

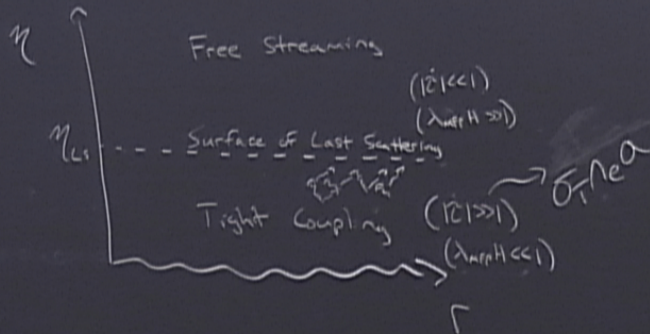
Title: 12/13 PSI - Explorations in Cosmology Lecture 7

Date: Apr 16, 2013 10:15 AM

URL: <http://pirsa.org/13040070>

Abstract:

Recap



Tight Coupling

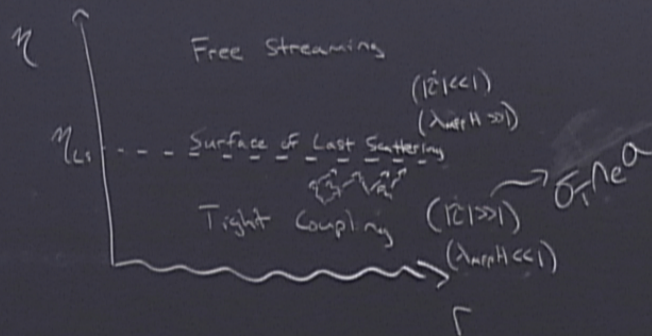
$$\left[\frac{\partial^2}{\partial \eta^2} + \frac{\dot{R}}{1+R} \frac{\partial}{\partial \eta} + k^2 c_s^2 \right] [\Theta_0 + \Phi] = \frac{k^2}{2} \left[\frac{1}{1+R} \Phi - \Psi \right]$$

$$c_s = \frac{1}{\sqrt{3(1+R)}}, \quad \Gamma_s = \int_0^\eta c_s(\eta') d\eta'$$

$$[\Theta_0(\eta) + \Phi(\eta)] = [\Theta_0(0) + \Phi(0)] \cos(k \Gamma_s)$$

$$+ \frac{k}{\sqrt{3}} \int_0^\eta d\eta' [\Phi(\eta') - \Gamma_s(\eta) - \Gamma_s(\eta')]$$

Recap



Tight Coupling

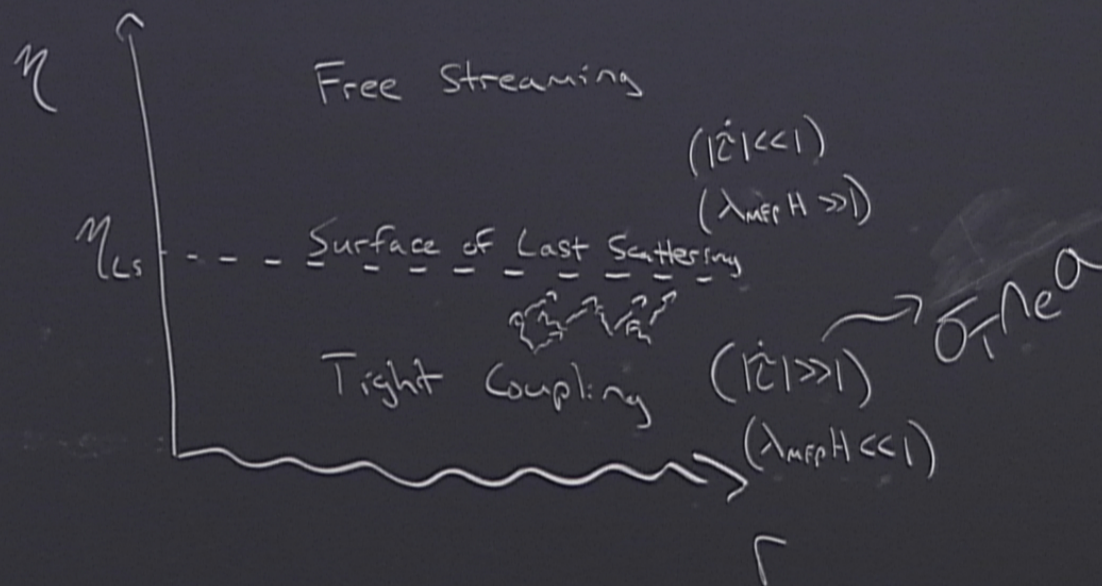
$$\left[\frac{\partial^2}{\partial \eta^2} + \frac{\dot{R}}{1+R} \frac{\partial}{\partial \eta} + k^2 c_s^2 \right] [\Theta_0 + \Phi] = \frac{k^2}{2} \left[\frac{1}{1+R} \Phi - \Psi \right]$$

$$c_s = \frac{1}{\sqrt{3(1+R)}}, \quad \Gamma_S = \int_0^\eta c_s(\eta') d\eta'$$

$$[\Theta_0(\eta) + \Phi(\eta)] = [\Theta_0(0) + \Phi(0)] \cos(k \Gamma_S)$$

$$+ \frac{k}{\sqrt{3}} \int_0^\eta d\eta' [\Phi(\eta') - \Gamma_S(\eta) - \Gamma_S(\eta')]$$

