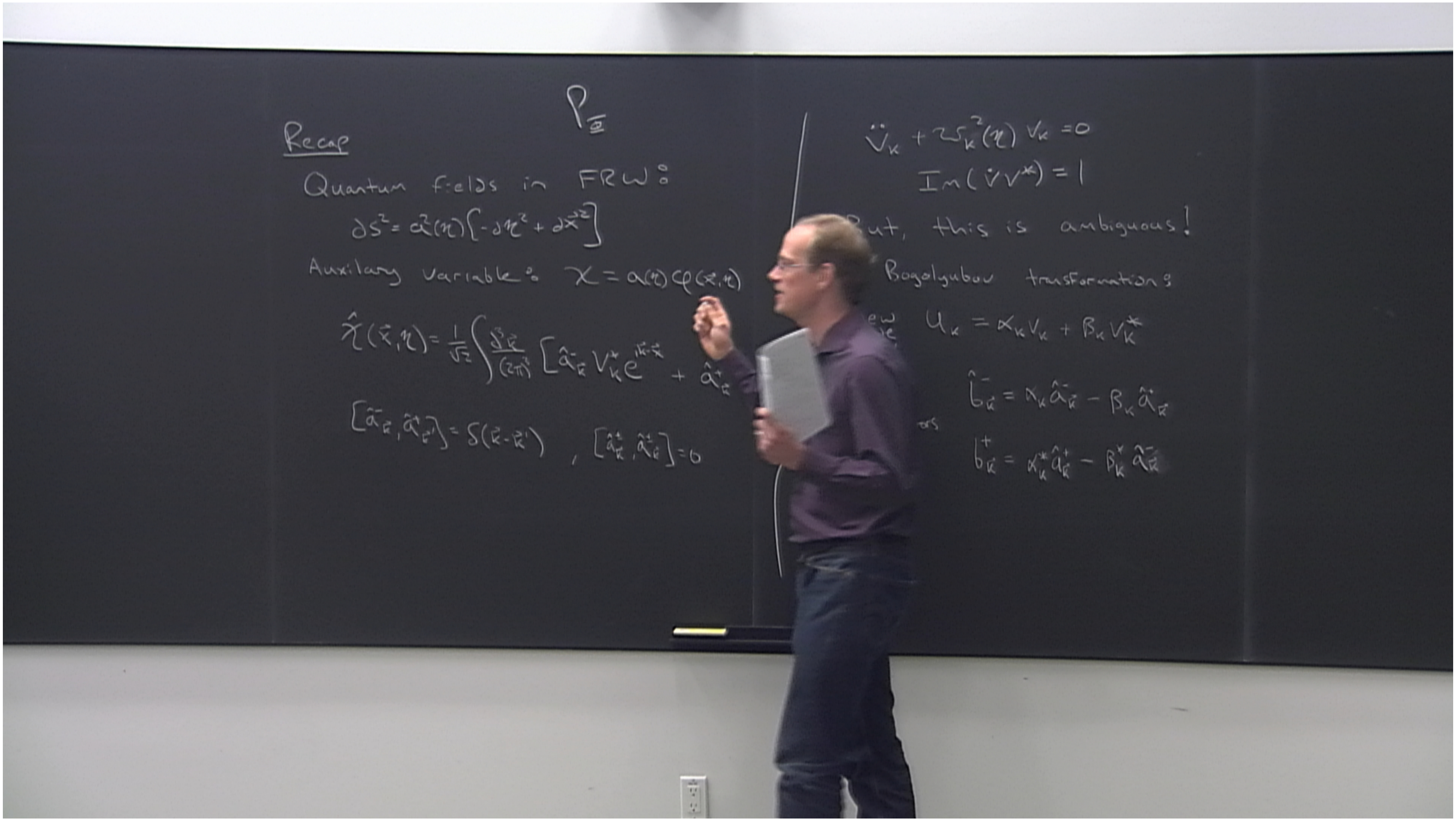
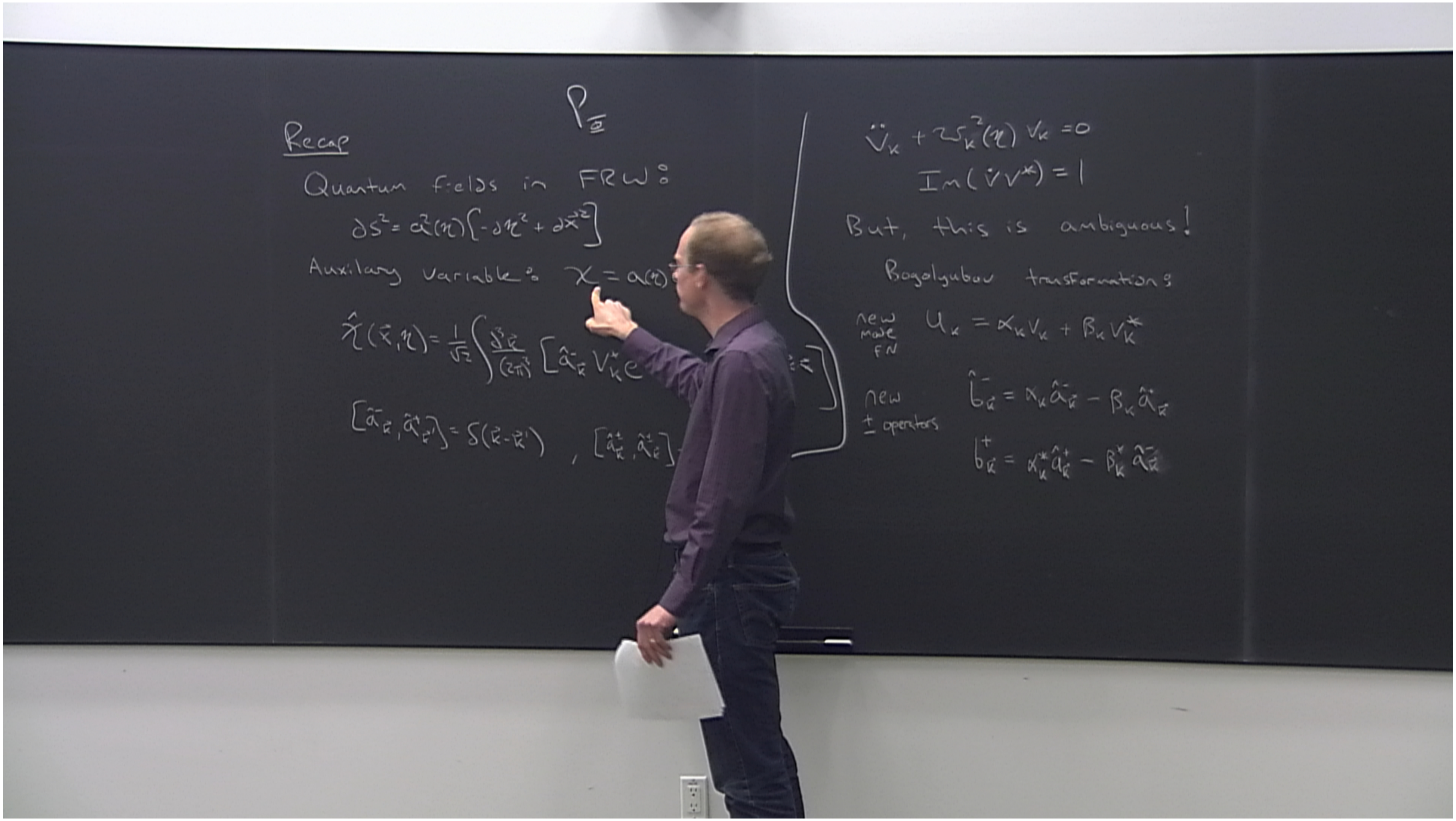
Title: 12/13 PSI - Explorations in Cosmology Lecture 11

Date: Apr 22, 2013 10:15 AM

URL: http://pirsa.org/13040076

Abstract:


Recap
Quantum fields in FRW:
$ds^2 = a^2(\eta)[-d\eta^2 + d\vec{x}^2]$
Auxilary variable: $\chi = a(\eta)\varphi(\vec{x},\eta)$
$\hat{\chi}(\vec{x},\eta) = \frac{1}{\sqrt{2}}\int \frac{d^3\vec{k}}{(2\pi)^3}[\hat{a}^-_{\vec{k}} V^*_k e^{i\vec{k}\cdot\vec{x}} + \hat{a}^+_{\vec{k}}$
$[\hat{a}^-_{\vec{k}}, \hat{a}^+_{\vec{k}'}] = \delta(\vec{k}-\vec{k}')$, $[\hat{a}^+_{\vec{k}}, \hat{a}^+_{\vec{k}'}] = 0$
$\ddot{V}_k + \omega_k^2(\eta) V_k = 0$
$\mathrm{Im}(\dot{v} v^*) = 1$
But, this is ambiguous!
Bogolyubov transformation:
$u_k = \alpha_k v_k + \beta_k v_k^*$
$\hat{b}^-_{\vec{k}} = \alpha_k \hat{a}^-_{\vec{k}} - \beta_k \hat{a}^+_{\vec{k}}$
$\hat{b}^+_{\vec{k}} = \alpha^*_k \hat{a}^+_{\vec{k}} - \beta^*_k \hat{a}^-_{\vec{k}}$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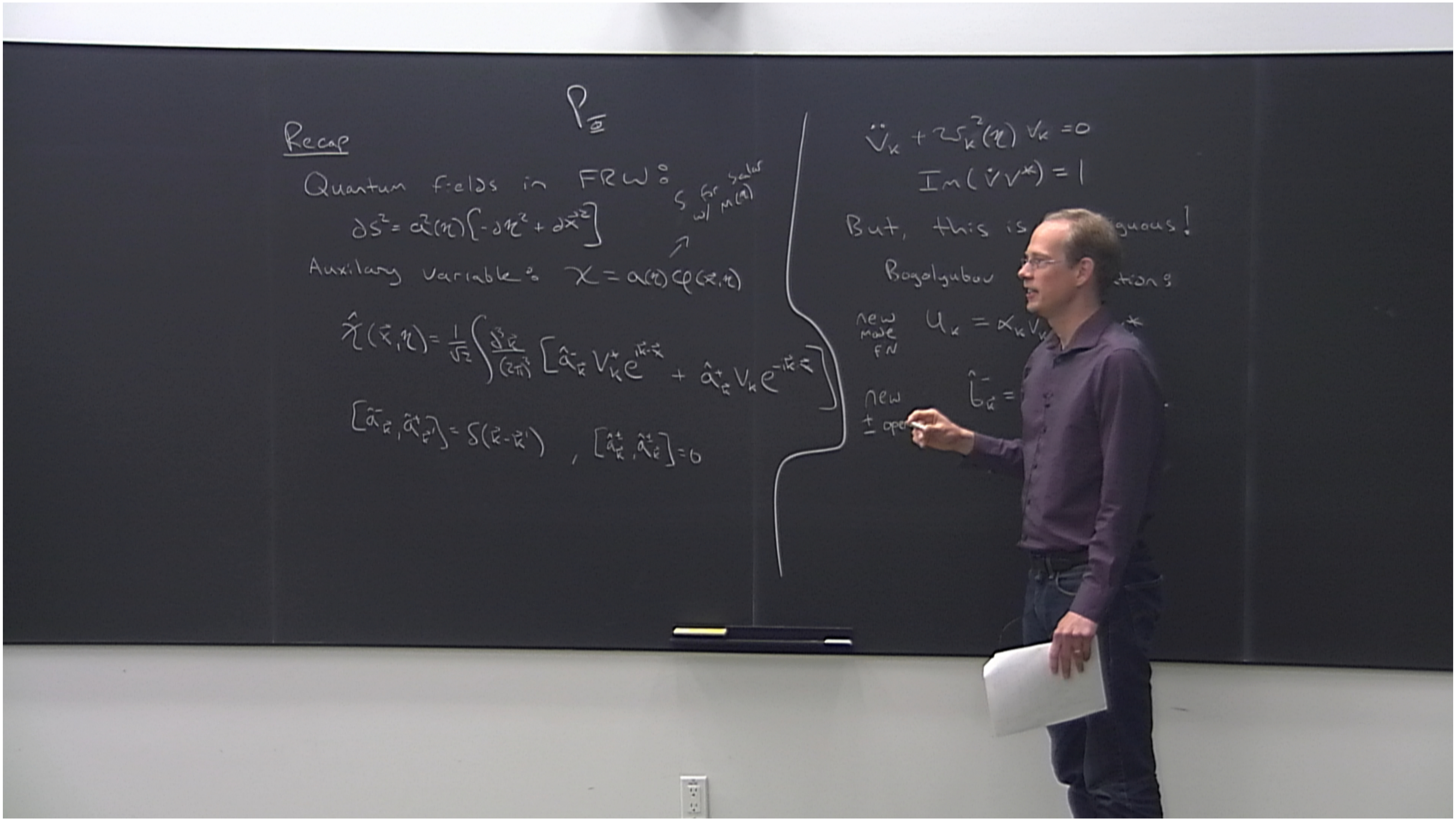
Quantum fields in FRW:
Auxilary variable:
But, this is ambiguous!
Bogolyubov transformation:
new mode FN
new ± operators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$u_k = \alpha_k v_k$
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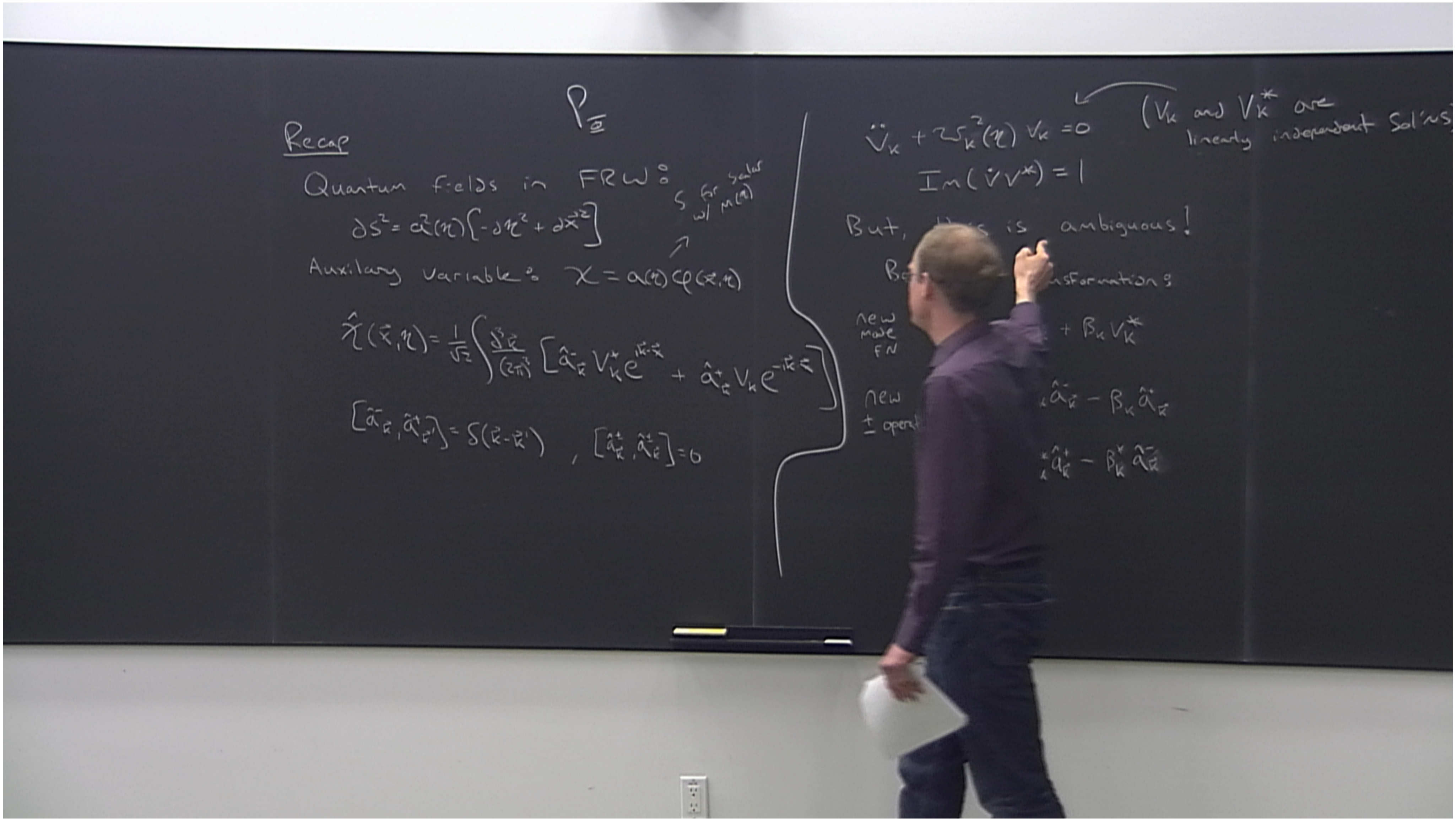
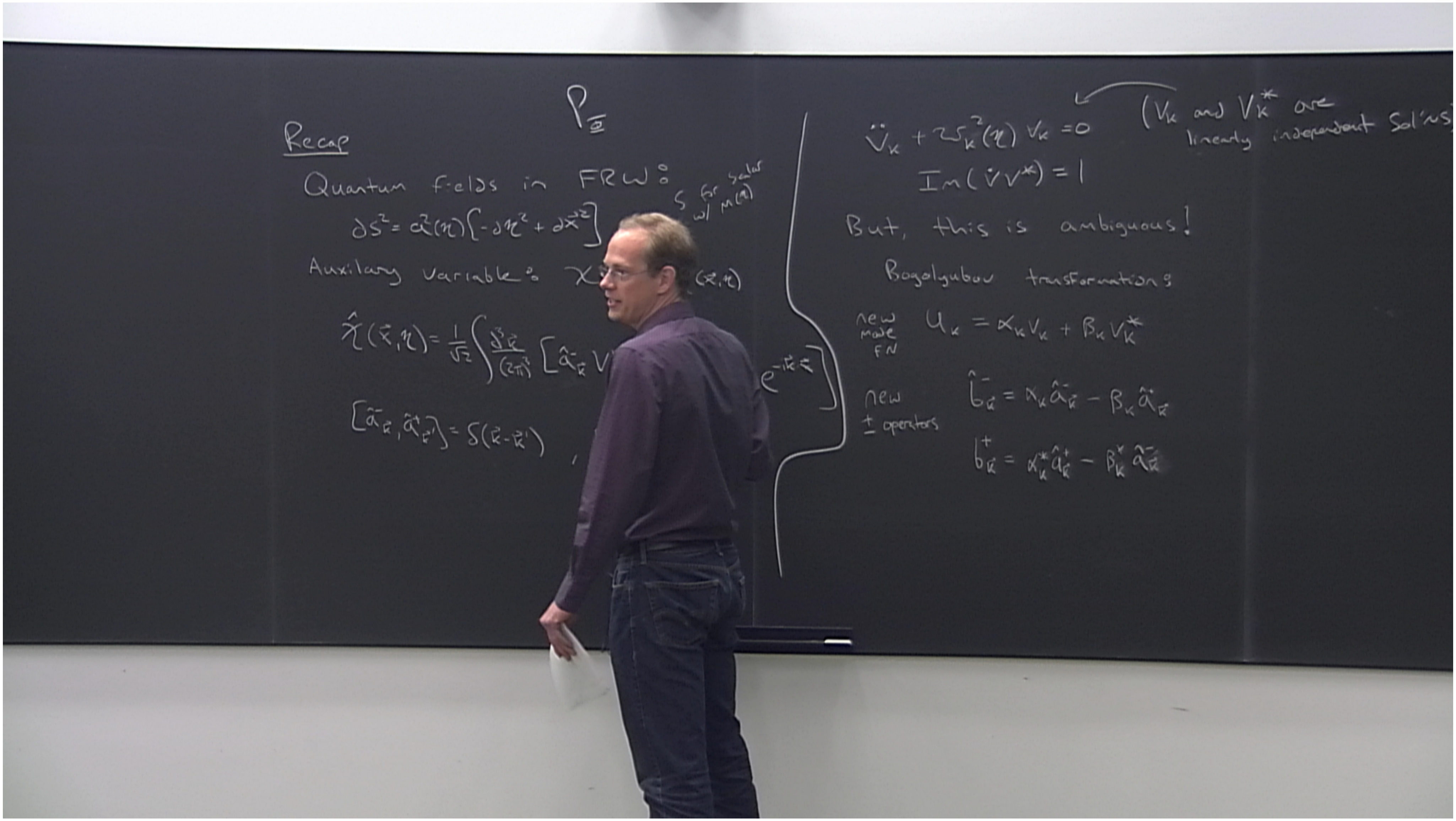
$\hat{b}^-_{\vec{k}} =$


