

Title: 12/13 PSI - Explorations in Cosmology Lecture 8

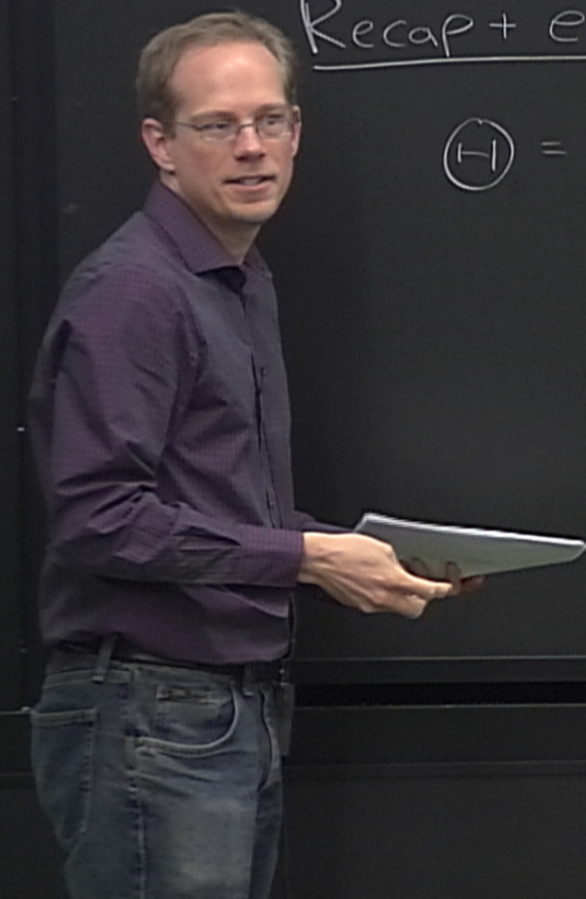
Date: Apr 17, 2013 10:15 AM

URL: <http://pirsa.org/13040071>

Abstract:

Recap + extensions

$$\langle H \rangle = \frac{S T}{T}(\theta, \varphi)$$



Recap + extensions

$$\langle \mathbf{H} \rangle = \frac{\mathcal{S}T}{T}(\theta, \varphi) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\theta, \varphi)$$

Recap + extensions

$$\Theta = \frac{\delta T}{T}(\theta, \varphi) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\theta, \varphi)$$

$$C_\ell = \langle a_{\ell m} a_{\ell m}^* \rangle$$

CMB-random field on celestial sphere

Recap + extensions

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CMB-random field on celestial sphere

$$\langle a_{\ell m} \rangle, \langle a_{\ell m} a_{\ell' m'} a_{\ell'' m''} \rangle, \dots$$

Recap + extensions

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CMB-random field on celestial sphere

$$\langle a_{\ell m} \rangle, \langle a_{\ell m} a_{\ell' m'} a_{\ell'' m''} \rangle \dots$$

$$\langle a_{\mathbf{k}m} a_{\mathbf{k}'m} \rangle \propto \int d\mathbf{k} d\mathbf{k}' \langle \Phi(\mathbf{k}) \Phi(\mathbf{k}') \rangle$$

$$\langle a_{\mathbf{k}m} a_{\mathbf{k}'m} \rangle \propto \int d\mathbf{k} d\mathbf{k}' \langle \Phi(\mathbf{k}) \Phi(\mathbf{k}') \rangle$$

Initial conditions

$$\langle a_{\mathbf{k}m} a_{\mathbf{k}'m} \rangle \propto \int d\mathbf{k} d\mathbf{k}' \langle \Phi(\mathbf{k}) \Phi(\mathbf{k}') \rangle$$

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$$\langle a_{\mathbf{k}m} \rangle \propto \int \langle \vec{\Phi}(\mathbf{k}) \rangle$$

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$$\langle a_{\mathbf{k}m} a_{\mathbf{k}'m} \dots \rangle \propto \int \langle \Phi(\mathbf{k}) \Phi(\mathbf{k}') \dots \rangle$$

If primordial perturbations
are gaussian $\rightarrow a_{\mathbf{k}m}$

$$\langle a_{\vec{k}m} a_{\vec{k}'m} \rangle \propto \int d\vec{k} d\vec{k}' \langle \Phi(\vec{k}) \Phi(\vec{k}') \rangle$$

Initial conditions

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$$\langle a_{\vec{k}m} a_{\vec{k}'m} \dots \rangle \propto \int \langle \Phi(\vec{k}) \Phi(\vec{k}') \dots \rangle$$

If primordial perturbations are gaussian $\rightarrow a_{\vec{k}m}$ will be drawn from a gaussian distribution

$$\langle a_{\vec{k}m} a_{\vec{k}'m} a_{\vec{k}''m} \rangle = 0$$

$$\langle a_{\mathbf{k}m} a_{\mathbf{k}'m} \rangle \propto \int d\mathbf{k} d\mathbf{k}' \langle \Phi(\mathbf{k}) \Phi(\mathbf{k}') \rangle$$

Initial conditions

$$\langle a_{\mathbf{k}m} \rangle \propto \int \langle \vec{\Phi}(\mathbf{k}) \rangle$$

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will be drawn from a
gaussian distribution

$$\langle a_{\mathbf{k}m} a_{\mathbf{k}'m} \rangle = 0$$

In this case c_{ℓ} is all there is

$$\langle a_{\mathbf{k}} a_{\mathbf{k}'} \rangle \propto \int d\mathbf{k} d\mathbf{k}' \langle \Phi_0(\mathbf{k}) \Phi_0^*(\mathbf{k}') \rangle$$

Initial conditions

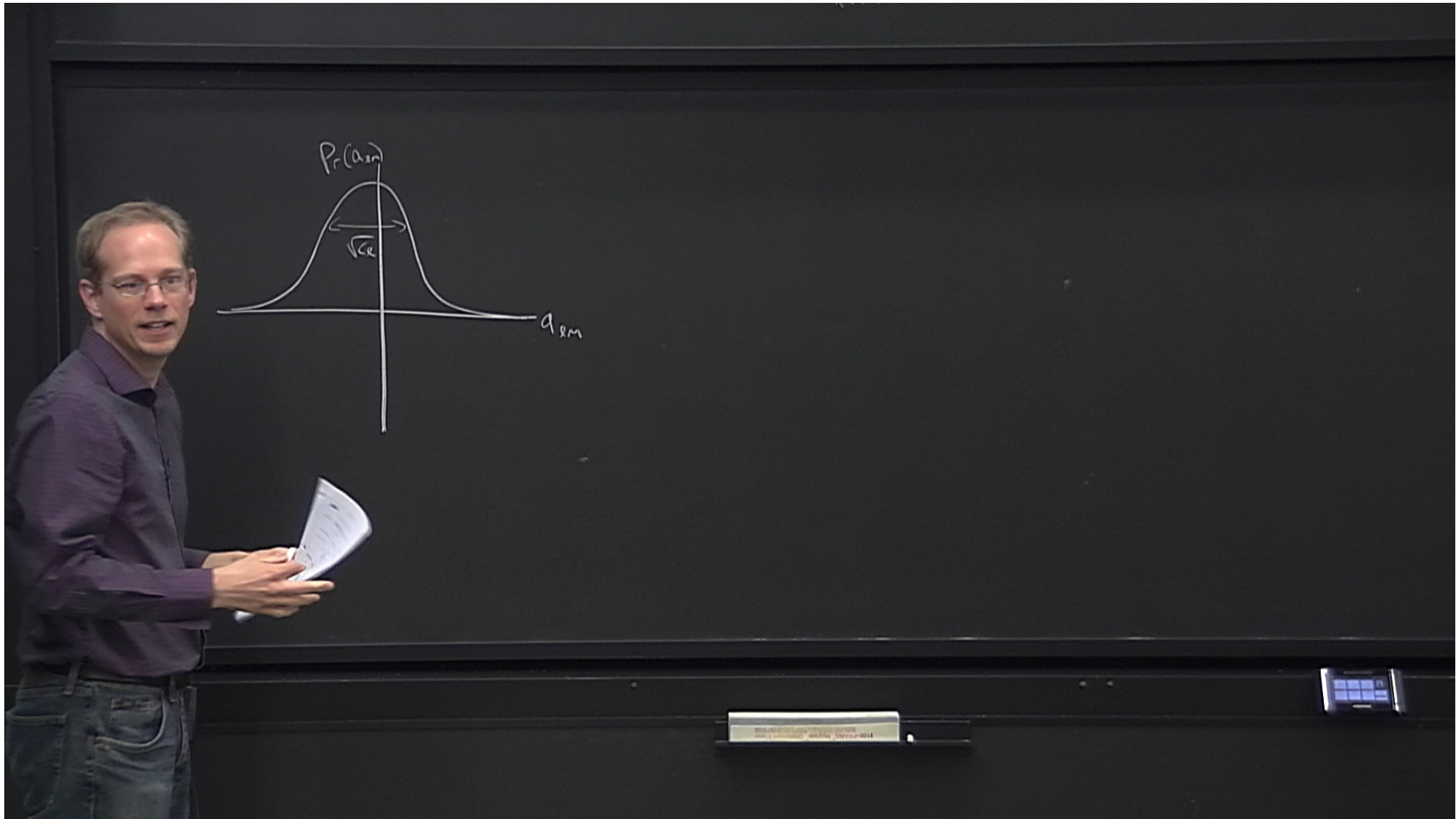
$$\langle a_{\mathbf{k}} \rangle \propto \int \langle \vec{\Phi}(\mathbf{k}) \rangle$$

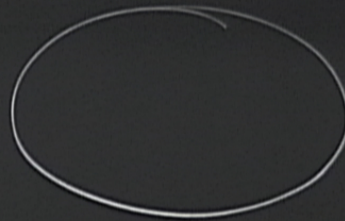
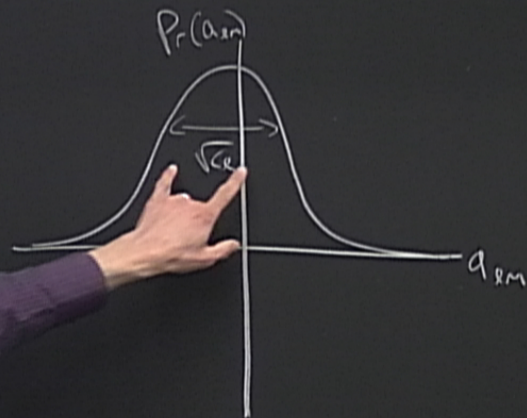
$$\langle a_{\mathbf{k}} a_{\mathbf{k}'} \dots \rangle \propto \int \langle \Phi_0(\mathbf{k}) \Phi_0^*(\mathbf{k}') \Phi_0(\mathbf{k}'') \dots \rangle$$

If primordial perturbations
are gaussian $\rightarrow a_{\mathbf{k}}$
will be drawn from a
gaussian distribution

$$\langle a_{\mathbf{k}} a_{\mathbf{k}'} a_{\mathbf{k}''} \rangle = 0$$

In this case ρ_c is all there is
In the real world we have already
constrained $\langle a_{\mathbf{k}} a_{\mathbf{k}'} a_{\mathbf{k}''} \rangle \ll 1$

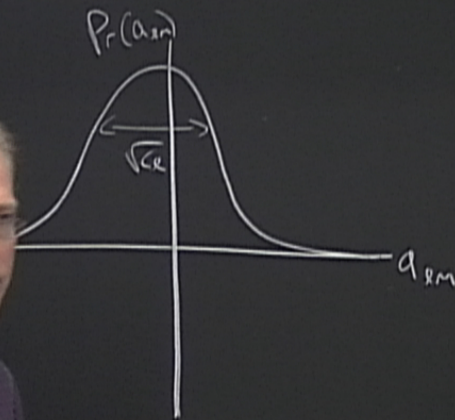




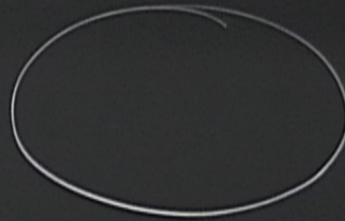
l -sky

For each l there are
only $2l+1$ "modes"

(different values of m)



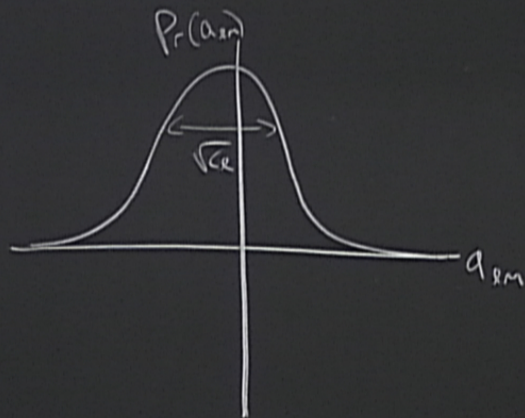
Cosmic Variance



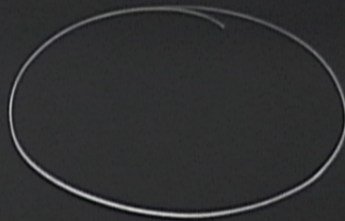
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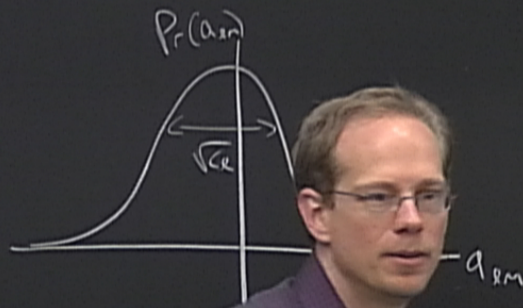


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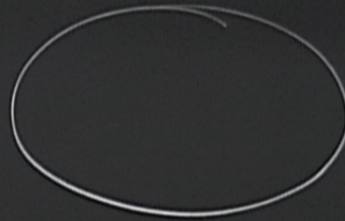
For each l there are
only $2l+1$ "modes"

(different values of

$$\frac{\Delta C_l}{C_l}$$



Cosmic Variance

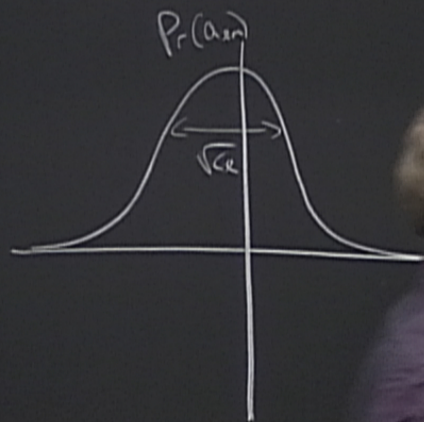


l-sky

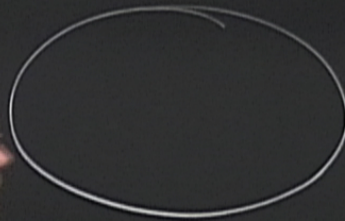
For each l there are
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(different values of m)

$$\frac{\Delta C_l}{C_l} = \frac{1}{\sqrt{2l+1}}$$



Cosmic Variance



l-sky

For each l there are
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(different values of m)

$$\frac{\Delta C_l}{C_l} = \frac{1}{\sqrt{2l+1}}$$

Cosmic-variance-limited
experiments—

have systematic + random
errors $\ll \frac{1}{\sqrt{2l+1}}$

$$\frac{\Delta C_\ell}{C_\ell} = \frac{1}{\sqrt{2\ell+1}}$$

Cosmic-variance-limited

experiments -

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Planck - C.V. limited to $\ell \sim 2000-3000$

