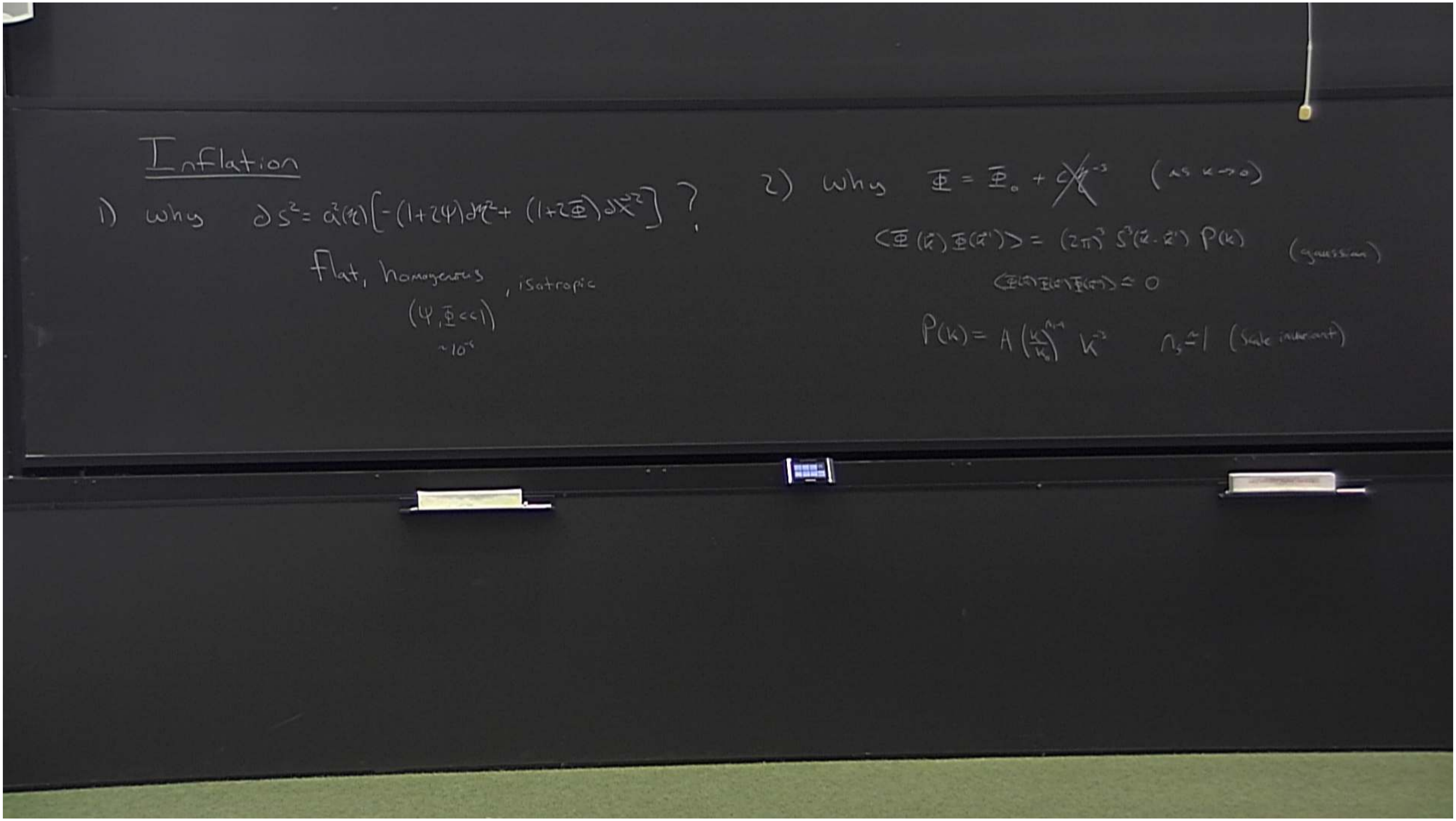
Title: 12/13 PSI - Explorations in Cosmology Lecture 9

