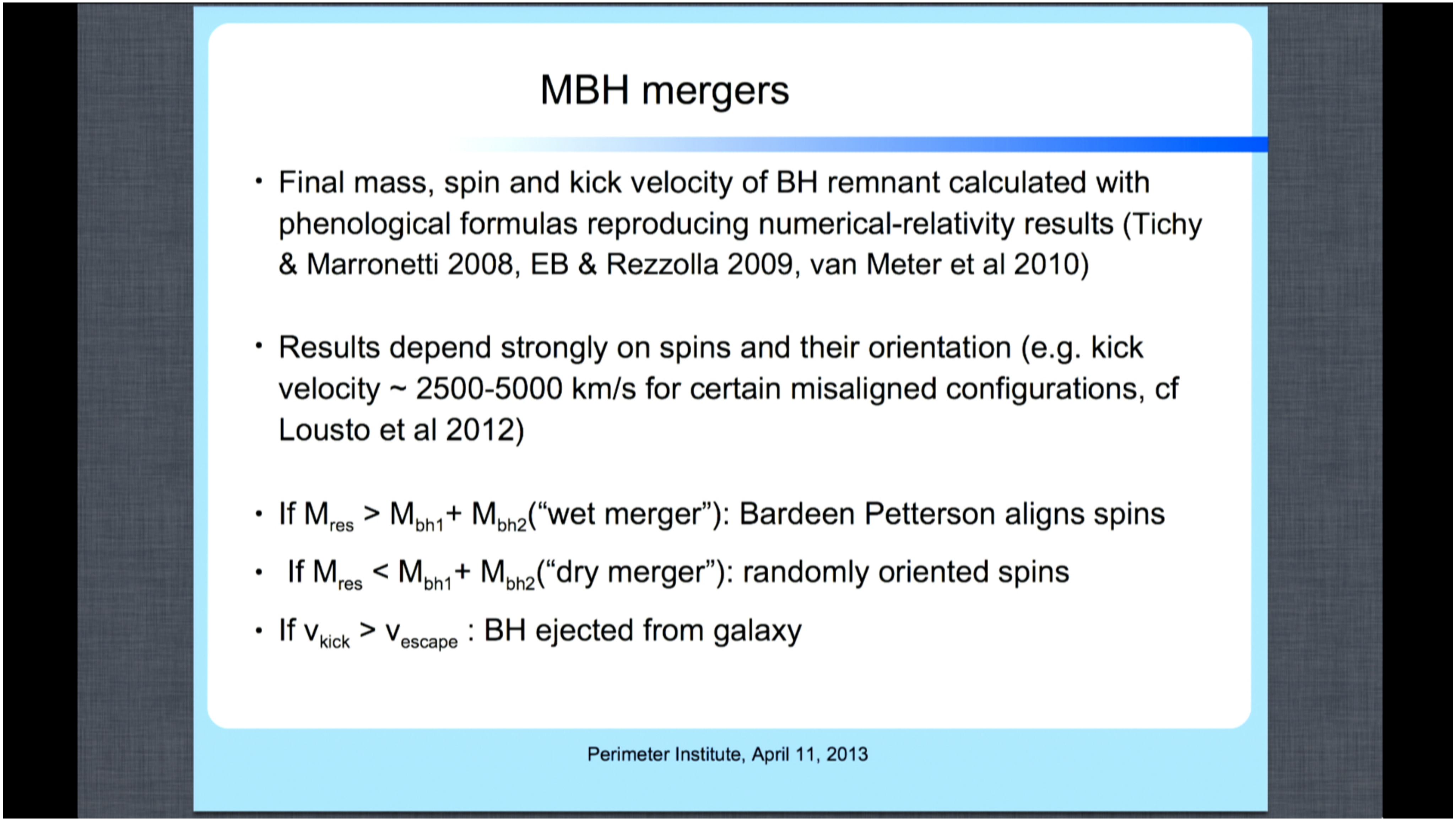

MBH mergers

- Final mass, spin and kick velocity of BH remnant calculated with phenological formulas reproducing numerical-relativity results (Tichy & Marronetti 2008, EB & Rezzolla 2009, van Meter et al 2010)

- Results depend strongly on spins and their orientation (e.g. kick velocity ~ 2500-5000 km/s for certain misaligned configurations, cf Lousto et al 2012)

- If $M_{res} > M_{bh1} + M_{bh2}$(“wet merger”): Bardeen Petterson aligns spins
- If $M_{res} < M_{bh1} + M_{bh2}$(“dry merger”): randomly oriented spins
- If $v_{kick} > v_{escape}$: BH ejected from galaxy

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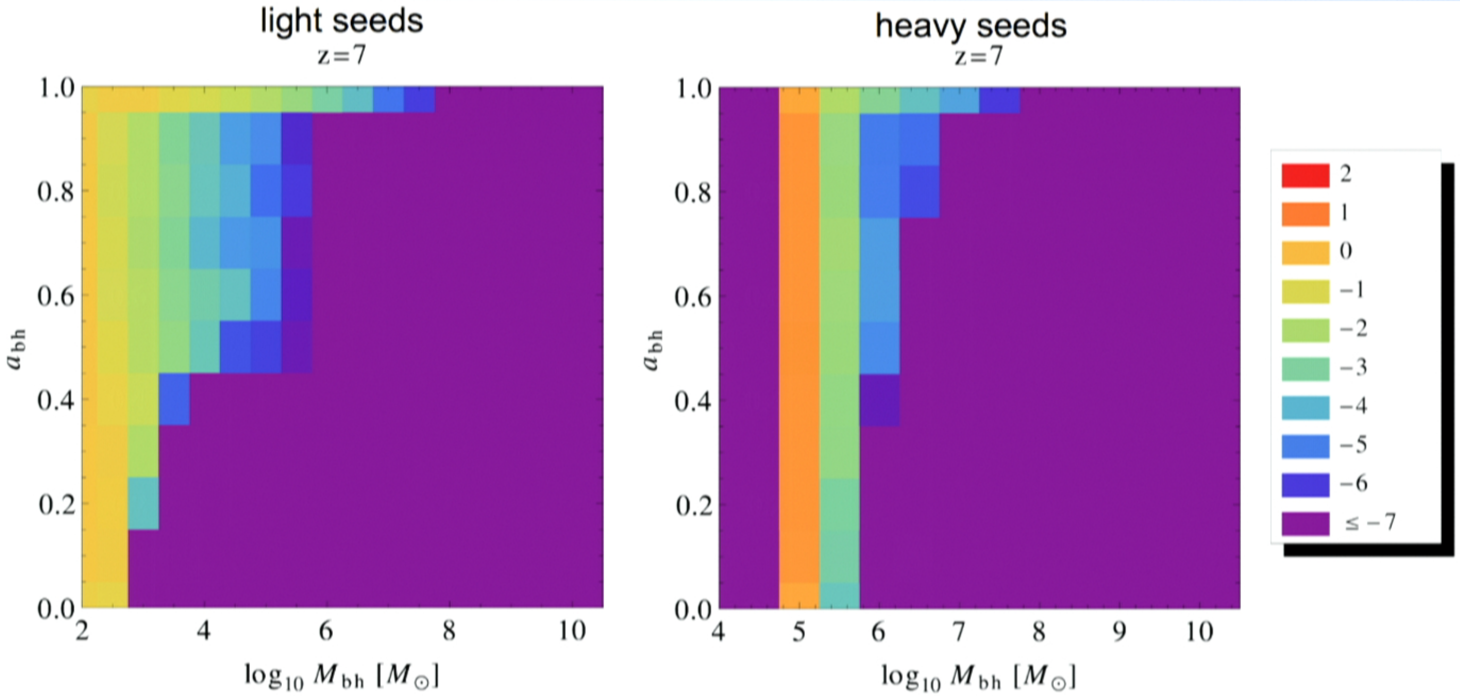
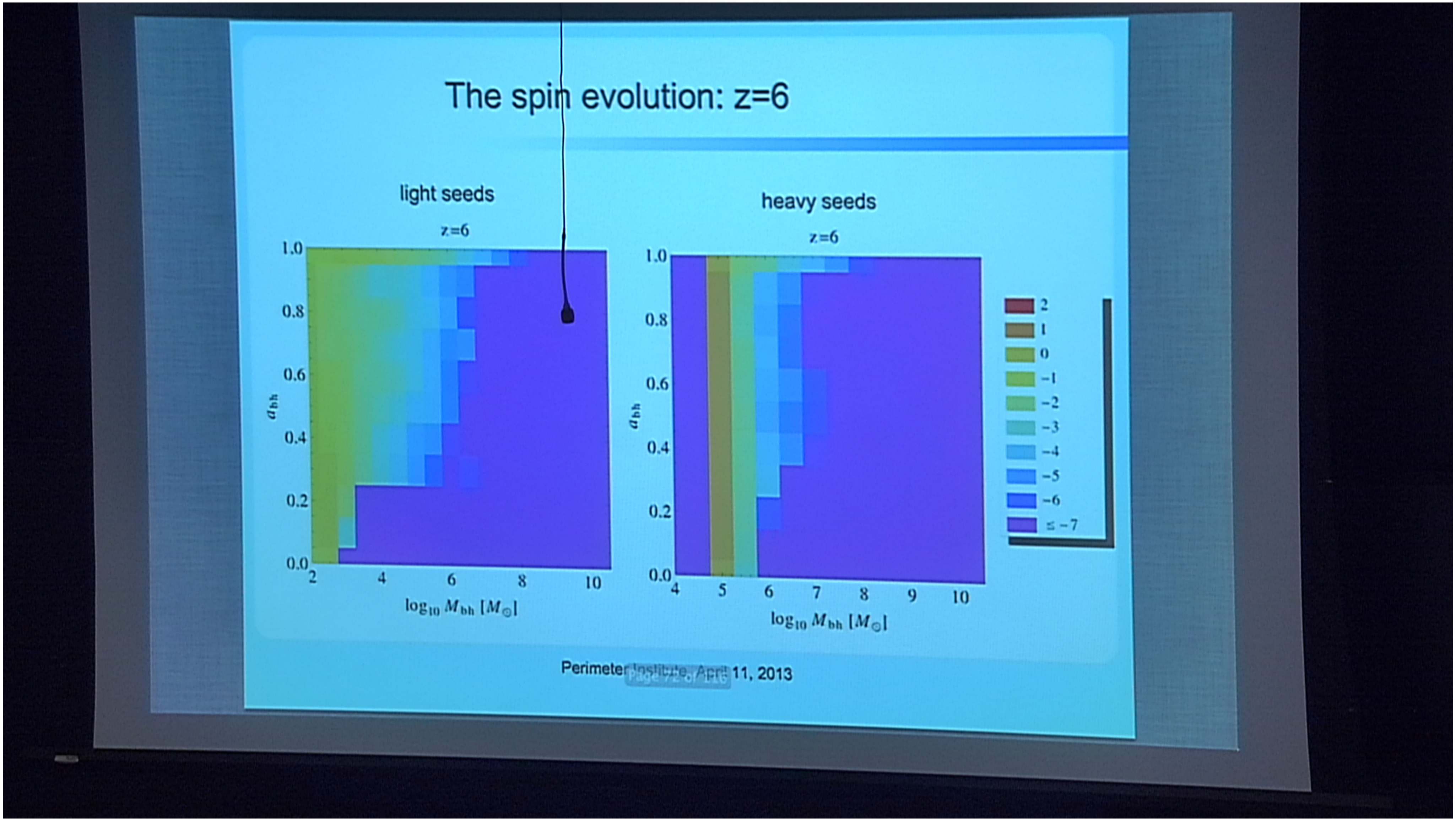
The spin evolution: z=7



