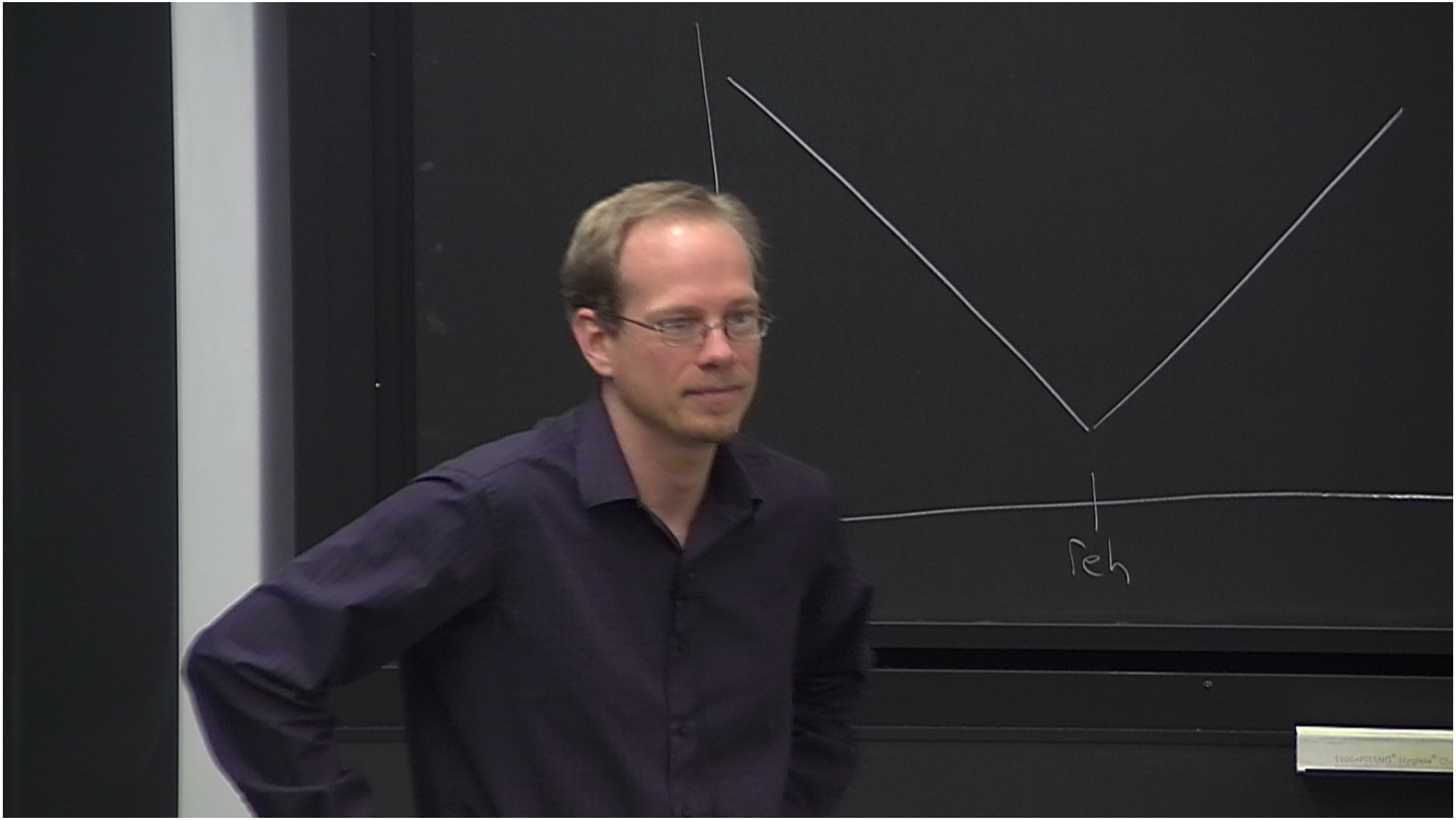
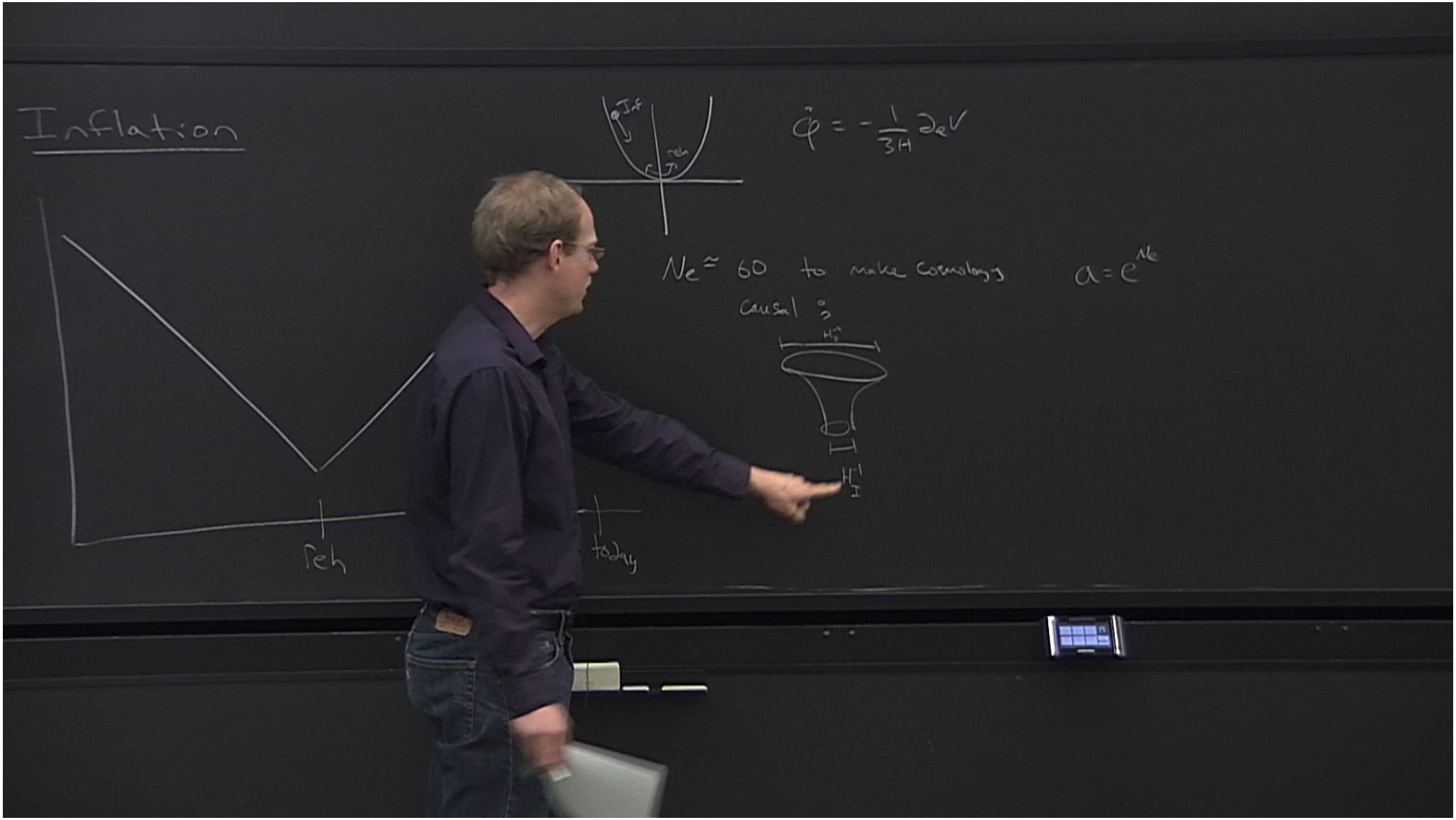
Title: 12/13 PSI - Explorations in Cosmology Lecture 15

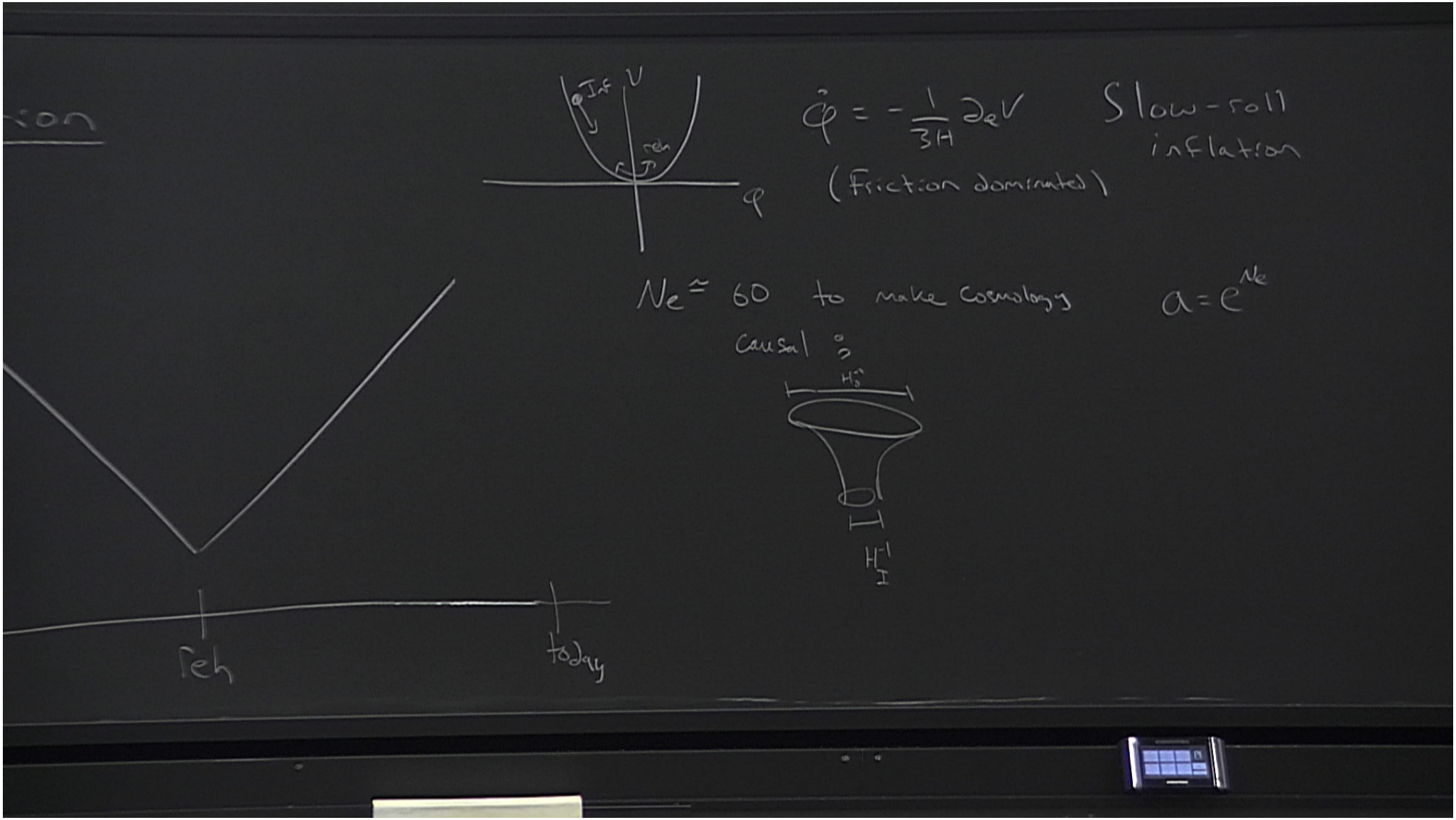
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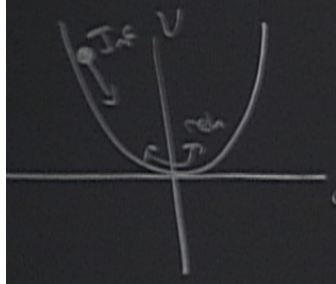
URL: http://pirsa.org/13040080