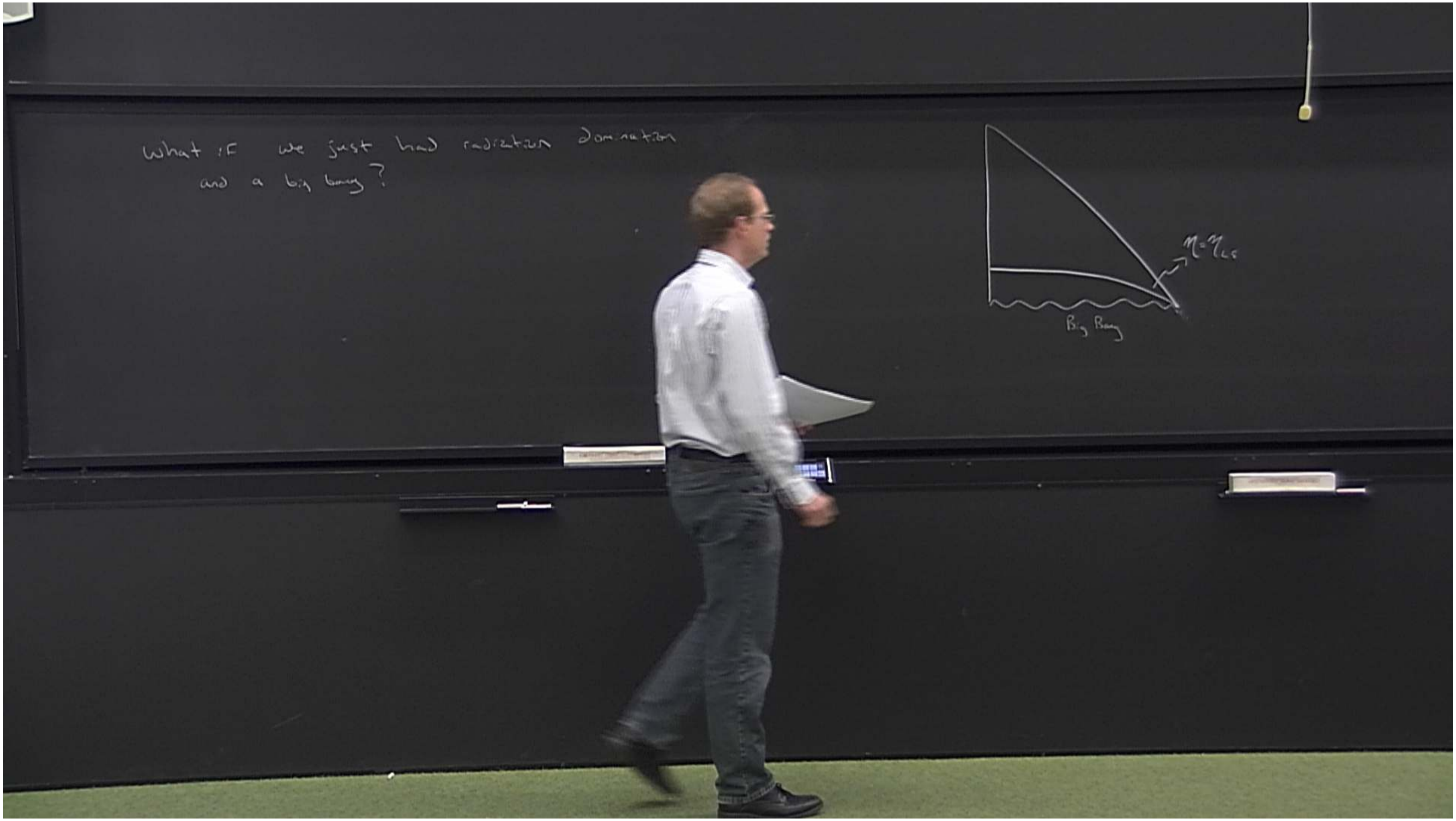
Date: Apr 18, 2013 10:15 AM

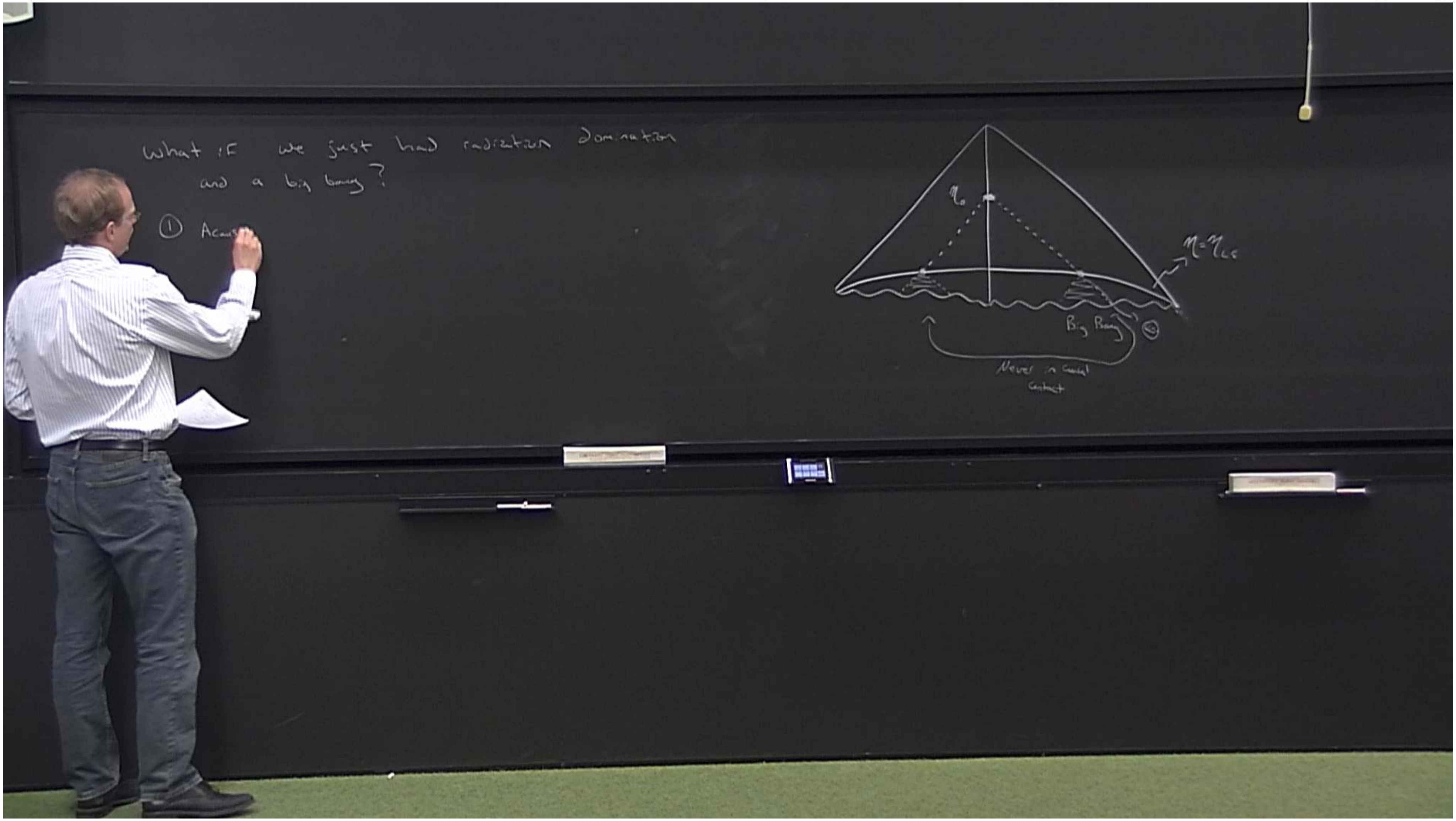
URL: http://pirsa.org/13040072

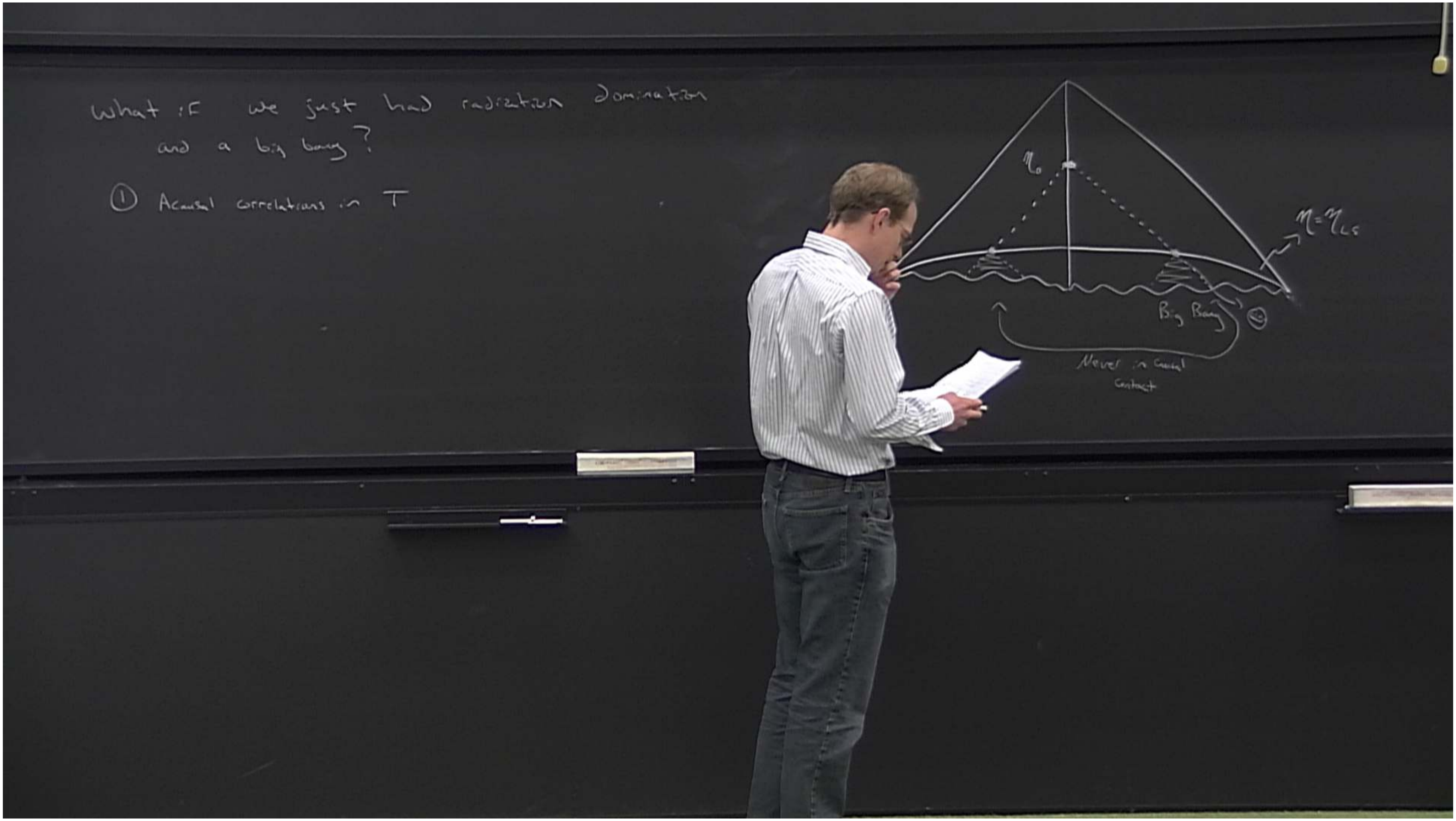
Abstract:

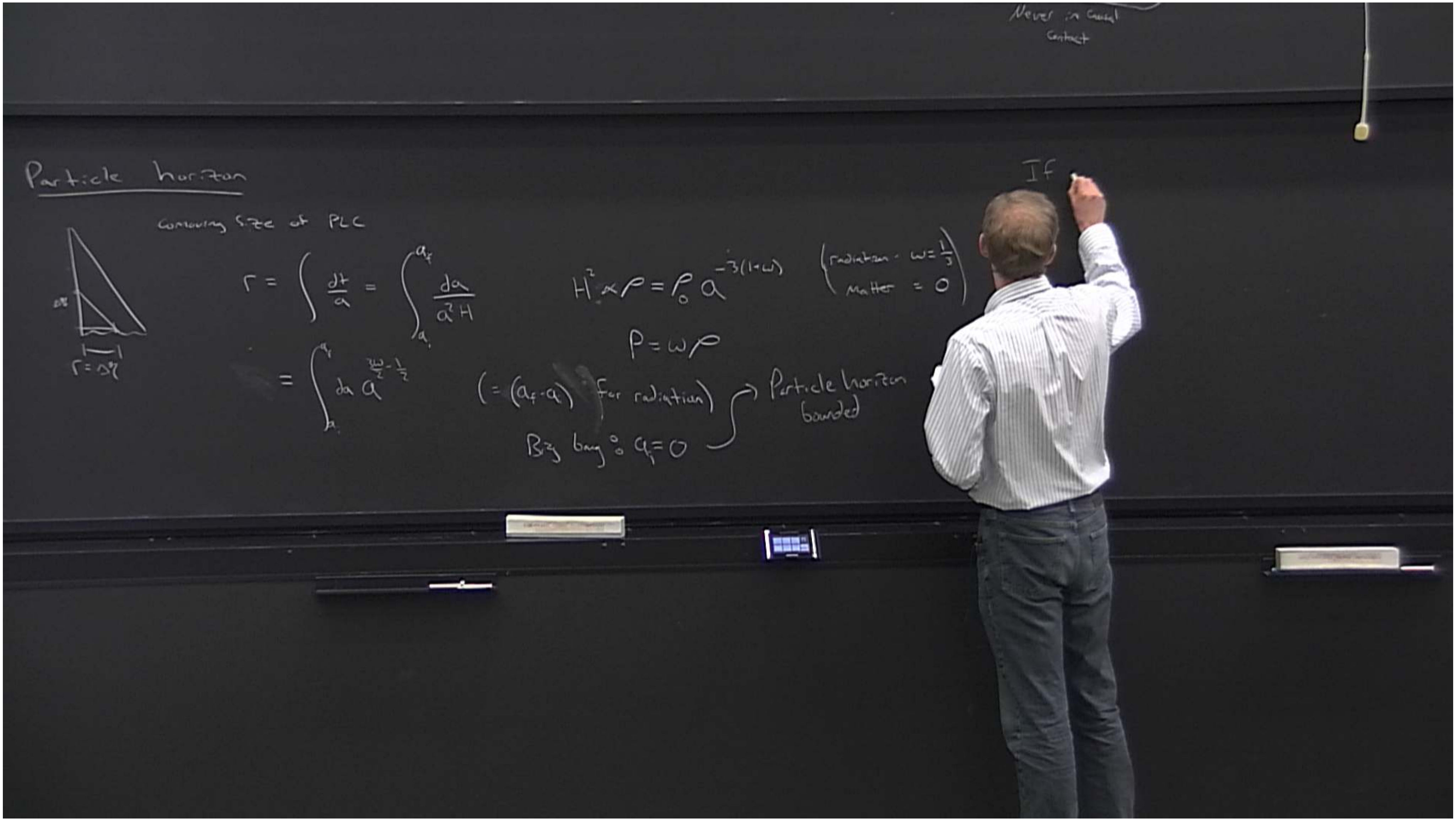

Inflation

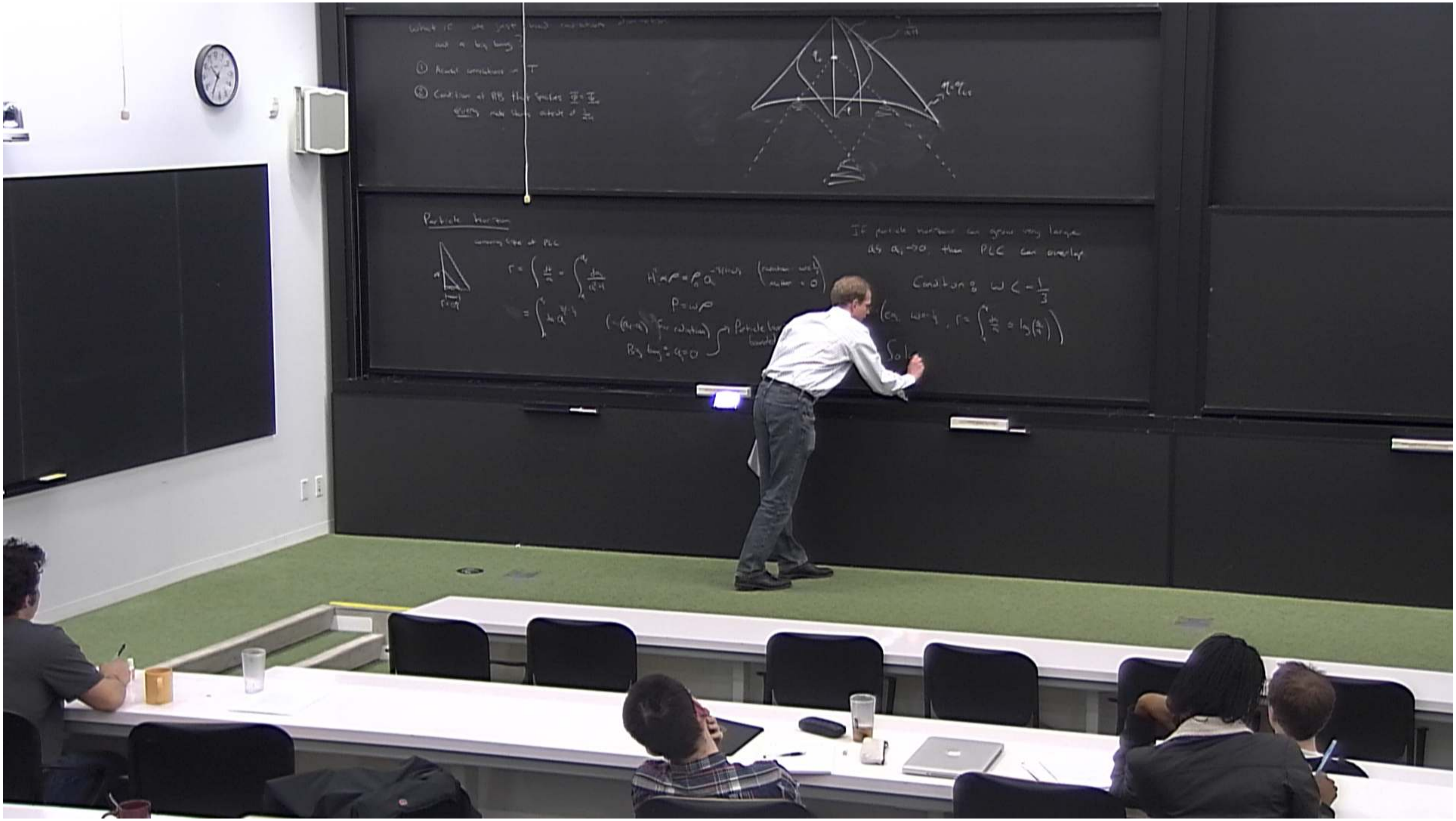

Inflation
1) why $ds^2 = a^2(\eta)[-(1+2\Psi)d\eta^2 + (1+2\Phi)d\vec{x}^2]$?
flat, homogeneous, isotropic
$(\Psi, \Phi \ll 1)$
$\sim 10^{-5}$
2) why $\Phi = \Phi_0 + c k^{-3}$ (as $k \to 0$)
$\langle \Phi(\vec{k}) \Phi(\vec{k}') \rangle = (2\pi)^3 \delta^3(\vec{k} - \vec{k}') P(k)$
(gaussian)
$\langle \Phi(\vec{k}) \Phi(\vec{k}') \Phi(\vec{k}'') \rangle \simeq 0$
$P(k) = A \left(\frac{k}{k_0}\right)^{n_s - 1} k^{-3}$
$n_s \simeq 1$ (scale invariant)

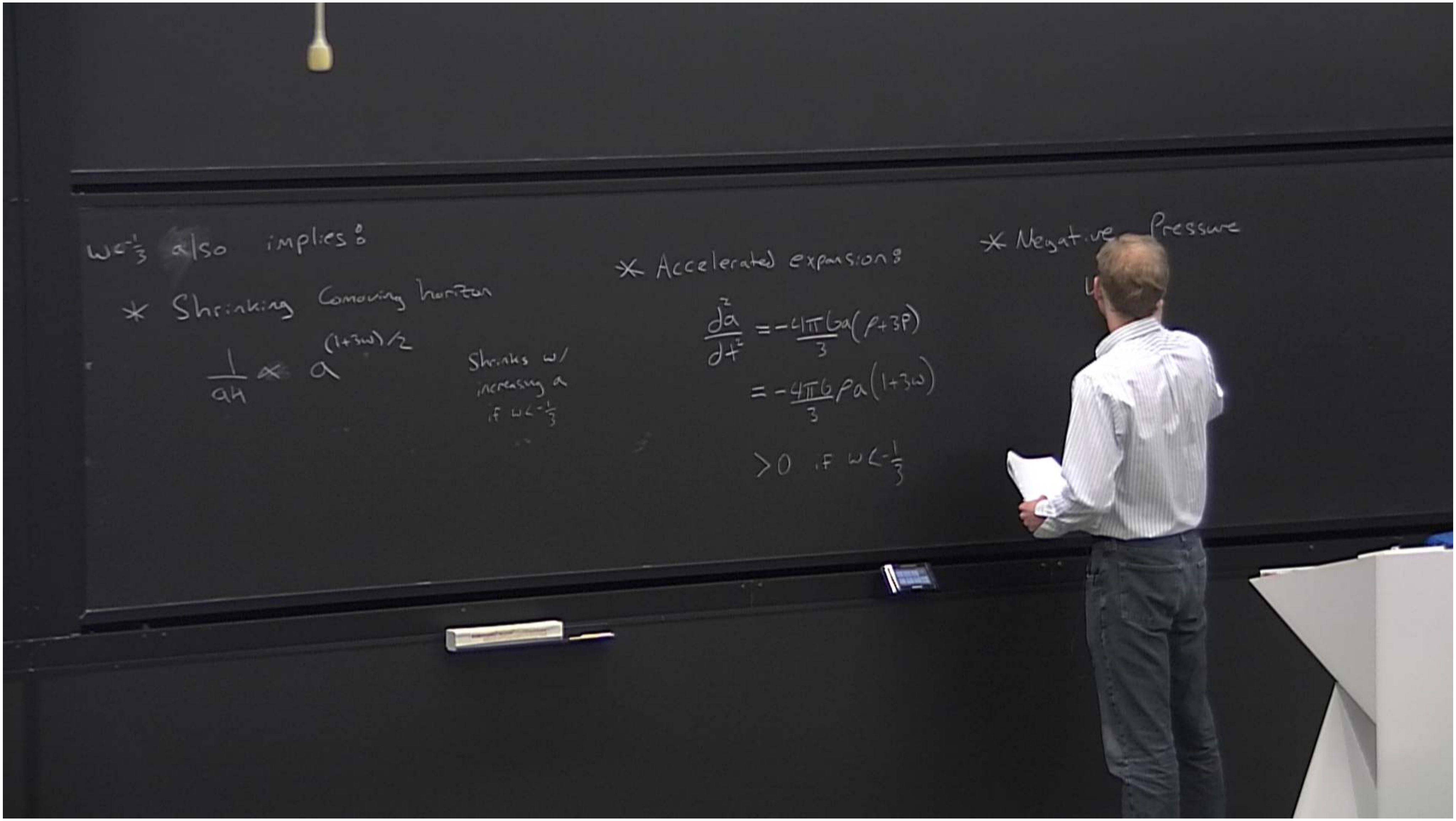

What if we just had radiation domination
and a big bang?
η=η_LS
Big Bang