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Sachs-Wolfe Plateau

$$= \frac{1}{\sqrt{2\ell+1}}$$

- variance limited

instruments -

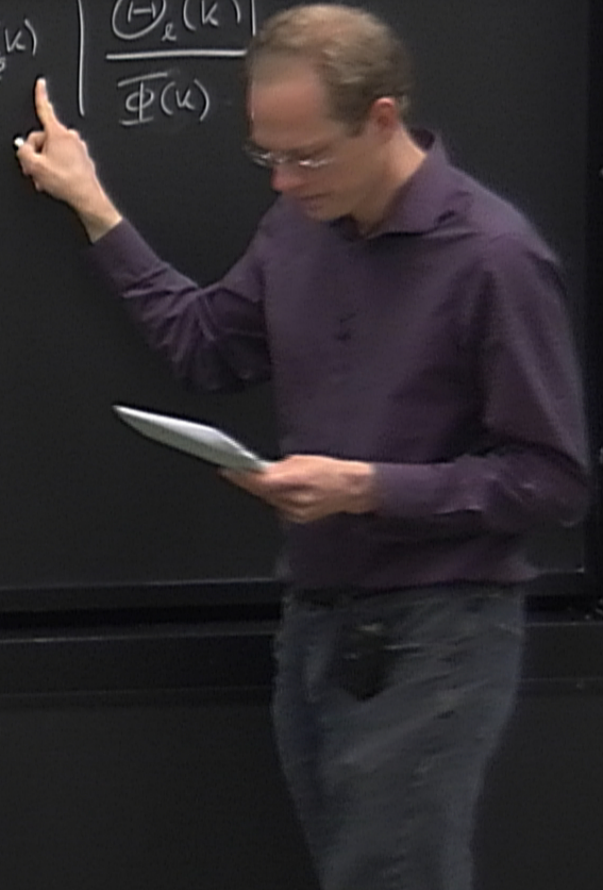
re systematic + random

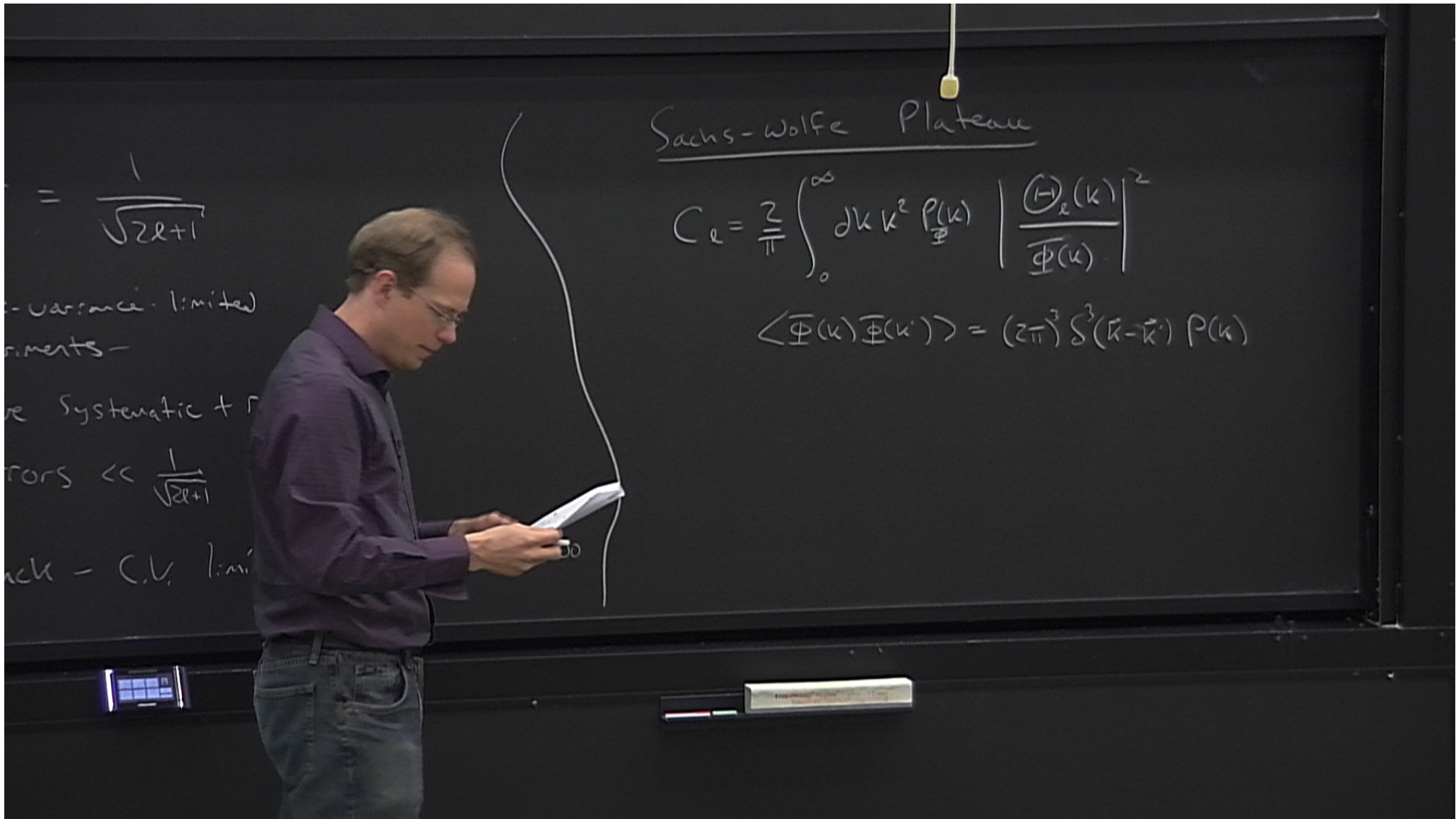
errors $\ll \frac{1}{\sqrt{2\ell+1}}$

ack - C.V. limited to $\ell \sim 2000-3000$

Sachs-Wolfe Plateau

$$C_\ell = \frac{2}{\pi} \int_0^\infty dk k^2 \frac{P(k)}{\Phi(k)} \left| \frac{(-1)_\ell(k)}{\Phi(k)} \right|^2$$





Sachs-Wolfe Plateau

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$$\langle \Phi(k) \Phi(k') \rangle = (2\pi)^3 \delta^3(\vec{k} - \vec{k}') P(k)$$

$$= \frac{1}{\sqrt{2\ell+1}}$$

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re systematic + r

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$$\langle \Phi(k) \Phi(k') \rangle = (2\pi)^3 \delta^3(\vec{k} - \vec{k}') P(k)$$

$$P_\Phi = A (k)^{n_s-1} k^{-3}$$

$n_s = 1$, Scale invariant

$$= \frac{1}{\sqrt{2\ell+1}}$$

- variance limited
experiments -

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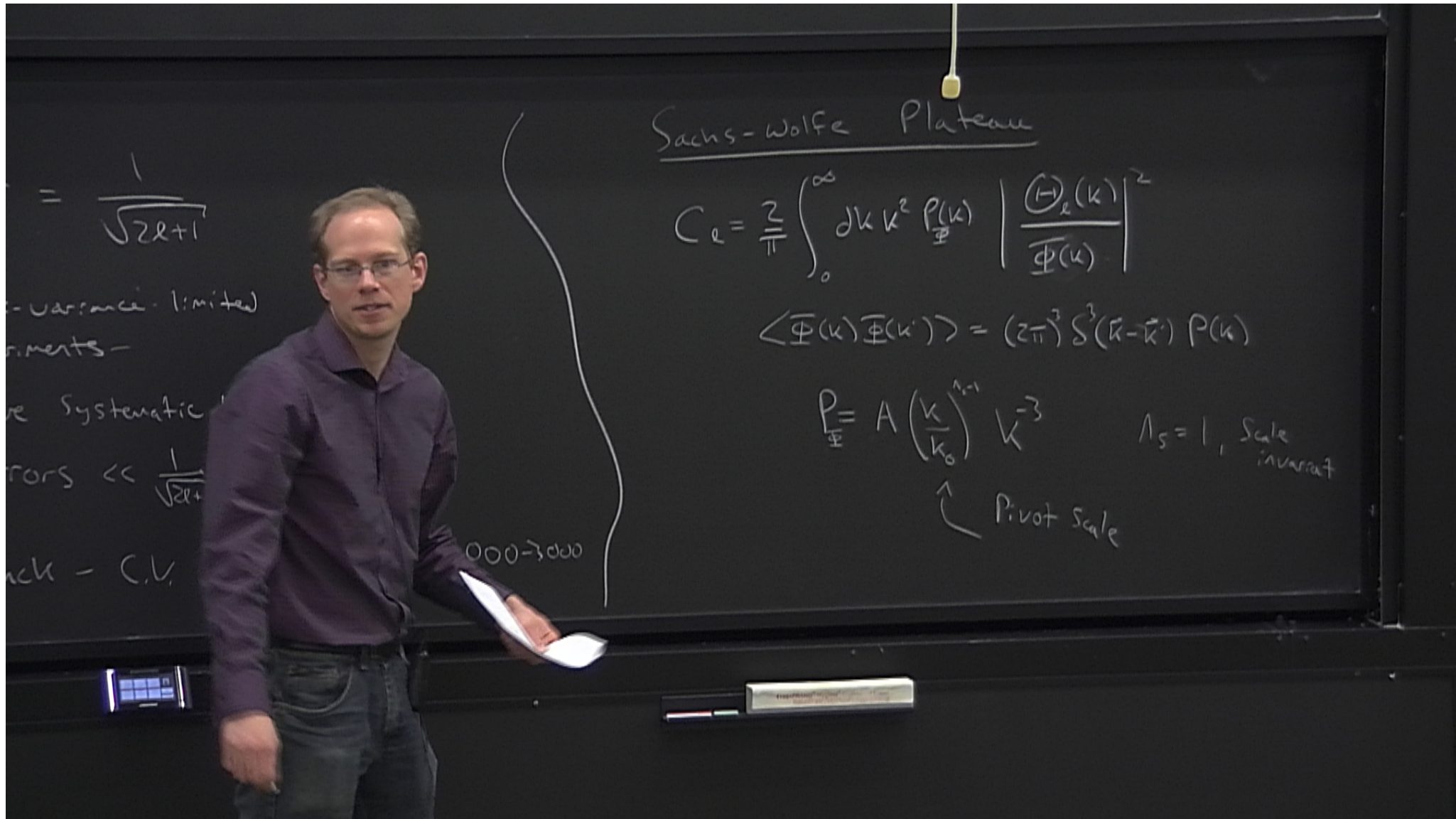
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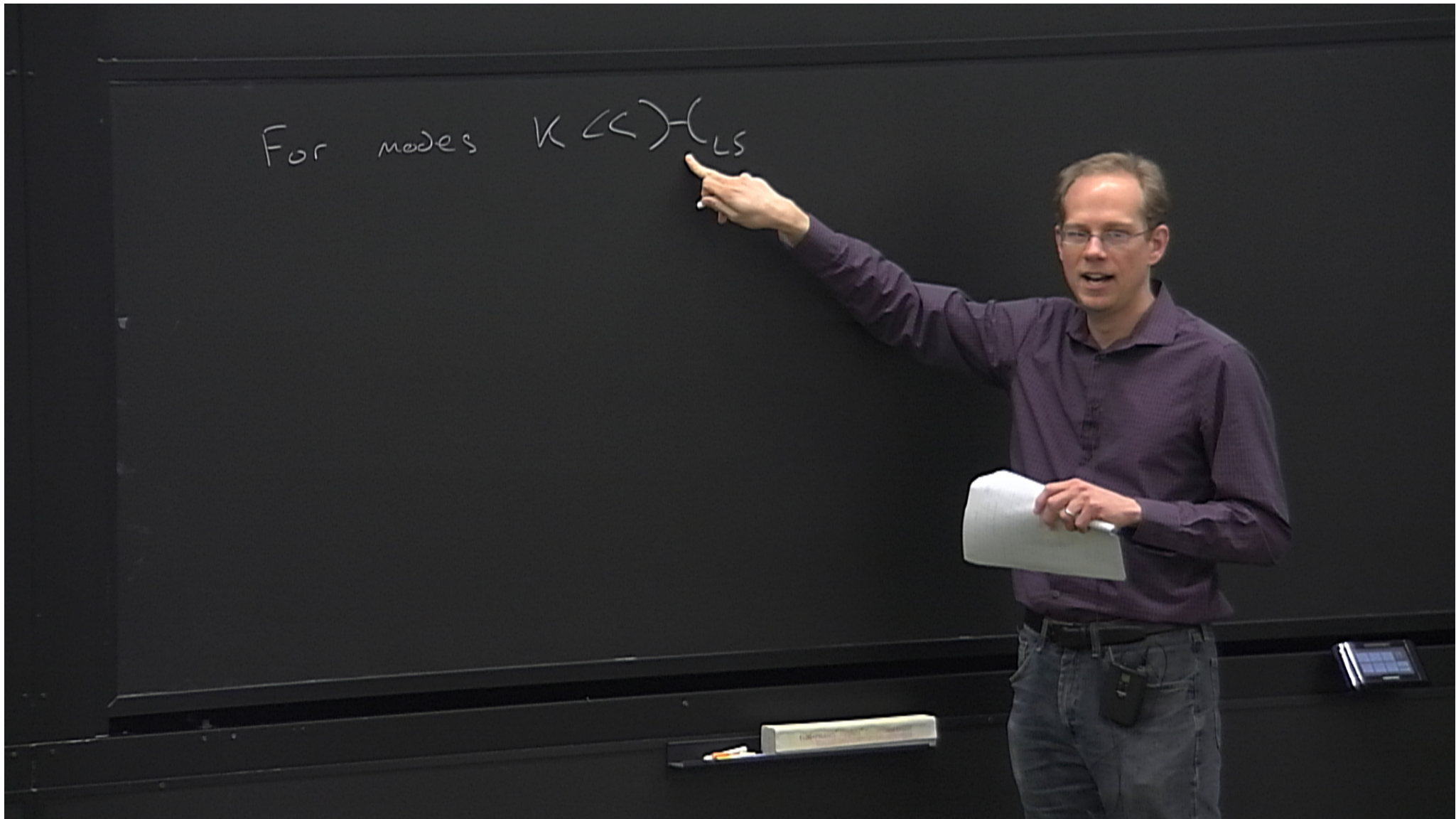
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↑ Pivot scale $k_0 \sim 0.05 \text{ Mpc}^{-1}$

For modes $k \ll k_{LS}$



For modes $k \ll \frac{1}{L_S}$

$$\overline{\Phi}(k, \eta_{L_S}) = \frac{9}{10} \overline{\Phi}(k, 0) = \frac{9}{10} \overline{\Phi}_0(k)$$

$$= \frac{1}{\sqrt{2\ell+1}}$$

variance-limited
ments -

Systematic + random

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k - C.V. limited to $\ell \sim 2000-3000$

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$$\overline{\Phi}(k, \eta_{L_S}) \rightarrow \overline{\Phi}_0, \quad P_{\Phi} \rightarrow P_{\Phi_0}$$

$$\textcircled{14} \quad \dot{\gamma}_\ell = -\frac{1}{3} \overline{\Phi}(k, \eta_{L_S}) \dot{\gamma}_\ell(k(\eta - \eta_{L_S}))$$