Color code = $\log_{10}$ of number density of MBHs per unit log-mass and unit spin, i.e

$$\log_{10}(\mathrm{d}\phi_{\rm bh}[\mathrm{Mpc}^{-3}]/\mathrm{d}a) = \log_{10}(\mathrm{d}^2 n_{\rm bh}[\mathrm{Mpc}^{-3}]/(\mathrm{d}\log_{10} M_{\rm bh}[M_\odot]\,\mathrm{d}a)))$$



The spin evolution: z=6

light seeds

heavy seeds

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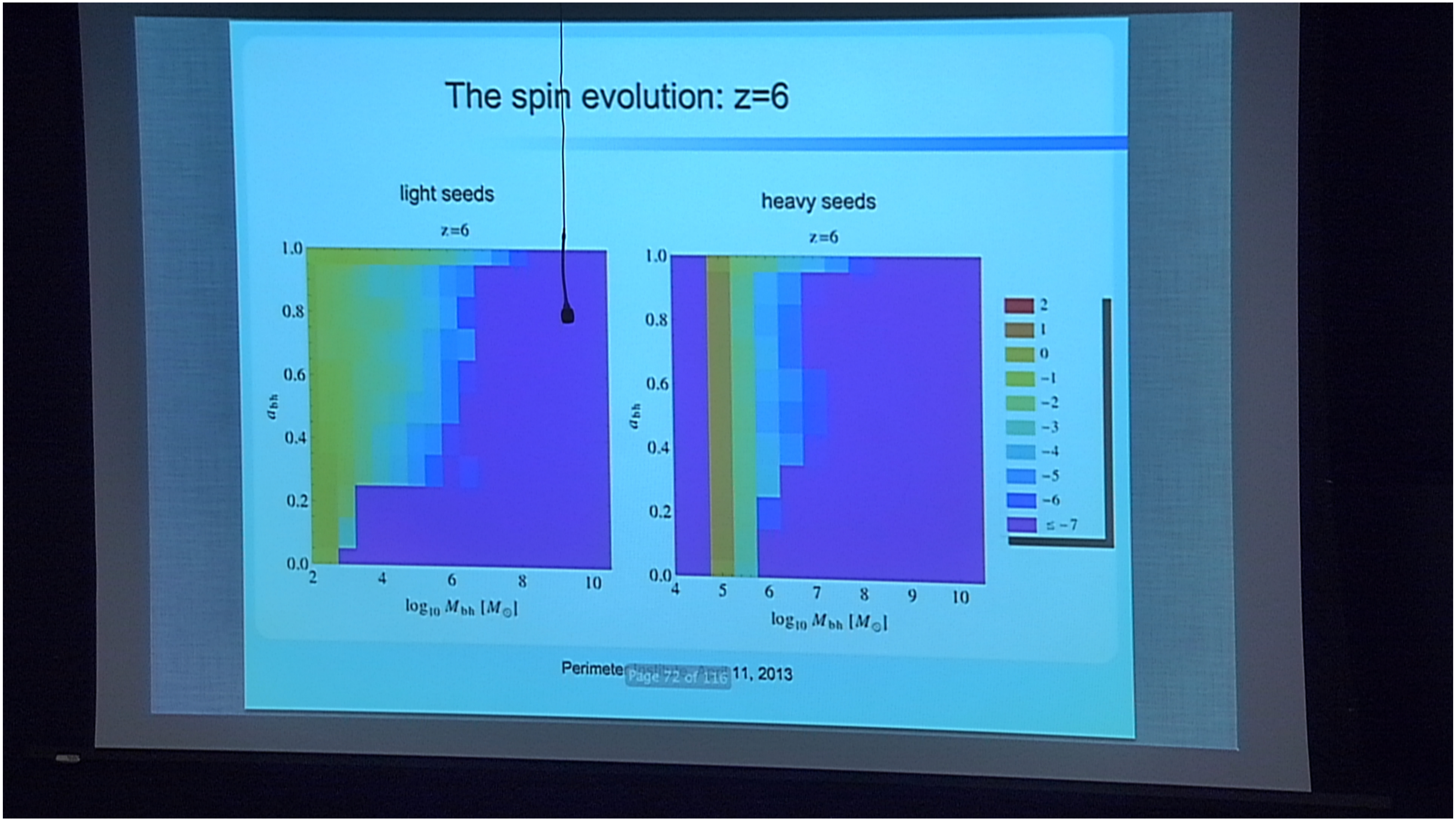

The spin evolution: z=6

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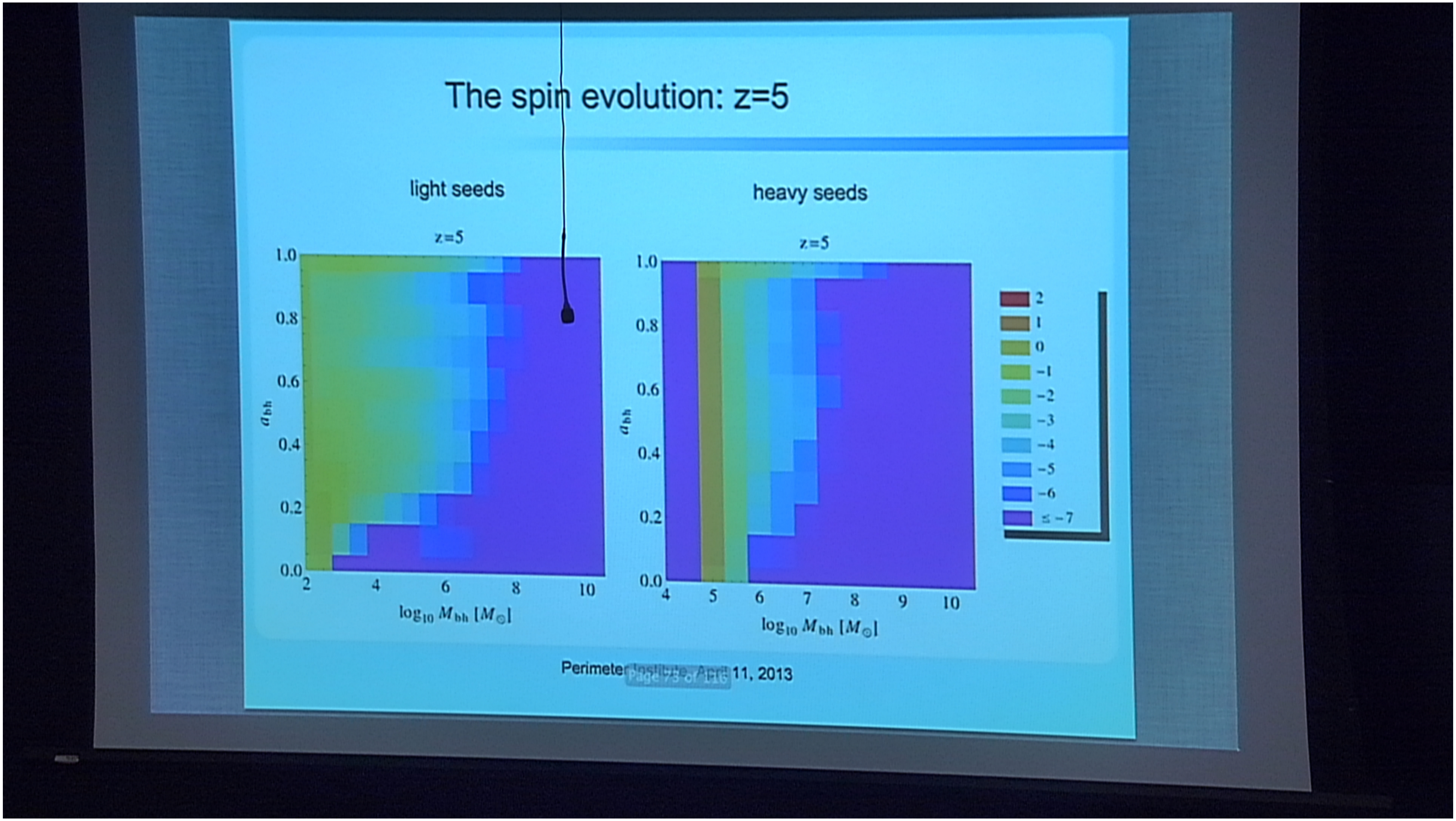

The spin evolution: z=5

light seeds

heavy seeds

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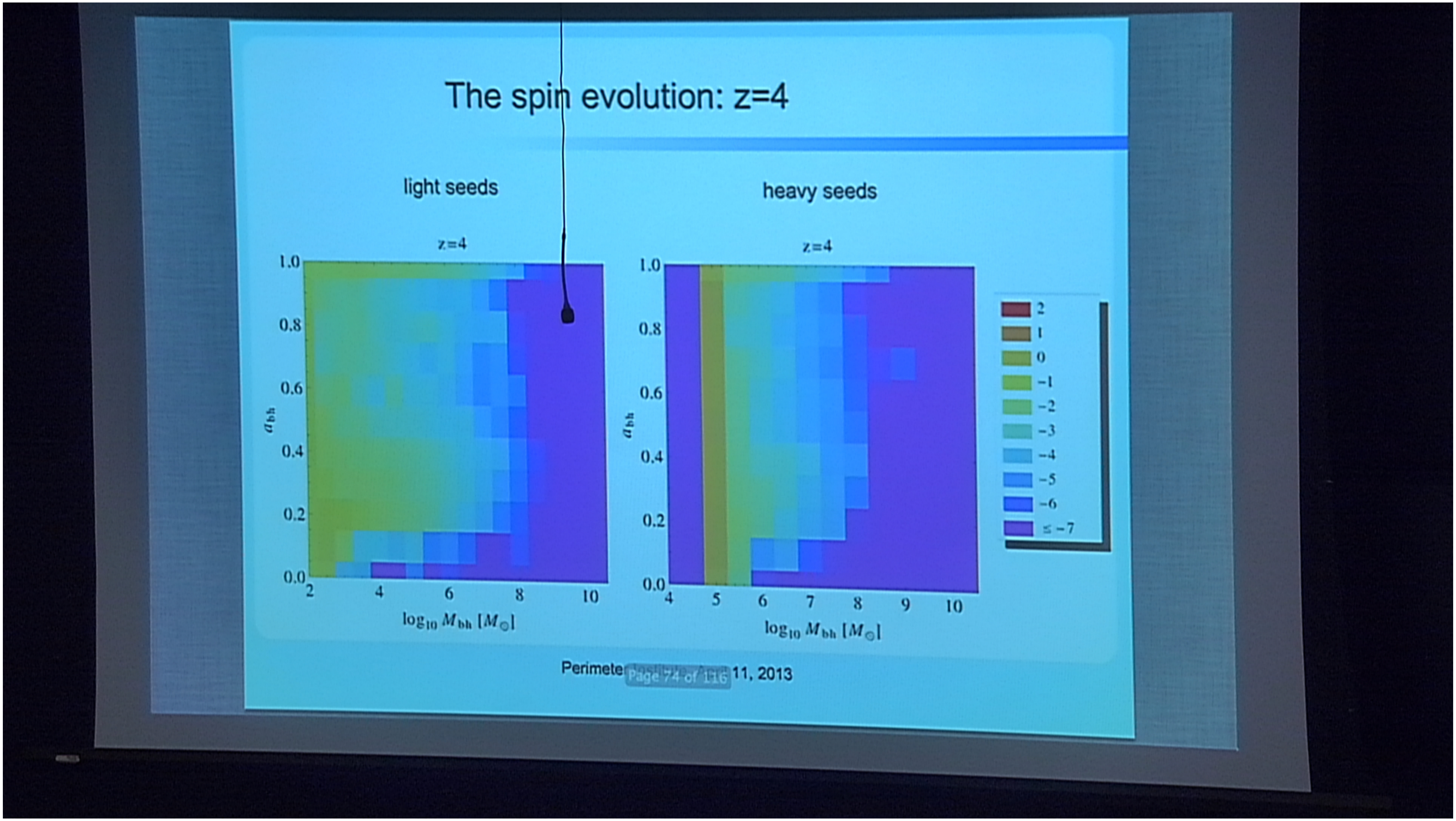

