Title: 12/13 PSI - Explorations in Cosmology Lecture 10

Date: Apr 19, 2013 10:15 AM

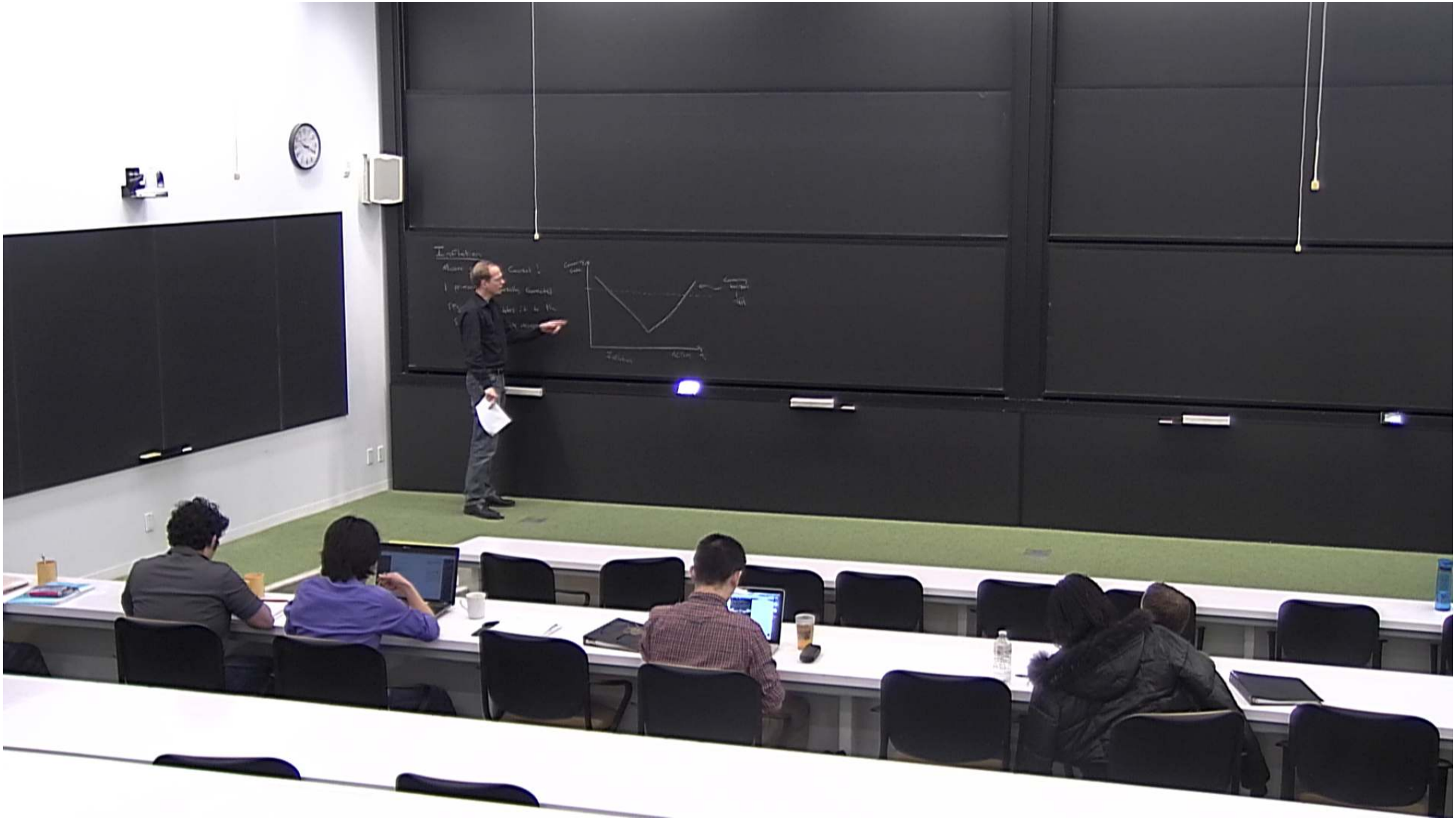
URL: http://pirsa.org/13040073

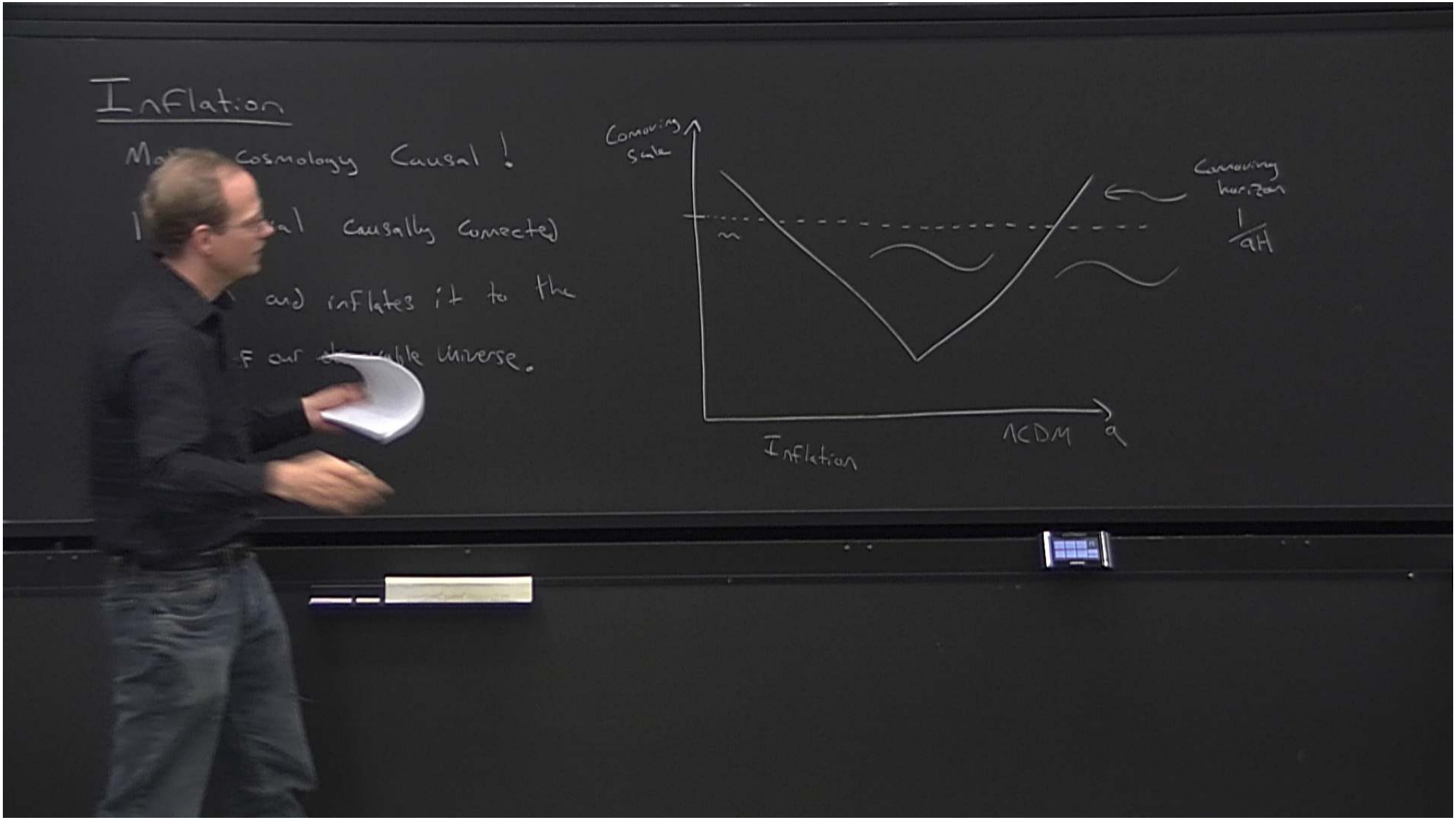
Abstract:

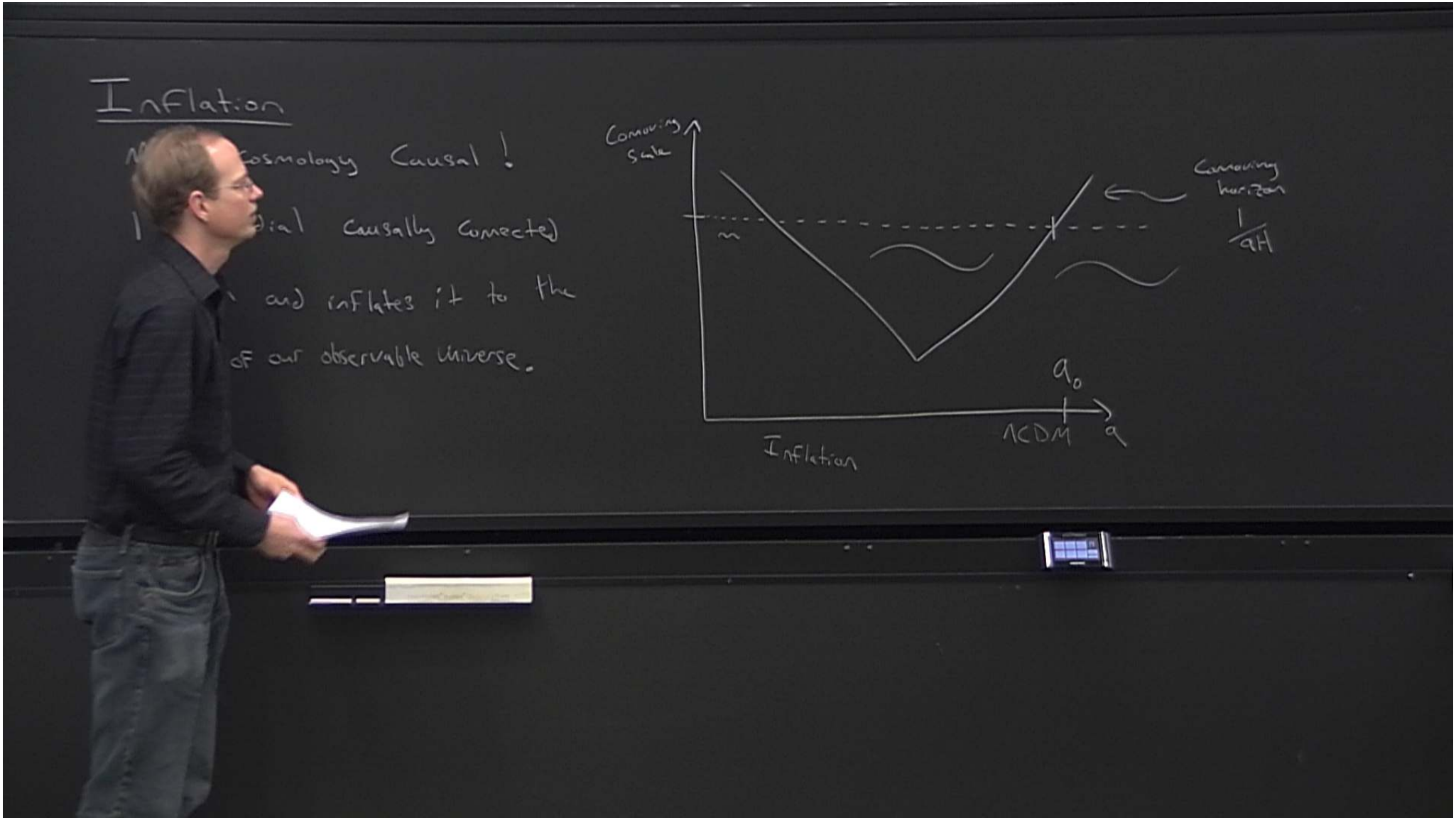
Inflation

Makes cosmology causal!

1 primordial causally connected region and inflates it to the size of our observable universe.




Inflation
Causal !
Causally Connected
and inflates it to the
Comoving scale
Comoving horizon
1/aH
Inflation
ΛCDM
a


Inflation
cosmology Causal !
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and inflates it to the
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Comoving scale
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1/aH
q_0
ΛCDM
q
Inflation

Inflation

Makes cosmology Causal !

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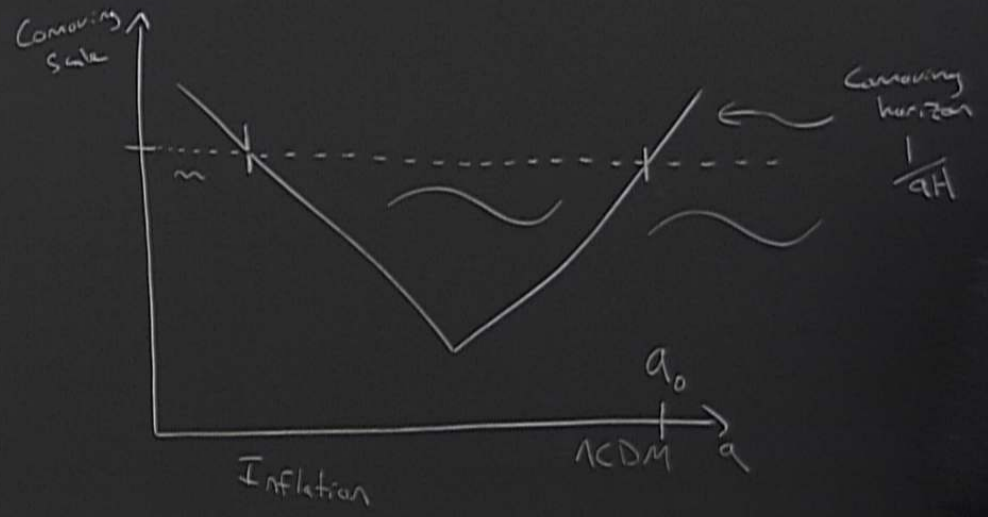
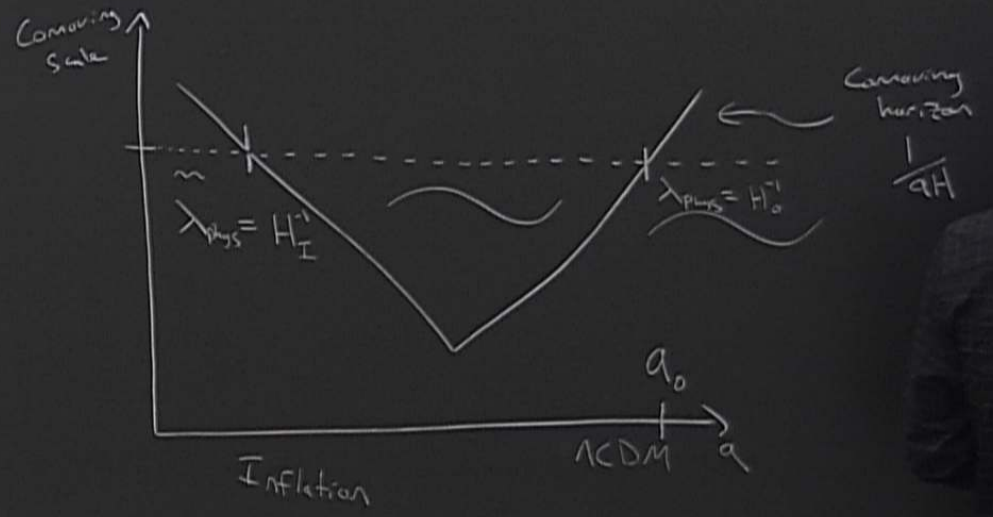
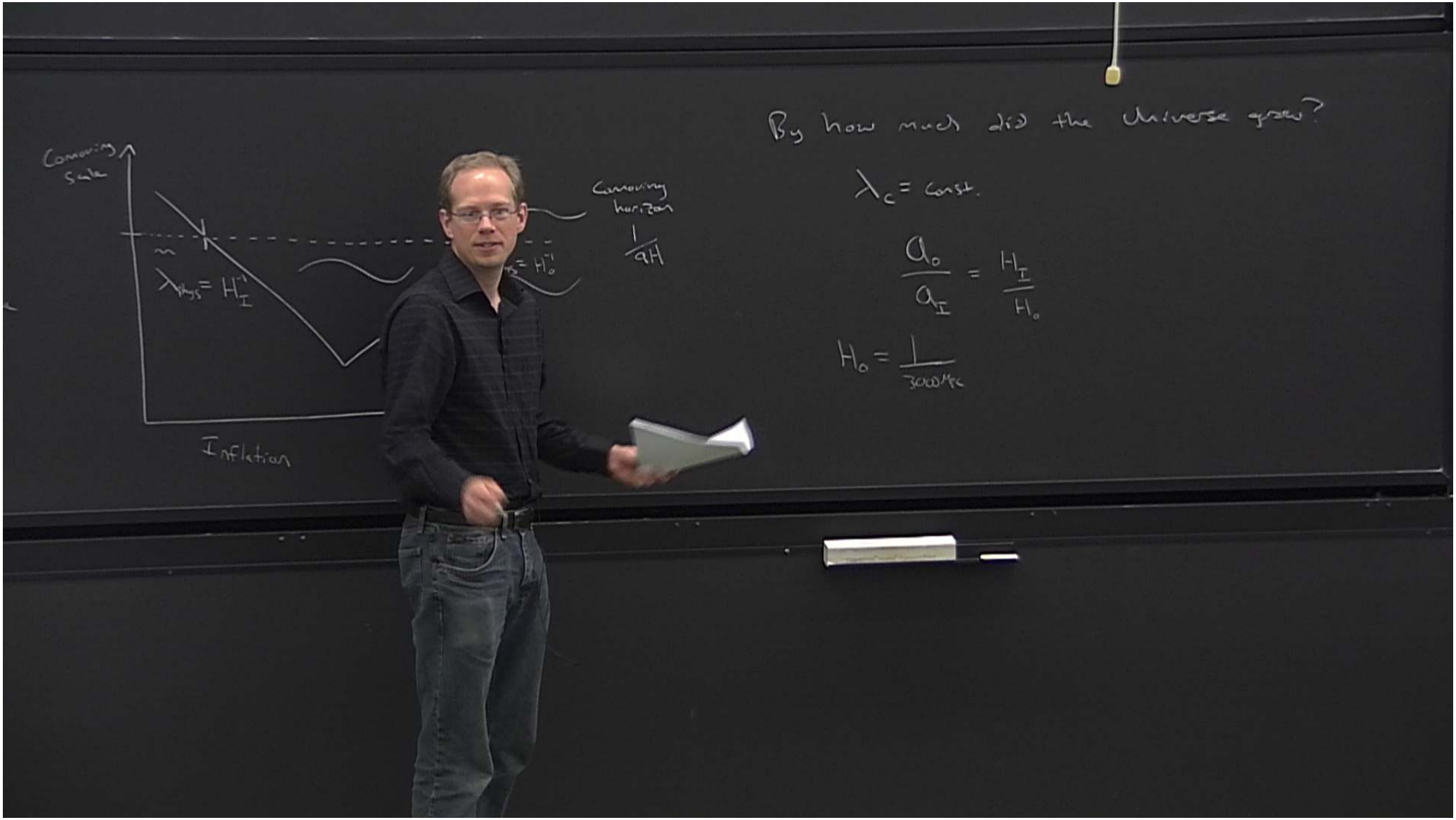


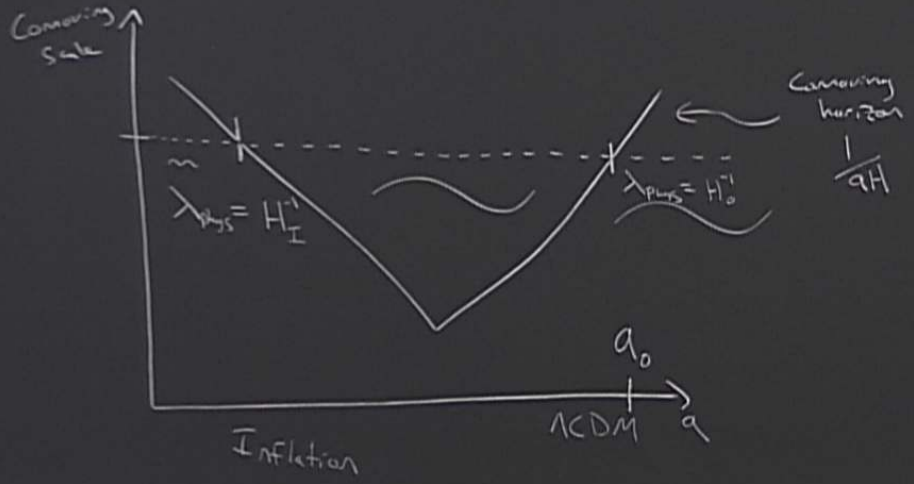
Inflation

Makes cosmology Causal!

1 primordial causally connected region and inflates it to the size of our observable universe.




By how much did the Universe grow?
λ_c = const.
$\frac{a_0}{a_I} = \frac{H_I}{H_0}$
$H_0 = \frac{1}{3000 Mpc}$
Comoving scale
Comoving horizon
$\frac{1}{aH}$
$\lambda_{phys} = H_I^{-1}$
$= H_0^{-1}$
Inflation



By how much did the Universe grow?

$\lambda_c = \text{const.}$

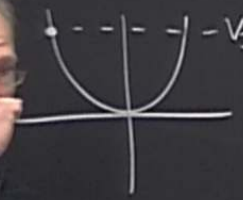
$$\frac{a_0}{a_I} = \frac{H_I}{H_0}$$

$$H_0 = \frac{1}{3000\,\text{Mpc}} = 10^{-26}\,\text{m}^{-1}$$

Inflation

Slow-roll inflation - Potential energy domination

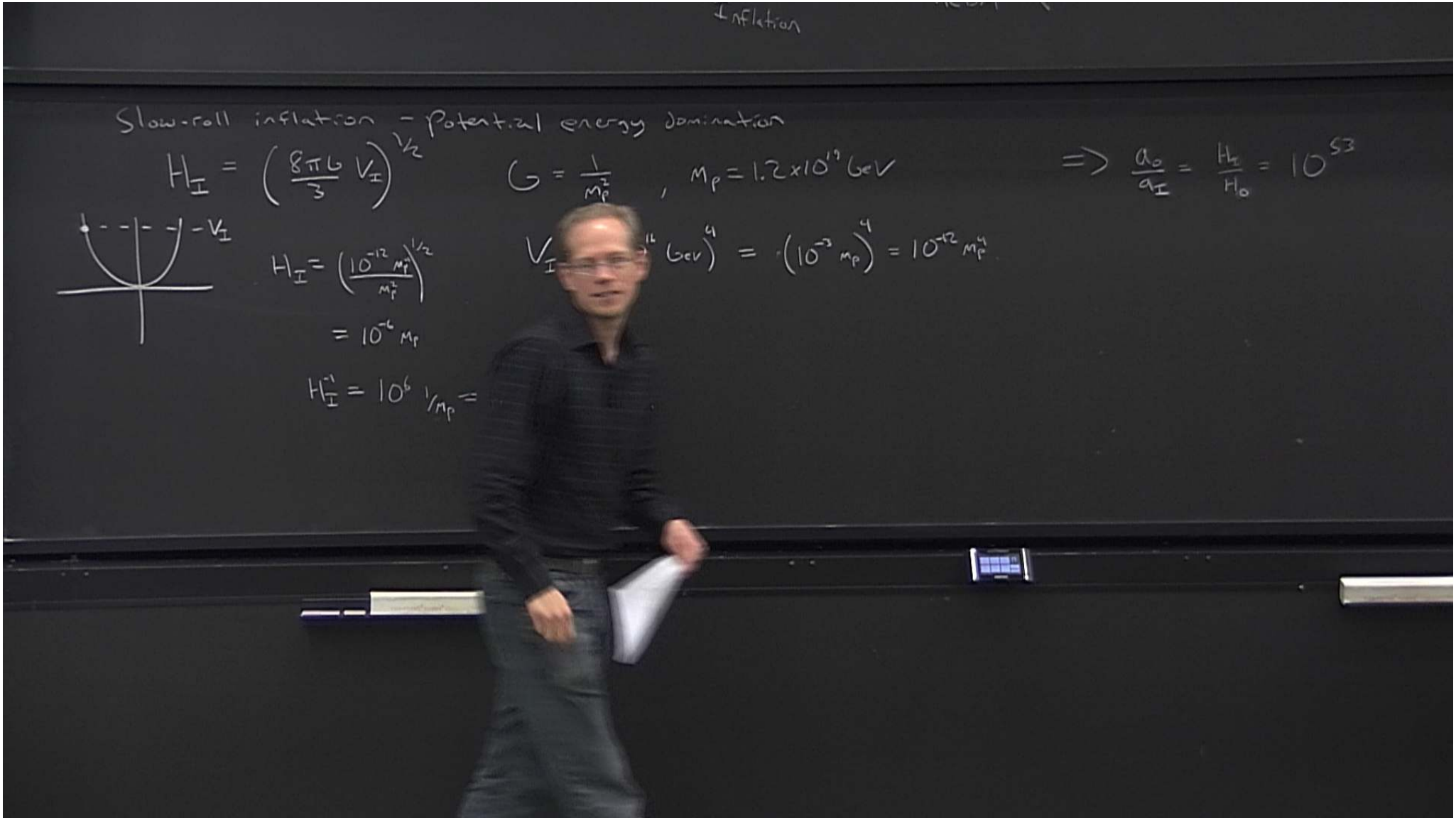
$$H_I = \left(\frac{8\pi G}{3} V_I\right)^{1/2} \qquad G = \frac{1}{M_P^2}, \quad M_P = 1.2 \times 10^{19}\ \text{GeV}$$



$$H_I = \left(\frac{10^{-12} M_P^4}{M_P^2}\right)^{1/2} \qquad V_I = \left(10^{16}\ \text{GeV}\right)^4 = \left(10^{-3} M_P\right)^4 = 10^{-12} M_P^4$$

$$= 10^{-6} M_P$$

$$H_I^{-1} = 10^6 \, 1/M_P = 10^6 \, l_P = 10^{-27}\ \text{m}$$


Inflation
Slow-roll inflation - Potential energy domination
$H_I = \left(\frac{8\pi G}{3} V_I\right)^{1/2}$
$G = \frac{1}{M_p^2}$, $M_p = 1.2 \times 10^{19}$ GeV
$\Rightarrow \frac{a_0}{a_I} = \frac{H_I}{H_0} = 10^{53}$
$-V_I$
$H_I = \left(\frac{10^{-12} M_p^2}{M_p^2}\right)^{1/2}$
$= 10^{-6} M_p$
$H_I^{-1} = 10^6 \, 1/M_p =$
V_I ... GeV)^4 = $(10^{-3} M_p)^4 = 10^{-12} M_p^4$

