

Title: 12/13 PSI - Explorations in Cosmology Lecture 13

Date: Apr 24, 2013 10:15 AM

URL: <http://pirsa.org/13040078>

Abstract:

Homework

Question 2, part 6:

$$\langle 0 | \hat{\chi}^2 | 0 \rangle_{(BD)}$$

Without radiation dominated era:

$$|V_k| \simeq \frac{1}{k^{3/2} |M|} e^{i k \eta} \simeq \frac{1}{k^{3/2} |M|}$$

With one:

$$u_k \simeq \frac{1}{k^{3/2} |M|} (\alpha_k + \beta_k) \rightarrow \text{Build } \hat{\chi}$$

From this

Homework

Question 2, part 6:

$$\langle 0 | \hat{x}^2 | 0 \rangle$$

Without radiation dominated era:

$$|v_k| \approx \frac{1}{k^2 |M|} e^{i k \eta} \approx \frac{1}{k^2 |M|}$$

With one:

$$u_k \approx \frac{1}{k^2 |M|} (k_k + p_k) \rightarrow B_{\text{eff}} \hat{x}$$

From this

Tutorial

$$n_{\text{ws}} = \frac{1}{e^{\frac{\pi}{2} \left(\frac{H}{2\pi} \right)} - 1}$$

$H = 1 \text{ min to}$

Cook a Hamburger

(Small Hamburger)

Recap

* Free scalars have a gaussian spectrum:

$$\langle 0 | \chi_k \chi_{k'}^\dagger | 0 \rangle = \frac{|v_k|^2}{2} \delta(k - k')$$

Homework

Question 2, part 6:

$$\langle \chi^2 \rangle = \langle 0 | \hat{\chi}^2 | 0 \rangle \quad \hat{\chi} = \int \frac{d^3k}{(2\pi)^3} \{ a_{\vec{k}}^\dagger v_{\vec{k}} e^{i\vec{k} \cdot \vec{r}} + a_{\vec{k}} \}$$

Without radiation dominated era:

$$|v_{\vec{k}}| \approx \frac{1}{k^{3/2} |M|} e^{i\vec{k} \cdot \vec{r}} \approx \frac{1}{k^{3/2} |M|} \quad a_{\vec{k}} |0\rangle_{\text{rad}} = 0 \quad H = 1_{\text{un}} \text{ to } H = \frac{H_1}{2\pi}$$

With one:

$$u_{\vec{k}} \approx \frac{1}{k^{3/2} |M|} (k_{\vec{k}} + p_{\vec{k}}) \rightarrow B_{\vec{k}} b_{\vec{k}} \hat{\chi} \quad \hat{\chi} = \int \frac{d^3k}{(2\pi)^3} [u_{\vec{k}} b_{\vec{k}} e^{i\vec{k} \cdot \vec{r}} + u_{\vec{k}}^\dagger b_{\vec{k}}^\dagger]$$

From this

Tutorial

$$n_{\text{2r}} = \frac{1}{-1}$$

Coord. a. h.

Recap

* Free scalars have a gaussian Spectrum:

$$\langle 0 | \chi_{\vec{k}} \chi_{\vec{k}}^\dagger | 0 \rangle = \frac{|v_{\vec{k}}|^2}{2} \delta(\vec{k} - \vec{k}')$$

Homework

ϕ^2 Question 2, part 6:

$$\langle \chi^2 \rangle = \langle 0 | \hat{\chi}^2 | 0 \rangle_{(BD)}$$

$$\hat{\chi} = \int \frac{d^3k}{(2\pi)^3} \left\{ \alpha_k^- v_k e^{ik\eta} + \alpha_k^+ \right\}$$

Without radiation dominated era:

$$|v_k| \approx \frac{1}{k^{3/2} |a|} e^{i k \eta} \approx \frac{1}{k^{3/2} |a|}$$

$$\alpha^- |0\rangle_{(BD)} = 0$$

With one:

$$u_k \approx \frac{1}{k^{3/2} |a|} (\alpha_k + \beta_k)$$

→ Build $\hat{\chi}$
From this

$$\hat{\chi} = \int \left[u_k b^- e^{ik\eta} + u_k^* b^+ \right] \quad (\text{Small})$$

Tutorial

$$n_{25} = \dots$$

$$\bar{H} = 1 \text{ um to}$$

Coor. a Han

Tutorial

$$\hat{\chi} = \int \frac{d^3k}{(2\pi)^3} \left[a_{\vec{k}} v_{\vec{k}} e^{i\vec{k}\cdot\vec{r}} + a_{\vec{k}}^\dagger \right]$$

$$\frac{1}{k^{3/2} m}$$

$$a|0\rangle_{(BD)} = 0$$

$$n_{\omega} = \frac{1}{e^{\omega/(H/2\pi)} - 1}$$

$$T = \frac{H}{2\pi}$$

$H = 1 \mu\text{m}$ to
Cook a Hamburger

→ Build $\hat{\chi}$
From this

$$\hat{\chi} = \int \left[u_{\vec{k}} b_{\vec{k}} e^{i\vec{k}\cdot\vec{r}} + u_{\vec{k}}^* b_{\vec{k}}^\dagger \right] \quad (\text{Small Hamburger})$$

