

Title: 12/13 PSI - Explorations in Cosmology Lecture 10

Date: Apr 19, 2013 10:15 AM

URL: <http://pirsa.org/13040073>

Abstract:

Inflation

Makes cosmology Causal!

1 primordial causally connected
region and inflates it to the
size of our observable universe.

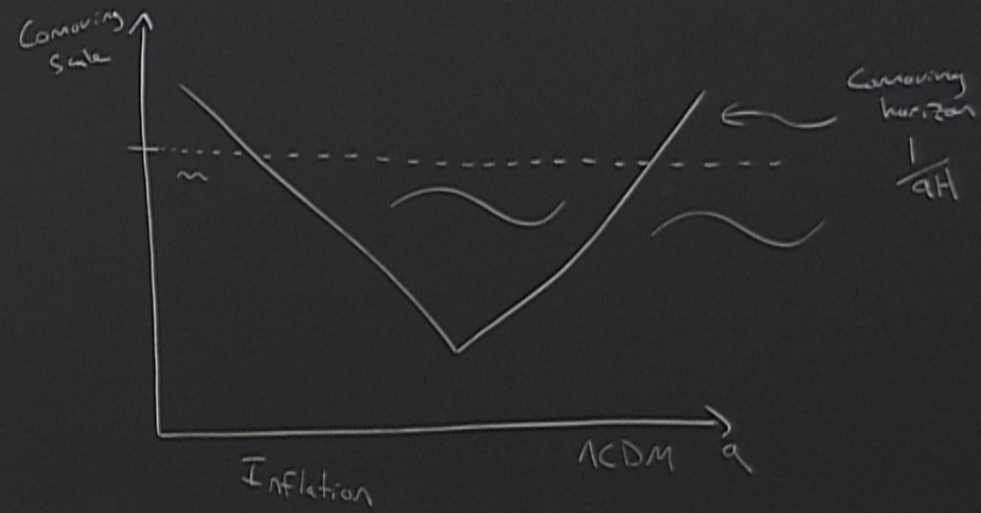


Inflation

Modern cosmology Causal!

1. Causally connected

and inflates it to the
size of our observable universe.

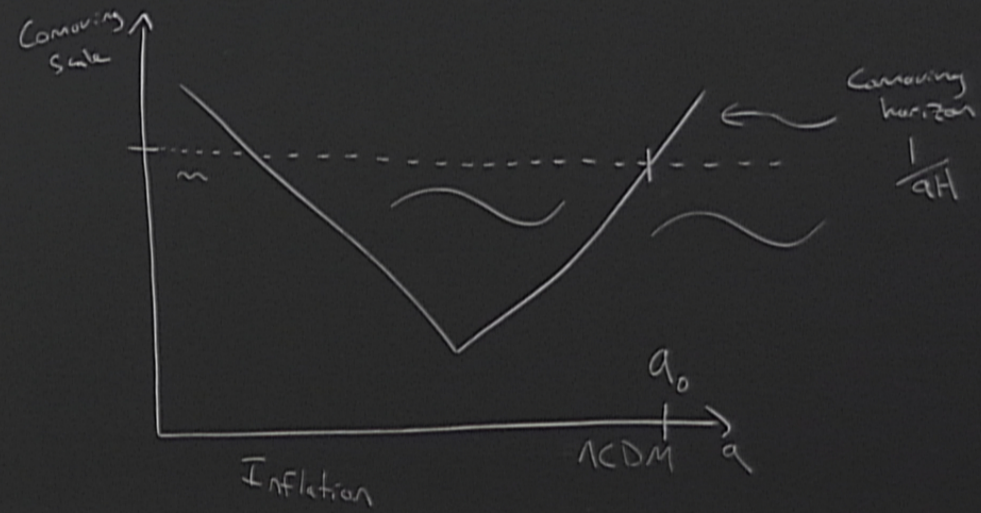


Inflation

Modern Cosmology Causal!