Recap



$$\Theta \simeq \Theta_0 + \Theta_1$$

Tight coupling

$$\left[\frac{\partial^2}{\partial \eta^2} + \right]$$

$$\left[\Theta_0(\eta) + \Theta_1(\eta) \right]$$

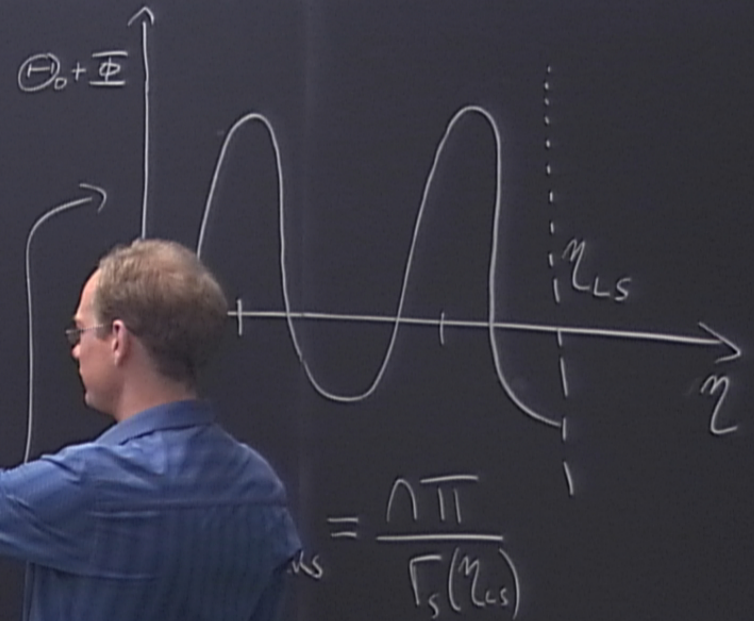
Tight coupling

$$\left[\frac{\partial^2}{\partial \eta^2} + \frac{\dot{R}}{1+R} \frac{\partial}{\partial \eta} + k^2 c_s^2 \right] [\Theta_0 + \Phi] = \frac{k^2}{3} \left[\frac{1}{1+R} \Phi - \Psi \right]$$

$$c_s = \frac{1}{\sqrt{3(1+R)}}, \quad R = \frac{3\rho_b}{4\rho_s}, \quad \Gamma_s = \int_0^\eta c_s(\eta') d\eta'$$

$$[\Theta_0(\eta) + \Phi(\eta)] = [\Theta_0(0) + \Phi(0)] \cos(k\Gamma_s)$$

$$+ \frac{k}{\sqrt{3}} \int_0^\eta d\eta' [\Phi(\eta') - \Psi(\eta')] \sin[k(\Gamma_s(\eta) - \Gamma_s(\eta'))]$$



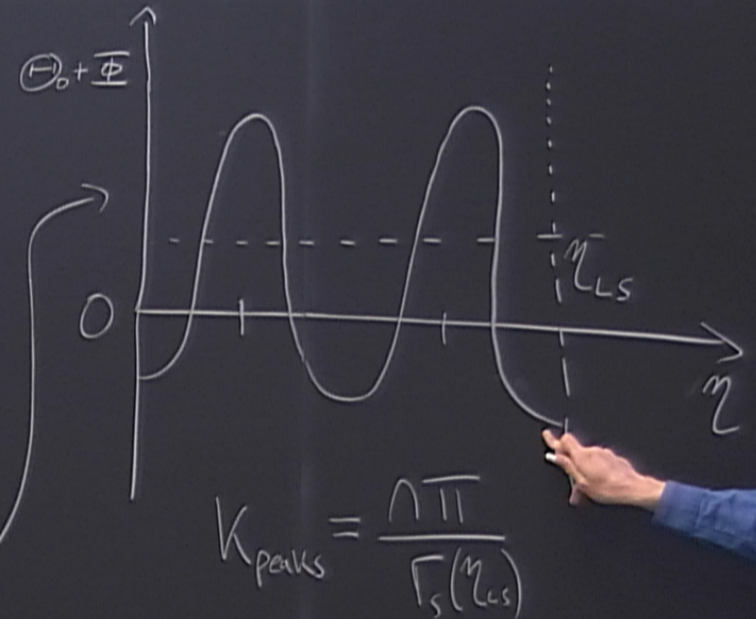
Tight coupling

$$\left[\frac{\partial^2}{\partial \eta^2} + \frac{\dot{R}}{1+R} \frac{\partial}{\partial \eta} + k^2 c_s^2 \right] [\Theta_0 + \Phi] = \frac{k^2}{3} \left[\frac{1}{1+R} \Phi - \Psi \right]$$

$$c_s = \frac{1}{\sqrt{3(1+R)}}, \quad R = \frac{3\rho_b}{4\rho_s}, \quad \Gamma_s = \int_0^\eta c_s(\eta') d\eta'$$

$$[\Theta_0(\eta) + \Phi(\eta)] = [\Theta_0(0) + \Phi(0)] \cos(k\Gamma_s) \quad \rightarrow \quad \frac{3}{4} \frac{\Omega_b}{\Omega_\gamma} a$$

$$+ \frac{k}{\sqrt{3}} \int_0^\eta d\eta' [\Phi(\eta') - \Psi(\eta')] \sin[k(\Gamma_s(\eta) - \Gamma_s(\eta'))]$$