Abstract:


feh


Inflation
Inf
reh
$\dot{\varphi} = -\frac{1}{3H}\partial_\varphi V$
$N_e \simeq 60$ to make Cosmology
causal
$a = e^{N_e}$
H_0^{-1}
H_I^{-1}
reh
today


V
φ
φ_Inf
φ_reh
$\dot{\varphi} = -\frac{1}{3H}\partial_\varphi V$
Slow-roll inflation
(friction dominated)
$N_e \simeq 60$ to make cosmology causal
$a = e^{N_e}$
H_0^{-1}
H_I^{-1}
reh
today



$\dot{\varphi} = -\frac{1}{3H}\partial_\varphi V$ (friction dominated)

Slow-roll inflation → Inflator decays gives rise SBB cosmology

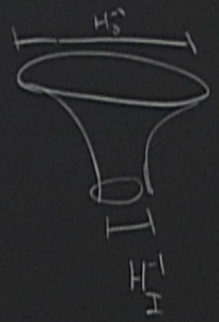
$N_e \simeq 60$ to make cosmology

$a = e^{N_e}$ → Source for primordial perturbations in

$\overline{\Phi}_0, \Theta_0, S_0$

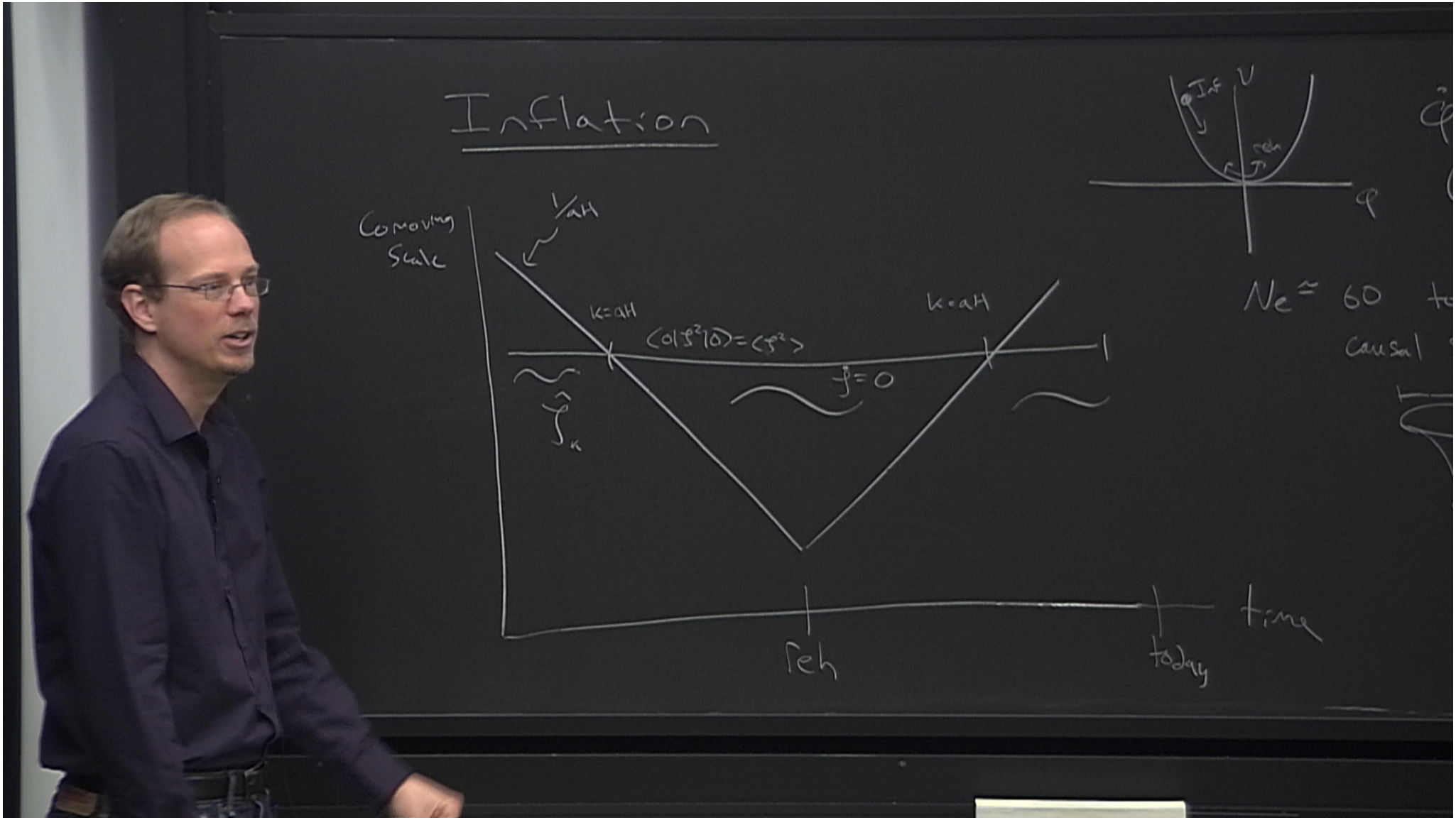
→ Structure + CMB

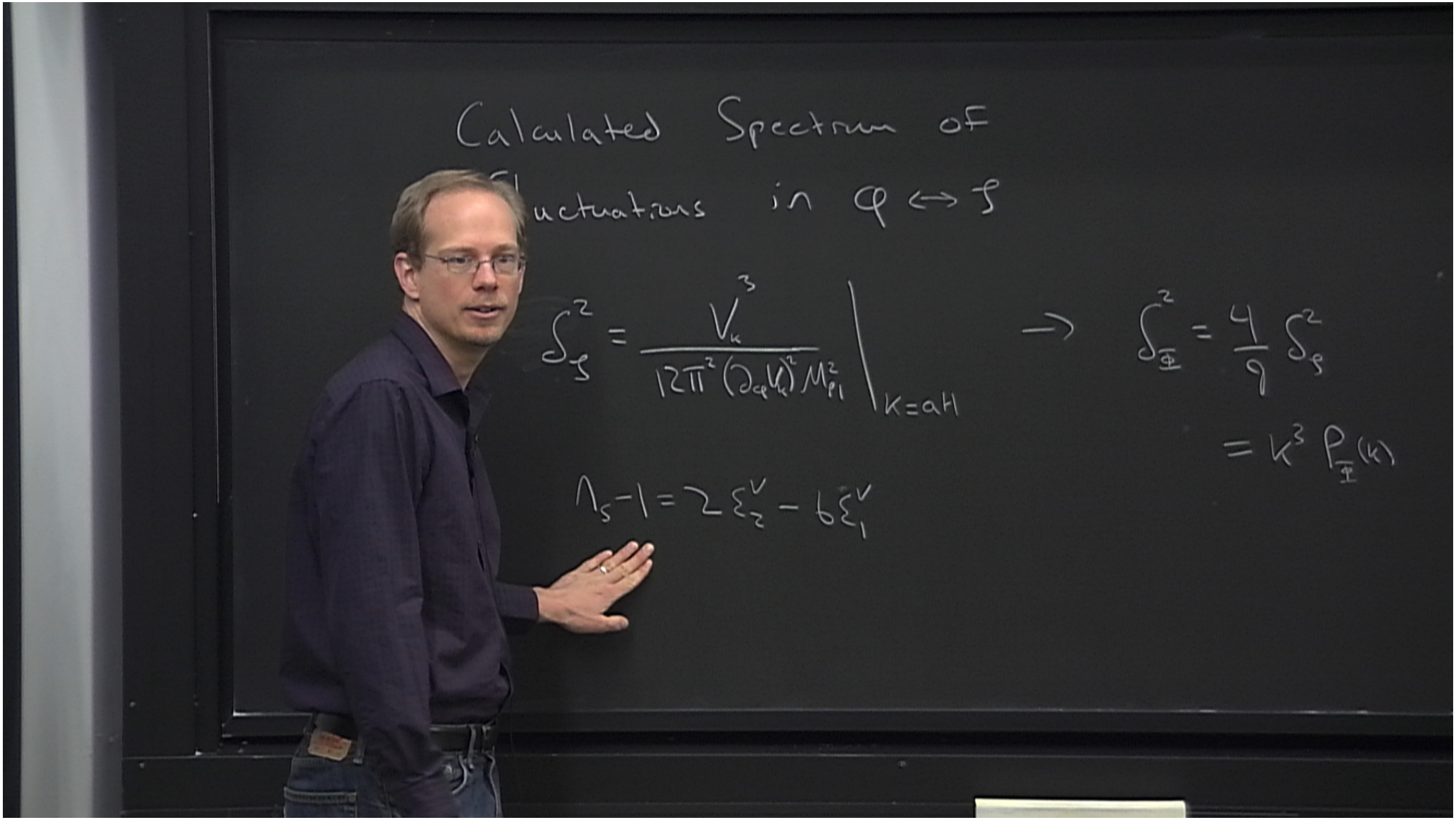
Causal ?