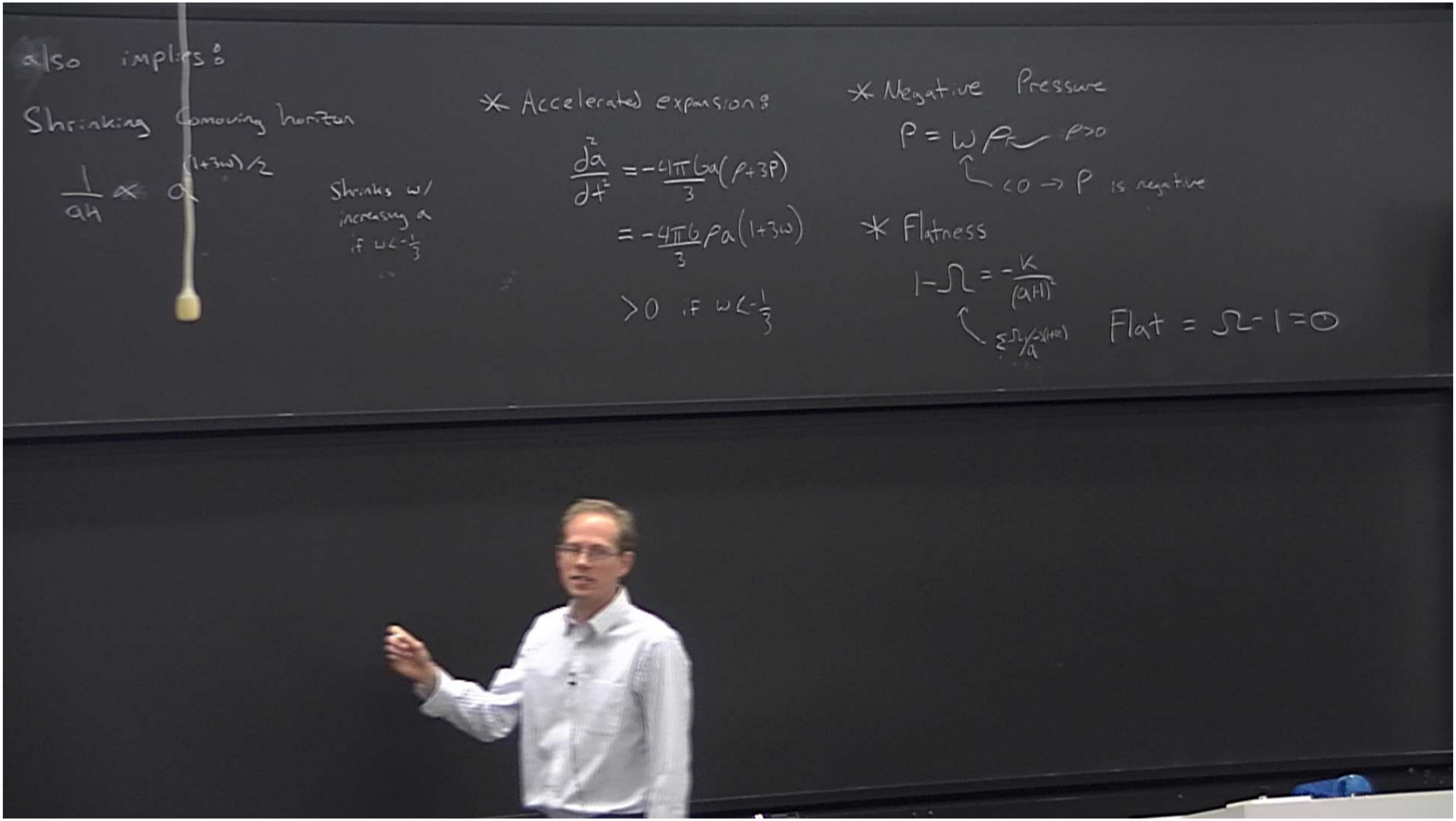

What if we just had radiation domination
and a big bang?
①
η_0
$\eta = \eta_{LS}$
Big Bang
Never in causal contact


What if we just had radiation domination
and a big bang?
① Acausal correlations in T
η0
η=ηLS
Big Bang
Never in causal
contact


Never in causal contact
Particle horizon
comoving size of PLC
r = ∫ dt/a = ∫ da/(a²H)
= ∫ da a^(3w/2 - 1/2)
H² ∝ ρ = ρ₀ a^(-3(1+w))
radiation w = 1/3
Matter = 0
P = wρ
(= (a_f - a_i) for radiation)
Big bang: a_i = 0
Particle horizon bounded
If


Particle horizon
If particle number can grow very large as $a_i \to 0$, then PLC can overlap
Condition: $w < -\frac{1}{3}$