1. Spatially Causally Connected

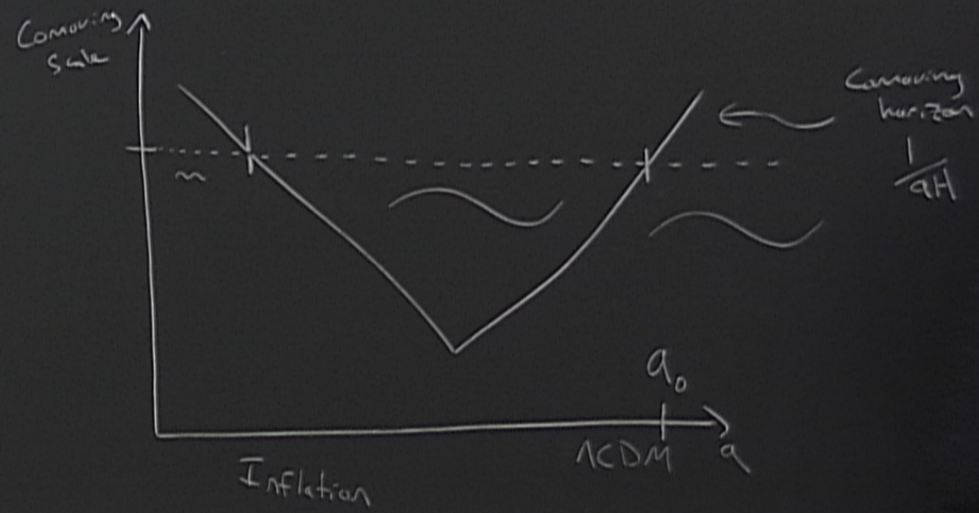
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Inflation

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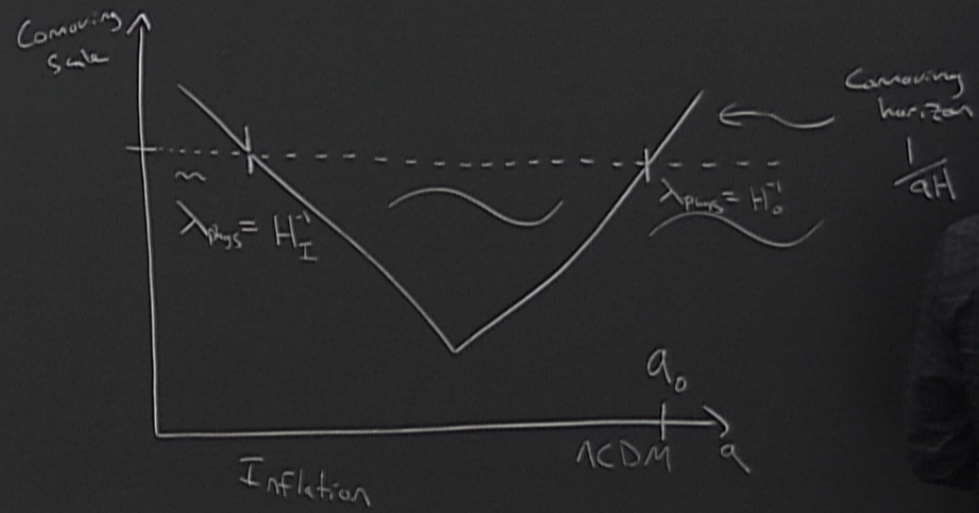
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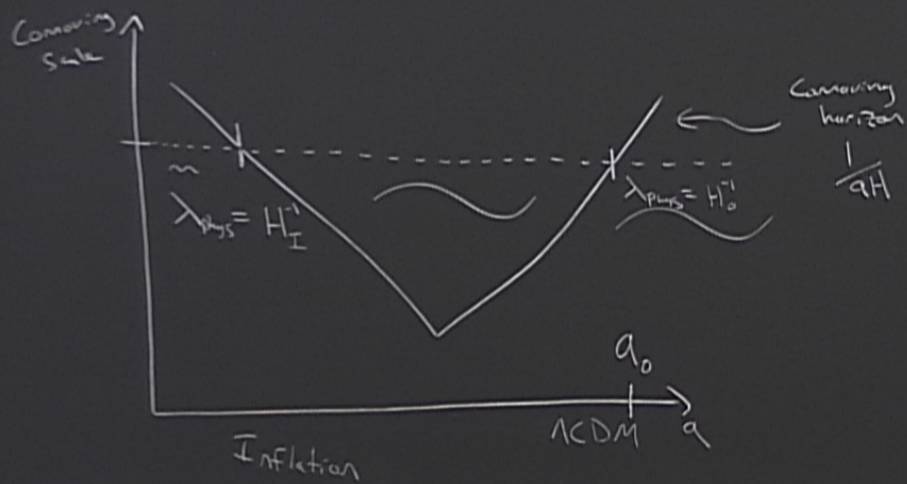
Inflation

Makes cosmology Causal!