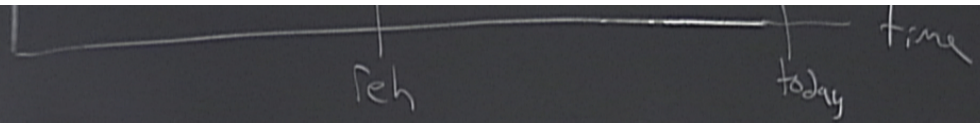


time

today


Inflation
Comoving Scale
1/aH
k=aH
k=aH
⟨0|δ²|0⟩=⟨δ²⟩
δ=0
δ_k
time
reh
today
V
φ
Ne ≃ 60


Calculated Spectrum of
fluctuations in φ ↔ ζ
$\delta_\zeta^2 = \frac{V_\kappa^3}{12\pi^2 (\partial_\varphi V_\kappa)^2 M_{Pl}^2}\Big|_{\kappa = aH}$
$\rightarrow$
$\delta_\Phi^2 = \frac{4}{9}\delta_\zeta^2$
$= k^3 P_\Phi(k)$
$n_s - 1 = 2\xi_2^V - 6\xi_1^V$



Calculated Spectrum of Fluctuations in $\varphi \leftrightarrow \sigma$

$$\delta_\sigma^2 = \left. \frac{V_\kappa^3}{12\pi^2 (\partial_\varphi V_\kappa)^2 M_{Pl}^2} \right|_{\kappa = aH}$$

$$\delta_\Phi^2 = \frac{4}{9}\delta_\sigma^2 = \kappa^3 P_{\mathcal{I}}(\kappa)$$

$$n_s - 1 = 2\xi_2^V - 6\xi_1^V$$

Tensor Modes

$$\delta_T^2 = \frac{2}{3\pi^2} \frac{V}{M_{Pl}^2} \Big|_{\kappa = aH}$$

$$n_T = -2\xi_1^V$$

Testing Models of Inflation



Testing Models of Inflation

$$V(\varphi) = \lambda M_{Pl}^4 \left(\frac{\varphi}{M_{Pl}}\right)^n$$

① Is cosmology causal? $(N_e \gtrsim 60)$

$$N_e = \int H dt = \frac{1}{M_{Pl}^2} \int_{\varphi_{end}}^{\varphi} \frac{V}{\partial_\varphi V} d\varphi$$

When does inflation end? $\epsilon_1^V = 1$

$$\epsilon_1^V = \frac{M_{Pl}^2}{2}\left(\frac{\partial_\phi V}{V}\right)^2$$

$$\epsilon_2^V = M_{Pl}^2\left(\frac{\partial_{\phi\phi} V}{V}\right)$$

$$\Rightarrow \epsilon_1^V = 1 \Rightarrow \phi_{end}^2 = \frac{1}{2}$$

$\approx 60)$

$\frac{V}{\partial_\phi V}\, d\phi$

nd

When does inflation end? $\epsilon_1^V = 1$, $\epsilon_2^V = 1$

$$\epsilon_1^V = \frac{M_{Pl}^2}{2}\left(\frac{\partial_\varphi V}{V}\right)^2 \qquad \frac{\partial_\varphi V}{V} = \frac{n\varphi^{n-1}/M_{Pl}^n}{\varphi^n/M_{Pl}^n}$$

$$\epsilon_2^V = M_{Pl}^2\left(\frac{\partial_{\varphi\varphi} V}{V}\right)$$

$$\Rightarrow \epsilon_1^V = 1 \Rightarrow \varphi_{end}^2 = \frac{n^2}{2}$$

$$\Rightarrow N_e = \frac{1}{M_{Pl}^2}\, n \int_{\varphi_{end}}^{\varphi} \varphi\, d\varphi = \frac{n}{M_{Pl}^2}\left(\frac{\varphi^2}{2} - \frac{\varphi_{end}^2}{2}\right) = 60 \Rightarrow$$

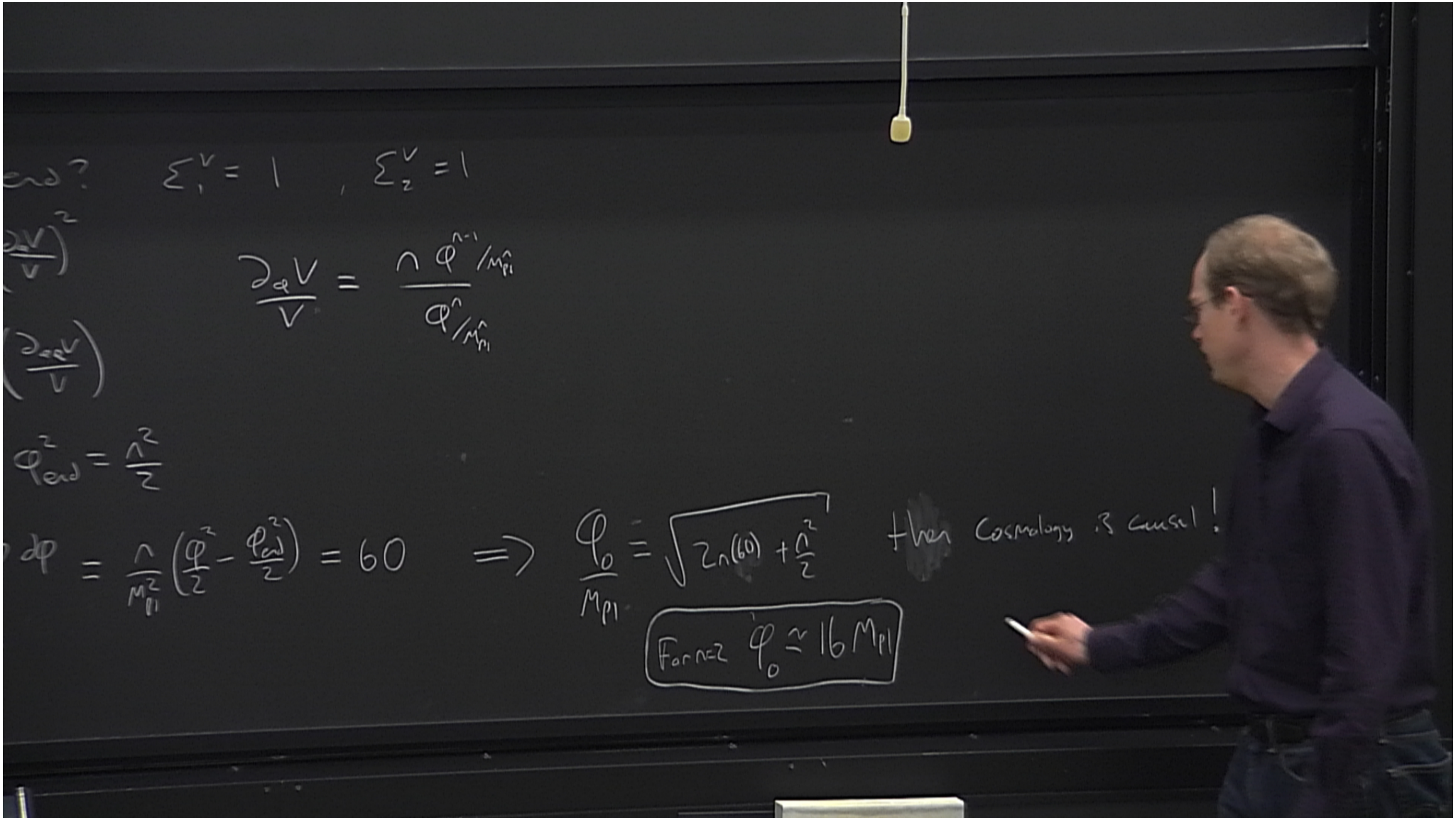
When does inflation end? $\epsilon_1^V = 1$, $\epsilon_2^V = 1$

$$\epsilon_1^V = \frac{M_{Pl}^2}{2}\left(\frac{\partial_\varphi V}{V}\right)^2 \qquad \frac{\partial_\varphi V}{V} = \frac{n\varphi^{n-1}/M_{Pl}^n}{\varphi^n/M_{Pl}^n}$$

$$\epsilon_2^V = M_{Pl}^2\left(\frac{\partial_{\varphi\varphi} V}{V}\right)$$

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$$\Rightarrow N_e = \frac{1}{M_{Pl}^2}\, n \int_{\varphi_{end}}^{\varphi} \varphi\, d\varphi = \frac{n}{M_{Pl}^2}\left(\frac{\varphi^2}{2} - \frac{\varphi_{end}^2}{2}\right) = 60 \Rightarrow \frac{\varphi_0}{M_{Pl}} = \sqrt{2n(60) + \frac{n^2}{2}}$$


end? $\epsilon_1^V = 1$, $\epsilon_2^V = 1$
$\frac{\partial_\varphi V}{V} = \frac{n\varphi^{n-1}/M_{Pl}^n}{\varphi^n/M_{Pl}^n}$
$\varphi_{end}^2 = \frac{n^2}{2}$
$N = \frac{1}{M_{Pl}^2}\left(\frac{\varphi^2}{2} - \frac{\varphi_{end}^2}{2}\right) = 60 \Rightarrow \frac{\varphi_0}{M_{Pl}} = \sqrt{2n(60) + \frac{n^2}{2}}$
then Cosmology is causal!
For n=2 $\varphi_0 \simeq 16 M_{Pl}$

end? $\epsilon_1^V = 1$, $\epsilon_2^V = 1$

$$\left(\frac{\partial V}{V}\right)^2$$

$$\frac{\partial_\varphi V}{V} = \frac{n\varphi^{n-1}/M_{Pl}^n}{\varphi^n/M_{Pl}^n}$$

$$\left(\frac{\partial_\varphi^2 V}{V}\right)$$

$$\varphi_{end}^2 = \frac{n^2}{2}$$

$$N = \frac{1}{M_{Pl}^2}\left(\frac{\varphi_0^2}{2} - \frac{\varphi_{end}^2}{2}\right) = 60 \quad \Rightarrow \quad \frac{\varphi_0}{M_{Pl}} = \sqrt{2n(60) + \frac{n^2}{2}}$$

then cosmology is causal!