$w < -\frac{1}{3}$ also implies:
* Shrinking Comoving horizon
$\frac{1}{aH} \propto a^{(1+3w)/2}$
Shrinks w/ increasing a if $w < -\frac{1}{3}$
* Accelerated expansion:
$\frac{d^2a}{dt^2} = -\frac{4\pi G a}{3}(\rho + 3P)$
$= -\frac{4\pi G}{3}\rho a(1+3w)$
> 0 if $w < -\frac{1}{3}$
* Negative Pressure


also implies:
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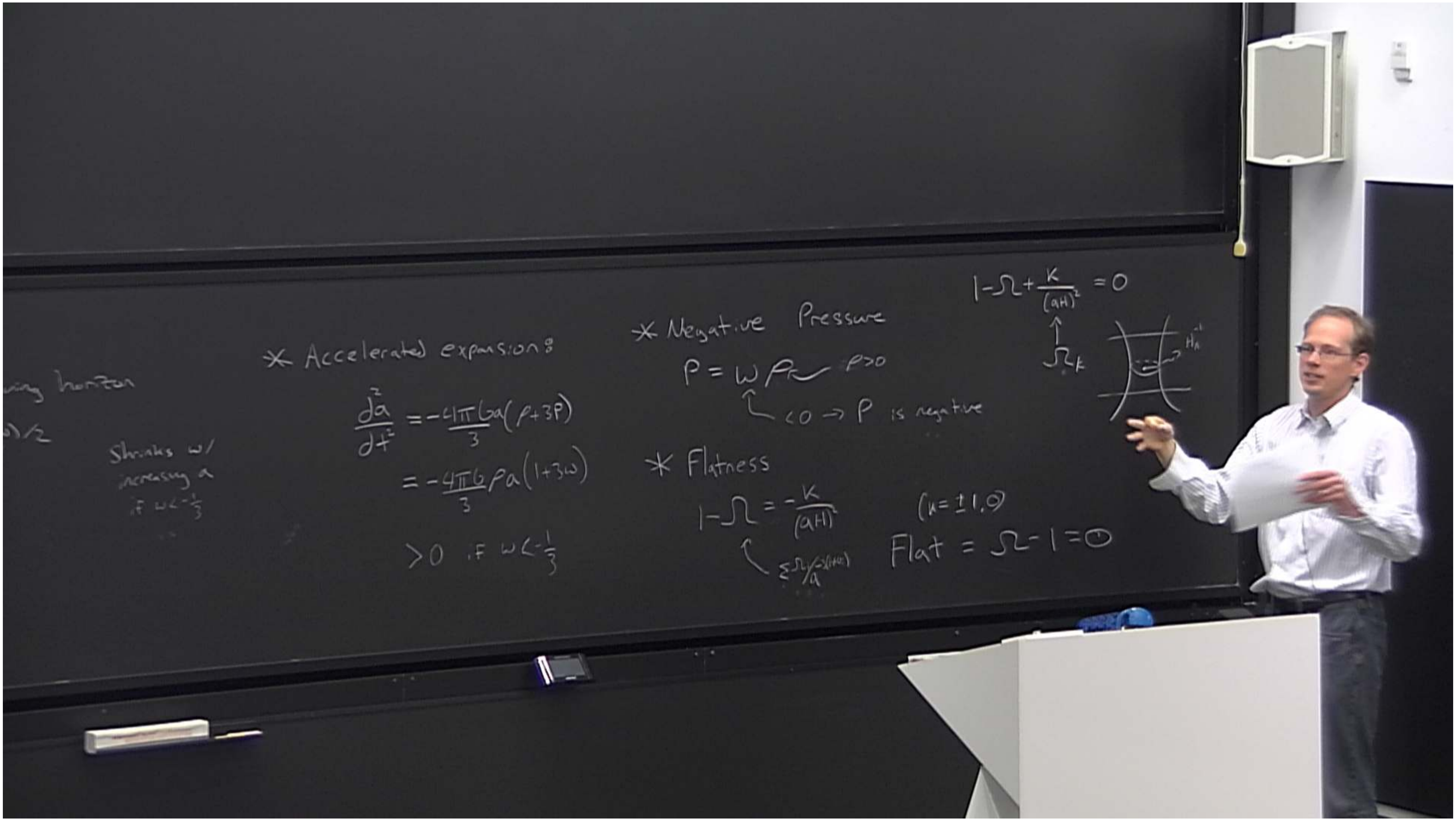
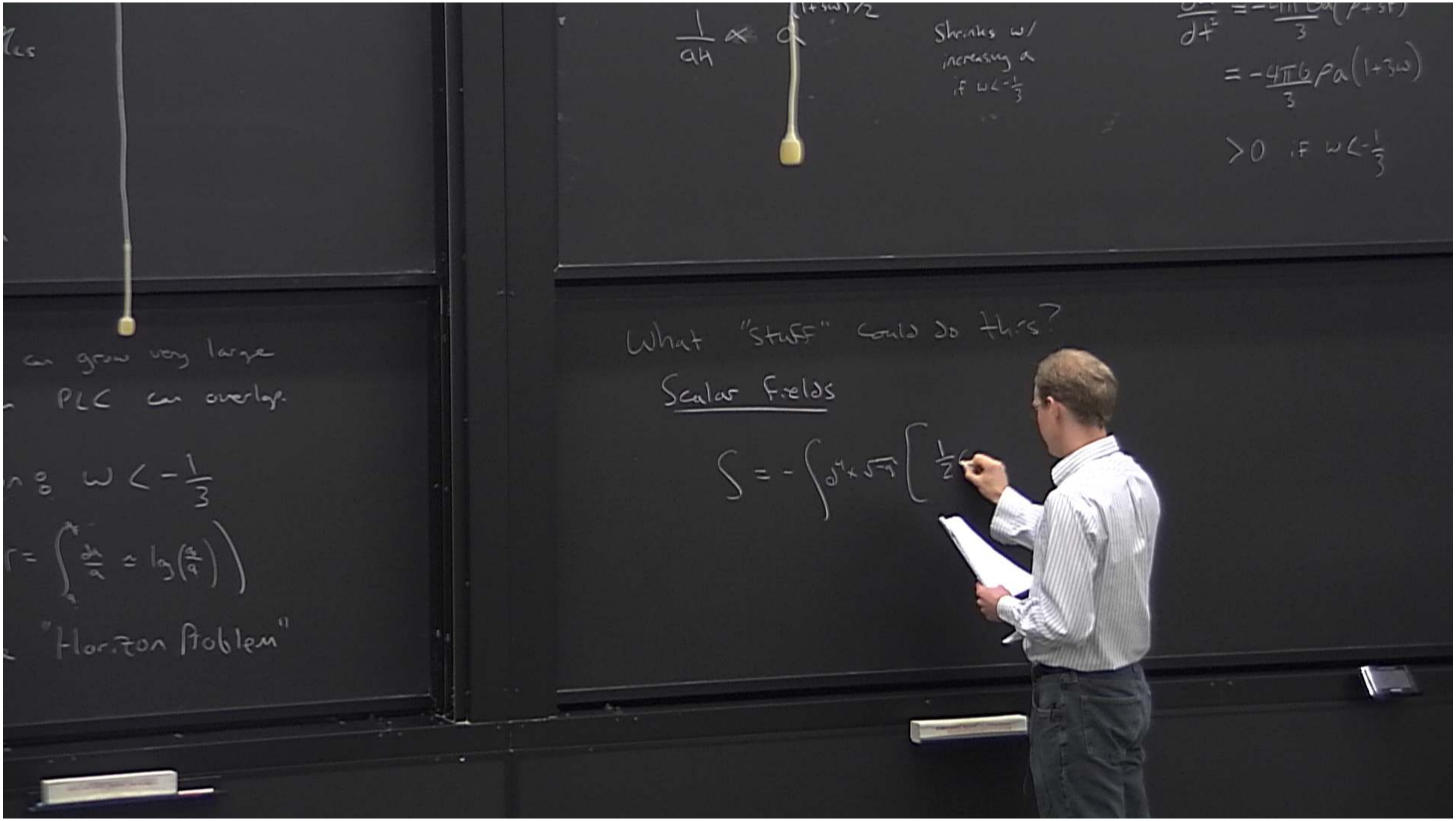
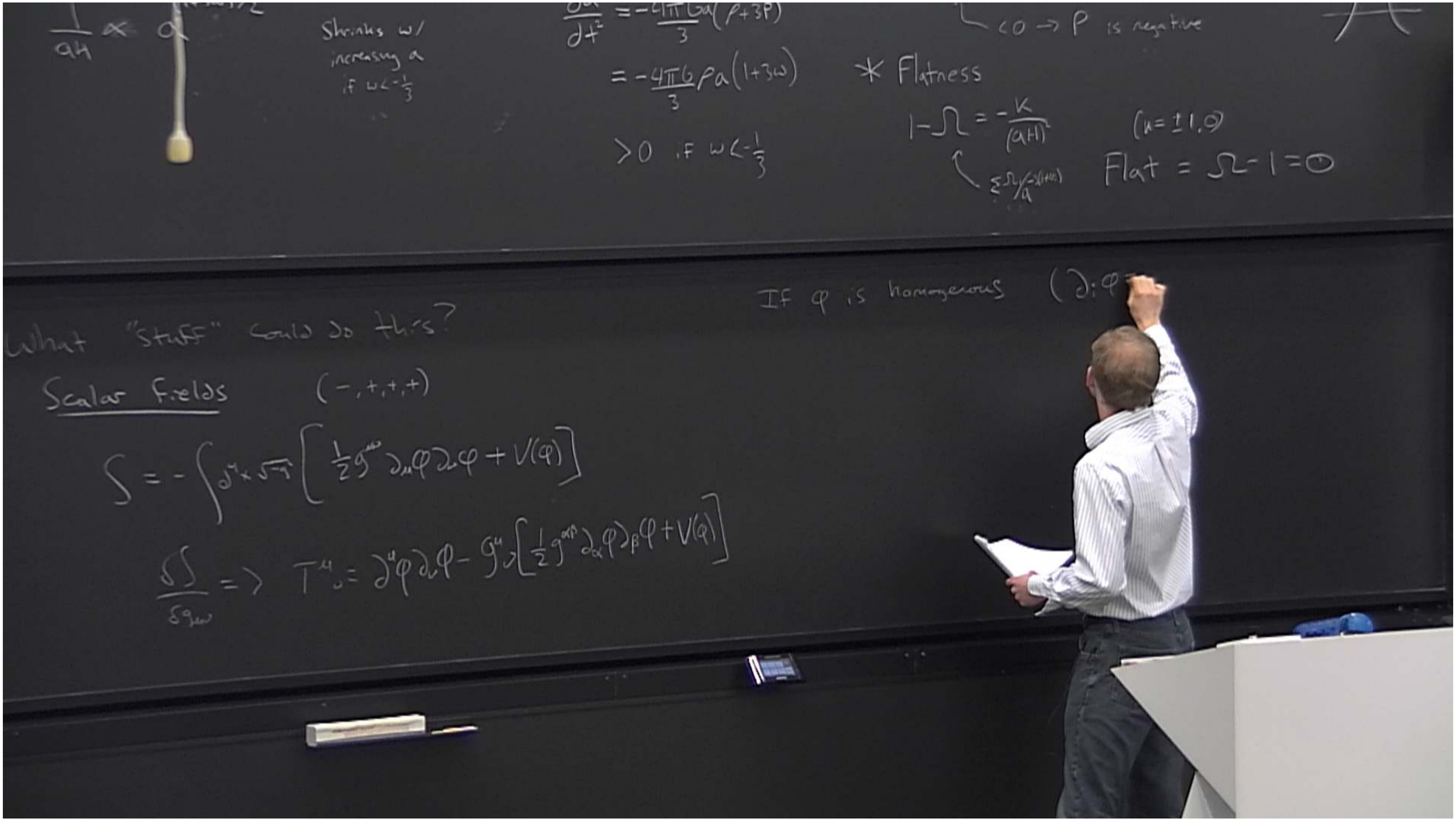
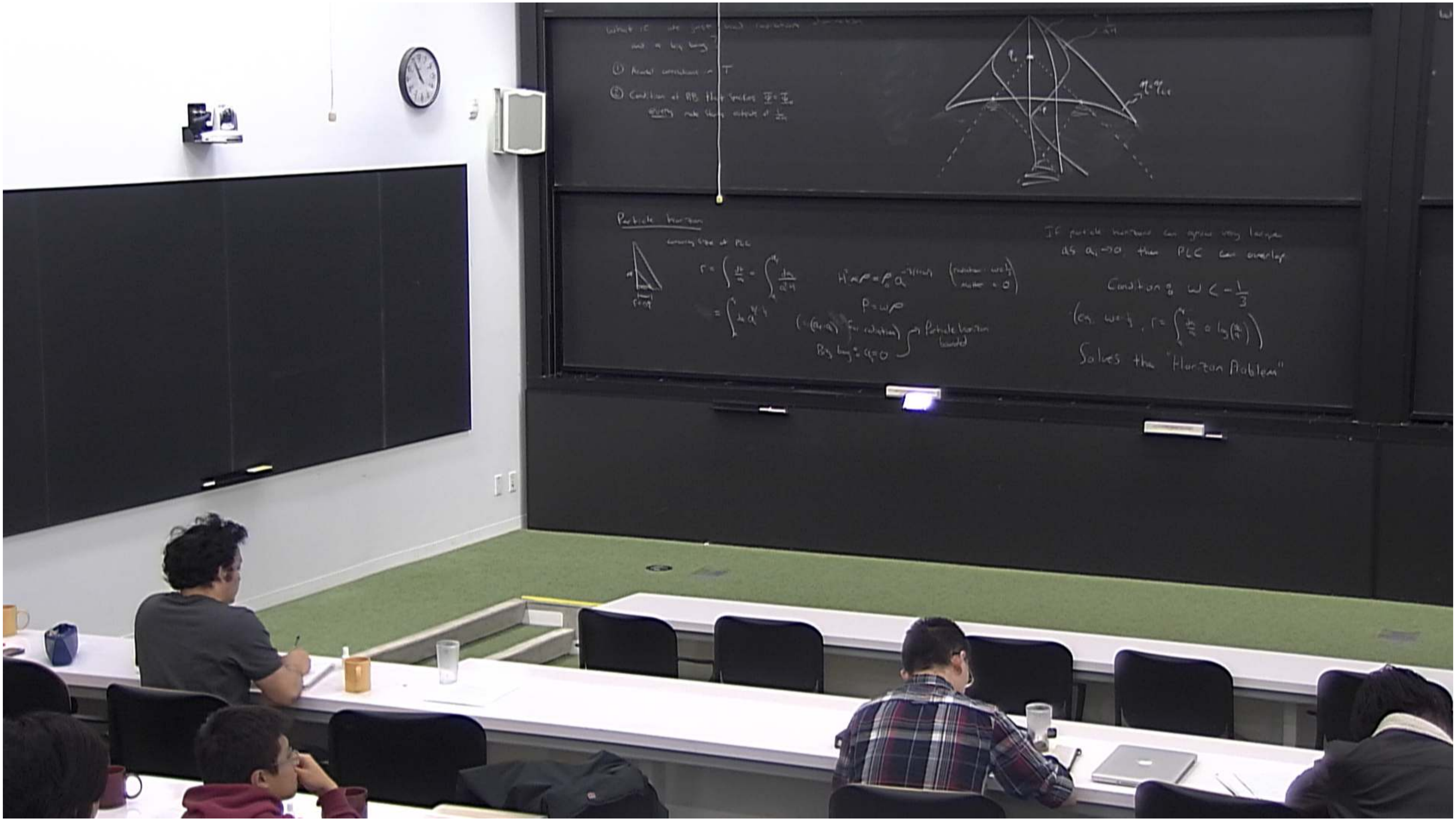
$\frac{d^2a}{dt^2} = -\frac{4\pi G a}{3}(\rho + 3P)$
$= -\frac{4\pi G}{3}\rho a(1+3w)$
> 0 if $w < -\frac{1}{3}$
* Negative Pressure
$P = w\rho$, $\rho > 0$
$< 0 \rightarrow P$ is negative
* Flatness
$1 - \Omega = -\frac{K}{(aH)^2}$
Flat = $\Omega - 1 = 0$