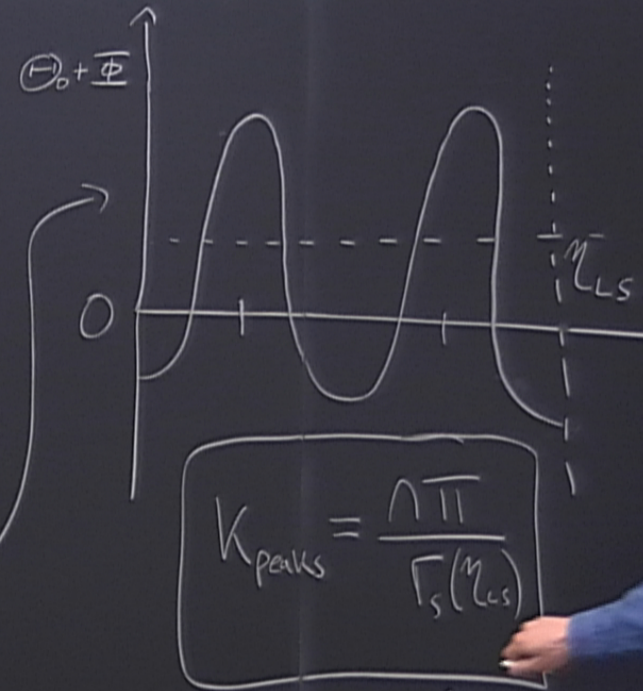
Tight coupling

$$\left[\frac{\partial^2}{\partial \eta^2} + \frac{\dot{R}}{1+R} \frac{\partial}{\partial \eta} + k^2 c_s^2 \right] [\Theta_0 + \Phi] = \frac{k^2}{3} \left[\frac{1}{1+R} \Phi - \Psi \right]$$

$$c_s = \frac{1}{\sqrt{3(1+R)}}, \quad R = \frac{3\rho_b}{4\rho_s}, \quad \Gamma_s = \int_0^\eta c_s(\eta') d\eta'$$

$$[\Theta_0(\eta) + \Phi(\eta)] = [\Theta_0(0) + \Phi(0)] \cos(k\Gamma_s) \quad \hookrightarrow \frac{3}{4} \frac{\Omega_b}{\Omega_\gamma} a$$

$$+ \frac{k}{\sqrt{3}} \int_0^\eta d\eta' [\Phi(\eta') - \Psi(\eta')] \sin[k(\Gamma_s(\eta) - \Gamma_s(\eta'))]$$

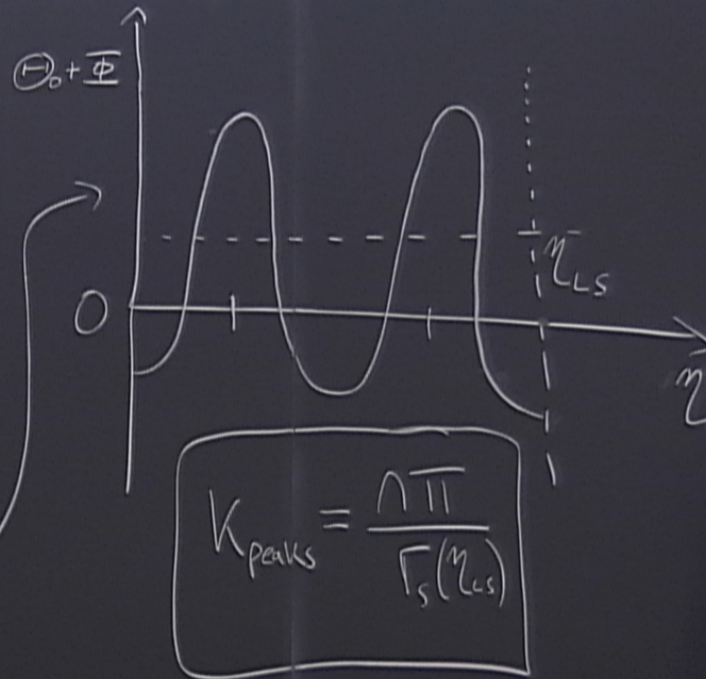


$$\frac{\partial}{\partial \eta} + k^2 C_s^2 \left[\Theta_0 + \Phi \right] = \frac{k^2}{3} \left[\frac{1}{1+R} \Phi - \Psi \right]$$

$$C_s = \frac{1}{\sqrt{3(1+R)}}, \quad R = \frac{3\rho_b}{4\rho_s}, \quad \Gamma_s = \int_0^\eta C_s(\eta') d\eta'$$

$$= \left[\Theta_0(0) + \Phi(0) \right] \cos(k\Gamma_s)$$

$$+ \frac{k}{\sqrt{3}} \int_0^\eta d\eta' \left[\Phi(\eta') - \Psi(\eta') \right] \sin \left[k(\Gamma_s(\eta) - \Gamma_s(\eta')) \right]$$