For modes $k \ll \eta_{LS}$

$$\overline{\Phi}(k, \eta_{LS}) = \frac{9}{10} \overline{\Phi}(k, 0) = \frac{9}{10} \overline{\Phi}_0(k)$$

$$\overline{\Phi}(k, \eta_{LS}) \rightarrow \overline{\Phi}_0, \quad P_{\Phi} \rightarrow P_{\Phi_0}$$

$$\textcircled{1} \quad \dot{\Phi}_L = -\frac{1}{3} \overline{\Phi}(k, \eta_{LS}) \dot{\eta}_L(k(1-\eta_{LS}))$$

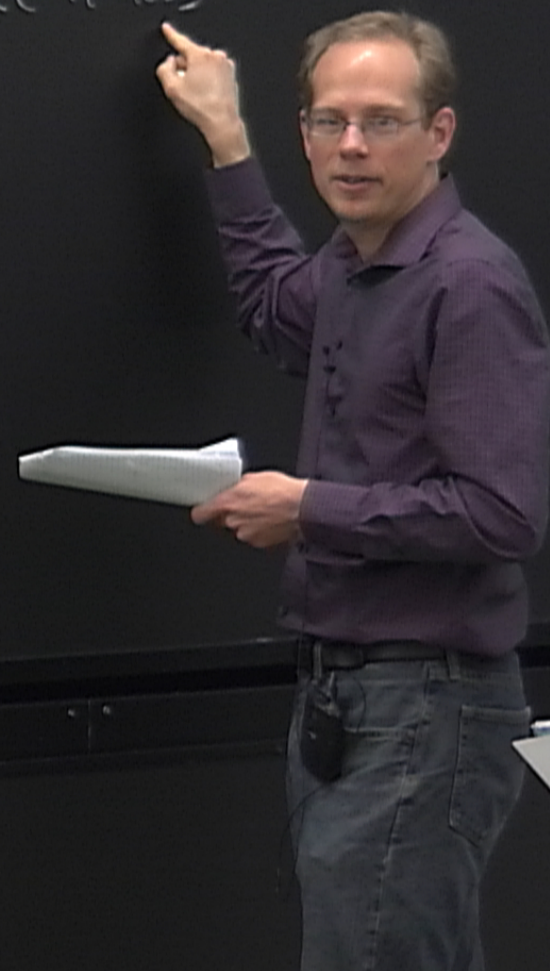
$$C_L = \frac{2A}{\pi k_0^{n_s-1}} \int dk k^{n_s-2} \dot{\eta}_L(k)$$

$$C_2 = \frac{2A}{\pi K_0^{n_s-1}} \int d^d k K^{n_s-2} \left[j_2(k(\eta-\eta_{cs})) \right]^2$$

$$(\vec{k}, 0) = \frac{9}{10} \overline{\Phi}_0(\vec{k})$$

$$\rightarrow \rho_{\overline{\Phi}_0}$$

$$(\eta - \eta_{cs})$$



$$(1 - \eta_{cs})$$

$$= \frac{2A}{\pi(k_0 \eta)}$$

χ_{LS}

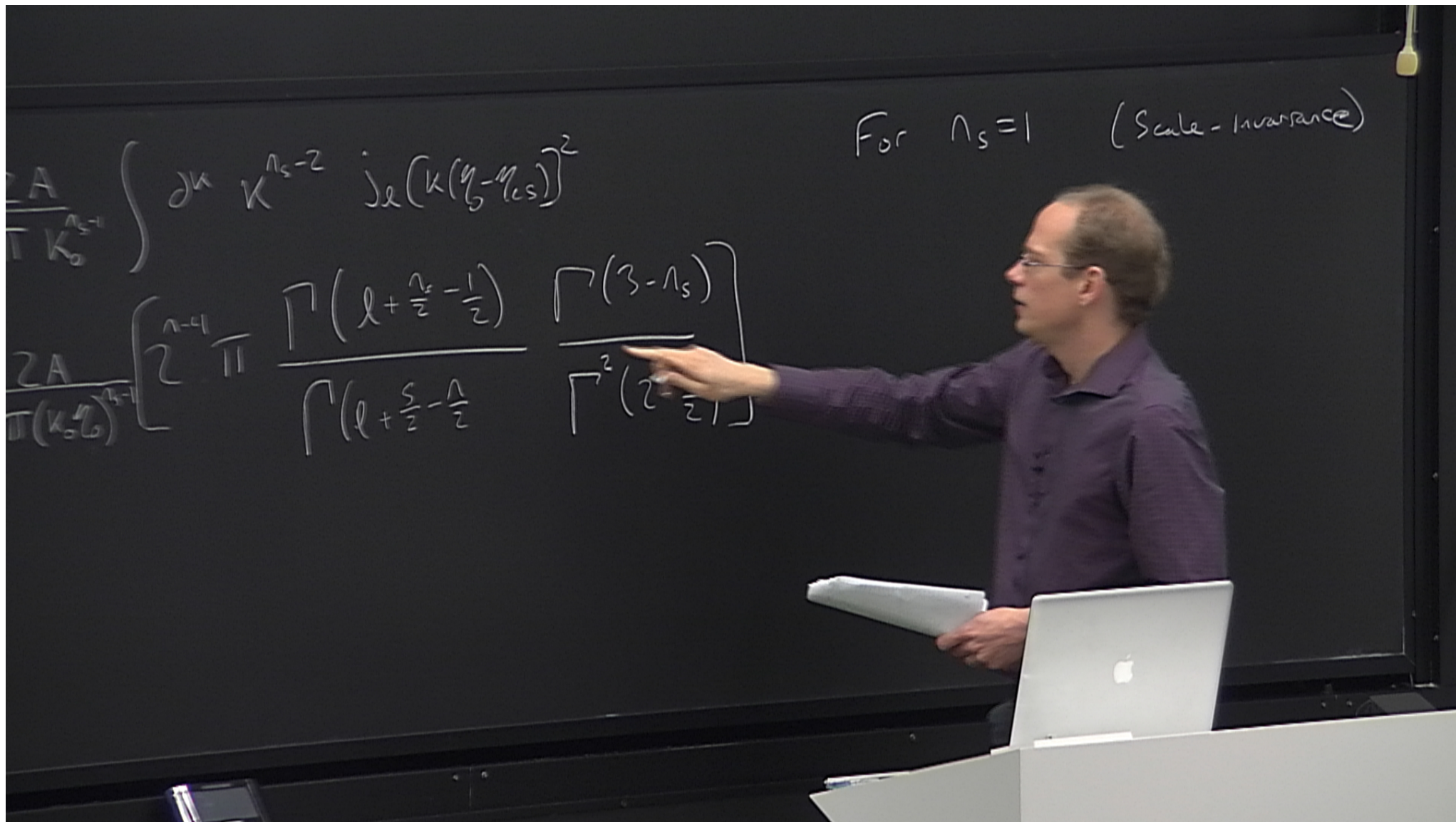
$$(\vec{k}, 0) = \frac{9}{10} \overline{\Phi}_0(\vec{k})$$

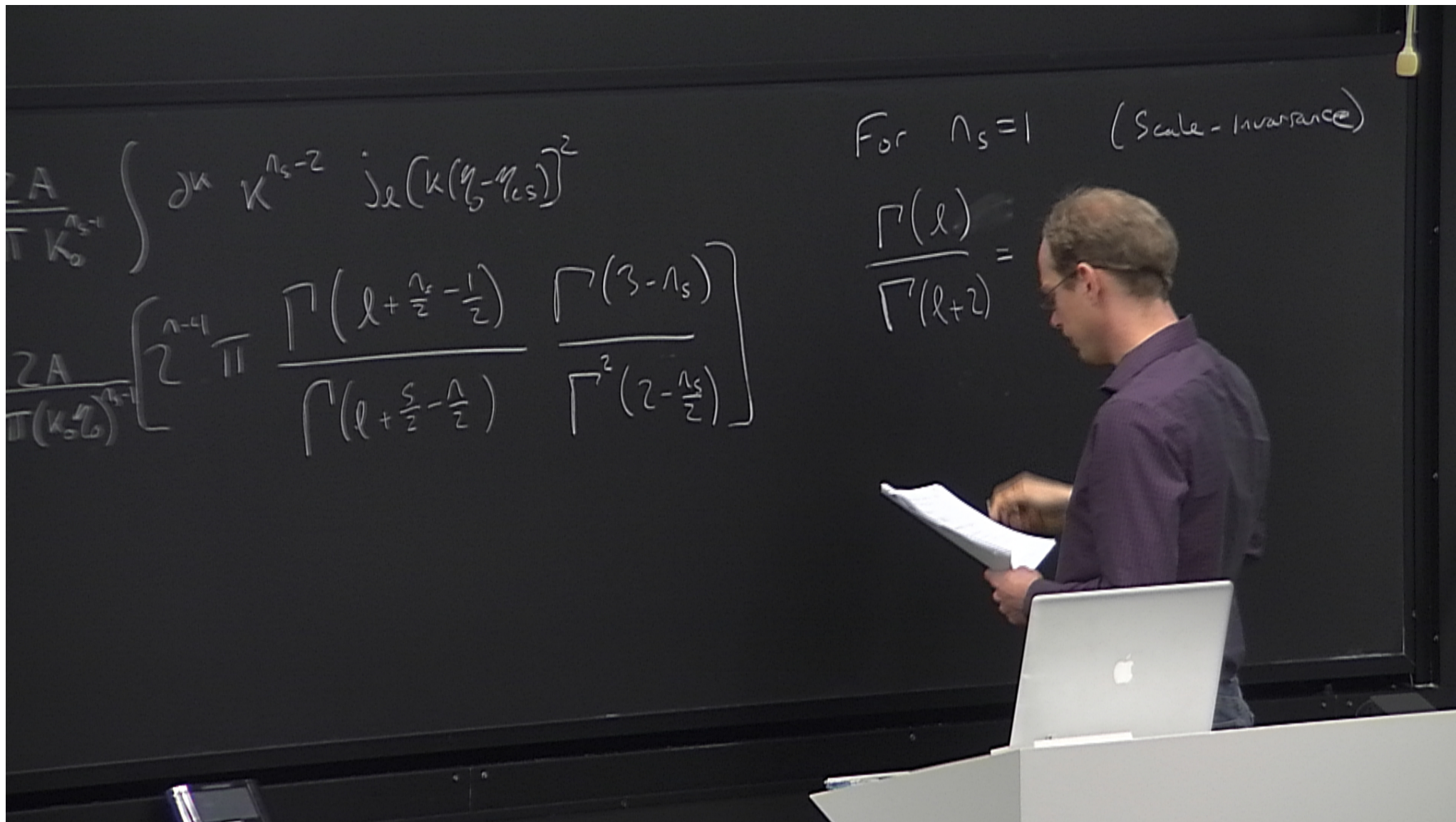
$\rightarrow \rho_{\overline{\Phi}_0}$

$\eta - \eta_{LS}$

$$C_Q(\eta) = \frac{2A}{\pi k_0^{n_s-1}} \int d^4 k k^{n_s-2} \left[\eta(k(\eta - \eta_{LS})) \right]^2$$

$$= \frac{2A}{\pi (k_0 \eta_0)^{n_s-1}} \left[2^{n-1} \pi \frac{\Gamma\left(l + \frac{n_s}{2} - \frac{1}{2}\right)}{\Gamma\left(l + \frac{5}{2} - \frac{n}{2}\right)} \frac{\Gamma(3-n_s)}{\Gamma^2\left(2 - \frac{n_s}{2}\right)} \right]$$



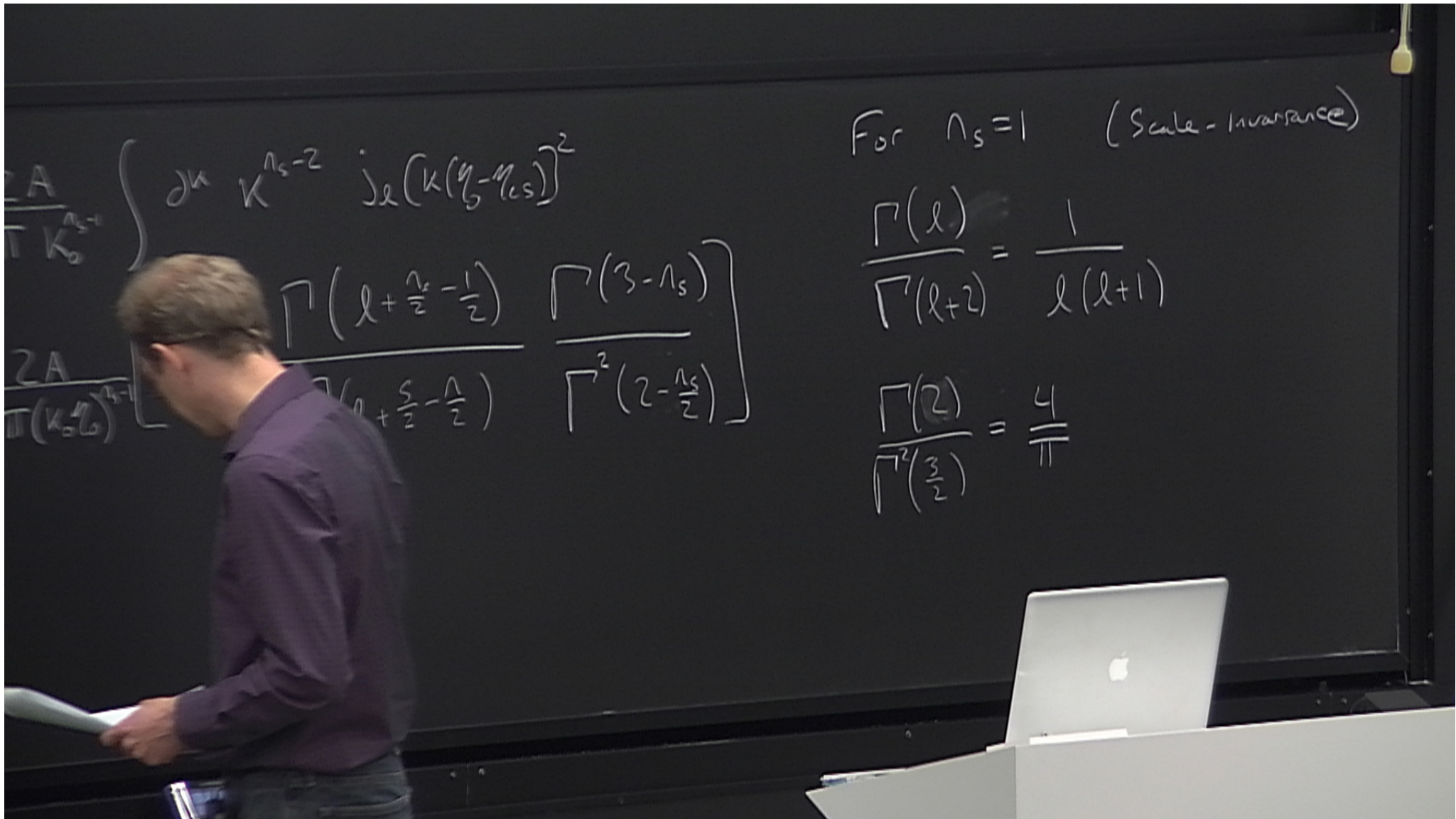


$$\frac{2A}{\pi K_0^{\frac{n_s-1}{2}}} \int d^d k \, K^{\frac{n_s-2}{2}} \, j_\ell(K(\eta_0 - \eta_{cs}))^2$$

$$\frac{2A}{\pi(K_0 \eta_0)^{\frac{n_s-1}{2}}} \left[2^{\frac{n_s-1}{2}} \pi \frac{\Gamma(\ell + \frac{n_s}{2} - \frac{1}{2})}{\Gamma(\ell + \frac{s}{2} - \frac{n_s}{2})} \frac{\Gamma(3-n_s)}{\Gamma^2(2-\frac{n_s}{2})} \right]$$