Recap
Quantum fields in FRW:
ds² = a²(η)[-dη² + dx²]
for scalar w/ m(η)
Auxilary variable: χ = a(η)φ(x,η)
χ̂(x,η) = 1/√2 ∫ d³k/(2π)³ [â⁻_k V*_k e^(ik·x) + â⁺_k V_k e^(-ik·x)]
[â⁻_k, â⁺_k'] = δ(k-k') , [â⁺_k, â⁺_k] = 0
V̈_k + ω_k²(η) V_k = 0
(V_k and V_k* are linearly independent Sol'ns)
Im(V̇V*) = 1
But, ... is ambiguous!
new mode FN
new ± operators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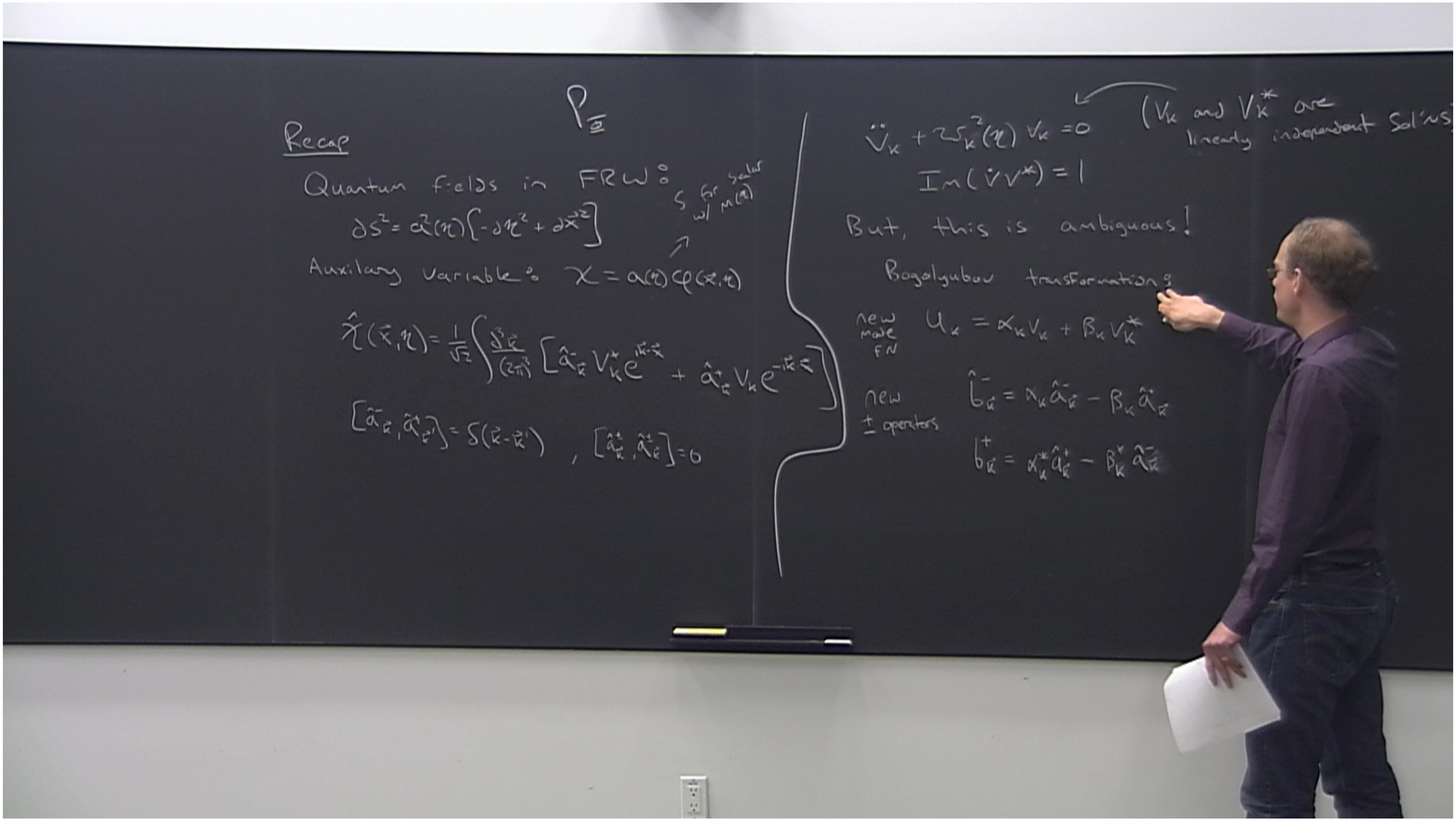
Recap
Quantum fields in FRW:
$ds^2 = a^2(\eta)[-d\eta^2 + d\vec{x}^2]$
S for scalar w/ $m(a)$
Auxilary variable: $\chi = a(\eta)\varphi(\vec{x},\eta)$
$\hat{\chi}(\vec{x},\eta) = \frac{1}{\sqrt{2}}\int \frac{d^3\vec{k}}{(2\pi)^{3/2}} [\hat{a}^-_{\vec{k}} v^*_k e^{i\vec{k}\cdot\vec{x}} + \hat{a}^+_{\vec{k}} v_k e^{-i\vec{k}\cdot\vec{x}}]$
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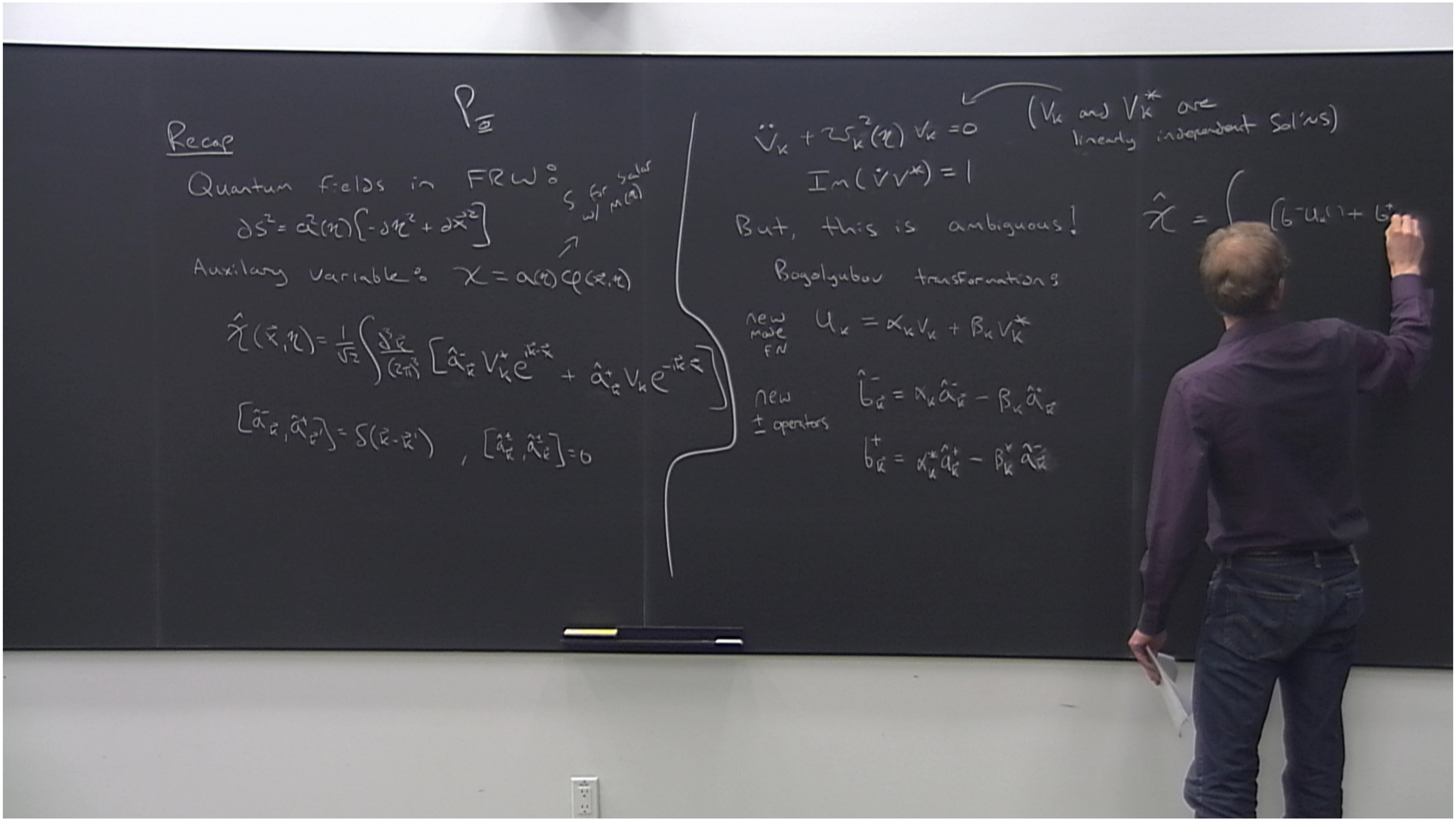
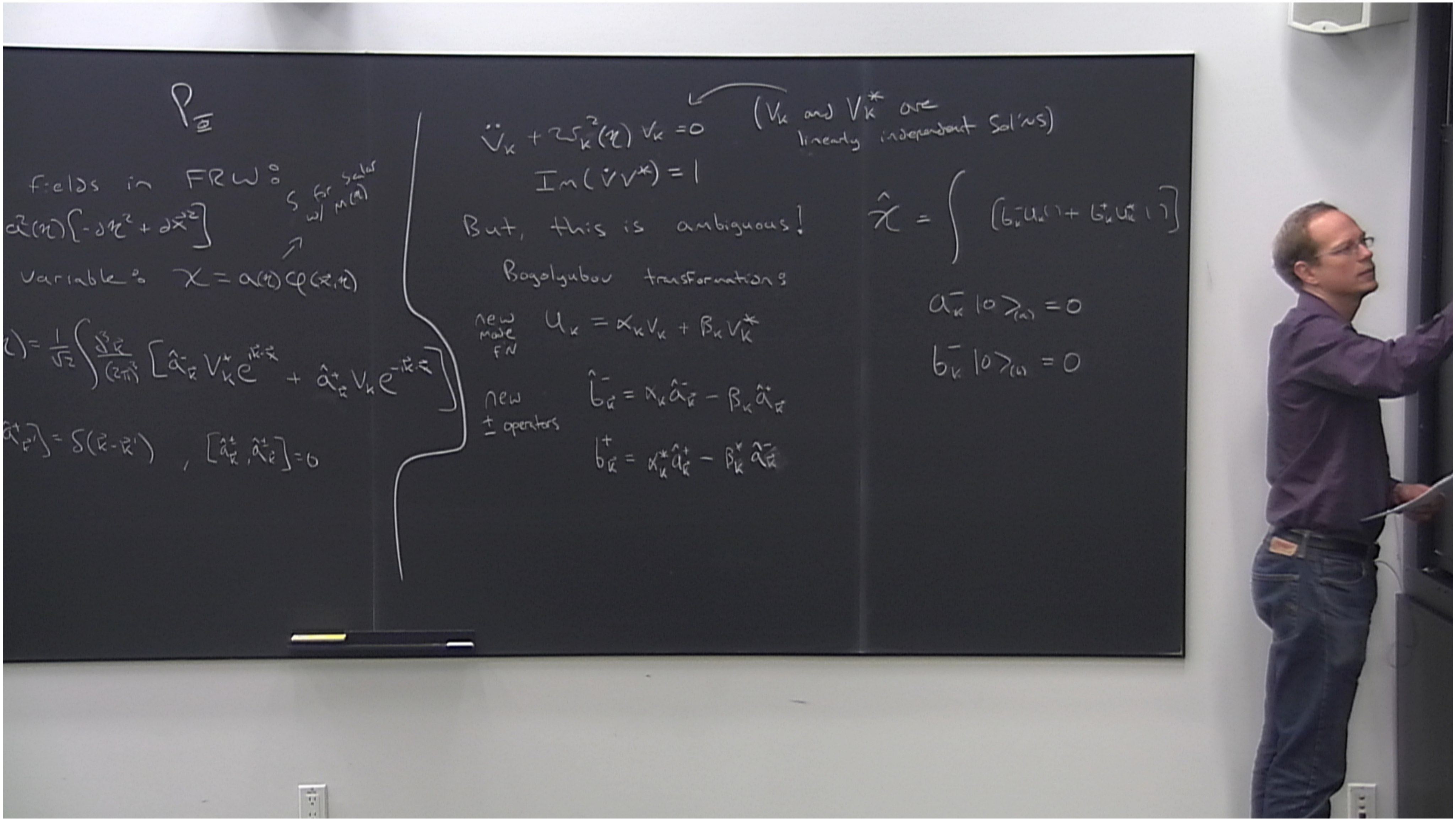
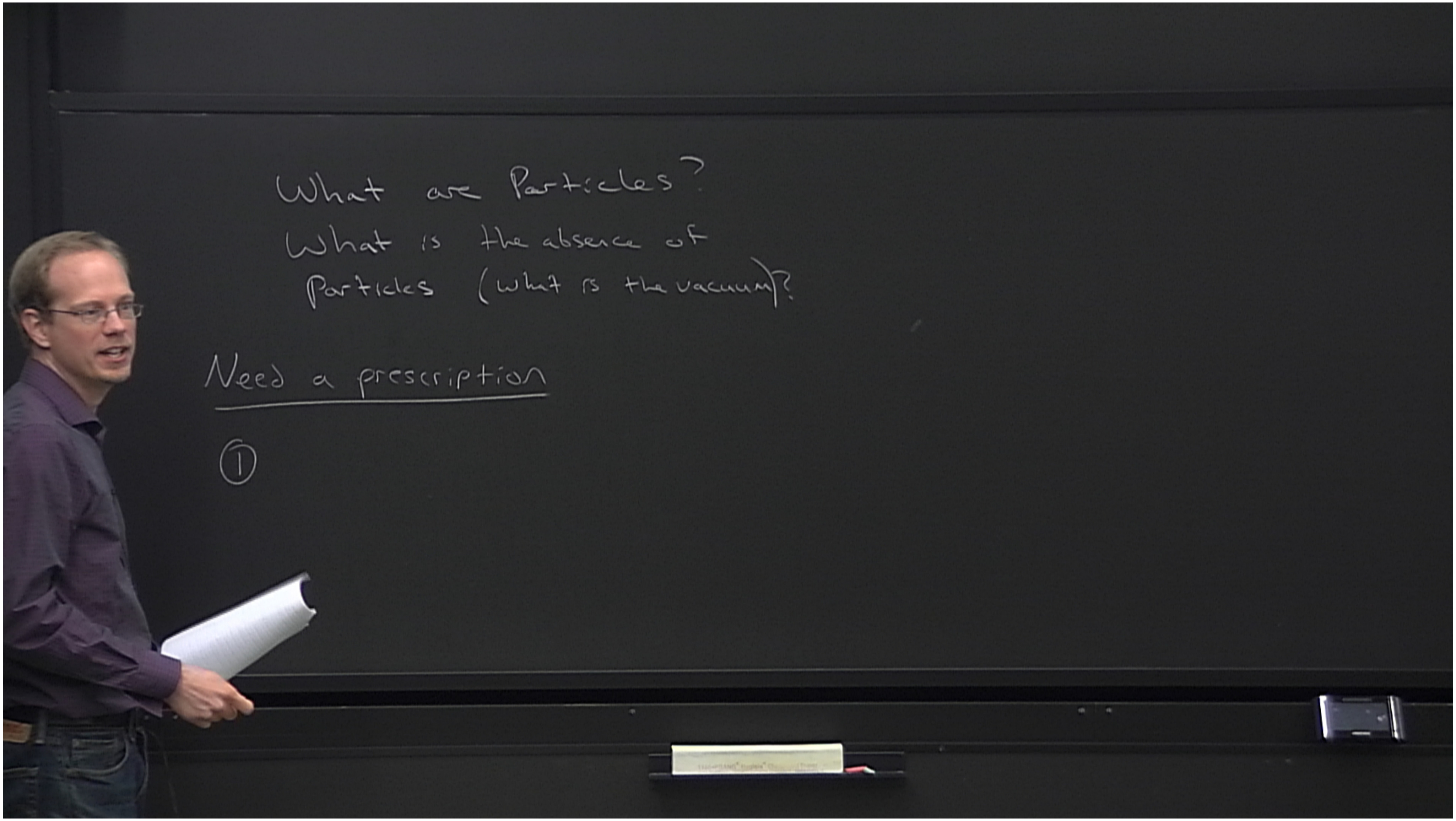
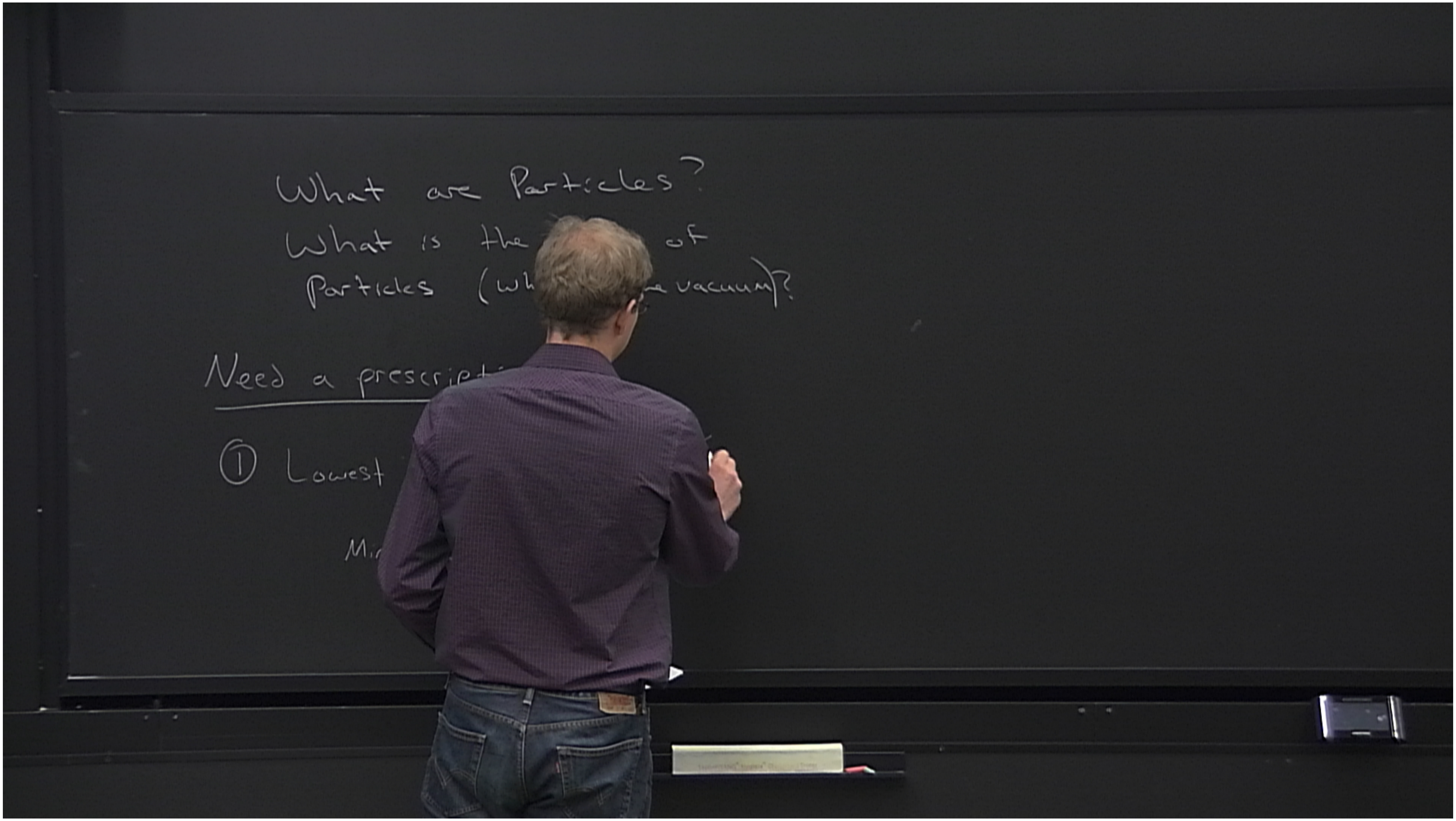
$\hat{b}^+_{\vec{k}} = \alpha^*_k \hat{a}^+_{\vec{k}} - \beta^*_k \hat{a}^-_{\vec{k}}$

