Title: 12/13 PSI - Explorations in Cosmology Lecture 12

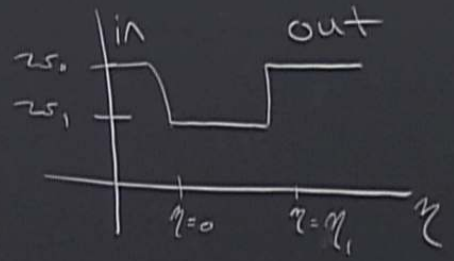
Date: Apr 23, 2013 10:15 AM

URL: http://pirsa.org/13040077

Abstract:

Recap

Particle production



$\omega^2 = k^2 + M^2_{eff}$ ← $M^2 a^2 - \frac{\ddot{a}}{a}$

this is what changed with η

$$\ddot{V}_k + \omega_k^2 V_k = 0$$

① Choose vacuum:

$$V_k^{(in)} = \frac{1}{\sqrt{2\omega_0}} e^{i\omega_0 \eta}, \quad \hat{a}_k |0\rangle_{(in)} = 0$$

② Evolve past η_1, require V_k and $\dot{V}_k$ are continuous

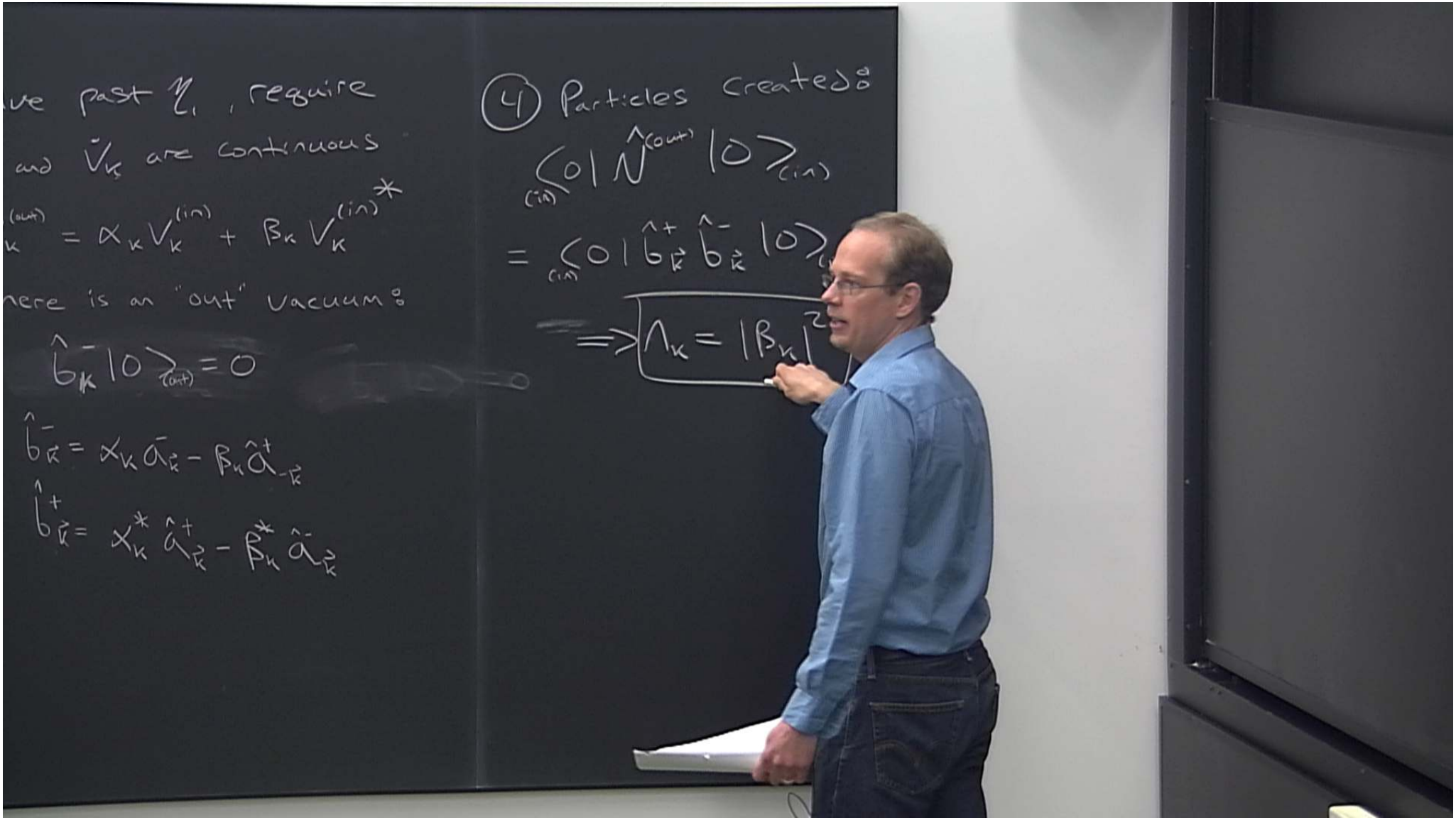
$$V_k^{(out)} = \alpha_k V_k^{(in)} + \beta_k V_k^{(in)*}$$

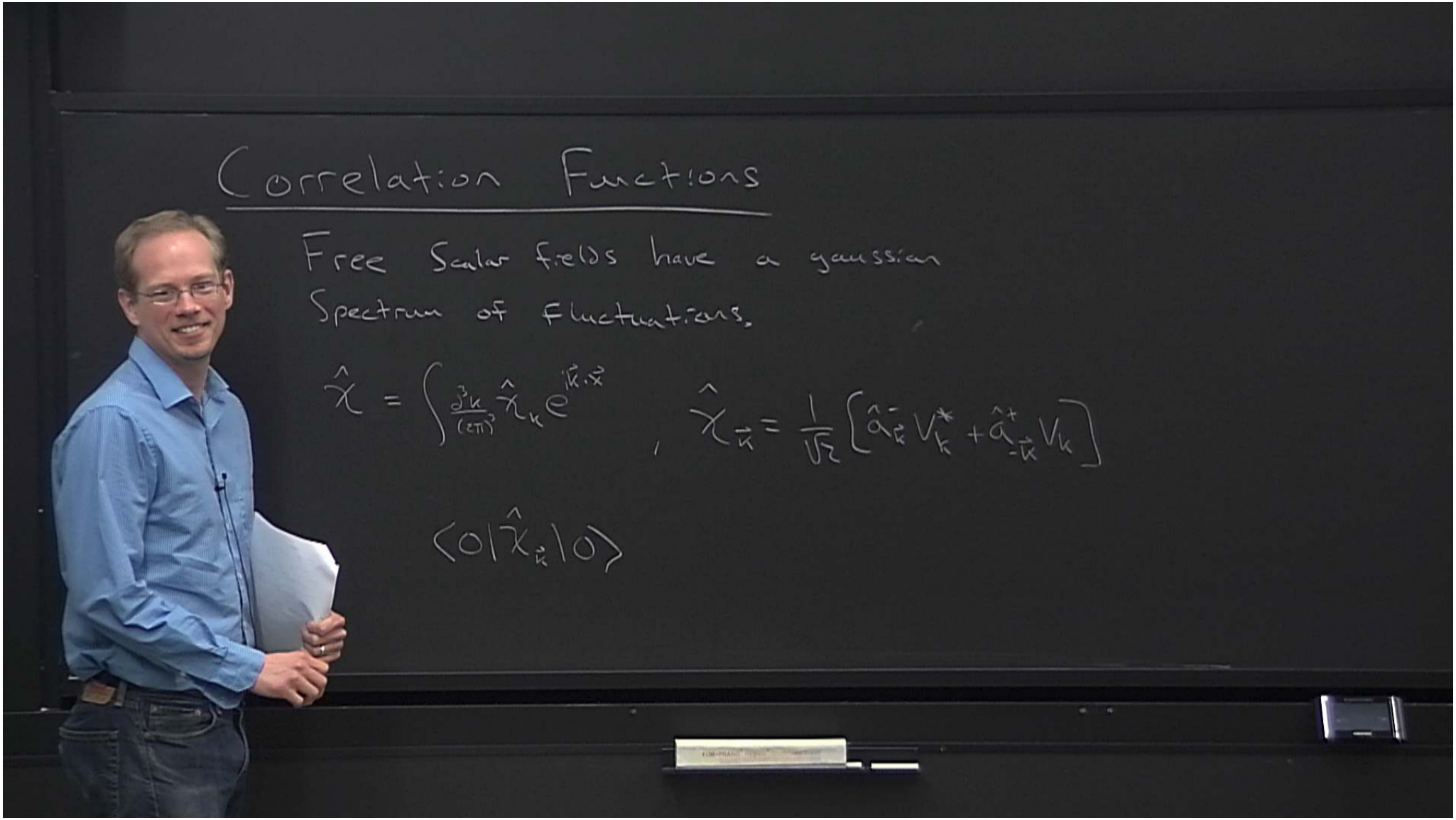
③ There is an "out" vacuum:

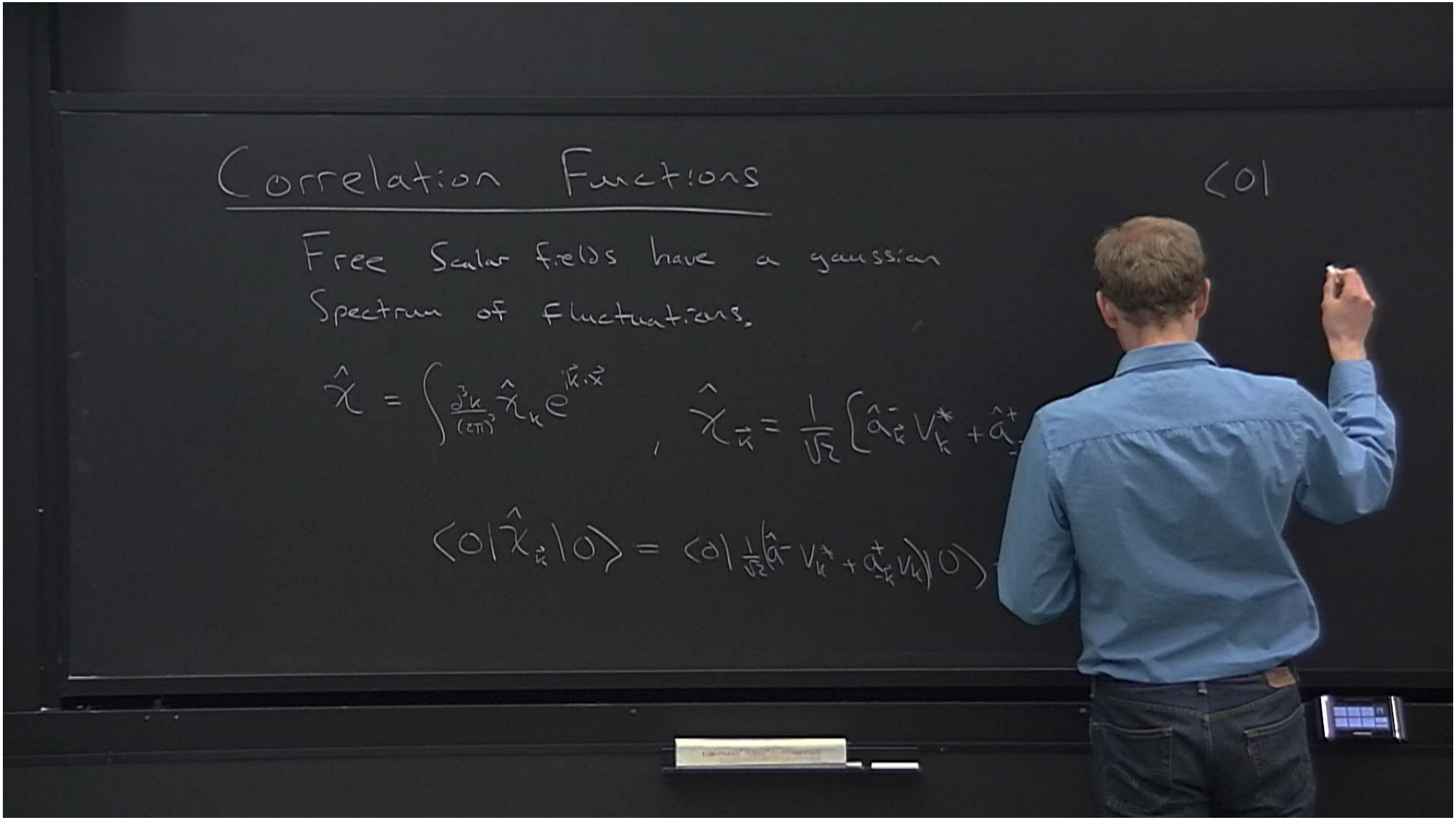
$$\hat{b}_k |0\rangle_{(out)} = 0$$

$$\hat{b}_{\vec{k}} = \alpha_k \hat{a}_{\vec{k}} - \beta_k \hat{a}^\dagger_{-\vec{k}}$$

$$\hat{b}^\dagger_{\vec{k}} = \alpha_k^* \hat{a}^\dagger_{\vec{k}} - \beta_k^* \hat{a}_{-\vec{k}}$$


ve past η_1, require
and V_k are continuous
$= \alpha_k V_k^{(in)} + \beta_k V_k^{(in)*}$
here is an "out" vacuum:
$\hat{b}_k^- |0\rangle_{(out)} = 0$
$\hat{b}_{\vec{k}}^- = \alpha_k \hat{a}_{\vec{k}}^- - \beta_k \hat{a}_{-\vec{k}}^+$
$\hat{b}_{\vec{k}}^+ = \alpha_k^* \hat{a}_{\vec{k}}^+ - \beta_k^* \hat{a}_{\vec{k}}^-$
(4) Particles created:
$\langle 0 | \hat{N}^{(out)} | 0 \rangle_{(in)}$
$= \langle 0 | \hat{b}_{\vec{k}}^+ \hat{b}_{\vec{k}}^- | 0 \rangle$
$\Rightarrow \Lambda_k = |\beta_k|^2$