For $n_s=1$ (Scale-Invariance)

$$\frac{\Gamma(\ell)}{\Gamma(\ell+2)} = \frac{1}{\ell(\ell+1)}$$



$$\frac{A}{\pi K_0^{\lambda_s-1}} \int d^d k \, K^{\lambda_s-2} \, j_\ell(k(\eta_0-\eta_{cs}))^2$$
$$\frac{2A}{\pi (K_0 \eta_0)^{\lambda_s-1}} \left[\frac{\Gamma(\ell + \frac{\lambda_s}{2} - \frac{1}{2})}{\Gamma(\ell + \frac{5}{2} - \frac{\lambda_s}{2})} \frac{\Gamma(3-\lambda_s)}{\Gamma^2(2-\frac{\lambda_s}{2})} \right]$$

For $\lambda_s=1$ (Scale-Invariance)

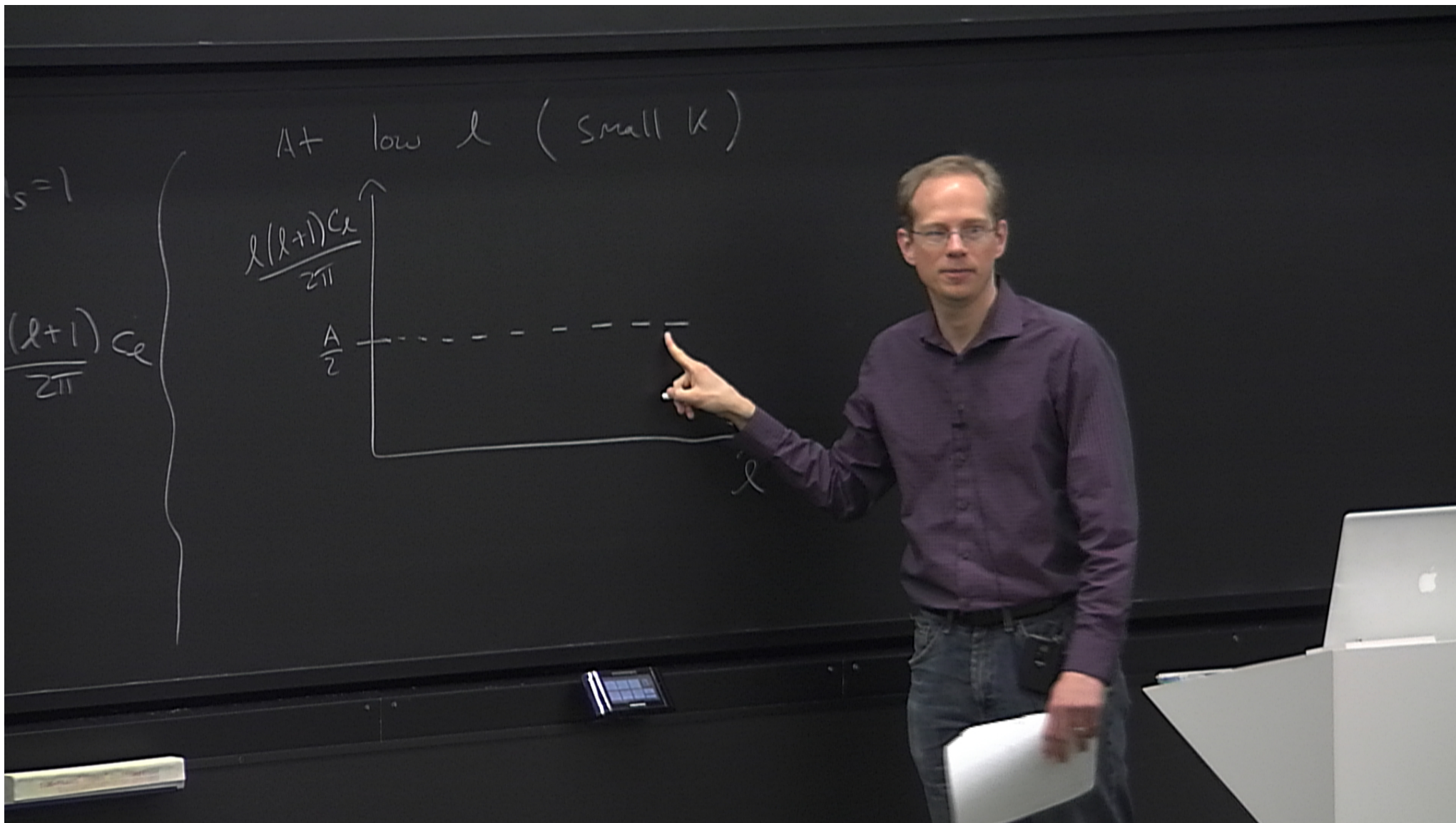
$$\frac{\Gamma(\ell)}{\Gamma(\ell+2)} = \frac{1}{\ell(\ell+1)}$$

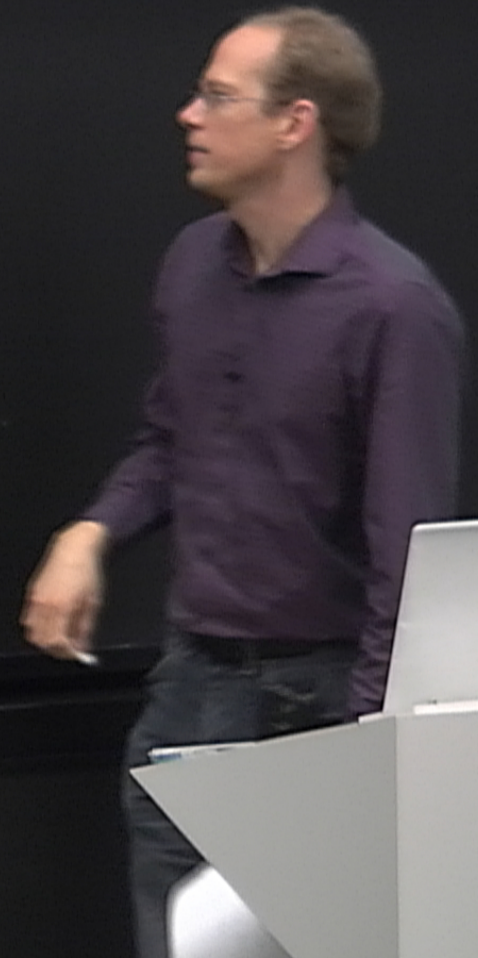
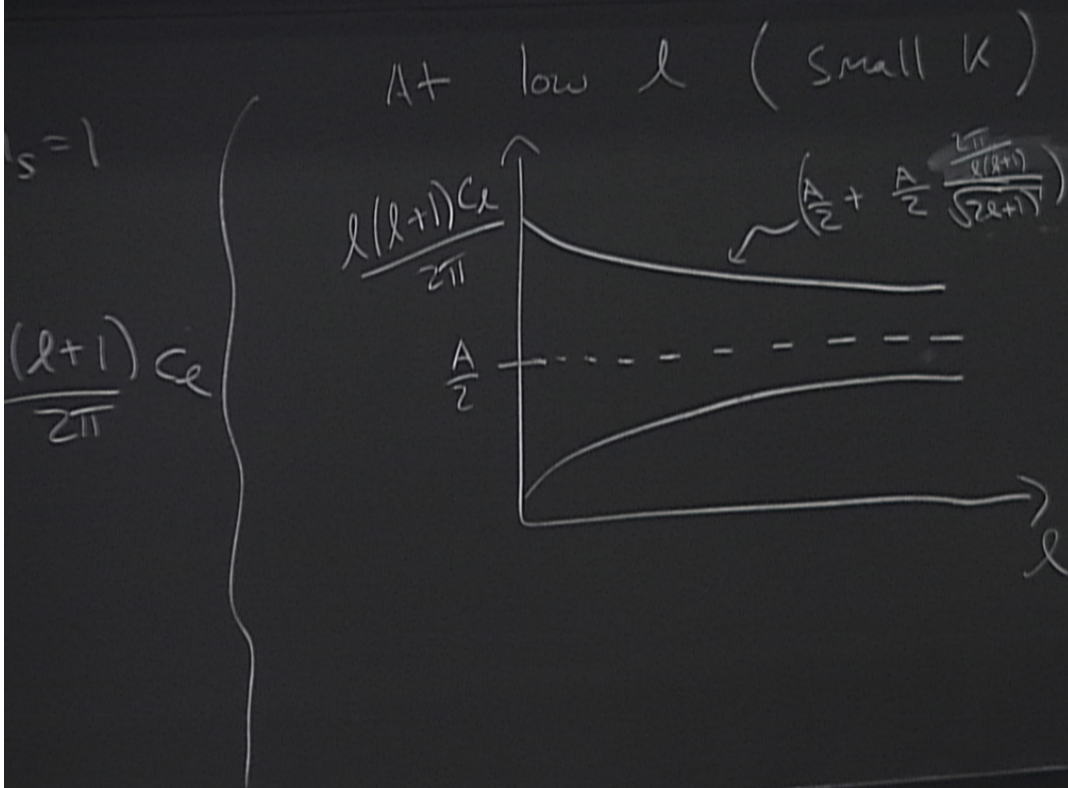
$$\frac{\Gamma(2)}{\Gamma^2(\frac{3}{2})} = \frac{4}{\pi}$$

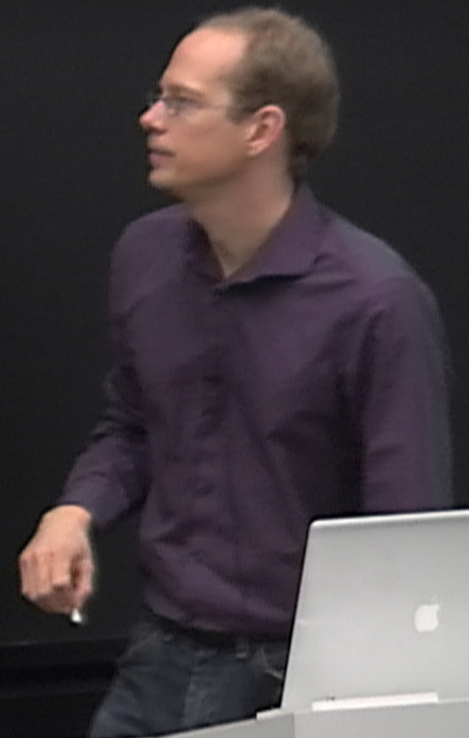
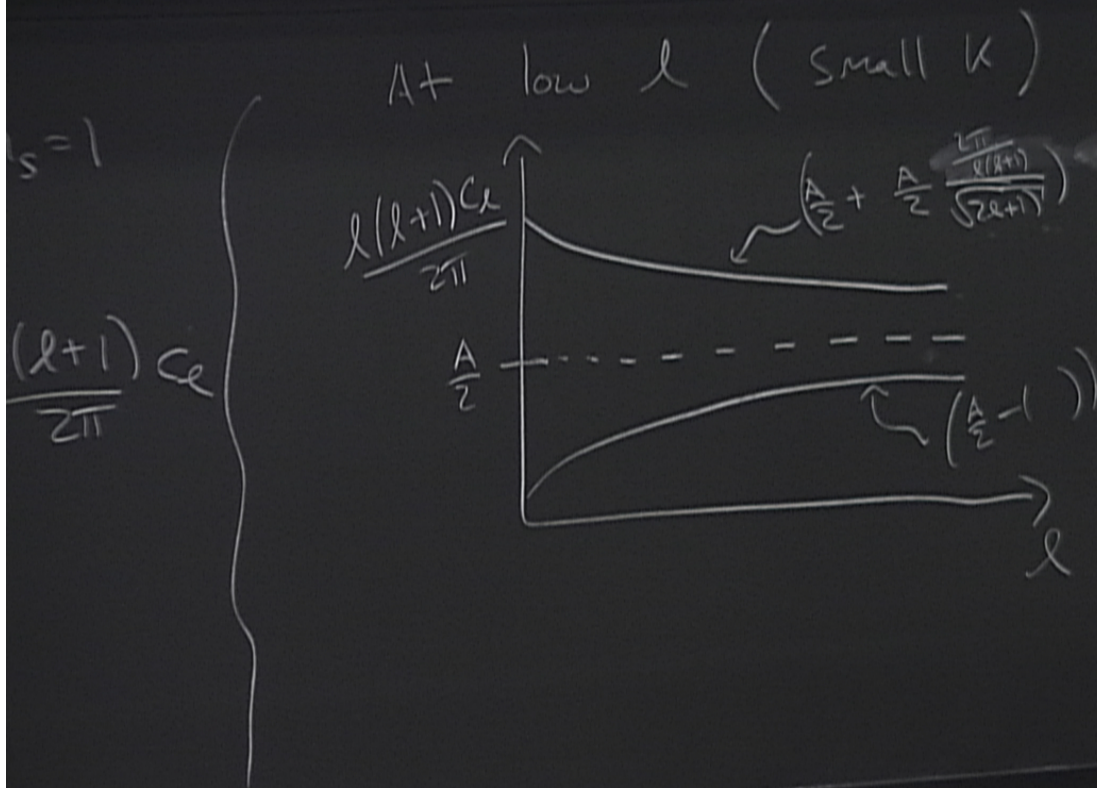
$$C_\ell = \frac{A}{\pi \ell(\ell+1)} \quad \text{for } n_s=1$$