Recap

* Free scalar
Spectrum

Tutorial

$$\{ \alpha_k^- v_k e^{i k y} + a_k^+ \}$$

$$\alpha^- |0\rangle_{(B)} = 0$$

on this

$$\hat{\chi} = \int [u_k b^- e^{i k y} + u_k^* b^+]$$

(Small Hamburger)

$$n_w = \frac{1}{e^{\frac{2\pi}{H}(\frac{H}{2\pi})} - 1}$$

$$H^- = 1 \text{ um to}$$

Coor. a Hamburger

$$T = \frac{H}{2\pi}$$



Recap

* Free scalars have a gauge spectrum:

$$\langle 0 | \chi_k \chi_k^* | 0 \rangle =$$

Horizontal
limit
S

Tutorial

$$\{a_k e^{ikx} + a_k^\dagger\}$$

$$a|0\rangle_{(BS)} = 0$$

$$n_w =$$

$$= \frac{1}{e^{\frac{\hbar \omega}{2\pi T}} - 1}$$

$$T = \frac{\hbar}{2\pi}$$

$$T_{\text{CMB}} = \frac{10^{-6} \text{ m}}{10^{-26} \text{ m}} \cdot 300 \text{ K} = 10^{-30} \text{ K}$$

$$\hbar = 1 \mu\text{m}$$

Cool. a Hamburger $\rightarrow 300 \text{ K}$

$$\hat{\chi} = \int [u_k \bar{b} e^{ikx} + u_k^* b^\dagger] \quad (\text{Small Hamburger})$$



Recap

* Free scalars have a Gaussian Spectrum:

$$\langle 0 | \chi_k \chi_k^\dagger | 0 \rangle =$$

Horizon is an intrinsic limit on size of any system.

$\Rightarrow$ limit on precision of any measurement

Recap

* Free scalars have a gaussian spectrum:

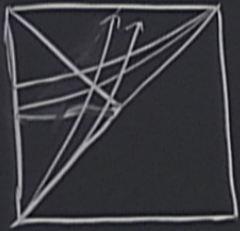
$$\langle 0 | \chi_k \chi_k^* | 0 \rangle = \frac{|V_k|^2}{2} \delta(\vec{k} - \vec{k}')$$

$$T_{\text{cur}} = \frac{10^{-6} \text{ m}}{10^{26} \text{ m}} \cdot 300 \text{ K} = 10^{-30} \text{ K}$$

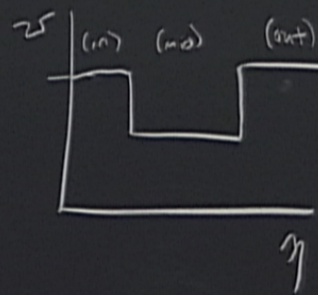
$$T = \frac{H}{2\pi}$$

Horizon is an intrinsic limit on size of any system.

$\Rightarrow$ limit on precision of any measurement



$\rightarrow$ BB transf.



$$V_k^{(\text{out})} = \alpha_k V_k^{(\text{in})} + \beta_k V_k^{(\text{in})*}$$

$$V_k^{(\text{mid})} =$$

Recap

* Free Scalars have a gaussian Spectrum:

$$\langle 0 | \hat{\chi}_k \hat{\chi}_{k'} | 0 \rangle = \frac{|v_k|^2}{2} \delta^3(\vec{k} - \vec{k}') \propto P(k)$$

Recap

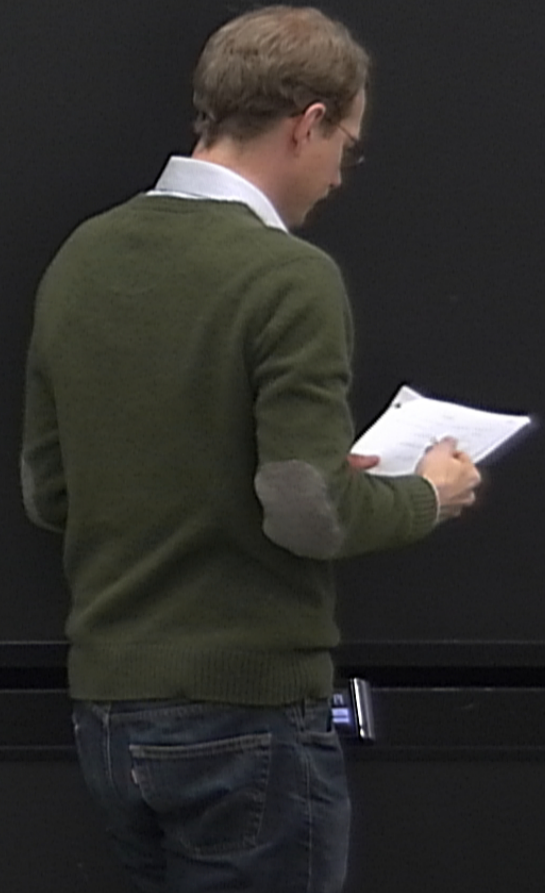
* Free Scalars have a gaussian spectrum

$$\langle 0 | \hat{\chi}_k \hat{\chi}_{k'} | 0 \rangle = \frac{|v_k|^2}{2} \delta^3(\vec{k} - \vec{k}')$$