1 primordial causally connected region and inflates it to the size of our observable universe.







By how much did the universe grow?

$$\lambda_c = \text{const.}$$

$$\frac{a_0}{a_I} = \frac{H_I}{H_0}$$

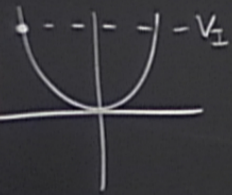
$$H_0 = \frac{1}{3000 \text{ Mpc}} = 10^{-26} \text{ m}^{-1}$$

inflation

Slow-roll inflation - potential energy domination

$$H_I = \left(\frac{8\pi G}{3} V_I \right)^{1/2}$$

$$G = \frac{1}{M_P^2}, \quad M_P = 1.2 \times 10^{19} \text{ GeV}$$



$$H_I = \left(\frac{10^{-12} M_P^4}{M_P^2} \right)^{1/2} = 10^{-6} M_P$$

$$V_I = (10^{16} \text{ GeV})^4 = (10^{-3} M_P)^4 = 10^{-12} M_P^4$$

$$H_I^{-1} = 10^6 \lambda_{M_P} = 10^6 \lambda_P = 10^{-27} \text{ m}$$

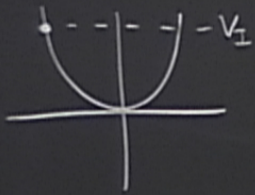
inflation

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$$\Rightarrow \frac{a_0}{a_I} = \frac{H_I}{H_0} = 10^{53}$$



$$H_I = \left(\frac{10^{-12} M_P^4}{M_P^2} \right)^{1/2} = 10^{-6} M_P$$

$$V_I \sim (10^4 \text{ GeV})^4 = (10^{-3} M_P)^4 = 10^{-12} M_P^4$$

$$H_I^{-1} = 10^6 \text{ } 1/M_P =$$

$$10^{13} \text{ GeV}$$

$$\Rightarrow \frac{a_0}{a_I} = \frac{H_I}{H_0} = 10^{53} \quad \text{From inflation to now}$$

$$\left(10^{-3} m_P\right)^4 = 10^{-12} m_P^4$$

How much did universe grow since end of inflation?
3000

$$\frac{a_0}{a_{\text{reh}}} = \frac{a_0}{a_{\text{eq}}} \frac{a_{\text{eq}}}{a_{\text{reh}}}$$

$$10^{12} \text{ GeV}$$

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How much did universe grow since end of inflation?

$$\frac{a_0}{a_{\text{reh}}} = \frac{a_0}{a_{\text{eq}}} \frac{a_{\text{eq}}}{a_{\text{reh}}} \quad \begin{array}{l} \text{3000} \\ \text{radiation} \\ \text{dominated} \\ \text{univ.} \\ a \propto 1/T \end{array}$$

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How much did universe grow since end of inflation?

$$\frac{a_0}{a_{\text{reh}}} = \frac{a_0}{a_{\text{eq}}} \frac{a_{\text{eq}}}{a_{\text{reh}}} \quad \begin{array}{l} \nearrow \text{Radiation} \\ \text{dominated} \\ \text{univ.} \\ a \propto 1/T \end{array}$$

$$= 3000 \frac{T_{\text{reh}}}{T_{\text{eq}}} \quad \begin{array}{l} \leftarrow 1 \text{ TeV} = 10^{12} \text{ GeV} \\ \leftarrow 1 \text{ eV} \end{array}$$

$$\Rightarrow \frac{a_{\text{reh}}}{a_{\text{I}}} \simeq 10^{23}$$

$$a_{\text{I}} \simeq e^{\textcircled{H+} N}$$

$$N = \log(10^{23}) \simeq 53$$

what you need to
make Univ Causal!