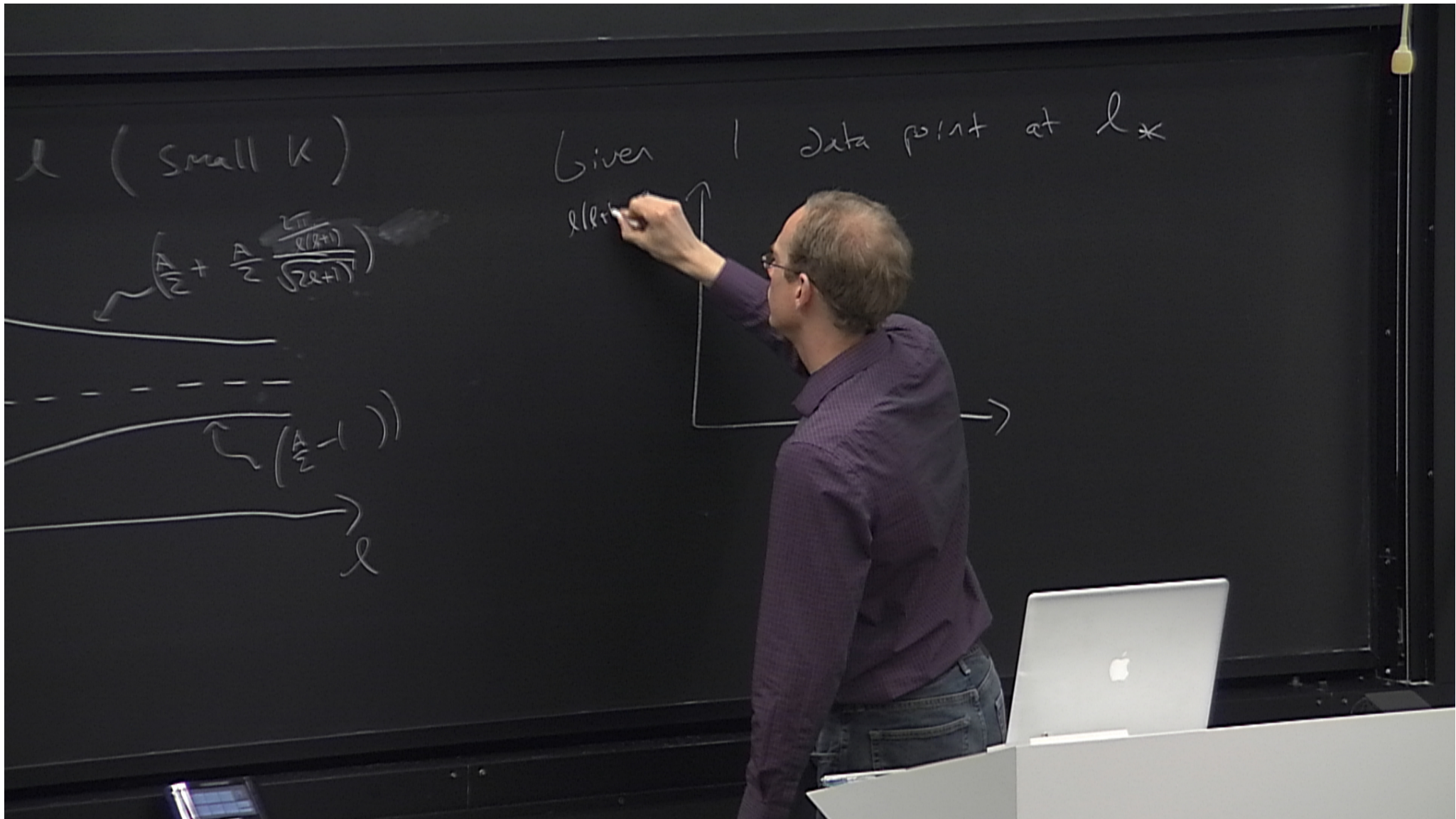
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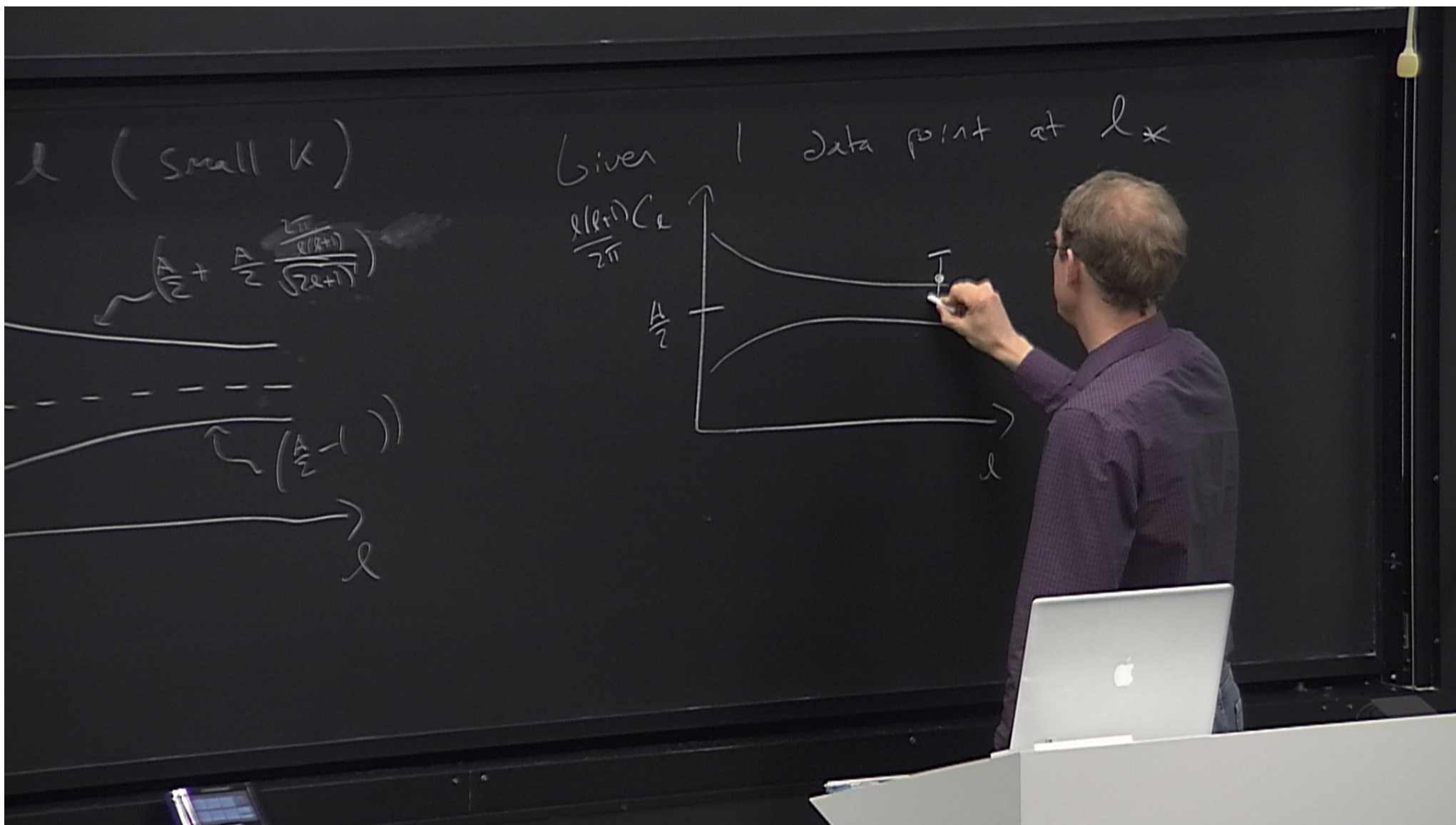
↗ why we plot $\frac{\ell(\ell+1)}{2\pi} C_\ell$

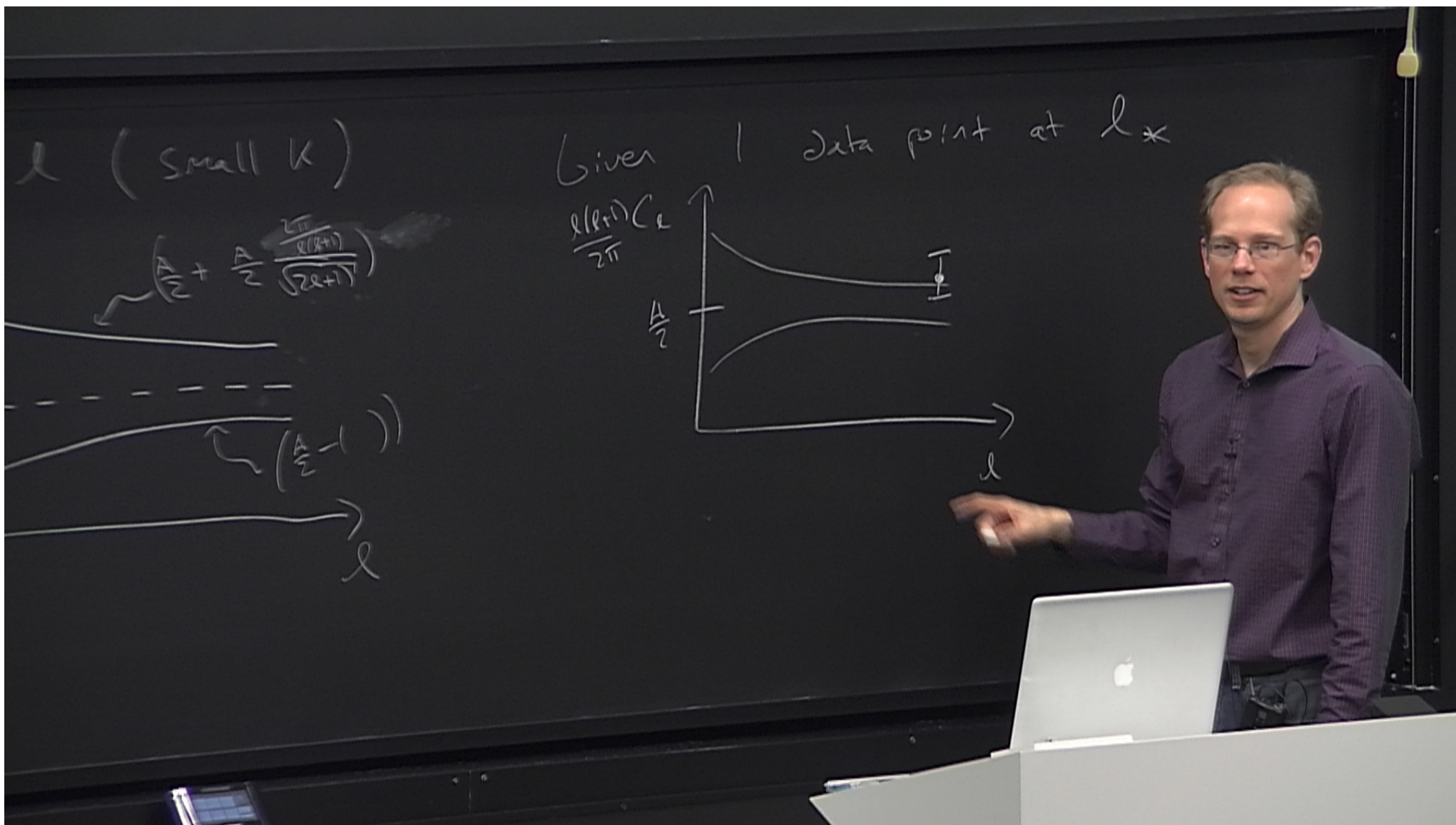










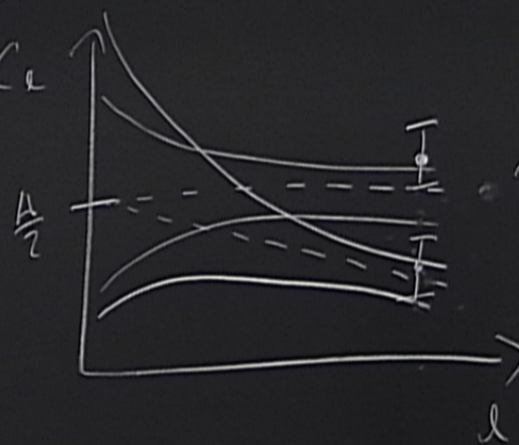


l (small k)

$$\sim \left(\frac{A}{2} + \frac{A}{2} \frac{\frac{2\pi}{l(l+1)}}{\sqrt{2l+1}} \right)$$



Given 1 data point at l_x

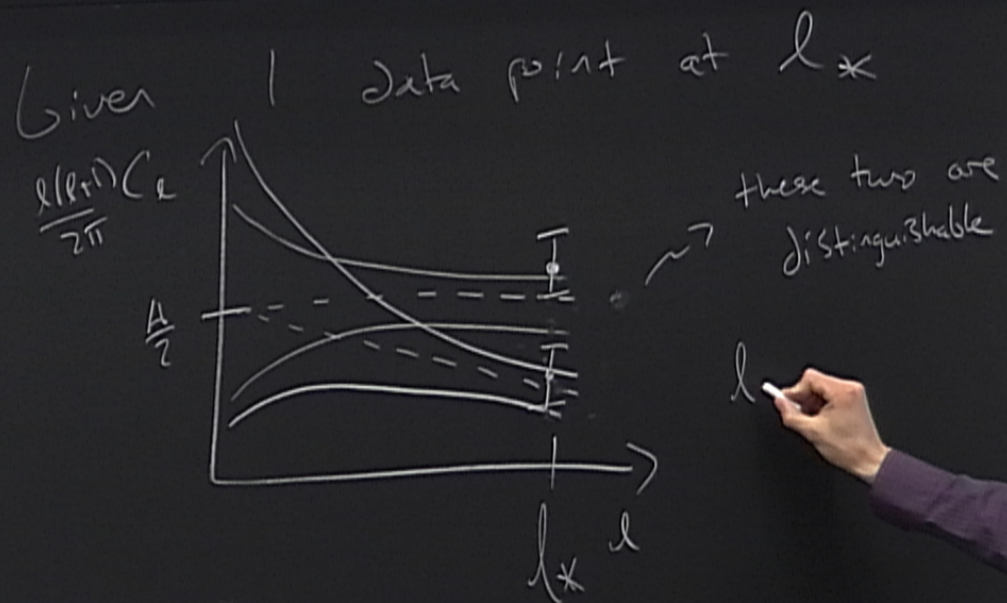


l (small k)

$$\sim \left(\frac{A}{2} + \frac{A}{2} \frac{\frac{2\pi}{l(l+1)}}{\sqrt{2l+1}} \right)$$


$\left(\frac{A}{2} - 1 \right)$

l

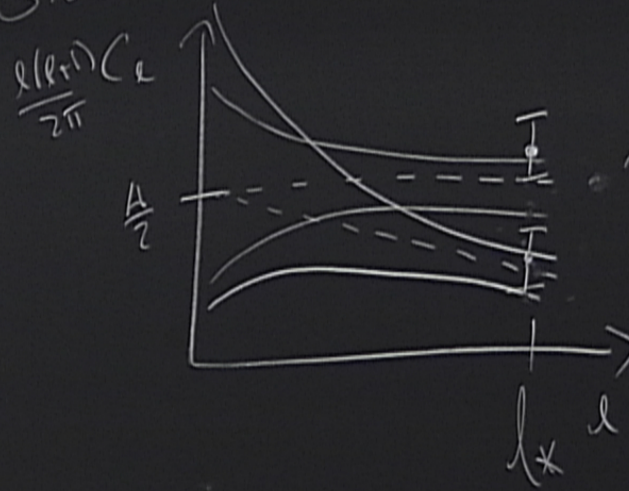


l (small k)

$$\sim \left(\frac{A}{2} + \frac{A}{2} \frac{\frac{2\pi}{k(k+1)}}{\sqrt{2k+1}} \right)$$



Given 1 data point at l_x



these two are distinguishable

$$l_x = 30$$

$$n_s \approx .75$$

$$n_s \approx 1.25$$

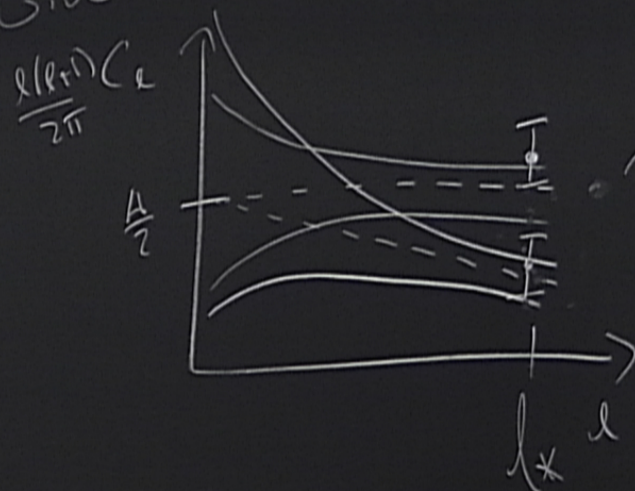
l (small k)

$$\sim \left(\frac{A}{2} + \frac{A}{2} \frac{\frac{2\pi}{k(k+1)}}{\sqrt{2l+1}} \right)$$