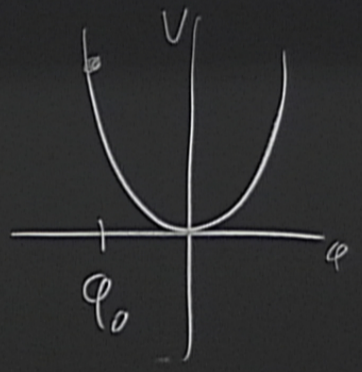
For $n=2$ $\varphi_0 \simeq 16 M_{Pl}$ large-field model

ation end? $\epsilon_1^V = 1$, $\epsilon_2^V = 1$

$$\frac{M_{Pl}^2}{2}\left(\frac{\partial_\varphi V}{V}\right)^2$$

$$= M_{Pl}^2\left(\frac{\partial_{\varphi\varphi} V}{V}\right)$$

$$\frac{\partial_\varphi V}{V} = \frac{\Lambda\, \varphi^{n-1}/M_{Pl}^n}{\varphi^n/M_{Pl}^n}$$

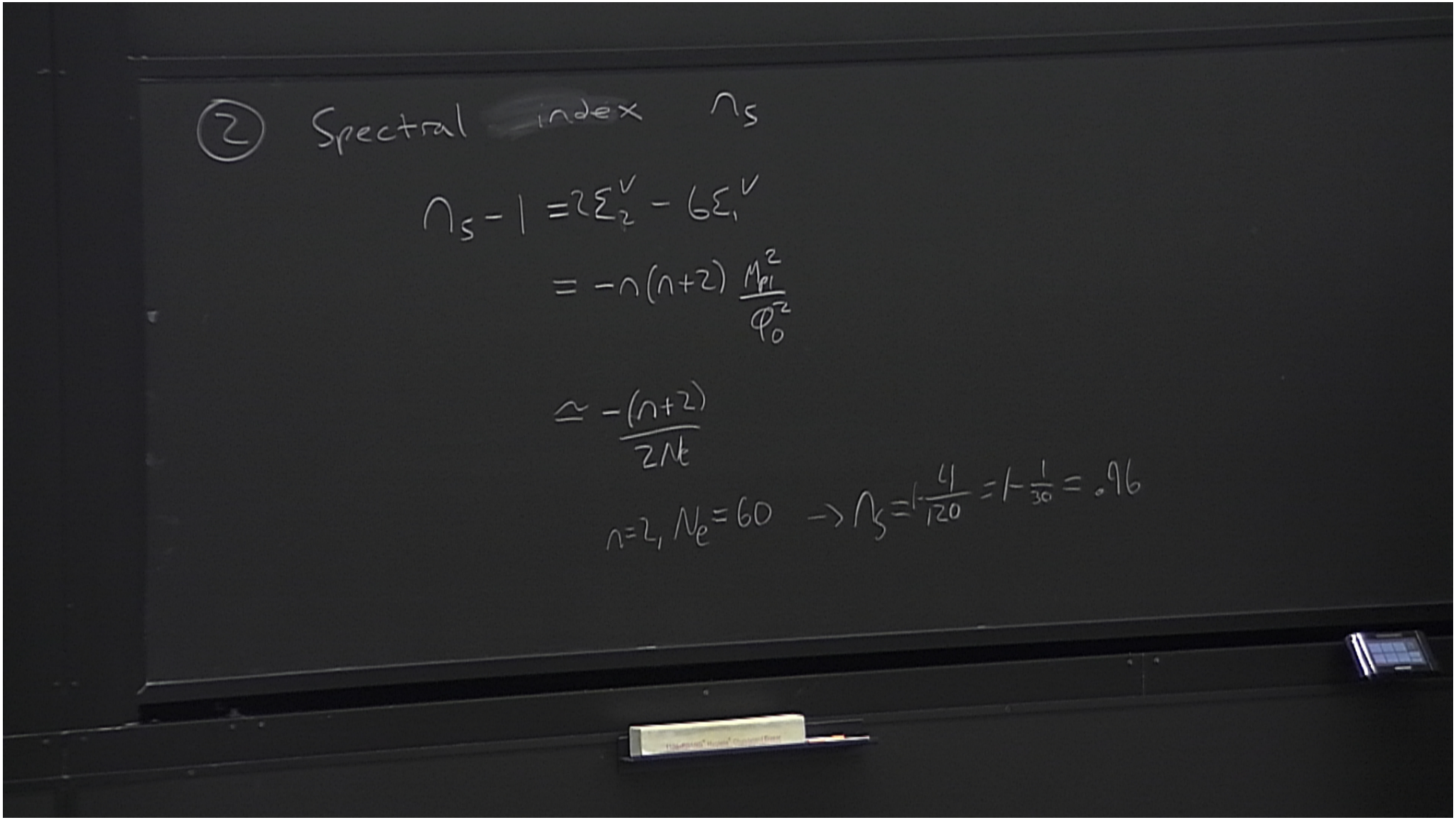


$$\Rightarrow \varphi_{end}^2 = \frac{n^2}{2}$$

$$\int_{\varphi_{end}}^{\varphi} \varphi\, d\varphi = \frac{\Lambda}{M_{Pl}^2}\left(\frac{\varphi^2}{2} - \frac{\varphi_{end}^2}{2}\right) = 60 \quad \Rightarrow \quad \frac{\varphi_0}{M_{Pl}} = \sqrt{2n(60) + \frac{n^2}{2}}$$

then Cosmology is causal!

For $n=2$ $\varphi_0 \simeq 16\, M_{Pl}$ large-field model


② Spectral index n_s
$n_s - 1 = 2\epsilon_2^V - 6\epsilon_1^V$
$= -n(n+2)\frac{M_{Pl}^2}{\phi_0^2}$
$\simeq -\frac{(n+2)}{2Nt}$
$n=2, N_e=60 \rightarrow n_s = 1 - \frac{4}{120} = 1 - \frac{1}{30} = .96$

Testing Models of Inflation

$$V(\varphi) = \lambda M_{Pl}^4 \left(\frac{\varphi}{M_{Pl}}\right)^n$$

① Is cosmology causal? $(N_e \gtrsim 60)$

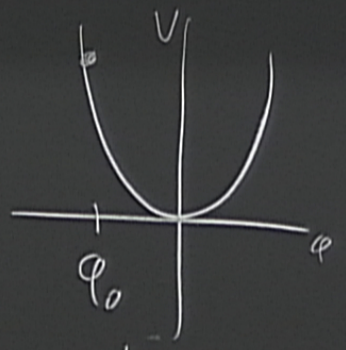
$$N_e = \int H\,dt = \frac{1}{M_{Pl}^2} \int_{\varphi_{end}}^{\varphi} \frac{V}{\partial_\varphi V}\,d\varphi$$

$$= \int \frac{H^{-1}\,d\varphi}{d\varphi/dt}$$

When does inflation end? $\epsilon_1^V = 1$, $\epsilon_2^V = 1$

$$\epsilon_1^V = \frac{M_{Pl}^2}{2}\left(\frac{\partial_\varphi V}{V}\right)^2 \qquad \frac{\partial_\varphi V}{V} = \frac{n\,\varphi^{n-1}/M_{Pl}^n}{\varphi^n/M_{Pl}^n}$$

$$\epsilon_2^V = M_{Pl}^2\left(\frac{\partial_\varphi^2 V}{V}\right)$$



$$\Rightarrow \epsilon_1^V = 1 \Rightarrow \varphi_{end}^2 = \frac{n^2}{2} M_{Pl}^2$$

$$\Rightarrow N_e = \frac{1}{M_{Pl}^2}\frac{1}{n}\int_{\varphi_{end}}^{\varphi} \varphi\, d\varphi = \frac{1}{n M_{Pl}^2}\left(\frac{\varphi^2}{2} - \frac{\varphi_{end}^2}{2}\right) = 60 \Rightarrow \frac{\varphi_0}{M_{Pl}} = \sqrt{2n(60) + \frac{n^2}{2}}$$

N_e

then Cosmology is causal!