$$\Rightarrow \frac{a_{\text{reh}}}{a_{\text{I}}} \simeq 10^{23}$$

$$a_{\text{I}} \simeq e^{(\text{H}^+) N}$$

$$N = \log(10^{23}) \simeq 53 \quad \text{what you need to make Univ Causal!}$$

$$\text{How long? } t \simeq 53 H_{\text{I}}^{-1} \simeq 53 \times 10^6 t_{\text{P}} \simeq 10^{-37} \text{ s}$$

$$\Rightarrow \frac{a_{\text{reh}}}{a_{\text{I}}} \simeq 10^{23}$$

$$a_{\text{I}} \simeq e^{(H^+) N}$$

$$N = \log(10^{23}) \simeq 53 \quad \text{what you need to make Univ Causal!}$$

$$\text{How long? } t \simeq 53 H_{\text{I}}^{-1} \simeq 53 \times 10^6 t$$

Q

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what you need to
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$$\text{How long? } t \simeq 53 H_{\text{I}}^{-1} \simeq 53 \times 10^6 t_{\text{P}} \simeq 10^{-37} \text{ s}$$

QFT

QFT in Curved Space

Why?

$$P_{\mathbb{I}}(k)$$

-37

S

QFT in Curved Space

Why?

$P_{\Phi}(k) \rightarrow$ from quantum fluctuations
in inflaton field + metric

-37

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* Scalar Field lives in a background (metric is fixed)

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* Quantize in fourier space.

-37

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QFT in Curved Space

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QFT in Curved Space

Why?

$P_{\Phi}(k) \rightarrow$ from quantum fluctuations
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* Scalar field lives in a background (metric is fixed)

* Quantize in Fourier space. $\rightarrow \infty$ # of SHO

* Need a vacuum state $\rightarrow$ what are particles?

* If we want P_{Φ} , need we need
to quantize $\varphi, \Phi, \psi, \dots$

$$* P_{\Phi} \rightarrow \langle \Phi^2 \rangle$$

* If we want P_{Ξ} , need we need
to quantize $\varphi, \Phi, \psi \dots$

* $P_{\Xi} \rightarrow \langle \alpha | \Xi | \alpha \rangle$, Quantum $\rightarrow$ Classical

Single massive scalar field in FRW

$$S = -\frac{1}{2} \int d^4x \sqrt{-g} \left[g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + m^2 \varphi^2 \right]$$

$$\partial S^2 = \bar{\alpha}(t) [-\partial \vec{t}^2 + \partial \vec{x}^2]$$

* If we want $P_{\pm}$, need we need
to quantize $\varphi, \Phi, \psi \dots$

* $P_{\pm} \rightarrow \langle \alpha | \Phi^2 | \alpha \rangle$, Quantum $\rightarrow$ Classical

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$$\partial S^2 = \vec{\alpha}(t) [-\dot{\varphi}^2 + \partial \vec{x}^2], \quad \sqrt{-g} = \alpha^4$$

* If we want P_{Ξ} , need we need
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$$S = -\frac{1}{2} \int dt d^3x \alpha^2 [-\partial_t^2 \varphi + (\nabla \varphi)^2]$$

Single massive scalar field in FRW

$$S = -\frac{1}{2} \int d^4x \sqrt{-g} \left[g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + m^2 \phi^2 \right]$$

$$ds^2 = a^2(\eta) [-d\eta^2 + d\vec{x}^2], \quad \sqrt{-g} = a^4$$

$$S = -\frac{1}{2} \int d\eta d^3\vec{x} a^2 \left[-(\partial_\eta \phi)^2 + (\nabla \phi)^2 + m^2 a^2 \phi^2 \right]$$

Single massive scalar field in FRW

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Define: $\chi = a\phi$

$$S = -\frac{1}{2} \int d^4x \partial \vec{x}^3 \left[-\partial_t^2 \chi + \nabla^2 \chi + m^2 \chi \right]$$

Single massive scalar field in FRW

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Define: $\chi = a\phi$ $\partial_t \chi = \dot{\chi}$

$$S = -\frac{1}{2} \int dt d^3x \left[-(\partial_t \chi)^2 + (\nabla \chi)^2 + \left(m^2 a^2 - \frac{\ddot{a}}{a} \right) \chi^2 \right]$$

Single massive scalar field in FRW

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$$\frac{\delta S}{\delta \chi} = \frac{\partial \mathcal{L}}{\partial \chi} - \frac{d}{dx^\mu} \frac{\partial \mathcal{L}}{\partial (\partial_\mu \chi)} = 0$$

$$\Rightarrow \ddot{\chi} - \nabla^2 \chi + \left(m^2 a^2 - \frac{\ddot{a}}{a} \right) \chi = 0$$