$\sim \left(\frac{A}{2} - 1 \right)$

l

Given 1 data point at l_x

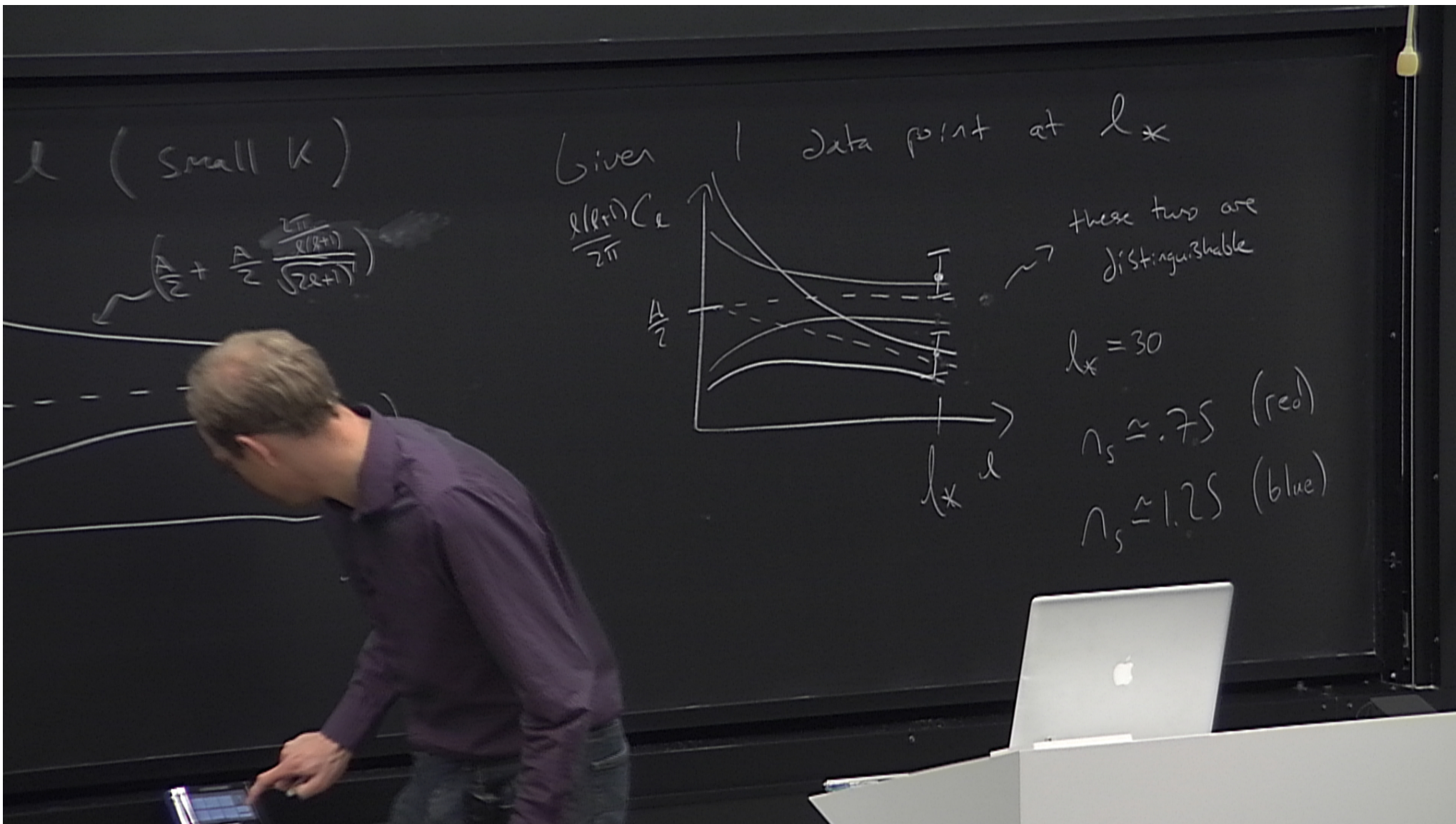


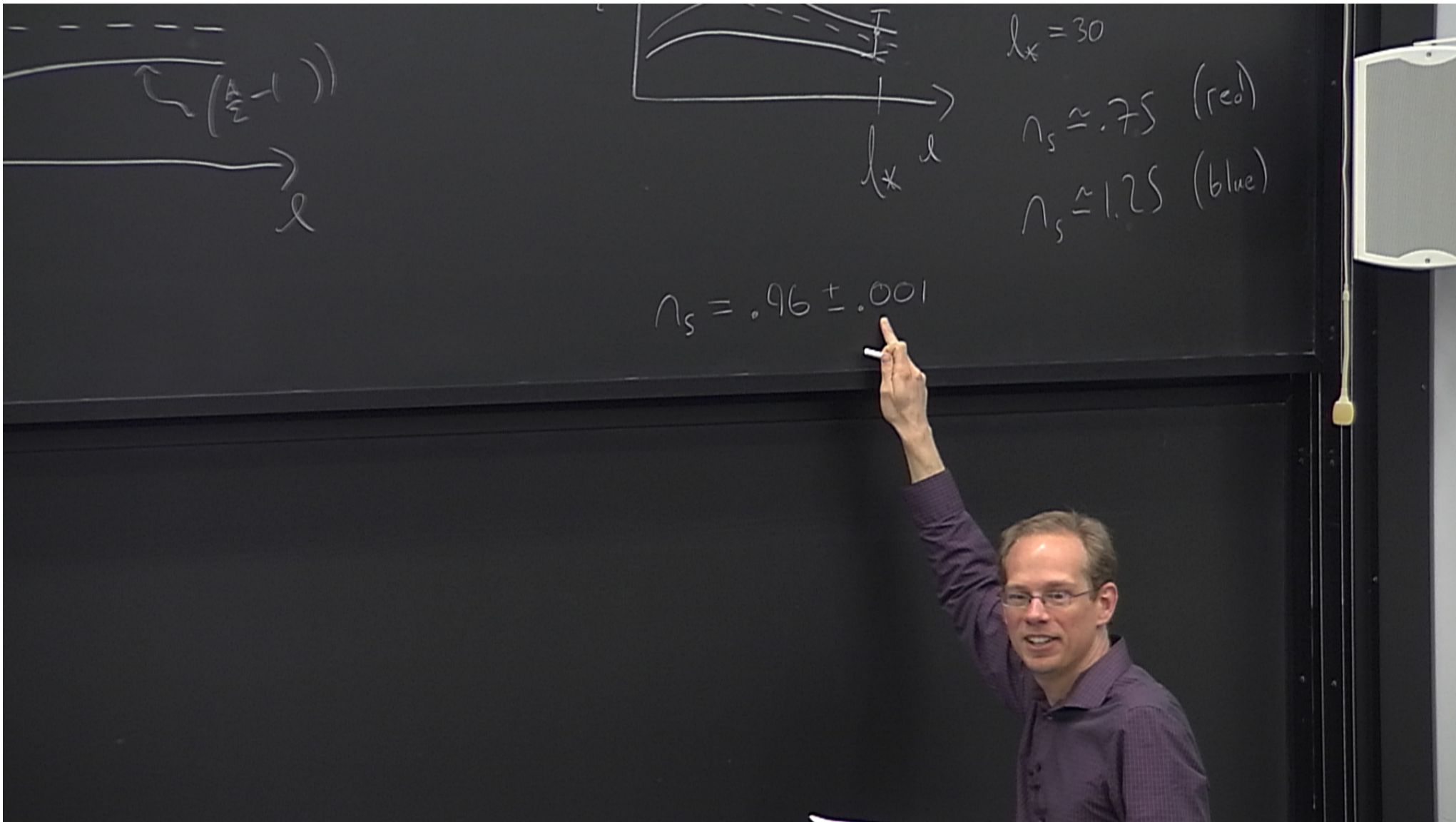
these two are dist.

$$l_x = 30$$

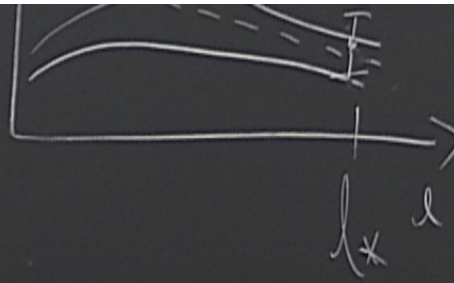
$$n_s \approx .75$$

$$n_s \approx 1.25$$





$$\left(\frac{A}{2} - 1\right)$$



$$l_x = 30$$

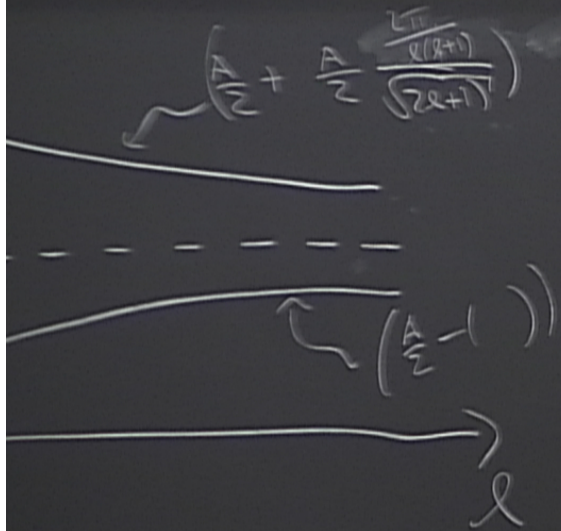
$$n_s \approx .75 \text{ (red)}$$

$$n_s \approx 1.25 \text{ (blue)}$$

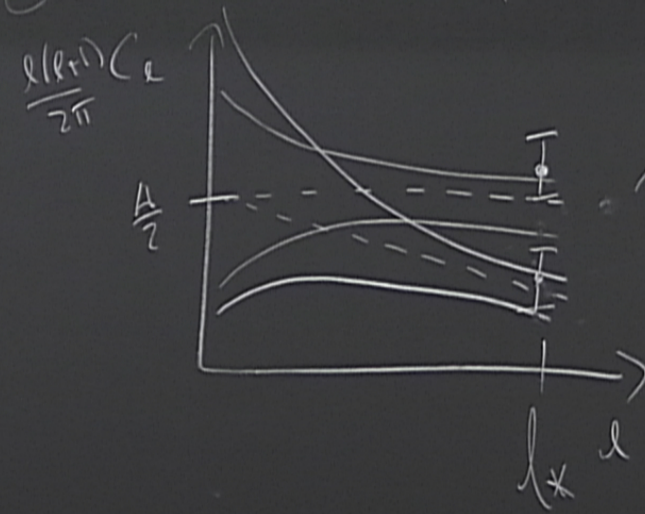
$$\underline{n_s = .96 \pm .001} \text{ (Planck)}$$

Beyond Sachs-Wolfe

l (small k)



Given | data point at l_*



these two are distinguishable

$$l_* = 30$$

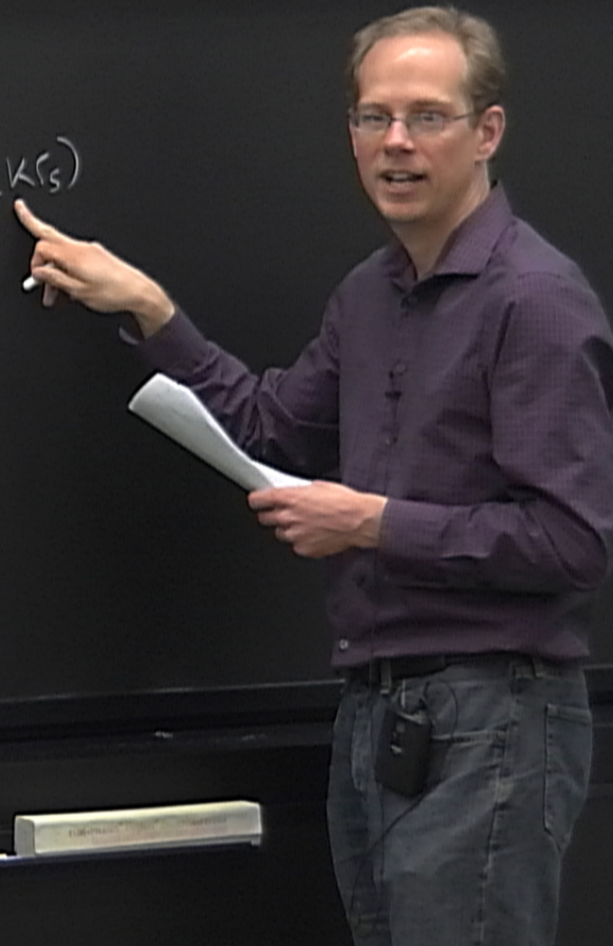
$$n_s = 0.75 \text{ (red)}$$

$$n_s = 1.25 \text{ (blue)}$$

$$n_s = 0.96 \pm 0.001 \text{ (Planck)}$$

Beyond Sachs-Wolfe

$$(\Theta)_\ell \simeq - \frac{[\Theta_0(0) + \Phi(0)]}{3} \cos(kr_s)$$



Beyond Sachs-Wolfe

$$- \frac{[\Psi_0(0) + \Phi(0)]}{3} \left(\cos(kr_s) + \dots \right) j_\ell(k(\eta_0 - \eta_{LS}))$$