The spin evolution: z=4

light seeds

heavy seeds

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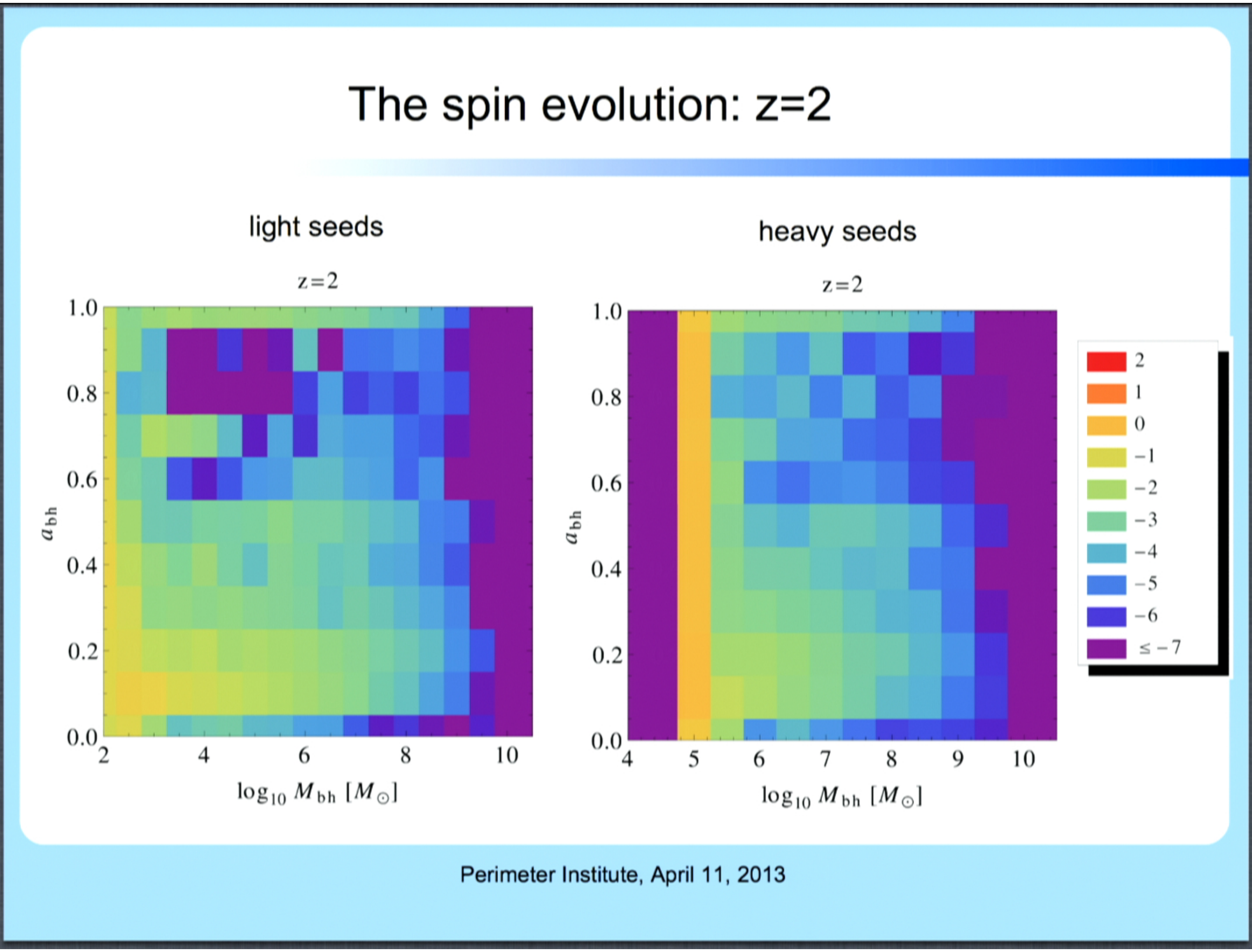

The spin evolution: z=2

light seeds

heavy seeds

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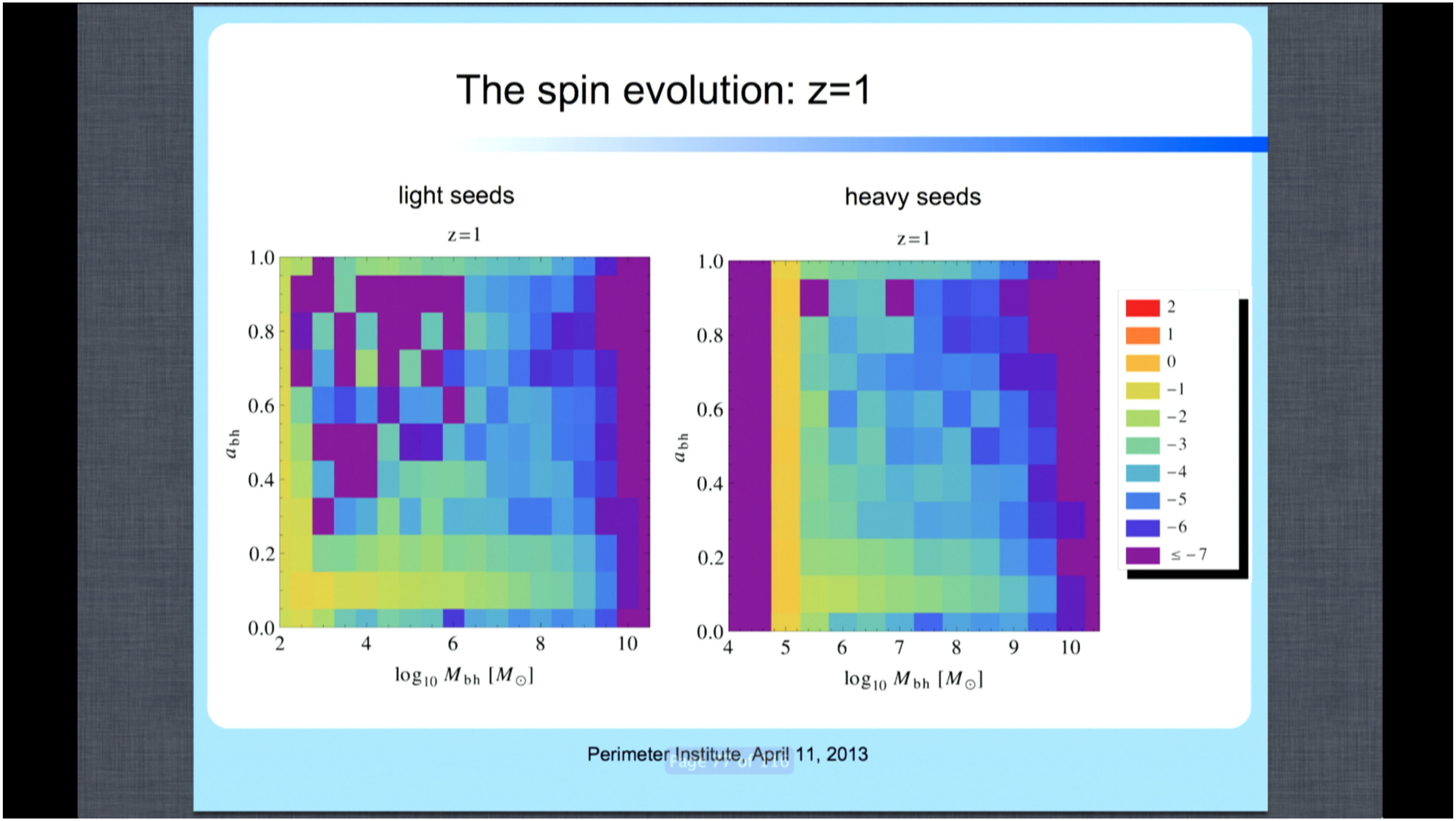