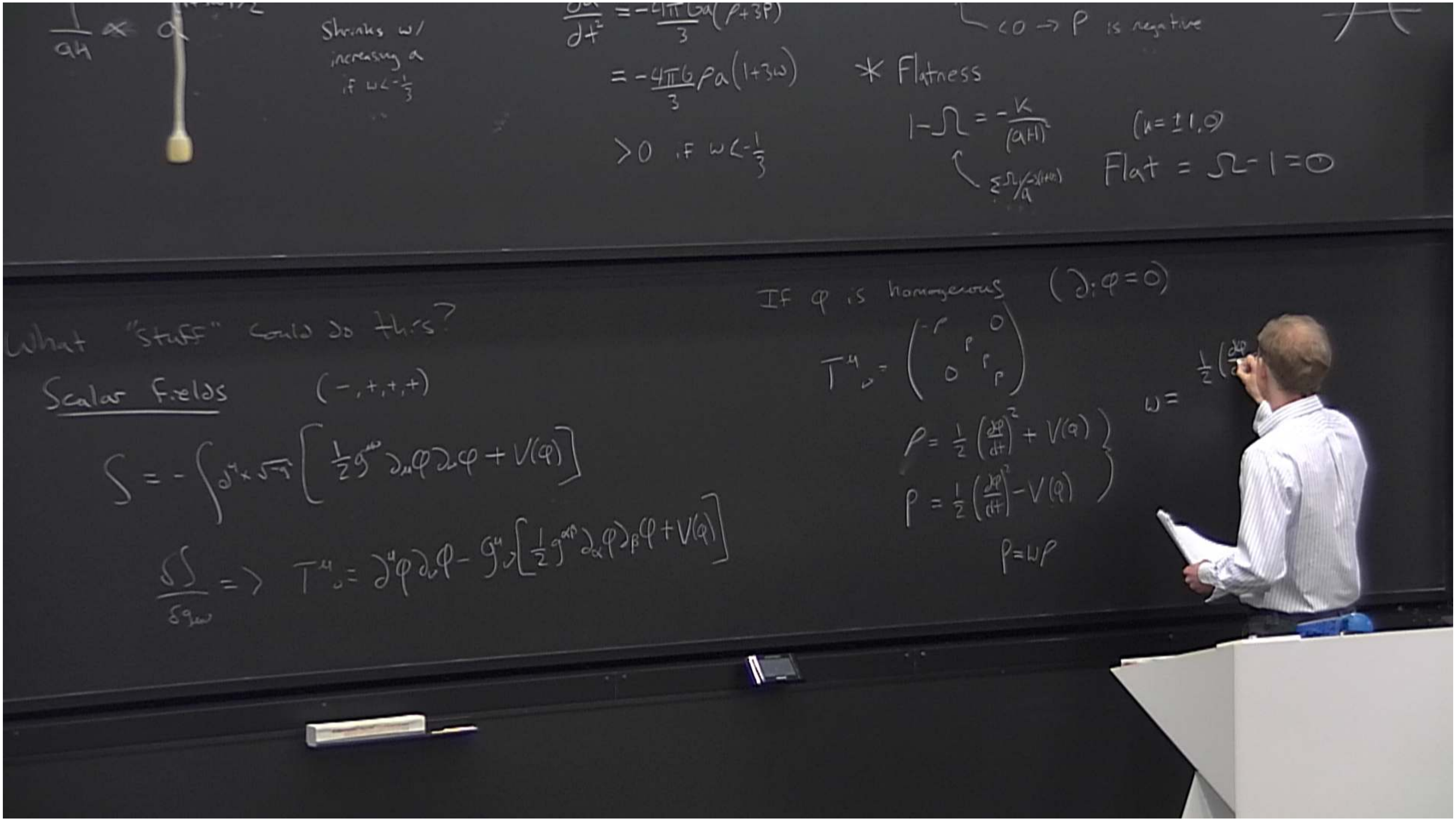


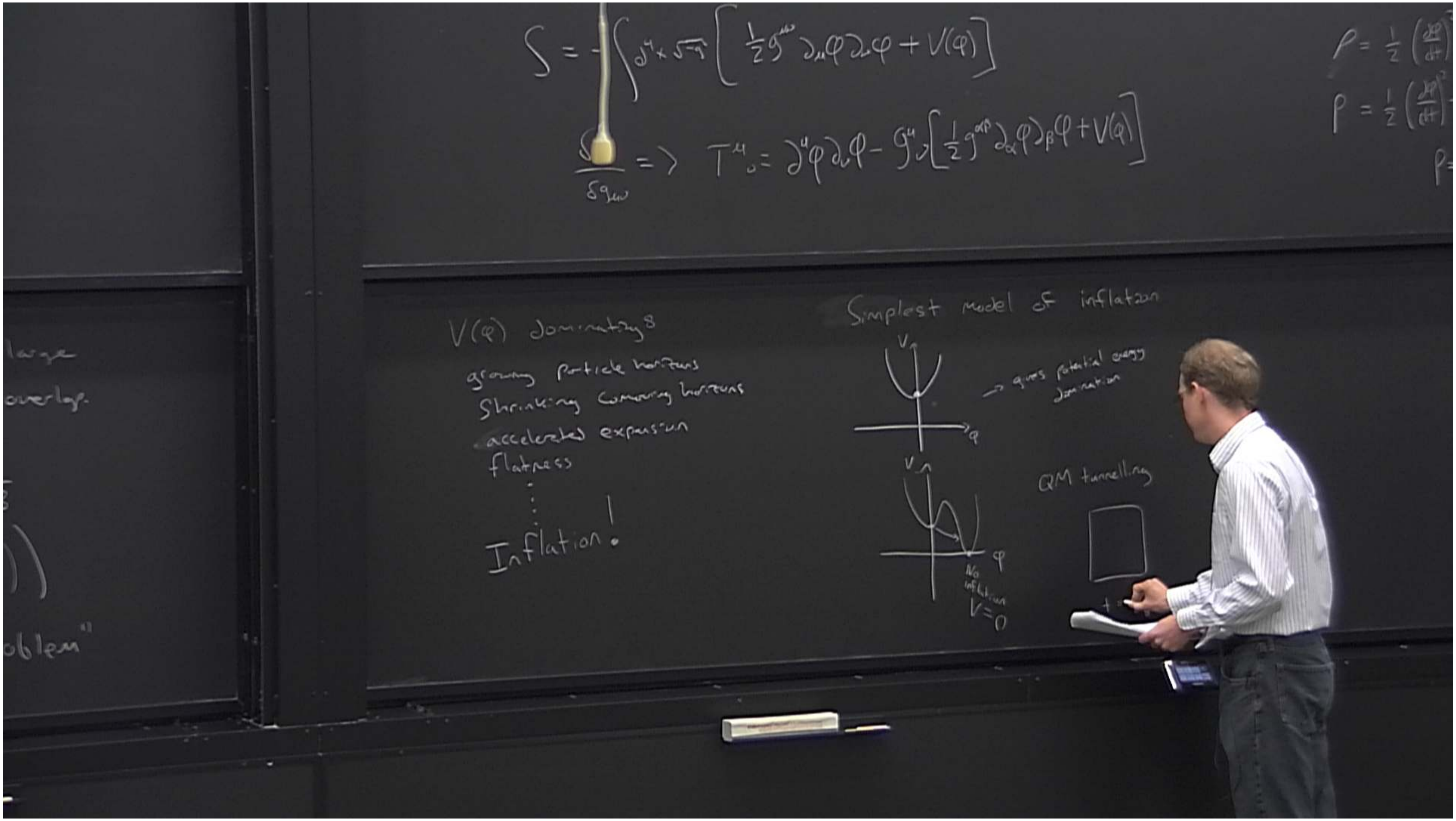
* Accelerated expansion:
Shrinks w/ increasing a if w < -1/3
>0 if w < -1/3
* Negative Pressure
ρ>0
<0 → P is negative
* Flatness
(k = ±1, 0)
Flat = Ω - 1 = 0

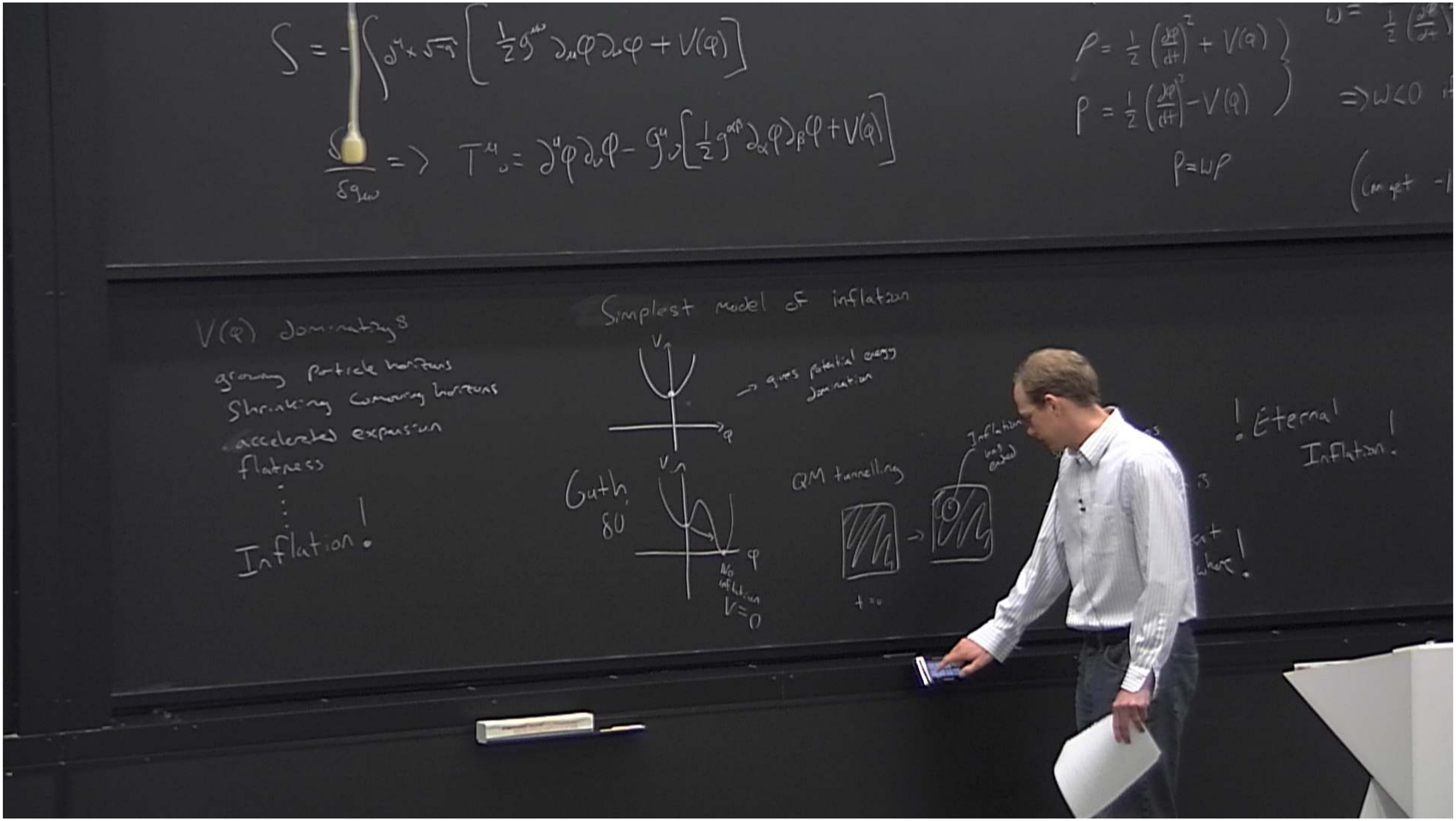

Shrinks w/
increasing a
if w < -1/3
> 0 if w < -1/3
can grow very large
PLC can overlap.
"Horizon Problem"
What "stuff" could do this?
Scalar Fields


Shrinks w/
increasing a
if w < -1/3
= -4πG/3 ρa(1+3w)
>0 if w < -1/3
< 0 → P is negative
* Flatness
1 - Ω = -k/(aH)²
(k = ±1, 0)
Flat = Ω - 1 = 0
If φ is homogeneous
What "stuff" could do this?
Scalar Fields
(-, +, +, +)
S = -∫d⁴x √-g [½ g^μν ∂_μφ ∂_νφ + V(φ)]
δS/δg_μν => T^μ_ν = ∂^μφ ∂_νφ - g^μ_ν [½ g^αβ ∂_αφ ∂_βφ + V(φ)]


Particle horizon
If particle horizons can grow very large
as $a_i \to 0$ then PLC can overlap
Condition: $w < -\frac{1}{3}$
Solves the "Horizon Problem"


Shrinks w/ increasing a if w < -1/3
= -4πG/3 ρa(1+3w)
> 0 if w < -1/3
< 0 → P is negative
* Flatness
1 - Ω = -k/(aH)²
(k = ±1, 0)
Flat = Ω - 1 = 0
What "stuff" could do this?
Scalar Fields
(-,+,+,+)
If φ is homogeneous (∂ᵢφ = 0)
ρ = ½(dφ/dt)² + V(φ)
P = ½(dφ/dt)² - V(φ)
P = wρ
w =


V(φ) dominating
growing particle horizons
Shrinking comoving horizons
accelerated expansion
flatness
Inflation!
Simplest model of inflation
gives potential energy domination
QM tunnelling
No inflation
V=0



ρ = ½ (dφ/dt)² + V(φ)
p = ½ (dφ/dt)² − V(φ)
p = wρ
⇒ w < 0
V(φ) dominating
growing particle horizons
Shrinking comoving horizons
accelerated expansion
flatness
Inflation!
Simplest model of inflation
gives potential energy domination
Guth 80
No inflation
V = 0
QM tunnelling
Inflation has ended
Eternal Inflation!

Scalar Fields $(-,+,+,+)$

$$S = -\int d^4x \sqrt{-g}\left[\frac{1}{2} g^{\mu\nu}\partial_\mu\varphi\,\partial_\nu\varphi + V(\varphi)\right]$$

$$\frac{\delta S}{\delta g_{\mu\nu}} \Rightarrow T^\mu{}_\nu = \partial^\mu\varphi\,\partial_\nu\varphi - g^\mu{}_\nu\left[\frac{1}{2} g^{\alpha\beta}\partial_\alpha\varphi\,\partial_\beta\varphi + V(\varphi)\right]$$

$T^\mu{}_\nu = \begin{pmatrix} \; & \; \\ 0 & P \end{pmatrix}$

$$\rho = \frac{1}{2}\left(\frac{d\varphi}{dt}\right)^2 + V(\varphi)$$

$$P = \frac{1}{2}\left(\frac{d\varphi}{dt}\right)^2 - V(\varphi)$$

$$P = w\rho$$

$w = \frac{\frac{1}{2}\left(\frac{d\varphi}{dt}\right) \ldots}{\frac{1}{2}\left(\frac{d\varphi}{dt}\right)^2 \ldots}$

$\Rightarrow w < 0$ if

(can get -1

$1 - \Omega + \frac{k}{(a \ldots}$

$w < -\frac{1}{3}$ also implies:

* Shrinking Comoving horizon

$$\frac{1}{aH} \propto a^{(1+3w)/2}$$

Shrinks w/ increasing a if $w < -\frac{1}{3}$

* Accelerated expansion:

$$\frac{d^2a}{dt^2} = -\frac{4\pi G a}{3}(\rho + 3P)$$

$$= -\frac{4\pi G}{3}\rho a(1+3w)$$

> 0 if $w < -\frac{1}{3}$

* Negative Pressure

$P = w\rho$ ← $\rho > 0$

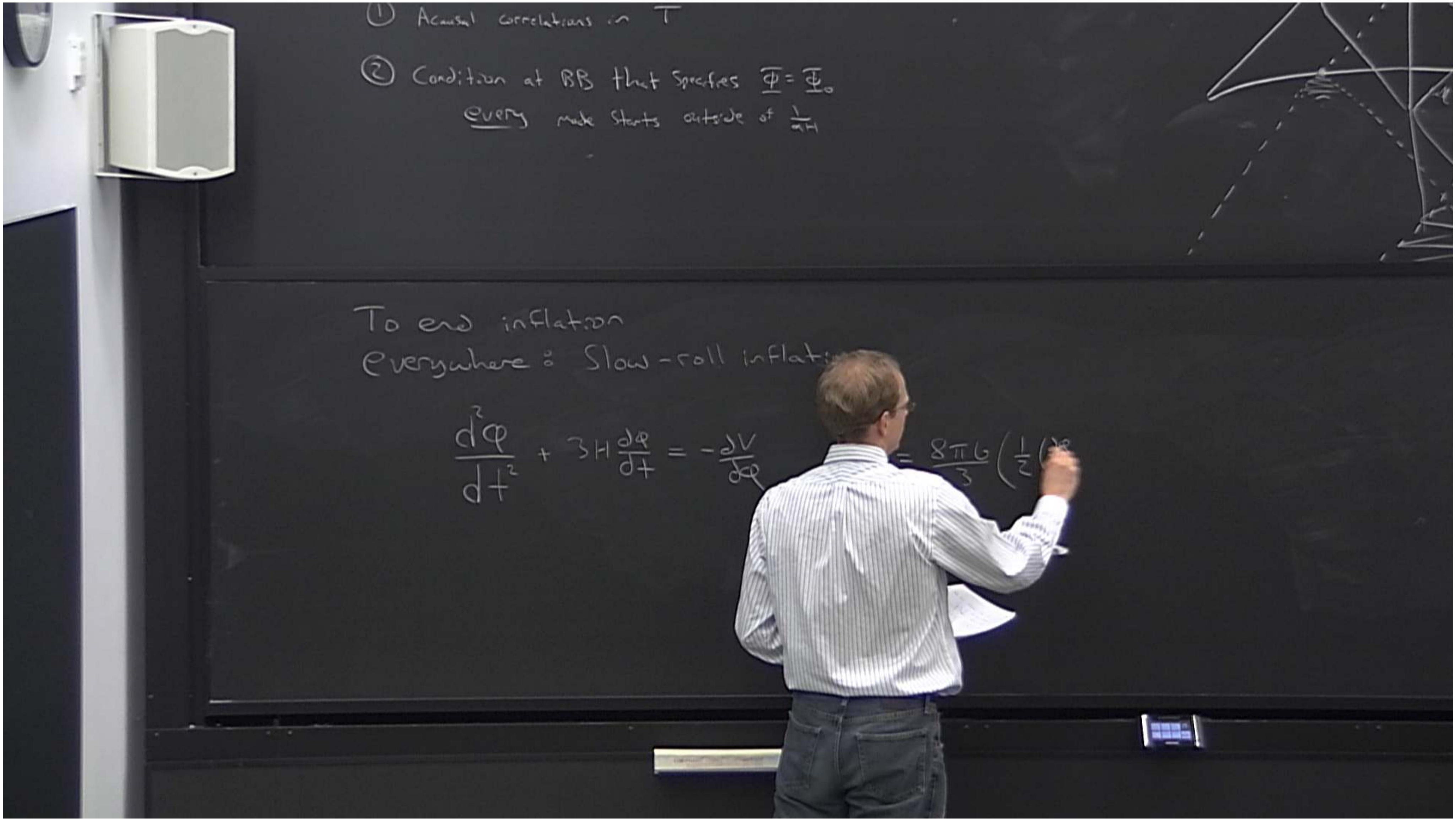
↑ $< 0 \rightarrow P$ is negative

* Flatness

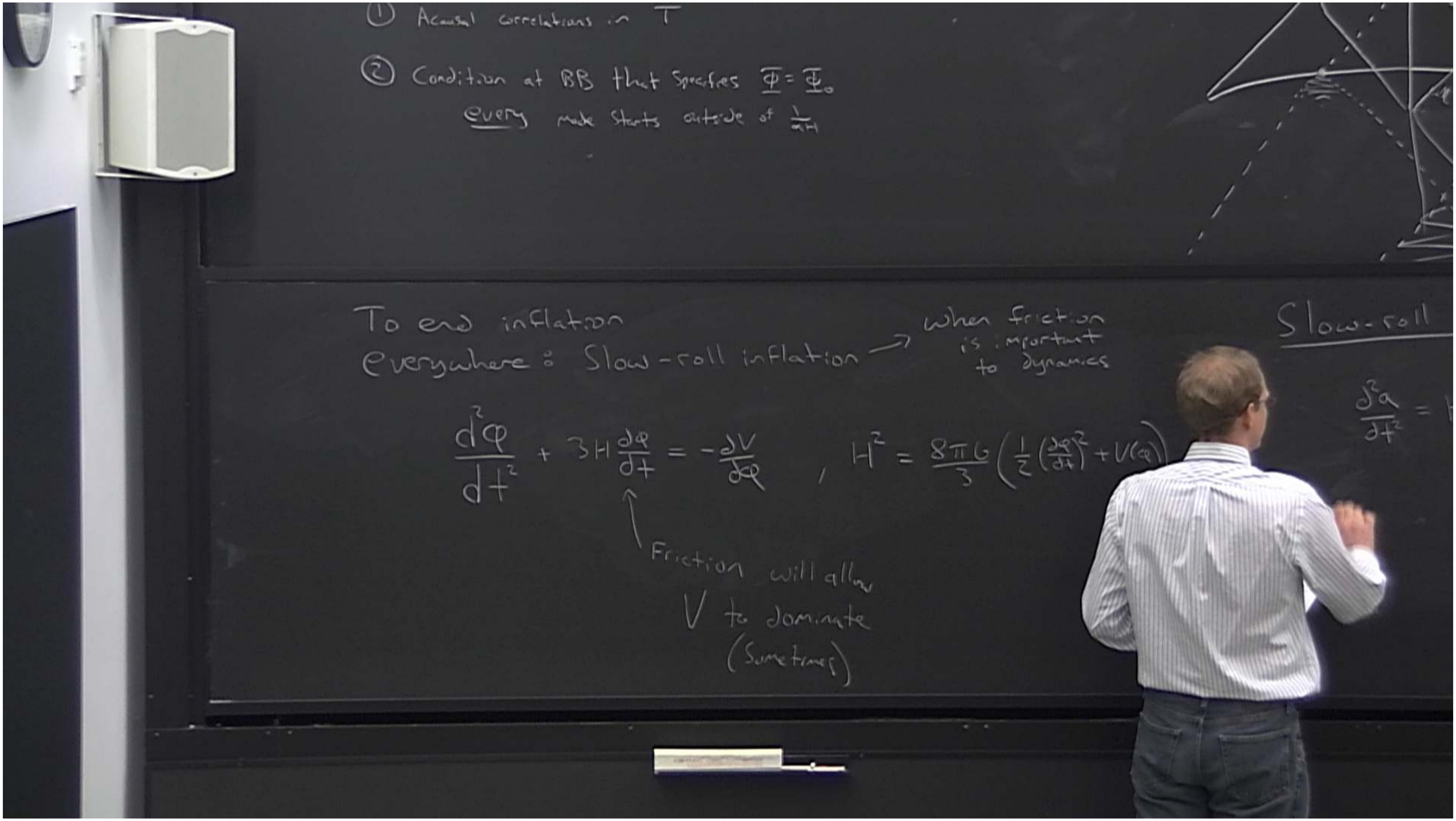
$$1 - \Omega = -\frac{k}{(aH)^2} \qquad (k = \pm 1, 0)$$