Beyond Sachs-Wolfe

$$\Theta_{\ell} \simeq - \frac{[\Theta_0(0) + \Phi(0)]}{3} \left(\cos(kr_s) + \dots \right) j_{\ell}(k(\eta_0 - \eta_{LS}))$$

$$\langle \Phi(k) \Phi(k') \rangle \propto T^2(k) \frac{D_L^2(k)}{a^2}$$

transfer
F.N
Growth
F.N

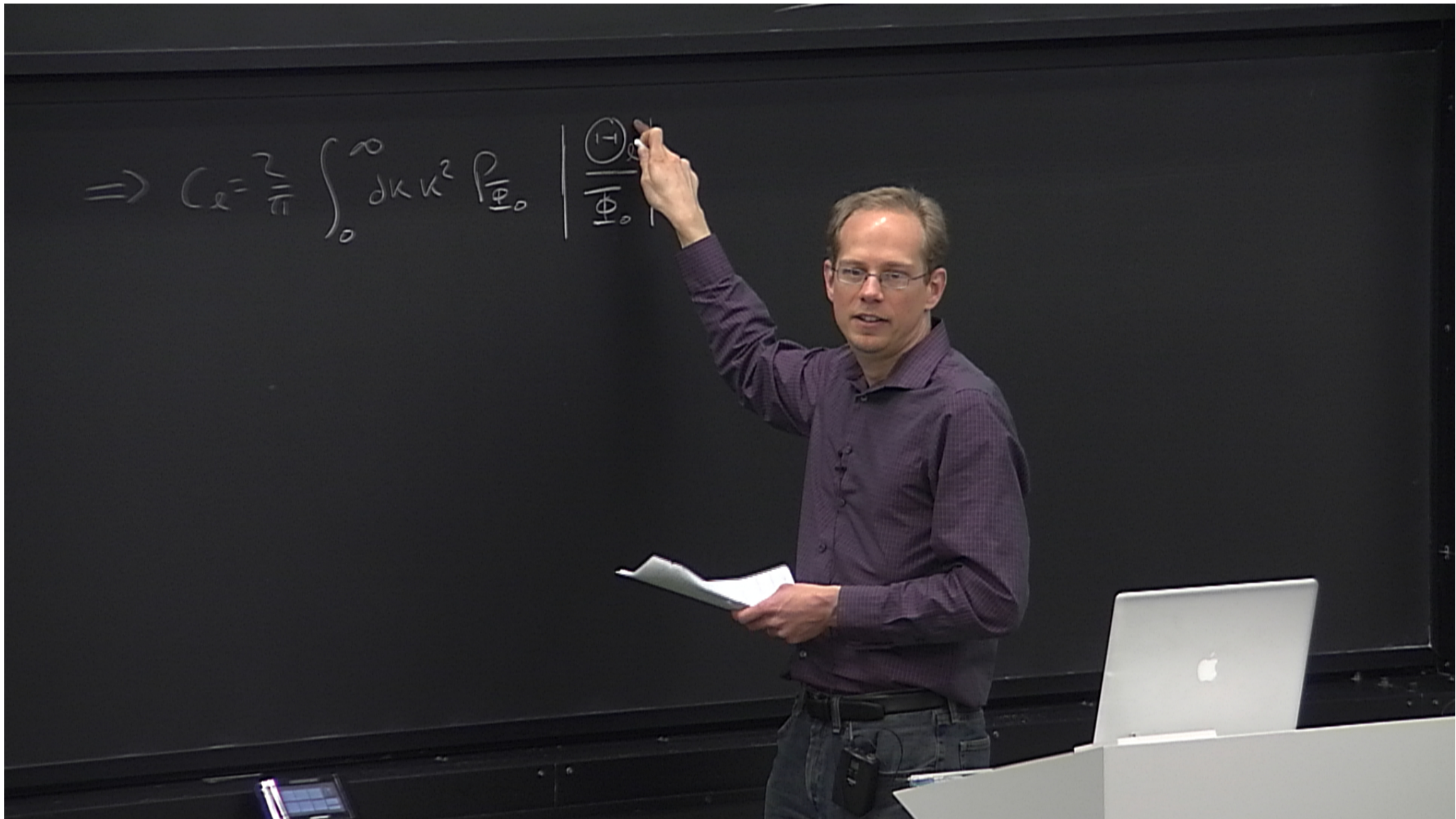
Beyond Sachs-Wolfe

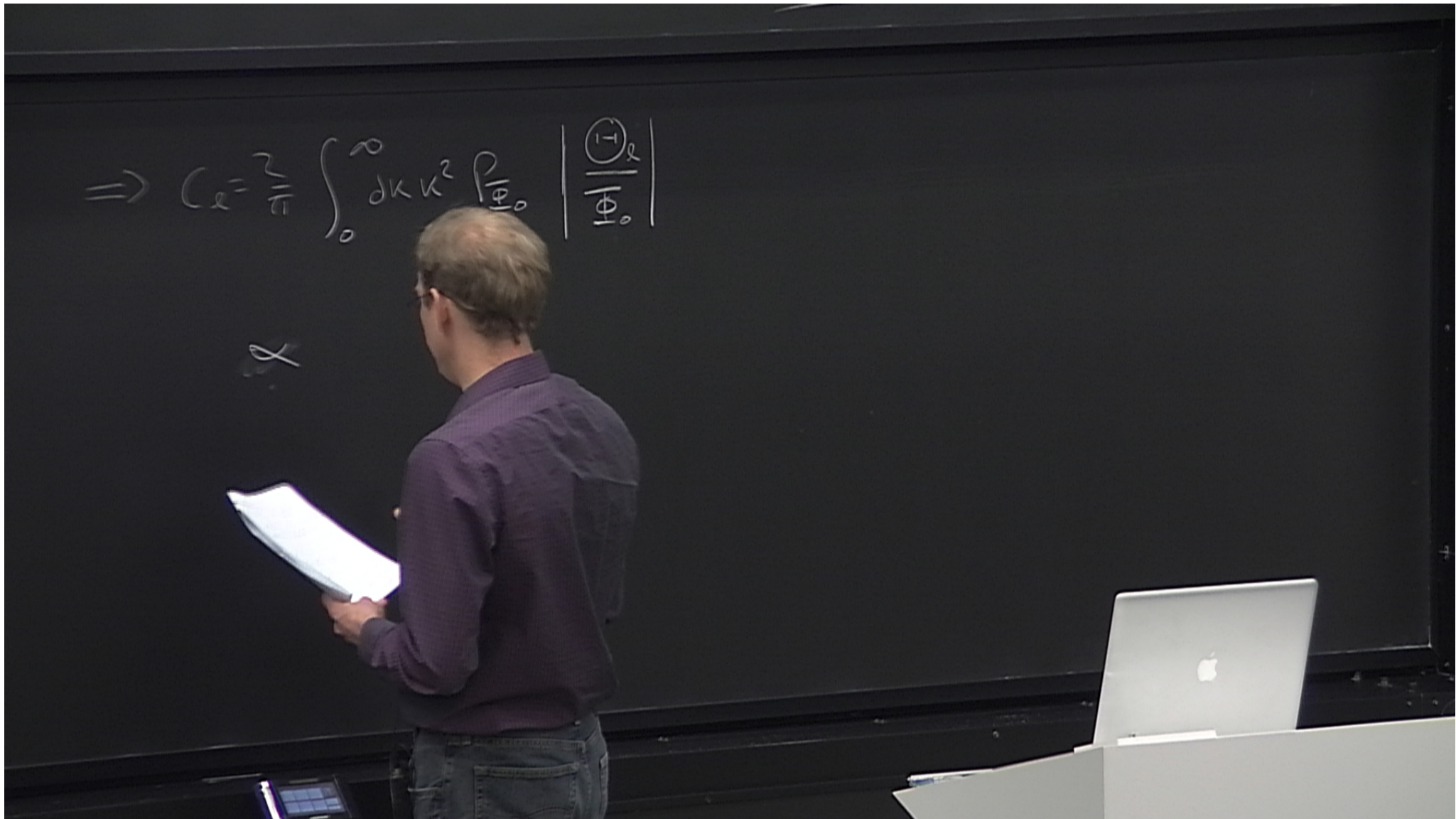
$$(\bar{\tau})_l \simeq - \frac{[\bar{\tau}_0(0) + \bar{\Phi}(0)]}{3} \left(\cos(kr_s) + \dots \right) j_l(k(\eta_0 - \eta_{LS}))$$

$$\langle \Phi(k) \Phi(k') \rangle \propto T^2(k) \frac{D^2(a)}{a^2}$$

transfer
FN
Growth
FN

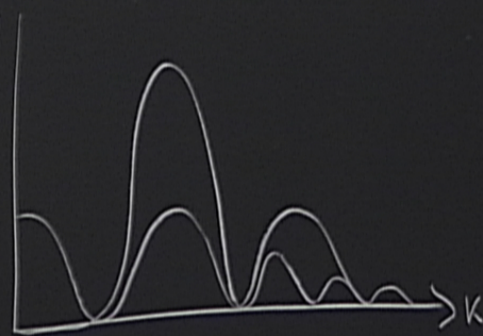
$$\bar{\Phi}(k)^2 \propto \quad \quad "$$





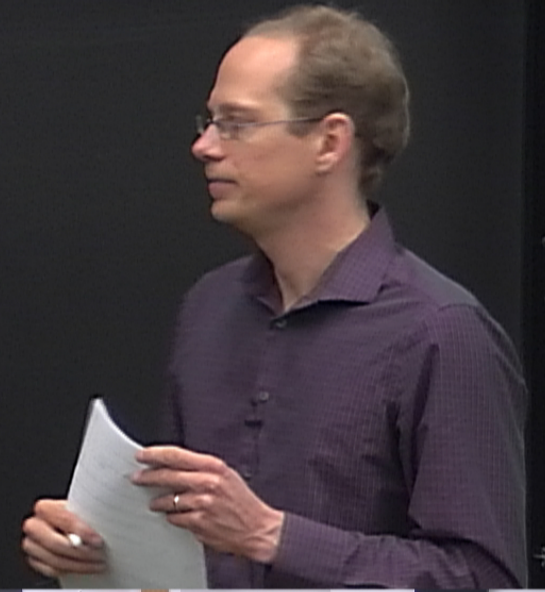
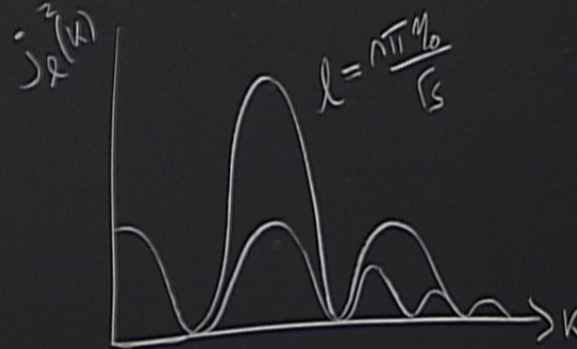
$$\Rightarrow C_2 = \frac{2}{\pi} \int_0^\infty dk k^2 P_{\Phi_0} \left| \frac{(1-i)}{\Phi_0} \right|$$

$$\propto \int_0^\infty dk k^{3-2} \cos^2[kr_s] j_2^2[k(\eta_0 - \eta_{cs})]$$



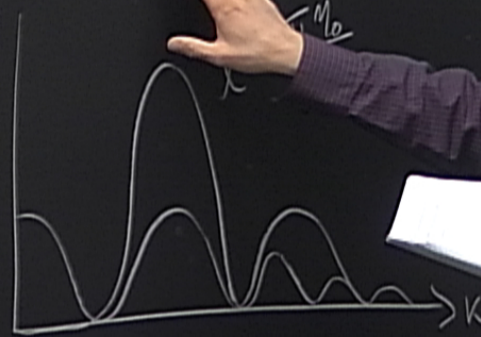
$$\Rightarrow C e^{-2} \frac{2}{\pi} \int_0^{\infty} dk k^2 P_{\Phi_0} \left| \frac{\Phi_0}{\Phi_0} \right|^2$$

$$\propto \int_0^{\infty} dk k^{3-2} \cos^2[k r_s] j_l^2[k(\eta_0 - \eta_{cs})]$$



$$\Rightarrow C_2 = \frac{2}{\pi} \int_0^\infty dk k^2 P_{\Phi_0} \left| \frac{\Phi_2}{\Phi_0} \right|^2$$

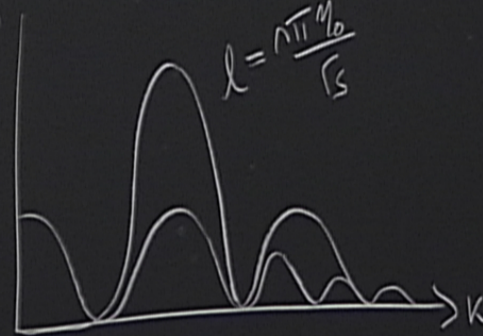
$$\propto \int_0^\infty dk k^{N_s-2} \cos^2[kr_s] j_2^2[k(\eta_0 - \eta_{cs})]$$



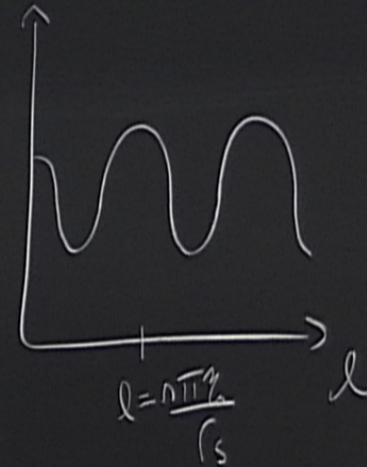
$$\Rightarrow C_2 = \frac{2}{\pi} \int_0^\infty dk k^2 P_{\Phi_0} \left| \frac{\Phi_2}{\Phi_0} \right|^2$$

$$\propto \int_0^\infty k^{N-2} \cos^2[k r_s] j_l[k(\eta_0 - \eta_{cs})]^2$$

$j_l^2(k)$



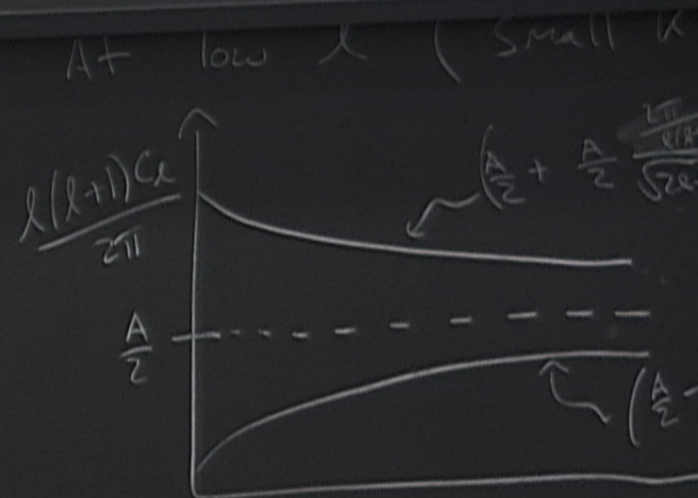
$\frac{l(l+1)}{2\pi} C_2$



In a matter dominated
Universe (close to ours)

$$\eta_0 = \frac{1}{2H_0} \quad l_p = \wedge$$

$$\Gamma_S \simeq c_s \eta_{LS}$$

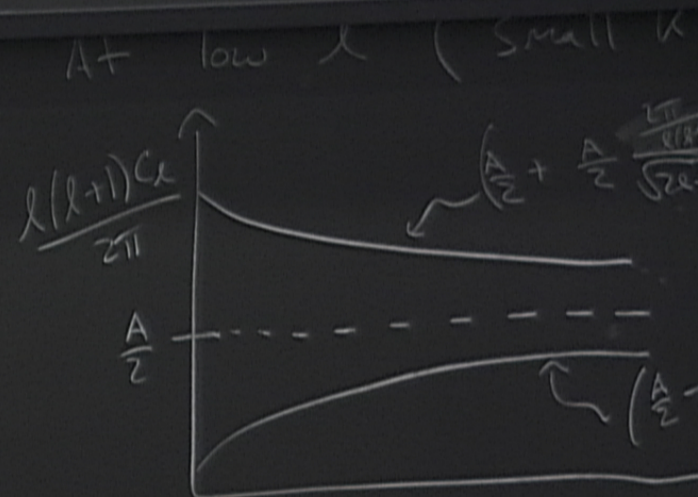


In a matter dominated
Universe (close to ours)

$$\eta_0 = \frac{1}{2H_0} \quad l_p = \Lambda$$

$$\Gamma_S \simeq c_s \eta_{LS}$$

$$\simeq \frac{c_s a_{LS}^{1/2}}{2H_0}$$



In a matter dominated
Universe (close to ows)