Correlation Functions
Free Scalar fields have a gaussian
Spectrum of fluctuations.
$\hat{\chi} = \int \frac{d^3k}{(2\pi)^3} \hat{\chi}_k e^{i\vec{k}\cdot\vec{x}}$
$\hat{\chi}_{\vec{k}} = \frac{1}{\sqrt{2}}\left[\hat{a}^-_{\vec{k}} v_k^* + \hat{a}^+_{-\vec{k}} v_k\right]$
$\langle 0|\hat{\chi}_{\vec{k}}|0\rangle$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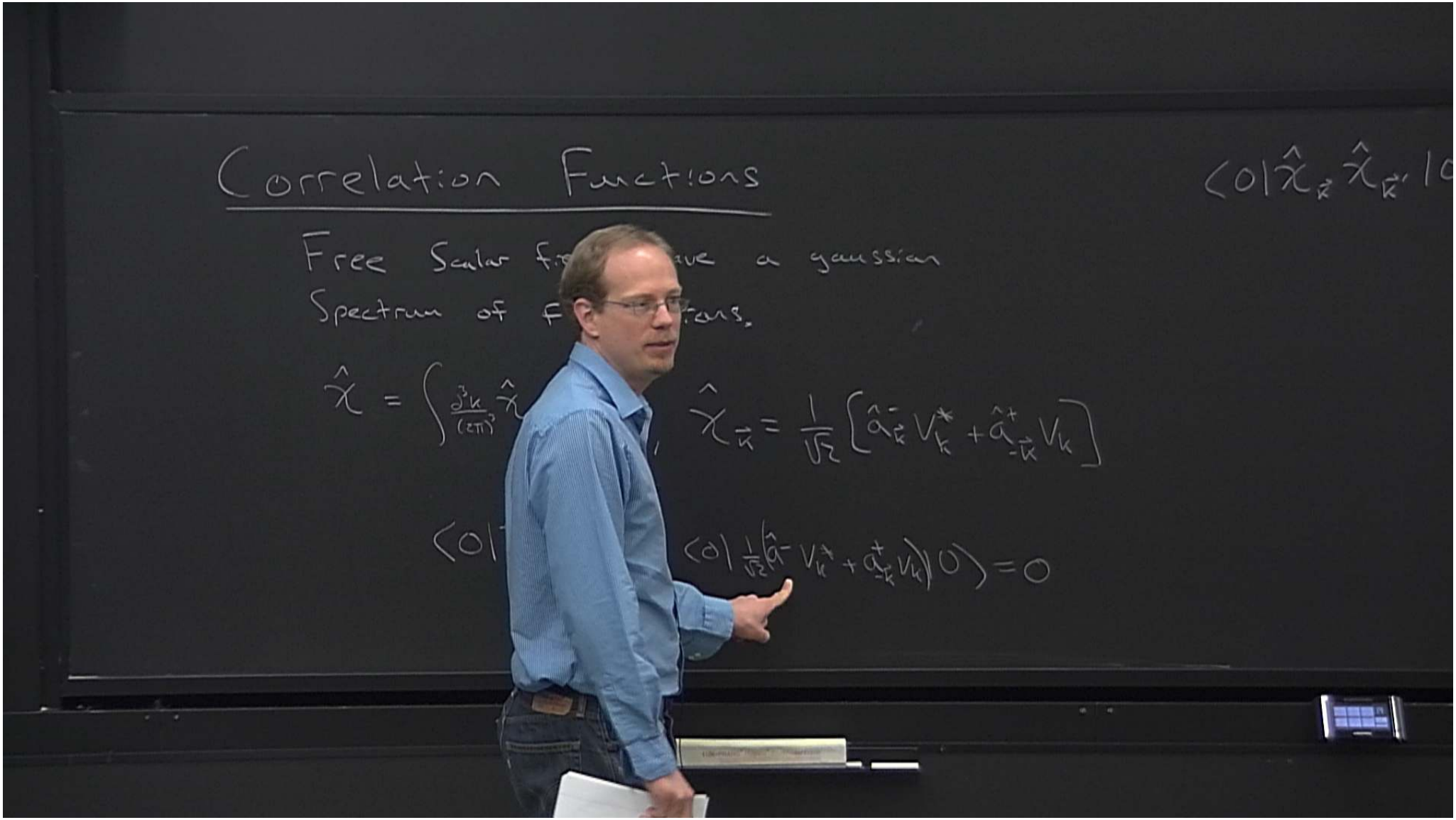
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Free Scalar f
ave a gaussian
Spectrum of f
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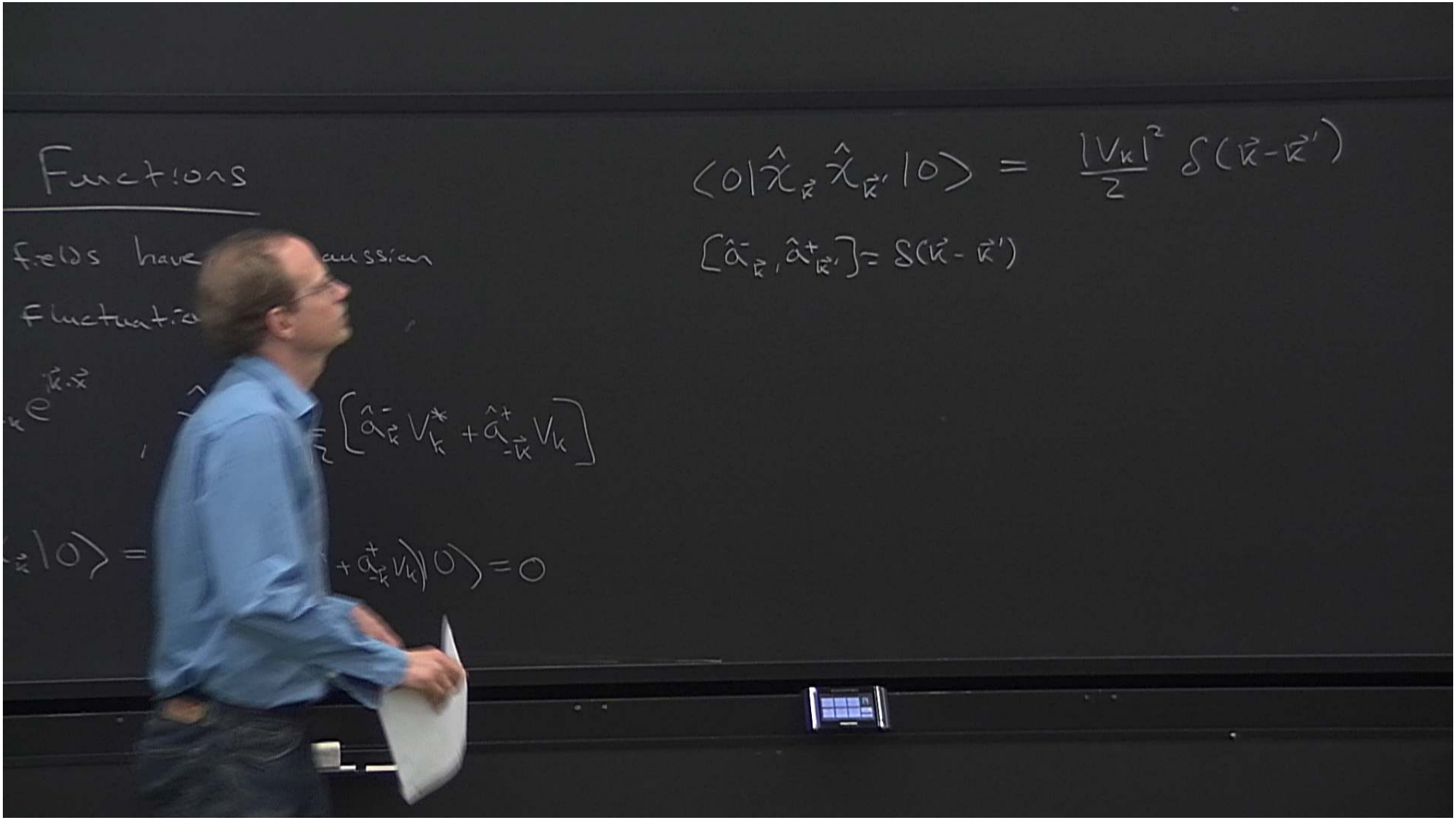
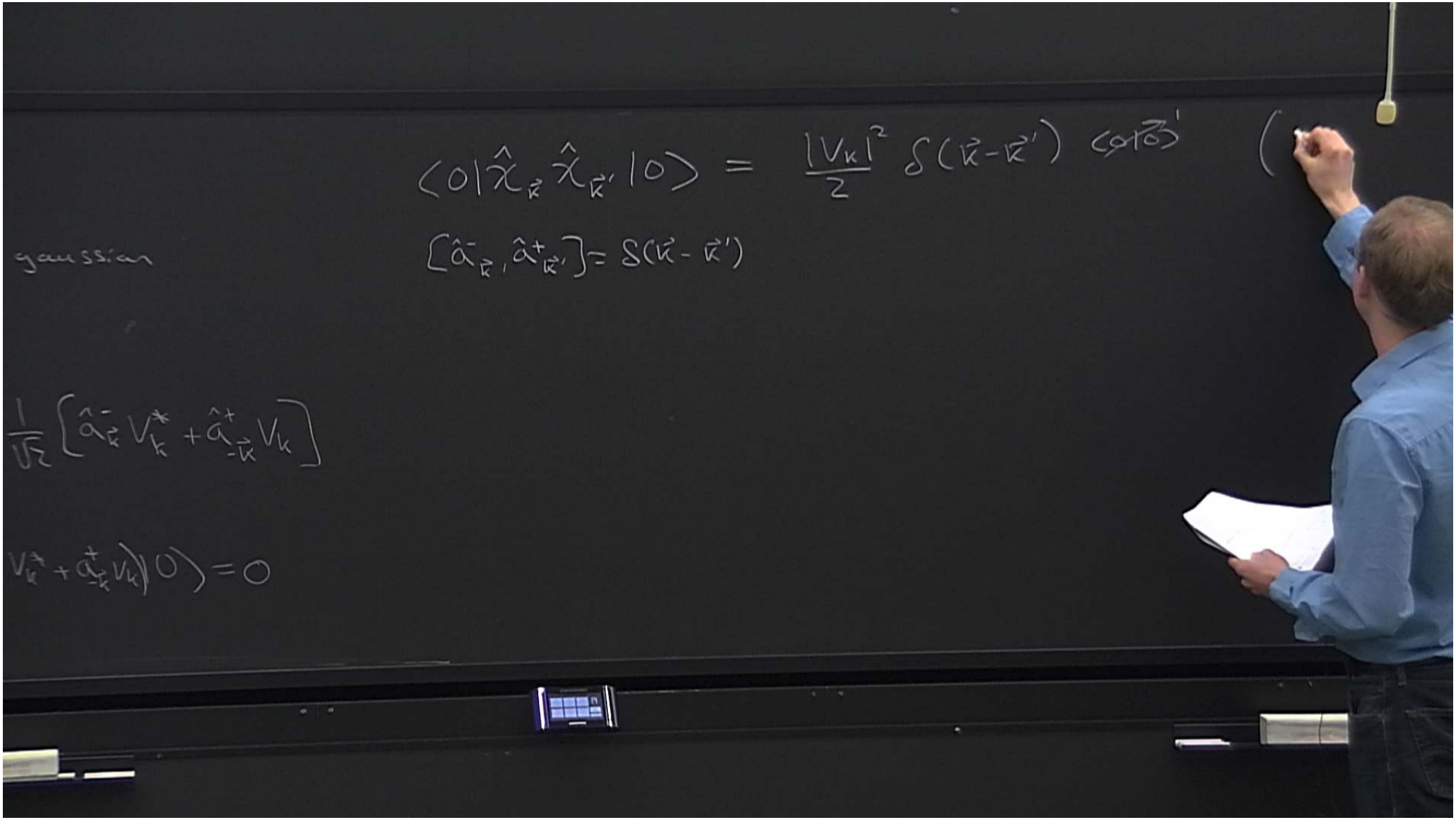
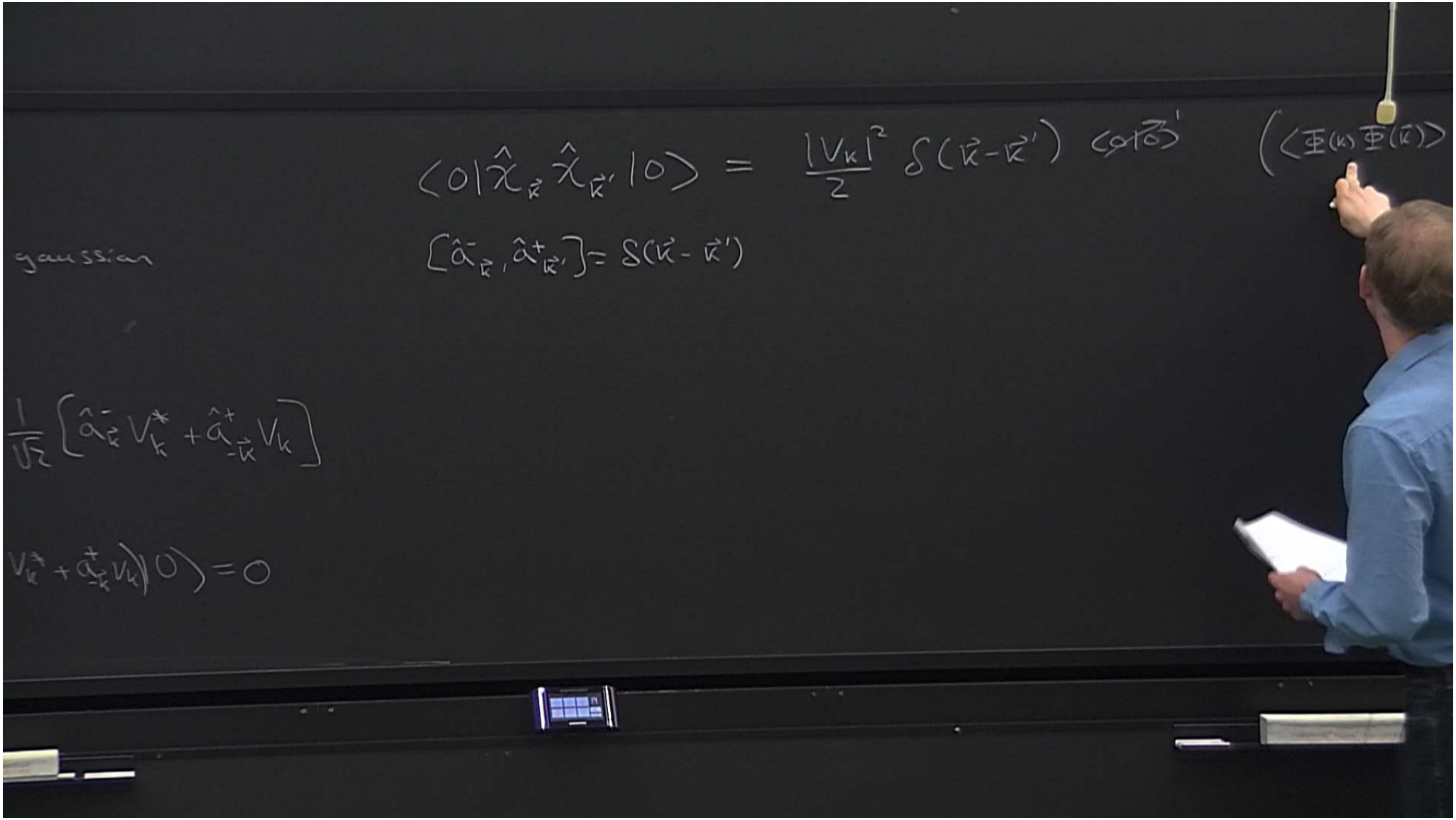
$\hat{\chi} = \int \frac{d^3k}{(2\pi)^3} \hat{\chi}$
$\hat{\chi}_{\vec{k}} = \frac{1}{\sqrt{2}} \left[\hat{a}^-_{\vec{k}} v^*_k + \hat{a}^+_{-\vec{k}} v_k \right]$
$\langle 0 | \hat{\chi}_{\vec{k}} \hat{\chi}_{\vec{k}'} | 0$
$\langle 0 | \frac{1}{\sqrt{2}} (\hat{a}^- v^*_k + a^+_{-k} v_k) | 0 \rangle = 0$


Functions
$\langle 0|\hat{x}_{\vec{k}}\hat{x}_{\vec{k}'}|0\rangle = \frac{|V_k|^2}{2}\delta(\vec{k}-\vec{k}')$
$[\hat{a}^-_{\vec{k}}, \hat{a}^+_{\vec{k}'}] = \delta(\vec{k}-\vec{k}')$