Scalar field with $m(\mathbf{r})$
→ particle production

Scalar field with $m(\eta)$

→ particle production

What are particles?

$$\chi(\vec{x}, \eta) = \int \frac{d^3k}{(2\pi)^3} \chi_{\vec{k}}(\eta) e^{i\vec{k} \cdot \vec{x}}$$

$$\Rightarrow \ddot{\chi}_{\vec{k}} + \omega_{\vec{k}}^2(\eta) \chi_{\vec{k}} = 0 \quad (\chi(\eta) = e^{i\eta})$$

$$\omega_{\vec{k}}^2 = k^2 + m^2 a^2 - \frac{\ddot{a}}{a}$$

χ is a real scalar

$$\chi = \chi^* \Rightarrow \chi_{\vec{k}}^*(\eta) = \chi_{-\vec{k}}(\eta)$$

Scalar field with $m(\eta)$

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1) χ is a real scalar

$$\chi = \chi^* \Rightarrow \chi_{\vec{k}}^*(\eta) = \chi_{-\vec{k}}$$

2) 2 linearly independent solutions

$$\chi_{\vec{k}}(\eta) = \frac{1}{\sqrt{2}} \left[a_{\vec{k}}^- v_{\vec{k}}^*(\eta) + a_{\vec{k}}^+ v_{\vec{k}}(\eta) \right]$$

Scalar field with $m(\eta)$

→ particle production

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$a_{\vec{k}}^-$ - integration constants
(depend on $\vec{k}$)

Scalar

$$\Rightarrow \chi_{\vec{k}}^*(\eta) = \chi_{-\vec{k}}(\eta)$$

$$V_k(\eta) - \text{satisfy} \quad \ddot{V}_k + 2\zeta_k^2 V_k = 0 \quad (\ddot{V}_k + 2\zeta_k^2 V_k = 0)$$

dependent solutions

only depend on η and $|\vec{k}|$

$$= \frac{1}{2} \left[a_{\vec{k}}^- V_k^*(\eta) + a_{\vec{k}}^+ V_k(\eta) \right]$$

$$-V_k \dot{V}_k^* \neq 0$$

$$= 2i \text{Im}(\dot{V} V^*)$$

$$\Rightarrow \chi(\vec{x}, \eta) = \frac{1}{\sqrt{2}} \int \frac{d^3 k}{(2\pi)^3} \left[a_{\vec{k}}^- V_k^* e^{i\vec{k} \cdot \vec{x}} + a_{\vec{k}}^+ V_k e^{-i\vec{k} \cdot \vec{x}} \right]$$

Quantize

$$\chi \rightarrow \hat{\chi}$$

$$-V_k \dot{V}_k^* \neq 0$$

$$= 2iI_k$$

$$\Rightarrow \chi(\vec{x}, \eta) = \frac{1}{\sqrt{2}} \int \frac{d^3 k}{(2\pi)^3} \left[a_{\vec{k}}^- V_k^* e^{i\vec{k} \cdot \vec{x}} + a_{\vec{k}}^+ V_k e^{-i\vec{k} \cdot \vec{x}} \right]$$

Quantize

$$\chi \rightarrow \hat{\chi}, \text{ conjugate momentum to } \chi : \pi = \dot{\hat{\chi}}$$

$$\pi \rightarrow \hat{\pi}$$

$$[\hat{\chi}(\vec{x}, t), \hat{\pi}(\vec{y}, t)] = i\delta(\vec{x} - \vec{y})$$

$$[\hat{x}(\vec{x}, t), \hat{\pi}(\vec{y}, t)] = i\delta(\vec{x} - \vec{y})$$

$$[\hat{x}, \hat{x}] = [\hat{\pi}, \hat{\pi}] = 0$$

$$[\hat{\chi}(\vec{x}, t), \hat{\pi}(\vec{y}, t)] = i\delta(\vec{x} - \vec{y})$$

$$[\hat{\chi}, \hat{\chi}] = [\hat{\pi}, \hat{\pi}] = 0$$

$$a_k^+ \rightarrow \hat{a}_k^+, \quad a_k^- \rightarrow \hat{a}_k^-$$

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Vacuum

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Vacuum

$$\hat{a}_k^- |0\rangle = 0$$

Particles: $a_k^+ |0\rangle = |1\rangle_k$

$$\Rightarrow \ddot{\chi}_{\vec{k}} + \omega_{\vec{k}}^2(\eta) \chi_{\vec{k}} = 0 \quad (\chi(\eta) = e^{i\vec{k}\cdot\vec{y}})$$

$$[\hat{\chi}(\vec{x}, \eta), \hat{\pi}(\vec{y}, \eta)] = i\delta(\vec{x} - \vec{y})$$