For $n=2$ $\varphi_0 \simeq 16\, M_{Pl}$ large-field Mode

② Spectral index n_s

Planck: $n_s = .9616 \pm .0094$

$$n_s - 1 = 2\xi_2^V - 6\xi_1^V$$

$$= -n(n+2)\frac{M_{Pl}^2}{\phi_0^2}$$

$$\simeq -\frac{(n+2)}{2N_k}$$

$n=2, N_e = 60 \rightarrow n_s = 1 - \frac{4}{120} = 1 - \frac{1}{30} = .96$

② Spectral index n_s

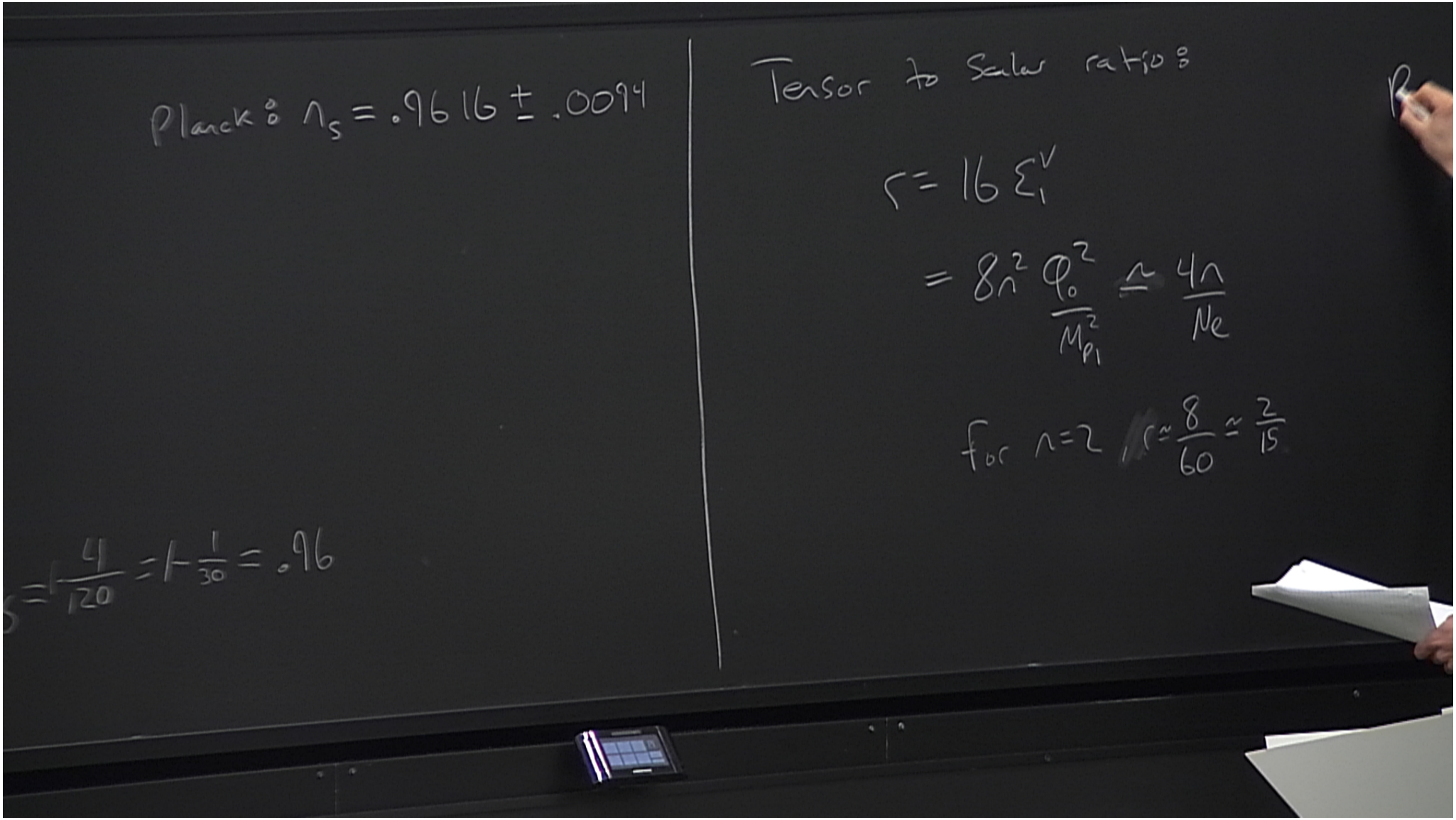
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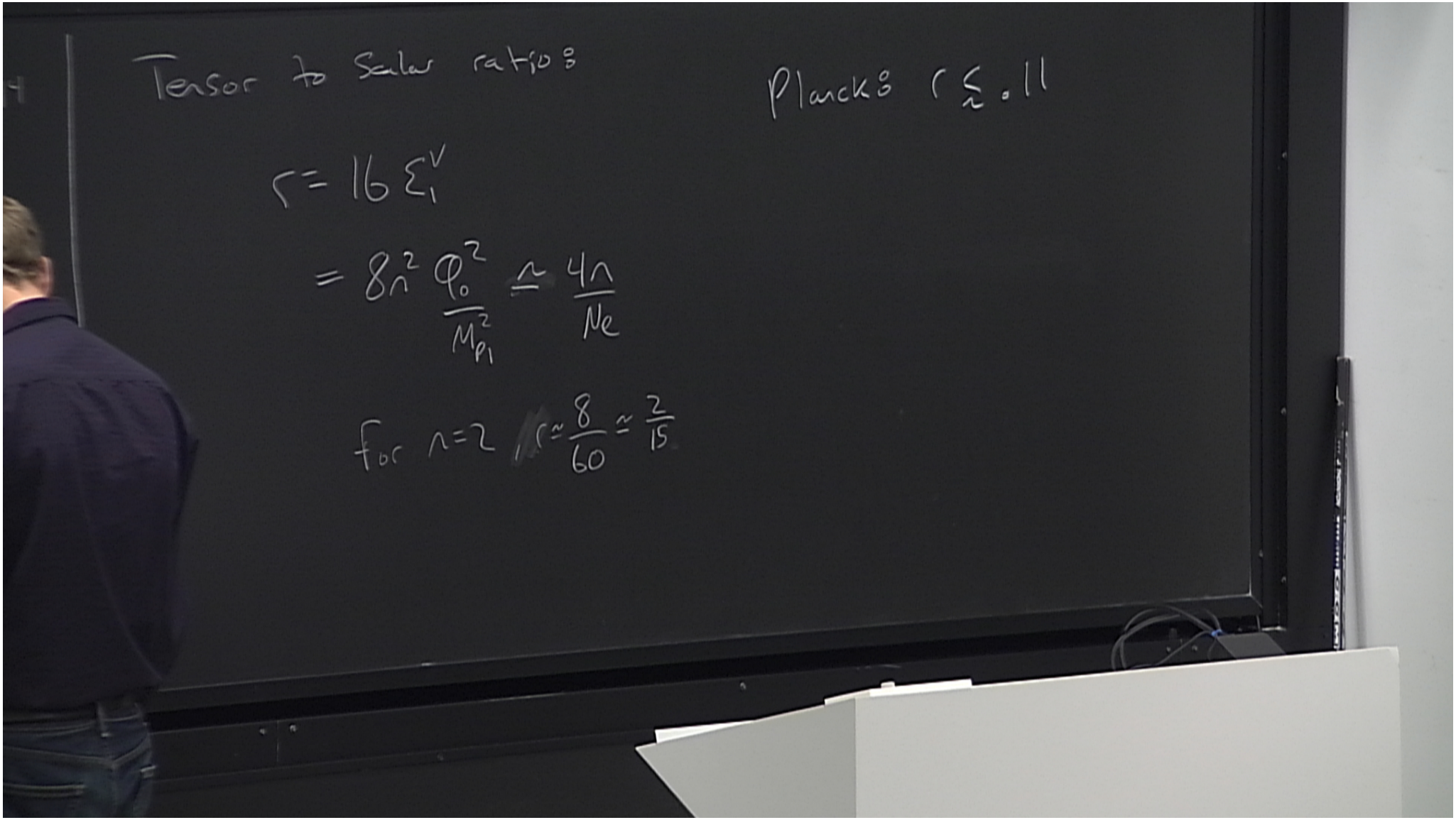
$$n_s - 1 = 2\Sigma_2^V - 6\Sigma_1^V$$