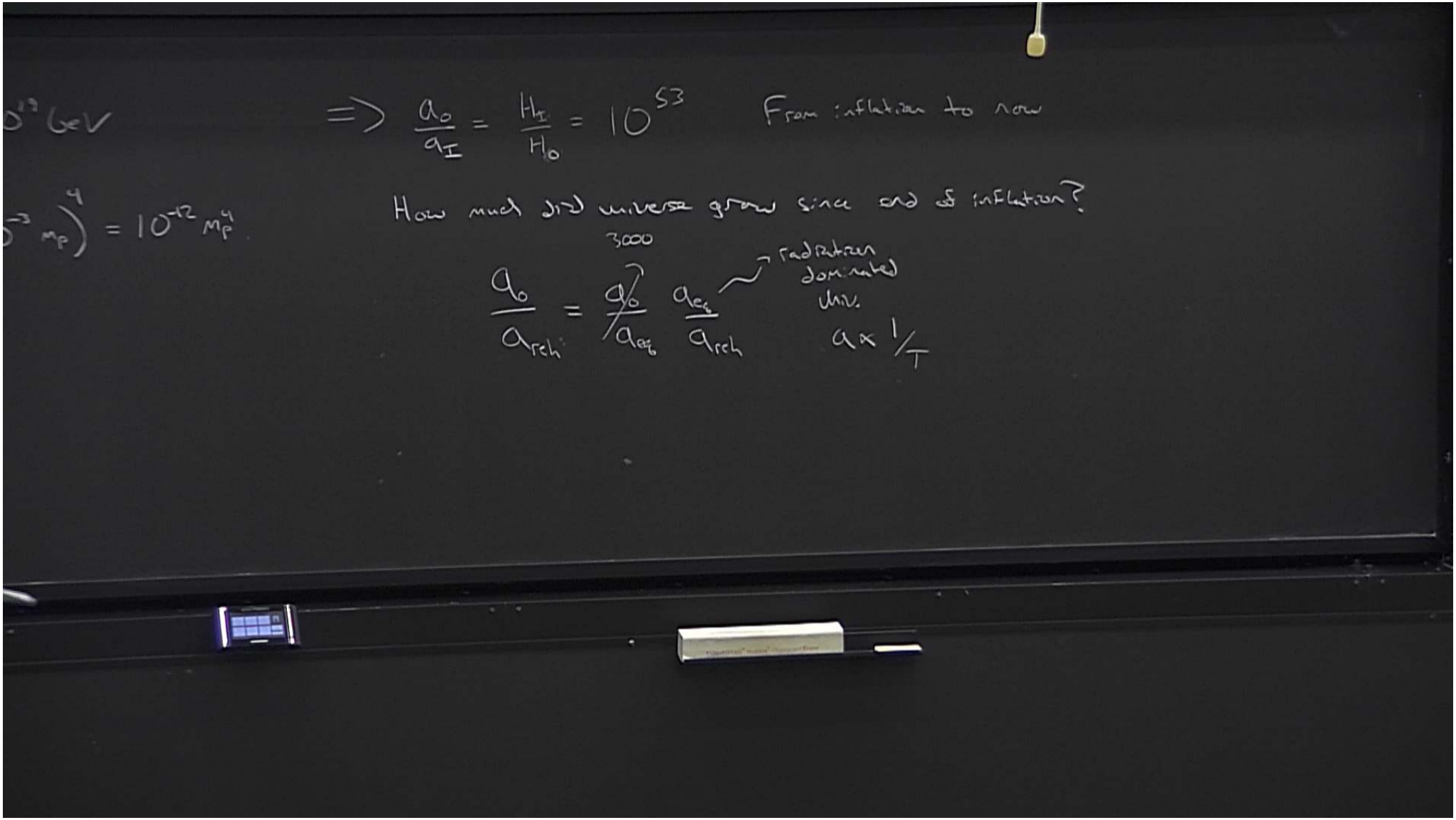
$\Rightarrow \frac{a_0}{a_I} = \frac{H_I}{H_0} = 10^{53}$ From inflation to now

$\left(10^{-3} M_p\right)^4 = 10^{-12} M_p^4$

How much did universe grow since end of inflation?

$$\frac{a_0}{a_{rch}} = \frac{a_0}{a_{eq}} \frac{a_{eq}}{a_{rch}}$$

$a_0 / a_{eq} \sim 3000$


$\Rightarrow \frac{a_0}{a_I} = \frac{H_I}{H_0} = 10^{53}$ From inflation to now
How much did universe grow since end of inflation?
3000
$\frac{a_0}{a_{rch}} = \frac{a_0}{a_{eq}} \frac{a_{eq}}{a_{rch}}$
radiation dominated univ.
$a \propto 1/T$

10^{19} GeV

$\left(\frac{}{M_p}\right)^4 = 10^{-12} M_p^4$

$$\Rightarrow \frac{a_0}{a_I} = \frac{H_I}{H_0} = 10^{53}$$ From inflation to now

How much did universe grow since end of inflation?

$$\frac{a_0}{a_{reh}} = \frac{a_0}{a_{eq}} \frac{a_{eq}}{a_{reh}}$$ (3000; a_{eq}/a_{reh}: radiation dominated univ., $a \propto 1/T$)

$$= 3000 \frac{T_{reh}}{T_{eq}}$$ ($T_{eq} \sim$ 1eV)

$\Rightarrow \frac{a_0}{a_I} = \frac{H_I}{H_0} = 10^{53}$ From inflation to now

$\left(\frac{}{M_p}\right)^4 = 10^{-12} M_p^4$

How much did universe grow since end of inflation?

$$\frac{a_0}{a_{reh}} = \frac{a_0}{a_{eq}} \frac{a_{eq}}{a_{reh}}$$

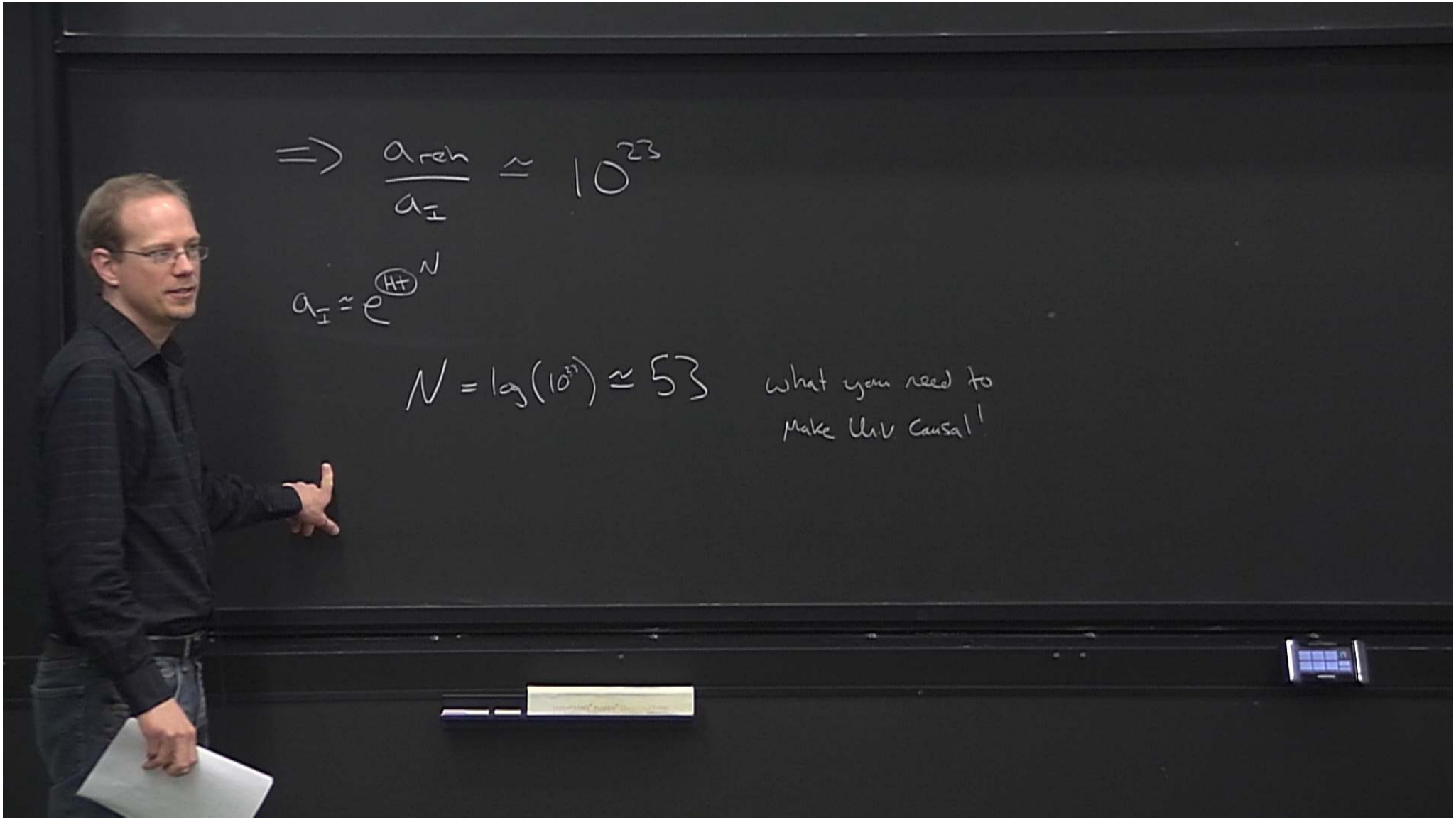
$\frac{a_0}{a_{eq}} \approx 3000$

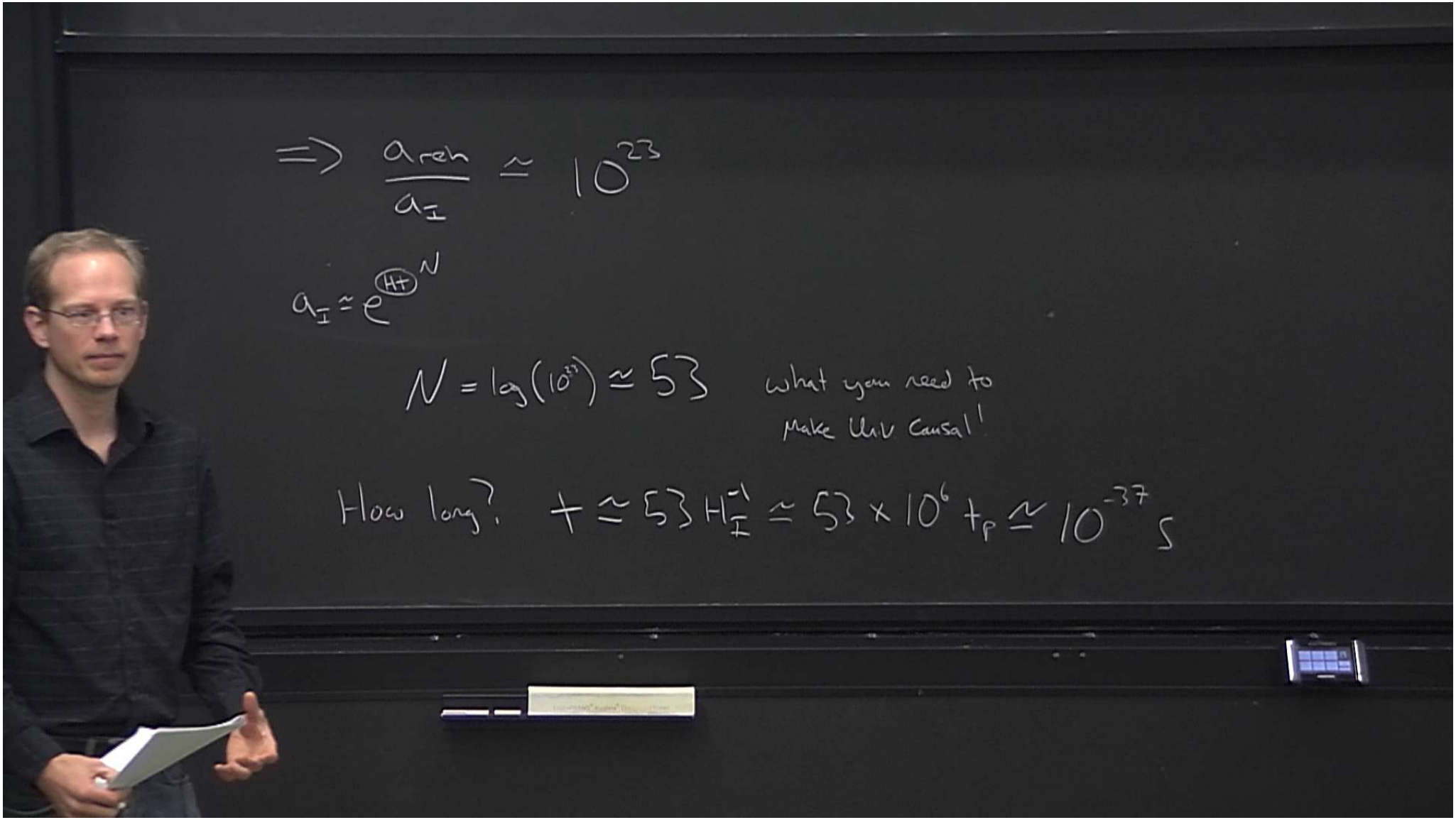
$\frac{a_{eq}}{a_{reh}}$ ~ radiation dominated univ. $a \propto 1/T$