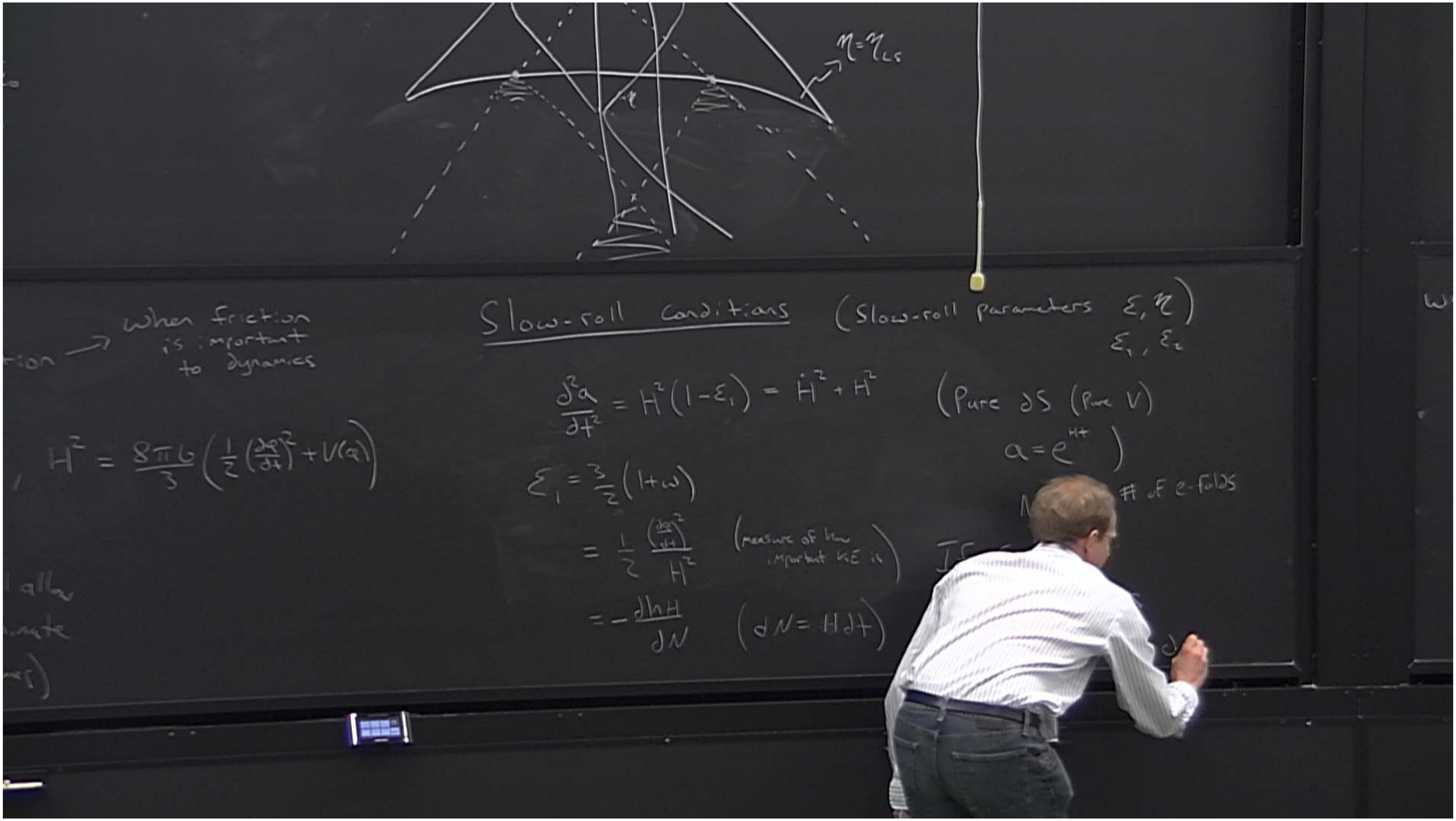
↑ $\Sigma\,\Omega_i / a^{-3(1+w)}$

Flat $= \Omega = 1$

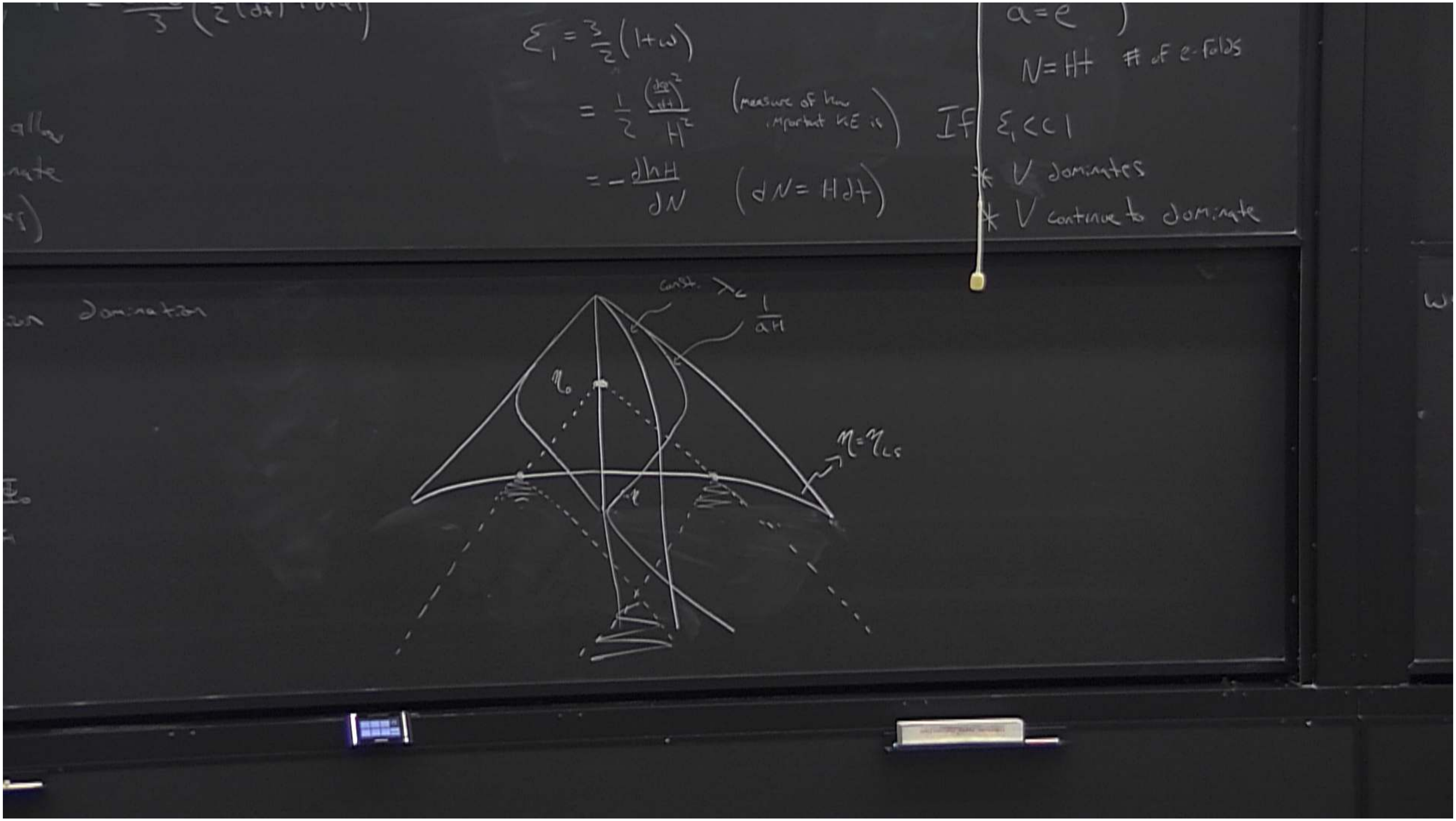


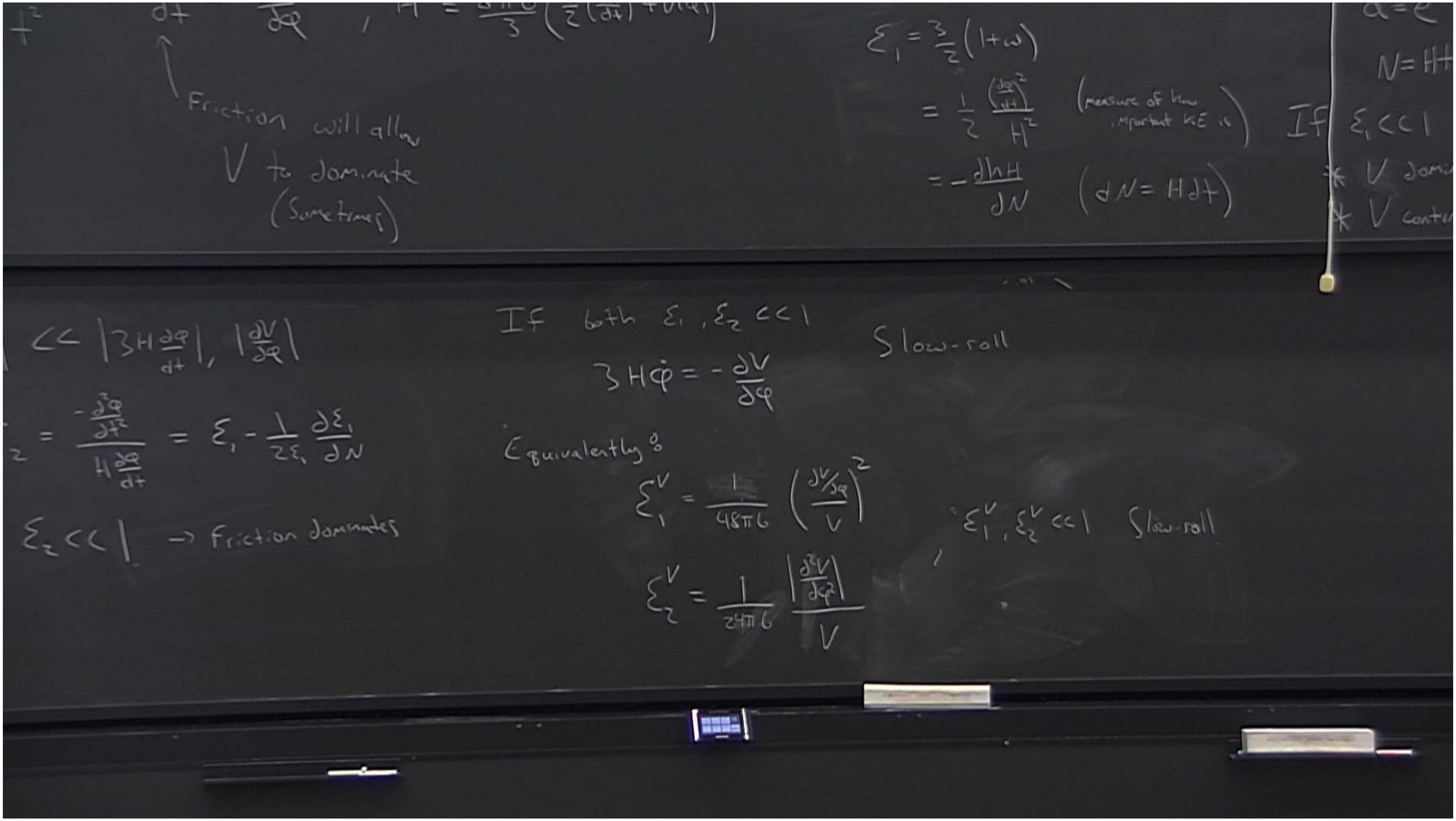
① Acausal correlations in T
② Condition at BB that specifies Φ = Φ₀
every mode starts outside of 1/aH
To end inflation
everywhere: Slow-roll inflat
d²φ/dt² + 3H dφ/dt = −∂V/∂φ
= 8πG/3 (1/2


① Acausal correlations in T
② Condition at BB that specifies Φ = Φ₀
every mode starts outside of 1/aH
To end inflation
everywhere : Slow-roll inflation → when friction is important to dynamics
$\frac{d^2\varphi}{dt^2} + 3H\frac{d\varphi}{dt} = -\frac{\partial V}{\partial\varphi}$, $H^2 = \frac{8\pi G}{3}\left(\frac{1}{2}\left(\frac{d\varphi}{dt}\right)^2 + V(\varphi)\right)$
Friction will allow V to dominate (Sometimes)
Slow-roll



$\eta = \eta_{LS}$
when friction
is important
to dynamics
Slow-roll conditions (Slow-roll parameters ε, η)
$\varepsilon_1, \varepsilon_2$
$H^2 = \frac{8\pi G}{3}\left(\frac{1}{2}\left(\frac{d\varphi}{dt}\right)^2 + V(\varphi)\right)$
$\frac{d^2a}{dt^2} = H^2(1-\varepsilon_1) = \dot{H}^2 + H^2$
(Pure dS (Pure V)
$a = e^{Ht}$)
# of e-folds
$\varepsilon_1 = \frac{3}{2}(1+w)$
$= \frac{1}{2}\frac{\left(\frac{d\varphi}{dt}\right)^2}{H^2}$
(measure of how important KE is)
$= -\frac{d\ln H}{dN}$
$(dN = H\,dt)$


ε₁ = 3/2 (1+ω)
= 1/2 (dφ/dt)² / H²
(measure of how important KE is)
= − d ln H / dN
(dN = H dt)
N = Ht
# of e-folds
If ε₁ << 1
* V dominates
* V continue to dominate
domination
const.
1/aH
η = η_LS



Friction will allow V to dominate (Sometimes)
ε₁ = 3/2 (1+ω)
= 1/2 (dφ/dt)² / H²
(measure of how important KE is)
= − d ln H / dN
(dN = H dt)
If ε₁ << 1
N = Ht
* V dom…
* V cont…
<< |3H dφ/dt|, |dV/dφ|
= (−d²φ/dt²) / (H dφ/dt) = ε₁ − 1/(2ε₁) dε₁/dN
ε₂ << 1 → Friction dominates
If both ε₁, ε₂ << 1
Slow-roll
3Hφ̇ = − ∂V/∂φ
Equivalently:
ε₁^V = 1/(48πG) ((∂V/∂φ)/V)²
ε₂^V = 1/(24πG) |∂²V/∂φ²| / V
ε₁^V, ε₂^V << 1 Slow-roll