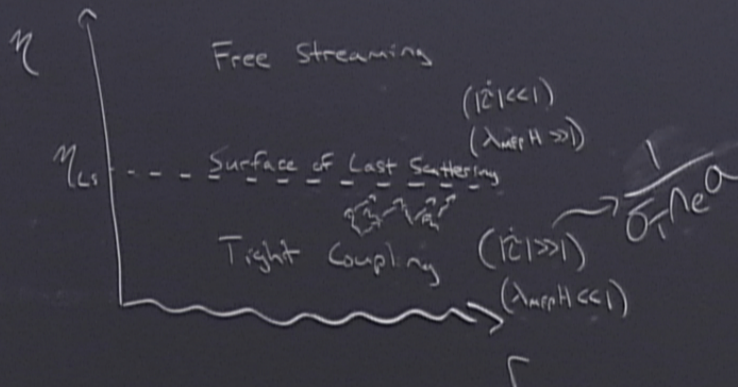
$$\Theta(m_{low}, \vec{X}_{here}, \hat{p}) = \left(\frac{\partial u}{\partial m} \right) \Theta(k)$$

$$\Theta_e(\cdot) = \int \frac{d\mu}{2} P_k \Theta(k)$$

$$\Theta_e(\eta_0, k) = -\frac{1}{3} \Phi(\eta_0, k) j_e(k(\eta_0 - \eta_{sc}))$$

$\uparrow$
 $[\Theta_0(\eta_0) + \Phi(\eta_0)]$

Recap



$$\Theta \simeq \Theta_0 + \Theta_1$$

Tight Coupling

$$\left[\frac{\partial^2}{\partial \eta^2} + \frac{R}{1+R} \frac{\partial}{\partial \eta} + k^2 C_s^2 \right] [\Theta_0 + \Phi] =$$

$$C_s = \frac{1}{\sqrt{3(1+R)}}, \quad R = \frac{3}{4} \frac{\rho_b}{\rho_r}$$

$$[\Theta_0(\eta) + \Phi(\eta)] = [\Theta_0(0) + \Phi(0)] \cos(k r_s)$$

$$+ \frac{k}{\sqrt{3}} \int_0^\eta d\eta' [\Phi(\eta') - \Psi(\eta')] S_{\text{...}}$$

$$\textcircled{1-1} (\eta_{\text{low}}, \vec{X}_{\text{here}}, \hat{p}) = \left(\frac{\partial^3 \kappa}{(\eta_1)^3} \right) \textcircled{1}(k)$$

$$\textcircled{1-1}(\quad) = \int \frac{\partial u}{\partial z} p_k \textcircled{1}(u)$$

$$\textcircled{1-1}(\eta_0, k) = -\frac{1}{3} \Phi(\eta_s, k) \textcircled{1}(k(\eta_0 - \eta_{cs}))$$