$\langle 0|\hat{x}_{\vec{k}}\,\hat{x}_{\vec{k}'}|0\rangle = \frac{|V_k|^2}{2}\,\delta(\vec{k}-\vec{k}')$ cartoon
gaussian
$[\hat{a}^-_{\vec{k}}, \hat{a}^+_{\vec{k}'}] = \delta(\vec{k}-\vec{k}')$
$\frac{1}{\sqrt{2}}\left[\hat{a}^-_{\vec{k}} V^*_k + \hat{a}^+_{-\vec{k}} V_k\right]$
$V^*_k + \hat{a}^+_{-k} V_k)|0\rangle = 0$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gaussian
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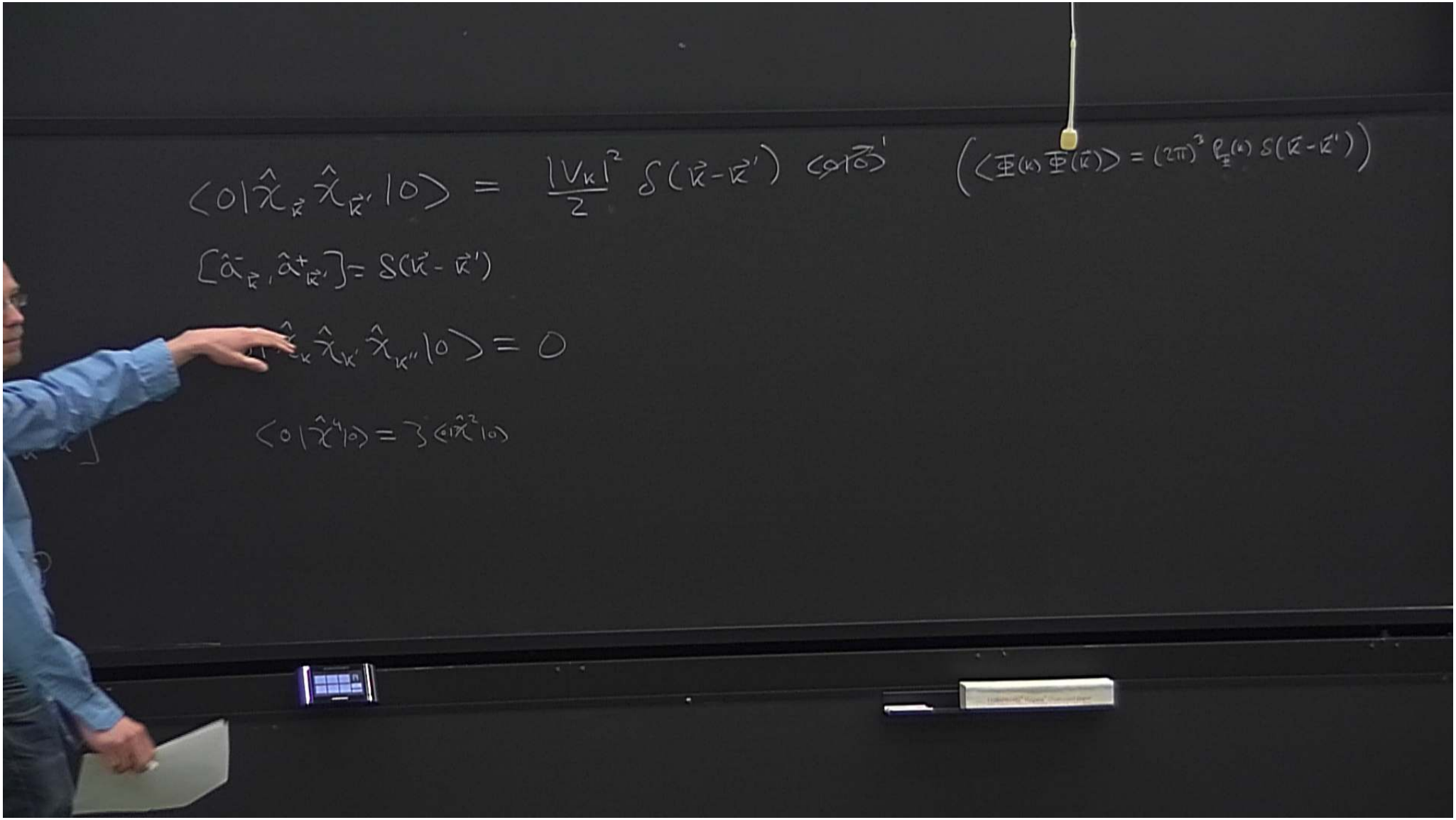
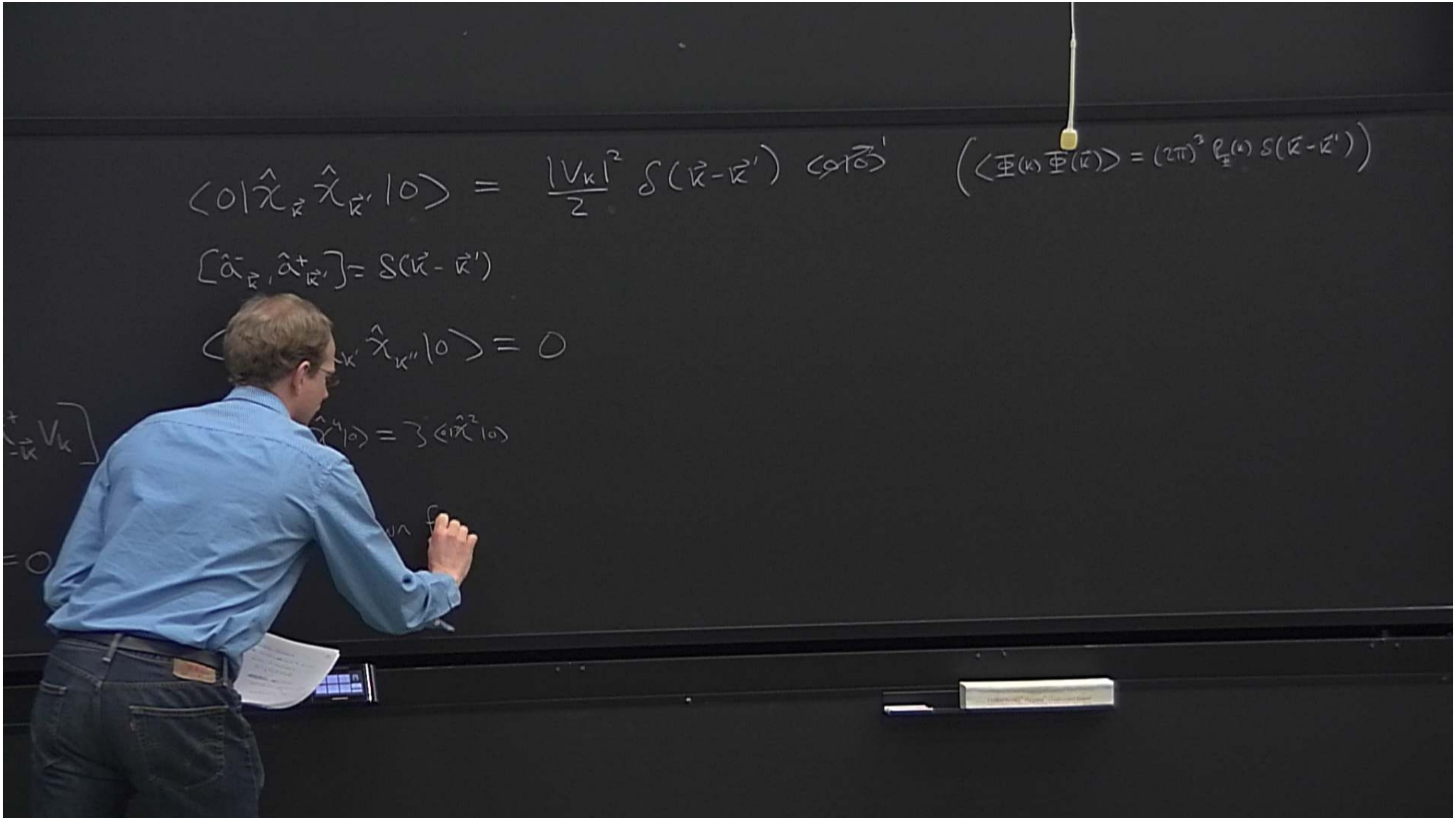
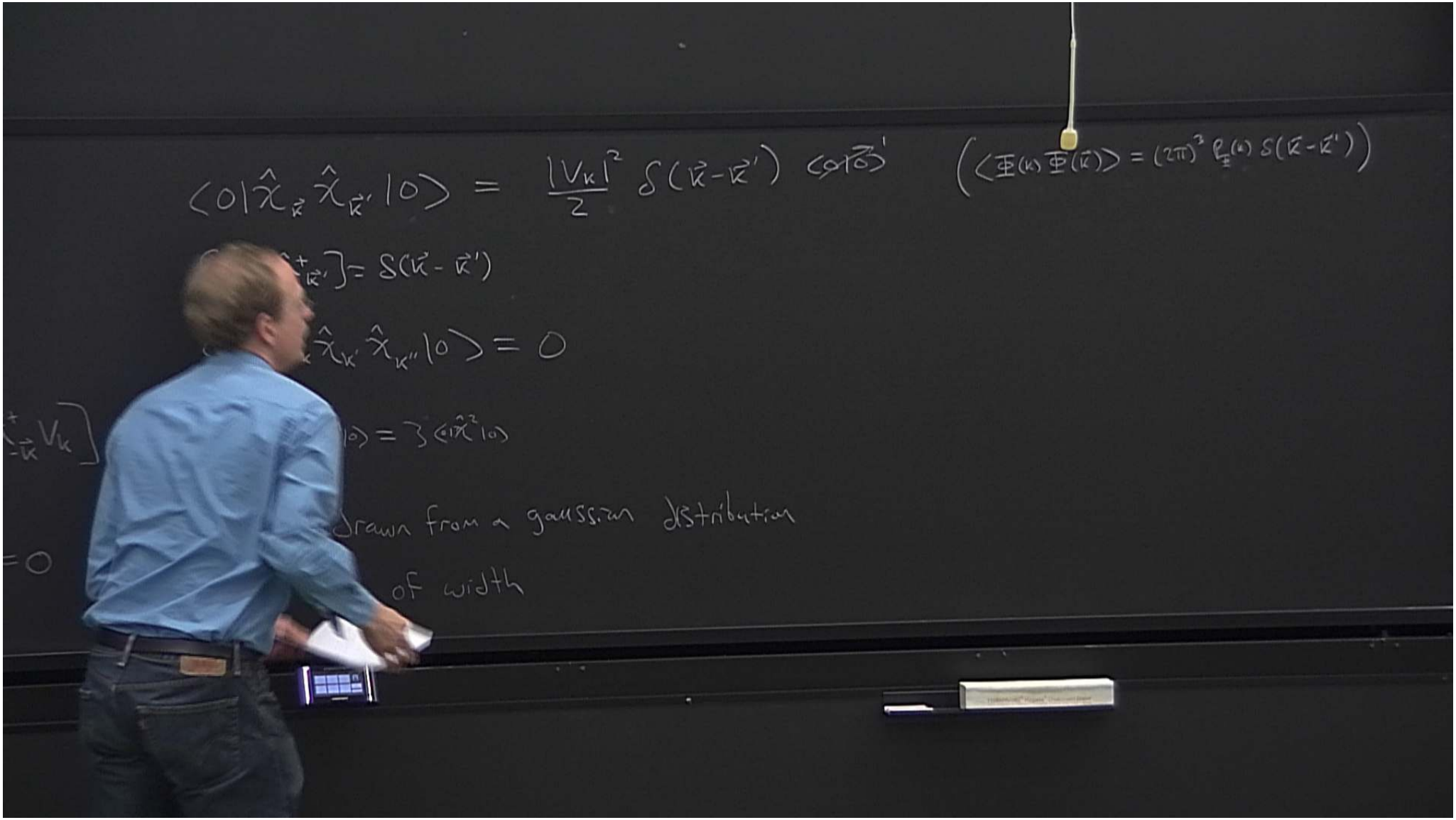
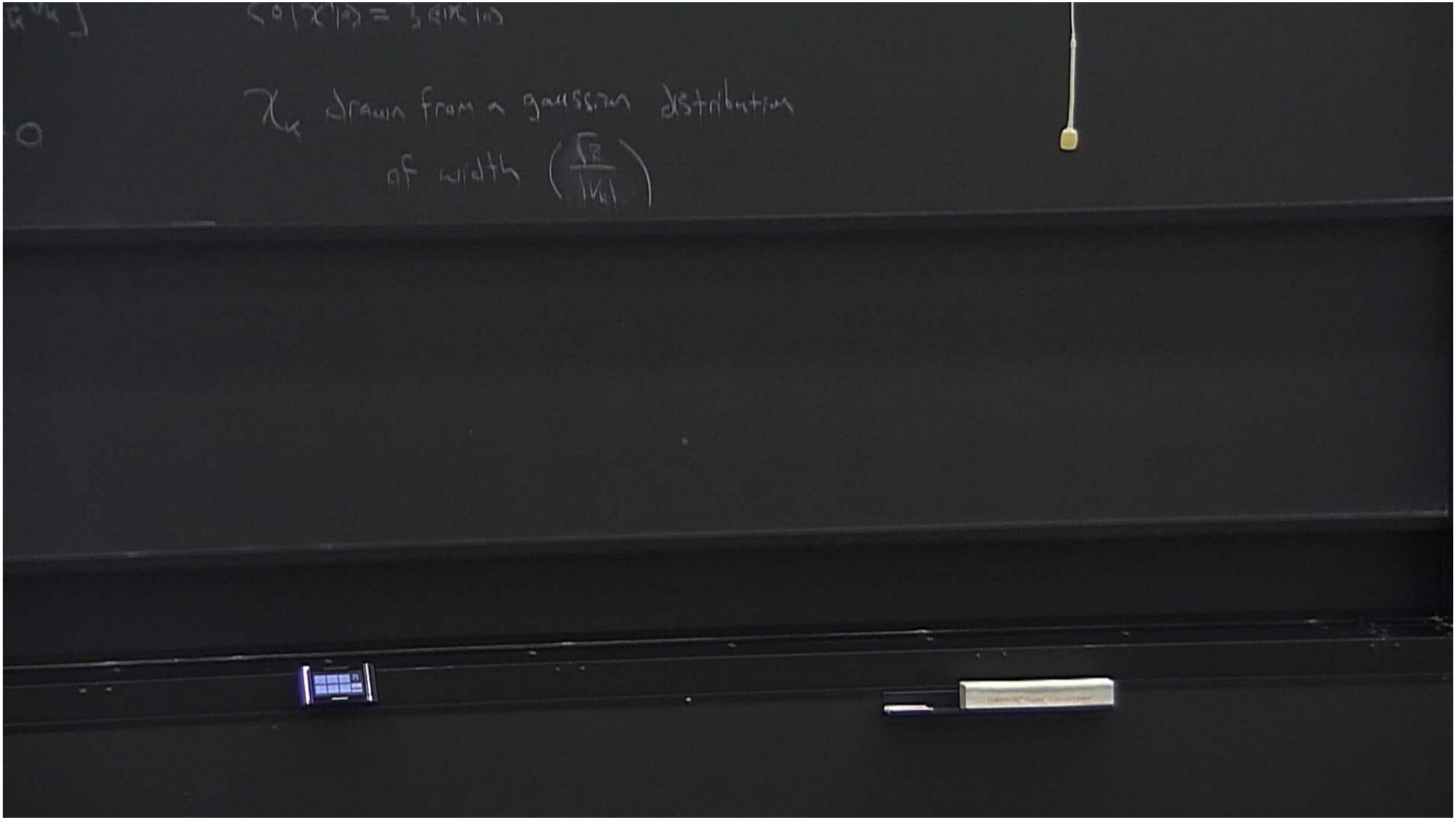
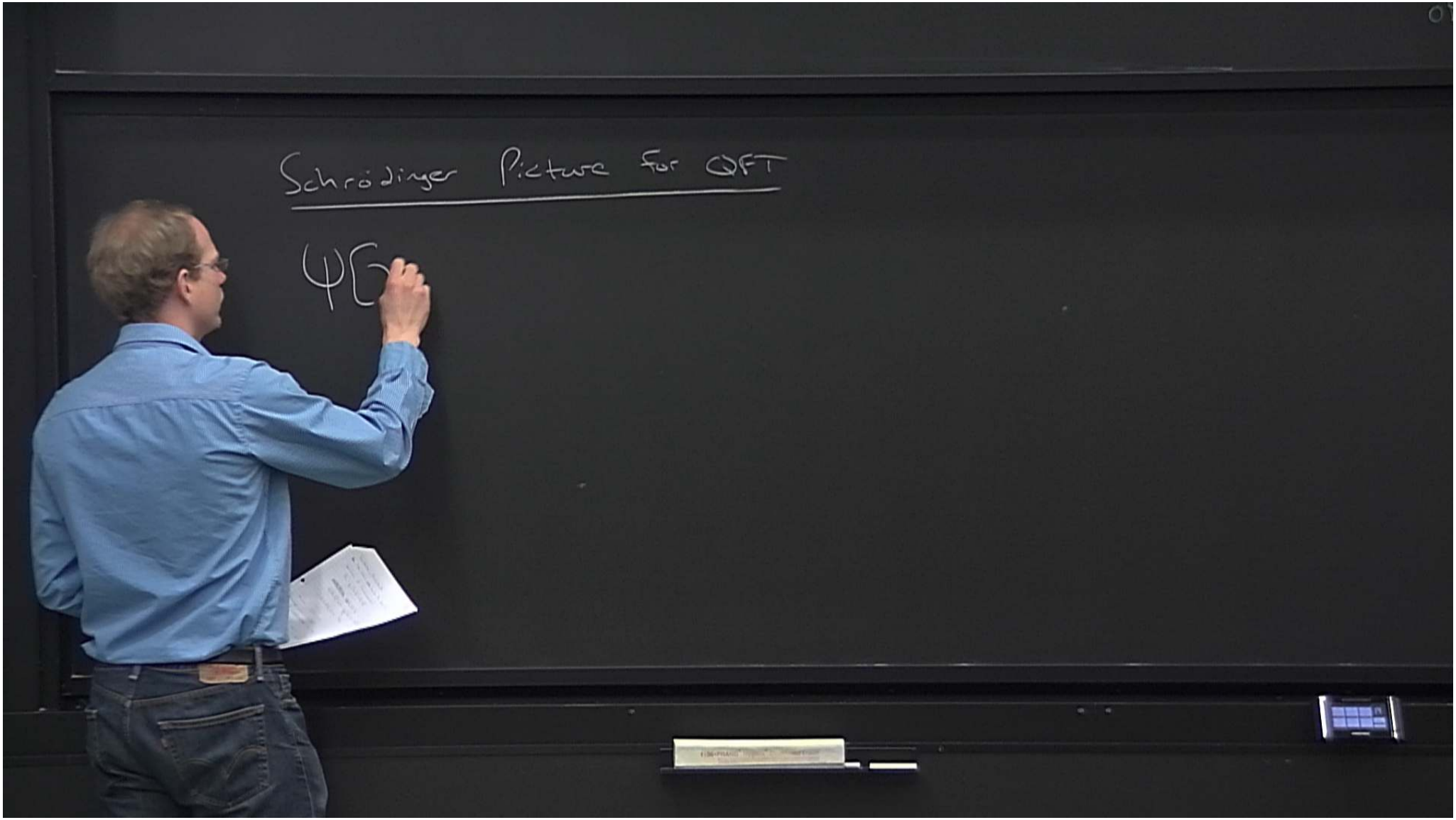
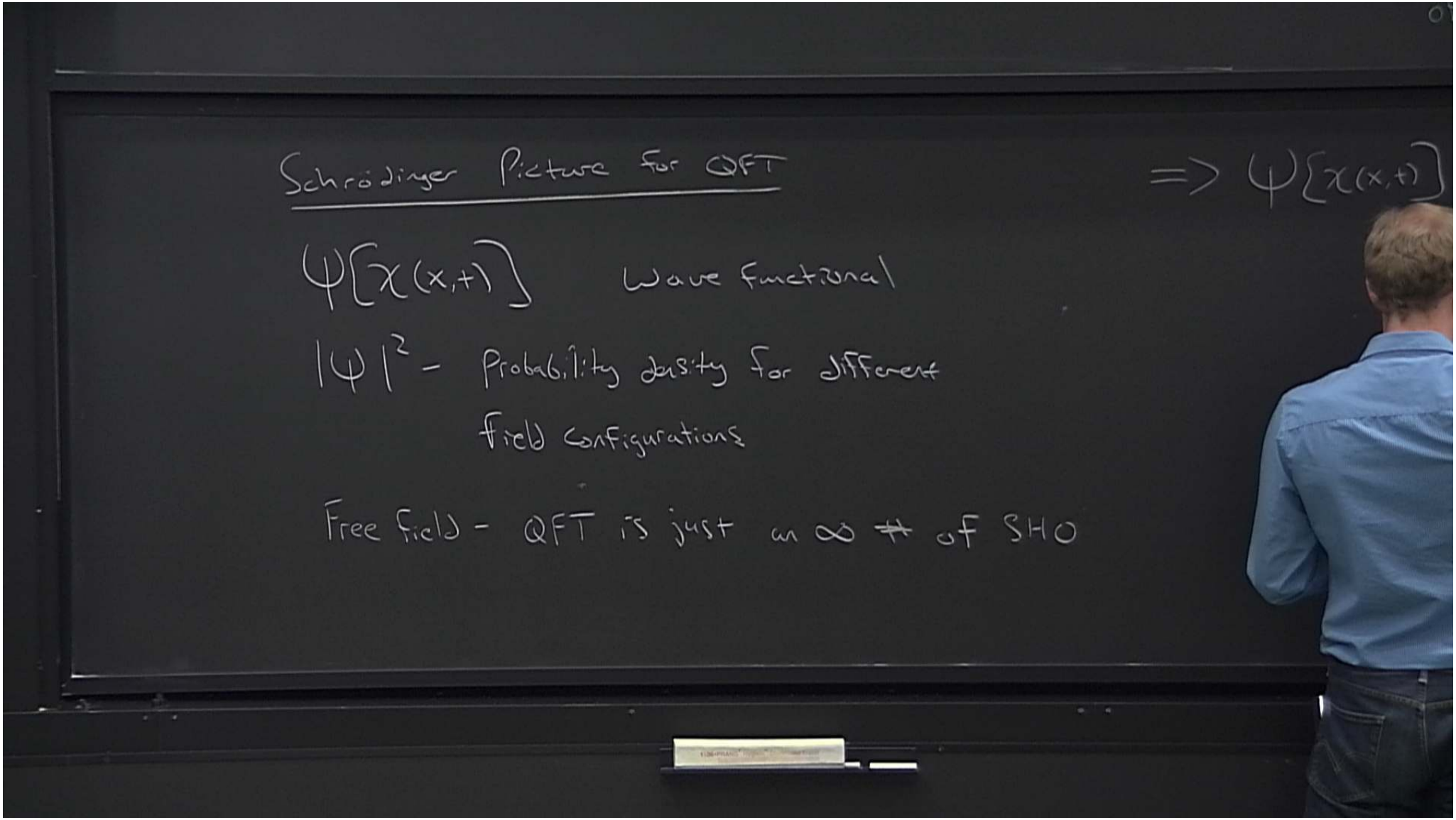