Why gaussian? $\rightarrow$ Moments
 $\rightarrow$ Wave functional

$\nwarrow \propto P(k)$

$$\langle 0 | \hat{\chi}(\vec{x}, t) \hat{\chi}(\vec{y}, t) | 0 \rangle =$$



$$\langle 0 | \hat{\chi}(\vec{x}, t) \hat{\chi}(\vec{y}, t) | 0 \rangle = \int_0^\infty \frac{d\kappa}{\kappa} S^2(\kappa) \frac{\sin(L\kappa)}{L\kappa}$$

$(\vec{k} - \vec{k}')$

$\propto p(\kappa)$

$$\langle 0 | \hat{\chi}(\vec{x}, t) \hat{\chi}(\vec{y}, t) | 0 \rangle = \int_0^\infty \frac{d\kappa}{\kappa} S^2(\kappa) \frac{\sin(L\kappa)}{L\kappa}$$

$\nearrow S^2 = \frac{\kappa^3 |V_\kappa|^2}{(2\pi)^2}, \quad L = |\vec{x} - \vec{y}|$

In ∂S^0 vacuum is Bunch Davies $\propto P(\kappa) \kappa^3$

$$V_\kappa(\eta \rightarrow \infty) = \frac{1}{\sqrt{\kappa}} e^{i\kappa\eta}, \quad \varphi = \frac{\chi}{a}$$

$$\Rightarrow \boxed{S_\varphi = \frac{H}{2\pi}}$$

$$\langle 0 | \hat{\chi}(\vec{x}, t) \hat{\chi}(\vec{y}, t) | 0 \rangle = \int_0^\infty \frac{d\kappa}{\kappa} S^2(\kappa) \frac{\sin(L\kappa)}{L\kappa}$$

$\nearrow \int^2 = \frac{\kappa^3 |V_\kappa|^2}{(2\pi)^2}, \quad L = |\vec{x} - \vec{y}|$

In ∂S^0 vacuum is Bunch Davies $\propto P(\kappa) \kappa^3$

$$V_\kappa(\eta \rightarrow \infty) = \frac{1}{\sqrt{\kappa}} e^{i\kappa\eta}, \quad \varphi = \frac{\chi}{a}$$

$$\Rightarrow \boxed{S_\varphi = \frac{H}{2\pi}} \quad \left(\text{Super-horizon scales, } a = -\frac{1}{H\eta} \Rightarrow \kappa|\eta| \ll 1 \right)$$

$$\langle 0 | \hat{\chi}(\vec{x}, t) \hat{\chi}(\vec{y}, t) | 0 \rangle = \int_0^\infty \frac{dk}{k} S^2(k) \frac{\sin(Lk)}{Lk}$$

$\nearrow S^2 = \frac{k^3 |V_k|^2}{(2\pi)^2}, \quad L = |\vec{x} - \vec{y}|$

In ∂S^0 vacuum is Bunch Davies $\propto P(k) k^3$

(all k satisfy $k|\eta| \gg 1$ as $\eta \rightarrow -\infty$)

$$V_k(\eta \rightarrow -\infty) = \frac{1}{\sqrt{k}} e^{ik\eta}, \quad \varphi = \frac{\chi}{a}$$

$$\Rightarrow \boxed{\mathcal{S}_\varphi = \frac{H}{2\pi}} \quad (\text{Super-horizon scales, } a = -\frac{1}{H\eta} \Rightarrow k|\eta| \ll 1)$$

Coincidence limit

$$\langle 0 | \hat{\phi}^2 | 0 \rangle = \frac{1}{a^2} \langle 0 | \hat{\chi}^2 | 0 \rangle$$

$$= \int_0^\infty \frac{dk}{k} S_q^2 = \int_0^{k_{\text{max}}} \frac{dk}{k} \left(\frac{H}{2\pi} \right)^2 + \langle 0 | \hat{\phi}^2 | 0 \rangle_{\text{UV}}$$

diverges as $k \rightarrow 0$

diverges as $k \rightarrow \infty$

If inflation beg

If inflation begins at some time t_0 , what is $\langle \hat{\varphi}^2 \rangle$?

$$k_{\min} = \eta_0^{-1}$$

Size of primordial
Hubble volume

$$k_{\max} = \eta^{-1}$$

Size of 1 Hubble volume
at η

If inflation begins at some time t_0 , what is $\langle \hat{\varphi}^2 \rangle$?