$\uparrow$
 $[\omega_0(\eta_{cs}) + \Phi(\eta_{cs})]$

$$\quad) (= \frac{1}{\alpha} \frac{\partial \alpha}{\partial \eta}$$

$$\frac{\partial \Phi}{\partial(ha)} + \frac{k}{\gamma} \Phi$$

Comoving horizon = $\frac{1}{H}$

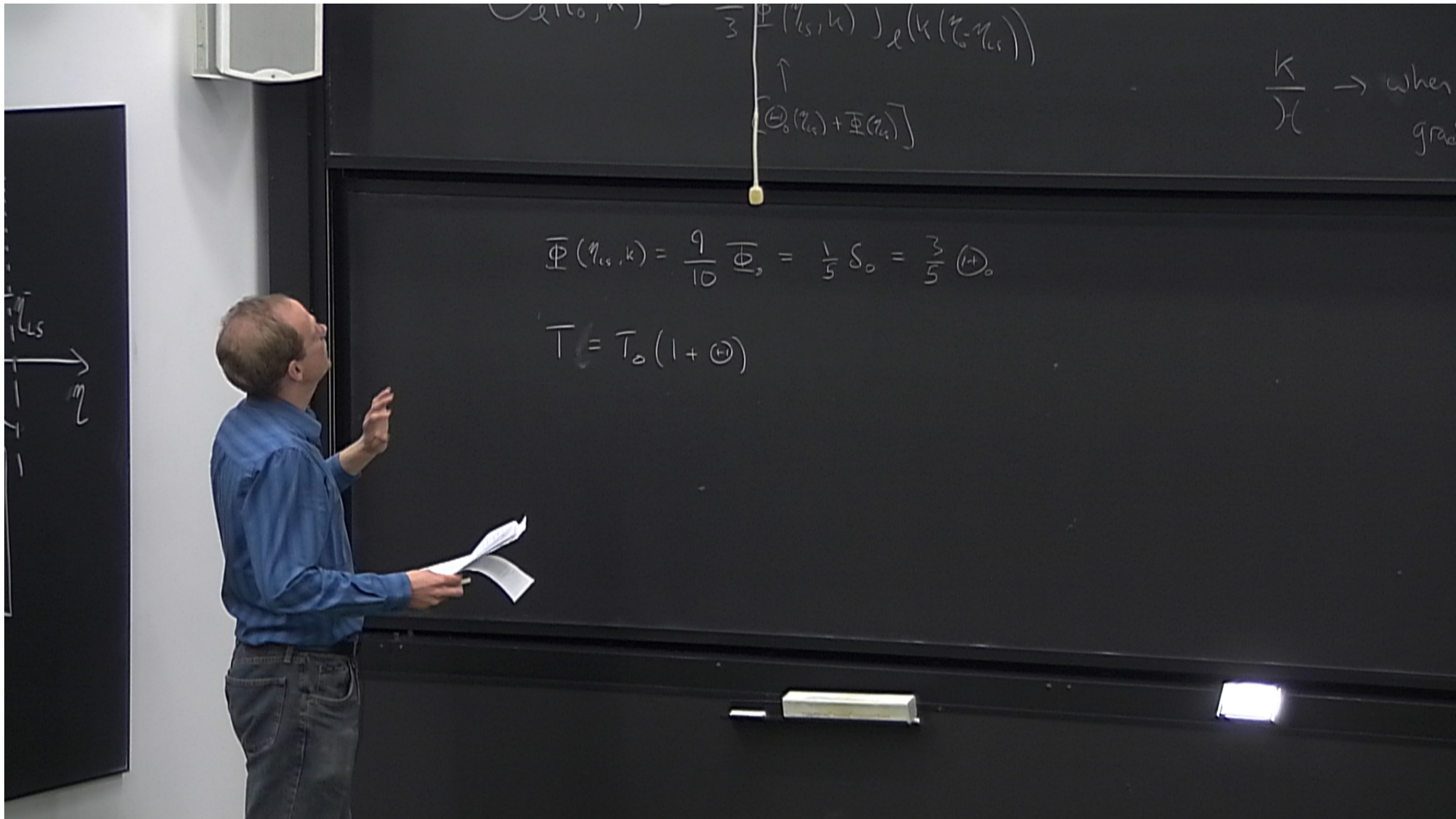
when $\gg 1$
gradient terms important

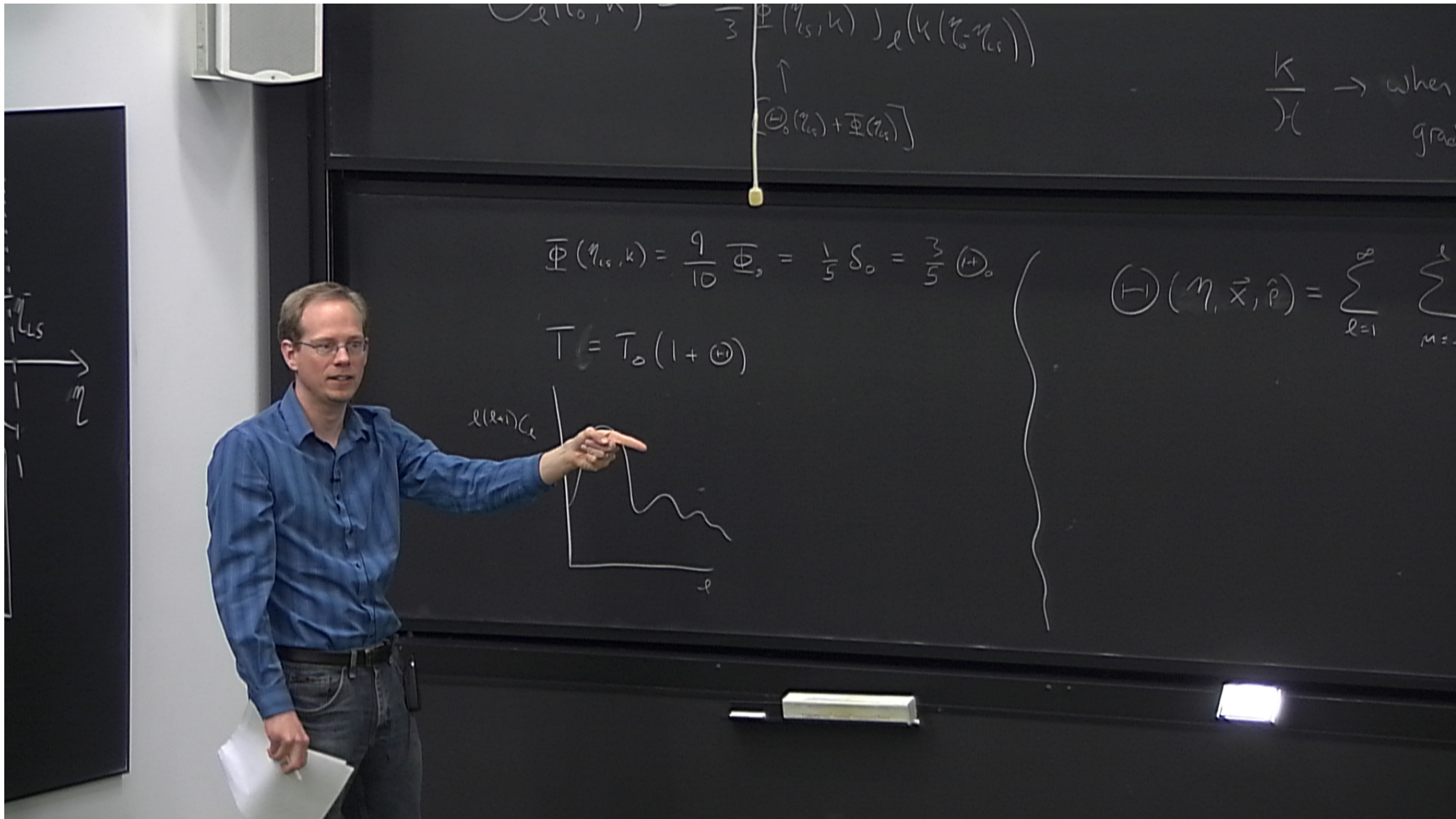
$$\textcircled{-}(\eta_{\text{low}}, \vec{X}_{\text{here}}, \hat{p}) = \left(\frac{\partial^3 \chi}{(\partial \eta^3)} \textcircled{-}(\eta) \right)$$