The spin evolution: z=1

light seeds

heavy seeds

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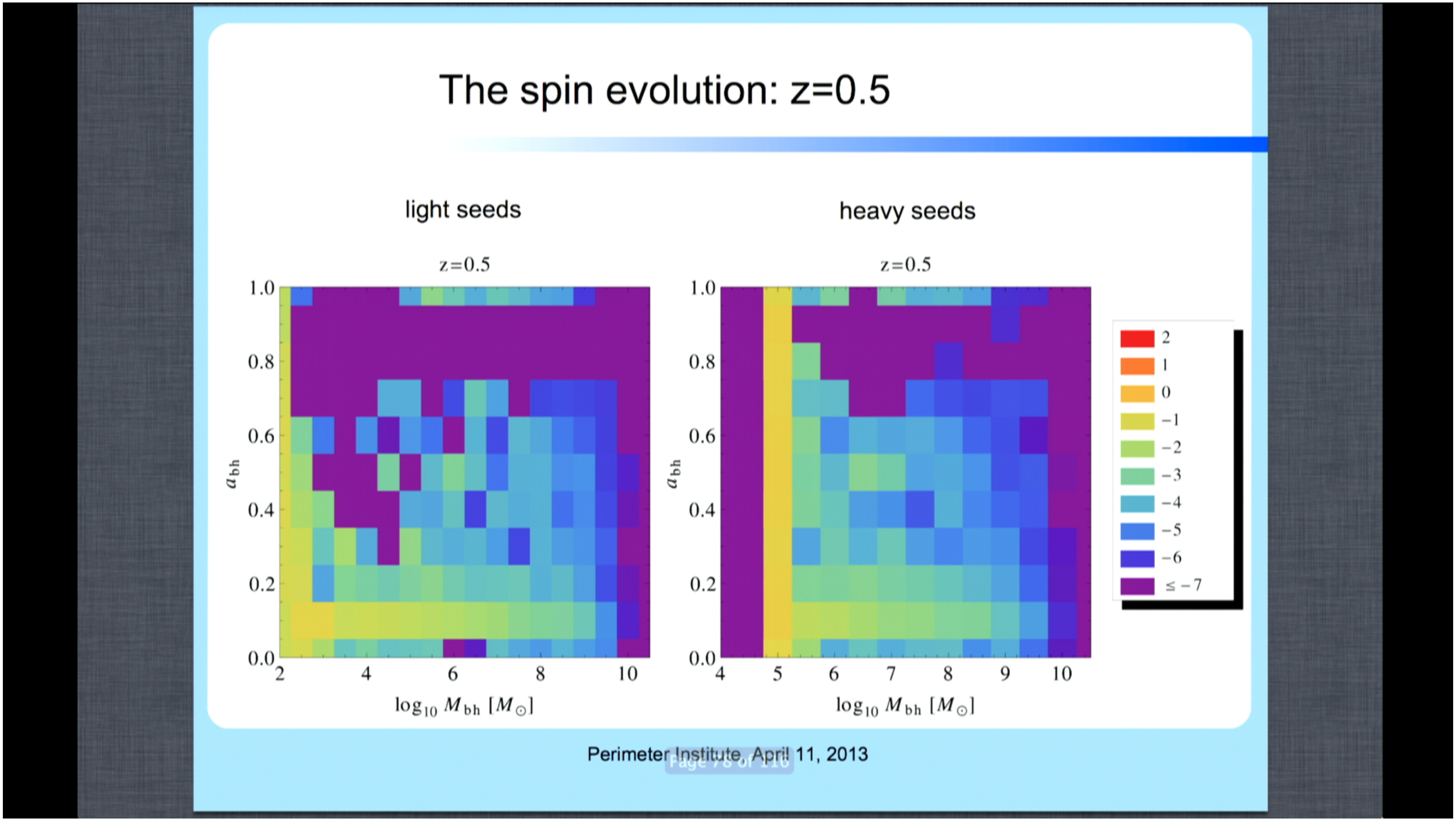
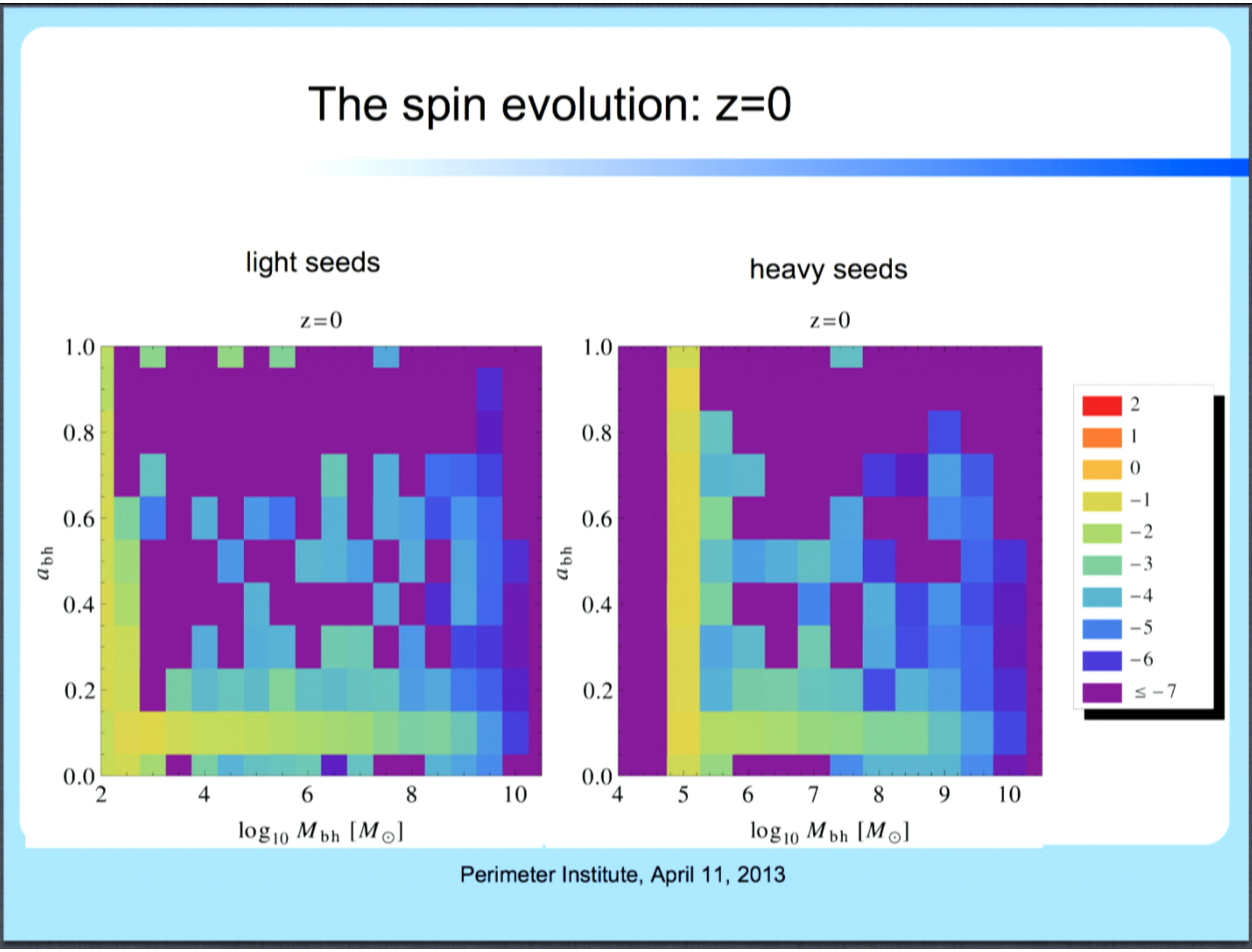


The spin evolution: z=0.5

light seeds

heavy seeds



The spin evolution: z=0

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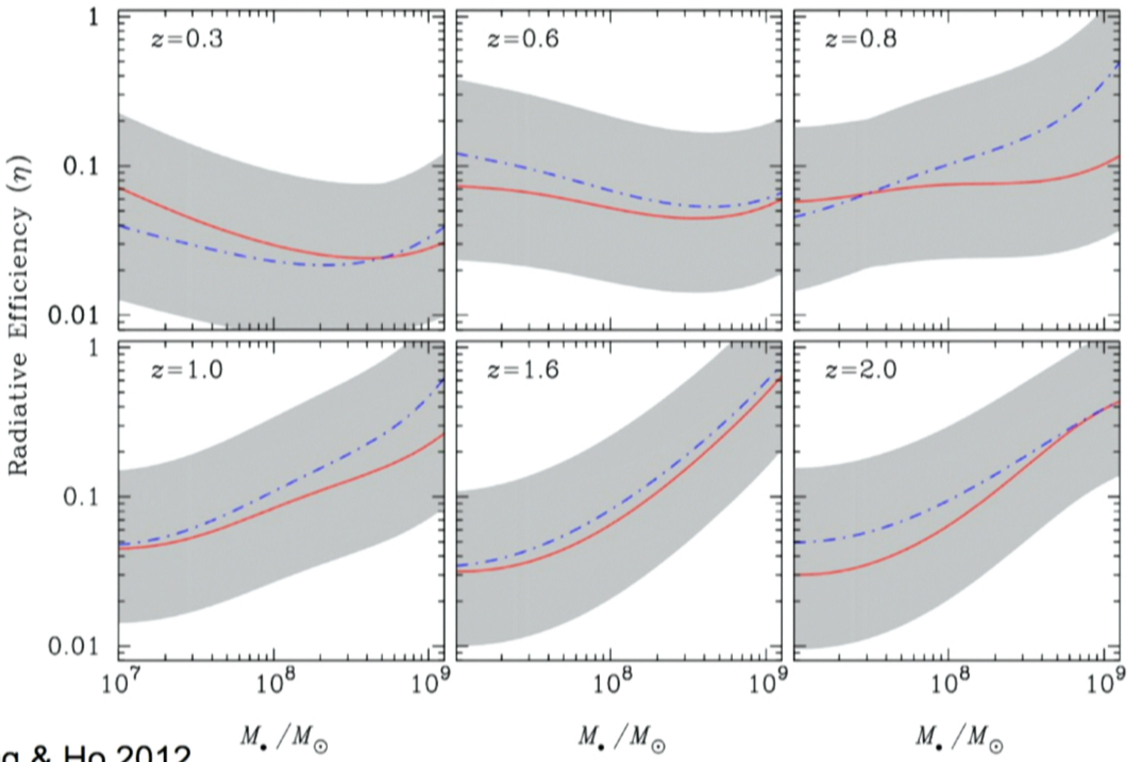
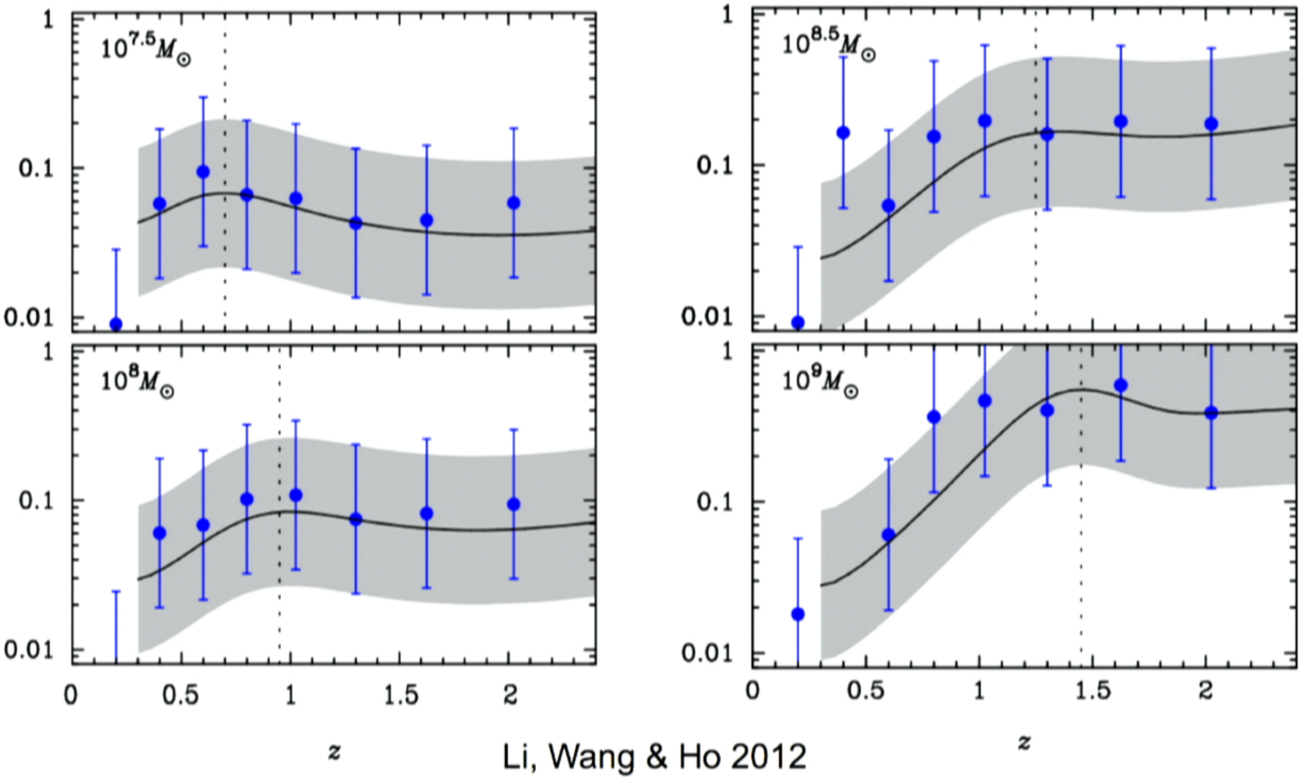
Consistent with data?



Li, Wang & Ho 2012

Consistent with data?