$$= 3000 \frac{T_{reh}}{T_{eq}}$$

$T_{reh} \gtrsim 1\,TeV - 10^{14}\,GeV$

$T_{eq} \lesssim 1\,eV$


$\Rightarrow \frac{a_{reh}}{a_I} \simeq 10^{23}$
$a_I \simeq e^{Ht\,N}$
$N = \log(10^{23}) \simeq 53$
what you need to
Make Univ Causal!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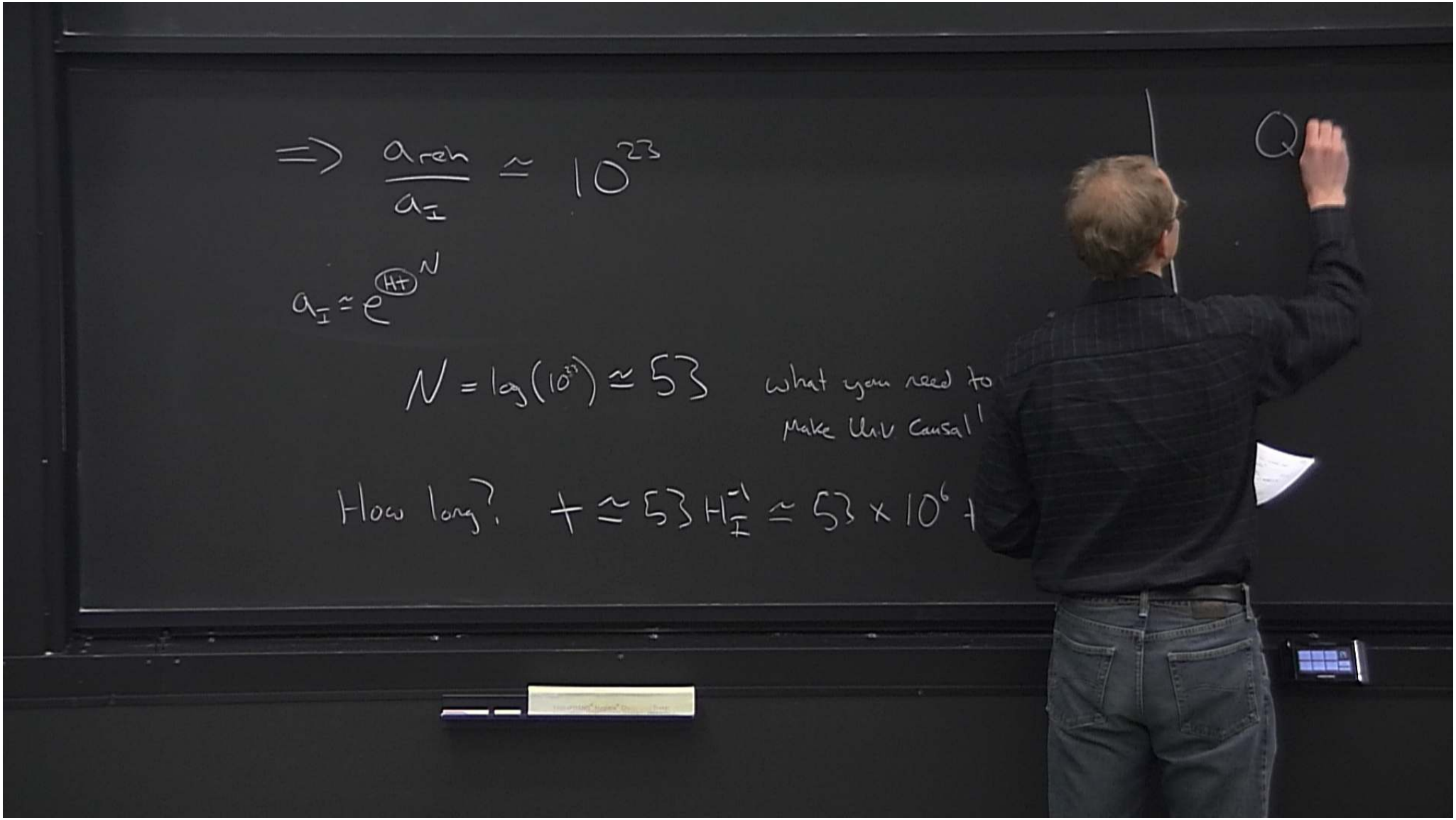
How long? $t \simeq 53\, H_I^{-1} \simeq 53 \times 10^6\, t_p \simeq 10^{-37}$ s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what you need to
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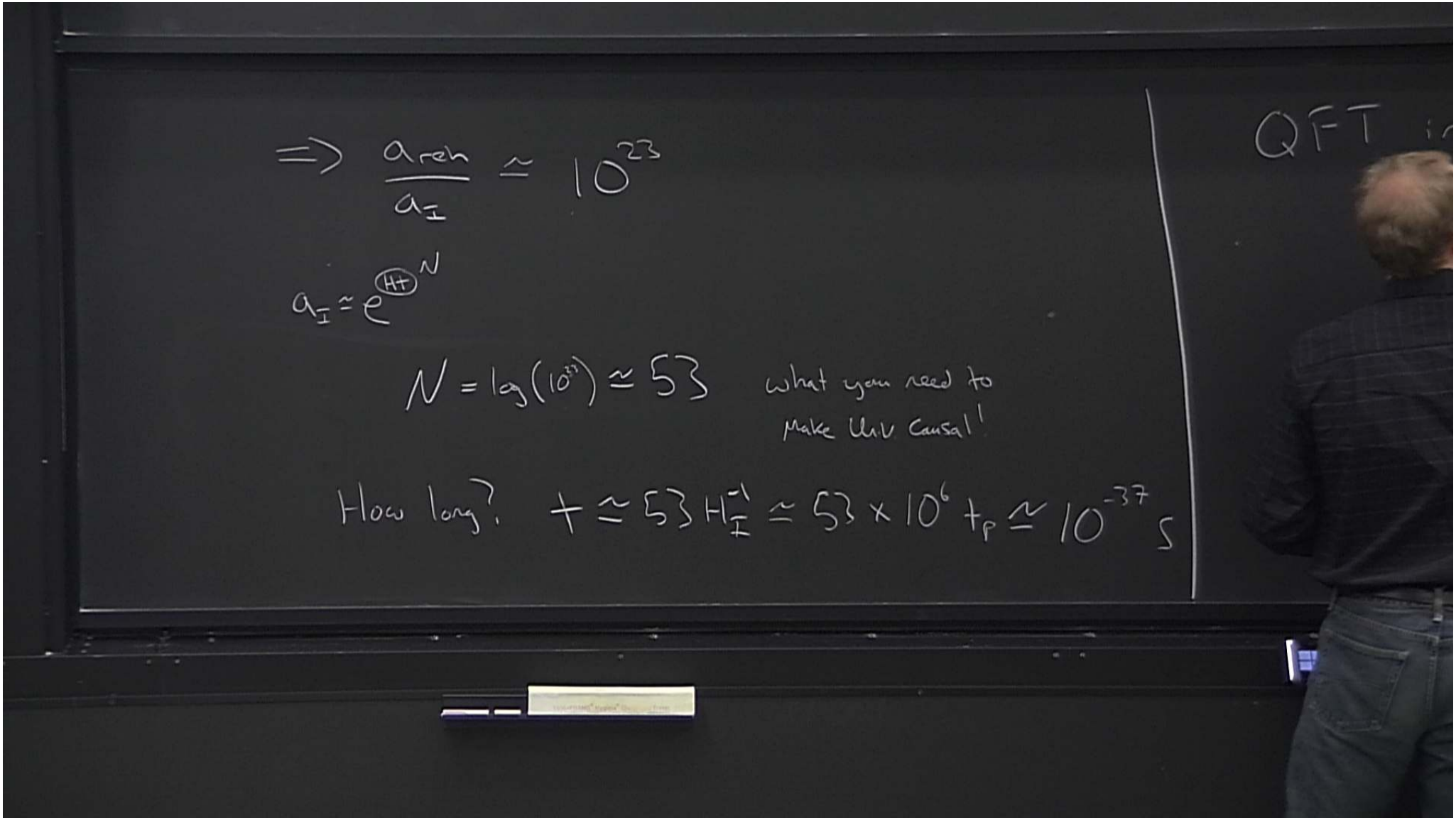
How long? $t \simeq 53\, H_I^{-1} \simeq 53 \times 10^6\, t$
Q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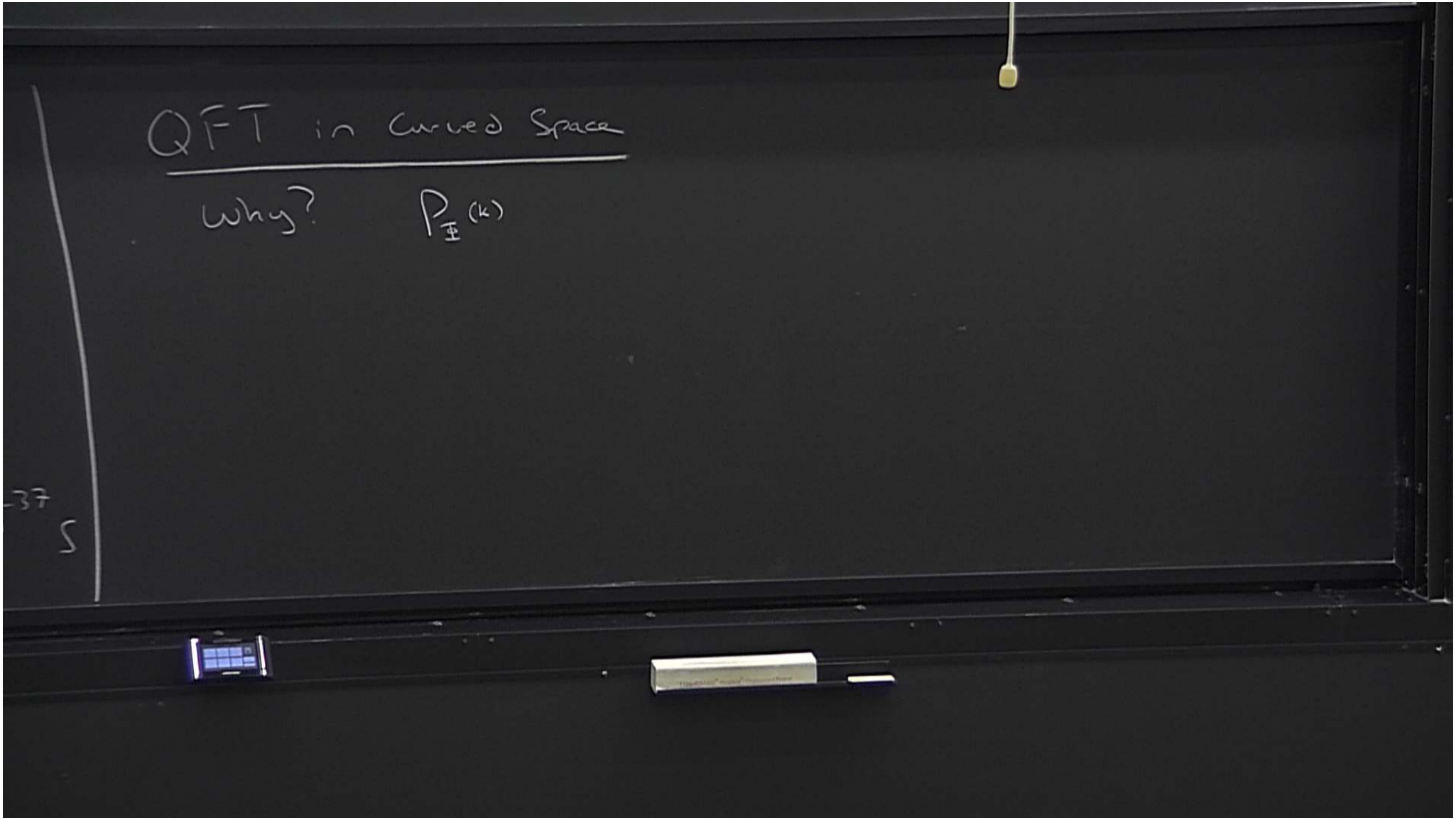
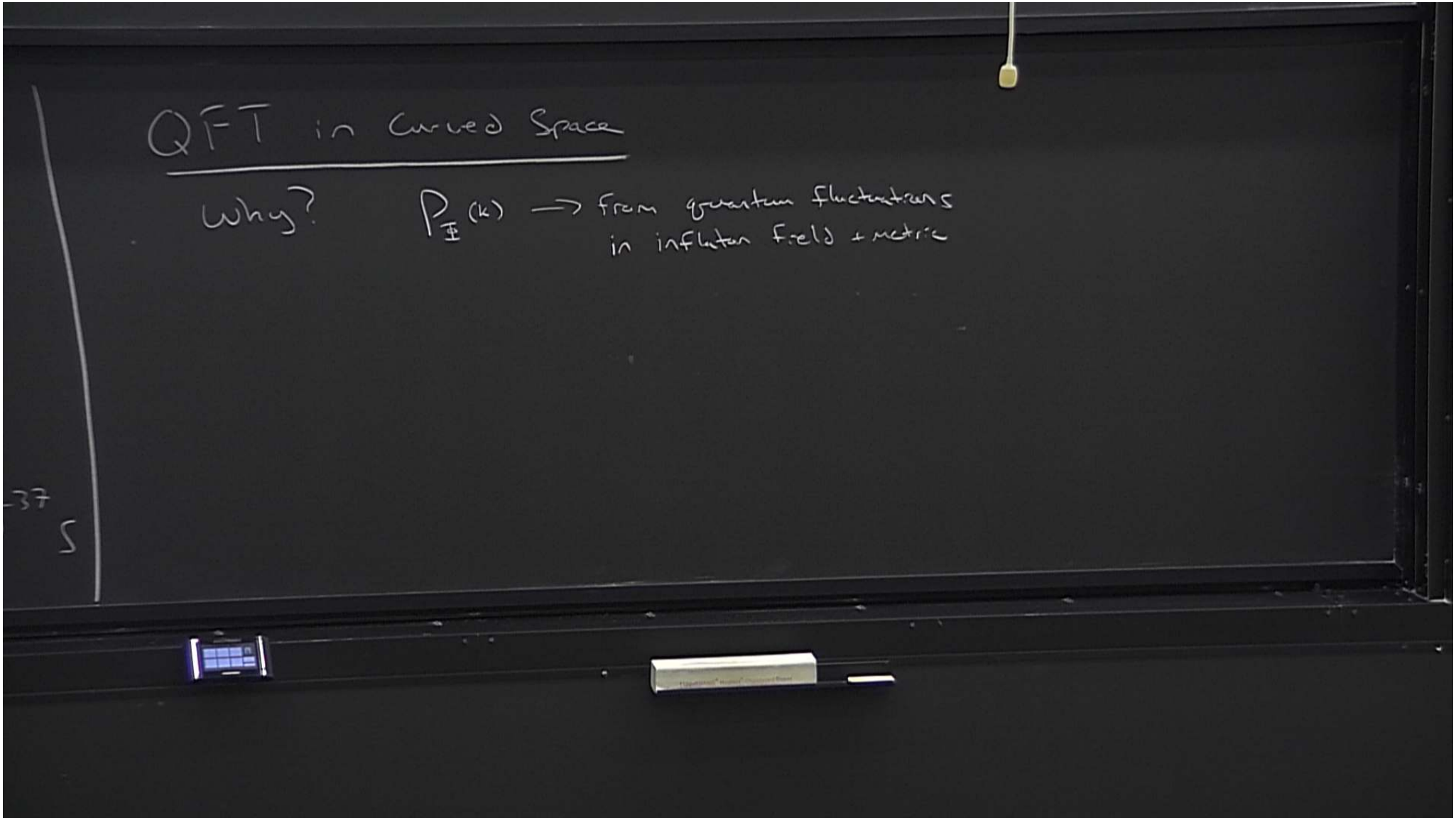
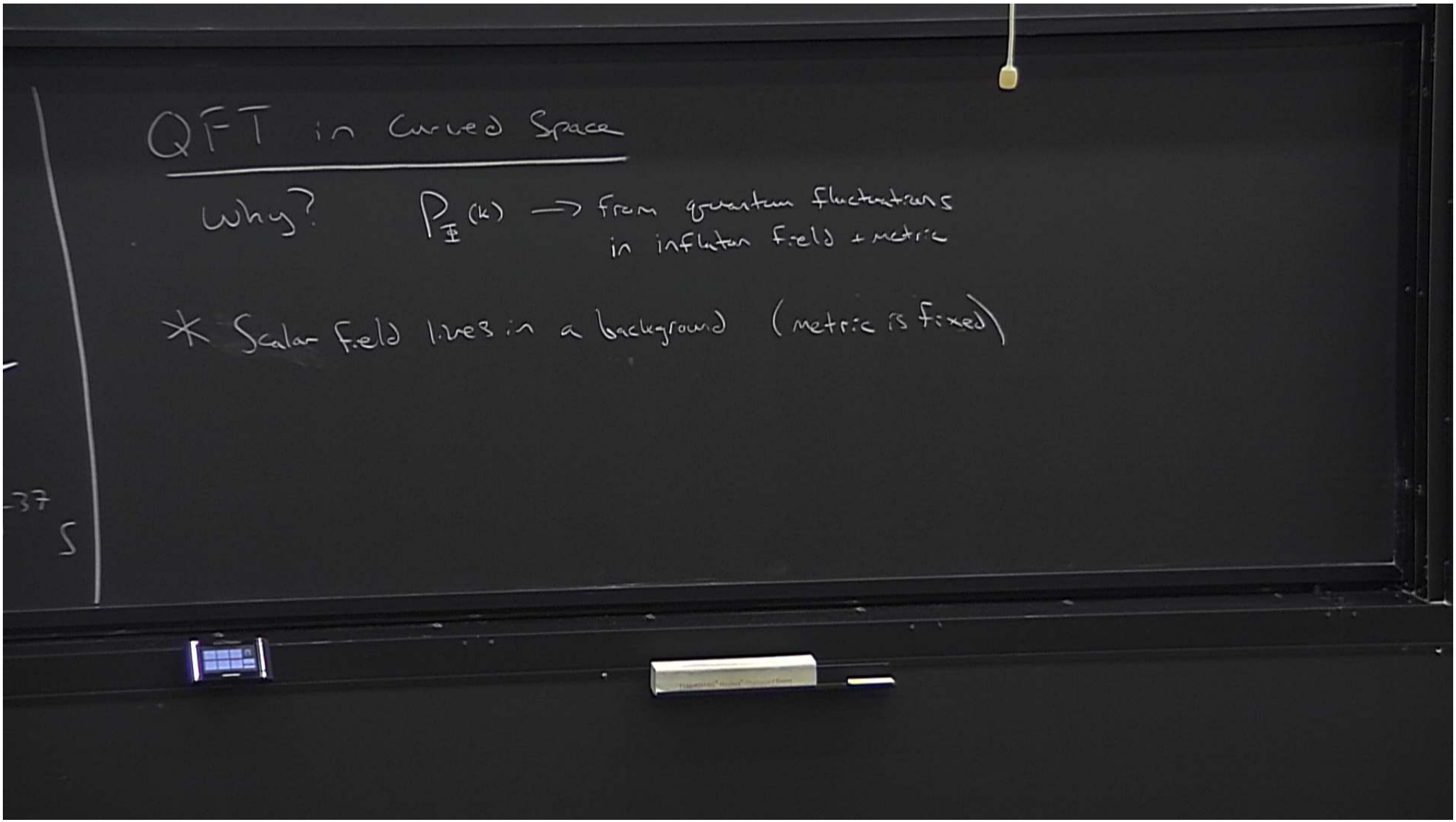
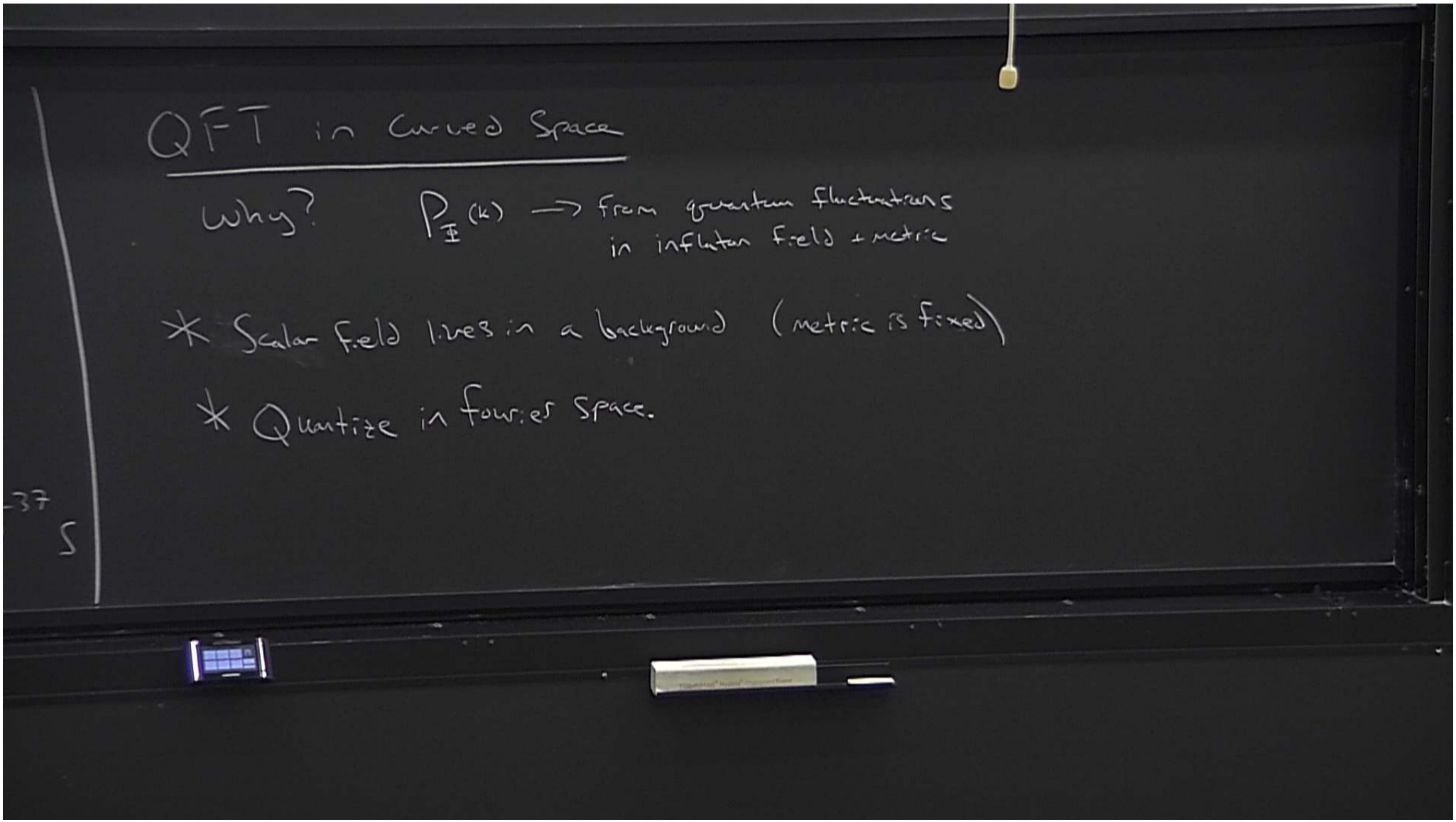
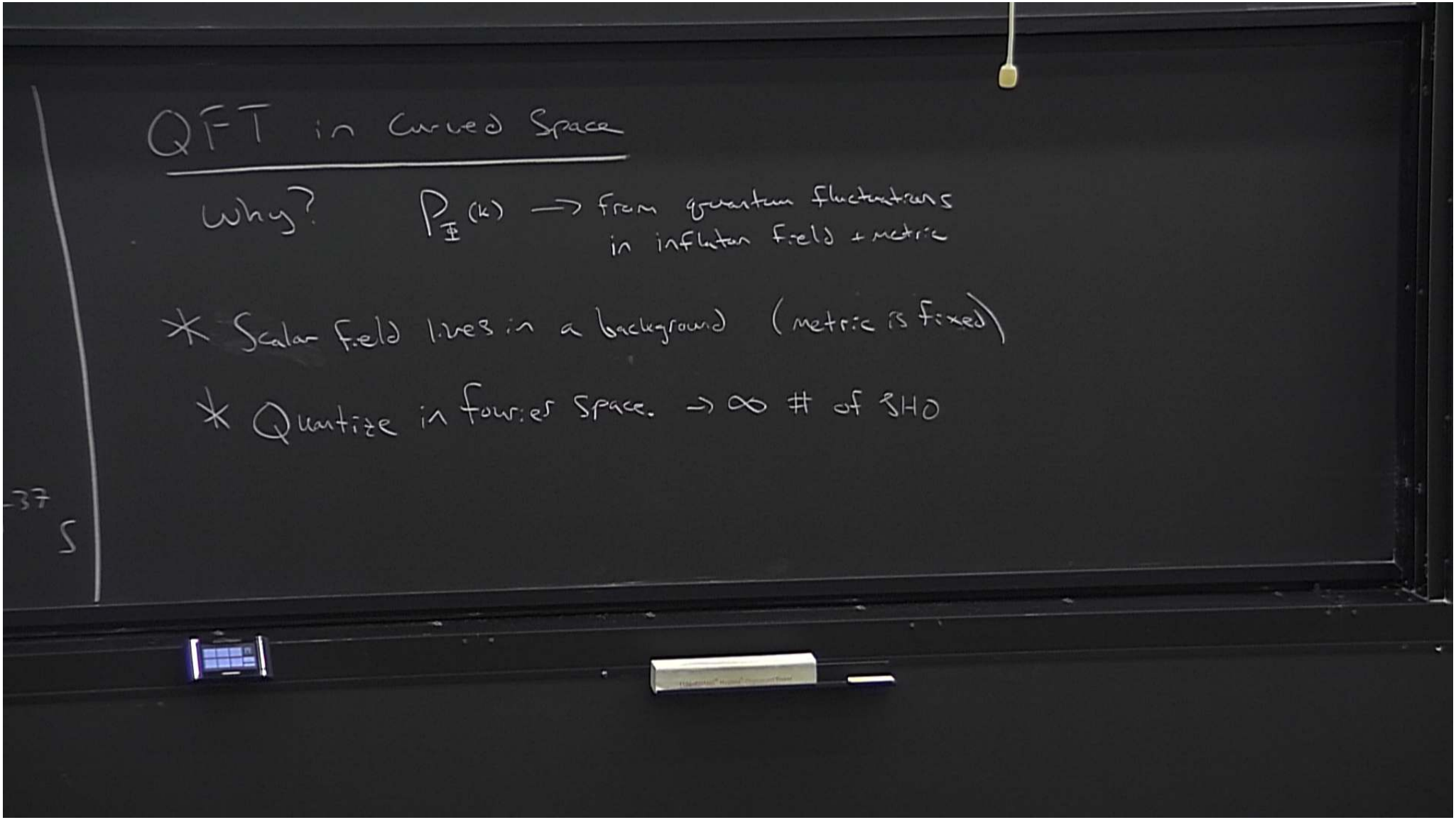
QFT


QFT in Curved Space
Why?
$P_{\mp}(k)$


QFT in Curved Space
Why?
$P_{\mathcal{I}}(k)$ → from quantum fluctuations in inflaton field + metric


QFT in Curved Space
Why? $P_{\mathcal{I}}(k)$ → from quantum fluctuations in inflaton field + metric
* Scalar field lives in a background (metric is fixed)


QFT in Curved Space
Why? $P_{\mathcal{I}}(k)$ → from quantum fluctuations in inflaton field + metric
* Scalar field lives in a background (metric is fixed)
* Quantize in fourier space.


QFT in Curved Space
Why? $P_{\mathcal{I}}(k)$ → from quantum fluctuations in inflaton field + metric
* Scalar field lives in a background (metric is fixed)
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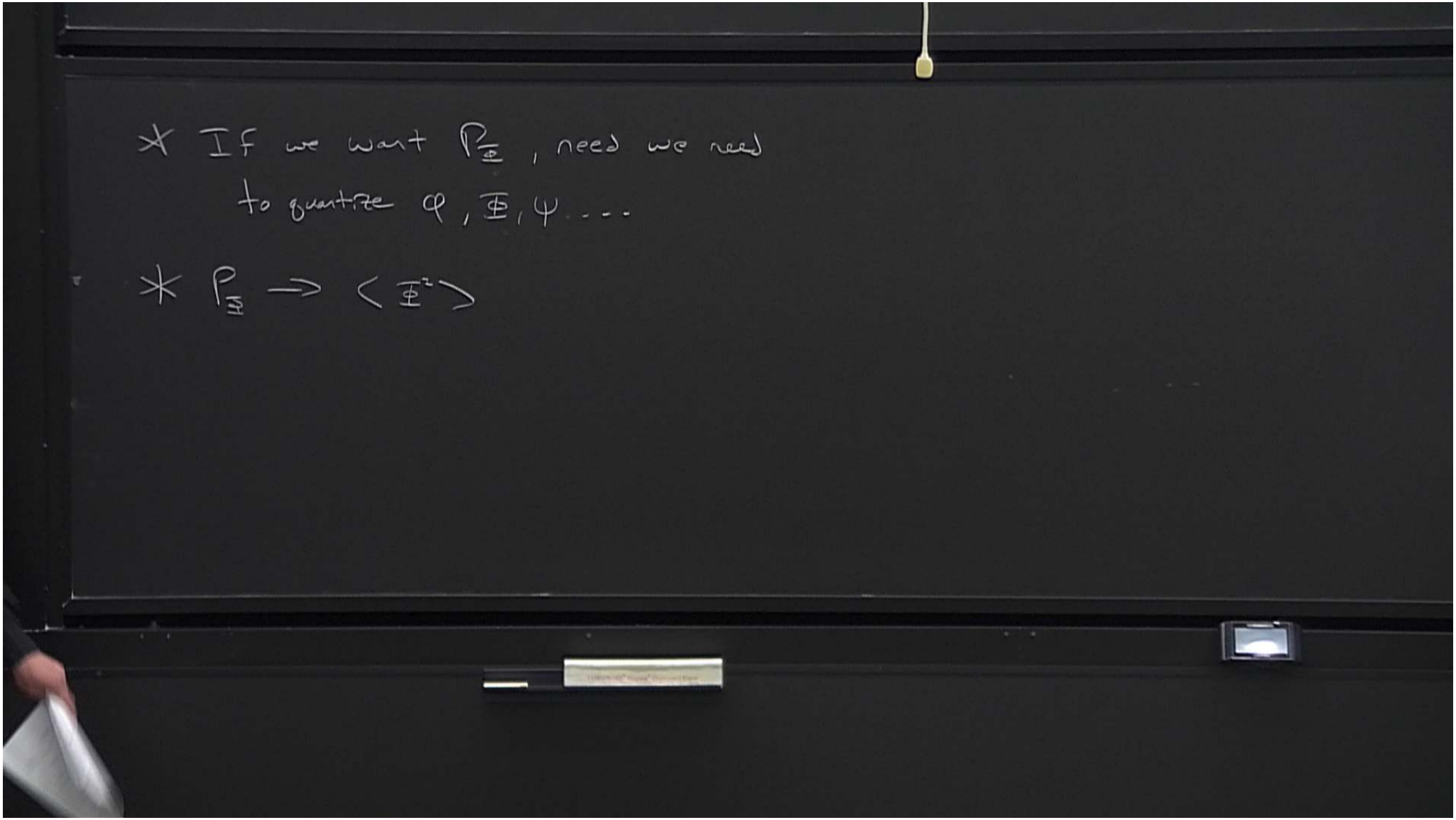
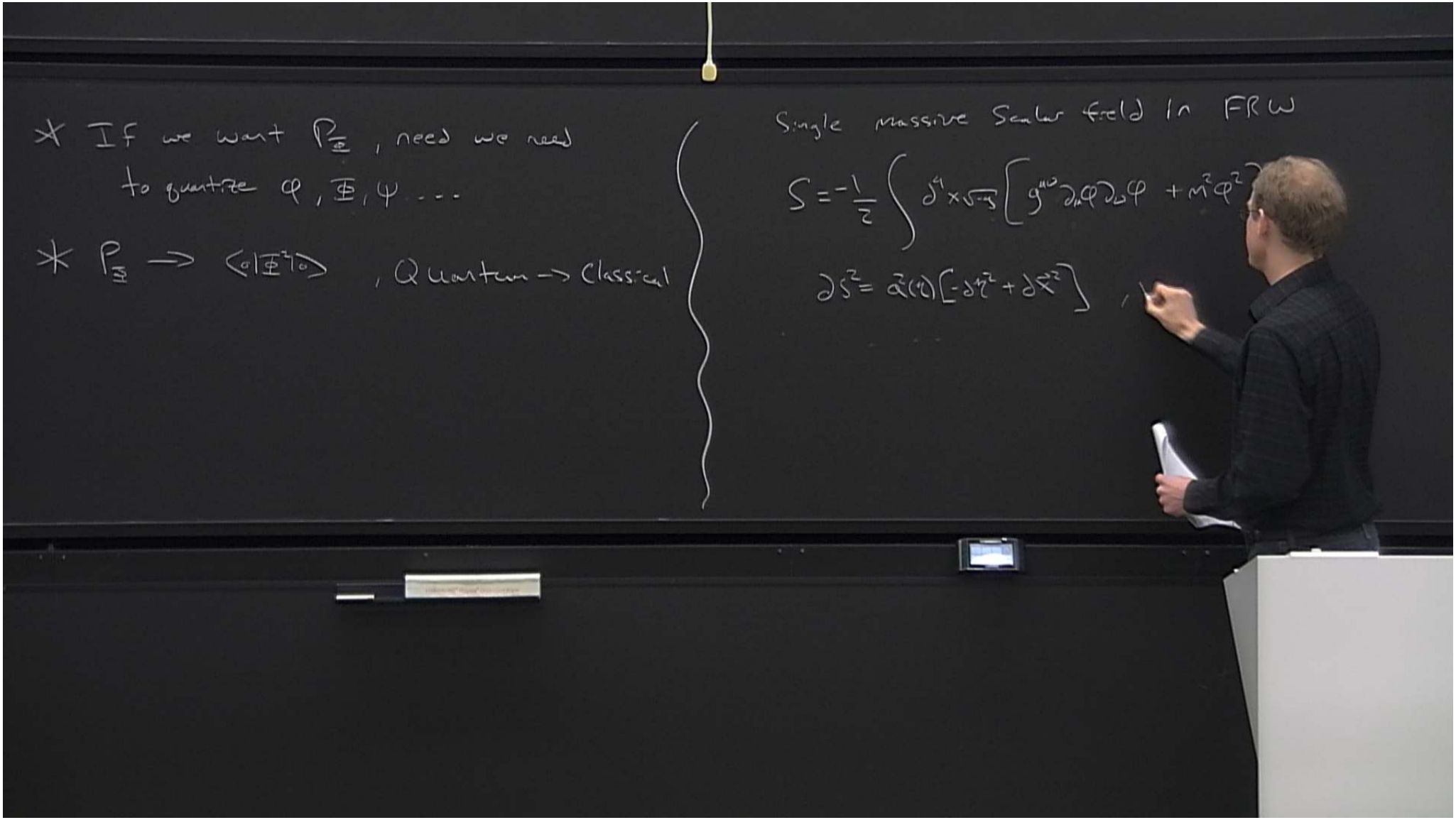
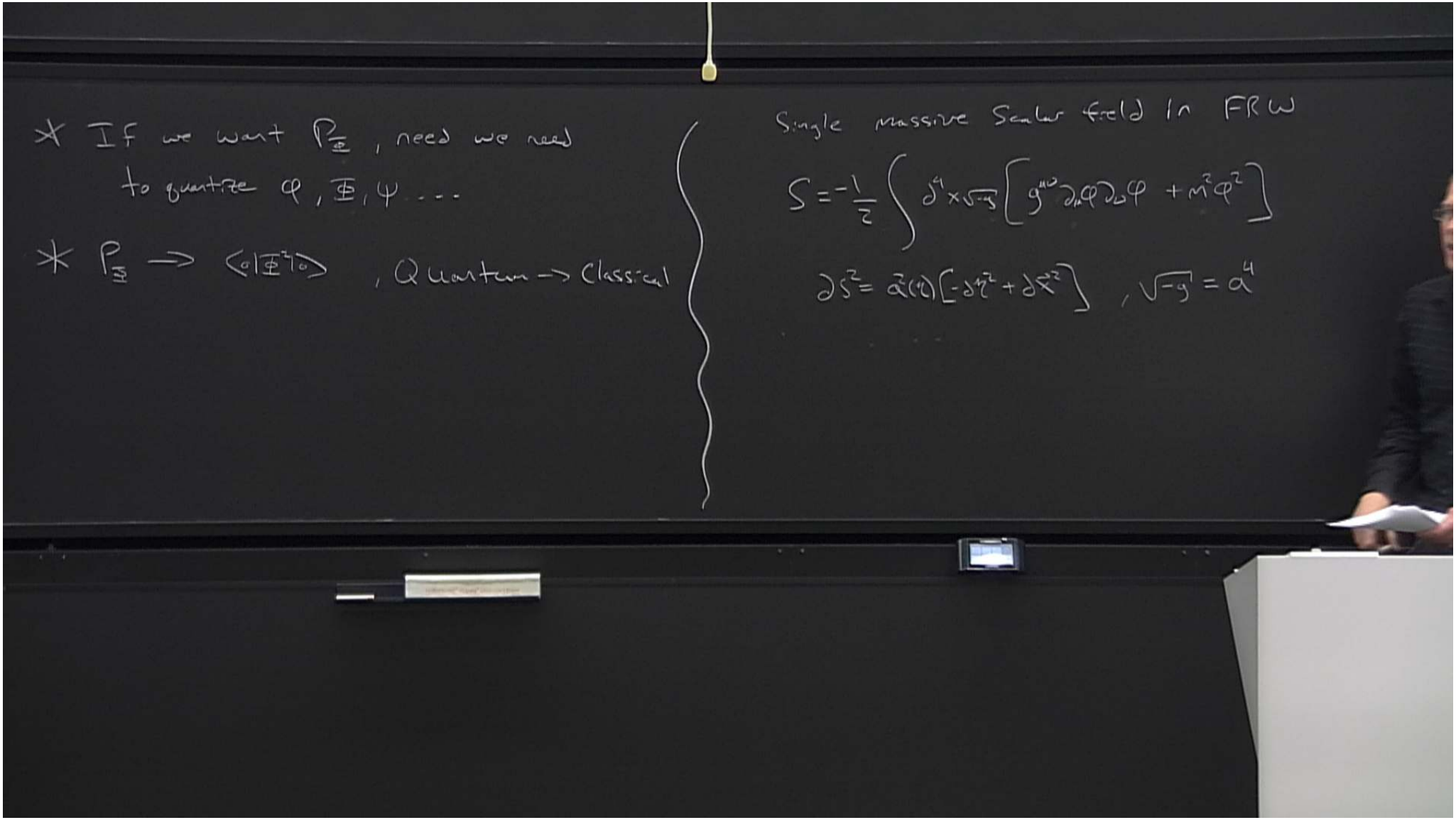
QFT in Curved Space