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at η

$$\langle 0 | \hat{\varphi} | 0 \rangle = \left(\frac{H}{2\pi} \right)^2 \int_{k_{\min}}^{k_{\max}} \frac{dk}{k} = \left(\frac{H}{2\pi} \right)^2 \log \left(\frac{\eta_0}{\eta} \right)$$

If inflation begins at some time t_0 , what is $\langle \hat{\varphi}^2 \rangle$?

$$k_{\min} = \eta_0^{-1}$$

Size of primordial
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Size of 1 Hubble volume
at η

$$\eta = -H^{-1} \dot{e}^{-Ht}$$

$$(ds^2 = -dt^2 + a^2 dx^2)$$

$$\langle 0 | \hat{\varphi}^2 | 0 \rangle = \left(\frac{H}{2\pi} \right)^2 \int_{k_{\min}}^{k_{\max}} \frac{dk}{k} = \left(\frac{H}{2\pi} \right)^2 \log \left(\frac{\eta_0}{\eta} \right)$$

$$= \frac{H^3}{4\pi^2} (t - t_0) \rightarrow$$

If inflation begins at some time t_0 , what is $\langle \hat{\varphi}^2 \rangle$?

$$k_{\min} = \eta_0^{-1}$$

Size of primordial
Hubble volume

$$k_{\max} = \eta^{-1}$$

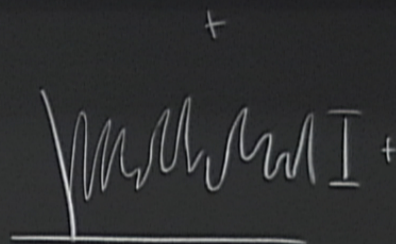
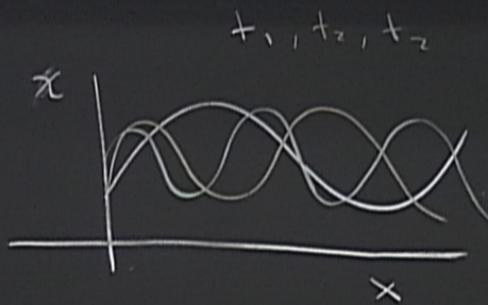
Size of 1 Hubble volume
at η