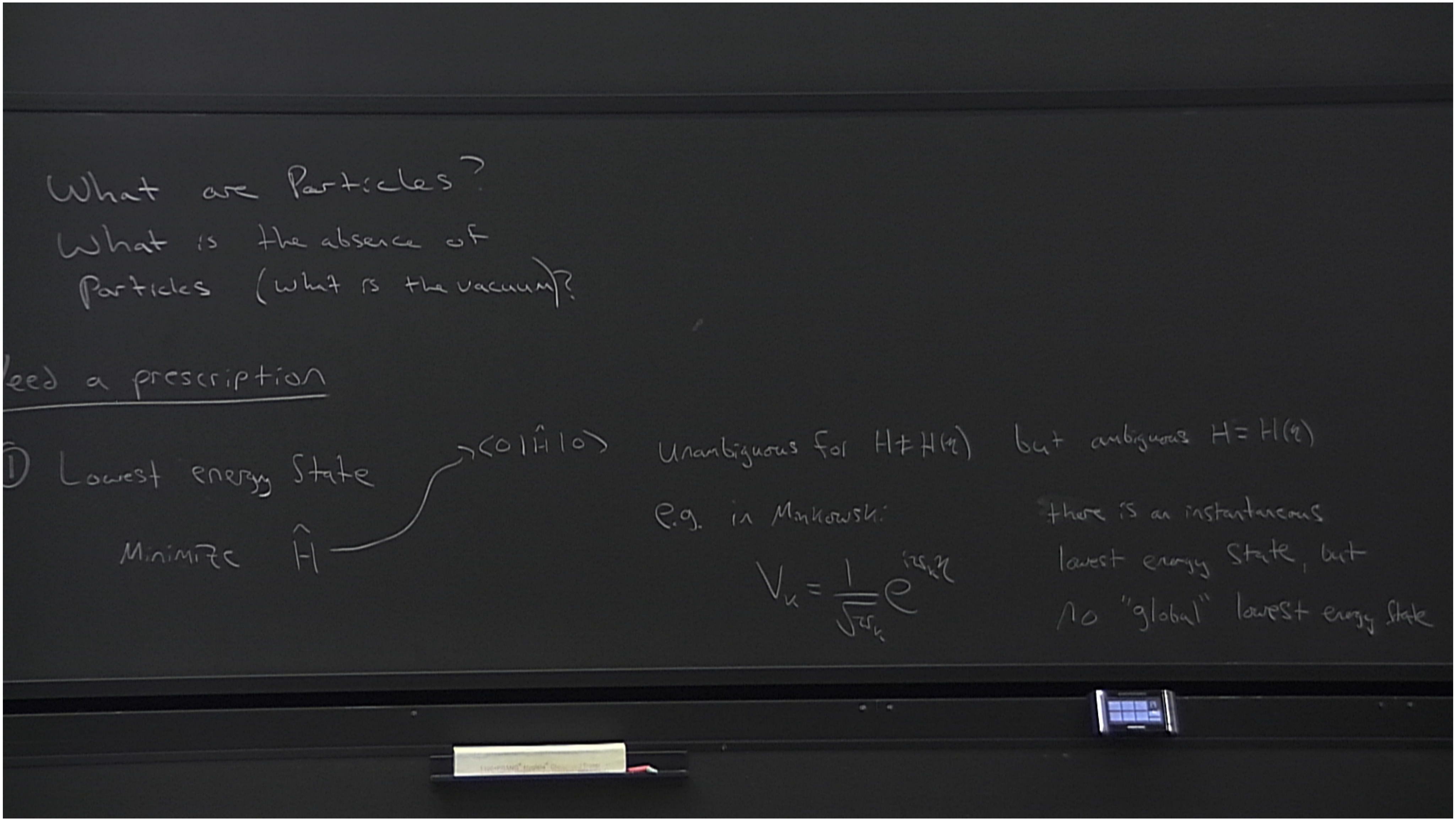

Recap
P_Φ
Quantum fields in FRW:
ds² = a²(η)[-dη² + dx⃗²]
S for scalar w/ m(η)
Auxilary variable: χ = a(η) φ(x⃗, η)
χ̂(x⃗, η) = 1/√2 ∫ d³k/(2π)³ [â⁻_k V*_k e^{ik·x} + â⁺_k V_k e^{-ik·x}]
[â⁻_k, â⁺_k'] = δ(k⃗ - k⃗') , [â±_k, â±_k'] = 0
V̈_k + ω_k²(η) V_k = 0
Im(V̇ V*) = 1
(V_k and V_k* are linearly independent Sol'ns)
But, this is ambiguous!
Bogolyubov transformation:
new mode FN
U_k = α_k V_k + β_k V_k*
new ± operators
b̂⁻_k = α_k â⁻_k − β_k â⁺_k
b̂⁺_k = α*_k â⁺_k − β*_k â⁻_k
χ̂ = ∫ (b⁻ U_k( ) + b⁺