Li, Wang & Ho 2012

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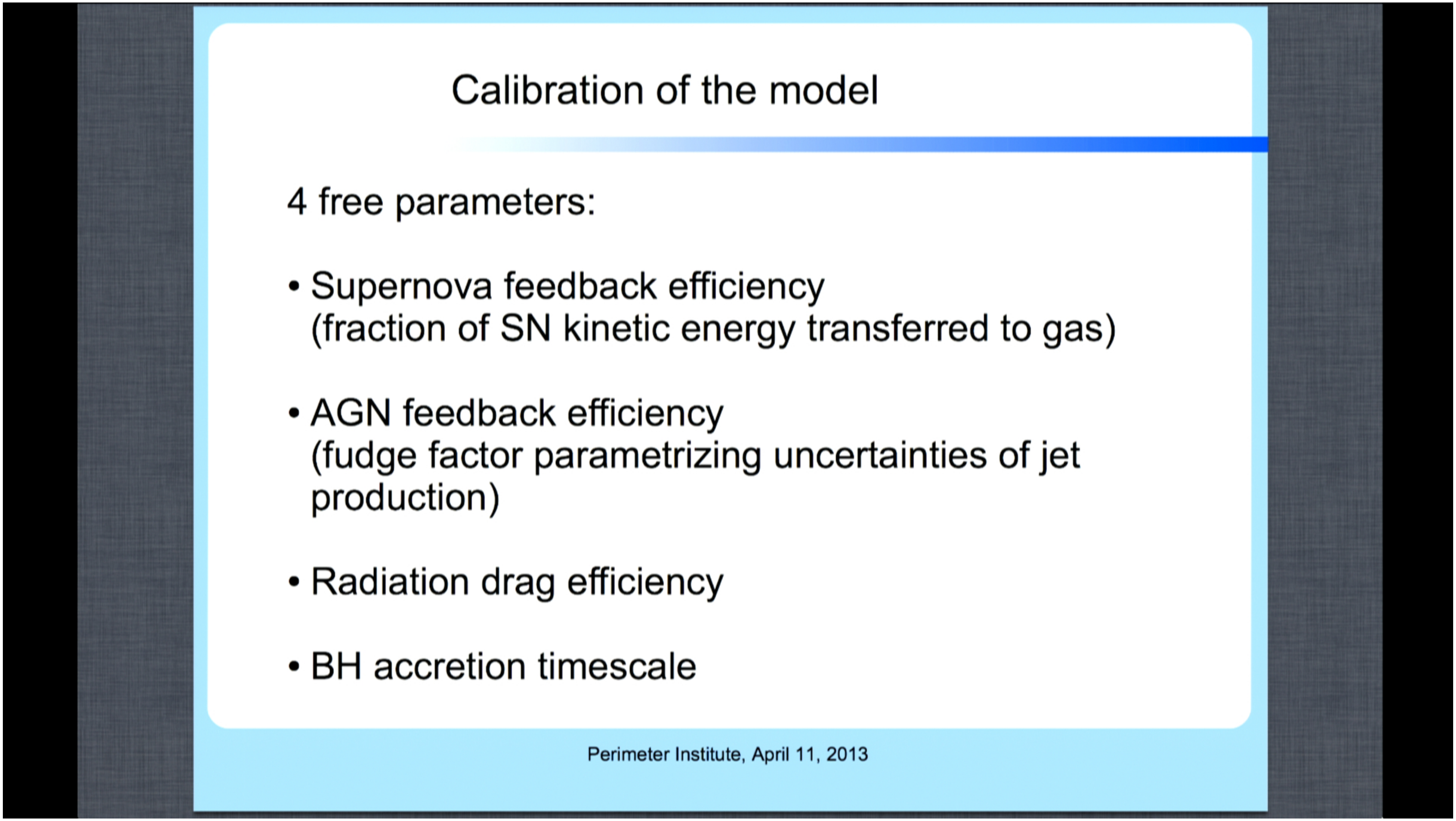
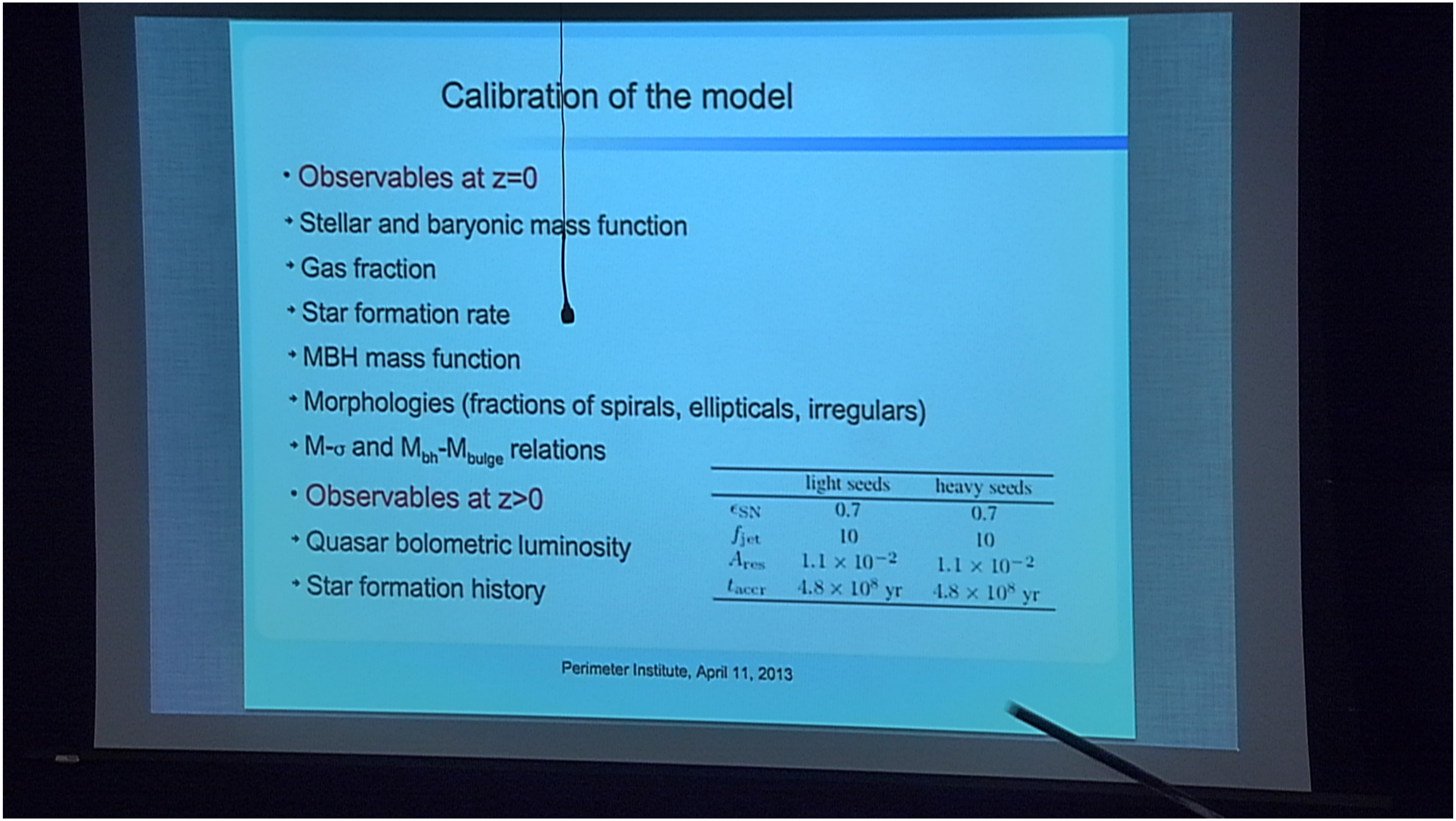


Calibration of the model

4 free parameters:

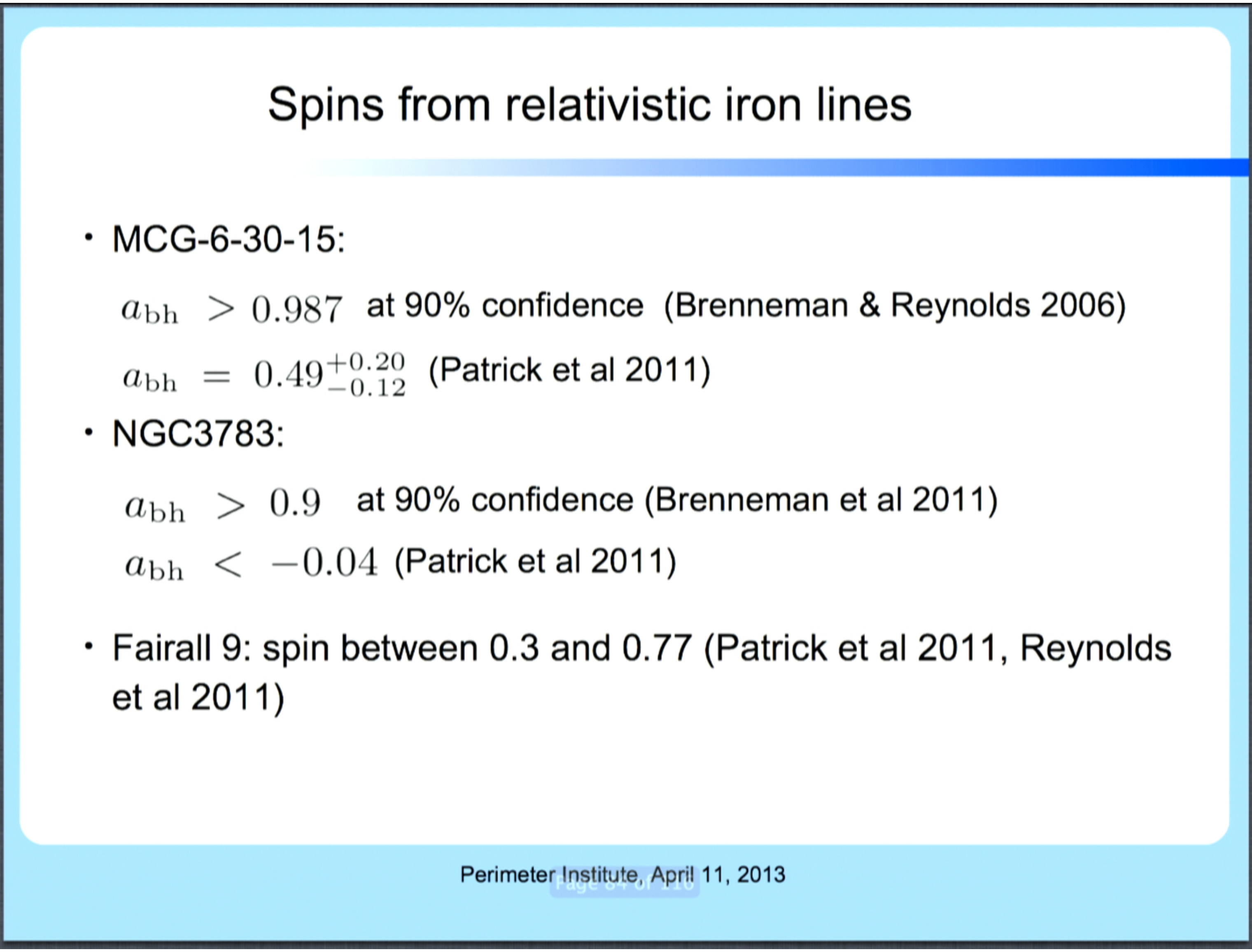
- Supernova feedback efficiency
  (fraction of SN kinetic energy transferred to gas)
- AGN feedback efficiency
  (fudge factor parametrizing uncertainties of jet production)
- Radiation drag efficiency
- BH accretion timescale