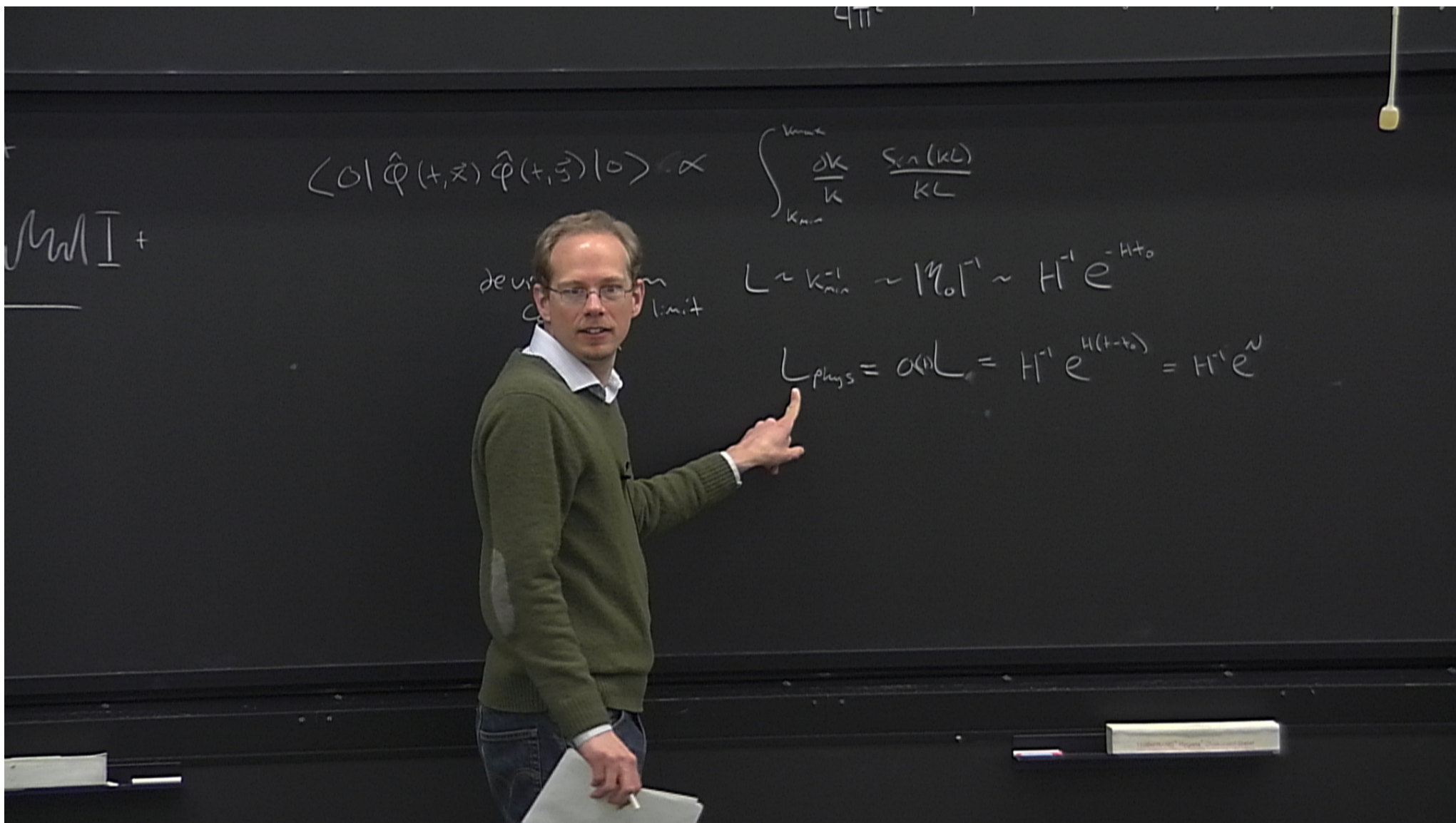
$$\eta = -H^{-1} \dot{e}^{-Ht}$$

$$(ds^2 = -dt^2 + a^2 dx^2)$$

$$\langle 0 | \hat{\varphi}^2 | 0 \rangle = \left(\frac{H}{2\pi} \right)^2 \int_{k_{\min}}^{k_{\max}} \frac{dk}{k} = \left(\frac{H}{2\pi} \right)^2 \log \left(\frac{\eta_0}{\eta} \right)$$

$$= \frac{H^3}{4\pi^2} (t - t_0) \rightarrow \text{long wavelength pert's p. long w.}$$





$\sqrt{M} \bar{I} +$

$$\langle 0 | \hat{\phi}(t, \vec{x}) \hat{\phi}(t, \vec{y}) | 0 \rangle \propto \int_{k_{\min}}^{k_{\max}} \frac{dk}{k} \frac{\sin(kL)}{kL}$$

deviates from
coincident limit
when

$$L \sim k_{\min}^{-1} \sim |H_0|^{-1} \sim H^{-1} e^{-Ht_0}$$

$$L_{\text{phys}} = a(t) L = H^{-1} e^{H(t-t_0)} = H^{-1} e^{\mathcal{N}}$$

Field correlated out to exponentially large physical distances.

$$\langle \hat{\phi}(t, \vec{x}) | 0 \rangle \propto$$

$$\int_{k_{\min}}^{k_{\max}} \frac{dk}{k} \frac{\sin(kL)}{kL}$$

deviates from
coincident limit
when

$$L \sim k_{\min}^{-1} \sim H^{-1} e^{-Ht_0}$$

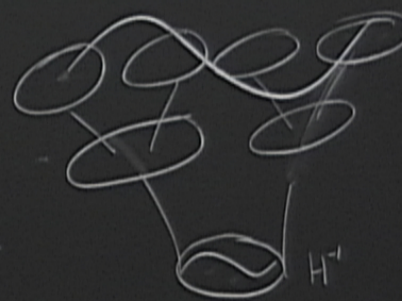
$$L \sim H^{-1} e^{H(t-t_0)} = H^{-1} e^N$$

Field correlated out

large physical distances.

Late- t

volumes share same ancestors



$$\langle \hat{\phi}(t, \vec{x}) | 0 \rangle \propto \int_{k_{\min}}^{k_{\max}} \frac{dk}{k} \frac{\sin(kL)}{kL}$$

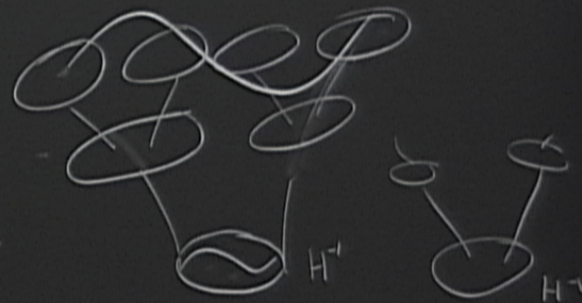
deviates from
coincident limit
when

$$L \sim k_{\min}^{-1} \sim |q_0|^{-1} \sim H^{-1} e^{-Ht_0}$$

$$L_{\text{phys}} = a(t) L = H^{-1} e^{H(t-t_0)} = H^{-1} e^N$$

Field correlated out to exponentially large physical distances.

Late-time hubble volumes share same ancestors



$$\langle \hat{\phi}(t, \vec{x}) | 0 \rangle \propto \int_{k_{\min}}^{k_{\max}} \frac{dk}{k} \frac{\sin(kL)}{kL}$$

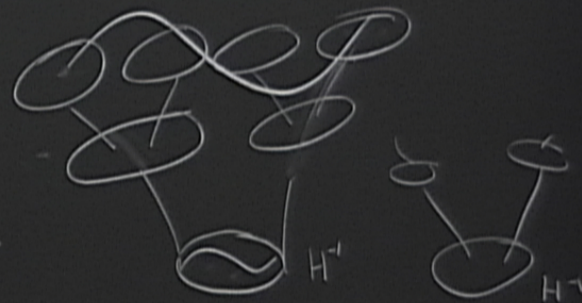
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Field correlated out to exponentially large physical distances.