Why? $P_{\mathcal{I}}(k)$ → from quantum fluctuations in inflaton field + metric

* Scalar field lives in a background (metric is fixed)
* Quantize in fourier space. → ∞ # of SHO
* Need a vacuum state → what are particles?


* If we want ρ_Φ, need we need
to quantize $\varphi, \Phi, \psi \ldots$
* $\rho_\Xi \rightarrow \langle \Xi^2 \rangle$


* If we want P_Φ, need we need to quantize $\varphi, \Phi, \Psi \ldots$
* $P_\Phi \rightarrow \langle 0|\Phi^2|0\rangle$, Quantum -> Classical
Single massive Scalar field in FRW
$S = -\frac{1}{2}\int d^4x\sqrt{-g}\left[g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + m^2\varphi^2\right]$
$ds^2 = a^2(\eta)\left[-d\eta^2 + dx^2\right]$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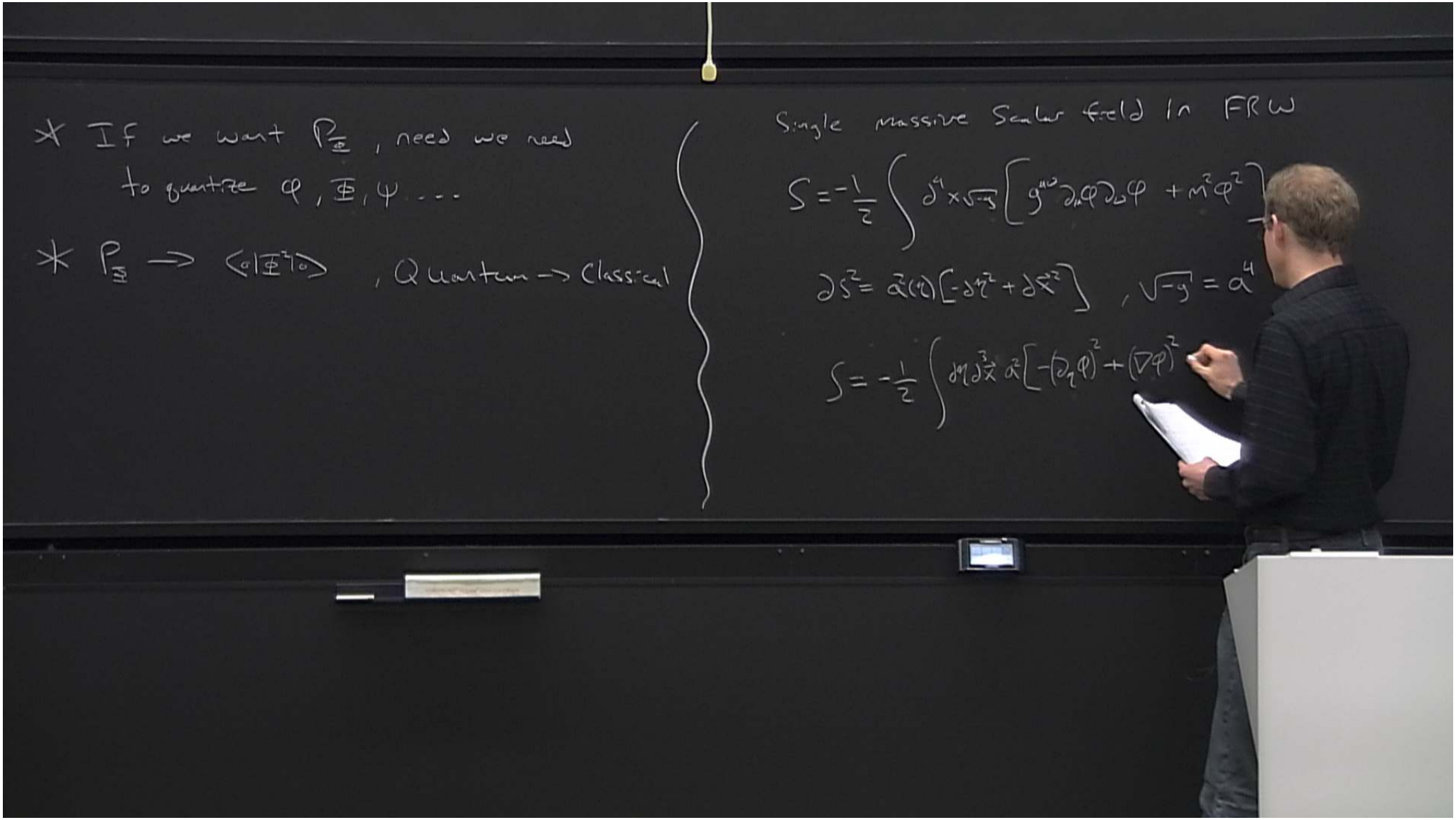
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Single massive Scalar field in FRW

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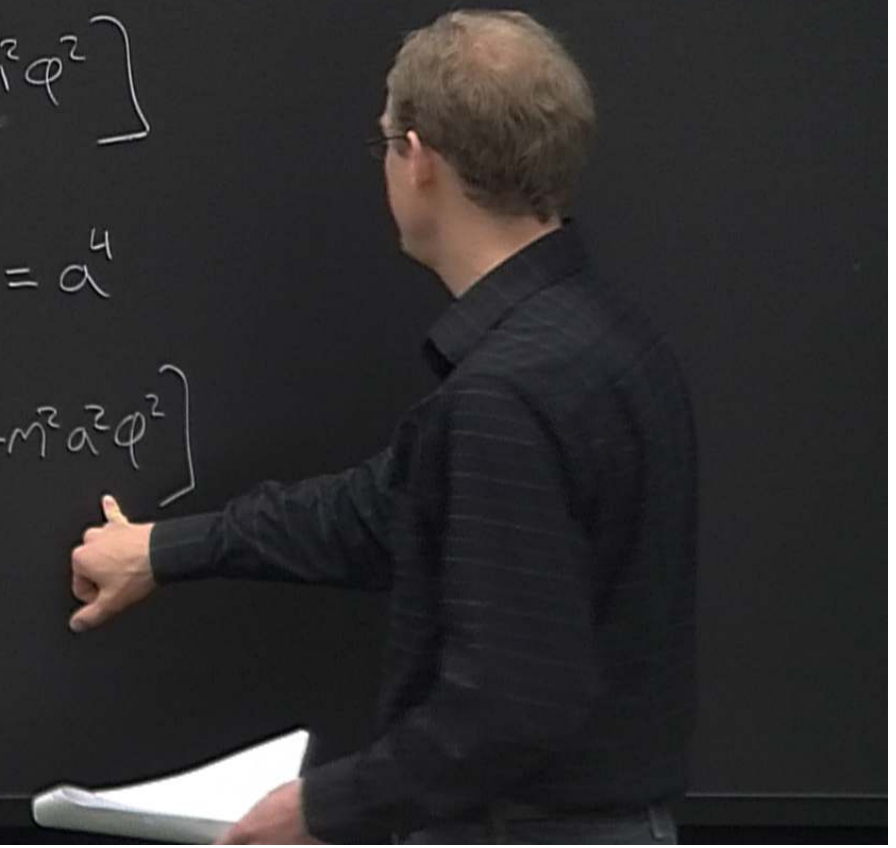

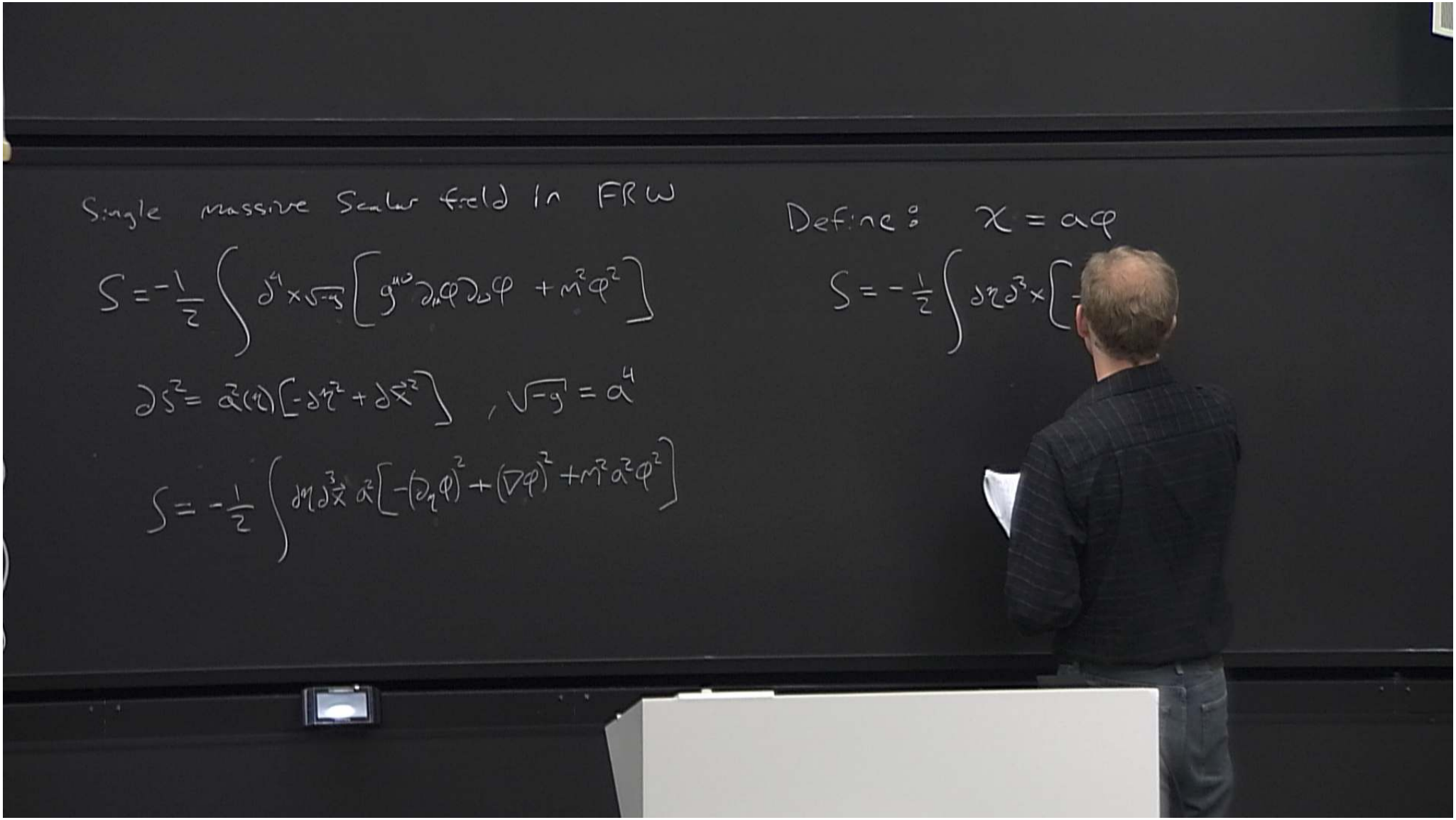
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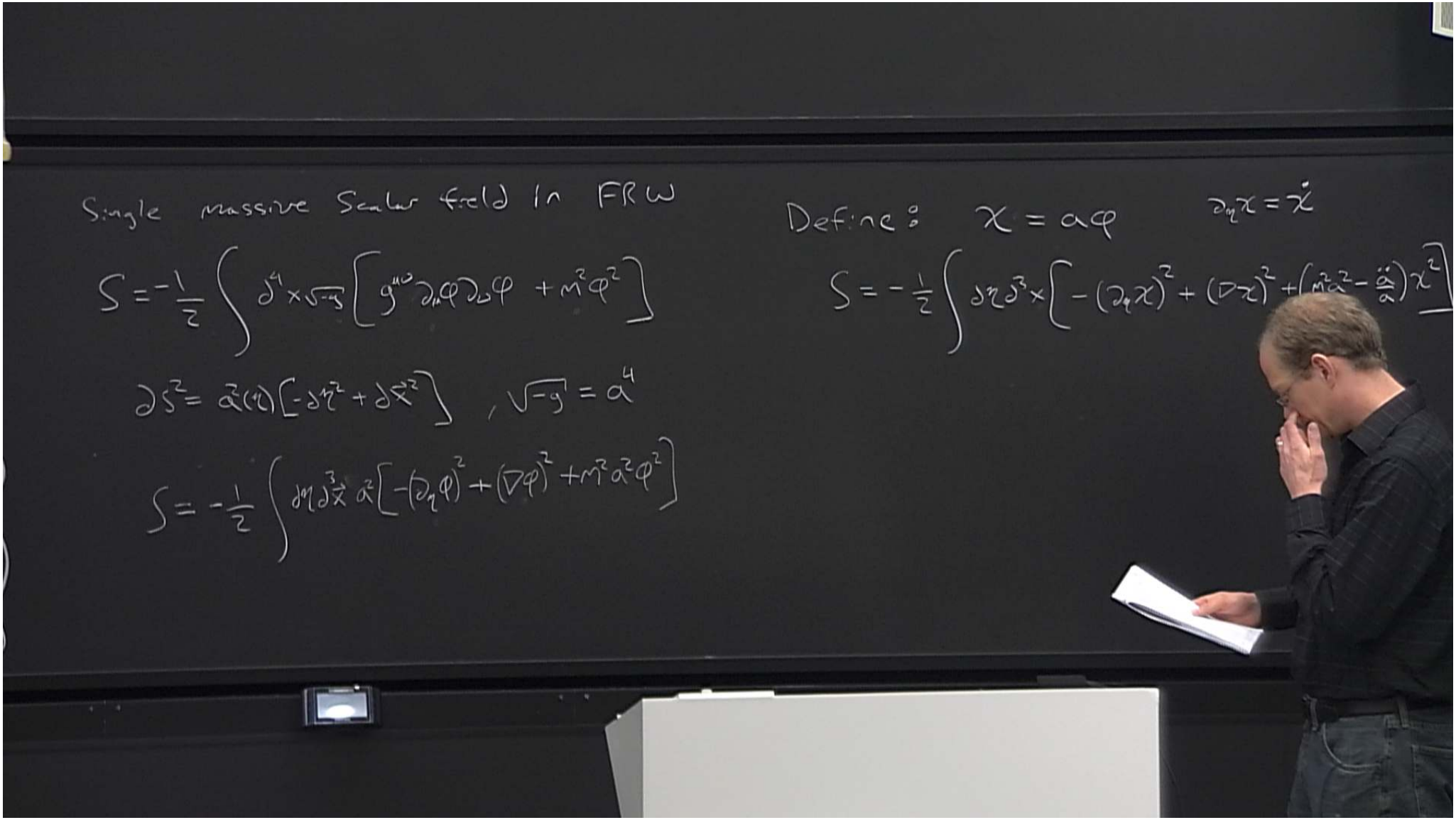
$S = -\frac{1}{2}\int d\eta d^3x\, a^2\left[-(\partial_\eta\varphi)^2 + (\nabla\varphi)^2 + m^2a^2\varphi^2\right]$
Define: $\chi = a\varphi$
$S = -\frac{1}{2}\int d\eta d^3x \left[$


Single massive Scalar field in FRW
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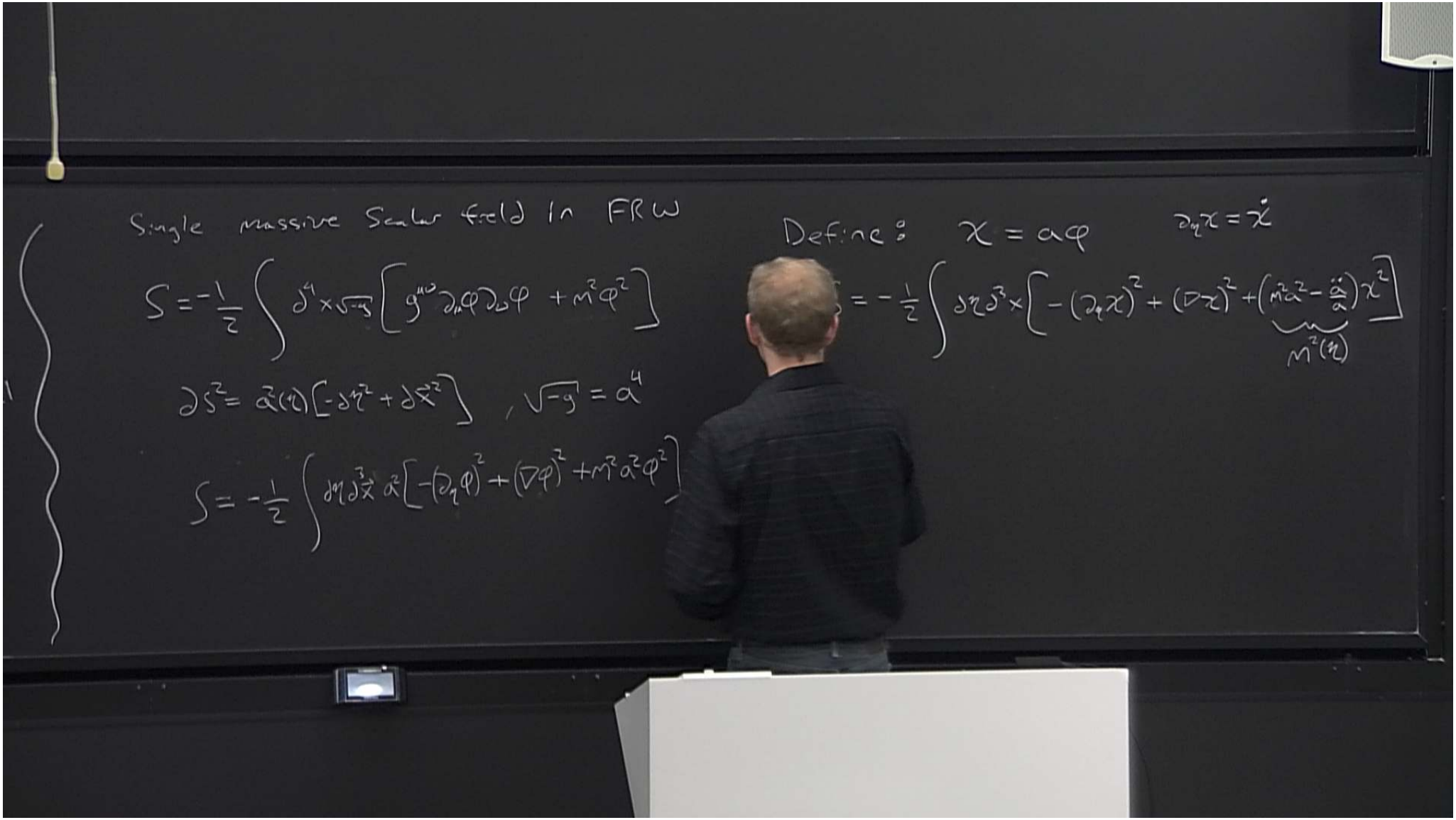
$S = -\frac{1}{2}\int d\eta d^3\vec{x}\, a^2\left[-(\partial_\eta\varphi)^2 + (\nabla\varphi)^2 + m^2a^2\varphi^2\right]$
Define: $\chi = a\varphi$ $\partial_\eta\chi = \dot{\chi}$
$S = -\frac{1}{2}\int d\eta d^3x\left[-(\partial_\eta\chi)^2 + (\nabla\chi)^2 + (m^2a^2 - \frac{\ddot{a}}{a})\chi^2\right]$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Define: $\chi = a\varphi$ $\partial_\eta\chi = \chi'$
$= -\frac{1}{2}\int d\eta d^3x\left[-(\partial_\eta\chi)^2 + (\nabla\chi)^2 + \underbrace{\left(m^2a^2 - \frac{a''}{a}\right)}_{M^2(\eta)}\chi^2\right]$

Single massive Scalar field in FRW

$$S = -\frac{1}{2}\int d^4x\sqrt{-g}\left[g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + m^2\varphi^2\right]$$

$$ds^2 = a^2(\eta)\left[-d\eta^2 + d\vec{x}^2\right], \quad \sqrt{-g} = a^4$$

$$S = -\frac{1}{2}\int d\eta d^3\vec{x}\, a^2\left[-(\partial_\eta\varphi)^2 + (\nabla\varphi)^2 + m^2a^2\varphi^2\right]$$

Define: $\chi = a\varphi$ $\quad \partial_\eta\chi = \chi'$

$$S = -\frac{1}{2}\int d\eta d^3x\left[-(\partial_\eta\chi)^2 + (\nabla\chi)^2 + \underbrace{\left(m^2a^2 - \frac{a''}{a}\right)}_{m^2(\eta)}\chi^2\right]$$

$$\frac{\delta S}{\delta\chi} = \frac{\partial\mathcal{L}}{\partial\chi} - \frac{d}{dx^\mu}\frac{\partial\mathcal{L}}{\partial(\partial_\mu\chi)} = 0$$

$$\Rightarrow \chi'' - \nabla^2\chi + \left(m^2a^2 - \frac{a''}{a}\right)\chi = 0$$