$$\textcircled{-}_\ell(\eta) = \int \frac{d\mu}{2} P_\ell \textcircled{-}(\mu)$$

$$\textcircled{-}_\ell(\eta_0, \mu) = -\frac{1}{3} \Phi(\eta_{\text{cs}}, \mu) j_\ell(\mu(\eta_0 - \eta_{\text{cs}}))$$

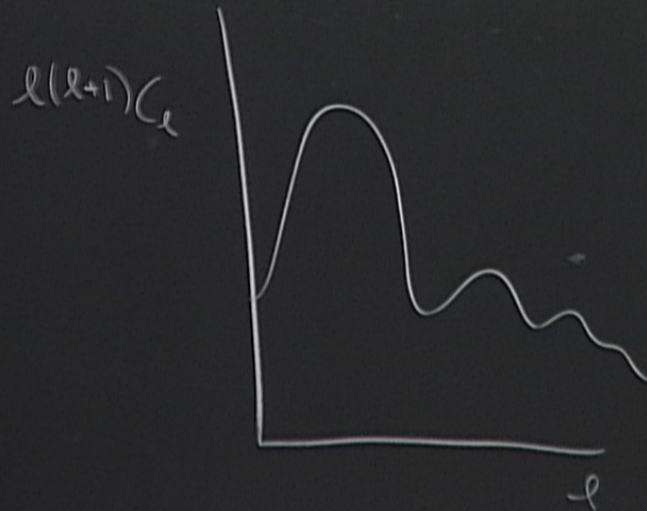
$\uparrow$
 $[\textcircled{-}_0(\eta_{\text{cs}}) + \Phi(\eta_{\text{cs}})]$





$$\overline{\Phi}(\eta_{ls}, k) = \frac{9}{10} \overline{\Phi}_0 = \frac{1}{5} \delta_0 = \frac{3}{5} \odot_0$$

$$\overline{T} = T_0 (1 + \odot)$$



$$\frac{1}{5} S_0 = \frac{3}{5} \textcircled{1+}_0 \quad / \quad \textcircled{+1}(\eta, \vec{x}, \hat{p}) = \sum_{l=1}^{\infty} \sum_{m=-l}^l a_{lm} Y_{lm}(\hat{p})$$

$$\langle a_{lm} a_{l'm'}^* \rangle = \delta_{l'l} \delta_{m'm} c_l$$

$$(1-1) (\vec{k}, \hat{p}, \eta) \propto \Phi_0(\vec{k})$$

$$\langle \Phi_0(\vec{k}) \Phi_0(\vec{k}') \rangle = (2\pi)^3 \delta(\vec{k} - \vec{k}') P_{\Phi}(k)$$

$$\langle \Phi_0(x)^2 \rangle = \int \frac{d^3k}{(2\pi)^3} P_{\Phi}(k) = \int d(\ln k) \frac{k^3 P_{\Phi}(k)}{2\pi^2}$$

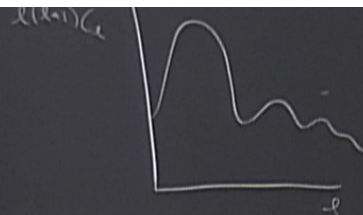
$$\langle a_{\mathbf{q}m} a_{\mathbf{q}'m'}^\dagger \rangle = \int \int$$

$$\langle \hat{\Phi}(\vec{k}, \hat{p}, \eta) \hat{\Phi}(\vec{k}', \hat{p}', \eta) \rangle$$

$$\rightarrow \propto \langle \Phi_0(k) \Phi_0(k') \rangle$$

$$\vec{k}') P_{\vec{k}}(k)$$

$$(k) \frac{V^3 P_{\vec{k}}(k)}{2\pi^2}$$



$$\langle a_{\ell m} a_{\ell' m'}^* \rangle = \delta_{\ell \ell'} \delta_{m m'} C_\ell$$

$$\textcircled{1-1} (\vec{k}, \hat{p}, \eta) \propto \Phi_0(\vec{k})$$

$$\langle a_{\ell m} a_{\ell' m'}^* \rangle = \left\{ \begin{array}{c} \end{array} \right\}$$

$$\langle \Theta(\vec{k}, \hat{p}, \eta) \Theta(\vec{k}', \hat{p}', \eta) \rangle$$