Late-time hubble volumes share same ancestors



For a massive scalar

$$\langle \phi^2 \rangle_{\text{IR}} = \frac{3H^4}{8\pi^2 M^2} \left[1 - e^{-\frac{2M^2}{3H^2} +} \right]$$

For a massive scalar

$$\langle \phi^2 \rangle_{\text{IR}} = \frac{3H^4}{8\pi^2 M^2} \left[1 - e^{-\frac{2M^2}{3H^2}t} \right]$$

No IR divergence as $t \rightarrow \infty$

For a massive scalar

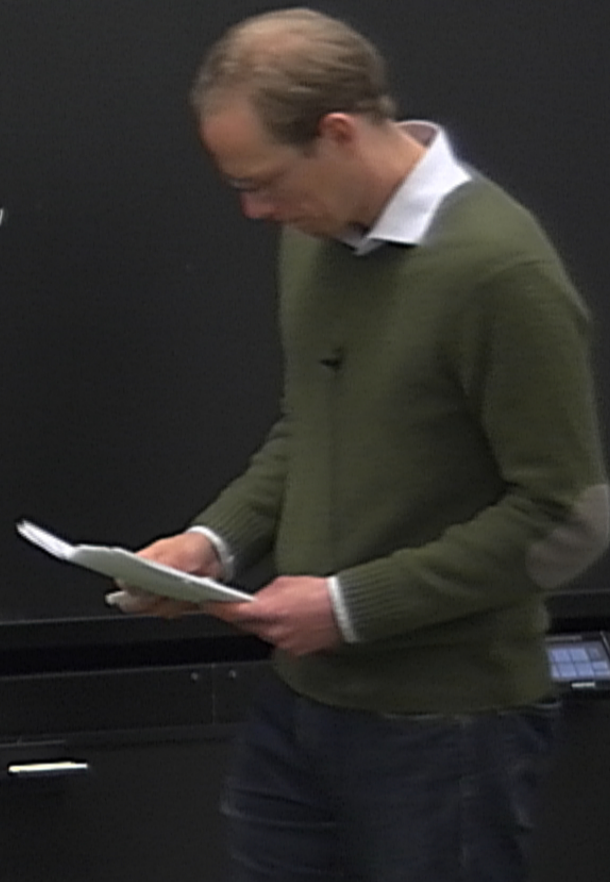
$$\langle \phi^2 \rangle_{\text{IR}} = \frac{3H^4}{8\pi^2 M^2} \left[1 - e^{-\frac{2M^2}{3H^2}t} \right]$$

No IR divergence as $t \rightarrow \infty$

$$V_k \propto |\eta|^{\frac{1}{2}-n}$$

$$\propto |\eta|^{-1+\frac{M^2}{3H^2}} \quad \left(\text{For } \frac{M^2}{H^2} \ll 1 \right)$$

$$n = \sqrt{\frac{9}{4} - \frac{M^2}{H^2}}$$



For a massive scalar

$$\langle \phi^2 \rangle_{\text{IR}} = \frac{3H^4}{8\pi^2 M^2} \left[1 - e^{-\frac{2M^2}{3H^2} t} \right]$$

No IR divergence as $t \rightarrow \infty$

$$V_k \propto |q|^{\frac{1}{2} - n}$$

$$n = \sqrt{\frac{9}{4} - \frac{M^2}{H^2}}$$

$$\propto |q|^{-1 + \frac{M^2}{3H^2}} \quad \left(\text{For } \frac{M^2}{H^2} \ll 1 \right)$$

$\Rightarrow \phi_k \propto |q|^{\frac{M^2}{3H^2}} \rightarrow \phi_k$ decays on Superhorizon Scales

For a massive scalar

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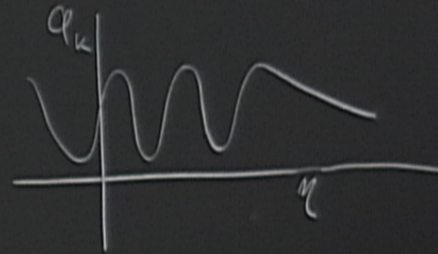
No IR divergence as $t \rightarrow \infty$