$\langle 0|\hat{\chi}_{\vec{k}}\hat{\chi}_{\vec{k}'}|0\rangle = \frac{|V_k|^2}{2}\delta(\vec{k}-\vec{k}')$ (a la BD)
$\left(\langle \Phi(k)\Phi(\vec{k})\rangle = (2\pi)^3 P_\Phi(k)\,\delta(\vec{k}-\vec{k}')\right)$
$[\hat{a}_{\vec{k}}, \hat{a}^+_{\vec{k}'}] = \delta(\vec{k}-\vec{k}')$
$\langle 0|\hat{\chi}_{\vec{k}}\hat{\chi}_{\vec{k}'}\hat{\chi}_{\vec{k}''}|0\rangle = 0$
$\langle 0|\hat{\chi}^4|0\rangle = 3\langle 0|\hat{\chi}^2|0\rangle$


$\langle 0|\hat{\chi}_{\vec{k}}\hat{\chi}_{\vec{k}'}|0\rangle = \frac{|V_k|^2}{2}\delta(\vec{k}-\vec{k}')$
$\left(\langle \Phi(\vec{k})\Phi(\vec{k})\rangle = (2\pi)^3 P_\Phi(k)\,\delta(\vec{k}-\vec{k}')\right)$
$[\hat{a}^-_{\vec{k}}, \hat{a}^+_{\vec{k}'}] = \delta(\vec{k}-\vec{k}')$
$\hat{\chi}_{k''}|0\rangle = 0$
$= 3\langle 0|\hat{\chi}^2|0\rangle$


$\langle 0|\hat{x}_{\vec{k}}\hat{x}_{\vec{k}'}|0\rangle = \frac{|V_k|^2}{2}\delta(\vec{k}-\vec{k}')$ (sort of)
$\left(\langle \Phi(\vec{k})\Phi(\vec{k})\rangle = (2\pi)^3 P_\Phi(k)\,\delta(\vec{k}-\vec{k}')\right)$
$= \delta(\vec{k}-\vec{k}')$
$= 0$
$= 3\langle 0|\hat{x}^2|0\rangle$
drawn from a gaussian distribution
of width


χ_k drawn from a gaussian distribution
of width $\left(\frac{\sqrt{2}}{|k|}\right)$

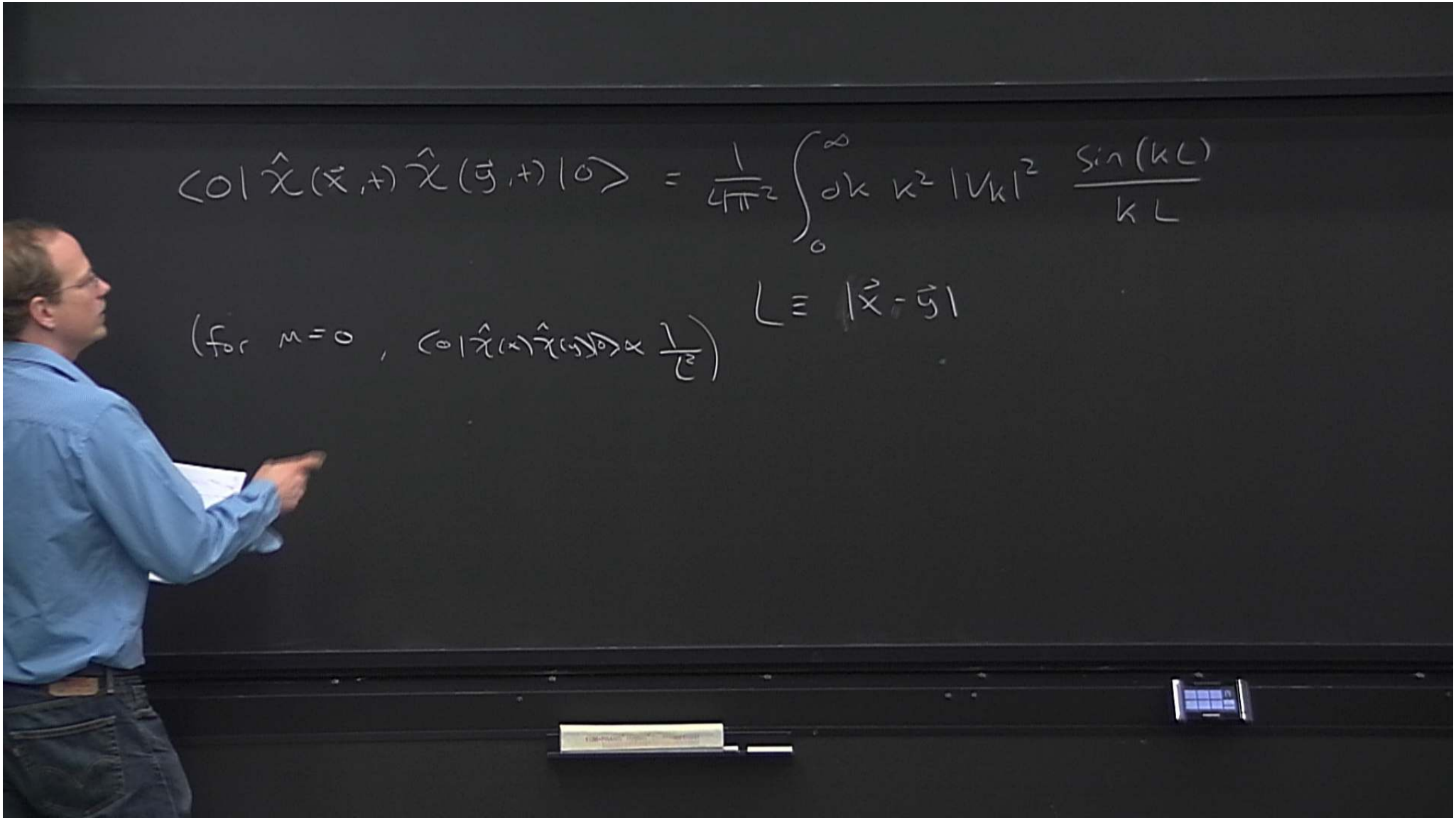

Schrödinger Picture for QFT


Schrödinger Picture for QFT
=> ψ[χ(x,t)]
ψ[χ(x,t)] Wave functional
|ψ|² - Probability density for different field configurations
Free field - QFT is just an ∞ # of SHO

of width $\left(\frac{1}{|k_i|}\right)$

$$\Rightarrow \psi[x(x,t)] = \prod_k e^{-|x_k|^2/2\sigma_k^2}$$

of SHO


$\langle 0|\hat{\chi}(\vec{x},t)\hat{\chi}(\vec{y},t)|0\rangle = \frac{1}{4\pi^2}\int_0^\infty dk\, k^2 |V_k|^2 \frac{\sin(kL)}{kL}$
$L \equiv |\vec{x}-\vec{y}|$
(for $m=0$, $\langle 0|\hat{\chi}(x)\hat{\chi}(y)|0\rangle \propto \frac{1}{L^2}$)

$$\langle 0|\hat{\chi}(\vec{x},t)\hat{\chi}^{*}(\vec{y},t)|0\rangle = \frac{1}{4\pi^2}\int_0^\infty dk\, k^2 |V_k|^2 \frac{\sin(kL)}{kL}$$