Scalar field with $M(t)$

→ particle production

Scalar field with $M(\eta)$

→ particle production

What are particles?

$$\omega_k^2 = k^2 + m^2 a^2 - \frac{\ddot{a}}{a}$$

χ is a real scalar

$$\chi = \chi^* \Rightarrow \chi_{\vec{k}}^*(\eta) = \chi_{-\vec{k}}(\eta)$$

$$\chi(\vec{x},\eta) = \int \frac{d^3k}{(2\pi)^3} \chi_{\vec{k}}(\eta) e^{i\vec{k}\cdot\vec{x}}$$

$$\Rightarrow \ddot{\chi}_{\vec{k}} + \omega_{\vec{k}}^2(\eta)\chi_{\vec{k}} = 0 \qquad (\chi(\eta) = e^{i\omega\eta})$$

Scalar field with $M(\eta)$

→ particle production

What are particles?

$$\chi(\vec{x},\eta) = \int \frac{d^3k}{(2\pi)^3} \chi_{\vec{k}}(\eta) e^{i\vec{k}\cdot\vec{x}}$$

$$\Rightarrow \ddot{\chi}_{\vec{k}} + \omega_{\vec{k}}^2(\eta)\chi_{\vec{k}} = 0 \qquad (\chi(\eta) = e^{i\omega\eta})$$

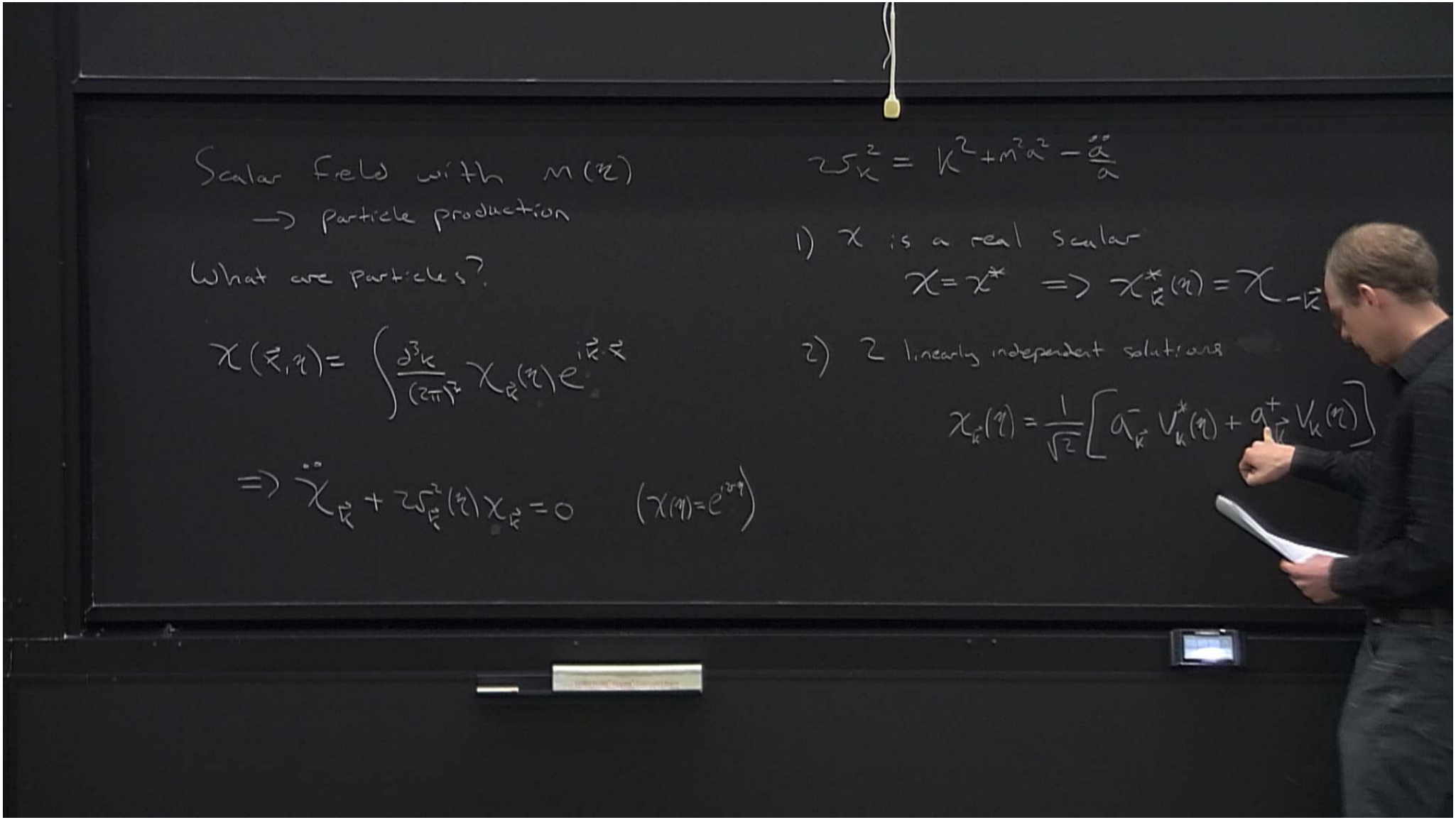
$$\omega_k^2 = k^2 + m^2a^2 - \frac{\ddot{a}}{a}$$

1) χ is a real scalar

$$\chi = \chi^* \Rightarrow \chi^*_{\vec{k}}(\eta) = \chi_{-\vec{k}}$$

2) 2 linearly independent solutions

$$\chi_{\vec{k}}(\eta) = \frac{1}{\sqrt{2}}\left[a^-_{\vec{k}} V^*_k(\eta) + a^+_{\vec{k}} V_k(\eta)\right]$$



Scalar field with $M(\eta)$

→ particle production

What are particles?

$$\chi(\vec{x},\eta) = \int \frac{d^3k}{(2\pi)^3} \chi_{\vec{k}}(\eta) e^{i\vec{k}\cdot\vec{x}}$$

$$\Rightarrow \ddot{\chi}_{\vec{k}} + \omega_{\vec{k}}^2(\eta)\chi_{\vec{k}} = 0 \qquad (\chi(\eta) = e^{i\omega\eta})$$

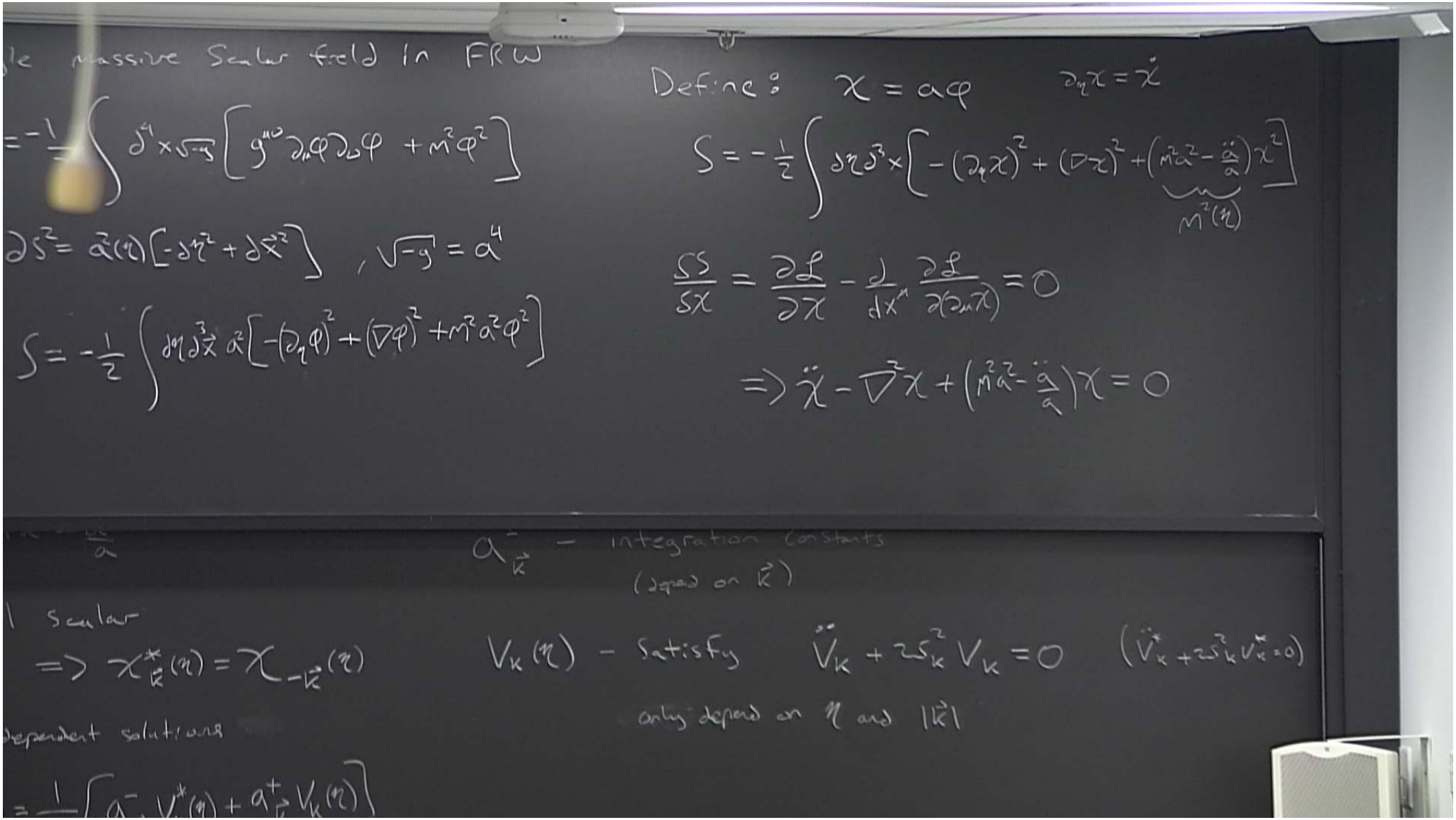
$$\omega_k^2 = k^2 + m^2a^2 - \frac{\ddot{a}}{a}$$

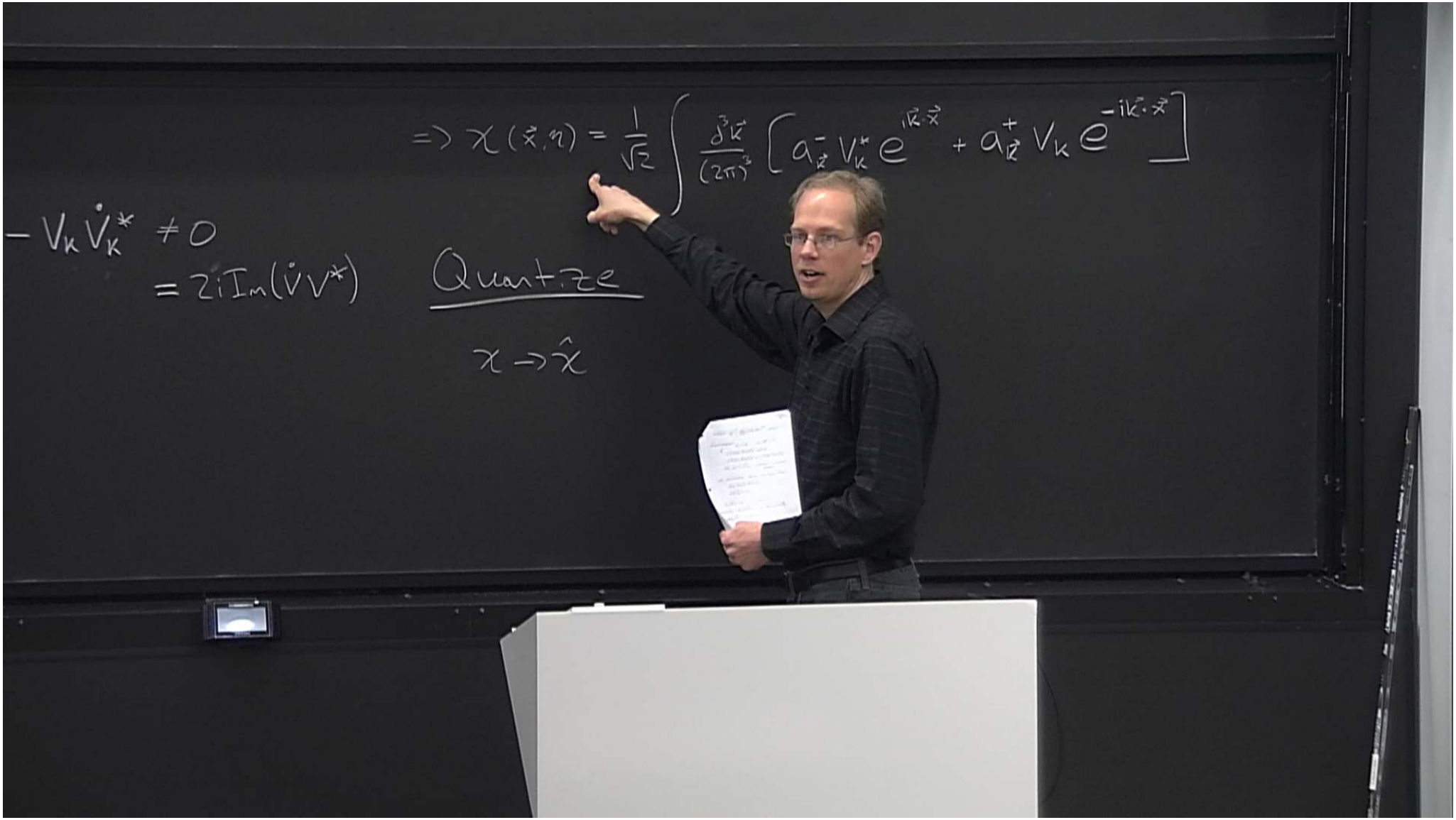
1) χ is a real scalar

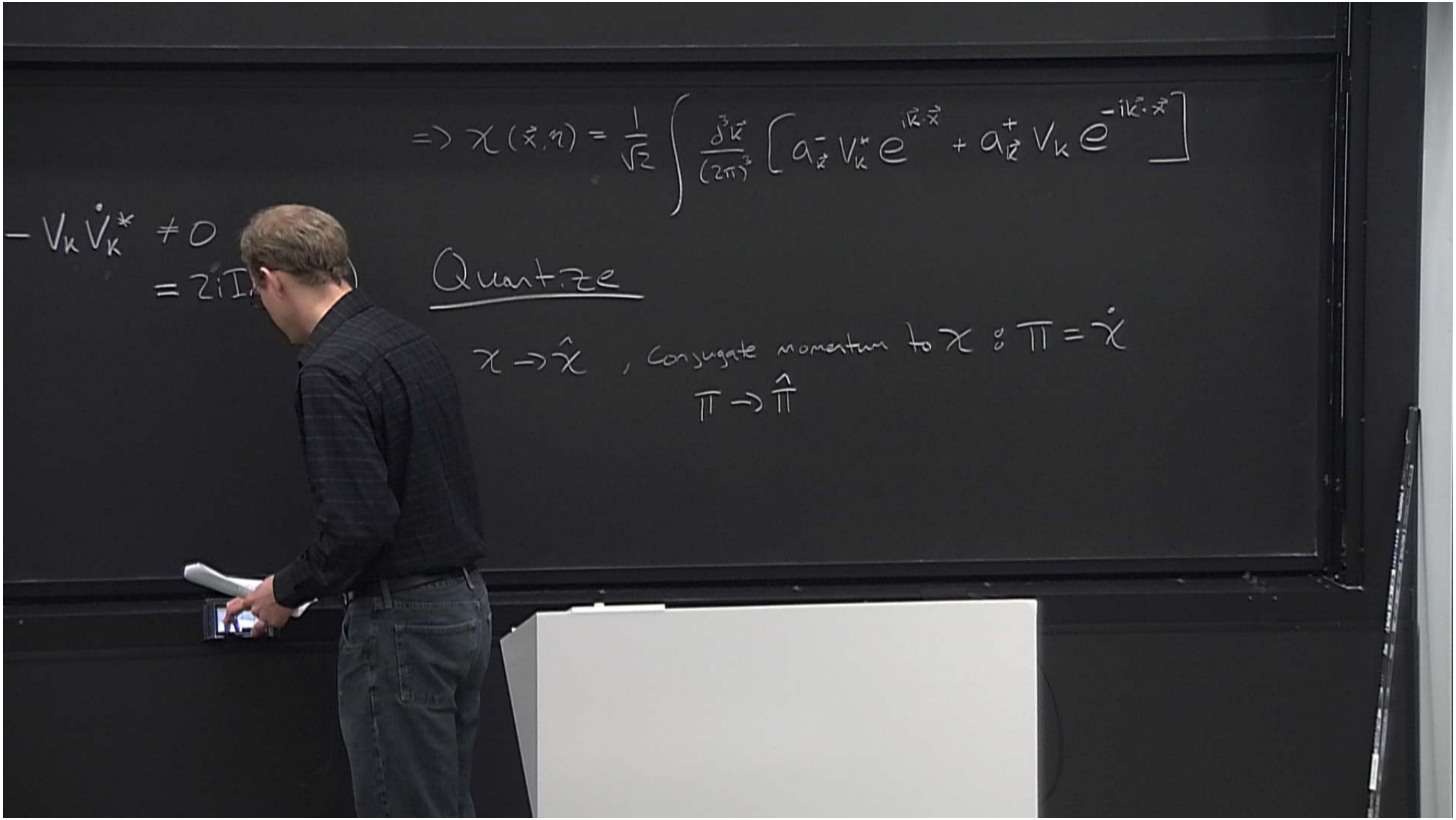
$$\chi = \chi^* \Rightarrow \chi^*_{\vec{k}}(\eta) = \chi_{-\vec{k}}(\eta)$$

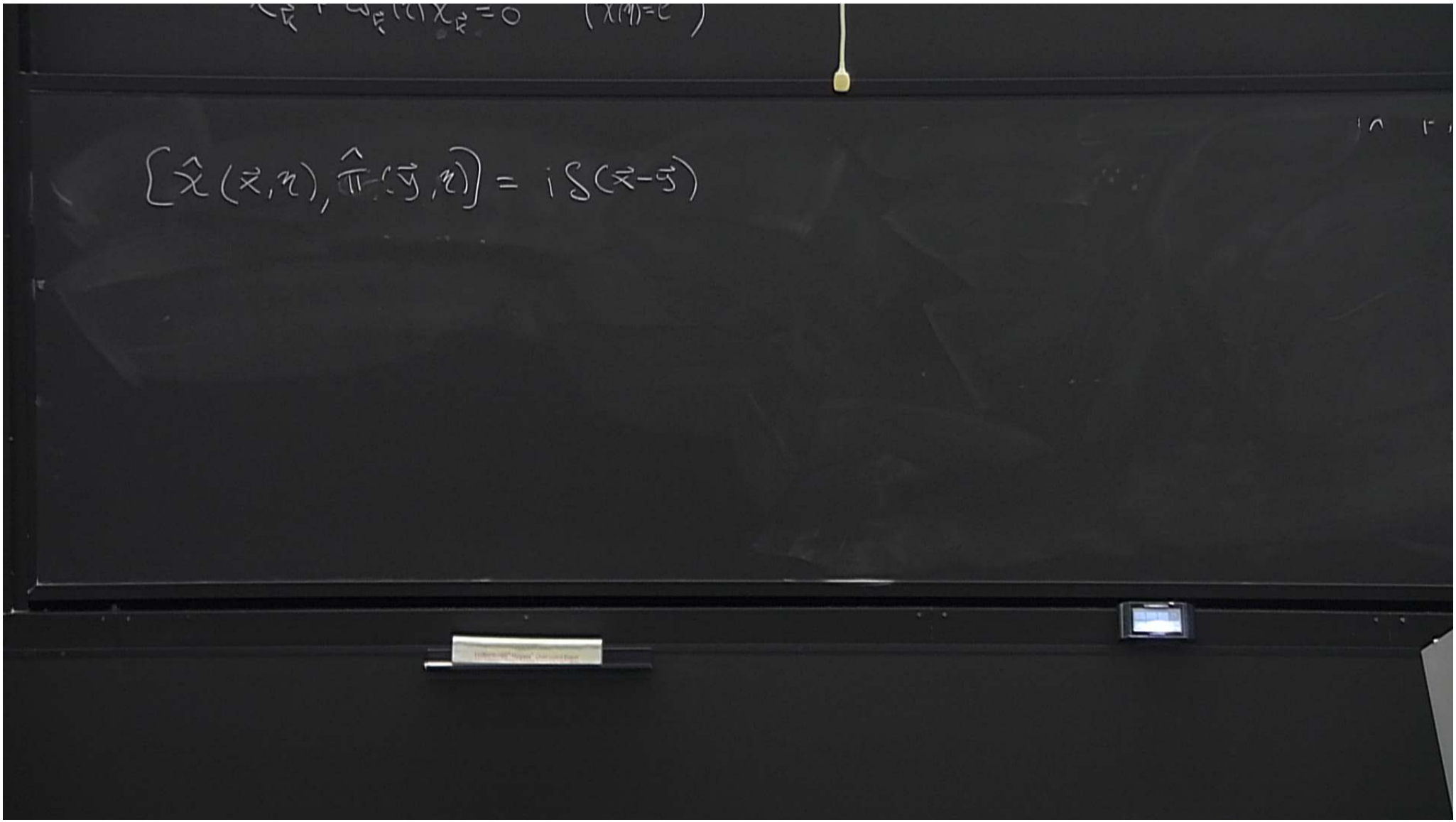
2) 2 linearly independent solutions

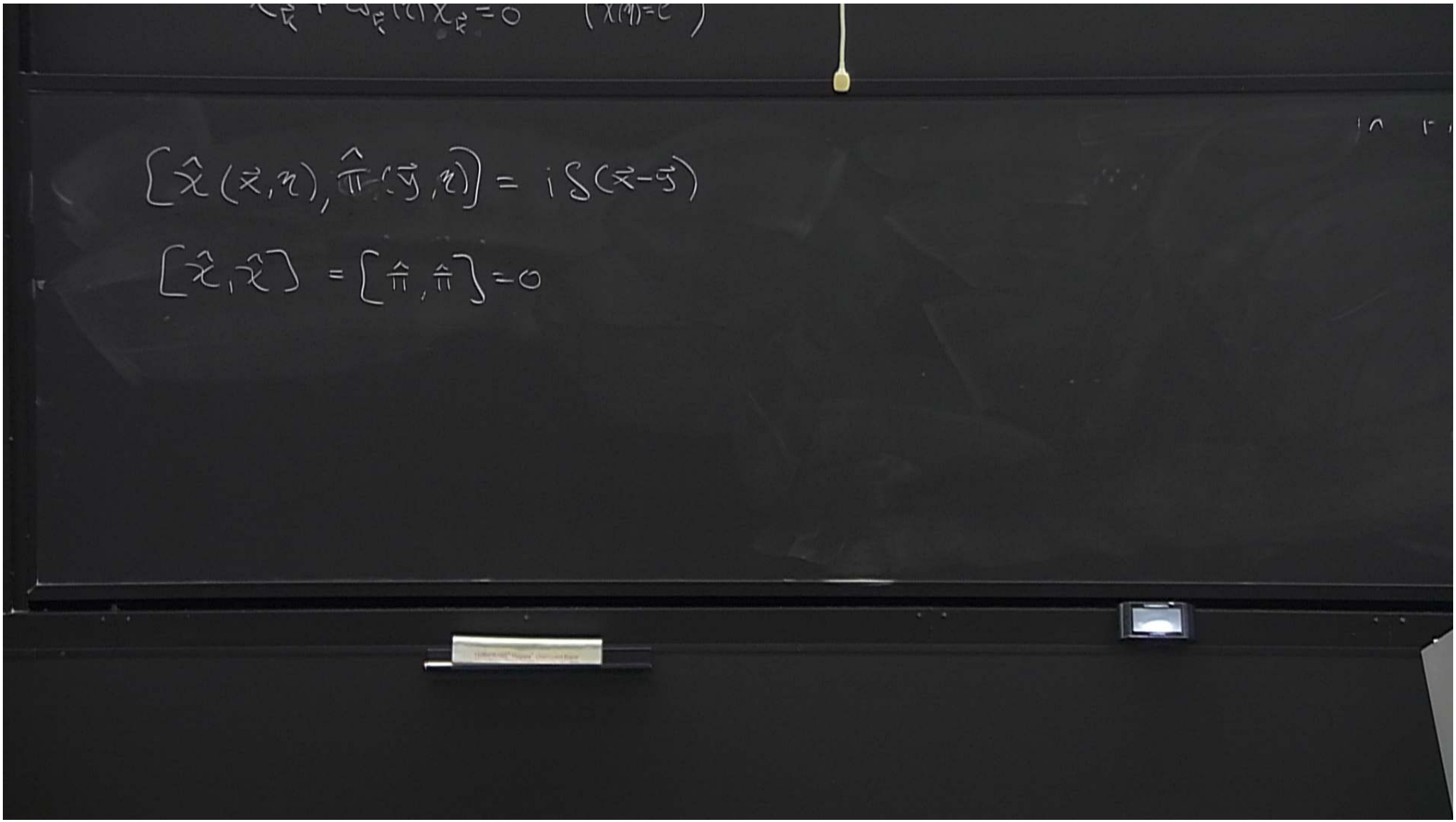
$$\chi_{\vec{k}}(\eta) = \frac{1}{\sqrt{2}}\left[a^-_{\vec{k}} V^*_k(\eta) + a^+_{\vec{k}} V_k(\eta)\right]$$