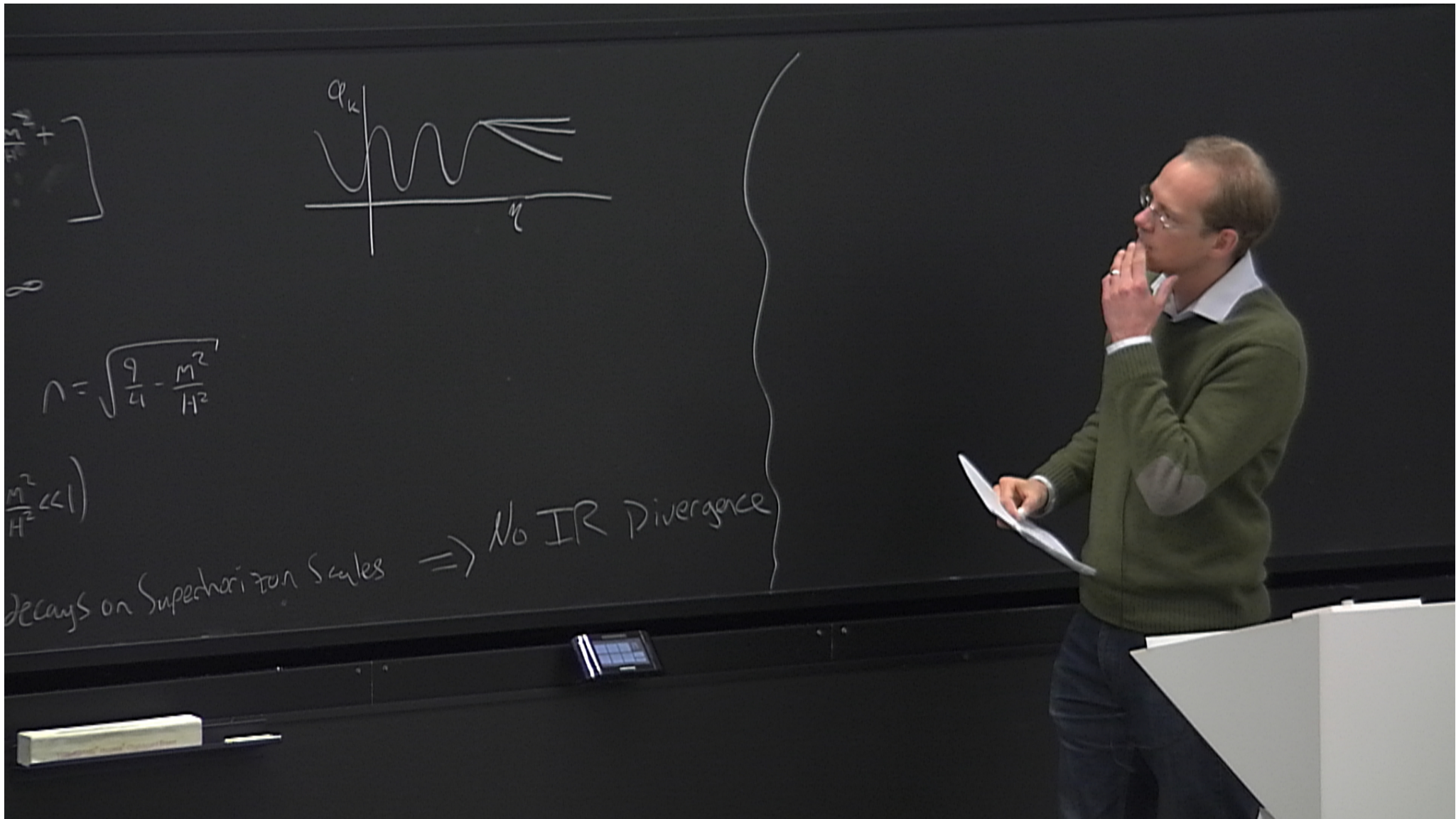
$$V_k \propto |\eta|^{\frac{1}{2} - n}$$

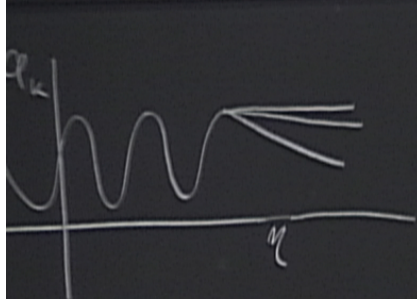
$$n = \sqrt{\frac{9}{4} - \frac{M^2}{H^2}}$$

$$\propto |\eta|^{-1 + \frac{M^2}{3H^2}} \quad \left(\text{For } \frac{M^2}{H^2} \ll 1 \right)$$

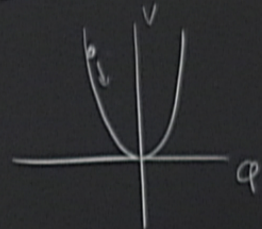
$\Rightarrow \phi_k \propto |\eta|^{\frac{M^2}{3H^2}} \rightarrow \phi_k$ decays on Superhorizon Scales







Inflation: $H \neq \text{constant}$



$$H^2 = \frac{8\pi G}{3} V(\phi)$$

$$\frac{\ddot{a}}{a} \neq \frac{2}{\eta}, \quad a \neq -\frac{1}{H^2}$$

no IR Divergence

$\rightarrow \phi_k \propto 1/k$ $\rightarrow \phi_k$ decays on super

Small k : $\ddot{V}_k - \frac{\ddot{a}}{a} V_k = 0$

(superhorizon)

$$V_k = \underbrace{A a}_{\text{growing}} + \underbrace{B a \int \frac{d\eta}{a^2}}_{\text{decaying}}$$