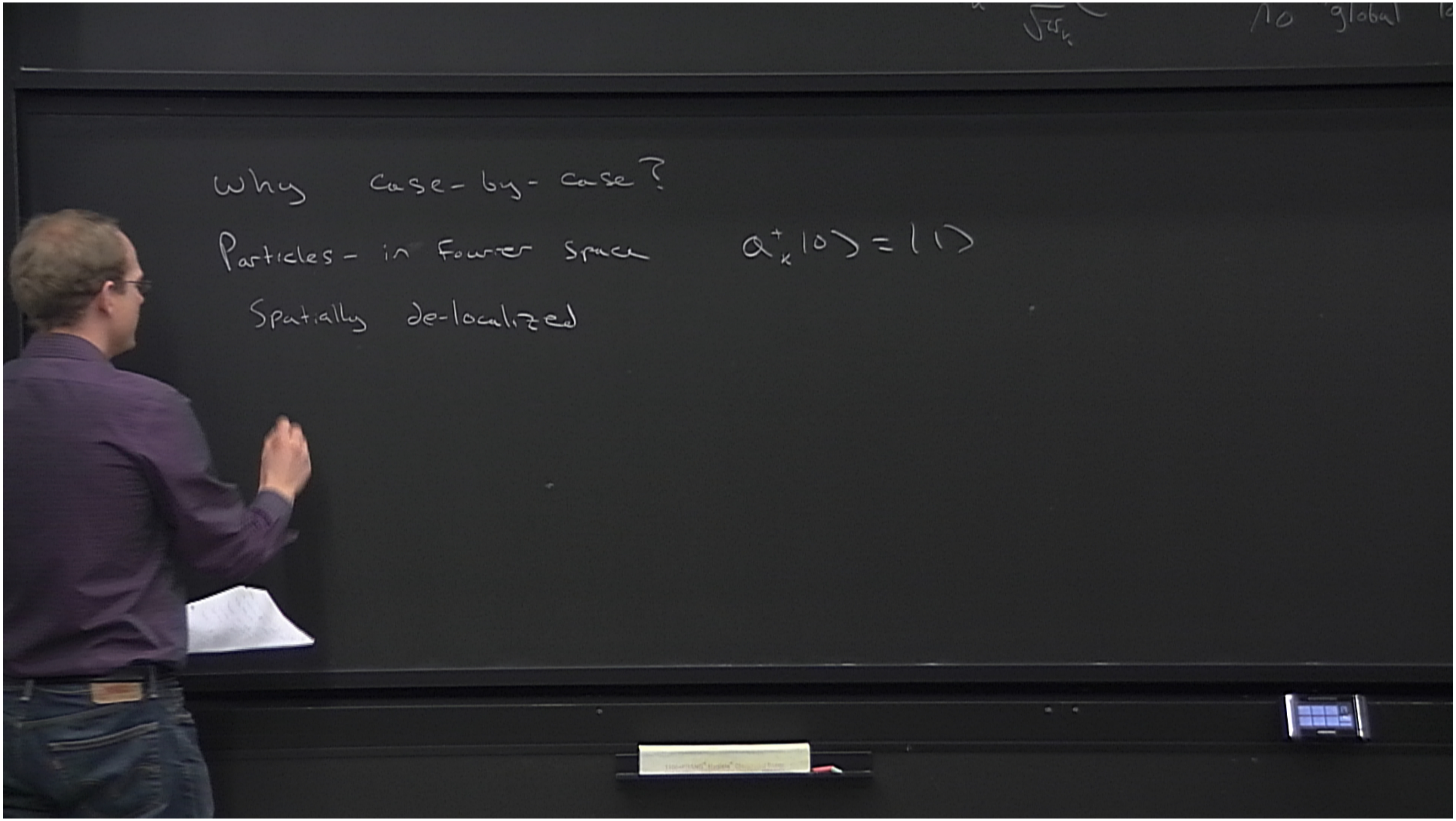


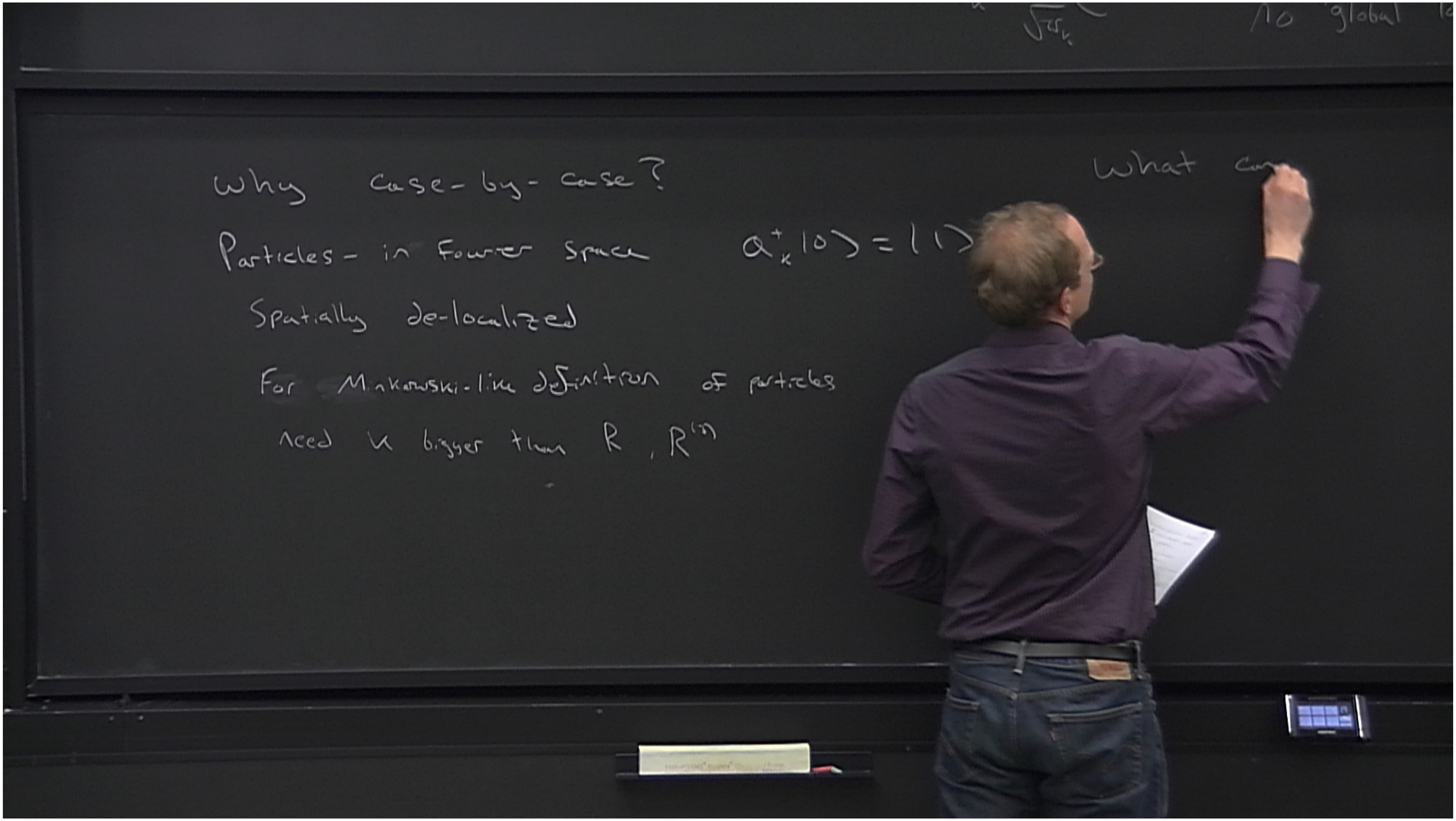
Fields in FRW:
Variable:
But, this is ambiguous!
Bogolyubov transformation:
new mode FN
new ± operators
(V_k and V_k^* are linearly independent Sol'ns)

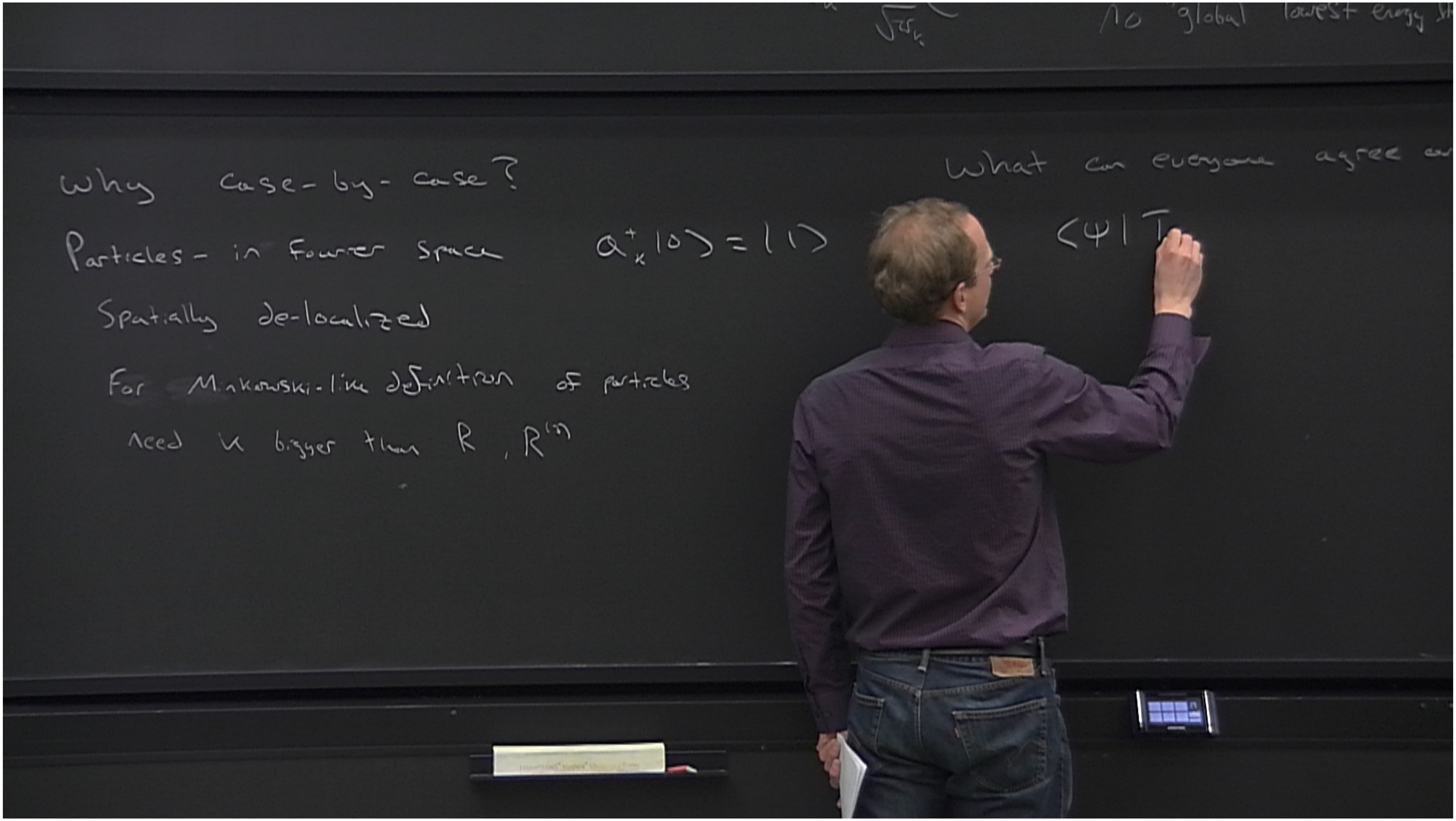

What are Particles?
What is the absence of Particles (what is the vacuum)?
Need a prescription
①