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Vacuum

$$\hat{a}_k^- |0\rangle = 0$$

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However, $\text{Im}(iV V^*) = 1$ has multiple solutions

(choose different V_k, V_k^*)

$$\Rightarrow \ddot{\chi}_{\vec{k}} + \omega_{\vec{k}}^2(\eta) \chi_{\vec{k}} = 0 \quad (\chi(\eta) = e^{i\vec{k}\cdot\vec{x}})$$

$$[\hat{\chi}(\vec{x}, \eta), \hat{\pi}(\vec{y}, \eta)] = i\delta(\vec{x} - \vec{y})$$

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$$a_k^+ \rightarrow \hat{a}_k^+, \quad a_k^- \rightarrow \hat{a}_k^-$$

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$$\hat{a}_k^- |0\rangle = 0$$

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However, $\text{Im}(iV_k^*) = 1$ has multiple solutions

(choose different V_k, V_k^*)

definition of $\hat{a}_k^\pm$
is not fixed

$$[\hat{a}_k^+, \hat{a}_k^-] = 0$$

How much freedom?

Bogolyubov transformation

$$U_k(\eta) = \alpha_k V_k(\eta) + \beta_k V_k^*$$

Satisfies everything if $|\alpha_k|^2 - |\beta_k|^2 = 1$

How much freedom?

Bogolyubov transformation

$$U_k(\eta) = \alpha_k V_k(\eta) + \beta_k V_k^*(\eta)$$

Satisfies everything if $|\alpha_k|^2 - |\beta_k|^2 = 1$

How much freedom?

Define:

Bogolyubov transformation

$$U_k(\eta) = \alpha_k V_k(\eta) + \beta_k V_k^\dagger(\eta)$$

Satisfies everything if $|\alpha_k|^2 - |\beta_k|^2 = 1$

How much freedom?

Bogolyubov transformation

$$U_k(\eta) = \alpha_k V_k(\eta) + \beta_k V_k^*(\eta)$$

Satisfies everything if $|\alpha_k|^2 - |\beta_k|^2 = 1$

Define:

$$\hat{b}_k^- = \alpha_k \hat{a}_k^- - \beta_k \hat{a}_k^+$$

$$\hat{b}_k^+ = \alpha_k^* \hat{a}_k^+ - \beta_k^* \hat{a}_k^-$$

Define :

$$\hat{b}_k^- = \alpha_k \hat{a}_k^- - \beta_k \hat{a}_k^+$$

$$\hat{b}_k^+ = \alpha_k^* \hat{a}_k^+ - \beta_k^* \hat{a}_k^-$$

$$\chi(\vec{x}, \eta) = \left[u_k \hat{b}_k^- e^{i\vec{k} \cdot \vec{x}} + u_k^* \hat{b}_k^+ e^{-i\vec{k} \cdot \vec{x}} \right]$$

$$|\alpha_k|^2 = 1$$

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$$R_k^2 =$$

$$\chi(\vec{x}, t) = \int \left[u_k \hat{b}_k^- e^{i\vec{k} \cdot \vec{x}} + u_k^* \hat{b}_k^+ e^{-i\vec{k} \cdot \vec{x}} \right]$$

So, many definitions of the vacuum

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$$|\beta_k|^2 = 1$$

$$\chi(\vec{x}, t) = \int \left[u_k \hat{b}_k^- e^{i\vec{k} \cdot \vec{x}} + u_k^* \hat{b}_k^+ e^{-i\vec{k} \cdot \vec{x}} \right]$$

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Which vacuum?

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$$\begin{aligned}\langle 0 | \hat{N}_k^{(\omega)} | 0 \rangle_{(b)} &= \langle 0 | \hat{a}_k^+ \hat{a}_k^- | 0 \rangle_{(b)} \\ &= |\beta_k|^2 \langle 0 | \hat{b}_k \hat{b}_k^+ | 0 \rangle_{(a)} \\ &= |\beta_k|^2 S^{(k)}\end{aligned}$$

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