$\rightarrow \phi_k \propto 1/k \rightarrow \phi_k$ decays on super

Small k : $\ddot{V}_k - \frac{\ddot{a}}{a} V_k = 0$
(superhorizon)

$$V_k = \boxed{Aa} + B a \int \frac{d\eta}{a^2}$$

growing decaying

$\phi_k = \frac{V_k}{a} \rightarrow \phi_k$ still freezes in
on large scales

Amplitude

$\rightarrow \phi_k \propto 1/l \rightarrow \phi_k$ decays on super

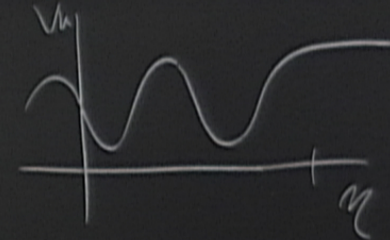
Small k : $\ddot{V}_k - \frac{\ddot{a}}{a} V_k = 0$
(superhorizon)

$$V_k = \boxed{Aa} + B a \int \frac{d\eta}{a^2}$$

growing decaying

$\phi_k = \frac{V_k}{a} \rightarrow \phi_k$ still freezes in on large scales

Amplitude at Freeze-in



$\rightarrow \phi_k \propto 1/k$ $\rightarrow \phi_k$ decays on super

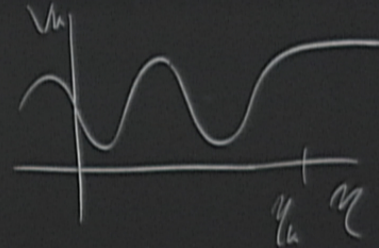
Small k : $\ddot{V}_k - \frac{\ddot{a}}{a} V_k = 0$

(super-horizon)

$$V_k = \underbrace{A a}_{\text{growing}} + \underbrace{B a \int \frac{d\eta}{a^2}}_{\text{decaying}}$$

$\phi_k = \frac{V_k}{a} \rightarrow \phi_k$ still freezes in on large scales

Amplitude at Freeze-in

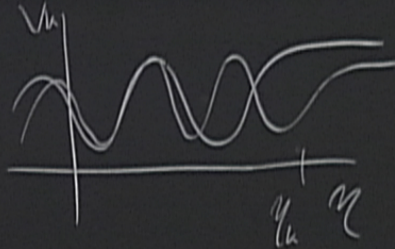


Horizon-crossing $k = a_k H_k$

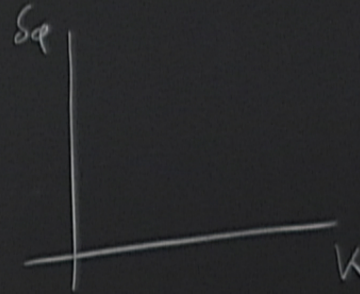
$$\frac{1}{\sqrt{k}} = A a_k \Rightarrow A = \frac{H_k}{k^{3/2}}$$

$$\Rightarrow S_\phi = \frac{H}{2}$$

Amplitude at Freeze-in



$$\Rightarrow S_q = \frac{H_k}{2\pi} \leftarrow H_k \text{ evaluated at horizon crossing}$$



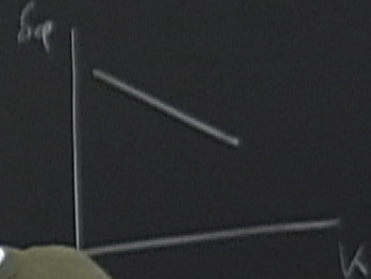
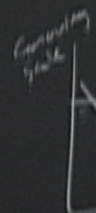
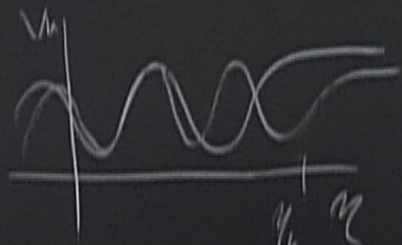
Horizon-crossing $k = a_k H_k$

$$\frac{1}{\sqrt{k}} = A a_k \Rightarrow A = \frac{H_k}{k^{3/2}}$$

Amplitude at Freeze-in

$$\Rightarrow \delta q = \frac{H_u}{2\pi} \leftarrow H_u \text{ evaluated at horizon crossing}$$

δq has red "tilt"



Horizon crossing $k = aH$

$$\frac{1}{\sqrt{k}} = A A_u \Rightarrow A = \frac{H_u}{k^{3/2}}$$

For a massive field, another correction

For a massive field, another correction

$$|\varphi_k| = |\eta|^{m^2/3H^2} \quad (m^2 \ll H^2)$$

$$\varphi = \frac{H_k}{2\pi} \left(\frac{\varphi_k^{\text{end}}}{\varphi_k^{H_k}} \right)$$

$$= \frac{H_k}{(2\pi)} \left(\frac{\eta_F}{\eta_k} \right)^{m^2/3H^2}$$

$$= \frac{H_k}{(2\pi)} \left(\frac{k}{H_{\text{end}} q_{\text{end}}} \right)^{m^2/3H^2}$$

$$\eta = -\frac{1}{aH}, \quad \eta_F = \frac{1}{a_{\text{end}} H_{\text{end}}}$$

$$\eta_k = k^{-1}$$

Before we had 3