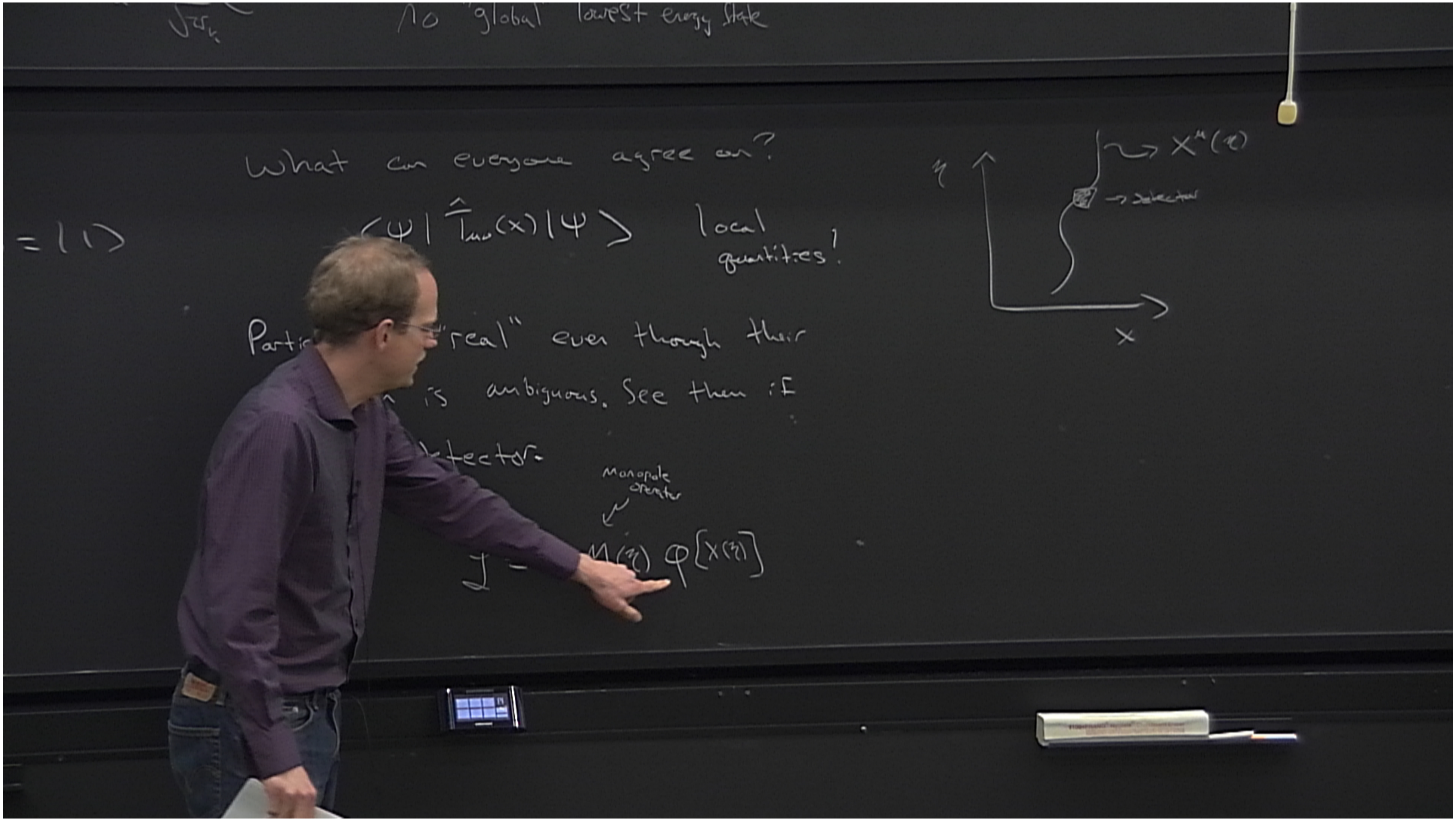

What are Particles?
What is the
of
Particles
vacuum)?
Need a
① Lowest

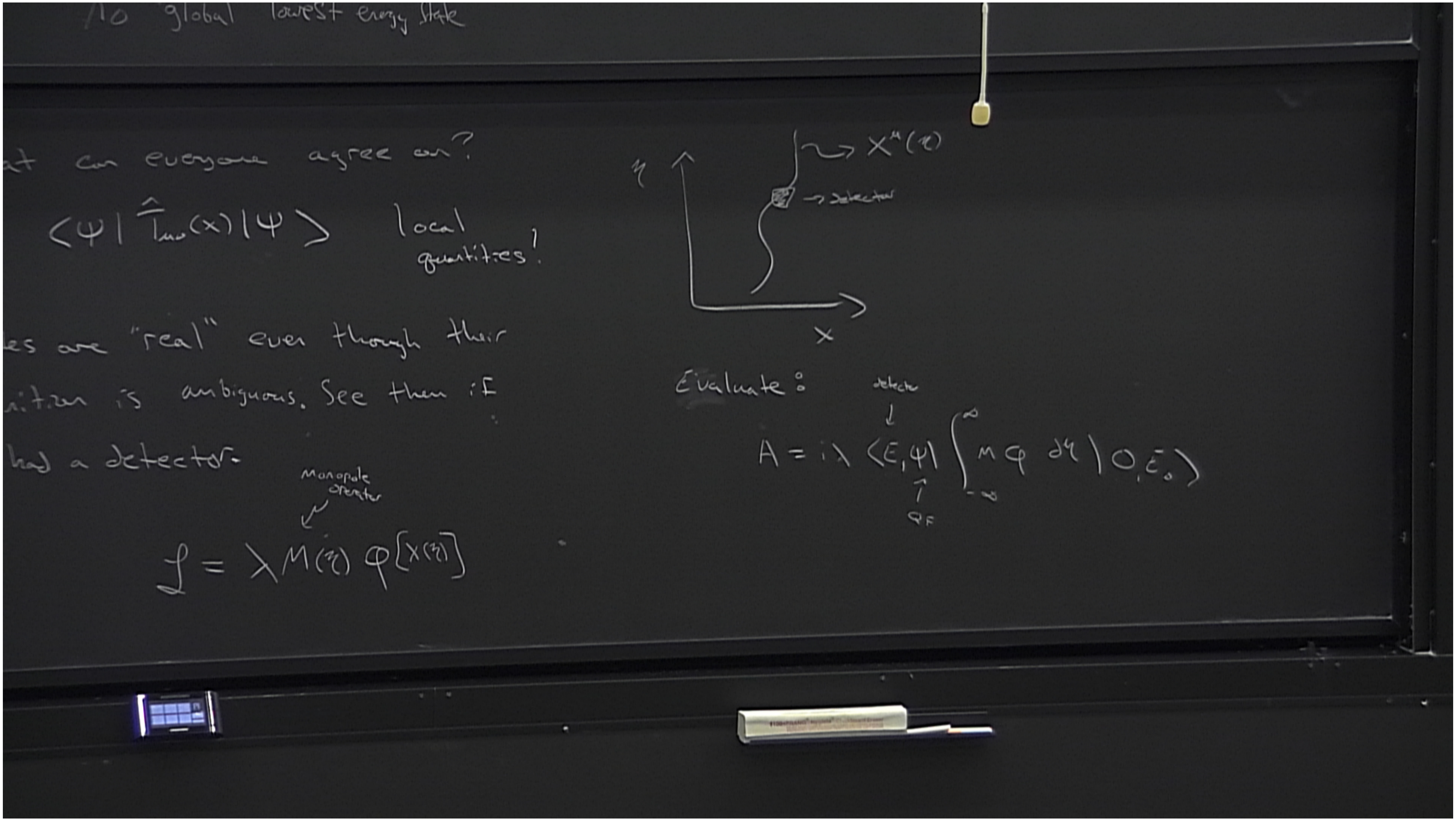

What are Particles?
What is the absence of Particles (what is the vacuum)?
Need a prescription
① Lowest energy State
Minimize $\hat{H}$ → $\langle 0|\hat{H}|0\rangle$
Unambiguous for $H \neq H(\eta)$ but ambiguous $H = H(\eta)$
e.g. in Minkowski:
$V_k = \frac{1}{\sqrt{2\omega_k}} e^{i\omega_k \eta}$
there is an instantaneous lowest energy State, but no "global" lowest energy State

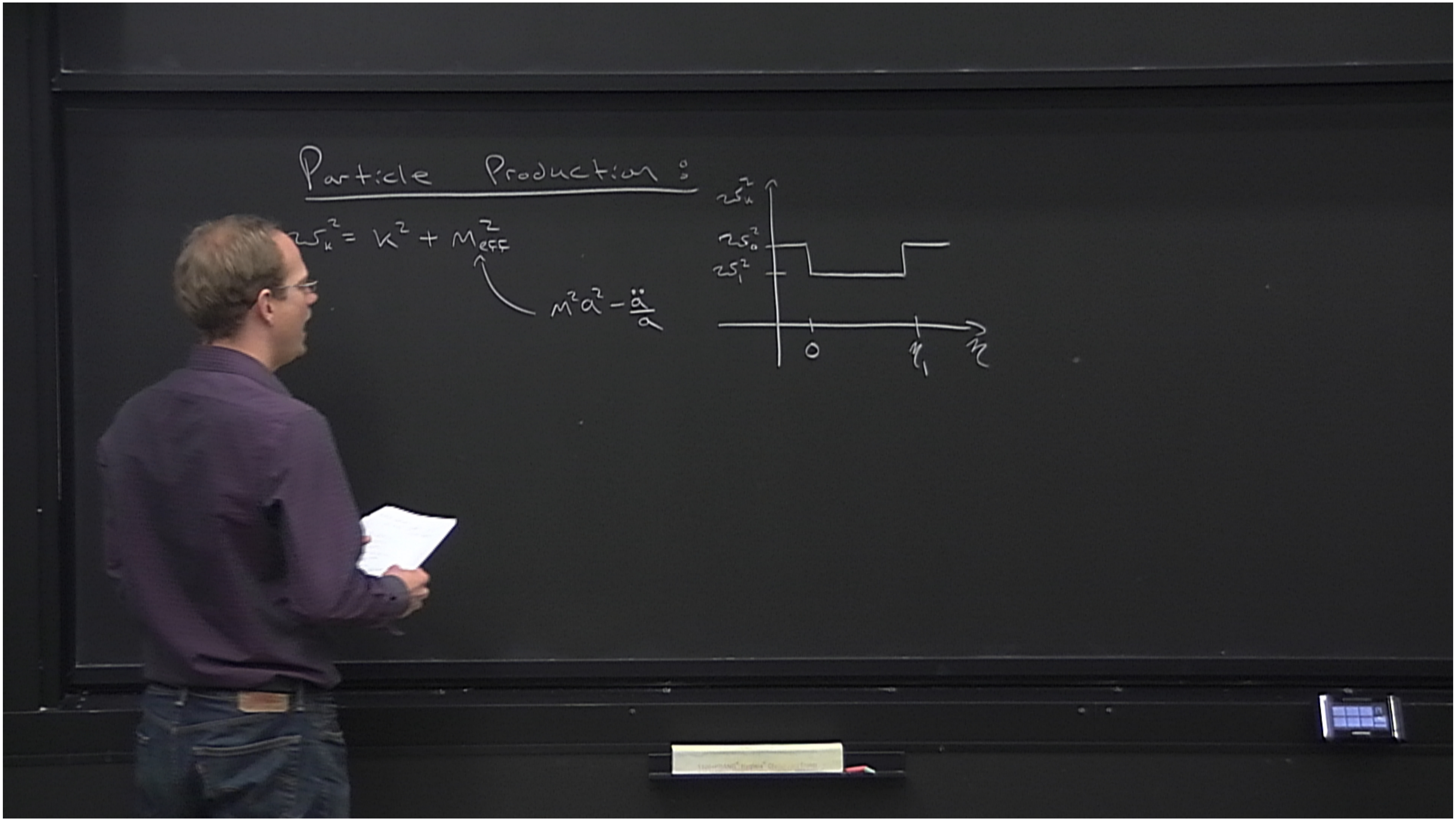

no global
why case-by-case?
Particles - in Fourier space
a+k |0> = |1>
Spatially de-localized

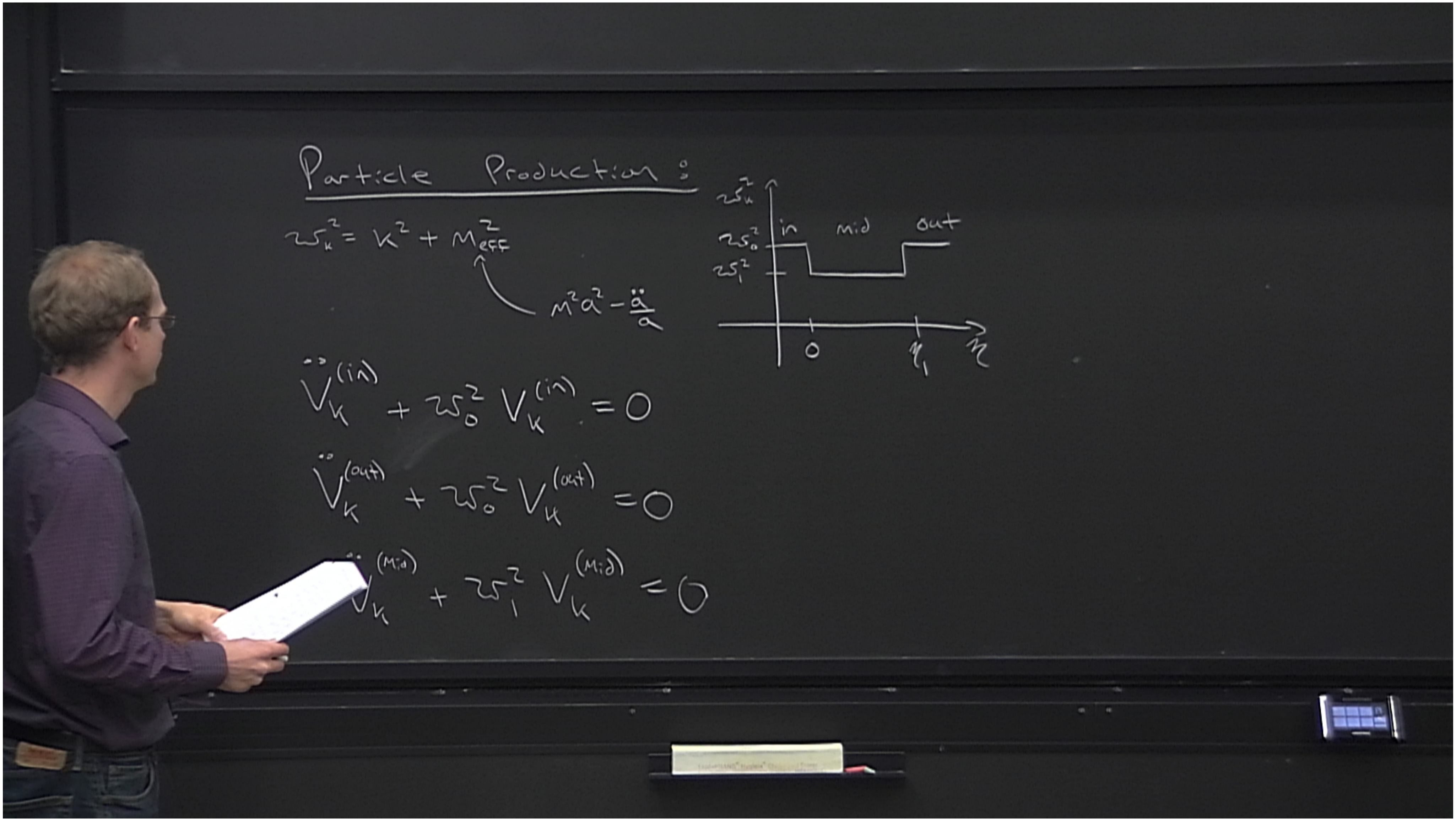

no global
Why case-by-case?
Particles - in Fourier space
a⁺ₖ|0⟩ = |1⟩
Spatially de-localized
For Minkowski-like definition of particles
need k bigger than R, R
What