Calibration of the model

- Observables at z=0
  - Stellar and baryonic mass function
  - Gas fraction
  - Star formation rate
  - MBH mass function
  - Morphologies (fractions of spirals, ellipticals, irregulars)
  - M-σ and M_{bh}-M_{bulge} relations
- Observables at z>0
  - Quasar bolometric luminosity
  - Star formation history

	light seeds	heavy seeds
ϵ_{SN}	0.7	0.7
f_{jet}	10	10
A_{res}	1.1×10^{-2}	1.1×10^{-2}
t_{accr}	4.8×10^{8} yr	4.8×10^{8} yr



Spins from relativistic iron lines

- MCG-6-30-15:

  $a_{\rm bh} > 0.987$ at 90% confidence (Brenneman & Reynolds 2006)

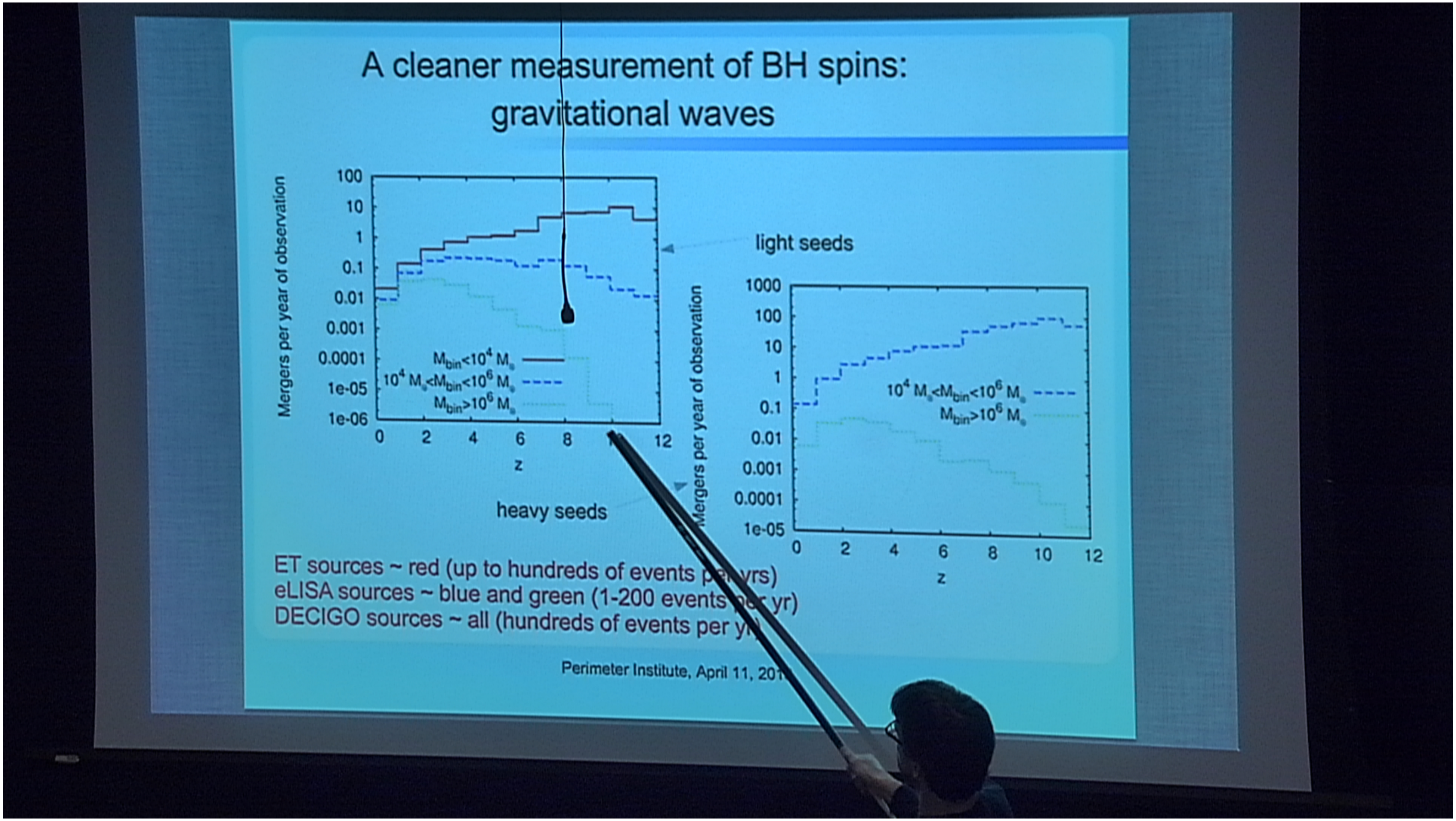
  $a_{\rm bh} = 0.49^{+0.20}_{-0.12}$ (Patrick et al 2011)

- NGC3783:

  $a_{\rm bh} > 0.9$ at 90% confidence (Brenneman et al 2011)

  $a_{\rm bh} < -0.04$ (Patrick et al 2011)

- Fairall 9: spin between 0.3 and 0.77 (Patrick et al 2011, Reynolds et al 2011)



A cleaner measurement of BH spins: gravitational waves

ET sources ~ red (up to hundreds of events per yrs)
eLISA sources ~ blue and green (1-200 events per yr)
DECIGO sources ~ all (hundreds of events per yr)

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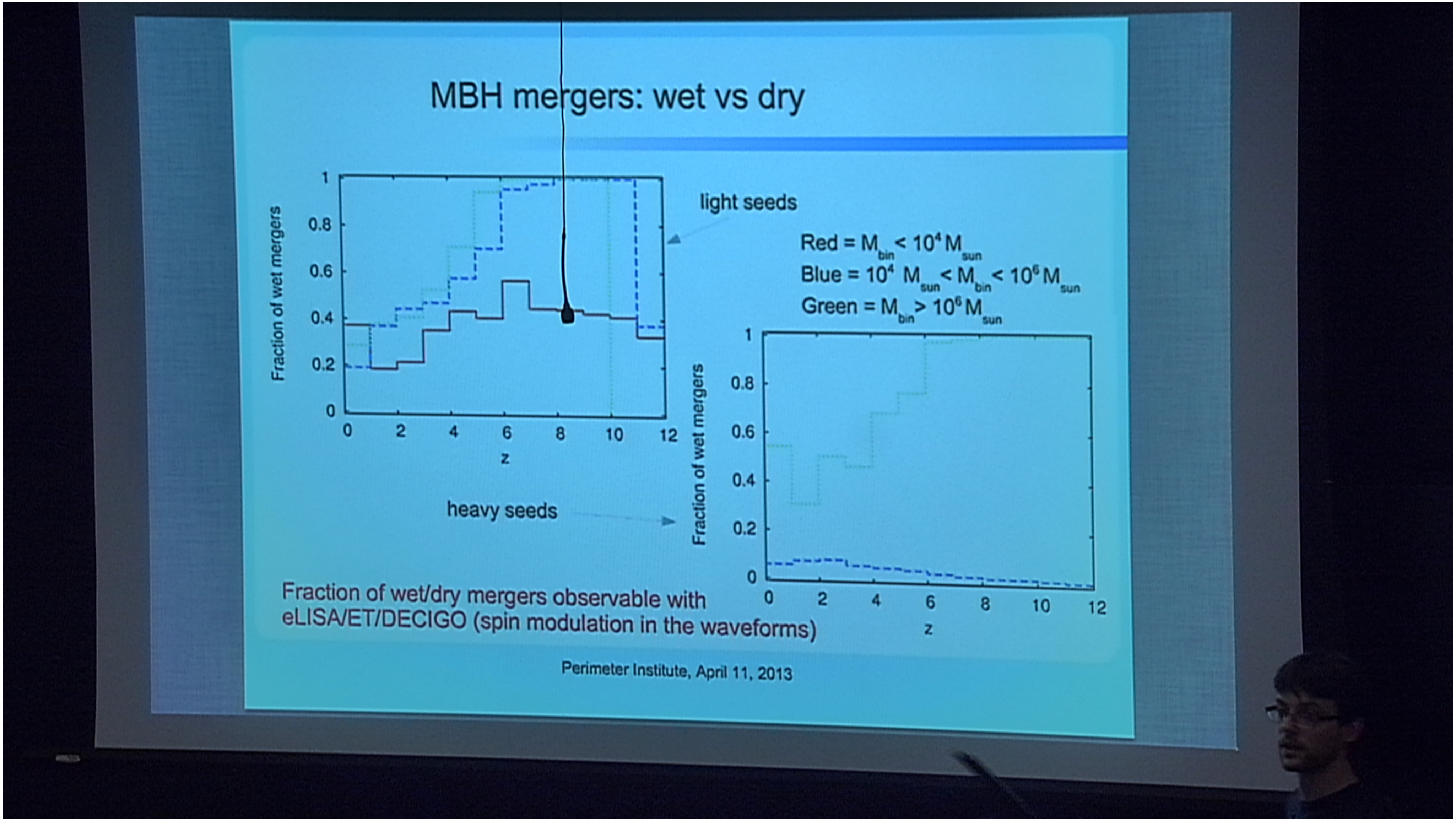

MBH mergers: wet vs dry

Red = $M_{bin} < 10^4 M_{sun}$

Blue = $10^4 M_{sun} < M_{bin} < 10^6 M_{sun}$

Green = $M_{bin} > 10^6 M_{sun}$

Fraction of wet/dry mergers observable with eLISA/ET/DECIGO (spin modulation in the waveforms)