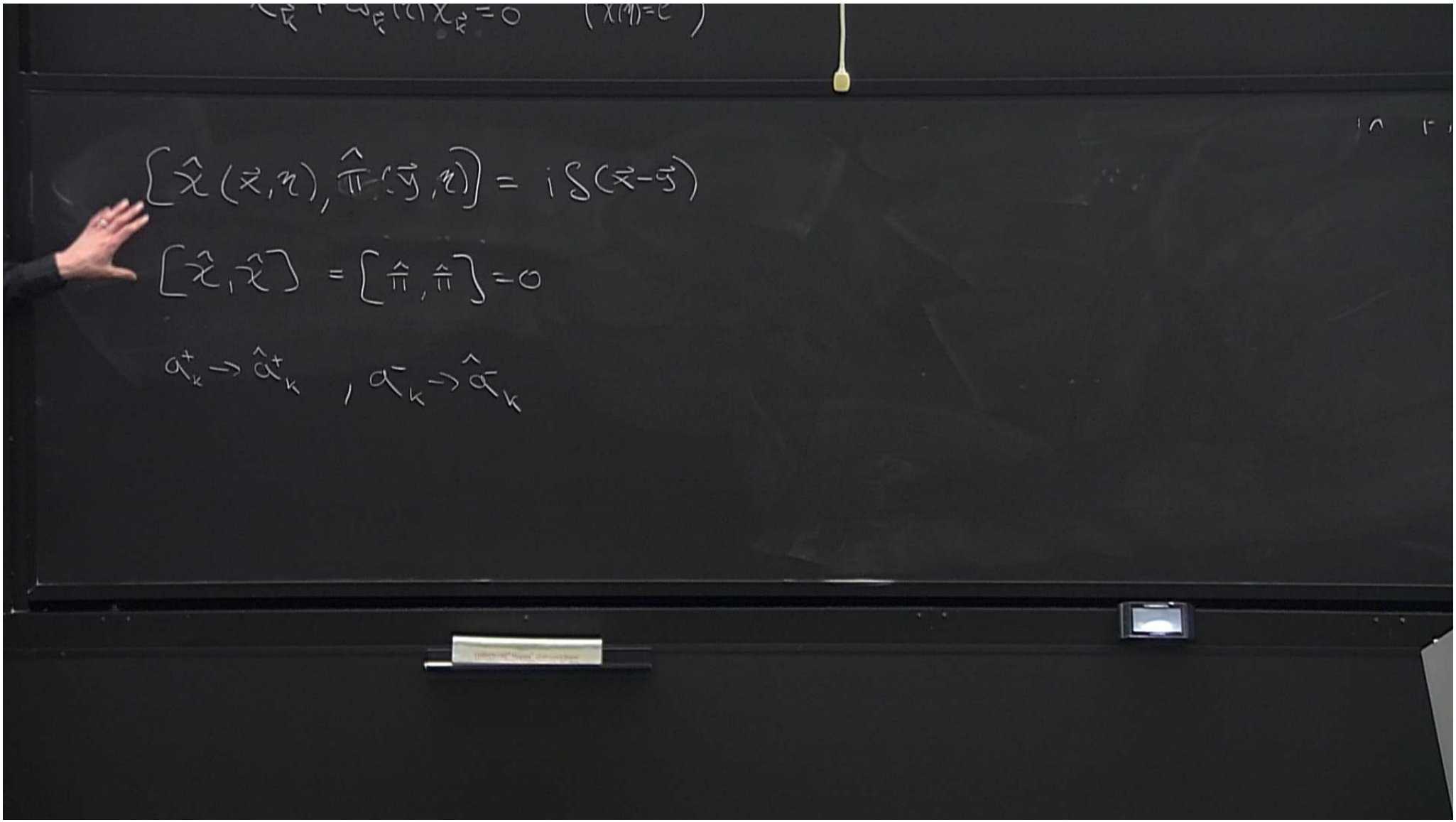

le massive Scalar field in FRW
$= -\frac{1}{2}\int d^4x\sqrt{-g}\left[g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + m^2\varphi^2\right]$
$ds^2 = a^2(\eta)\left[-d\eta^2 + d\vec{x}^2\right]$, $\sqrt{-g} = a^4$
$S = -\frac{1}{2}\int d\eta d^3\vec{x}\, a^2\left[-(\partial_\eta\varphi)^2 + (\nabla\varphi)^2 + m^2a^2\varphi^2\right]$
Define: $\chi = a\varphi$ $\partial_\eta\chi = \dot{\chi}$
$S = -\frac{1}{2}\int d\eta d^3x\left[-(\partial_\eta\chi)^2 + (\nabla\chi)^2 + \underbrace{\left(m^2a^2 - \frac{\ddot{a}}{a}\right)}_{m^2(\eta)}\chi^2\right]$
$\frac{\delta S}{\delta\chi} = \frac{\partial\mathcal{L}}{\partial\chi} - \frac{d}{dx^\mu}\frac{\partial\mathcal{L}}{\partial(\partial_\mu\chi)} = 0$
$\Rightarrow \ddot{\chi} - \nabla^2\chi + \left(m^2a^2 - \frac{\ddot{a}}{a}\right)\chi = 0$
$\frac{\ddot{a}}{a}$
$a^-_{\vec{k}}$ – integration constants
(depend on $\vec{k}$)
l scalar
$\Rightarrow \chi^*_{\vec{k}}(\eta) = \chi_{-\vec{k}}(\eta)$
$V_k(\eta)$ – satisfy $\ddot{V}_k + 2\omega_k^2 V_k = 0$ $\left(\ddot{V}^*_k + 2\omega_k^2 V^*_k = 0\right)$
only depend on η and $|\vec{k}|$
dependent solutions
$= \frac{1}{}\left[a^-_{\vec{k}} V^*_k(\eta) + a^+_{\vec{k}} V_k(\eta)\right]$

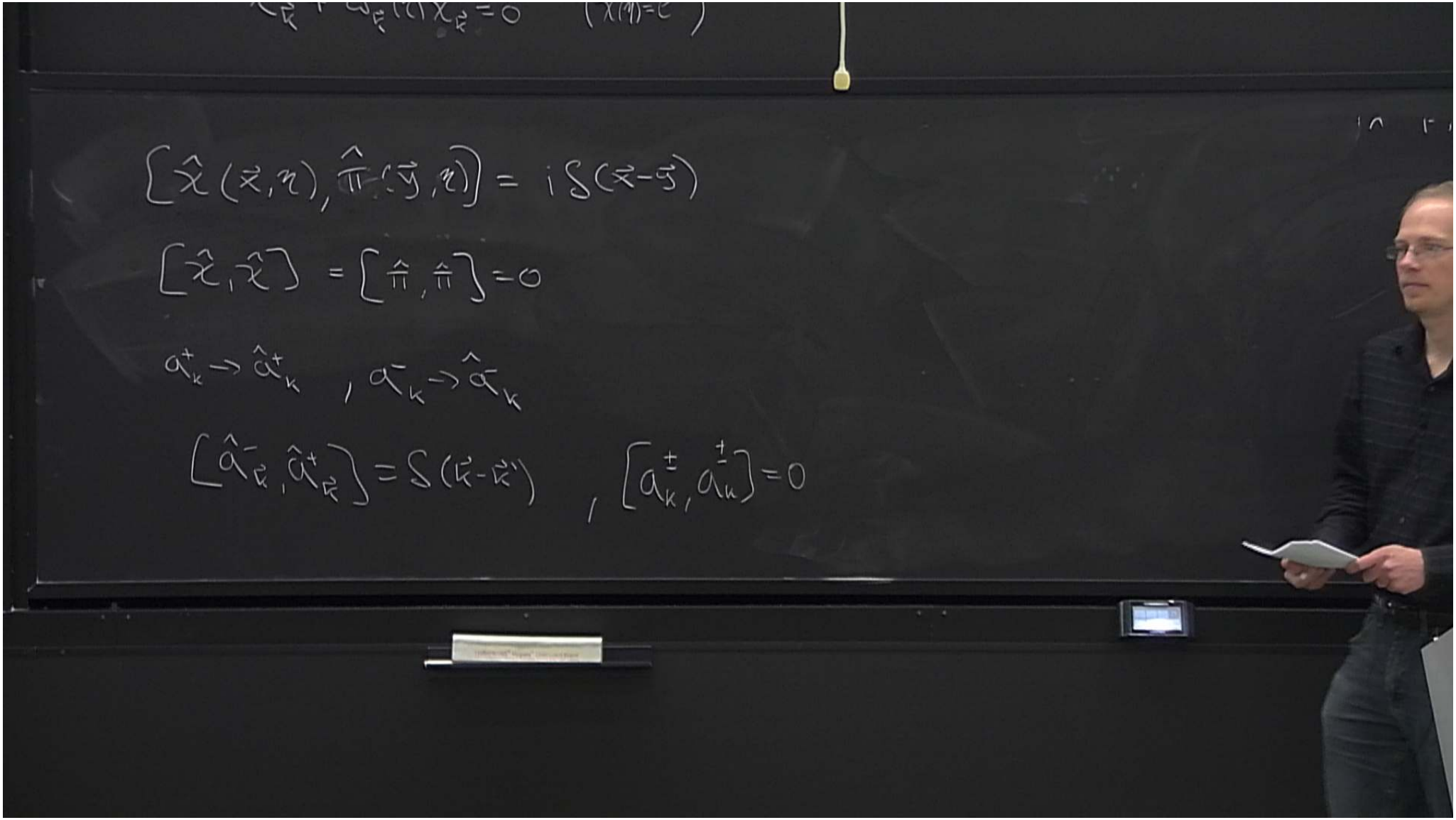

$\Rightarrow \chi(\vec{x},\eta) = \frac{1}{\sqrt{2}} \int \frac{d^3\vec{k}}{(2\pi)^3} \left[a_{\vec{k}}^- V_k^* e^{i\vec{k}\cdot\vec{x}} + a_{\vec{k}}^+ V_k e^{-i\vec{k}\cdot\vec{x}} \right]$
$-V_k \dot{V}_k^* \neq 0$
$= 2i\,\mathrm{Im}(\dot{V}V^*)$
Quantize
$\chi \to \hat{\chi}$

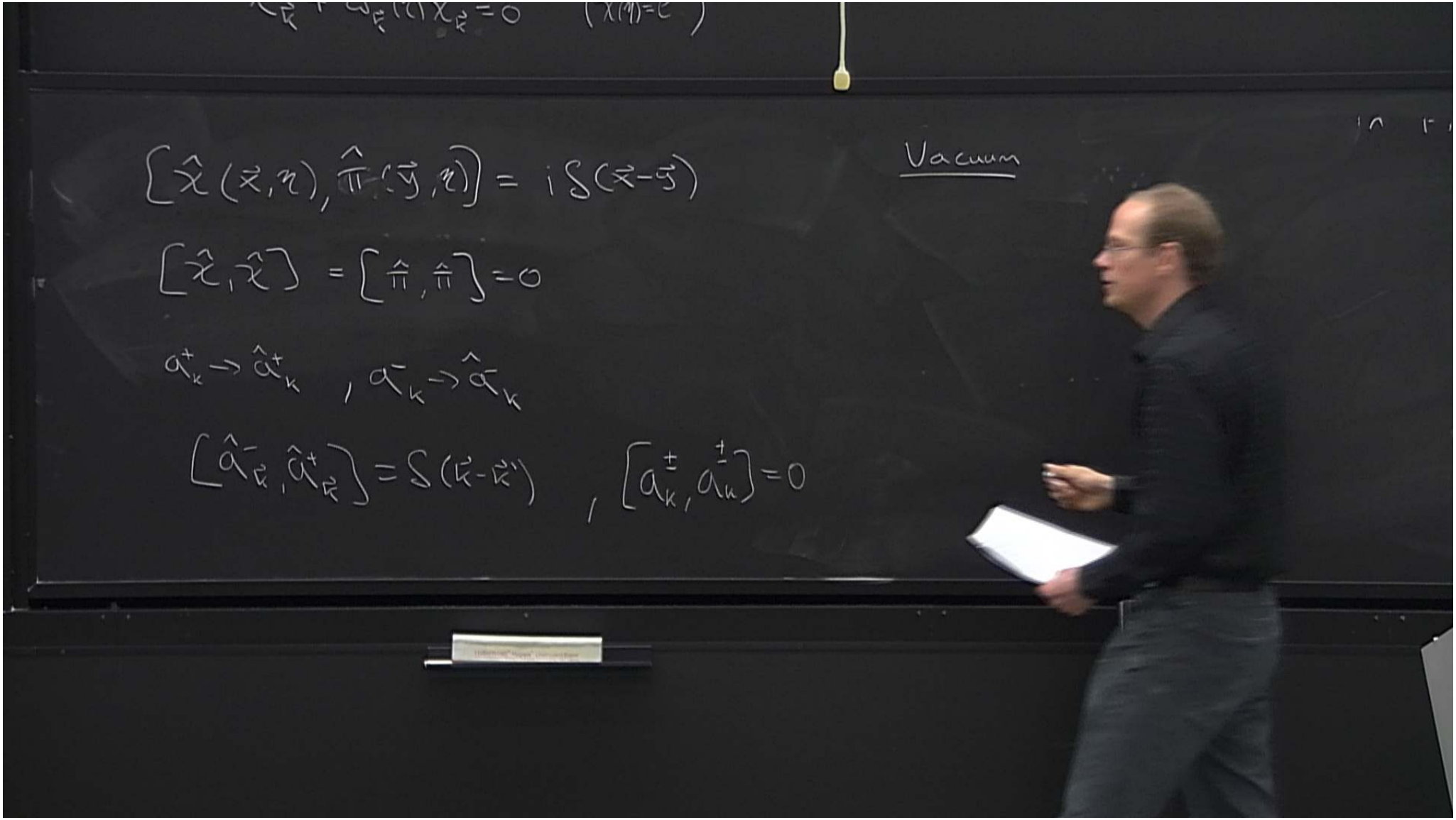

$\Rightarrow \chi(\vec{x},\eta) = \frac{1}{\sqrt{2}} \int \frac{d^3\vec{k}}{(2\pi)^3} \left[a^-_{\vec{k}} v^*_k e^{i\vec{k}\cdot\vec{x}} + a^+_{\vec{k}} v_k e^{-i\vec{k}\cdot\vec{x}} \right]$
$- v_k \dot{v}^*_k \neq 0$
$= 2iI$
Quantize
$\chi \to \hat{\chi}$, conjugate momentum to χ : $\pi = \dot{\chi}$
$\pi \to \hat{\pi}$


$\left[\hat{x}(\vec{x},t), \hat{\pi}(\vec{y},t)\right] = i\delta(\vec{x}-\vec{y})$


$\left[\hat{x}(\vec{x},t), \hat{\pi}(\vec{y},t)\right] = i\delta(\vec{x}-\vec{y})$
$\left[\hat{x},\hat{x}\right] = \left[\hat{\pi},\hat{\pi}\right] = 0$


$\left[\hat{x}(\vec{x},t), \hat{\pi}(\vec{y},t)\right] = i\delta(\vec{x}-\vec{y})$
$\left[\hat{x},\hat{x}\right] = \left[\hat{\pi},\hat{\pi}\right] = 0$
$a_k^+ \to \hat{a}_k^+$, $a_k^- \to \hat{a}_k$


$\ddot{\chi}_k + \omega_k^2 \chi_k = 0 \quad (\chi(\eta) = e^{\dots})$
$[\hat{\chi}(\vec{x},\eta), \hat{\pi}(\vec{y},\eta)] = i\delta(\vec{x}-\vec{y})$
$[\hat{\chi},\hat{\chi}] = [\hat{\pi},\hat{\pi}] = 0$
$a_k^+ \to \hat{a}_k^+ \;,\; a_k^- \to \hat{a}_k^-$
$[\hat{a}_k^-, \hat{a}_{k'}^+] = \delta(\vec{k}-\vec{k}') \;,\; [a_k^\pm, a_{k'}^\pm] = 0$


$[\hat{\chi}(\vec{x},\eta), \hat{\pi}(\vec{y},\eta)] = i\delta(\vec{x}-\vec{y})$
$[\hat{\chi},\hat{\chi}] = [\hat{\pi},\hat{\pi}] = 0$
$a^+_k \to \hat{a}^+_k \quad , a^-_k \to \hat{a}^-_k$
$[\hat{a}^-_{\vec{k}}, \hat{a}^+_{\vec{k}}] = \delta(\vec{k}-\vec{k}') \quad , [a^\pm_k, a^\pm_k] = 0$
Vacuum

$$\chi_{\vec{k}} + \omega_{\vec{k}}(\eta)\chi_{\vec{k}} = 0 \qquad (\chi(\eta) = e^{\ldots})$$

$$[\hat{\chi}(\vec{x},\eta), \hat{\pi}(\vec{y},\eta)] = i\delta(\vec{x}-\vec{y})$$

$$[\hat{\chi},\hat{\chi}] = [\hat{\pi},\hat{\pi}] = 0$$

$$a^+_k \to \hat{a}^+_k \quad , \quad a^-_k \to \hat{a}^-_k$$

$$[\hat{a}^-_{\vec{k}}, \hat{a}^+_{\vec{k}}] = \delta(\vec{k}-\vec{k}') \quad , \quad [a^\pm_k, a^\pm_k] = 0$$

Vacuum

$$\hat{a}^-_k |0\rangle = 0$$

Particles: $a^+_k |0\rangle = |1\rangle_k$

$$\Rightarrow \ddot{\chi}_k + \omega_k^2(\eta)\chi_k = 0 \qquad (\chi(\eta) = e^{i\omega\eta})$$

$$\left[\hat{\chi}(\vec{x},\eta), \hat{\pi}(\vec{y},\eta)\right] = i\delta(\vec{x}-\vec{y})$$

$$\left[\hat{\chi},\hat{\chi}\right] = \left[\hat{\pi},\hat{\pi}\right] = 0$$

$$a_k^+ \to \hat{a}_k^+ \,, \quad a_k^- \to \hat{a}_k$$

$$\left[\hat{a}_k^-, \hat{a}_k^+\right] = \delta(\vec{k}-\vec{k}') \,, \quad \left[a_k^\pm, a_k^\pm\right] = 0$$

Vacuum

$$\hat{a}_k^- |0\rangle = 0$$

Particles: $a_k^+ |0\rangle = |1\rangle_k$

However, $\mathrm{Im}(\dot{V}V^*) = 1$ has multiple solutions (choose different V_k, V_k^*)

$\Rightarrow \ddot{\chi}_k + \omega_k^2(\eta)\chi_k = 0 \quad (\chi(\eta) = e^{i\omega\eta})$

$[\hat{\chi}(\vec{x},\eta), \hat{\pi}(\vec{y},\eta)] = i\delta(\vec{x}-\vec{y})$

$[\hat{\chi},\hat{\chi}] = [\hat{\pi},\hat{\pi}] = 0$

$a_k^+ \to \hat{a}_k^+ \,, \; a_k^- \to \hat{a}_k$

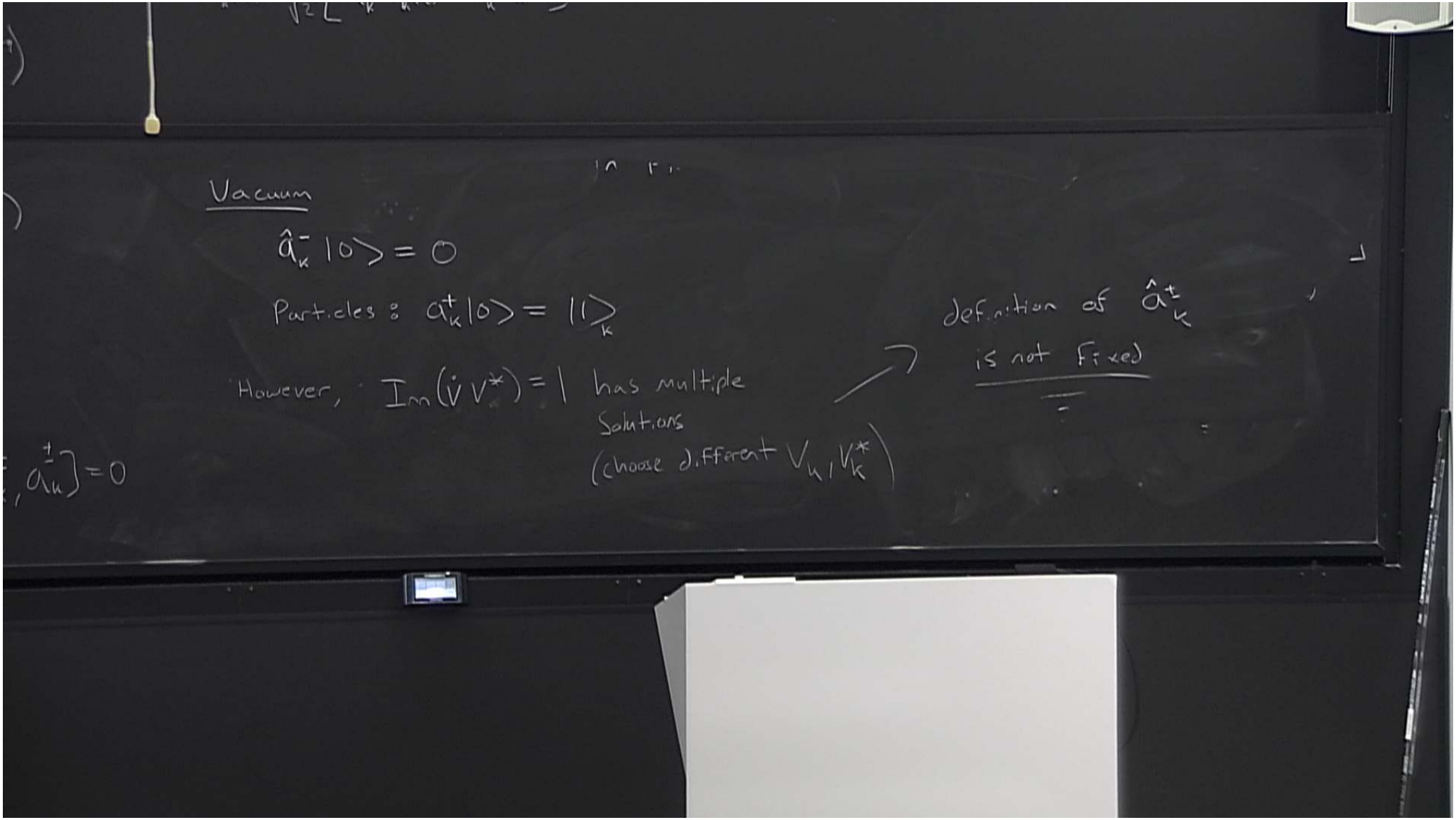
$[\hat{a}_{\vec{k}}, \hat{a}^+_{\vec{k}'}] = \delta(\vec{k}-\vec{k}') \,, \; [a_k^\pm, a_k^\pm] = 0$