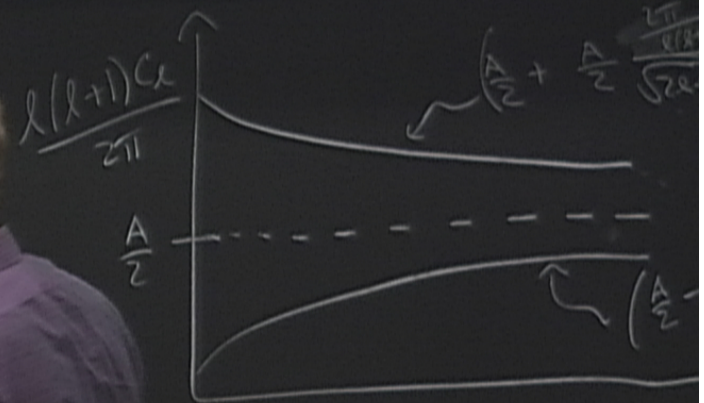
$$\eta_0 = \frac{1}{2H_0} \quad l_p = \frac{\Lambda \pi \eta_0}{\Gamma_S} = \Lambda \pi \frac{\sqrt{\eta_0}}{c_s a_{LS} \frac{1}{2H_0}}$$

$$\Gamma_S \simeq c_s \eta_{LS}$$

$$\simeq \frac{c_s a_{LS}^{1/2}}{2H_0}$$

$$= \frac{\Lambda \pi}{c_s a_{LS}^{1/2}} = \frac{\Lambda \pi}{\sqrt{3} (10^{10})^{1/2}} \sim$$

At low l (small k)



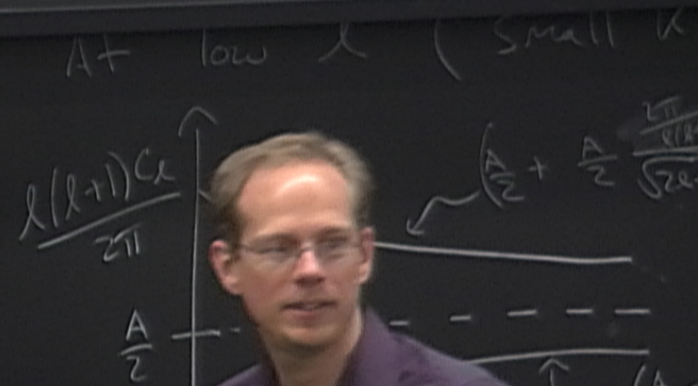
In a matter dominated
Universe (close to ours)

$$\eta_0 = \frac{1}{2H_0} \quad l_p = \frac{\Lambda \pi \eta_0}{\Gamma_S} = \Lambda \pi \frac{\frac{1}{2H_0}}{\frac{c_s a_{LS}^{1/2}}{2H_0}}$$

$$\Gamma_S \approx c_s \eta_{LS}$$

$$\approx \frac{c_s a_{LS}^{1/2}}{2H_0}$$

$$= \frac{\Lambda \pi}{c_s a_{LS}^{1/2}} = \frac{\Lambda \pi}{\sqrt{3} (10^{10})^{-1/2}} \approx 170$$



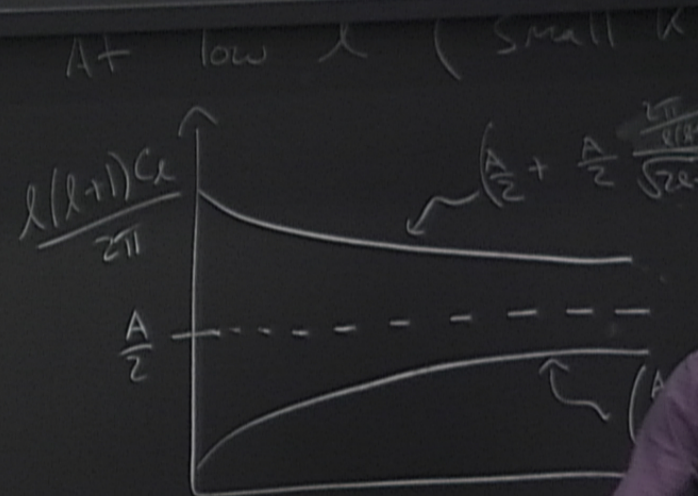
In a matter dominated
Universe (close to zeros)

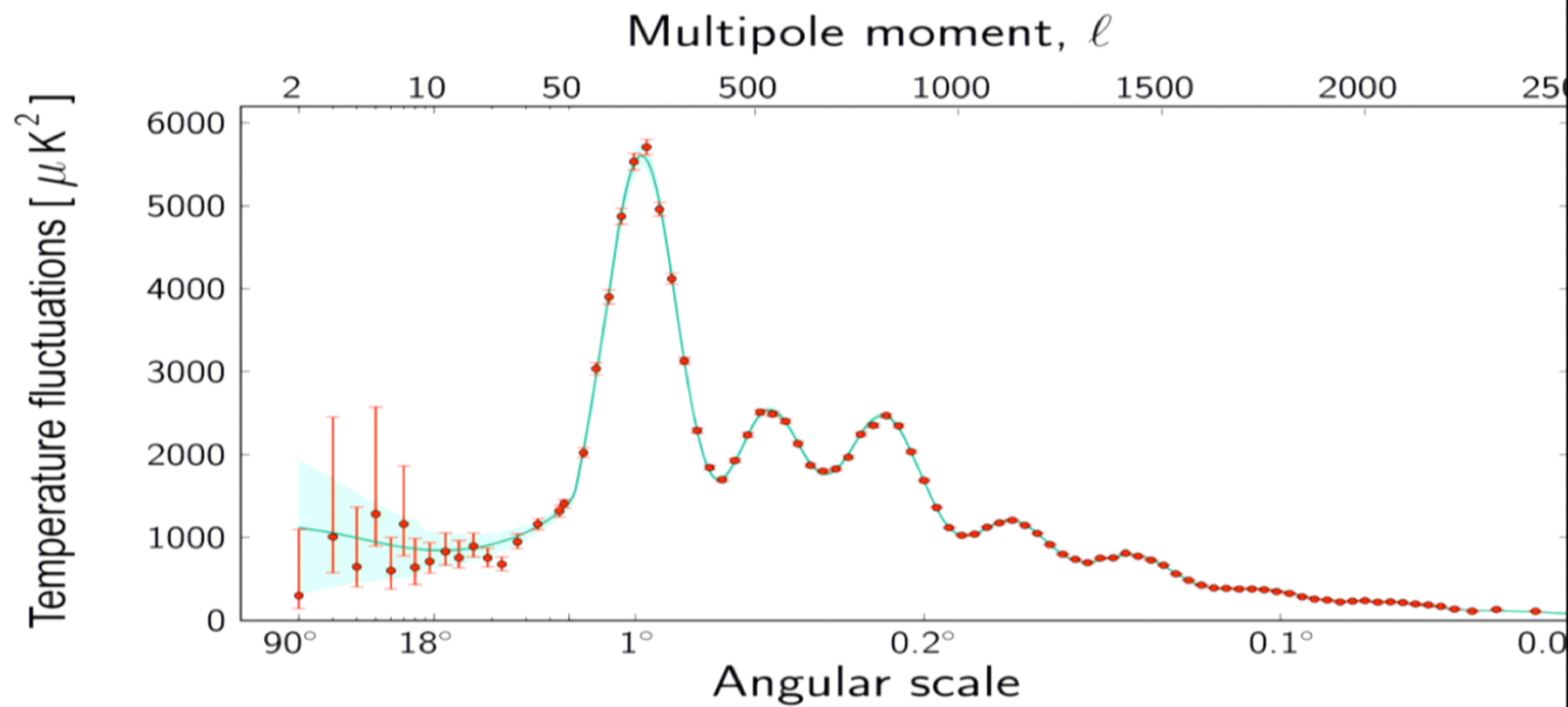
$$\eta_0 = \frac{1}{2H_0} \quad l_p = \frac{\Lambda \pi \eta_0}{\Gamma_S} = \Lambda \pi \frac{\sqrt{\eta_0}}{c_s a_{LS}^{1/2} \frac{1}{2H_0}}$$

$$\Gamma_S \approx c_s \eta_{LS}$$

$$\approx \frac{c_s a_{LS}^{1/2}}{2H_0}$$

$$= \frac{\Lambda \pi}{c_s a_{LS}^{1/2}} = \frac{\Lambda \pi}{\sqrt{3} (10^{10})^{1/2}} \approx 170 \Lambda$$







$$\ell \leftrightarrow k \approx \lambda_D^{-1}$$

$$C_2(\eta) = \frac{2A}{\pi k_0^{n_s-1}} \int_0^\infty k^{n_s-2} \left[\eta \left(k(\eta - \eta_{cs}) \right)^2 \right]$$

$$= \frac{2A}{\pi (k_0 \eta_0)^{n_s-1}} \left[2 \frac{\pi}{\pi} \frac{\Gamma\left(\ell + \frac{n_s-1}{2}\right)}{\Gamma\left(\ell + \frac{n_s}{2}\right)} \frac{\Gamma(3-n_s)}{\Gamma^2\left(2-\frac{n_s}{2}\right)} \right]$$

For $n_s = 1$

$$\frac{\Gamma(\ell)}{\Gamma(\ell+2)} = \frac{1}{\ell(\ell+1)}$$

$$\frac{\Gamma(2)}{\Gamma^2\left(\frac{3}{2}\right)} = \frac{4}{\pi}$$

$$\ell_p = \frac{\eta_0}{\Gamma_s}$$

At low ℓ (small k)

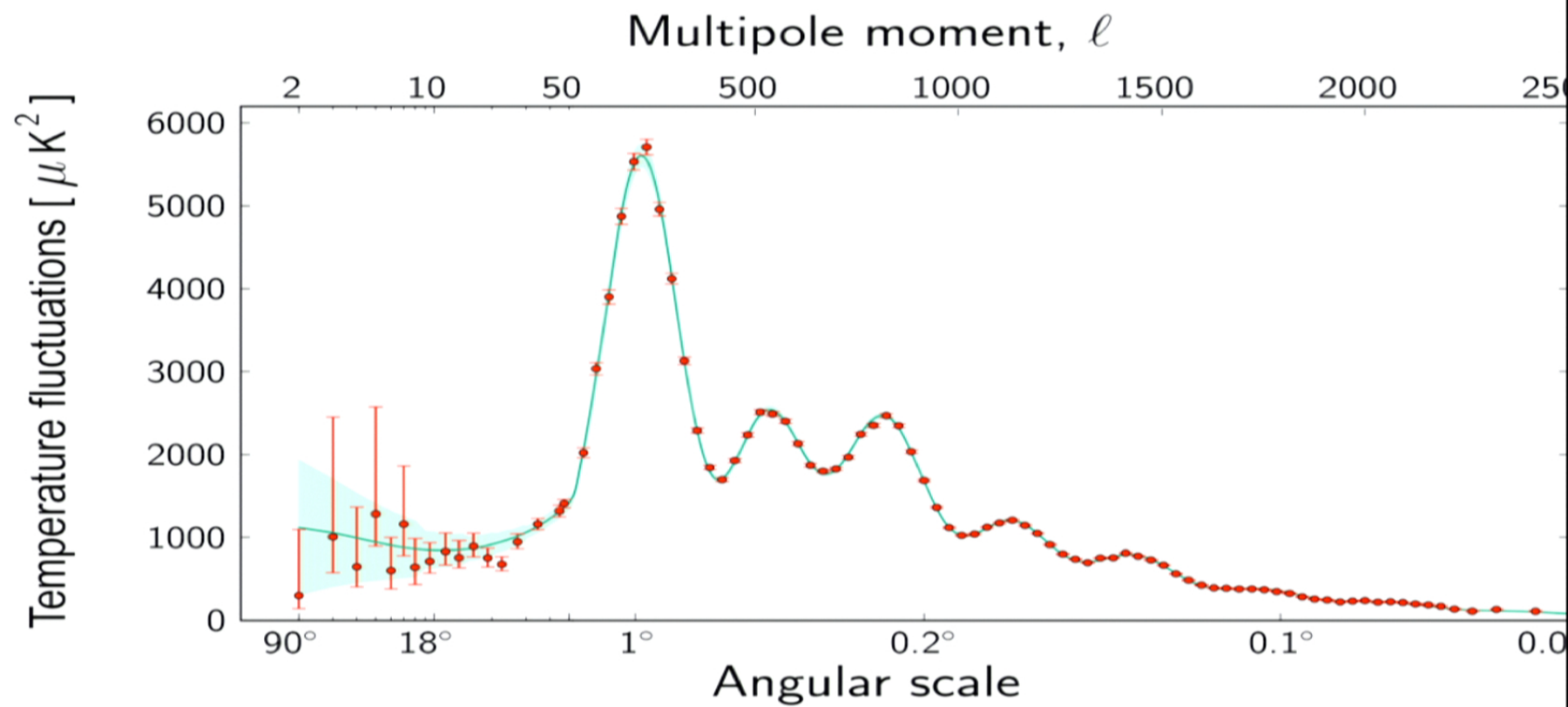
Given data point at

Λ CDM

Parameter	<i>Planck</i>		<i>Planck+lensing</i>		<i>Planck+WP</i>	
	Best fit	68% limits	Best fit	68% limits	Best fit	68% limits
$\Omega_b h^2$	0.022068	0.02207 ± 0.00033	0.022242	0.02217 ± 0.00033	0.022032	0.02205 ± 0.00028
$\Omega_c h^2$	0.12029	0.1196 ± 0.0031	0.11805	0.1186 ± 0.0031	0.12038	0.1199 ± 0.0027
$100\theta_{MC}$	1.04122	1.04132 ± 0.00068	1.04150	1.04141 ± 0.00067	1.04119	1.04131 ± 0.00063
τ	0.0925	0.097 ± 0.038	0.0949	0.089 ± 0.032	0.0925	$0.089^{+0.012}_{-0.014}$
n_s	0.9624	0.9616 ± 0.0094	0.9675	0.9635 ± 0.0094	0.9619	0.9603 ± 0.0073
$\ln(10^{10} A_s)$	3.098	3.103 ± 0.072	3.098	3.085 ± 0.057	3.0980	$3.089^{+0.024}_{-0.027}$

Λ CDM

$\Omega_b h^2$	Early thermal history: particle physics
$\Omega_c h^2$	
$100\theta_{MC}$	Cosmological constant problem
τ	Astrophysics
n_s	Primordial initial conditions: Inflation
$\ln(10^{10} A_s)$	



$$C_l(\eta_0) = \frac{C_l}{\pi K_0^{\eta_0-1}} \left(\frac{\eta_0}{K} \right)^{K-1} \eta_l(K(\eta_0 - \eta_{ls}))$$

$$= \frac{2A}{\pi(K_0 \eta_0)^{\eta_0-1}} \left[2^{\eta_0-1} \pi \frac{\Gamma\left(l + \frac{\eta_0}{2} - \frac{1}{2}\right)}{\Gamma\left(l + \frac{\eta_0}{2} - \frac{1}{2}\right)} \frac{\Gamma(3-\eta_0)}{\Gamma^2\left(2 - \frac{\eta_0}{2}\right)} \right]$$

$$\frac{\Gamma(l)}{\Gamma(l+2)} = \frac{1}{l(l+1)}$$

$$\frac{\Gamma(2)}{\Gamma^2\left(\frac{3}{2}\right)} = \frac{4}{\pi}$$

$$l_p = \frac{\eta_0}{\eta_s}$$

A+ low l (small K)



given $l_p C_l$

Data point at l_*