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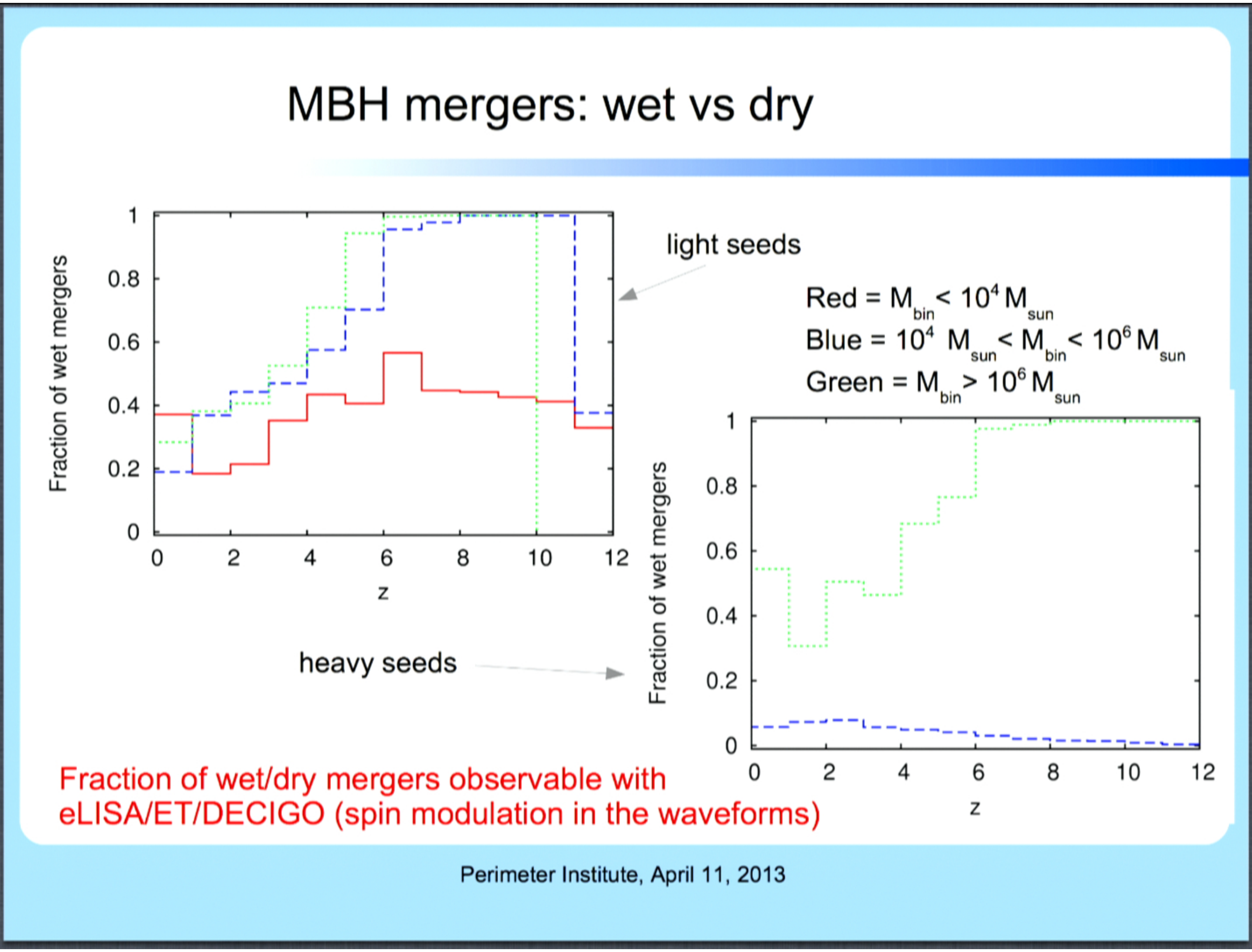

MBH mergers: wet vs dry

light seeds

Red = $M_{bin} < 10^4 M_{sun}$

Blue = $10^4 M_{sun} < M_{bin} < 10^6 M_{sun}$

Green = $M_{bin} > 10^6 M_{sun}$

heavy seeds

Fraction of wet/dry mergers observable with eLISA/ET/DECIGO (spin modulation in the waveforms)

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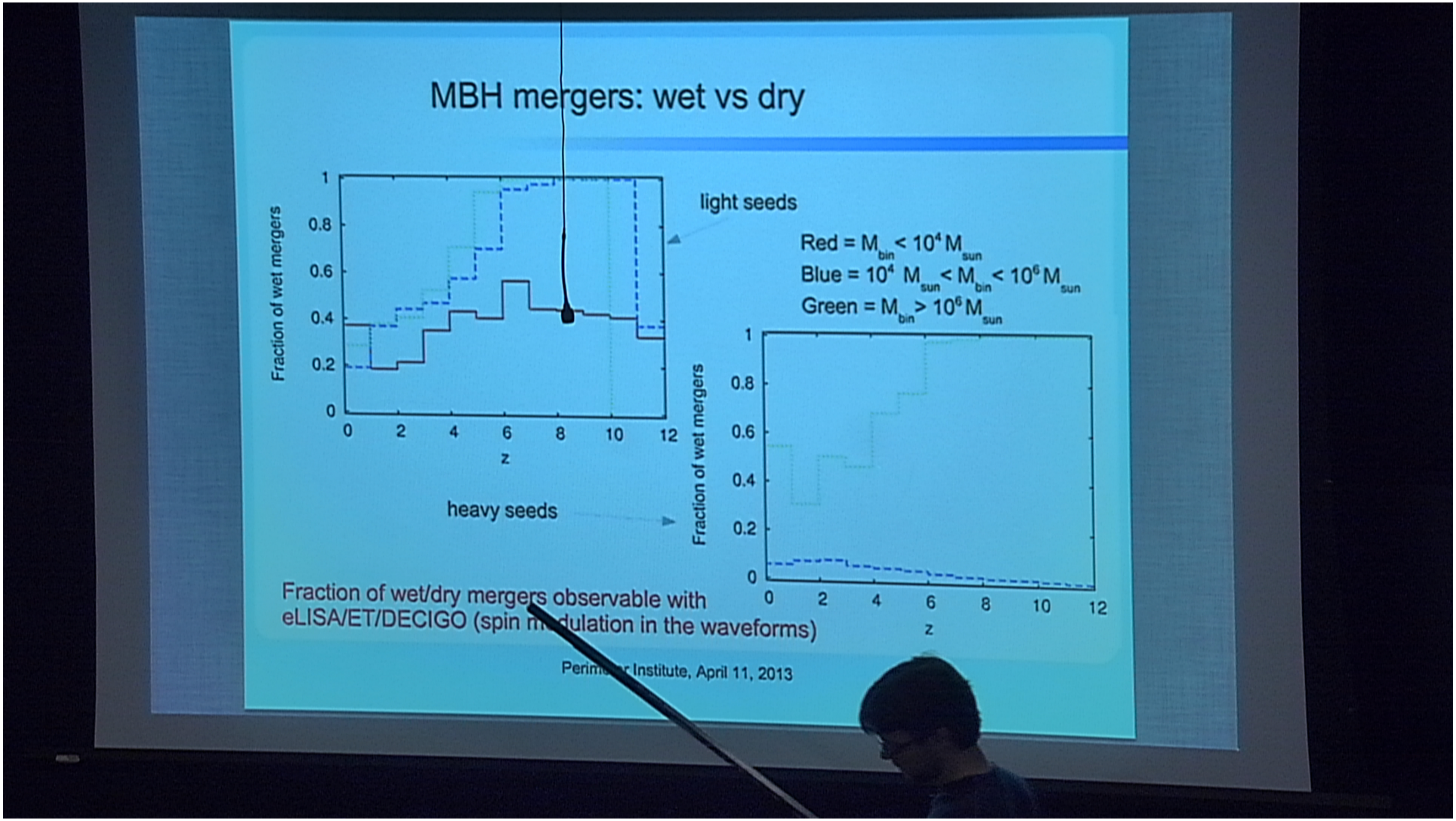

MBH mergers: wet vs dry

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Fraction of wet/dry mergers observable with eLISA/ET/DECIGO (spin modulation in the waveforms)

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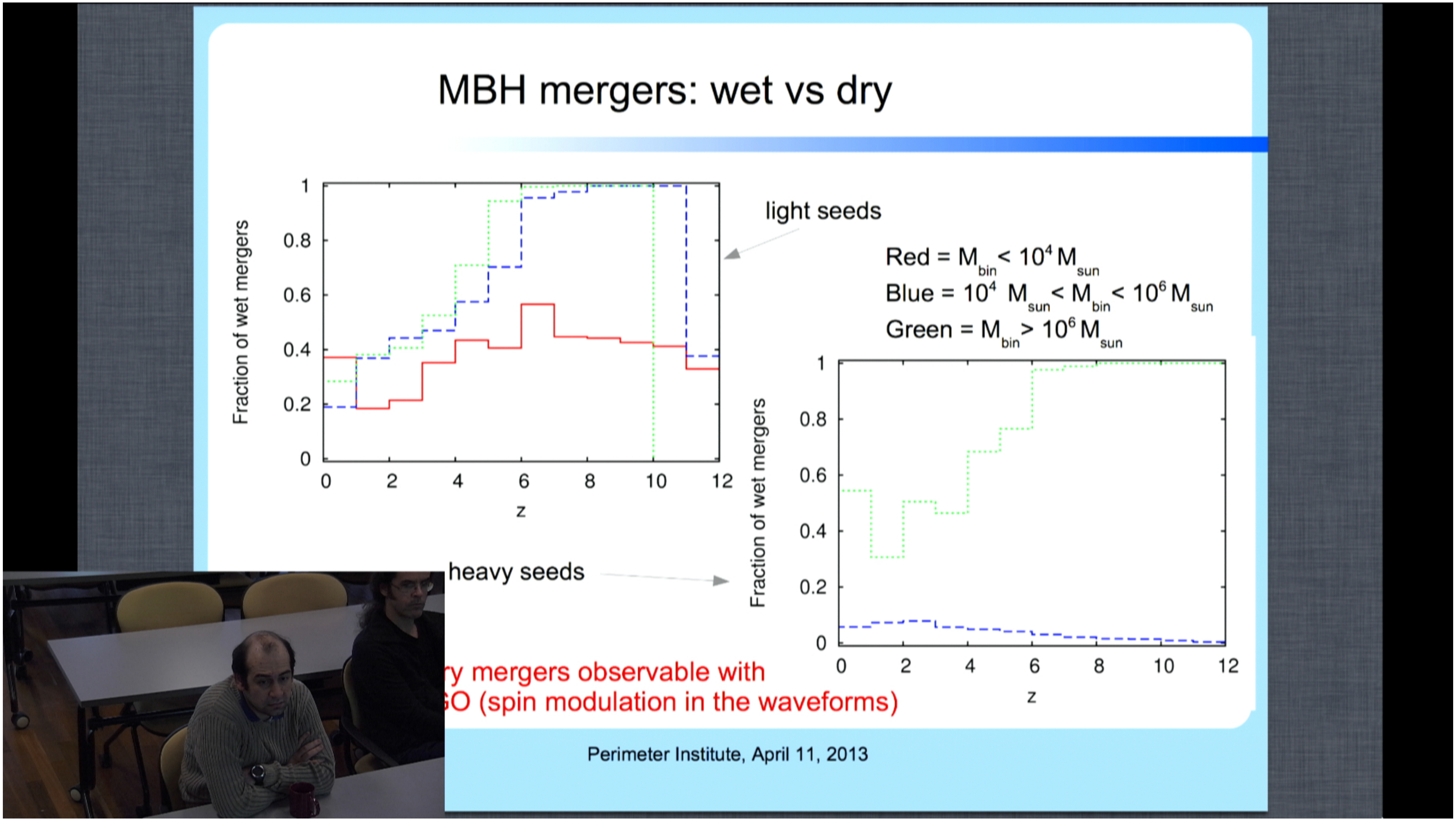