$$\rightarrow \propto \langle \Phi_0(k) \Phi_0(k') \rangle$$

$$\langle \Phi_0(\vec{k}) \Phi_0(\vec{k}') \rangle = (2\pi)^3 \delta(\vec{k} - \vec{k}') P_\Phi(k)$$

$$C_\ell = \frac{2}{\pi} \int_0^\infty dk k^2 P_\Phi(k) \left| \frac{\Theta_1}{\Phi_0} \right|^2$$

$$\langle \Phi_0(x)^2 \rangle = \int \frac{d^3 k}{(2\pi)^3} P_\Phi(k) = \int d \ln k$$

$$\langle a_{\mathbf{e}m} a_{\mathbf{e}'m'}^* \rangle = \left(\begin{array}{c} \\ \end{array} \right)$$

$$\langle \hat{\mu}(\vec{k}', \hat{p}, \eta) \hat{\mu}(\vec{k}', \hat{p}', \eta) \rangle$$

$$\rightarrow \propto \langle \bar{\Phi}_0(k) \bar{\Phi}_0(k') \rangle$$

$$C_\ell = \frac{2}{\pi} \int_0^\infty dk k^2 P_\Phi(k) \left| \frac{\hat{\mu}_\ell}{\bar{\Phi}_0} \right|^2$$

Sachs-Wolfe limit

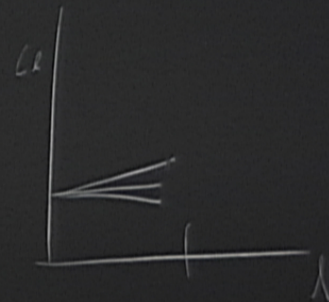
largest scales (Scales that are
superhorizon at
LS)

$$\Theta_l = -\frac{3}{10} \Phi_0 j_l[k(\eta - \eta_{LS})]$$

$$C_l = \frac{24}{1100} \int_0^\infty dk k^2 j_l[k(\eta - \eta_{LS})]^2 \frac{P(k)}{k}$$

$$P_{\frac{P}{k}}(k) = A k^{n_s-1}$$

$$\Rightarrow C_l = () A \frac{\Gamma(l + \frac{n_s}{2} - 1) \Gamma(4 - n_s)}{\Gamma(l + \frac{7}{2} - \frac{n_s}{2}) \Gamma(\frac{5}{2} - \frac{n_s}{2})}$$



Sachs-Wolfe limit

largest scales (Scales that are
superhorizon at
LS)

$$\Theta_l = -\frac{3}{10} \Phi_0 j_l[k(\eta - \eta_{LS})]$$

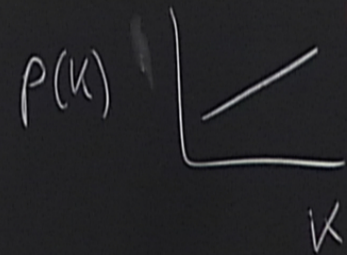
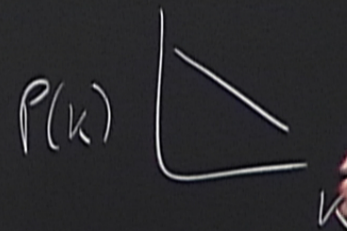
$$C_l = \frac{9}{50\pi} \int_0^\infty dk k^2 j_l[k(\eta - \eta_{LS})]^2 \frac{P(k)}{k^3}$$

$\gamma_s = 1$ Scale invariant

$$P(k) = A$$

$\gamma_s < 1$ red tilt

$\gamma_s > 1$ blue tilt



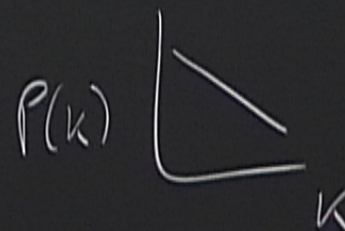
$n_s = 1$ Scale invariant (Harrison-Zeldovich)

$$\frac{4-n_s}{2}$$

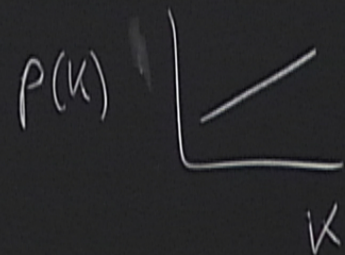
$$\frac{5-n_s}{2}$$

$$P(k) = A$$

$n_s < 1$ red tilt



$n_s > 1$ blue tilt



$$C_l = \frac{9}{50\pi} \int_0^{\infty} dk k^2 j_l[k(\eta - \eta_{LS})] \frac{P(k)}{k^3}$$

Subhorizon Scales

$$\bar{\Phi}(\eta_{LS}, \vec{k}) \neq \frac{1}{10} \bar{\Phi}_0(\eta_0, \vec{k})$$