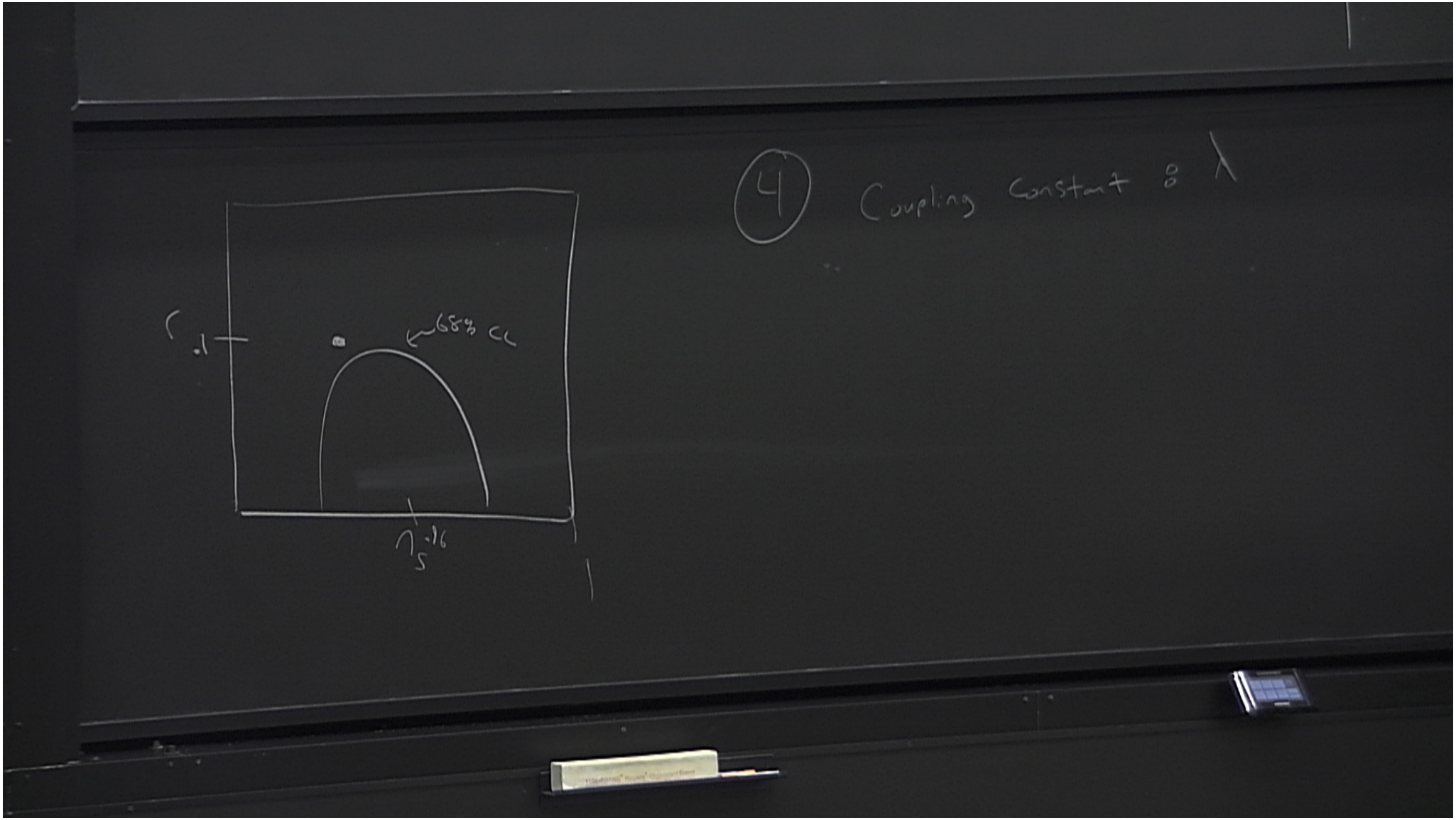
$$= -n(n+2)\frac{M_{Pl}^2}{\phi_0^2}$$

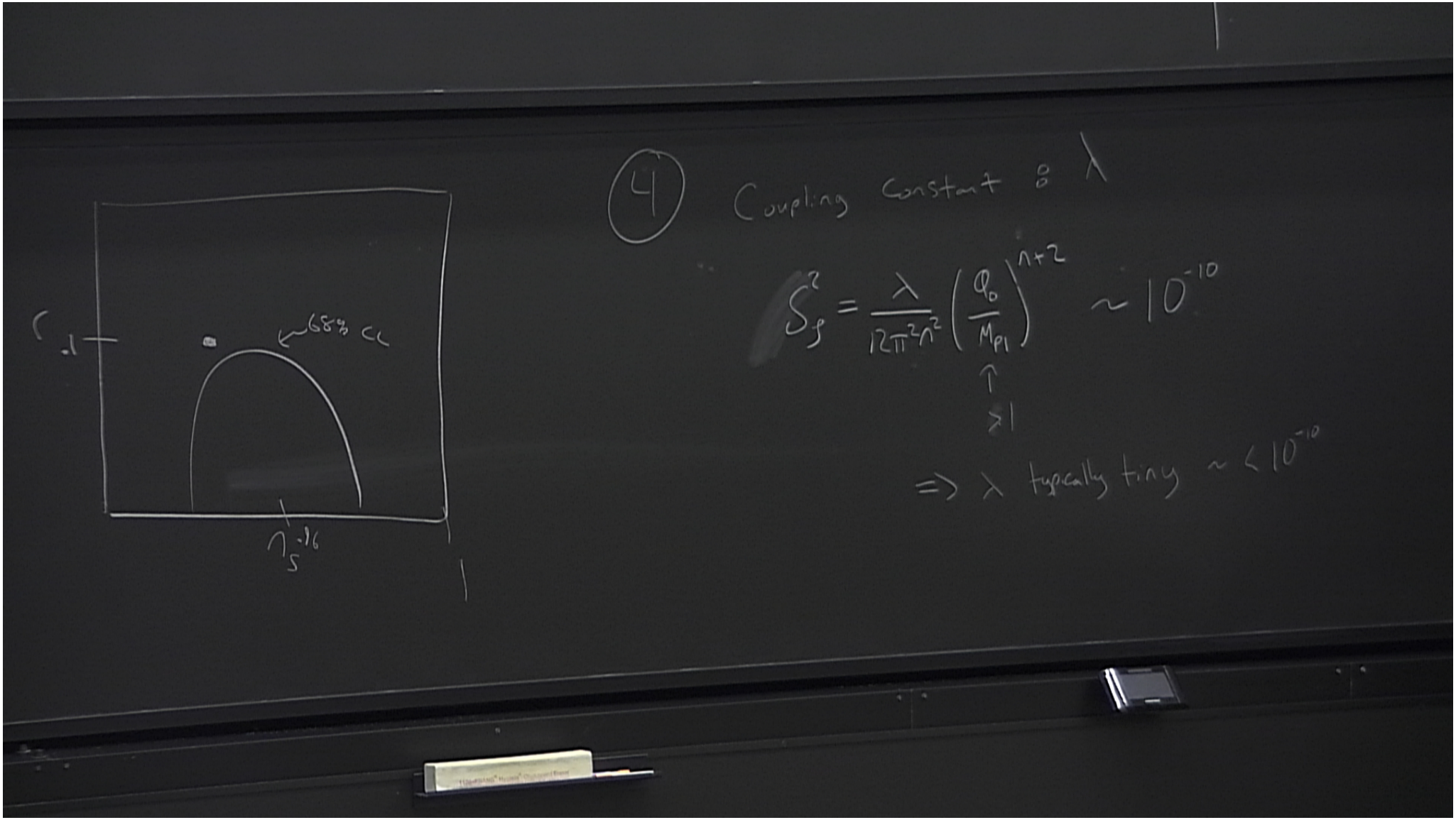
$$\simeq -\frac{(n+2)}{2N_e}$$

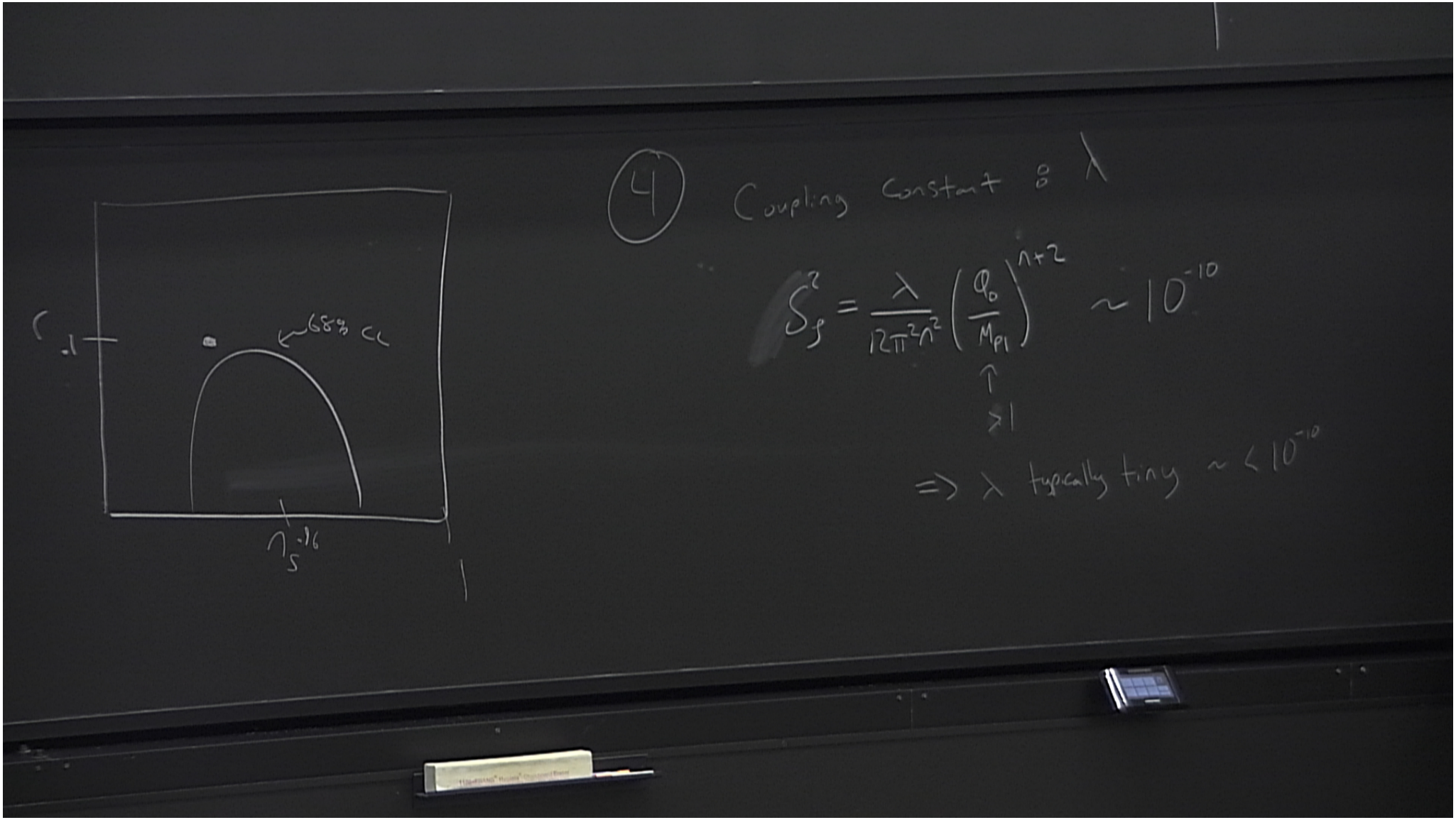
$n=2, N_e = 60 \rightarrow n_s = 1 - \frac{4}{120} = 1 - \frac{1}{30} = .96$

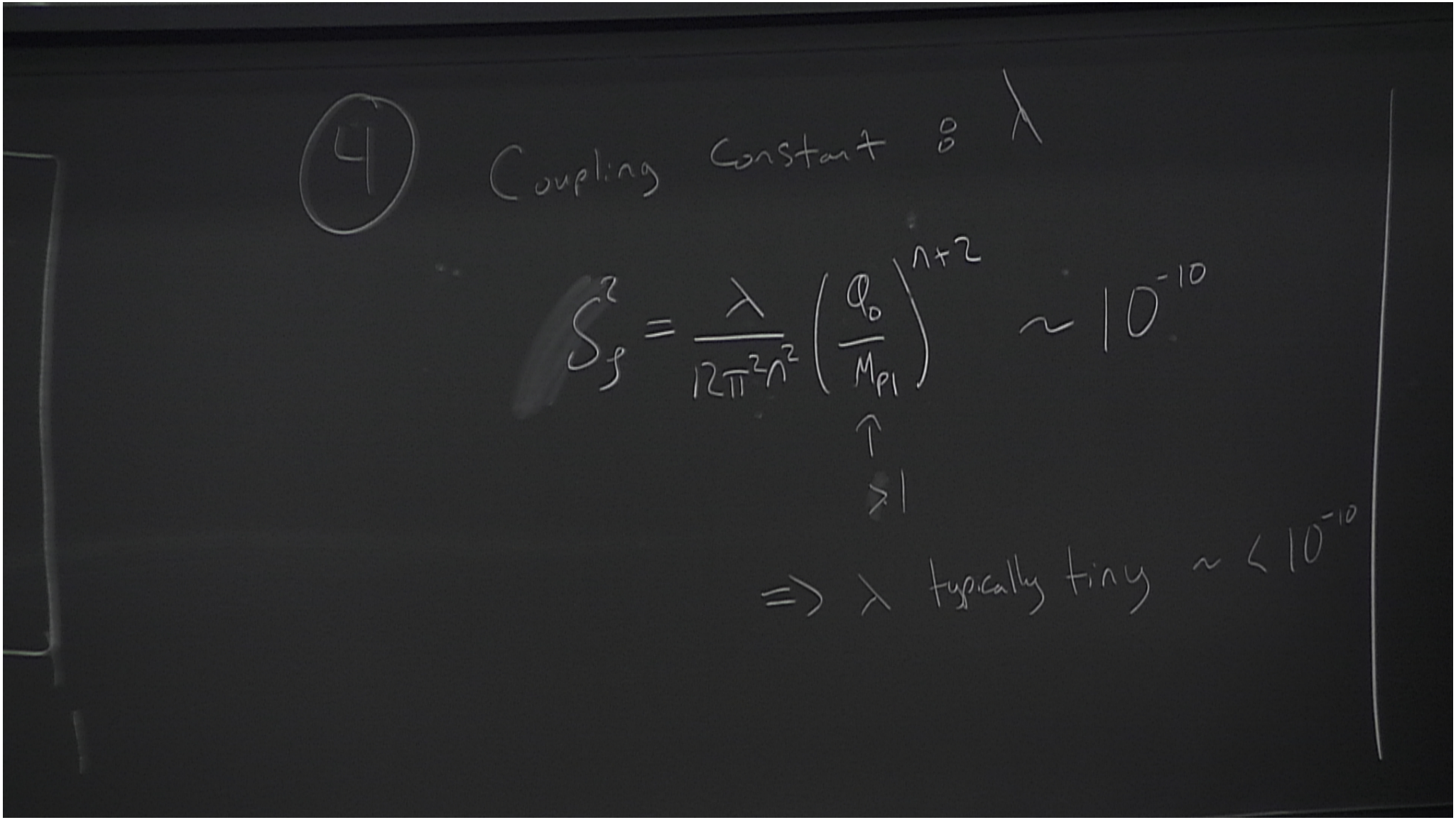

Planck: $n_s = .9616 \pm .0094$
Tensor to Scalar ratio:
$r = 16\epsilon^V$
$= 8n^2 \frac{\phi_0^2}{M_{Pl}^2} \simeq \frac{4n}{Ne}$
for $n=2$ $r \simeq \frac{8}{60} \simeq \frac{2}{15}$
$= 1 - \frac{4}{120} = 1 - \frac{1}{30} = .96$


Tensor to Scalar ratio:
Planck: $r \lesssim .11$
$r = 16\,\epsilon_1^V$
$= 8N^2 \frac{\varphi_0^2}{M_{Pl}^2} \simeq \frac{4N}{Ne}$
for $N=2$ $r \simeq \frac{8}{60} \simeq \frac{2}{15}$


4
Coupling Constant
λ
.1
68% CL
.16
1


(4) Coupling Constant : λ
$\delta_s^2 = \frac{\lambda}{12\pi^2 n^2}\left(\frac{\phi_0}{M_{Pl}}\right)^{n+2} \sim 10^{-10}$
≥ 1
$\Rightarrow \lambda$ typically tiny $\sim < 10^{-10}$
.1
68% CL
n_s
.96
1


(4) Coupling Constant λ
$\delta_s^2 = \frac{\lambda}{12\pi^2 n^2}\left(\frac{\phi_0}{M_{Pl}}\right)^{n+2} \sim 10^{-10}$
≥ 1
$\Rightarrow \lambda$ typically tiny $\sim < 10^{-10}$
~68% CL
.1
n_s .96
1


4
Coupling Constant : λ
$\delta_s^2 = \frac{\lambda}{12\pi^2 n^2}\left(\frac{\phi_0}{M_{Pl}}\right)^{n+2} \sim 10^{-10}$
≥ 1
$\Rightarrow \lambda$ typically tiny $\sim < 10^{-10}$

What makes a good Model?