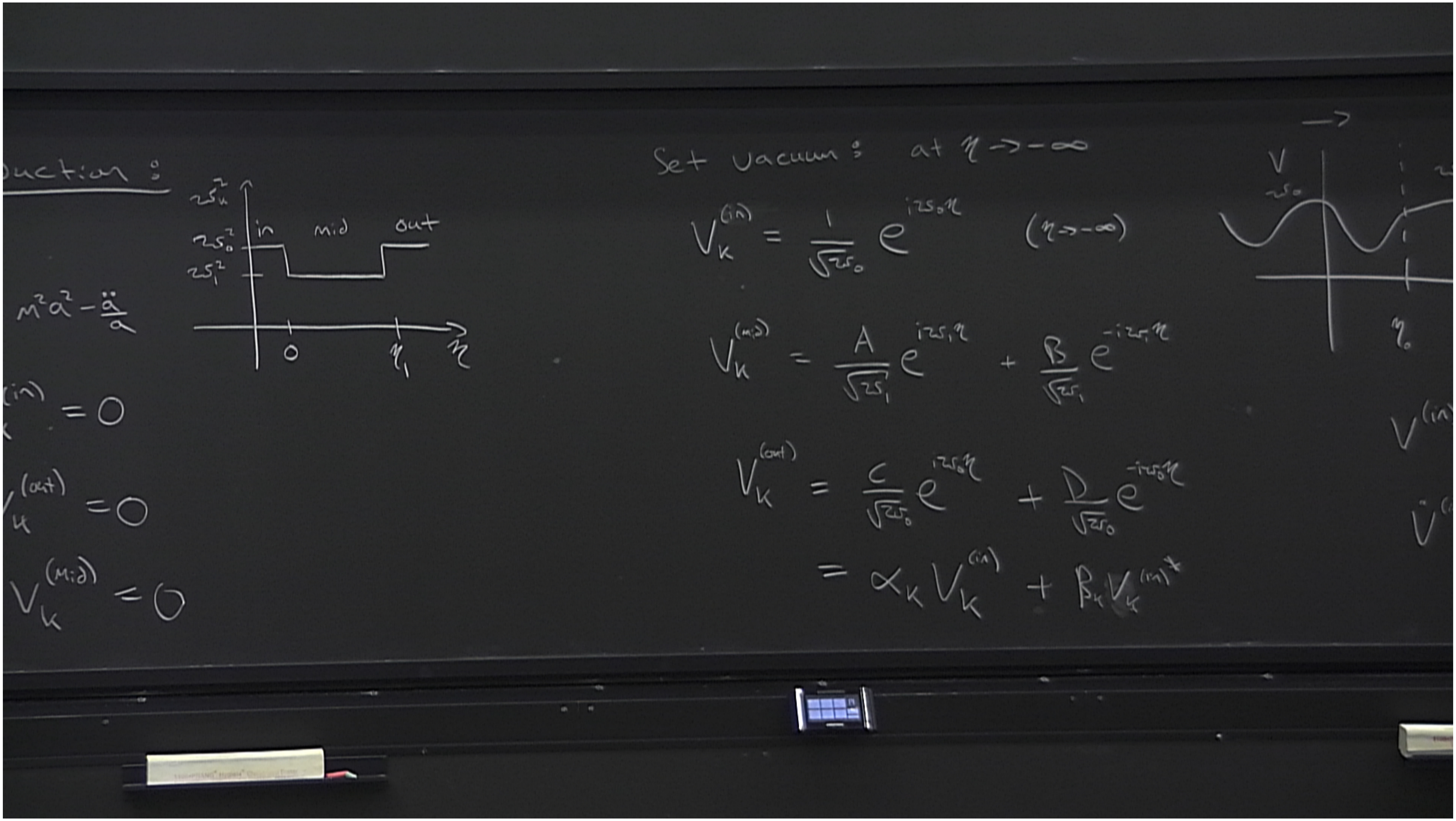

no 'global' lowest energy
Why case-by-case?
Particles - in Fourier space $a^+_k|0\rangle = |1\rangle$
Spatially de-localized
For Minkowski-like definition of particles
need k bigger than R , $R^{(n)}$
What can everyone agree
$\langle\psi|T$


no "global" lowest energy state
What can everyone agree on?
local quantities!
ambiguous. See them if
Monopole operator
x

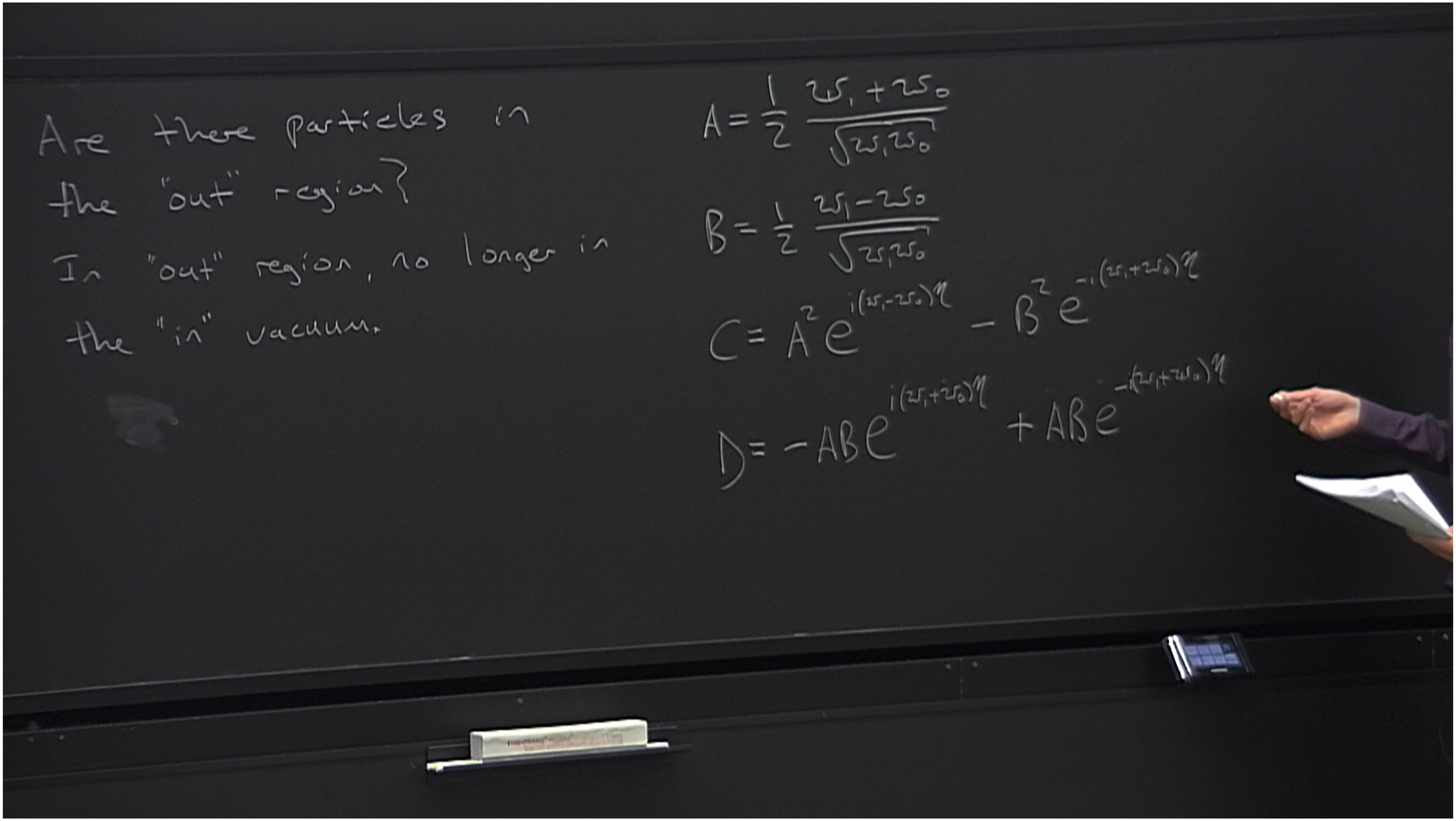

No global lowest energy state
at can everyone agree on?
⟨ψ| T̂μν(x) |ψ⟩ local quantities!
es are "real" even though their
nition is ambiguous. See them if
had a detector.
Monopole operator
L = λ M(τ) φ[x(τ)]
η
x
x^μ(τ)
detector
Evaluate:
detector
A = iλ ⟨E, ψ| ∫ M φ dτ |0, E₀⟩
∞
-∞
φF

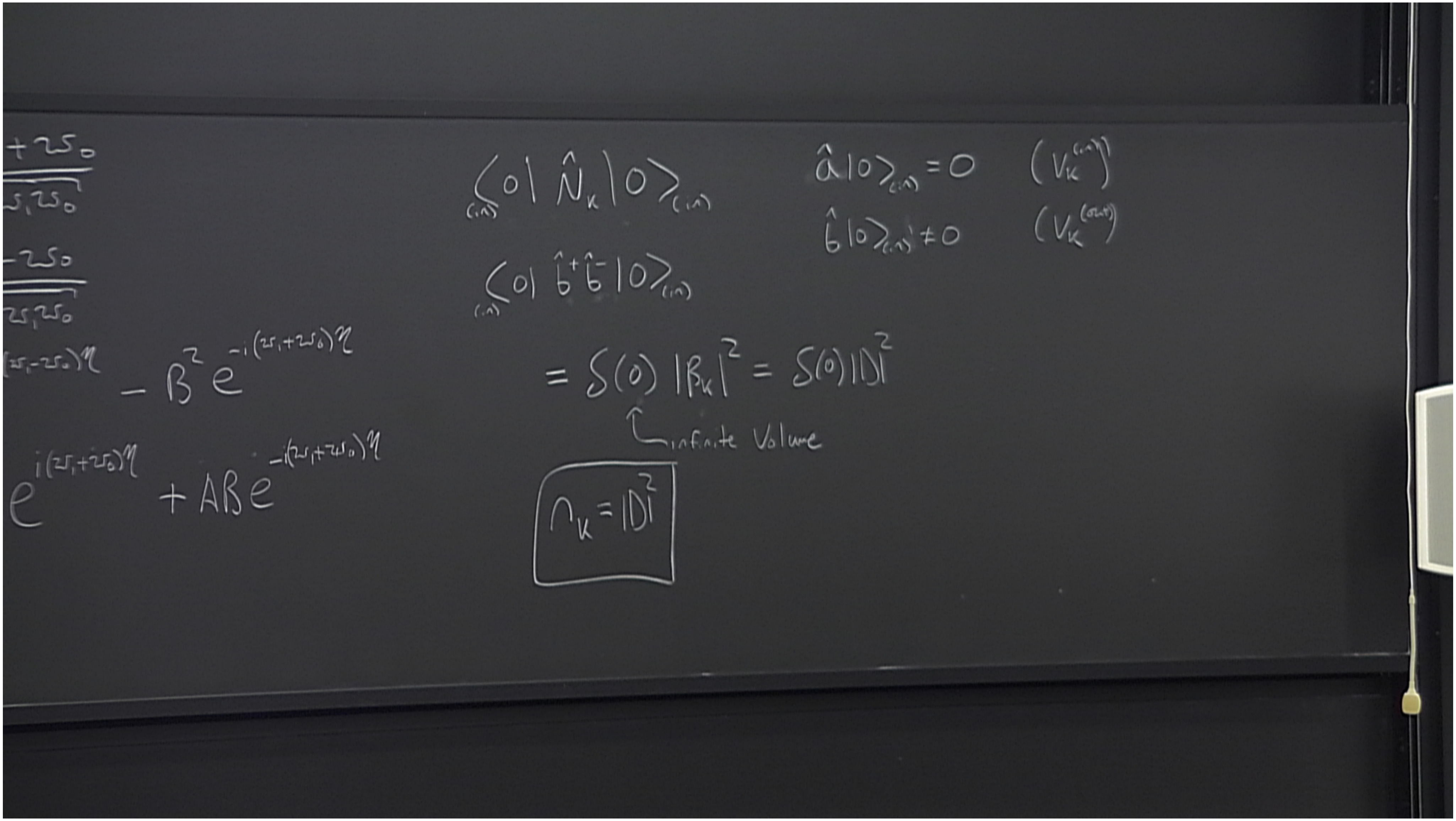

Particle Production:
$\omega_k^2 = k^2 + M_{eff}^2$
$m^2a^2 - \frac{\ddot{a}}{a}$
ω_k^2
ω_0^2
ω_1^2
0
η_1
η

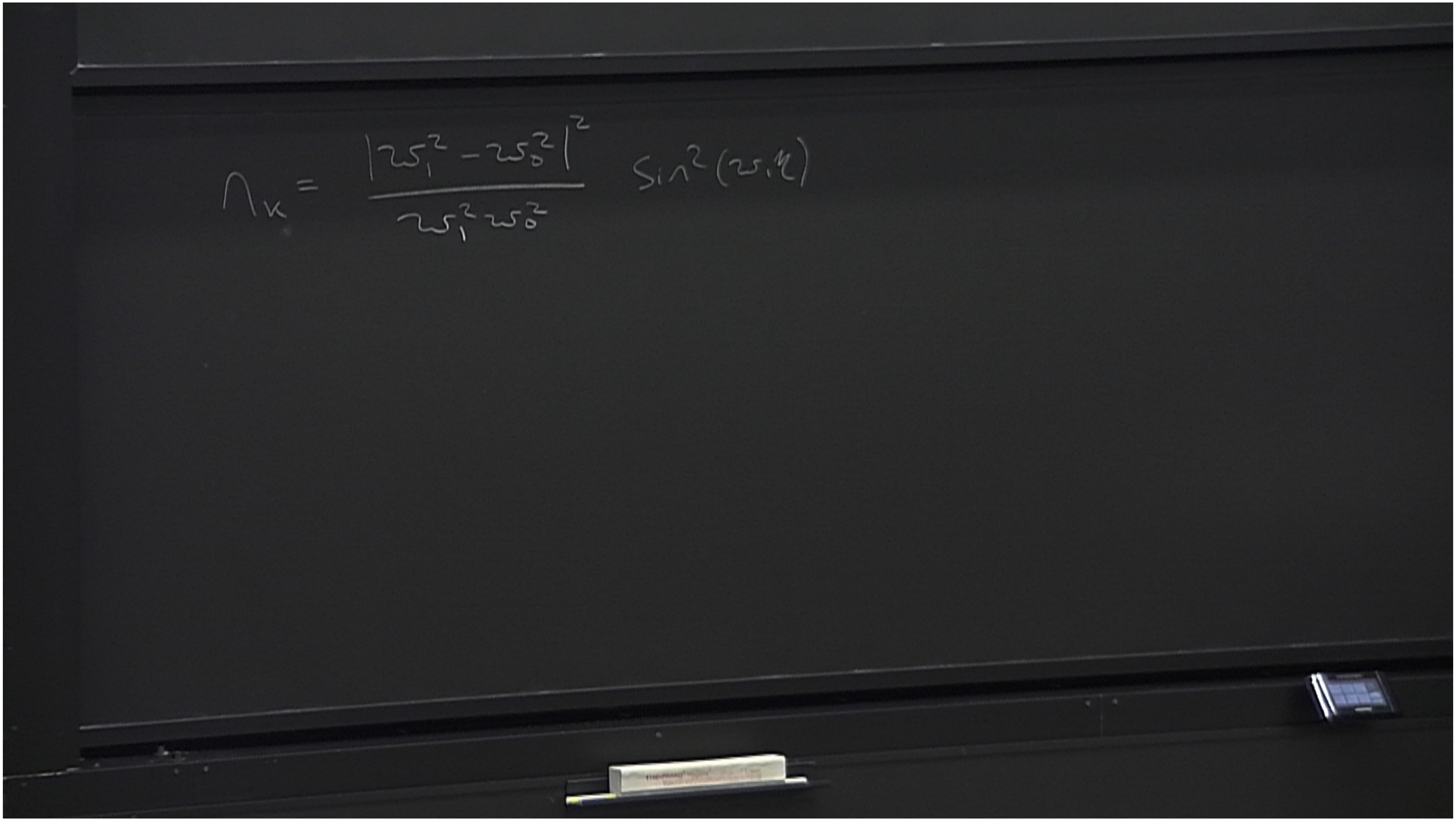

Particle Production:
$\omega_k^2 = k^2 + M_{eff}^2$
$m^2 a^2 - \frac{\ddot{a}}{a}$
$\ddot{V}_k^{(in)} + \omega_0^2 V_k^{(in)} = 0$
$\ddot{V}_k^{(out)} + \omega_0^2 V_k^{(out)} = 0$
$\ddot{V}_k^{(Mid)} + \omega_1^2 V_k^{(Mid)} = 0$
ω_k^2
ω_0^2
ω_1^2
in
mid
out
0
η_1
η