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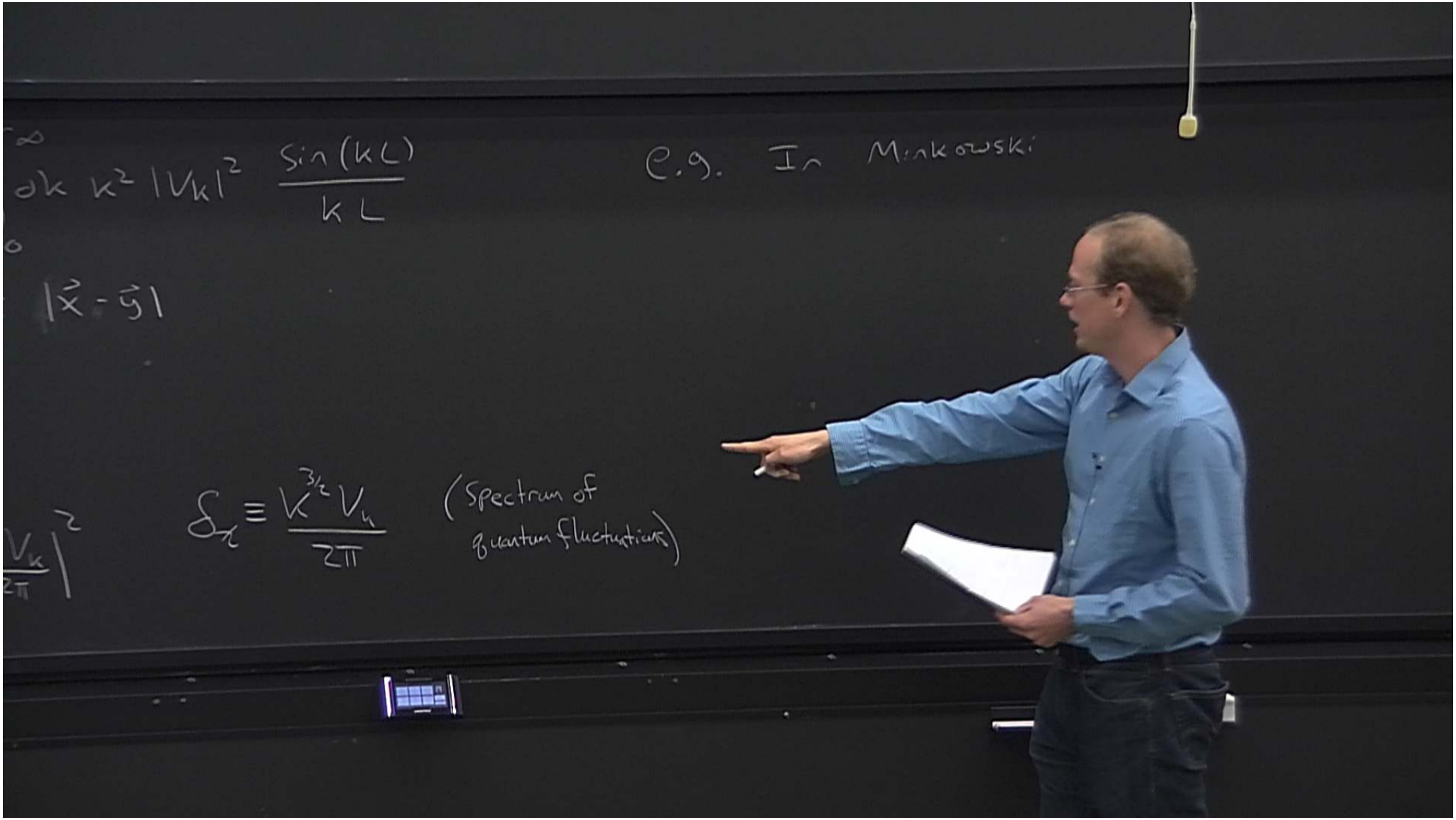
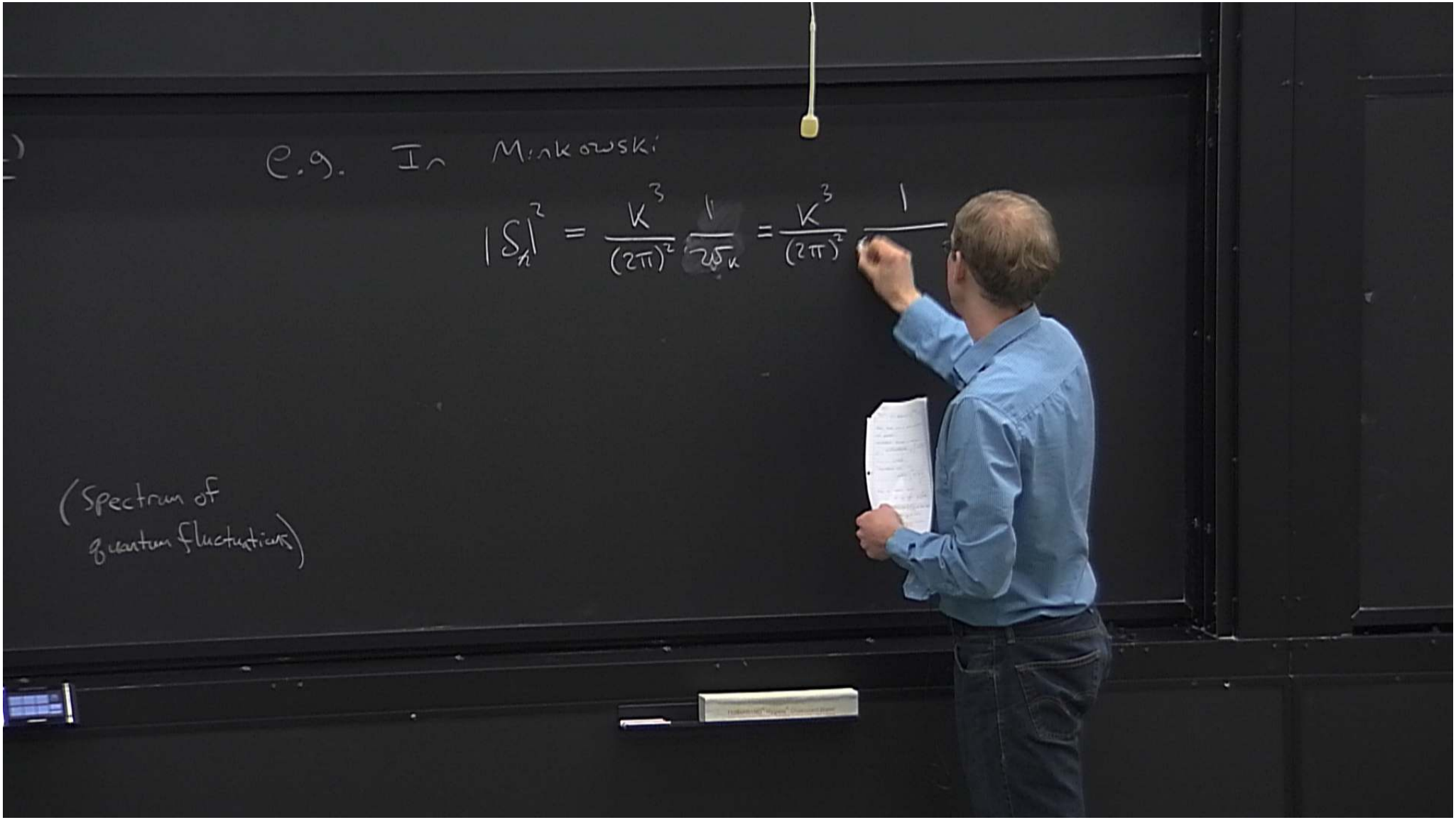
Coincidence limit ($L=0$)

$$\langle 0|\hat{\chi}(x,t)^2|0\rangle = \int_0^\infty \frac{dk}{k}\left|\frac{k^{3/2}V_k}{2\pi}\right|^2$$


$\int_0^\infty dk\, k^2 |V_k|^2 \frac{\sin(kL)}{kL}$
e.g. In Minkowski
$|\vec{x} - \vec{y}|$
$\left|\frac{V_k}{2\pi}\right|^2$
$\delta_\chi \equiv \frac{k^{3/2} V_k}{2\pi}$
(Spectrum of quantum fluctuations)


e.g. In Minkowski
$|\delta_k|^2 = \frac{k^3}{(2\pi)^2}\frac{1}{2\omega_k} = \frac{k^3}{(2\pi)^2}\frac{1}{}$
(Spectrum of
quantum fluctuations)

e.g. In Minkowski

$$|\delta_k|^2 = \frac{k^3}{(2\pi)^2}\frac{1}{2\omega_k} = \frac{k^3}{(2\pi)^2}\frac{1}{(k^2+M^2)^{1/2}}$$

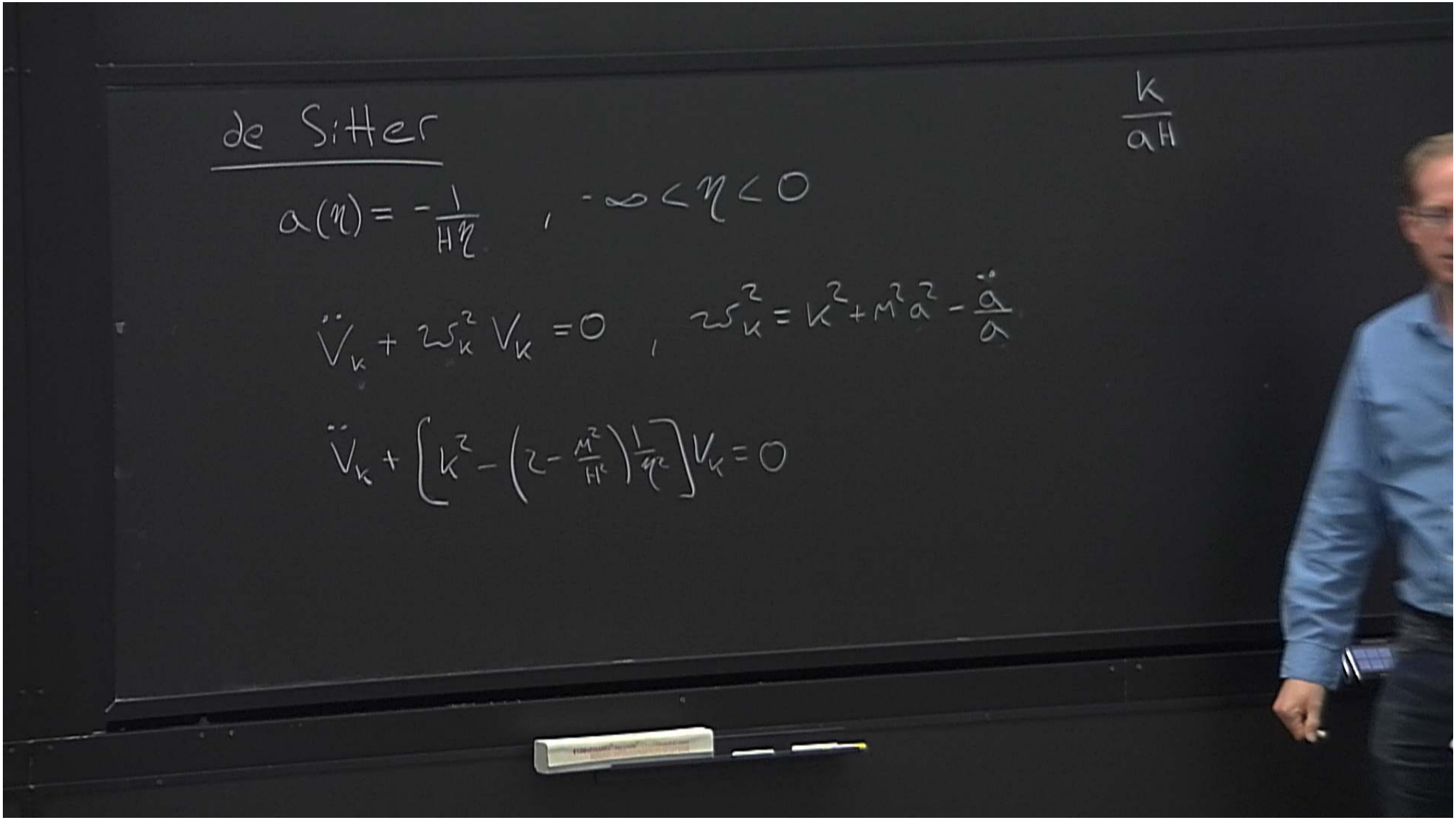
fluct. small on large scales

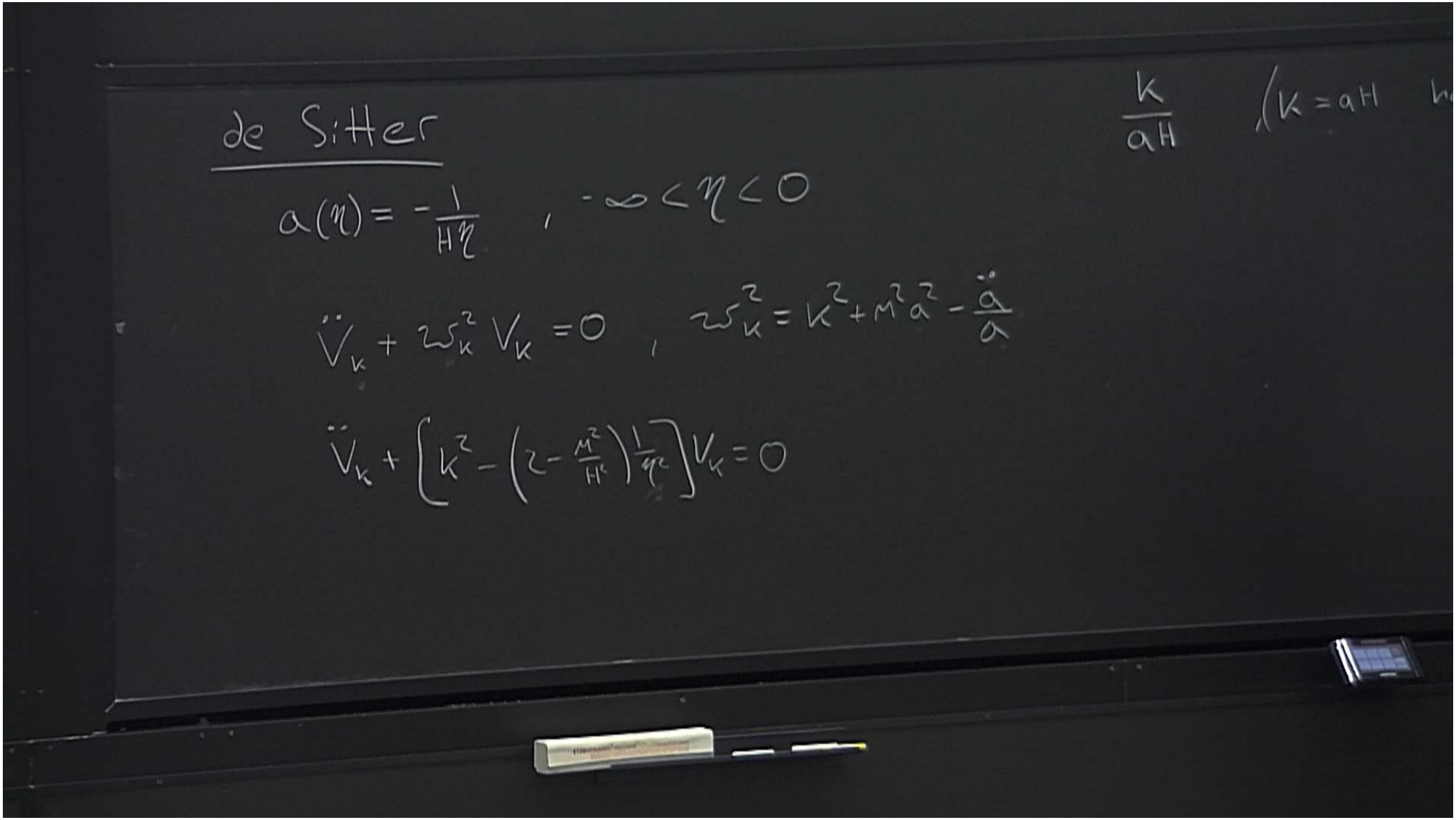
" "large" " small" "

$$\langle 0|\hat{\chi}(\vec{x},t)\hat{\chi}^\dagger(\vec{y},t)|0\rangle = \frac{1}{2\pi^2}\frac{M}{L}K_1(ML)$$

(Spectrum of quantum fluctuations)