MBH mergers: wet vs dry

light seeds

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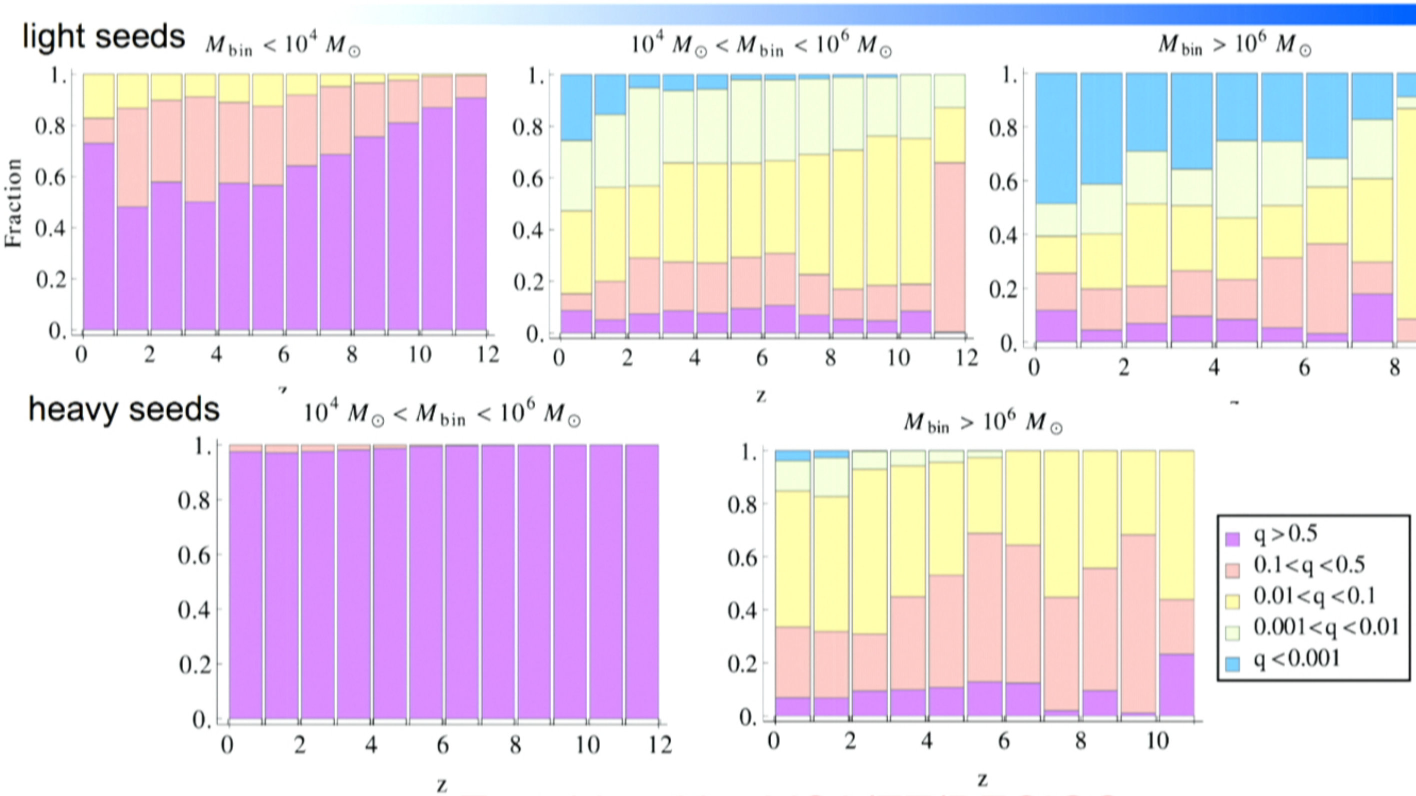
Blue = $10^4 M_{sun} < M_{bin} < 10^6 M_{sun}$

Green = $M_{bin} > 10^6 M_{sun}$

heavy seeds

ry mergers observable with
O (spin modulation in the waveforms)

MBH mergers: the mass ratios



Testable with eLISA/ET/DECIGO

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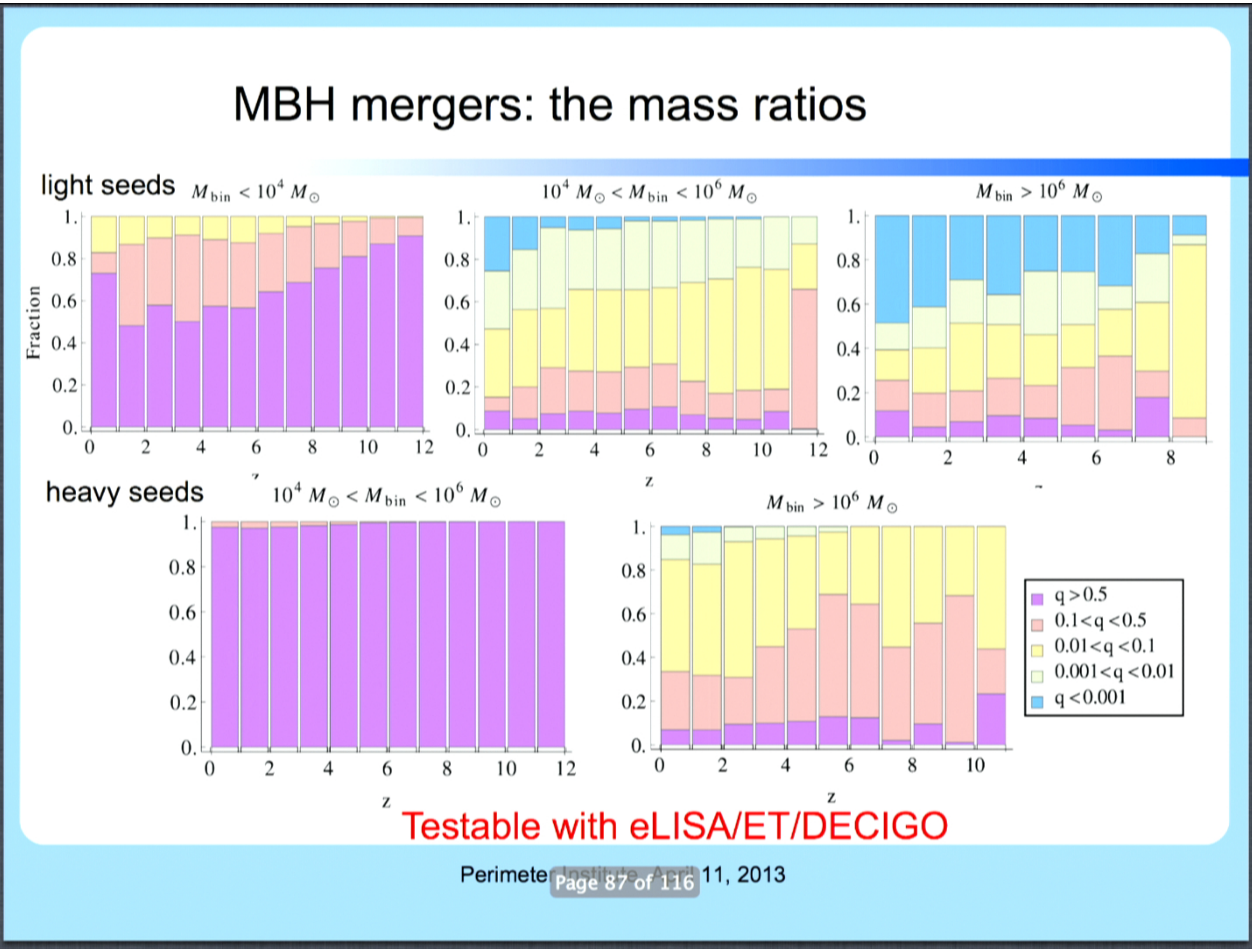

MBH mergers: the mass ratios

light seeds

heavy seeds

Testable with eLISA/ET/DECIGO

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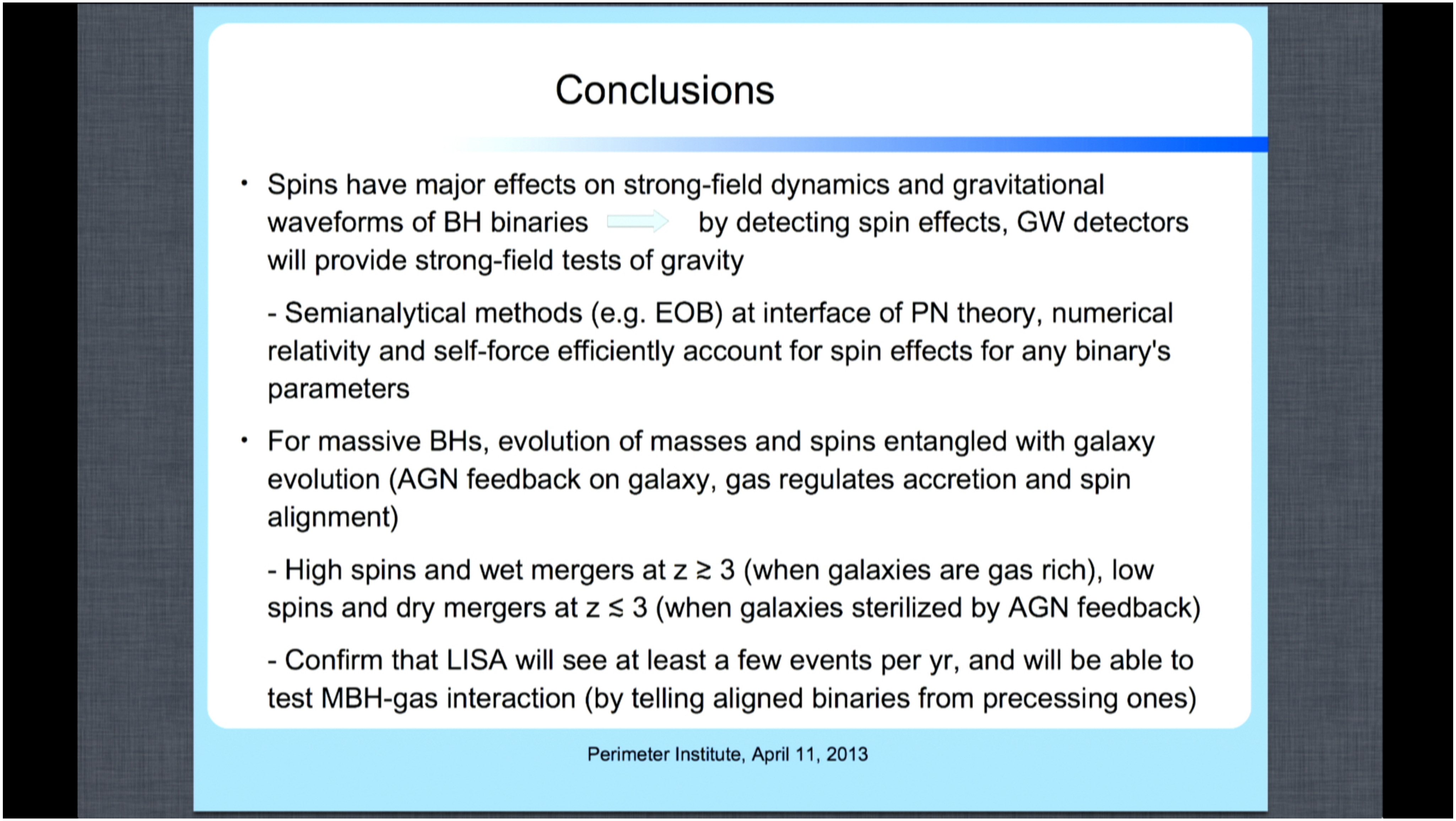

Conclusions

- Spins have major effects on strong-field dynamics and gravitational waveforms of BH binaries → by detecting spin effects, GW detectors will provide strong-field tests of gravity

  - Semianalytical methods (e.g. EOB) at interface of PN theory, numerical relativity and self-force efficiently account for spin effects for any binary's parameters

- For massive BHs, evolution of masses and spins entangled with galaxy evolution (AGN feedback on galaxy, gas regulates accretion and spin alignment)

  - High spins and wet mergers at $z \gtrsim 3$ (when galaxies are gas rich), low spins and dry mergers at $z \lesssim 3$ (when galaxies sterilized by AGN feedback)

  - Confirm that LISA will see at least a few events per yr, and will be able to test MBH-gas interaction (by telling aligned binaries from precessing ones)

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