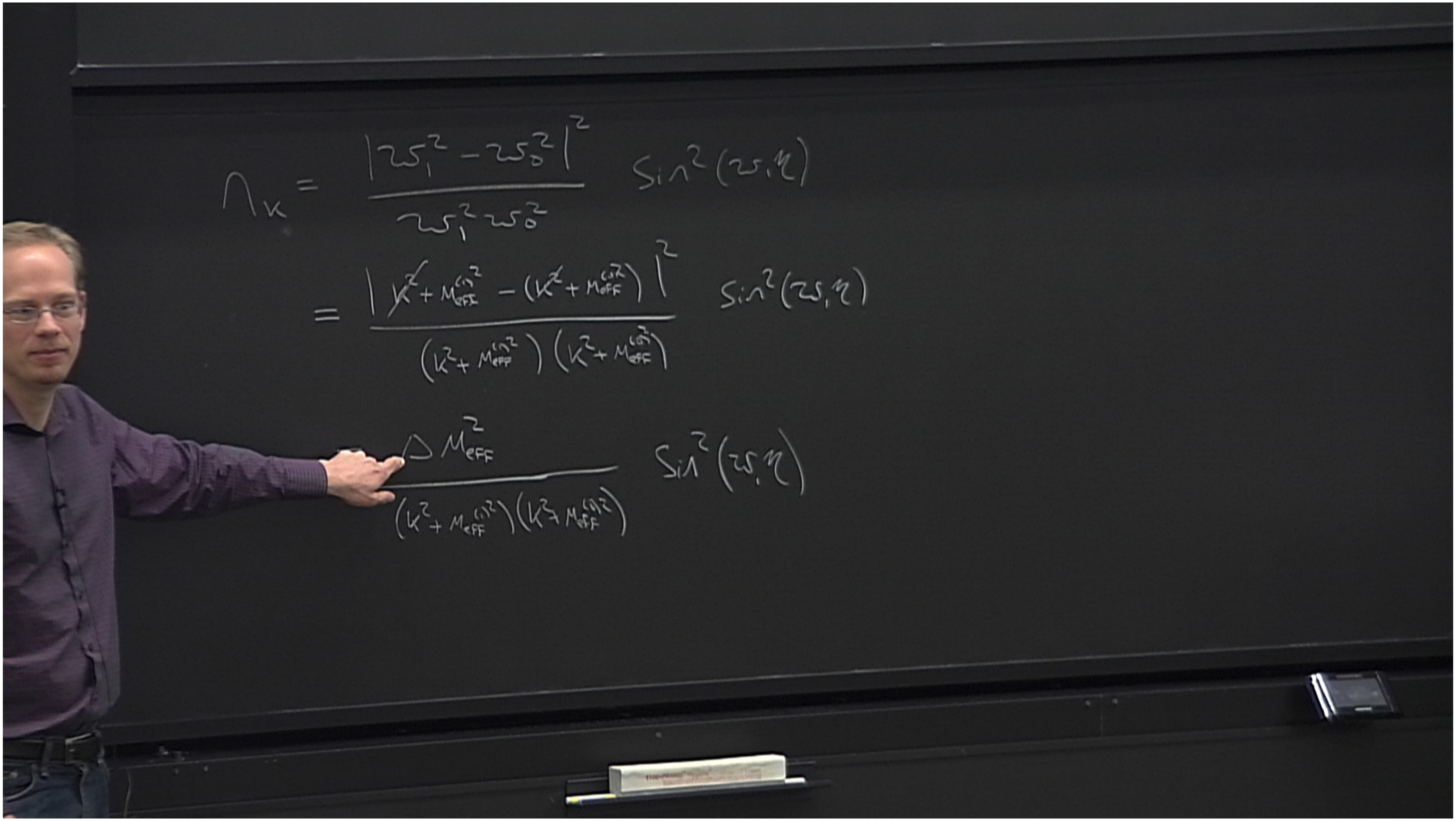


uction:
$2S_k^2$
$2S_0^2$
$2S_1^2$
in
mid
out
0
η_1
η
$m^2a^2 - \frac{\ddot{a}}{a}$
$V_k^{(in)} = 0$
$V_k^{(out)} = 0$
$V_k^{(mid)} = 0$
Set vacuum: at $\eta \to -\infty$
$V_k^{(in)} = \frac{1}{\sqrt{2S_0}} e^{i2S_0\eta} \quad (\eta \to -\infty)$
$V_k^{(mid)} = \frac{A}{\sqrt{2S_1}} e^{i2S_1\eta} + \frac{B}{\sqrt{2S_1}} e^{-i2S_1\eta}$
$V_k^{(out)} = \frac{C}{\sqrt{2S_0}} e^{i2S_0\eta} + \frac{D}{\sqrt{2S_0}} e^{-i2S_0\eta}$
$= \alpha_k V_k^{(in)} + \beta_k V_k^{(in)*}$
V
η_0

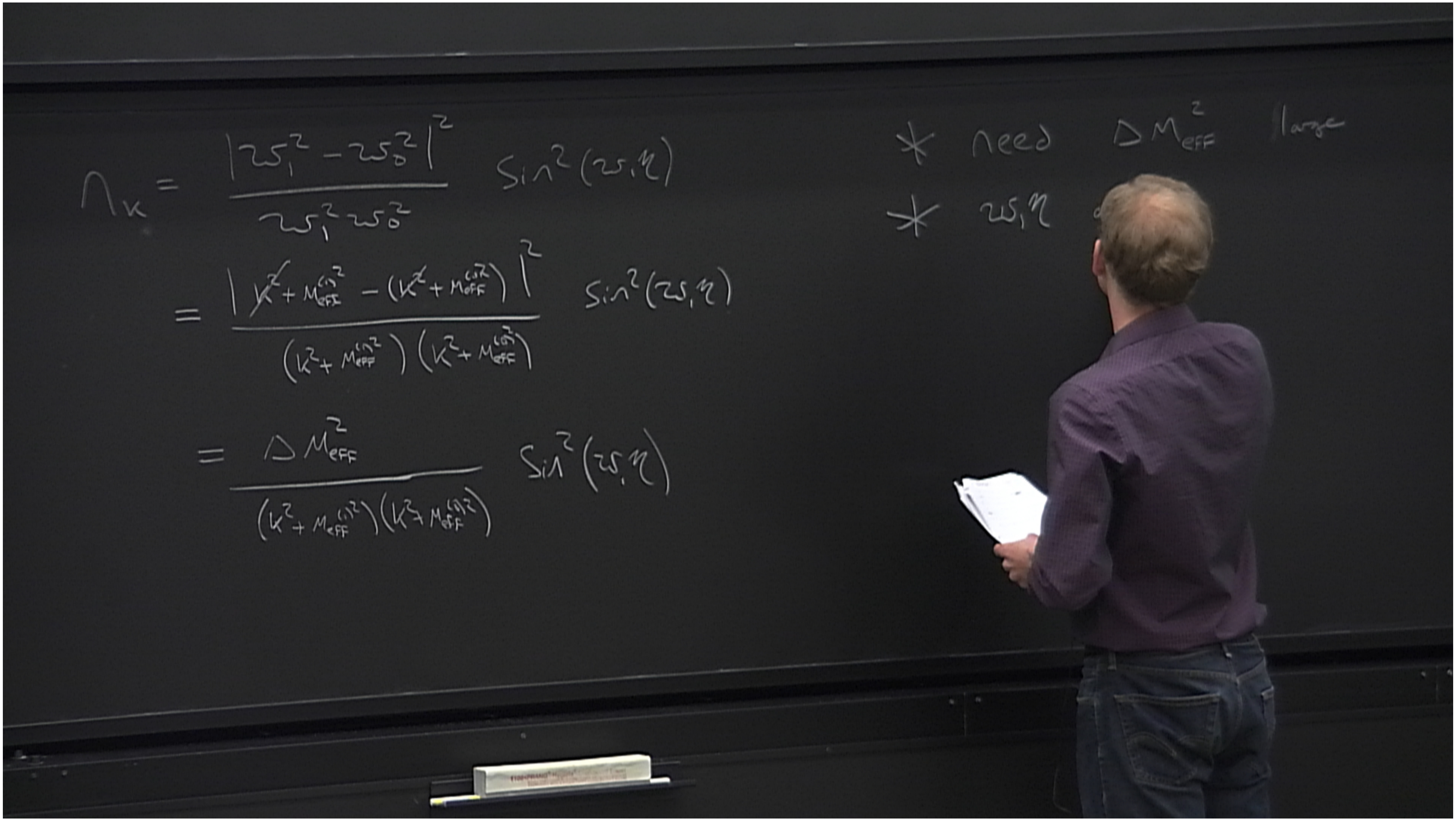

Are there particles in the "out" region?
In "out" region, no longer in the "in" vacuum.
$A = \frac{1}{2}\frac{2\omega_1 + 2\omega_0}{\sqrt{2\omega_1 2\omega_0}}$
$B = -\frac{1}{2}\frac{2\omega_1 - 2\omega_0}{\sqrt{2\omega_1 2\omega_0}}$
$C = A^2 e^{i(2\omega_1 - 2\omega_0)\eta} - B^2 e^{-i(2\omega_1 + 2\omega_0)\eta}$
$D = -AB e^{i(2\omega_1 + 2\omega_0)\eta} + AB e^{-i(2\omega_1 + 2\omega_0)\eta}$

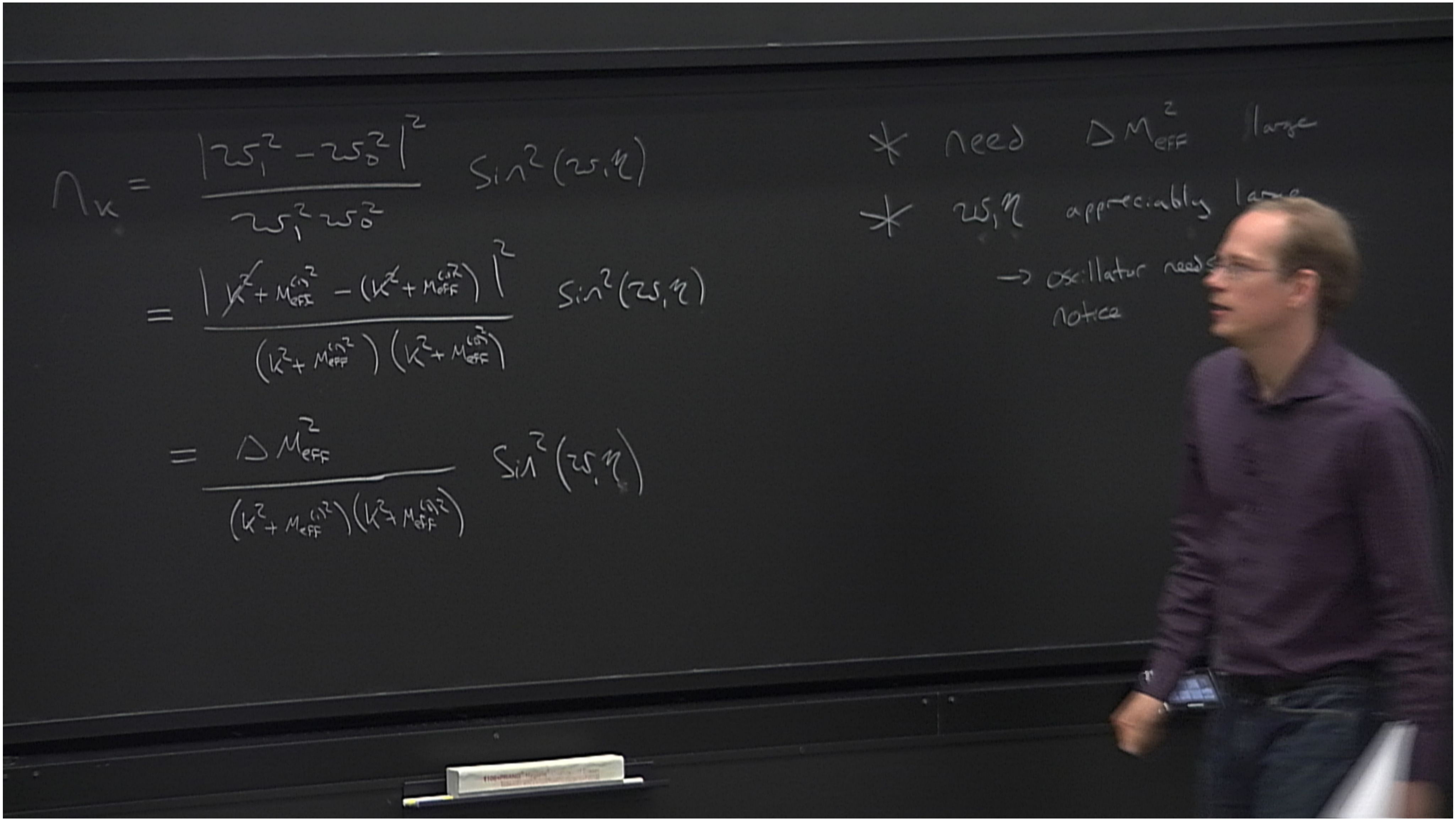

⟨0| N̂_k |0⟩ (in)
â|0⟩(in) = 0 (V_k^(in))
b̂|0⟩(in) ≠ 0 (V_k^(out))
⟨0| b̂⁺ b̂⁻ |0⟩ (in)
= δ(0) |β_k|² = δ(0) |D|²
infinite Volume
n_k = |D|²
− β² e^{−i(ω₁+ω₀)η}
+ Aβ e^{−i(ω₁+ω₀)η}