$M \to 0 \to \frac{1}{L^2}$


de Sitter
$a(\eta) = -\frac{1}{H\eta}$, $-\infty < \eta < 0$
$\ddot{V}_k + \omega_k^2 V_k = 0$, $\omega_k^2 = k^2 + M^2 a^2 - \frac{\ddot{a}}{a}$
$\ddot{V}_k + \left[k^2 - \left(2 - \frac{M^2}{H^2}\right)\frac{1}{\eta^2}\right] V_k = 0$
$\frac{k}{aH}$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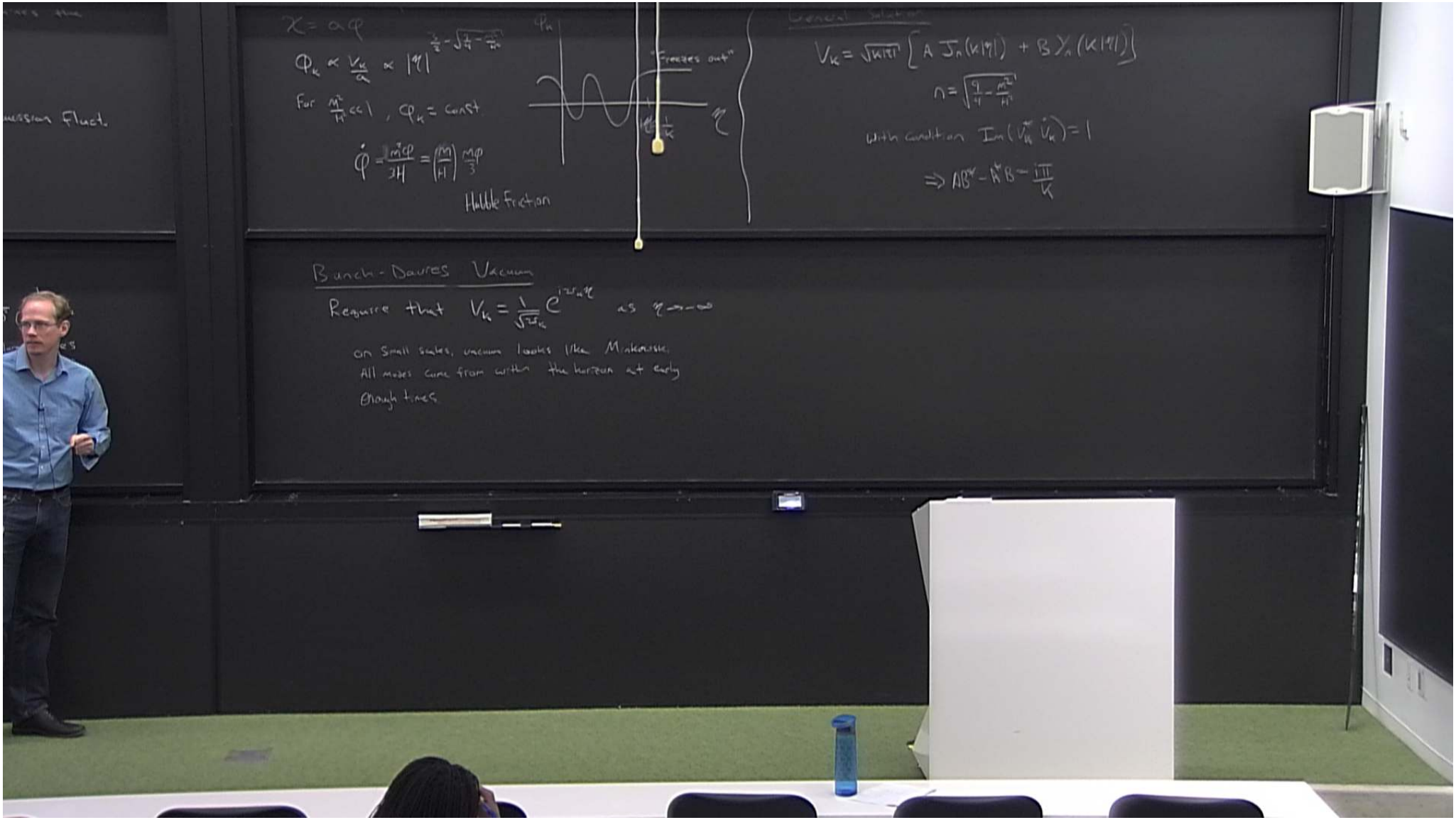
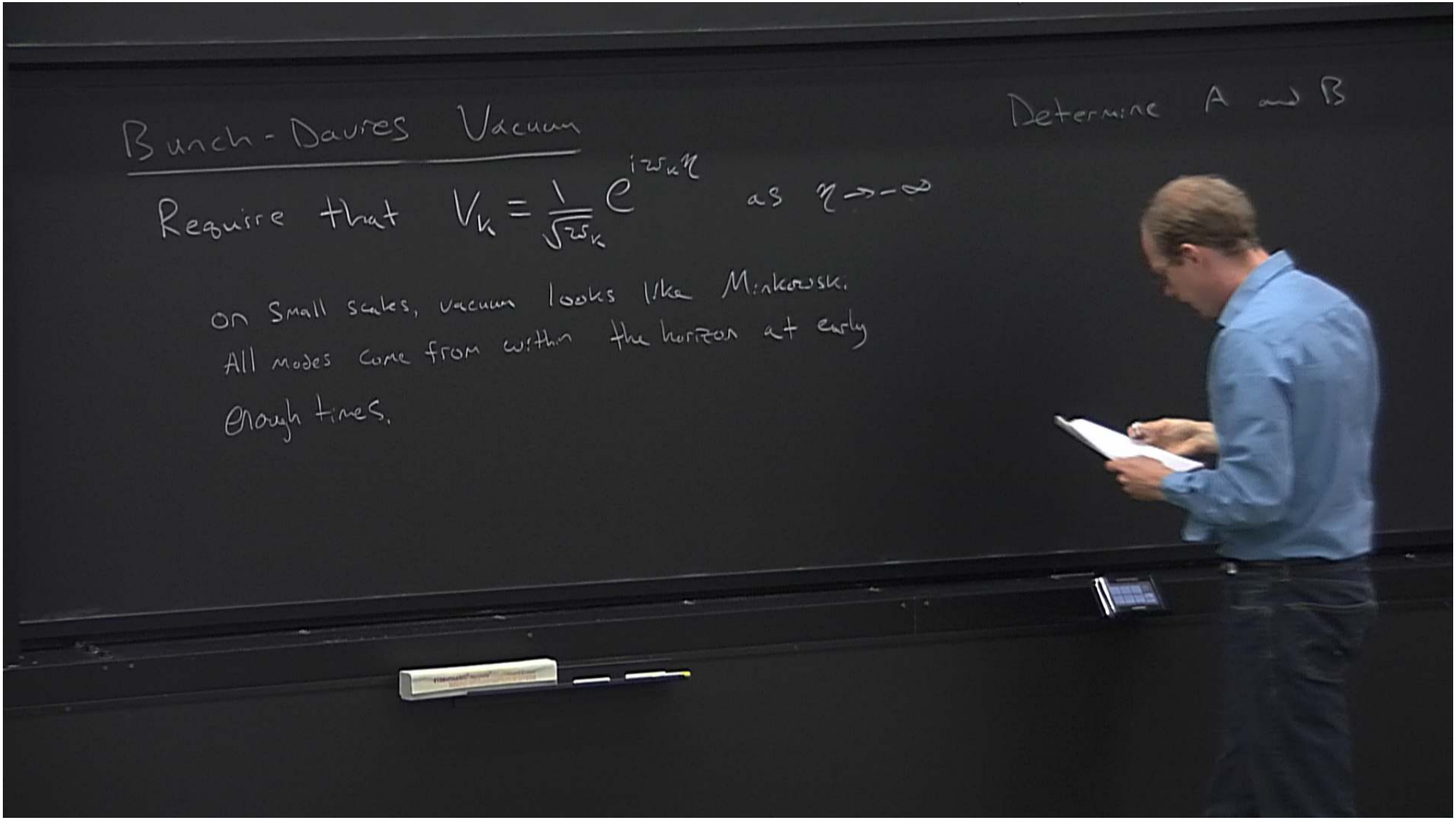
$\frac{k}{aH}$ (k = aH


$\chi = a\varphi$
$\varphi_k \propto \frac{v_k}{a} \propto |\eta|^{\frac{3}{2} - \sqrt{\frac{1}{4} - \frac{m^2}{H^2}}}$
For $\frac{m^2}{H^2} \ll 1$, $\varphi_k = \text{const}$
$\dot{\varphi} = \frac{m^2 \varphi}{3H} = \left(\frac{m}{H}\right) \frac{m\varphi}{3}$
Hubble Friction
"freezes out"
$\frac{1}{k}$
η
General solution
$v_k = \sqrt{|\eta|} \left[A J_n(k|\eta|) + B Y_n(k|\eta|) \right]$
$n = \sqrt{\frac{9}{4} - \frac{m^2}{H^2}}$
With condition $\mathrm{Im}(v_k^* \dot{v}_k) = 1$
$\Rightarrow AB^* - A^* B = \frac{i\pi}{4}$
Bunch-Davies Vacuum
Require that $v_k = \frac{1}{\sqrt{2\omega_k}} e^{-i\omega_k \eta}$ as $\eta \to -\infty$
on small scales, vacuum looks like Minkowski.
All modes come from within the horizon at early enough times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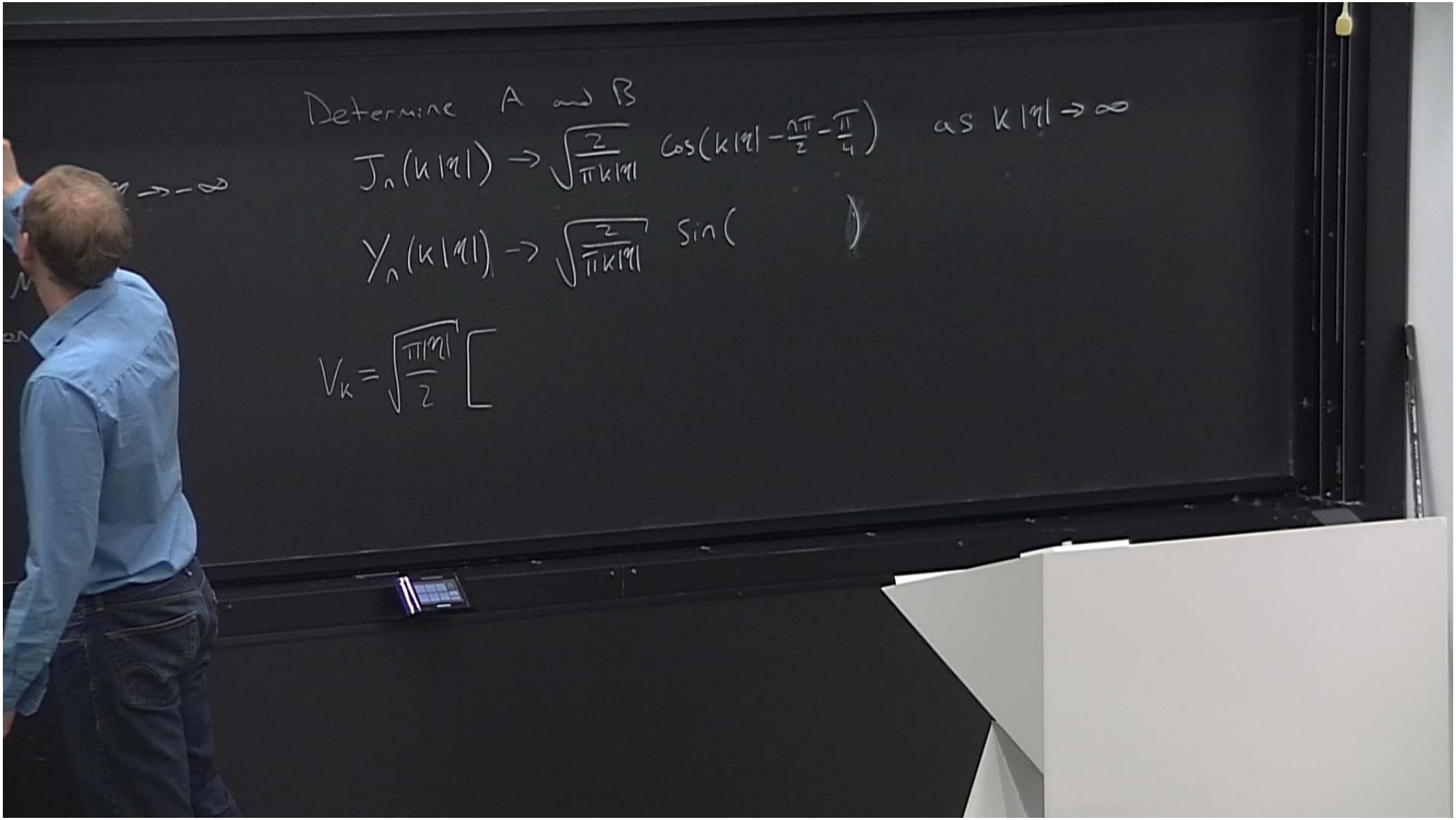
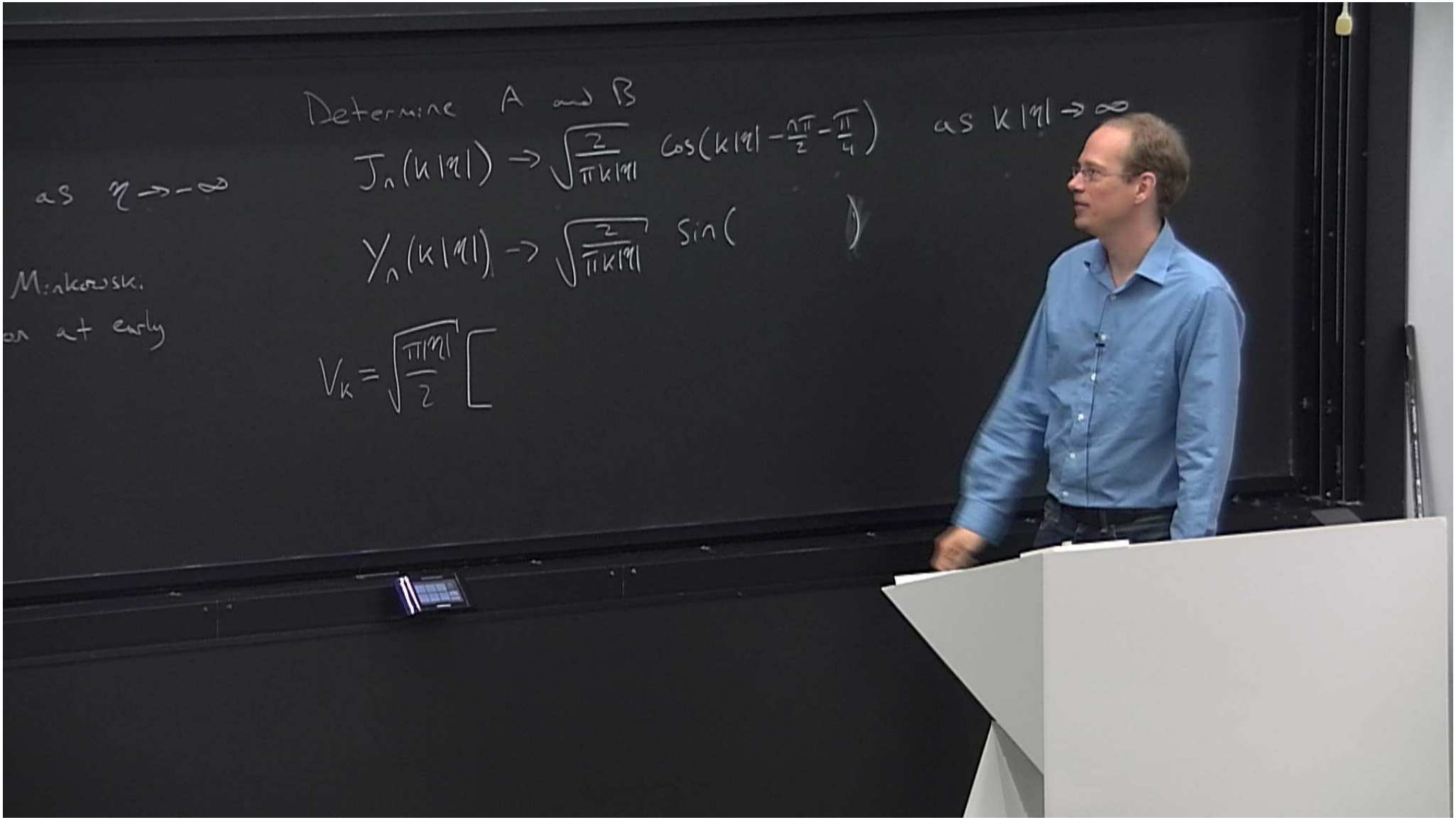
Determine A and B