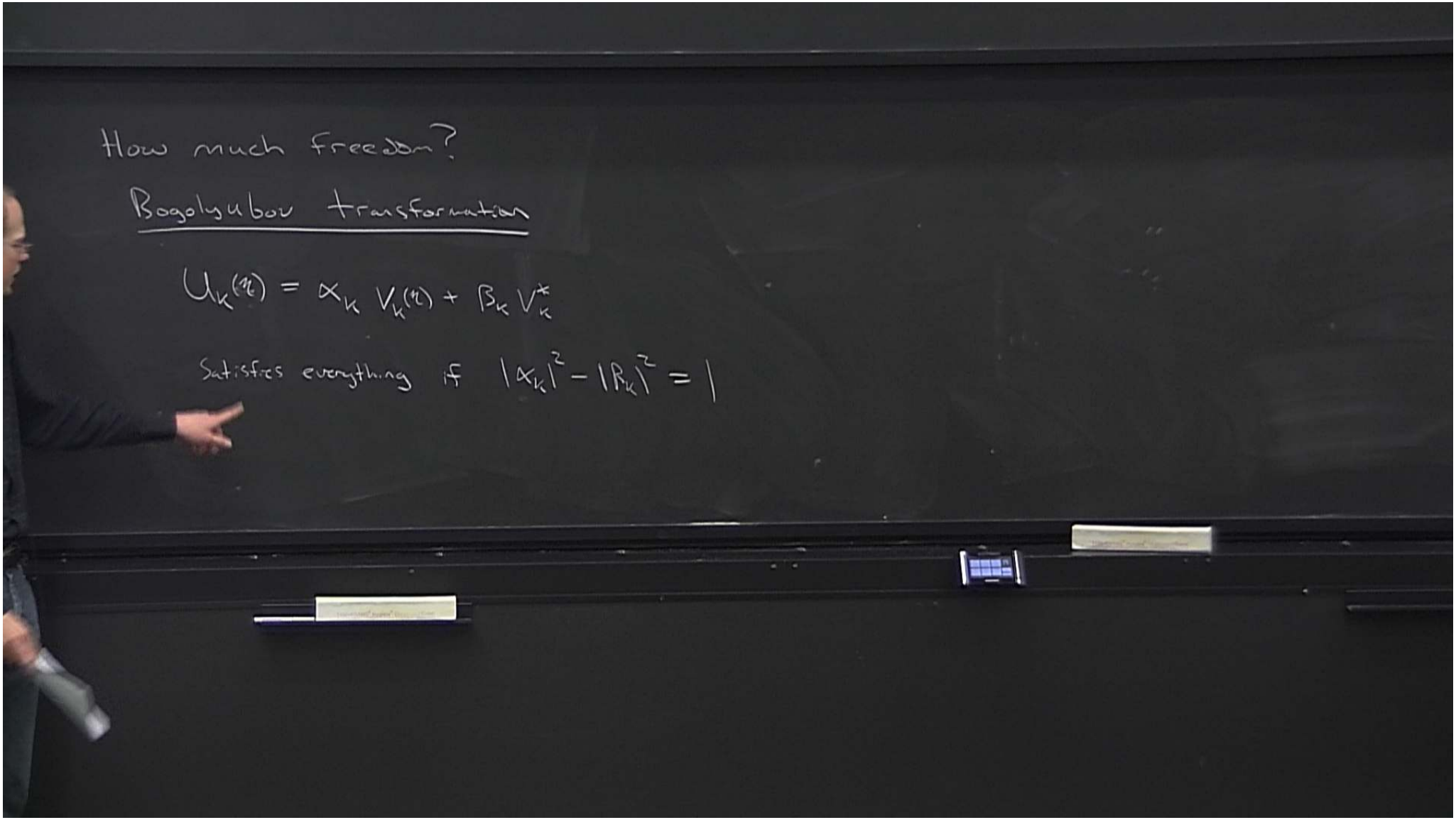
Vacuum

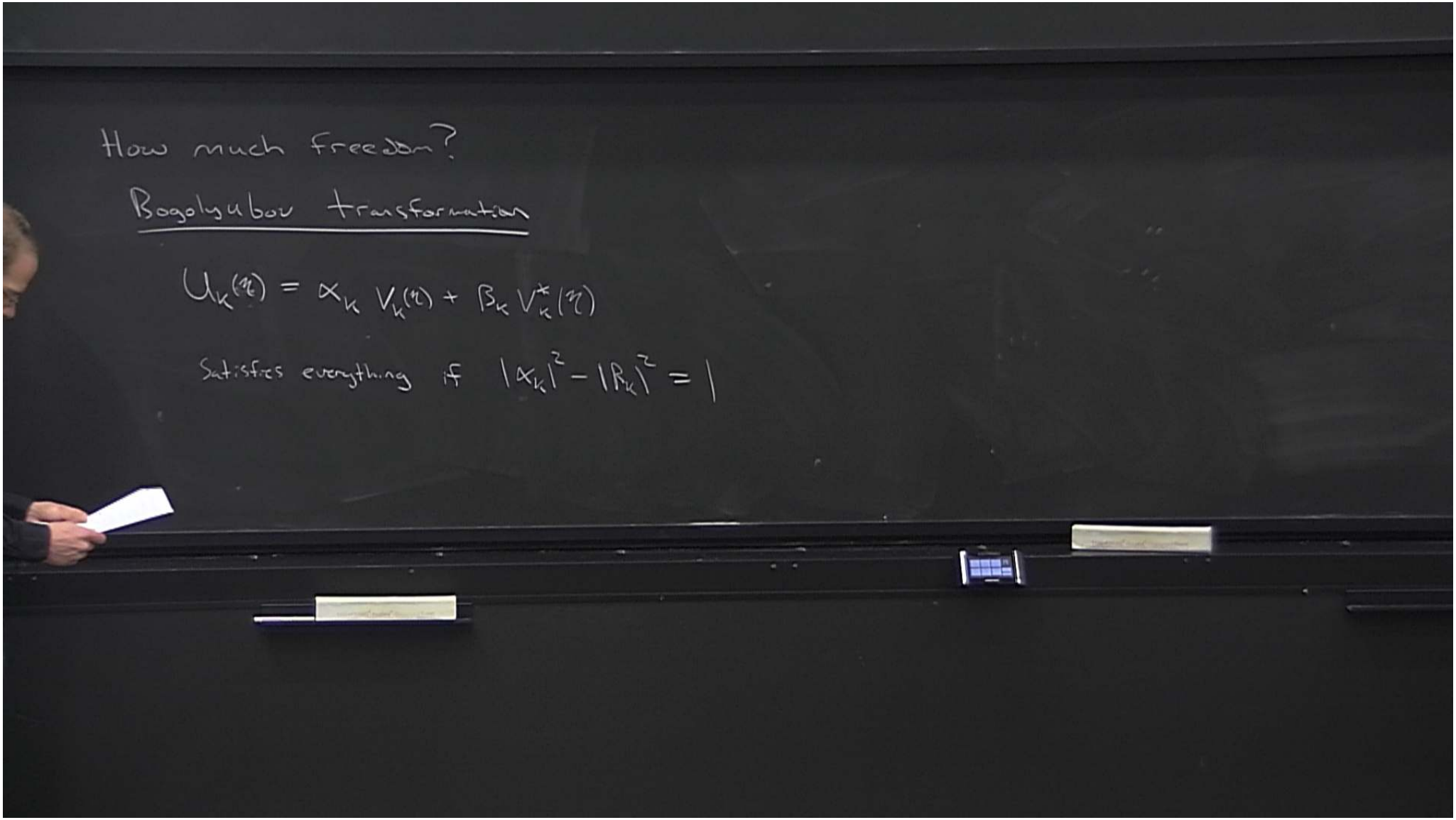
$\hat{a}_k^- |0\rangle = 0$

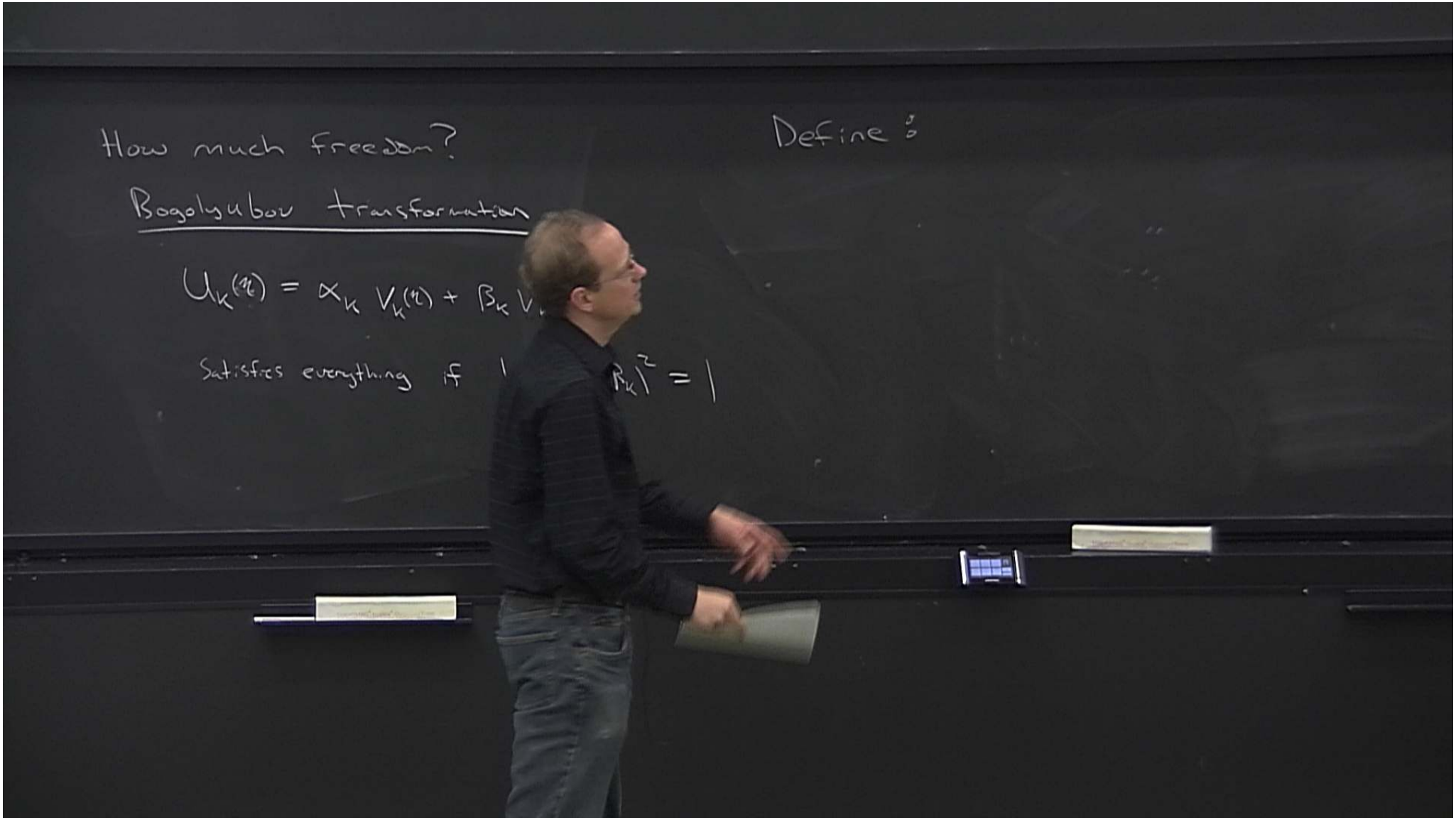
Particles: $a_k^+ |0\rangle = |1\rangle_k$

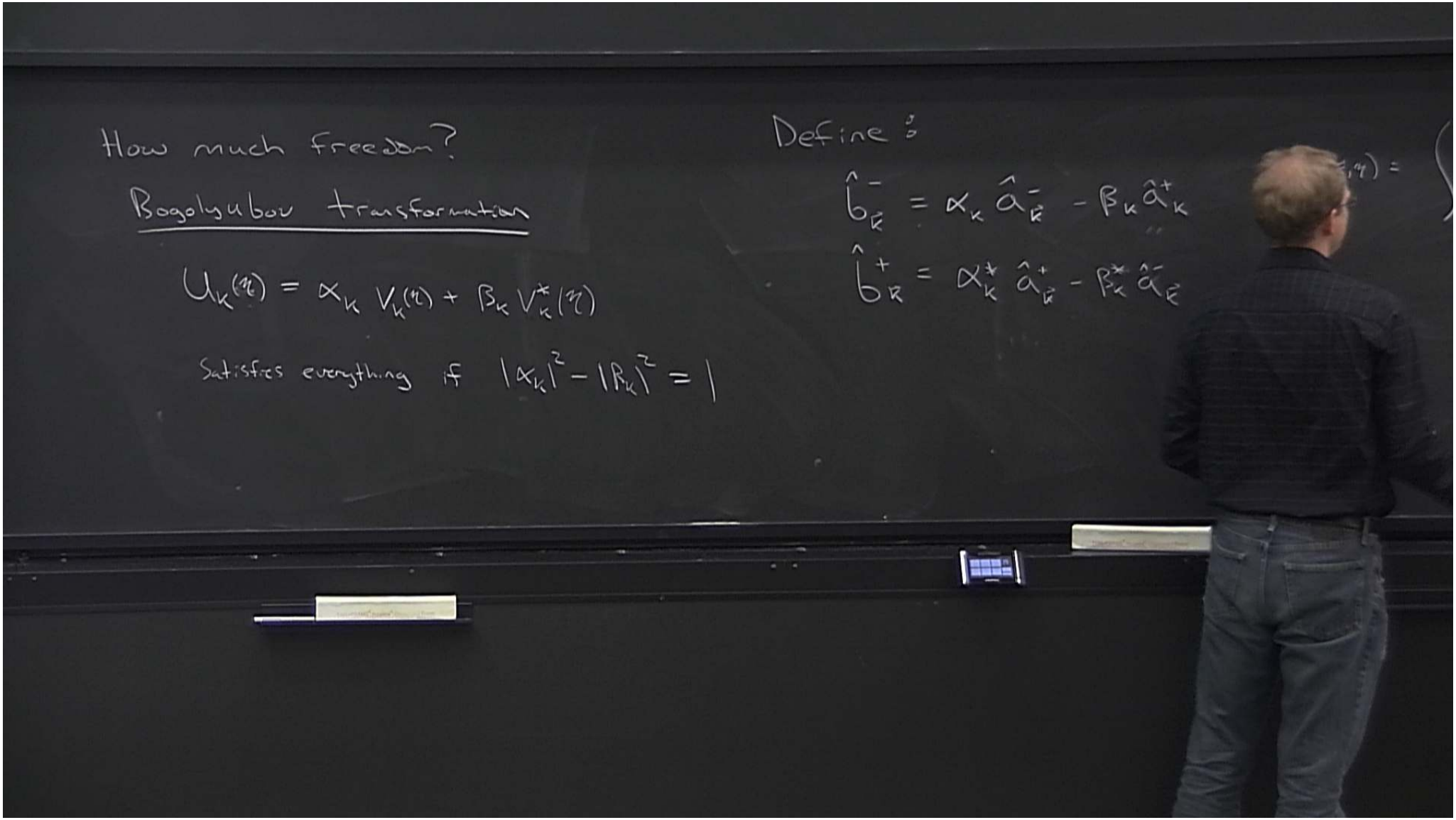
However, $\mathrm{Im}(\dot{V}V^*) = 1$ has multiple solutions (choose different V_k, V_k^*)

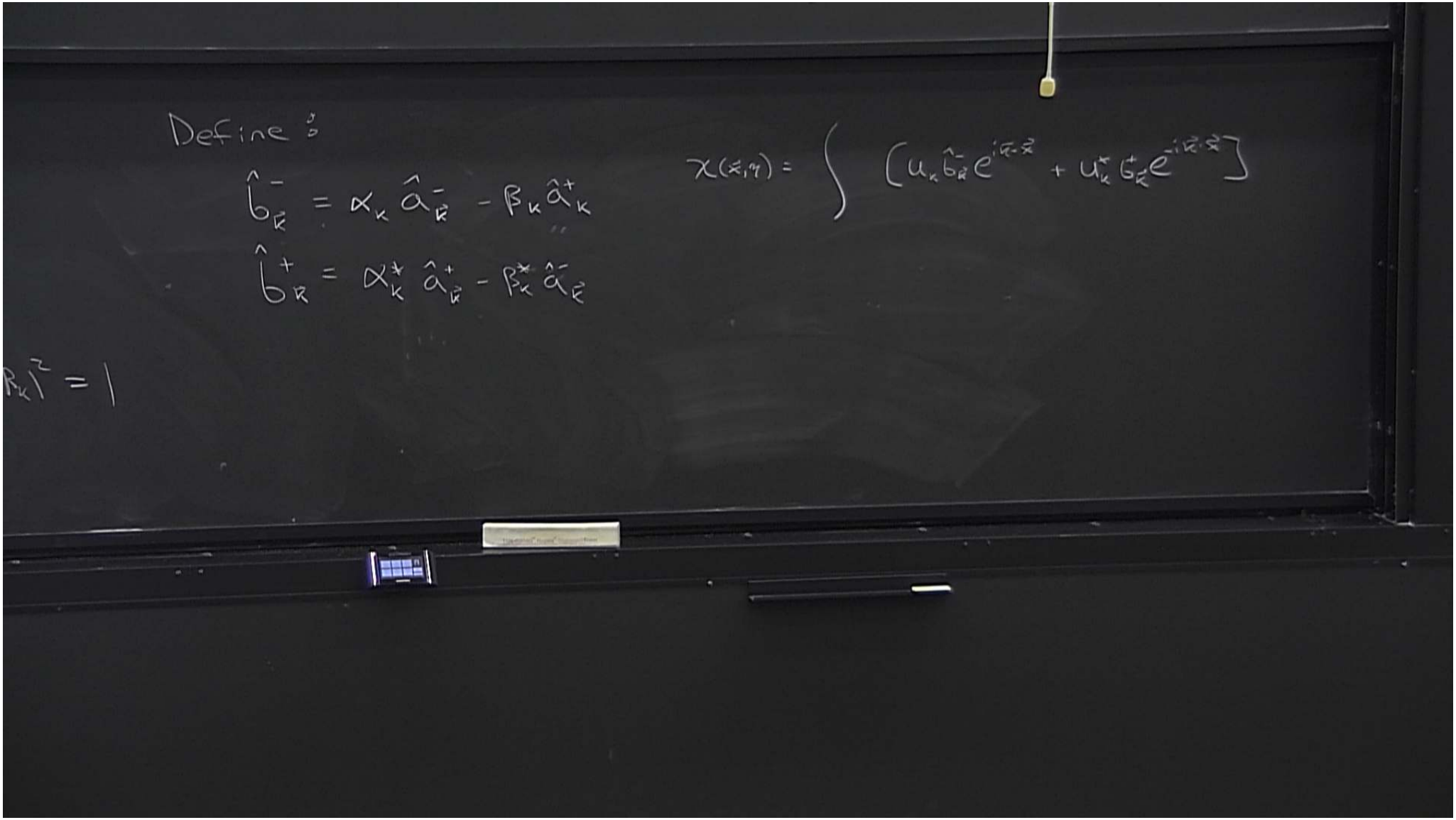

Vacuum
$\hat{a}_k^- |0\rangle = 0$
Particles: $a_k^+ |0\rangle = |1\rangle_k$
However, $Im(\dot{v} v^*) = 1$ has multiple solutions (choose different v_k, v_k^*)
definition of $\hat{a}_k^\pm$ is not fixed
$a_u^+] = 0$

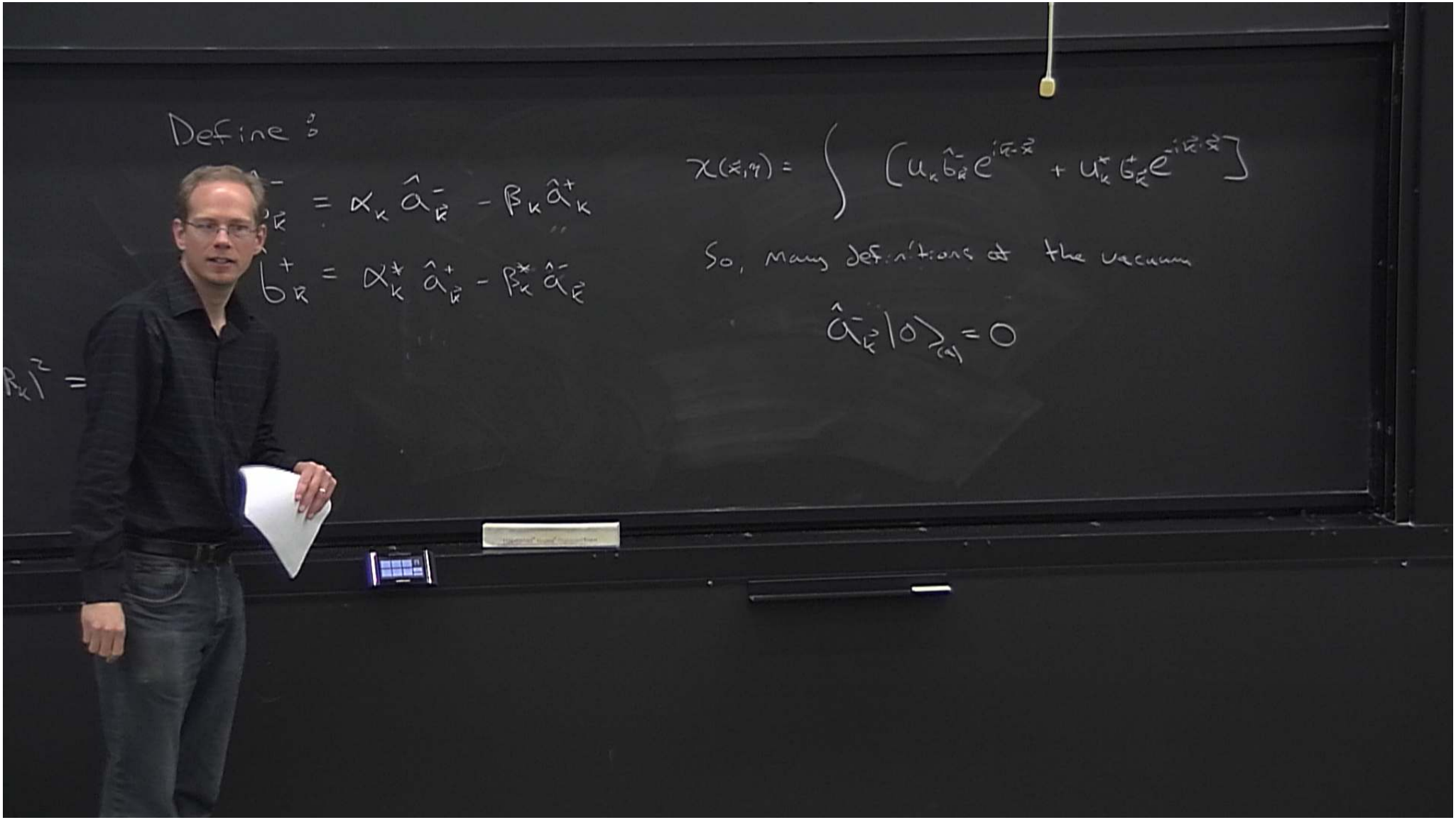

How much freedom?
Bogolyubov transformation
$U_k(\eta) = \alpha_k V_k(\eta) + \beta_k V_k^*$
Satisfies everything if $|\alpha_k|^2 - |\beta_k|^2 = 1$


How much freedom?
Bogolyubov transformation
$U_k(\eta) = \alpha_k V_k(\eta) + \beta_k V_k^*(\eta)$
Satisfies everything if $|\alpha_k|^2 - |\beta_k|^2 = 1$


How much freedom?
Define!
Bogolyubov transformation
$U_k(\eta) = \alpha_k V_k(\eta) + \beta_k V$
Satisfies everything if $|$ $\beta_k|^2 = |$


How much freedom?
Bogolyubov transformation
$u_k(\eta) = \alpha_k v_k(\eta) + \beta_k v_k^*(\eta)$
Satisfies everything if $|\alpha_k|^2 - |\beta_k|^2 = 1$
Define:
$\hat{b}_k^- = \alpha_k \hat{a}_k^- - \beta_k \hat{a}_k^+$
$\hat{b}_k^+ = \alpha_k^* \hat{a}_k^+ - \beta_k^* \hat{a}_k^-$