On smaller scales, $\bar{\Phi}(\eta_{LS}, \vec{k})$

Peaks for $k_1 = \frac{\pi n}{r_s(\eta_{LS})}$

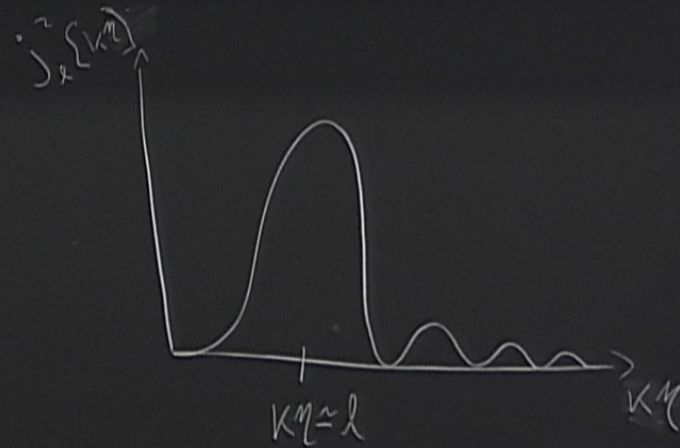
$$C_l = \frac{9}{50\pi} \int_0^\infty dk k^2 j_l[k(\eta - \eta_{LS})] \frac{P(k)}{k^3}$$

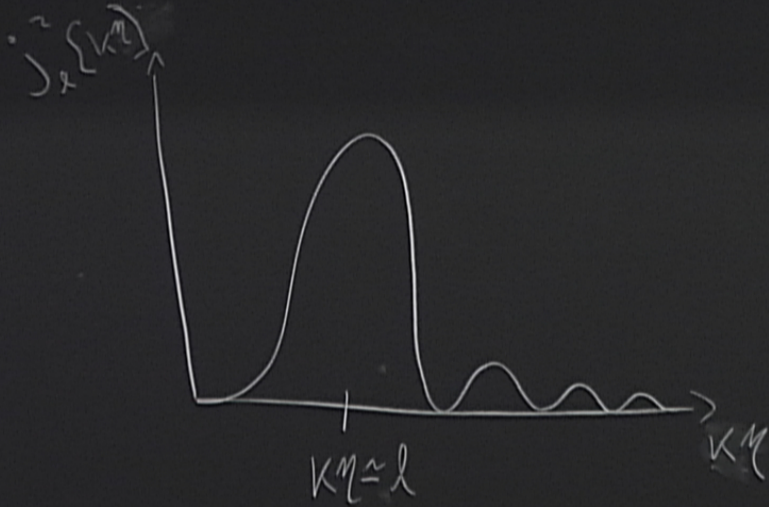
Subhorizon Scales

$$\Phi(\eta_{LS}, \vec{k}) \neq \frac{\eta}{\eta_0} \Phi_0(\eta_0, \vec{k})$$

On smaller scales, $\Phi(\eta_{LS}, \vec{k})$

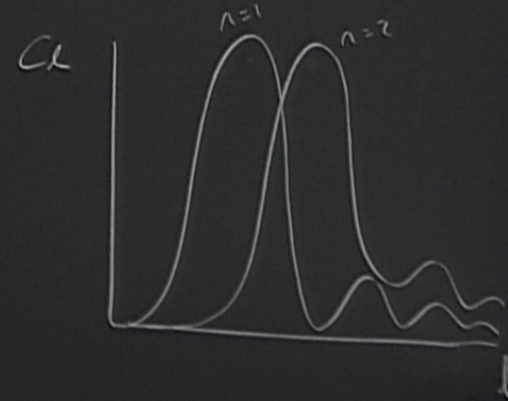
Peaks for $k_1 = \frac{\pi n}{\Gamma_S(\eta_{LS})}$





Extra power on Scales

$$l \approx \frac{n\pi}{r_s} (\eta_0 - \eta_{LS})$$



Position of peaks
related to r_s and $(\eta_0 - \eta_{LS})$

$\downarrow$ $\downarrow$

$\frac{P_0}{P_s}$ P_m, P_n

$$\Theta_l(k, \eta_0) = \int_0^{\eta_0} d\eta \, g(\eta) \left[\Theta_0(k, \eta) + \Psi(k, \eta) \right] J_L[k(\eta_0 - \eta)] \quad \text{Sachs Wolfe}$$

$$g(\eta) = -\dot{\tau} e^{-\tau}$$

Visibility $\equiv \int_0^{\eta} d\eta g(\eta) = 1$

$$\int_0^{\eta} d\eta g(\eta) = 1$$

Previous limit - $g(\eta) = \delta(\eta - \eta_{LS})$

$$- \int d\eta \, g(\eta) \frac{1}{k} \frac{\partial}{\partial \eta} \left[J_L[k(\eta_0 - \eta)] \right] \quad \text{dop}$$

$$\begin{aligned}
&= \int_0^{\eta_0} d\eta \, g(\eta) \left[\Phi_0(k, \eta) + \Psi(k, \eta) \right] j_\ell[k(\eta_0 - \eta)] \quad \text{Sachs-Wolfe} \\
&\quad - \int d\eta \, g(\eta) \frac{iv_L(k, \eta)}{k} \frac{\partial}{\partial \eta} \left[j_\ell[k(\eta_0 - \eta)] \right] \quad \text{Doppler} \quad (v_b \propto \Phi_1) \\
&\quad + \int d\eta \, e^{-\hat{\tau}} \left[\dot{\Psi} - \dot{\Phi} \right] j_\ell[k(\eta - \eta_0)] \quad \text{Integrated Sachs-Wolfe}
\end{aligned}$$

$g(\eta) = \delta(\eta - \eta_{LS})$