Determine A and B
$J_n(k|x|) \to \sqrt{\frac{2}{\pi k|x|}} \cos\left(k|x| - \frac{n\pi}{2} - \frac{\pi}{4}\right)$ as $k|x| \to \infty$
$Y_n(k|x|) \to \sqrt{\frac{2}{\pi k|x|}} \sin($
$V_k = \sqrt{\frac{\pi|x|}{2}}[$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as $\eta \to -\infty$
Minkowski
on at early
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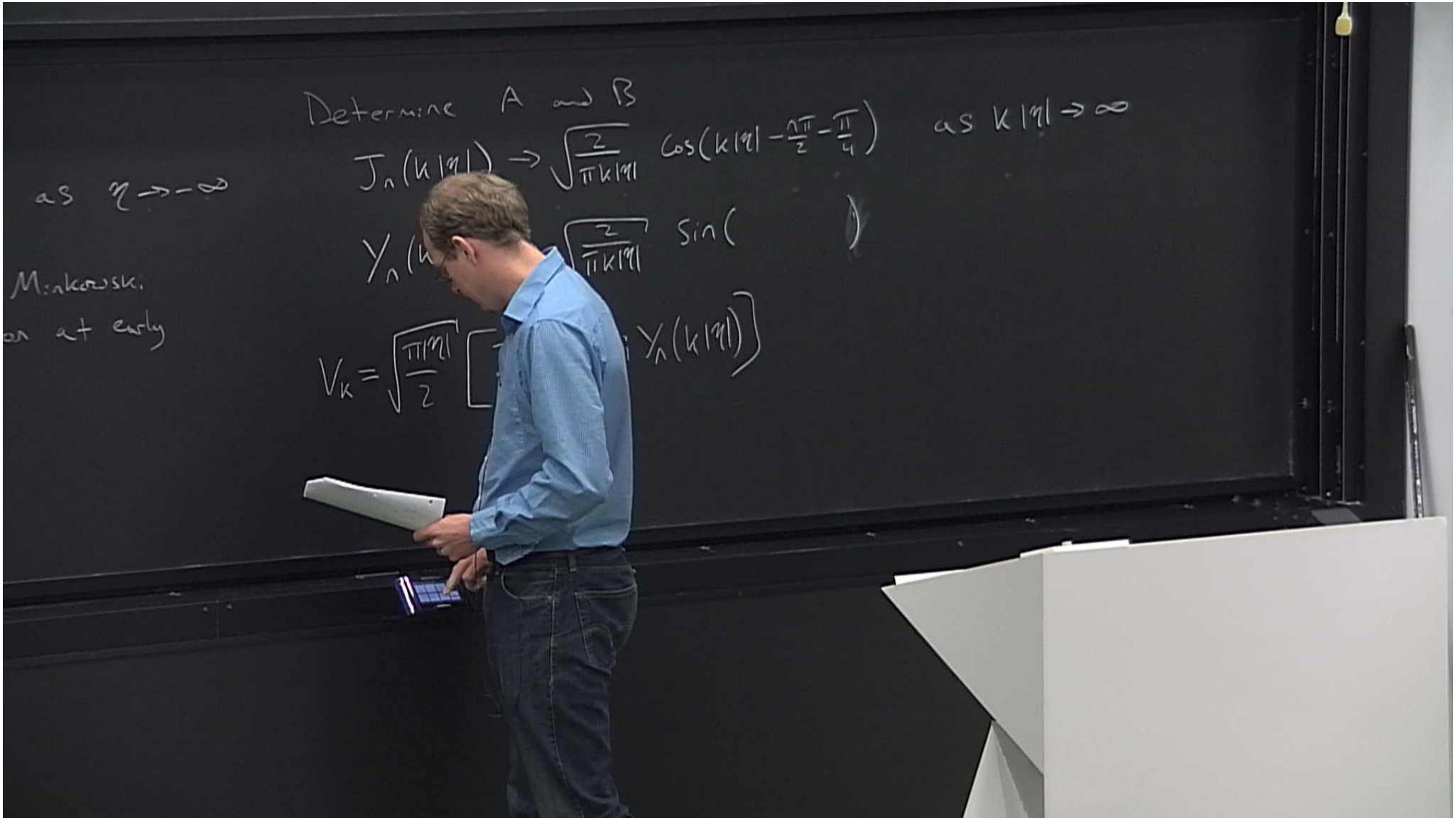
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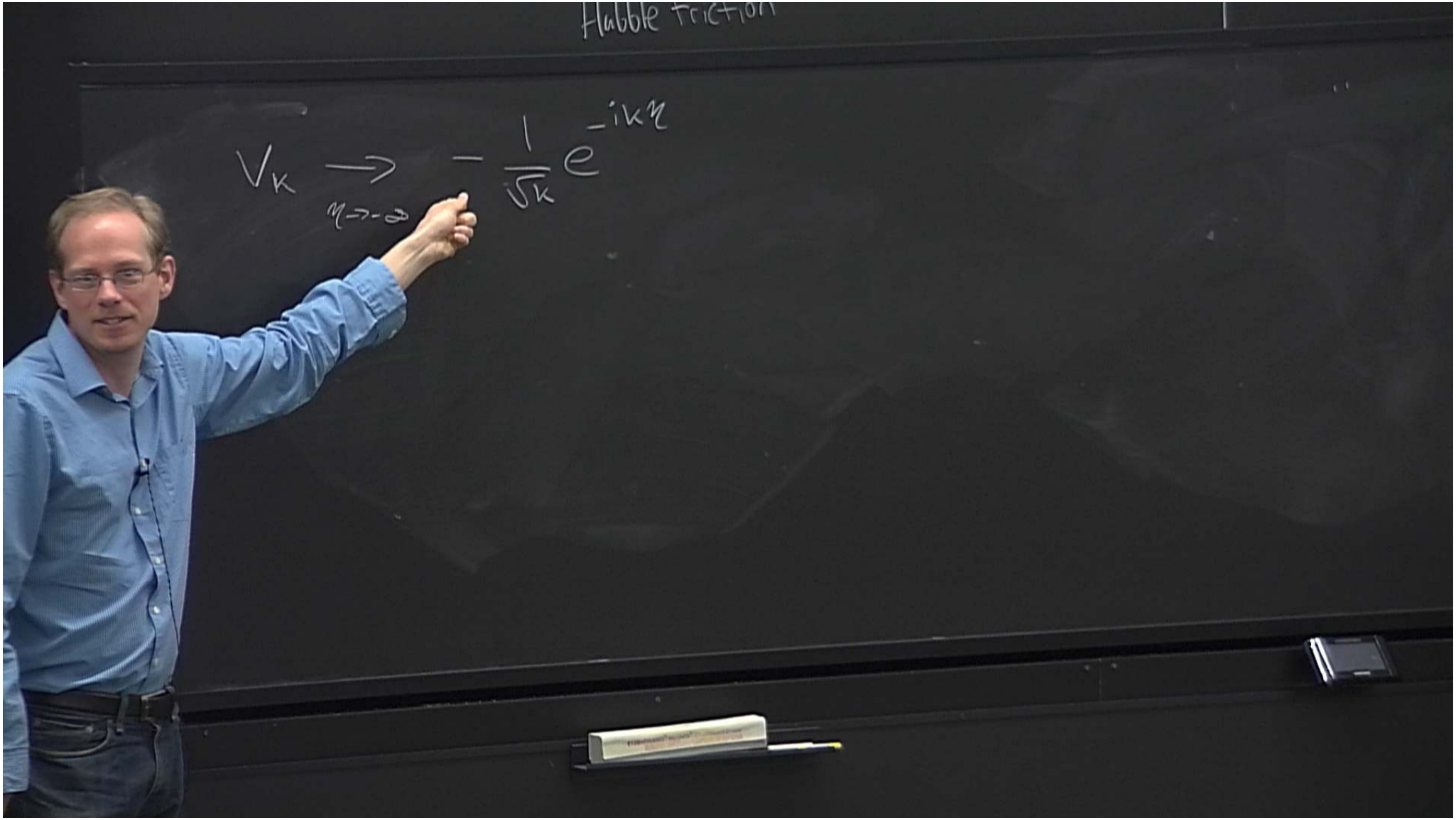
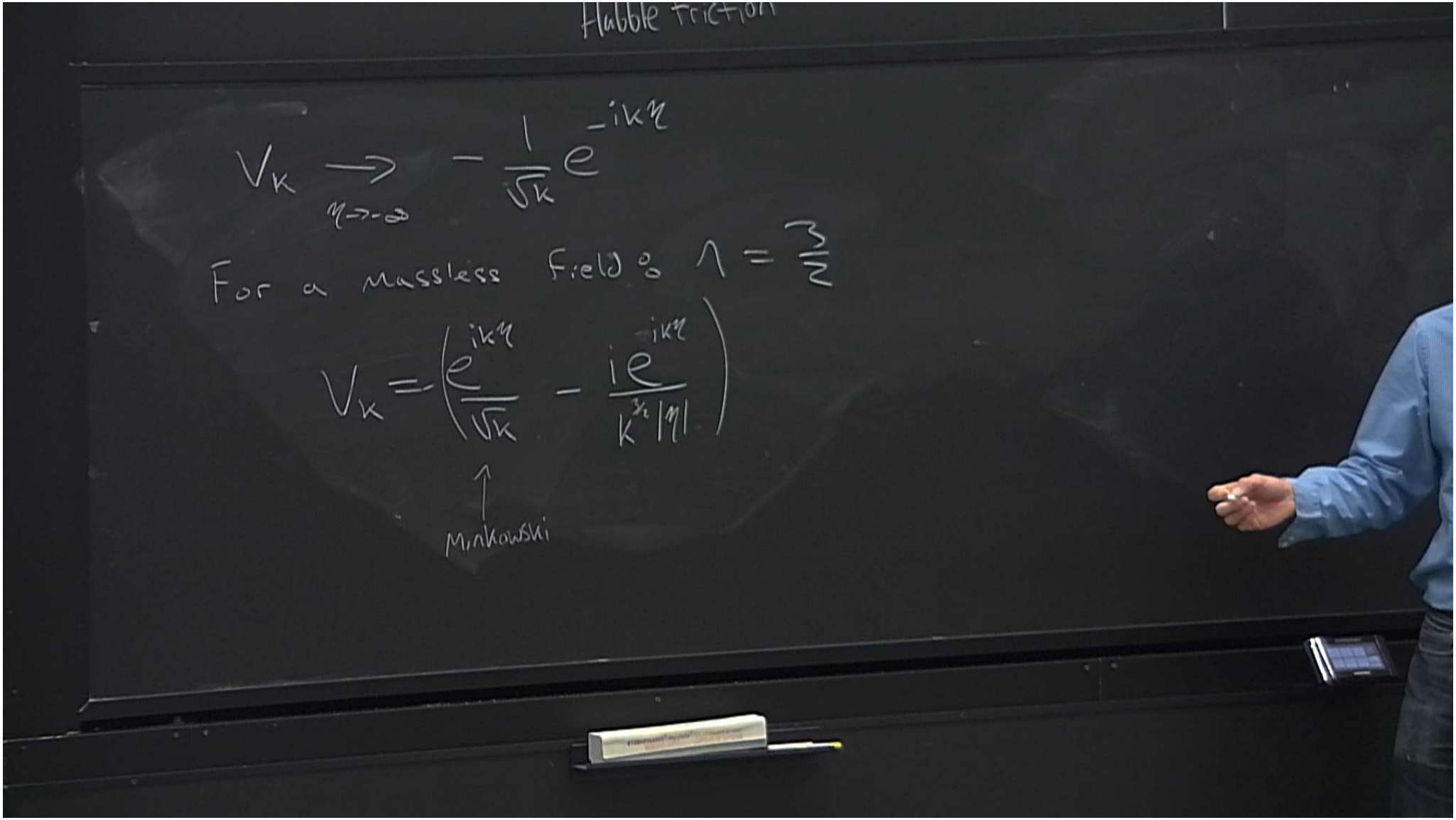
as $\eta \to -\infty$
$Y_n(k$
$\sqrt{\frac{2}{\pi k|\eta|}}$ sin(
Minkowski
on at early
$V_k = \sqrt{\frac{\pi|\eta|}{2}}$ [$Y_n(k|\eta|)$]


Hubble friction
$v_k \to -\frac{1}{\sqrt{k}} e^{-ik\eta}$
$\eta \to -\infty$


Hubble friction
$v_k \xrightarrow[\eta \to -\infty]{} -\frac{1}{\sqrt{k}} e^{-ik\eta}$
For a massless field $\Delta = \frac{3}{2}$
$v_k = -\left(\frac{e^{ik\eta}}{\sqrt{k}} - \frac{ie^{ik\eta}}{k^{3/2}|\eta|}\right)$
Minkowski

Hubble friction

$$v_k \xrightarrow[\eta \to -\infty]{} -\frac{1}{\sqrt{k}} e^{-ik\eta}$$

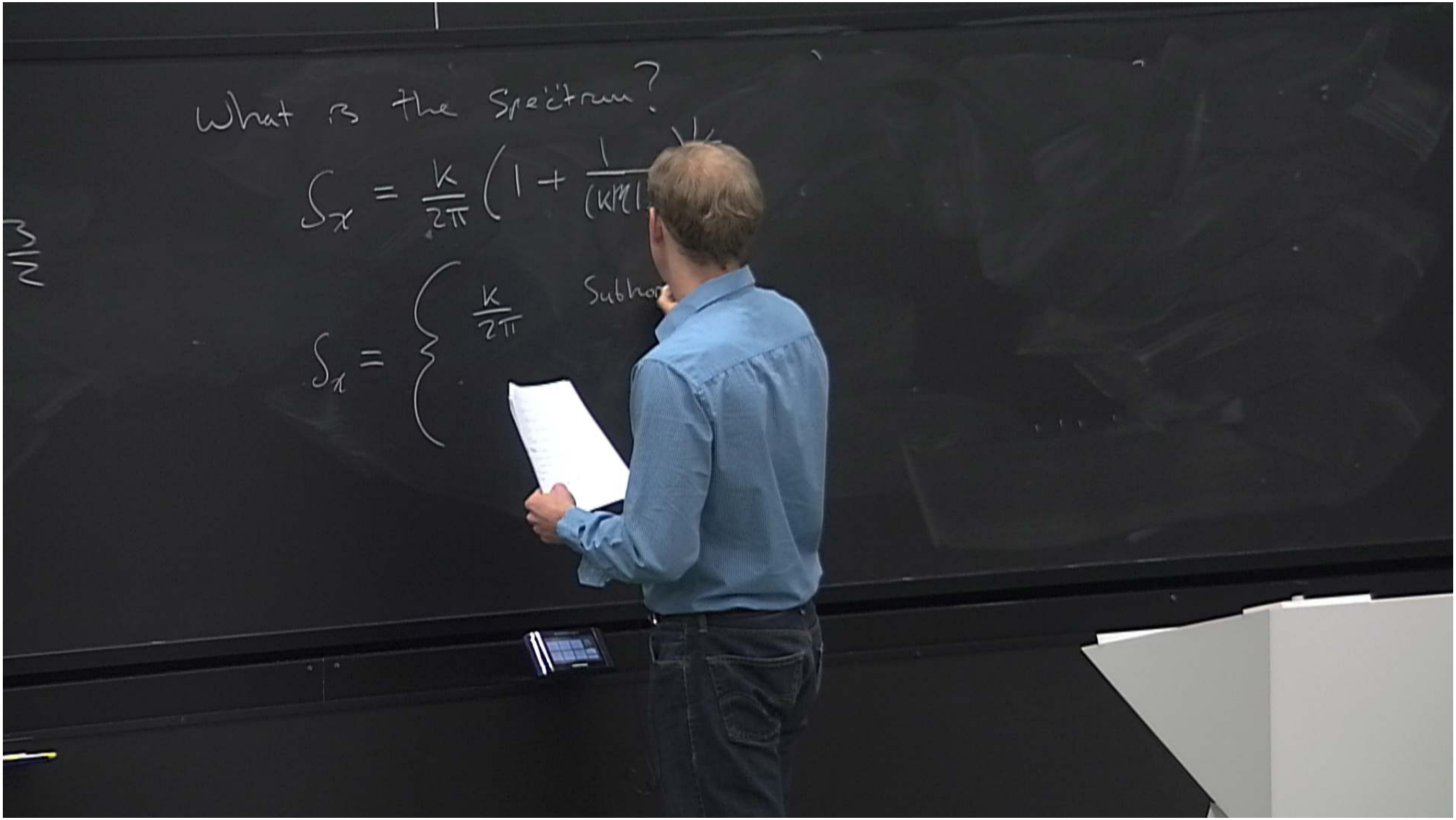
What is the spectrum?