Why is λ so small?

Symmetry: Shift-Symmetry

$\varphi \to \varphi + \text{const.}$

$(\partial_\mu \varphi)^2$ respects Symmetry

$\lambda M_{Pl}^4 \left(\frac{\varphi}{M_{Pl}}\right)^n$ doesn't

λ small from UV completion

$< 10^{-10}$



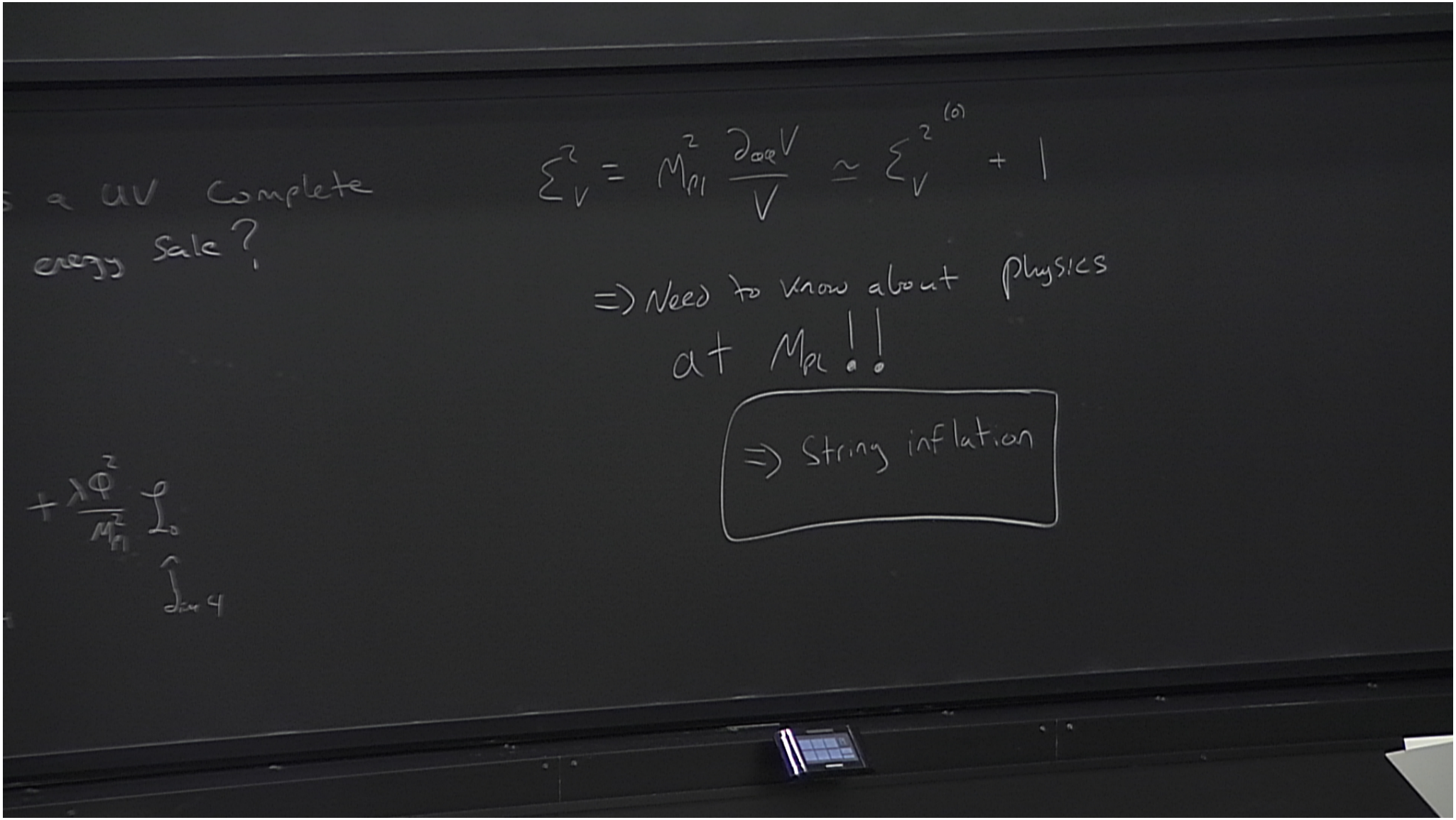
Inflation requires a UV Complete theory. At what energy scale?

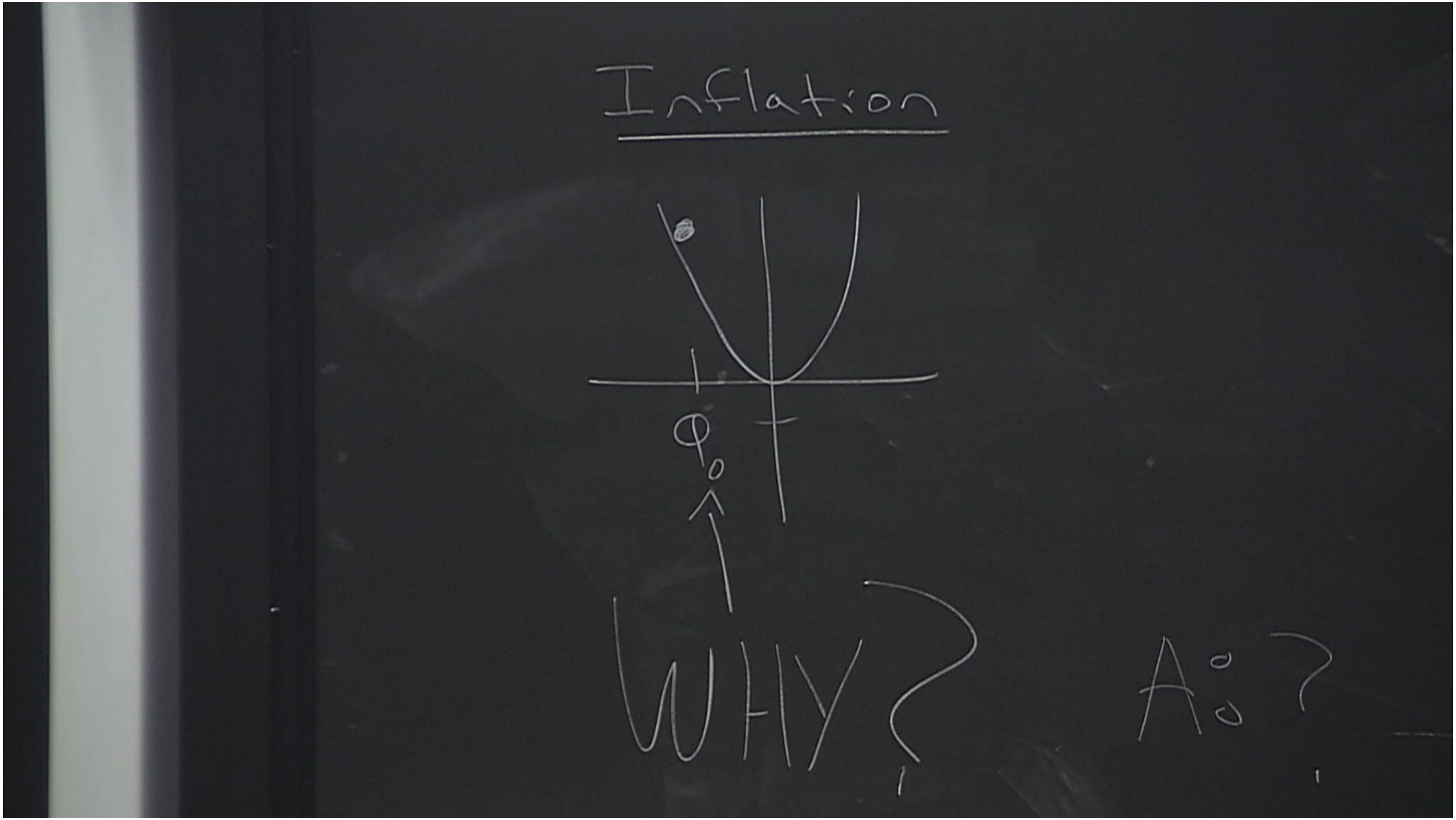
$$\Sigma_V^2 = M_{Pl}^2 \frac{\partial}{}$$

$$\Lambda \gtrsim M_{Pl}$$

$$\mathcal{L} = \mathcal{L}_0 + \frac{\phi^2}{M_{Pl}^2}\mathcal{L}_0$$

$\mathcal{L}_0$: dim 4 $\quad$ $\mathcal{L}_0$: dim 4


a UV Complete
energy Sale?
$\xi_V^2 = M_{Pl}^2 \frac{\partial_{\phi\phi} V}{V} \simeq \xi_V^{2\,(0)} + 1$
⇒ Need to know about physics
at M_{Pl}!!
⇒ String inflation
$+\frac{\lambda\phi^2}{M_{Pl}^2}\int_0$
dim 4


Inflation
WHY?
A?

Traded Solving initial conditions for another problem in SBB (ΛCDM)