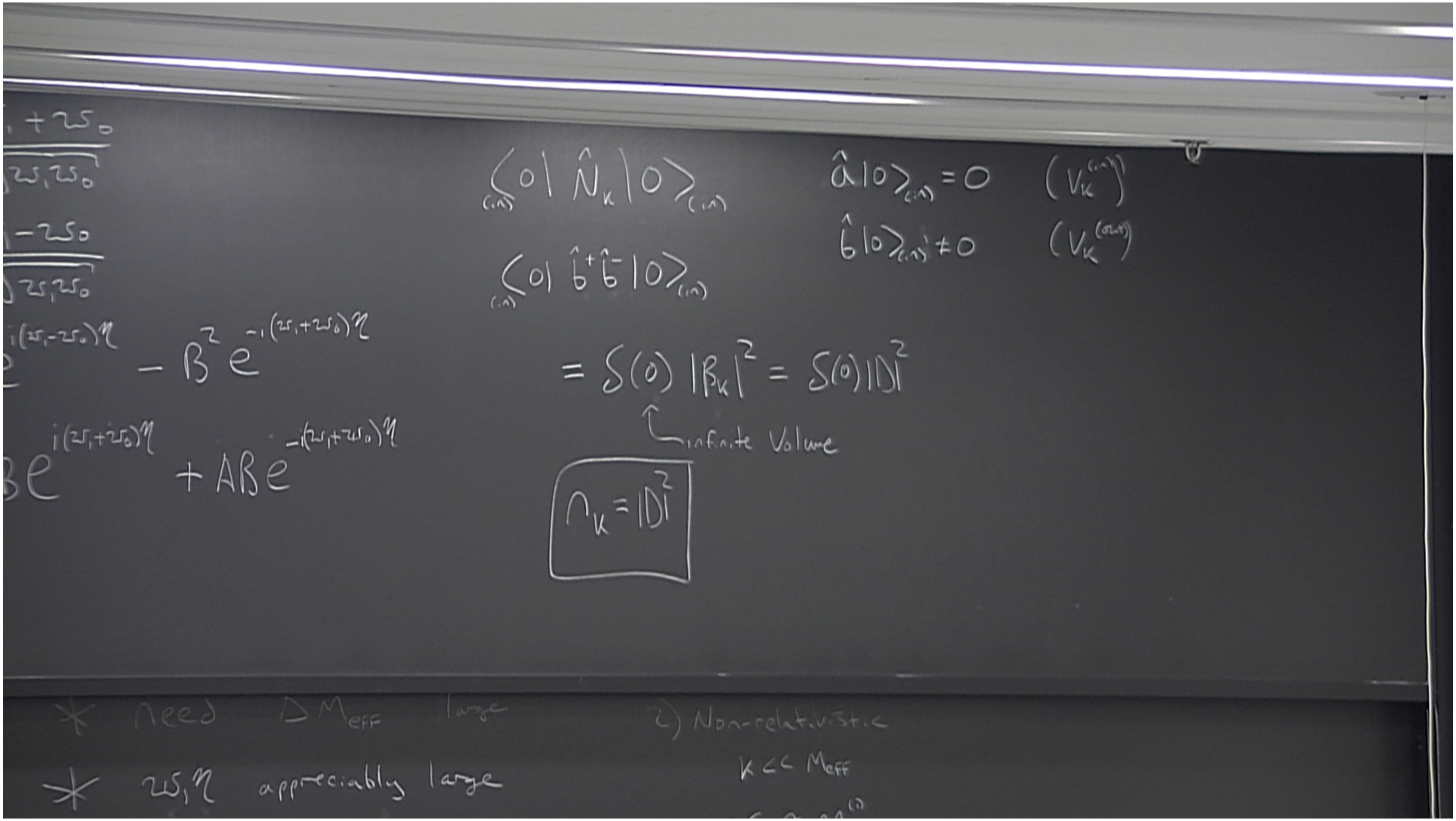

$\Lambda_k = \frac{|\varpi_1^2 - \varpi_0^2|^2}{\varpi_1^2 \varpi_0^2} \sin^2(\varpi_1 \eta)$



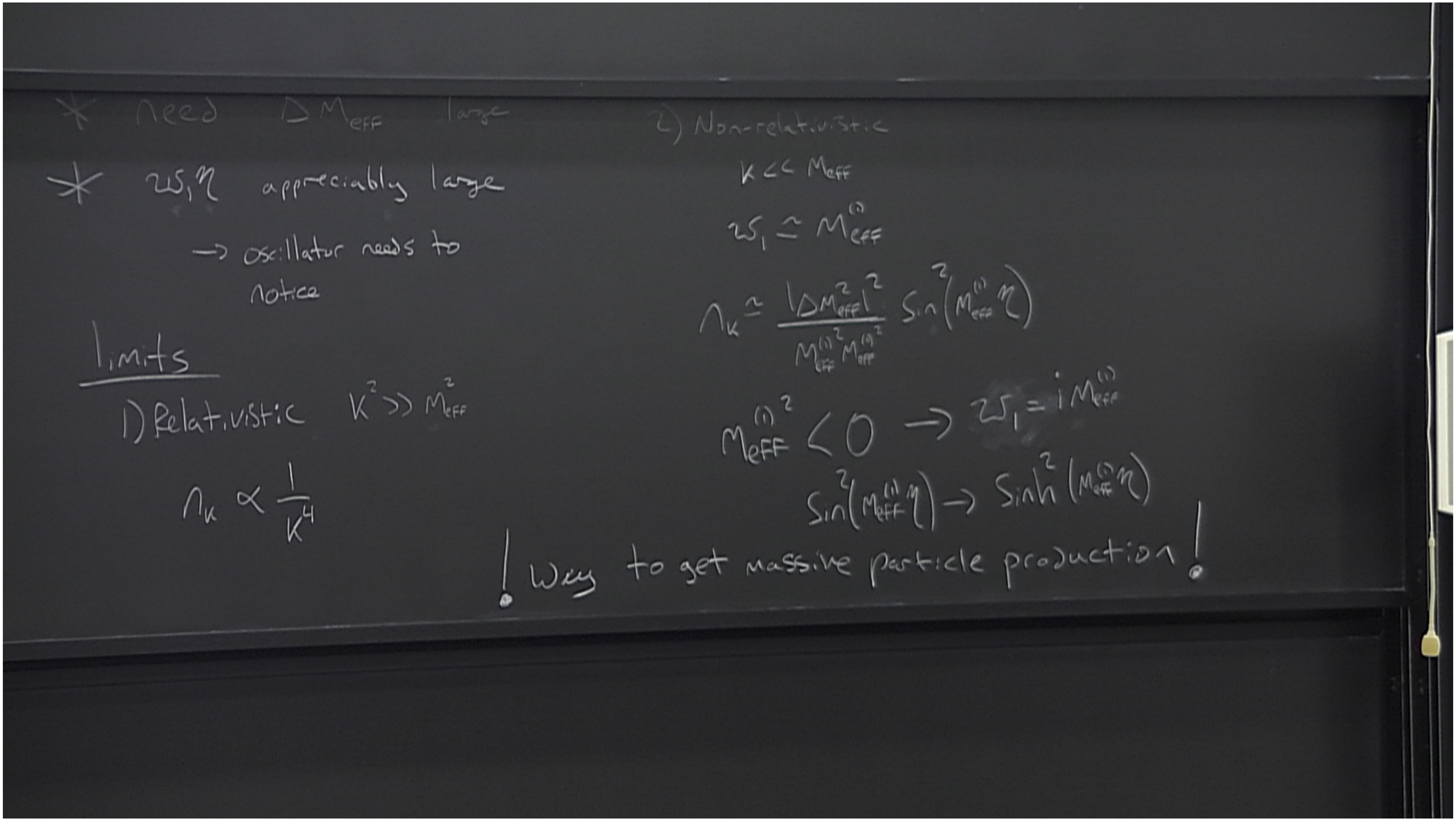
$\Lambda_k = \frac{|\omega_1^2 - \omega_0^2|^2}{\omega_1^2 \omega_0^2} \sin^2(\omega_1 \eta)$
$= \frac{|k^2 + M_{eff}^{(in)2} - (k^2 + M_{eff}^{(out)2})|^2}{(k^2 + M_{eff}^{(in)2})(k^2 + M_{eff}^{(out)2})} \sin^2(\omega_1 \eta)$
$= \frac{\Delta M_{eff}^2}{(k^2 + M_{eff}^{(i)2})(k^2 + M_{eff}^{(o)2})} \sin^2(\omega_1 \eta)$

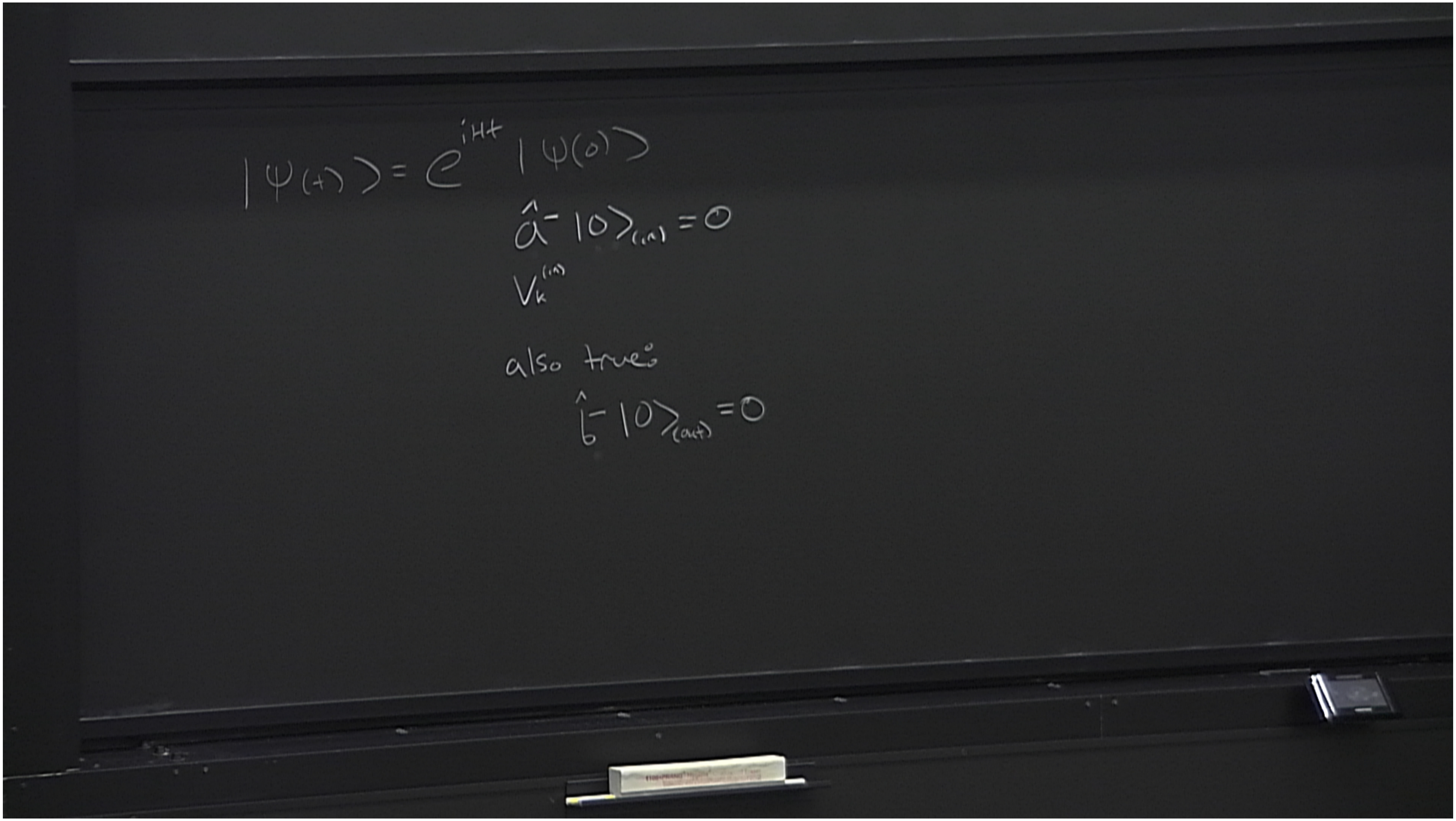

$n_k = \frac{|\omega_1^2 - \omega_0^2|^2}{\omega_1^2 \omega_0^2} \sin^2(\omega_1 \eta)$
$= \frac{|k^2 + M_{eff}^{(1)2} - (k^2 + M_{eff}^{(0)2})|^2}{(k^2 + M_{eff}^{(1)2})(k^2 + M_{eff}^{(0)2})} \sin^2(\omega_1 \eta)$
$= \frac{\Delta M_{eff}^2}{(k^2 + M_{eff}^{(1)2})(k^2 + M_{eff}^{(0)2})} \sin^2(\omega_1 \eta)$
* need ΔM_{eff}^2 large
* $\omega_1 \eta$


$\Omega_k = \frac{|\omega_1^2 - \omega_0^2|^2}{\omega_1^2 \omega_0^2} \sin^2(\omega_1 \eta)$
$= \frac{|k^2 + M_{eff}^{(i)2} - (k^2 + M_{eff}^{(i)2})|^2}{(k^2 + M_{eff}^{(i)2})(k^2 + M_{eff}^{(i)2})} \sin^2(\omega_1 \eta)$
$= \frac{\Delta M_{eff}^2}{(k^2 + M_{eff}^{(i)2})(k^2 + M_{eff}^{(i)2})} \sin^2(\omega_1 \eta)$
* need ΔM_{eff}^2 large
* $\omega_1 \eta$ appreciably
→ oscillator
notice



${}_{(in)}\langle 0 | \hat{N}_k | 0 \rangle_{(in)}$
${}_{(in)}\langle 0 | \hat{b}^{+} \hat{b}^{-} | 0 \rangle_{(in)}$
$= \delta(0) |\beta_k|^2 = \delta(0) |D|^2$
infinite Volume
$n_k = |D|^2$
$\hat{a} |0\rangle_{(in)} = 0$
$(V_k^{(in)})$
$\hat{b} |0\rangle_{(in)} \neq 0$
$(V_k^{(out)})$
$- \beta^2 e^{-i(2\sigma_1 + 2\sigma_0)\eta}$
$+ A\beta e^{-i(2\sigma_1 + 2\sigma_0)\eta}$
* need ΔM_{eff} large
* $2\sigma_1 \eta$ appreciably large
2) Non-relativistic
$k << M_{eff}$


* need ΔM_eff large
* ϖ₁η appreciably large
→ oscillator needs to notice
Limits
1) Relativistic k² >> M²_eff
n_k ∝ 1/k⁴
2) Non-relativistic
k << M_eff
ϖ₁ ≃ M_eff^(1)
n_k ≃ |ΔM²_eff|² / (M_eff^(1)² M_eff^(2)²) sin²(M_eff^(1) η)
M_eff^(1)² < 0 → ϖ₁ = i M_eff^(1)
sin²(M_eff^(1) η) → sinh²(M_eff^(1) η)
! Way to get massive particle production !


|ψ(t)> = e^{iHt} |ψ(0)>
â⁻ |0>_(in) = 0
V_k^(in)
also true:
b̂⁻ |0>_(out) = 0