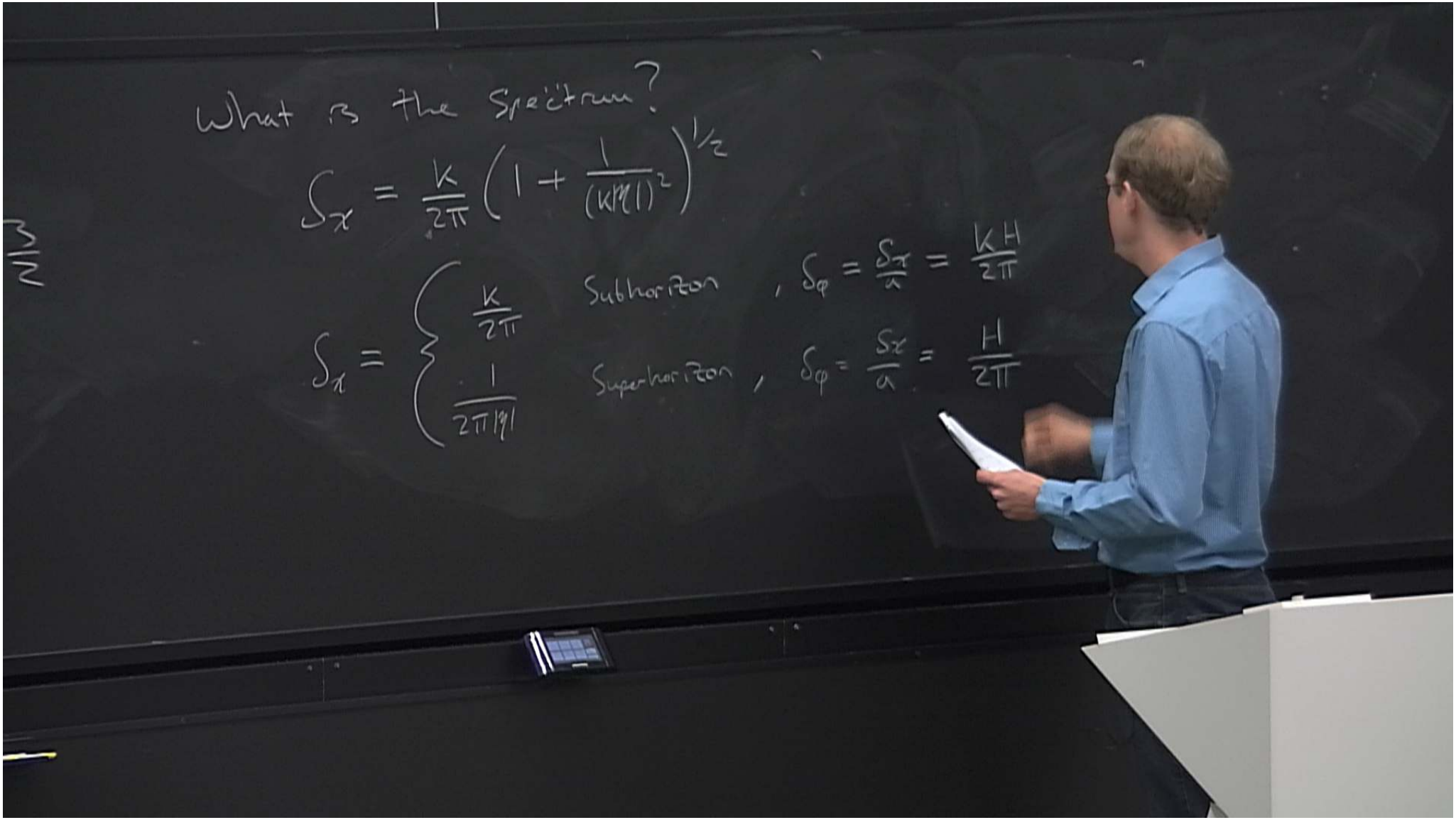
For a massless field: $\Lambda = \frac{3}{2}$

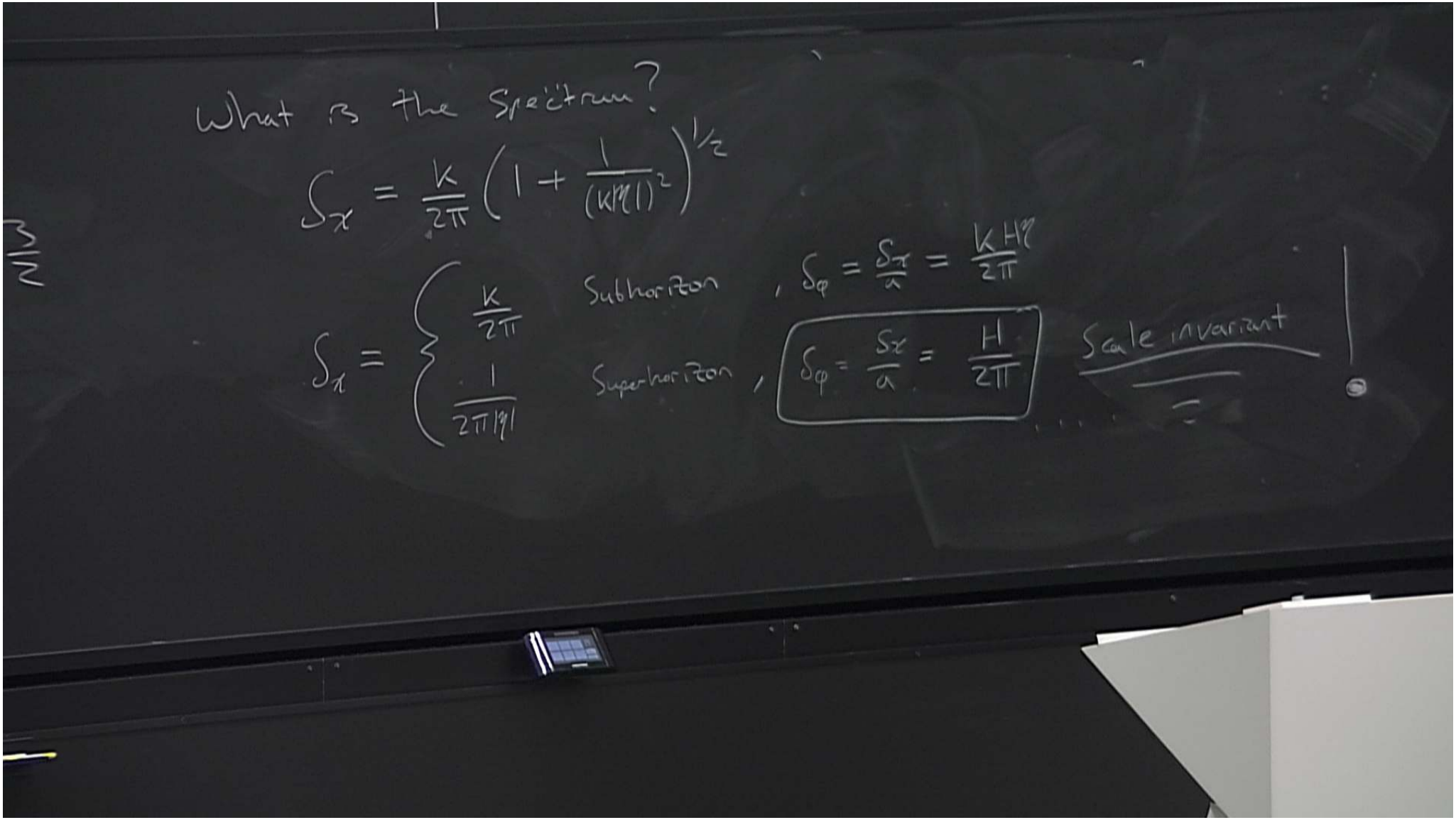
$$v_k = -\left(\frac{e^{ik\eta}}{\sqrt{k}} - \frac{i e^{ik\eta}}{k^{3/2} |\eta|} \right)$$

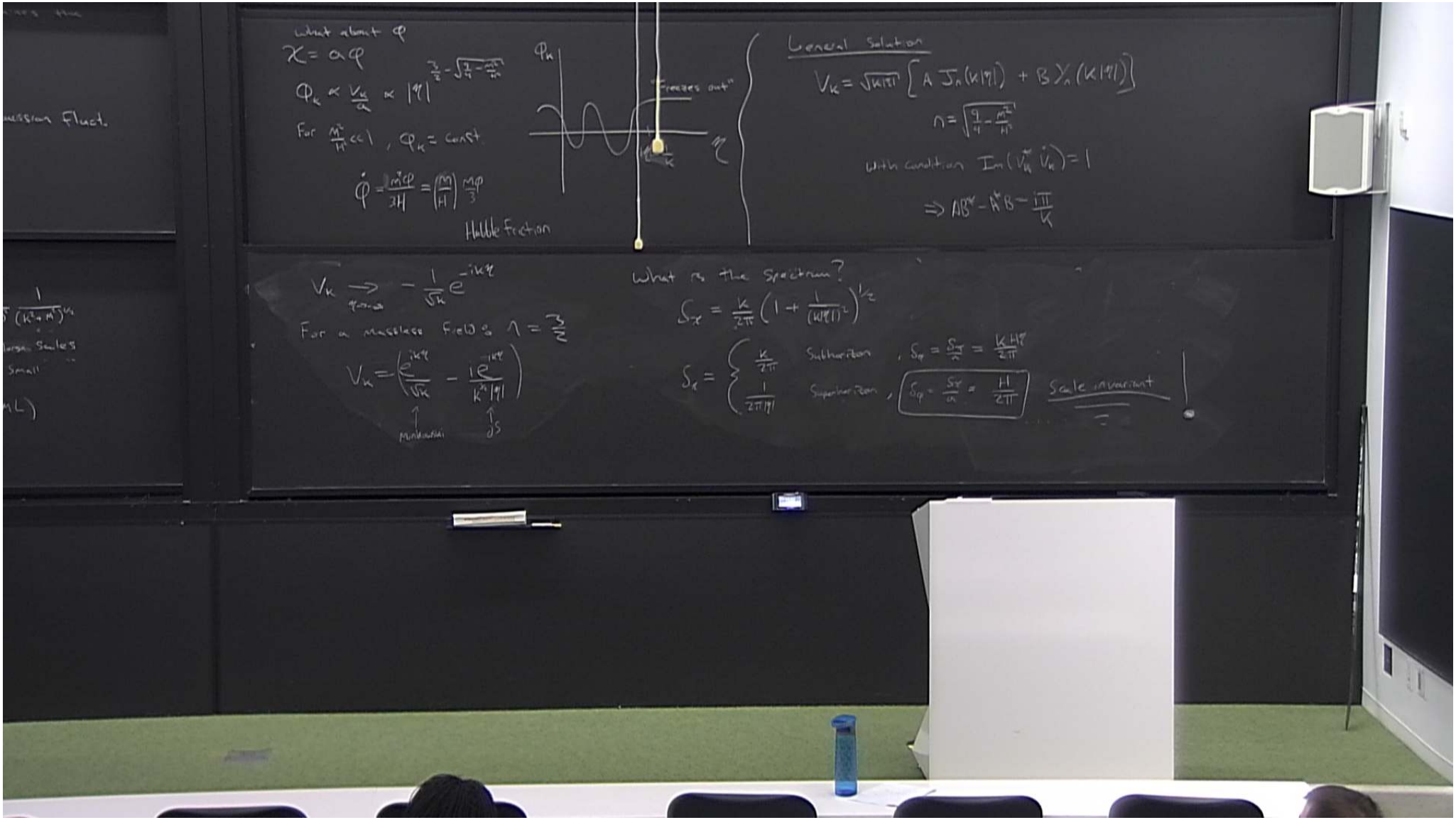
Minkowski

dS


What is the Spectrum?
$\delta_x = \frac{k}{2\pi}\left(1 + \frac{1}{(kM($
$\delta_x = \begin{cases} \frac{k}{2\pi} & \text{Subhor} \end{cases}$


What is the Spectrum?
$\delta_x = \frac{k}{2\pi}\left(1+\frac{1}{(k\eta(1)^2}\right)^{1/2}$
$\delta_x = \begin{cases} \frac{k}{2\pi} & \text{Subhorizon} \\ \frac{1}{2\pi|\eta|} & \text{Superhorizon} \end{cases}$
$\delta_\varphi = \frac{\delta_x}{a} = \frac{kH}{2\pi}$
$\delta_\varphi = \frac{\delta_x}{a} = \frac{H}{2\pi}$


What is the Spectrum?
$\delta_\chi = \frac{k}{2\pi}\left(1+\frac{1}{(k\eta(1)^2}\right)^{1/2}$
$\delta_\chi = \begin{cases} \frac{k}{2\pi} & \text{Subhorizon} \\ \frac{1}{2\pi|\eta|} & \text{Superhorizon} \end{cases}$
$\delta_\varphi = \frac{\delta_\chi}{a} = \frac{kH\eta}{2\pi}$
$\delta_\varphi = \frac{\delta_\chi}{a} = \frac{H}{2\pi}$
Scale invariant


What about φ
$\chi = a\varphi$
$\varphi_k \propto \frac{v_k}{a} \propto |\eta|^{\frac{3}{2}-\sqrt{\frac{9}{4}-\frac{m^2}{H^2}}}$
For $\frac{m^2}{H^2} \ll 1$, $\varphi_k = \text{const}$
$\dot{\varphi} = -\frac{m^2\varphi}{3H} = \left(\frac{m}{H}\right)\frac{m\varphi}{3}$
Hubble Friction
"Freezes out"
General Solution
$v_k = \sqrt{|k\eta|}\left[A J_n(k|\eta|) + B Y_n(k|\eta|)\right]$
$n = \sqrt{\frac{9}{4}-\frac{m^2}{H^2}}$
With condition $\mathrm{Im}(v_k^* \dot{v}_k) = 1$
$\Rightarrow AB^* - A^*B = \frac{i\pi}{k}$
$v_k \xrightarrow[\text{quantum}]{} -\frac{1}{\sqrt{k}} e^{-ik\eta}$
For a massless Field, $n = \frac{3}{2}$
$v_k = \left(\frac{e^{-ik\eta}}{\sqrt{k}} - \frac{ie^{-ik\eta}}{k^{3/2}|\eta|}\right)$
Minkowski
dS
What is the Spectrum?
$\mathcal{S}_\chi = \frac{k}{2\pi}\left(1+\frac{1}{(k|\eta|)^2}\right)^{1/2}$
$\mathcal{S}_\chi = \begin{cases} \frac{k}{2\pi} & \text{Subhorizon} \\ \frac{1}{2\pi|\eta|} & \text{Superhorizon} \end{cases}$, $\mathcal{S}_\varphi = \frac{\mathcal{S}_\chi}{a} = \frac{kH\eta}{2\pi}$
$\mathcal{S}_\varphi = \frac{\mathcal{S}_\chi}{a} = \frac{H}{2\pi}$
Scale invariant !

$\frac{n(kL)}{kL}$

e.g. In Minkowski

$$|S_k|^2 = \frac{k^3}{(2\pi)^2}\frac{1}{2\omega_k} = \frac{k^3}{(2\pi)^2}\frac{1}{(k^2+M^2)^{1/2}}$$

fluct. small on large scales

" "large" " small "

$$\langle 0|\hat{\chi}(\vec{x},t)\hat{\chi}^\dagger(\vec{y},t)|0\rangle = \frac{1}{2\pi^2}\frac{M}{L}K_1(ML)$$

$\frac{k^2 V_4}{2\pi}$ (Spectrum of quantum fluctuations)

$M \to 0 \to \frac{1}{L^2}$