T → S

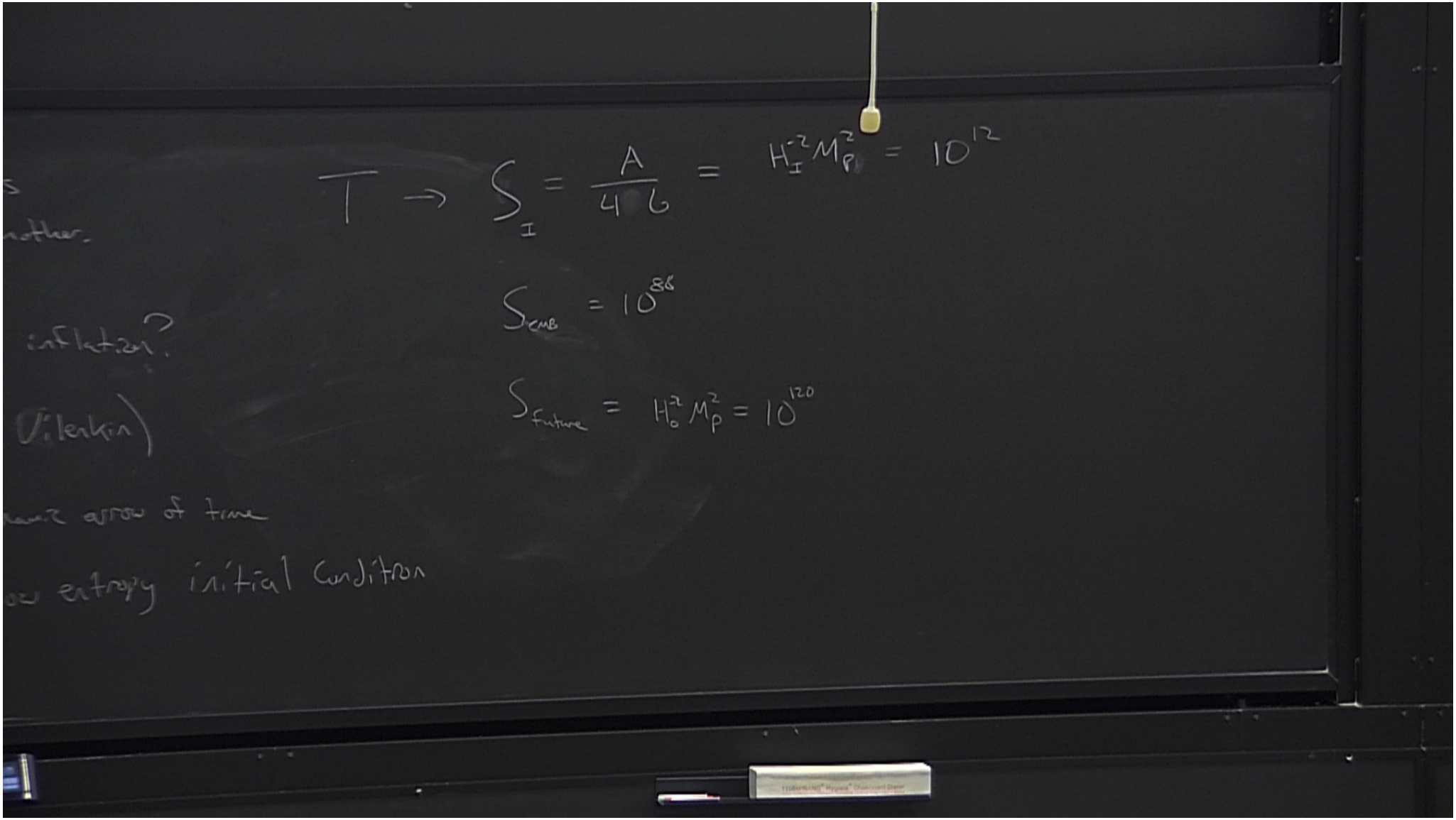
① Was there a Singularity before inflation?

Yes! (Guth, Borde, Vilenkin)

② In observable Universe → thermodynamic arrow of time

Inflation was a very low entropy initial condition

?


$T \to S_I = \frac{A}{4G} = H_I^{-2} M_p^2 = 10^{12}$
$S_{CMB} = 10^{88}$
$S_{future} = H_0^{-2} M_p^2 = 10^{120}$
inflation?
(Vilenkin)
arrow of time
entropy initial condition

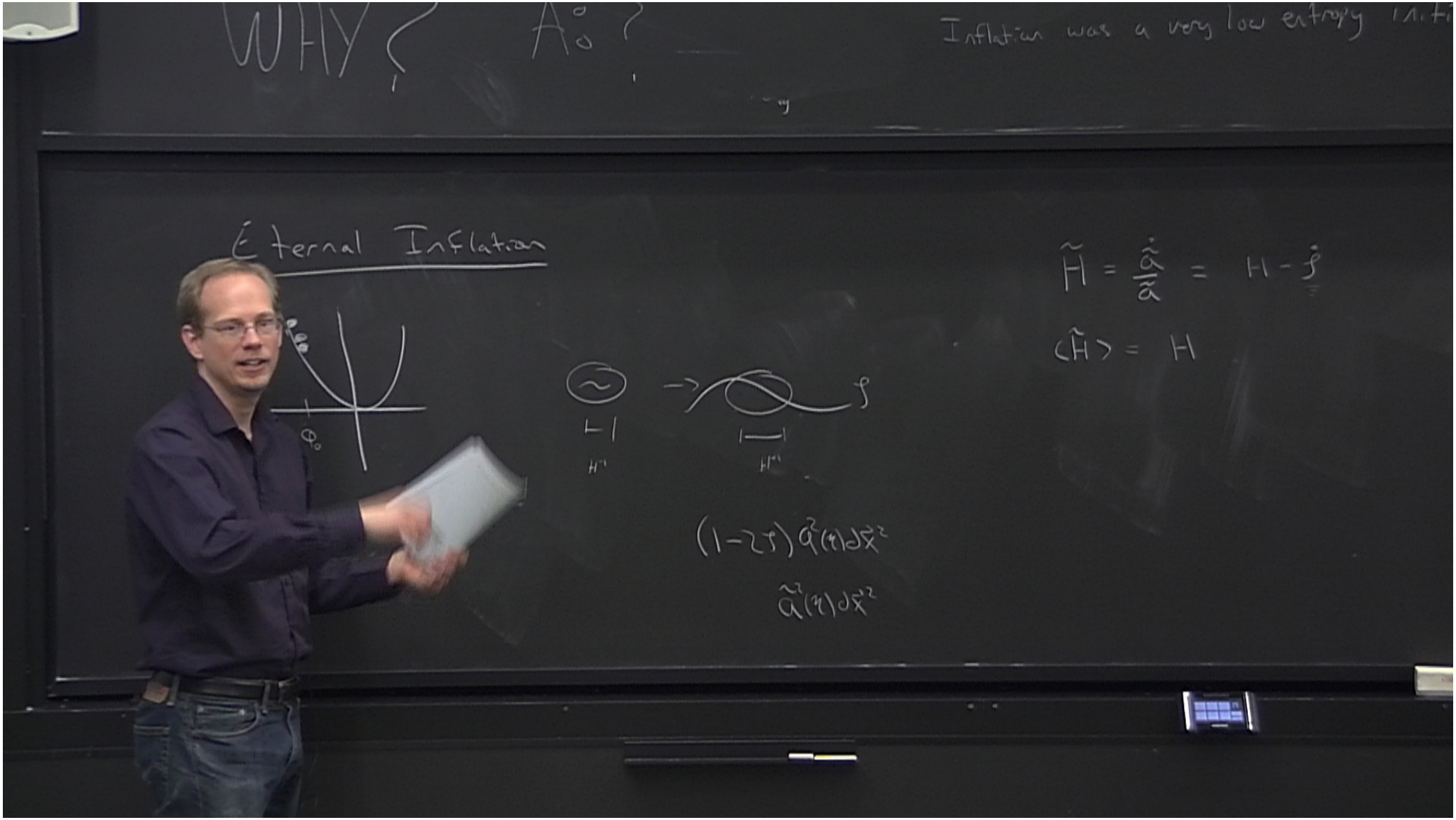
$$T \to S_I = \frac{A}{4G} = H_I^{-2} M_P^2 = 10^{12}$$

$$S_{CMB} = 10^{88}$$

$$S_{future} = H_0^{-2} M_P^2 = 10^{120}$$

of time

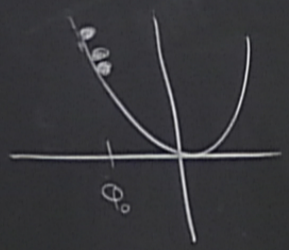
initial condition

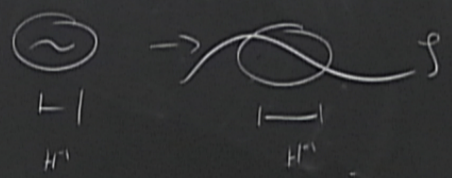

WHY?
Inflation was a very low entropy
Eternal Inflation
$\tilde{H} = \frac{\dot{\tilde{a}}}{\tilde{a}} = H - \dot{\rho}$
$\langle \tilde{H} \rangle = H$
$(1-2\rho)\, a^2(\eta) d\vec{x}^2$
$\tilde{a}^2(\eta) d\vec{x}^2$

WHY? A_0?

Inflation was a very low entropy initial condition

Eternal Inflation





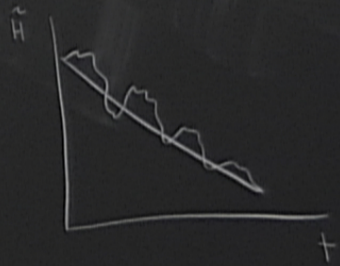
$$\tilde{H} = \frac{\dot{\tilde{a}}}{\tilde{a}} = H - \dot{f}$$

$$\langle \tilde{H} \rangle = H$$

$$\langle \tilde{H}^2 \rangle = H^2 + \langle \dot{f}^2 \rangle$$

$$(1-2f)\,a^2(\eta)\,dx^2$$

$$\tilde{a}^2(\eta)\,dx^2$$

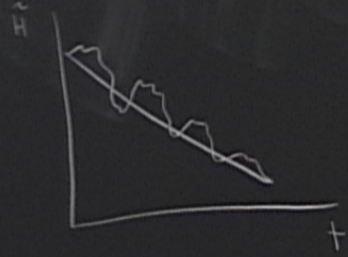


flation was a very low entropy initial condition

$\tilde{H} = \frac{\dot{\tilde{a}}}{\tilde{a}} = H - \dot{\zeta}$

$\langle \tilde{H} \rangle = H$

$\langle \tilde{H}^2 \rangle = H^2 + \langle \dot{\zeta}^2 \rangle$



Scalar field in background

Classical motion $|\Delta\phi|$ in $t = H^{-1}$

Quantum fluctuations $\sqrt{\langle \phi^2 \rangle}$ in $t = H^{-1}$

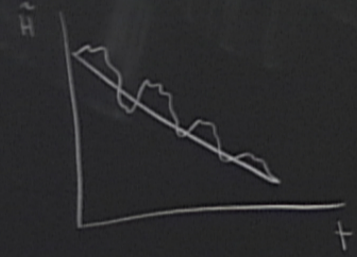
$\frac{H^3}{4\pi^2} t$ in $t = H^{-1} = \left(\frac{H}{2\pi}\right)^2$

flation was a very low entropy initial condition

$\tilde{H} = \frac{\dot{\tilde{a}}}{\tilde{a}} = H - \dot{\delta}$

$\langle \tilde{H} \rangle = H$

$\langle \tilde{H}^2 \rangle = H^2 + \langle \dot{\delta}^2 \rangle$



Scalar field in background

Classical motion $|\Delta\phi|$ in $t = H^{-1}$

Quantum fluctuations $\sqrt{\langle \phi^2 \rangle}$ in $t = H^{-1}$

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$\mathcal{E}'_V \propto \left(\frac{\rho_{qV}}{V}\right)^2$ → decreases as you go up the Potential

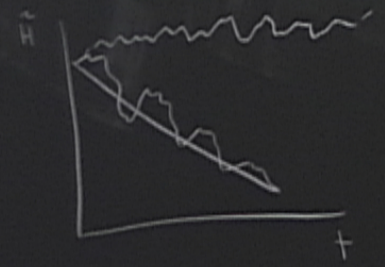
⇒ Far enough up Quantum Fluct. dominate Classical

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