Define :
$\hat{b}_k^- = \alpha_k \hat{a}_k^- - \beta_k \hat{a}_k^+$
$\hat{b}_k^+ = \alpha_k^* \hat{a}_k^+ - \beta_k^* \hat{a}_k^-$
$\chi(x,y) = \int [u_k \hat{b}_k^- e^{i\vec{k}\cdot\vec{x}} + u_k^* \hat{b}_k^+ e^{-i\vec{k}\cdot\vec{x}}]$
$|\alpha_k|^2 = 1$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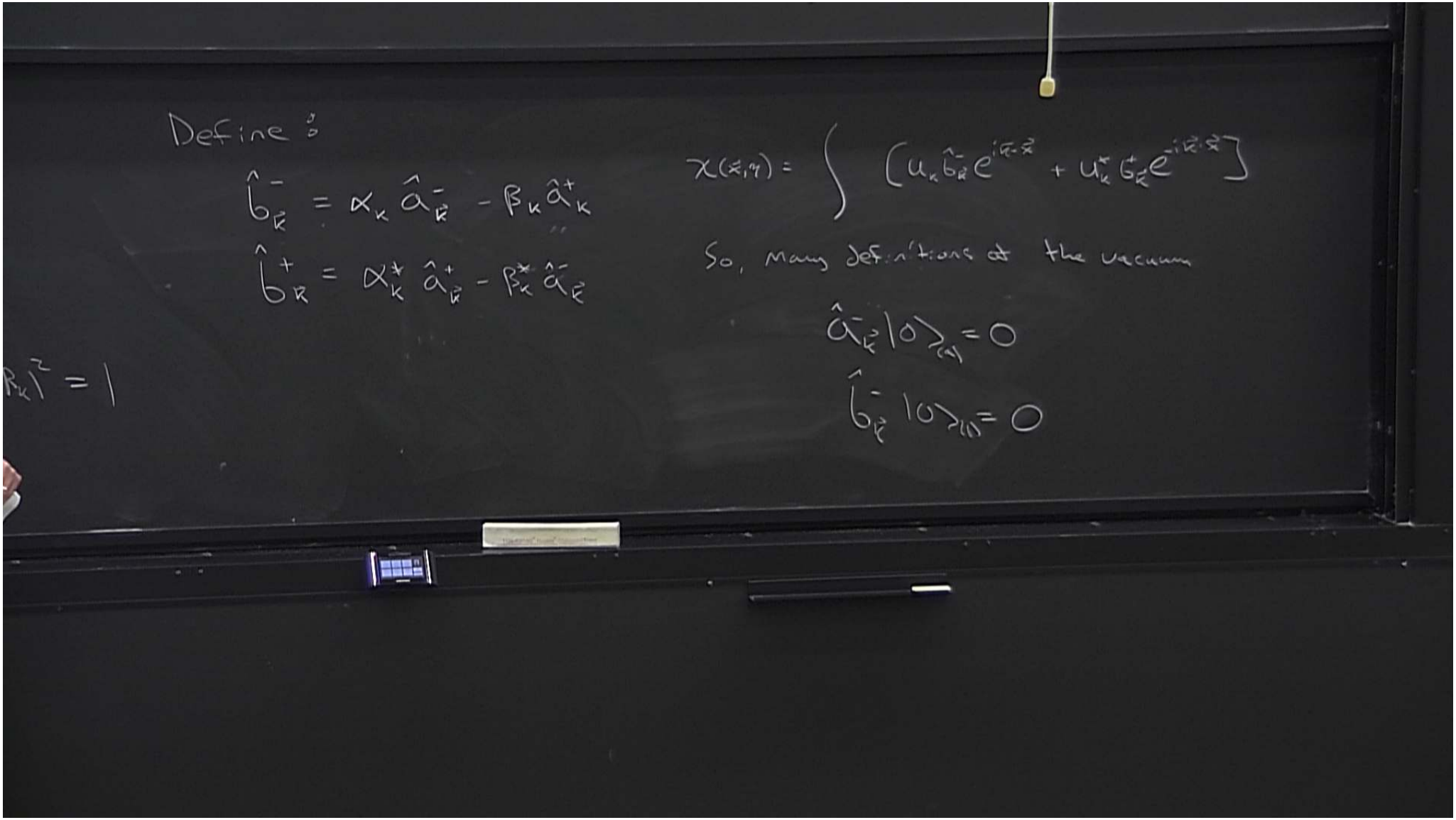
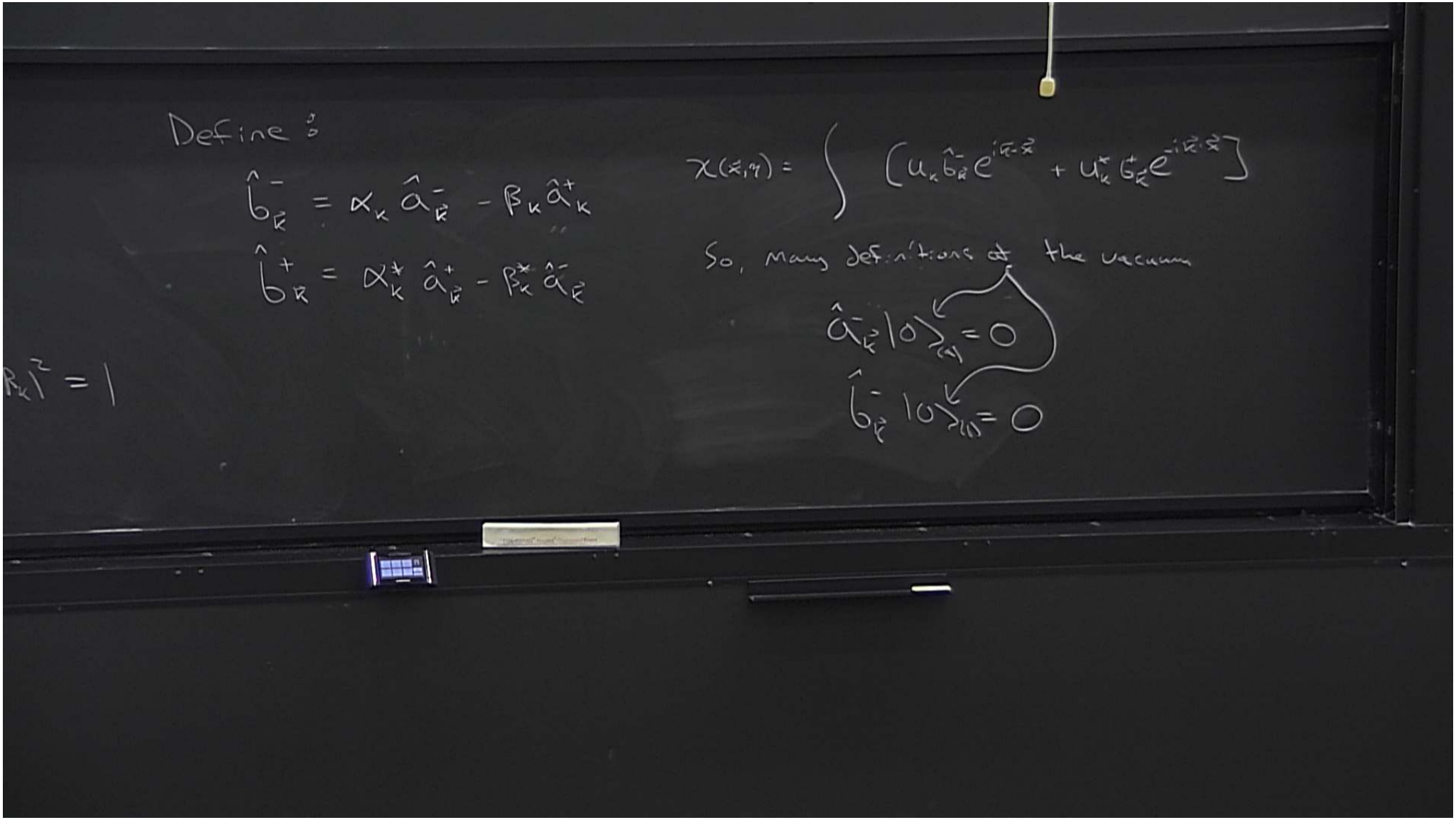
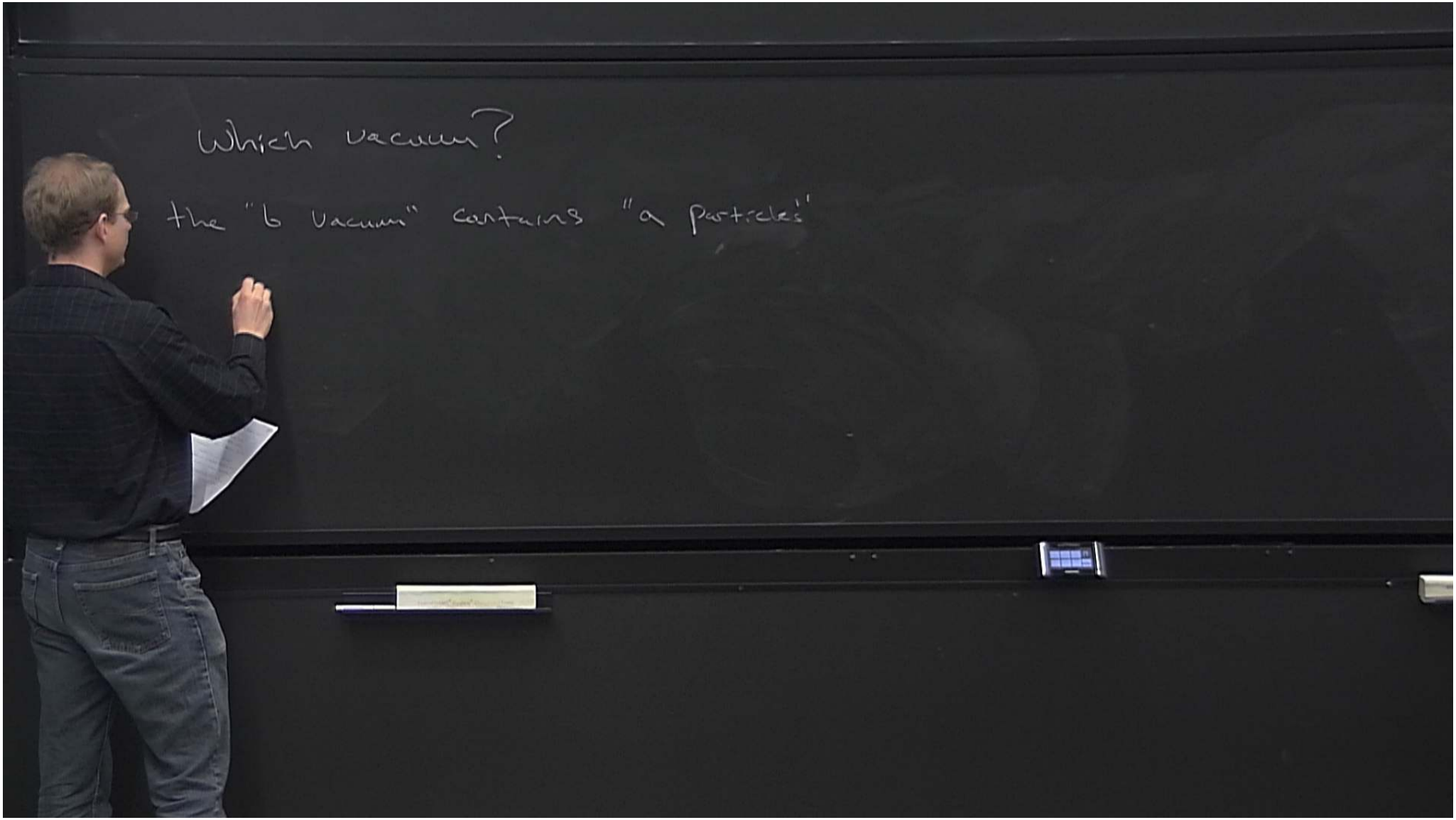
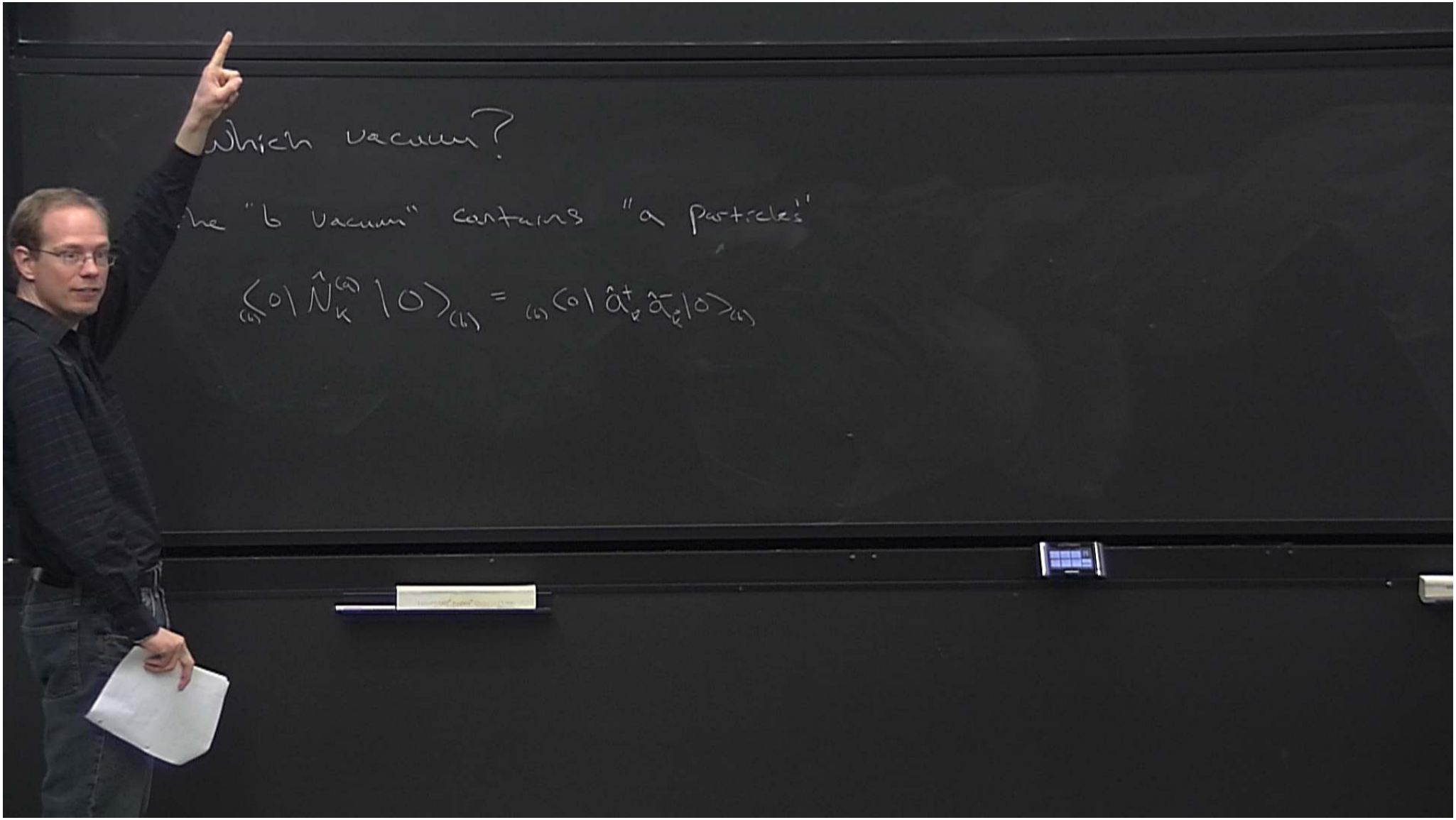
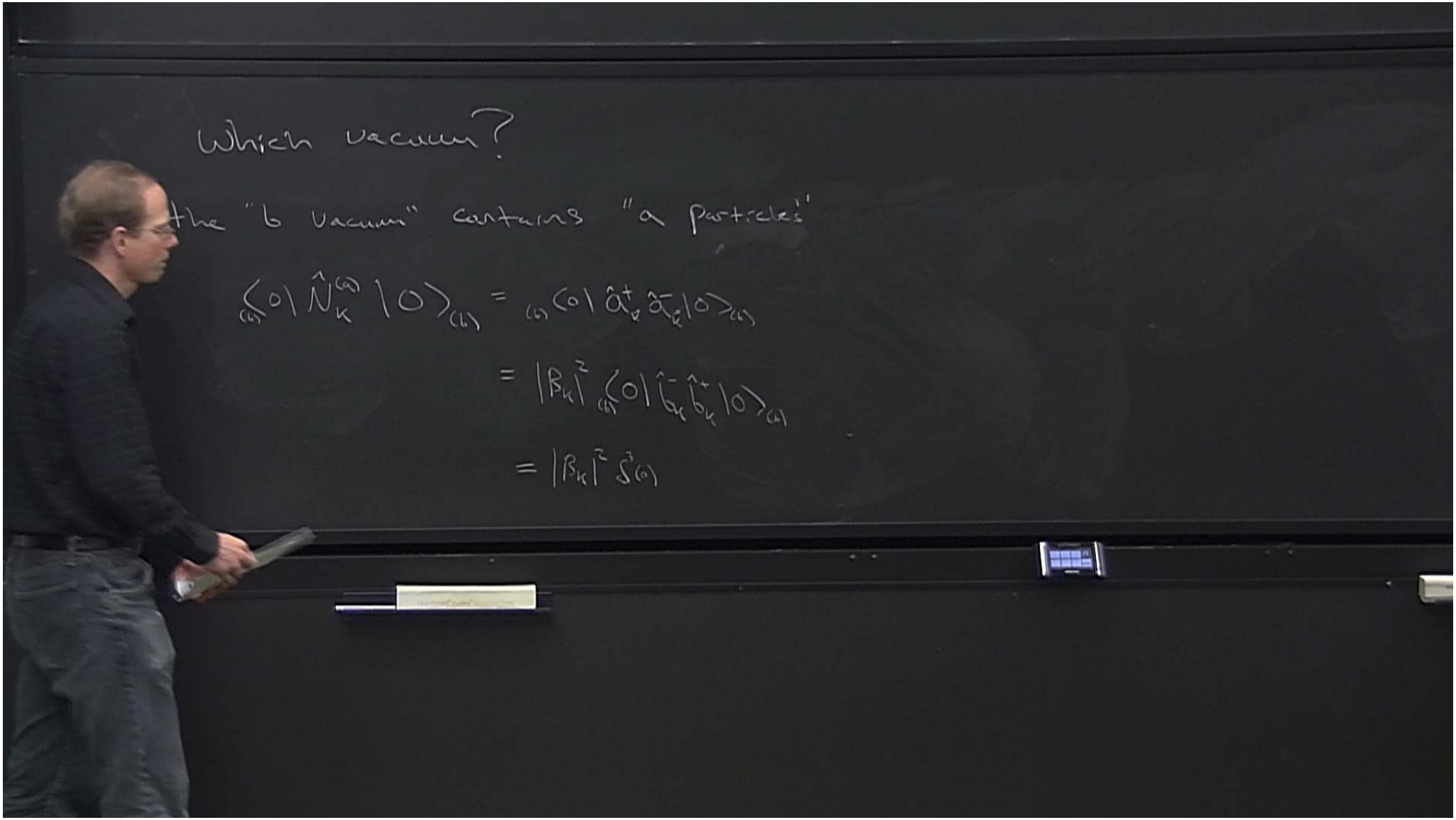
$\hat{b}_k^- = \alpha_k \hat{a}_k^- - \beta_k \hat{a}_k^+$
$b_k^+ = \alpha_k^* \hat{a}_k^+ - \beta_k^* \hat{a}_k^-$
$\chi(x,y) = \int [u_k \hat{b}_k^- e^{i\vec{k}\cdot\vec{x}} + u_k^* \hat{b}_k^+ e^{-i\vec{k}\cdot\vec{x}}]$
So, many definitions of the vacuum
$\hat{a}_k^- |0\rangle_{(a)} = 0$

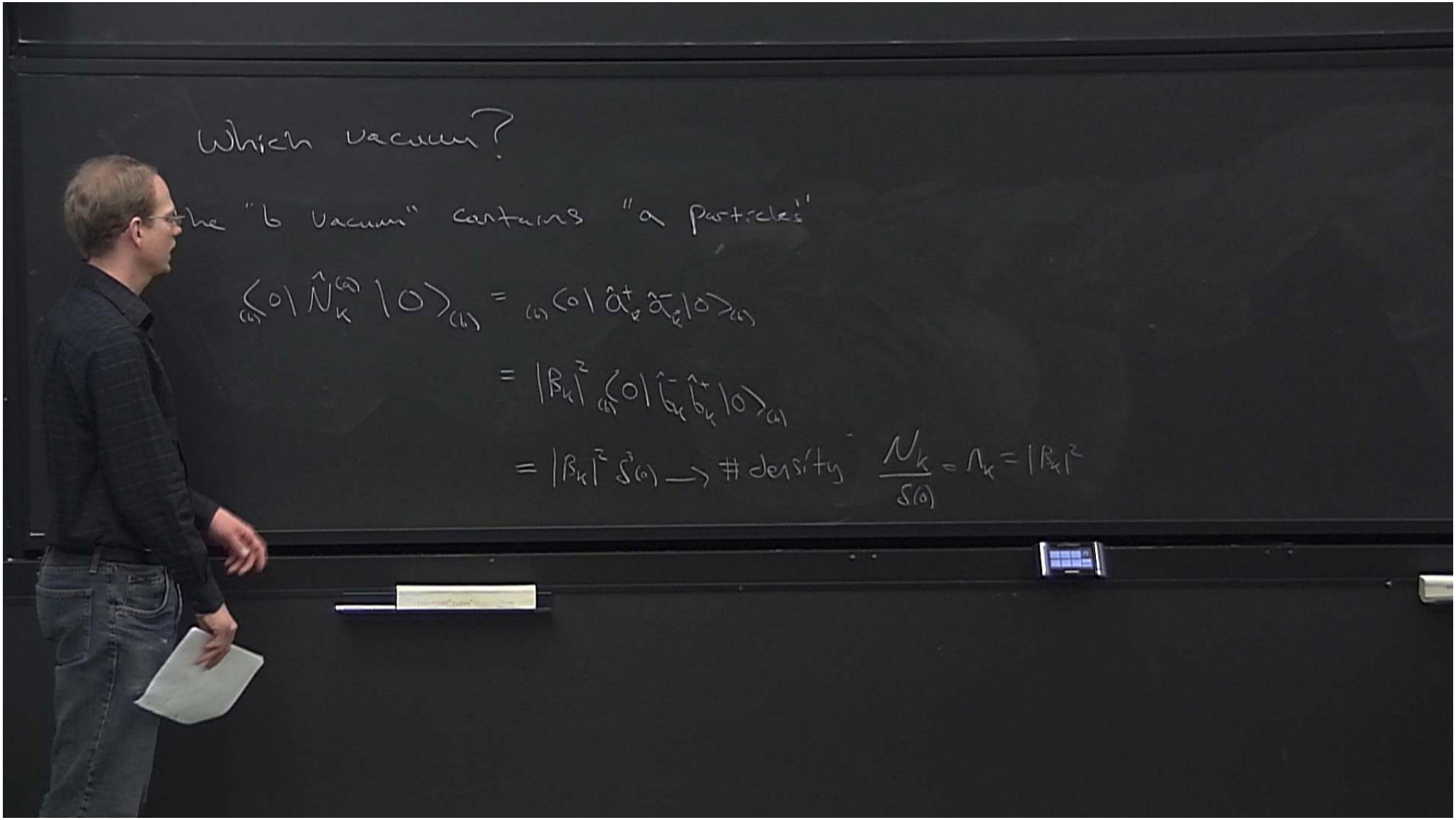

Define:
$\hat{b}_k^- = \alpha_k \hat{a}_k^- - \beta_k \hat{a}_k^+$
$\hat{b}_k^+ = \alpha_k^* \hat{a}_k^+ - \beta_k^* \hat{a}_k^-$
$|\beta_k|^2 = 1$
$\chi(x,y) = \int [u_k \hat{b}_k^- e^{i\vec{k}\cdot\vec{x}} + u_k^* \hat{b}_k^+ e^{-i\vec{k}\cdot\vec{x}}]$
So, many definitions of the vacuum
$\hat{a}_k^- |0\rangle_{(a)} = 0$
$\hat{b}_k^- |0\rangle_{(b)} = 0$


Define:
$\hat{b}_k^- = \alpha_k \hat{a}_k^- - \beta_k \hat{a}_k^+$
$\hat{b}_k^+ = \alpha_k^* \hat{a}_k^+ - \beta_k^* \hat{a}_k^-$
$|\alpha_k|^2 = 1$
$\chi(x,y) = \int [u_k \hat{b}_k^- e^{ik\cdot x} + u_k^* \hat{b}_k^+ e^{-ik\cdot x}]$
So, many definitions of the vacuum
$\hat{a}_k^- |0\rangle_{(a)} = 0$
$\hat{b}_k^- |0\rangle_{(b)} = 0$


Which vacuum?
the "b vacuum" contains "a particles"


Which vacuum?
The "b vacuum" contains "a particles"
$\langle 0|\hat{N}_k^{(a)}|0\rangle_{(b)} = {}_{(b)}\langle 0|\hat{a}_k^{\dagger}\hat{a}_k|0\rangle_{(b)}$


Which vacuum?
the "b vacuum" contains "a particles"
$\langle 0|\hat{N}_k^{(a)}|0\rangle_{(b)} = {}_{(b)}\langle 0|\hat{a}_k^\dagger \hat{a}_k|0\rangle_{(b)}$
$= |\beta_k|^2 \, {}_{(b)}\langle 0|\hat{b}_k^- \hat{b}_k^+|0\rangle_{(b)}$
$= |\beta_k|^2 \, \delta^3(0)$


Which vacuum?
the "b vacuum" contains "a particles"
$\langle 0|\hat{N}_k^{(a)}|0\rangle_{(a)} = {}_{(a)}\langle 0|\hat{a}_k^\dagger \hat{a}_k|0\rangle_{(a)}$
$= |\beta_k|^2 \langle 0|\hat{b}_k^- \hat{b}_k^+|0\rangle_{(a)}$
$= |\beta_k|^2 \delta(0) \longrightarrow$ # density
$\frac{N_k}{\delta(0)} = n_k = |\beta_k|^2$

Which vacuum?

the "b vacuum" contains "a particles"

$$ {}_{(a)}\langle 0|\hat{N}_k^{(a)}|0\rangle_{(a)} = {}_{(a)}\langle 0|\hat{a}_k^{\dagger}\hat{a}_k^{-}|0\rangle_{(a)} $$

$$ = |\beta_k|^2\, {}_{(a)}\langle 0|\hat{b}_k^{-}\hat{b}_k^{+}|0\rangle_{(a)} $$

$$ = |\beta_k|^2 \delta(0) \longrightarrow \#\ \text{density} \quad \frac{\mathcal{N}_k}{\delta(0)} = n_k = |\beta_k|^2 $$